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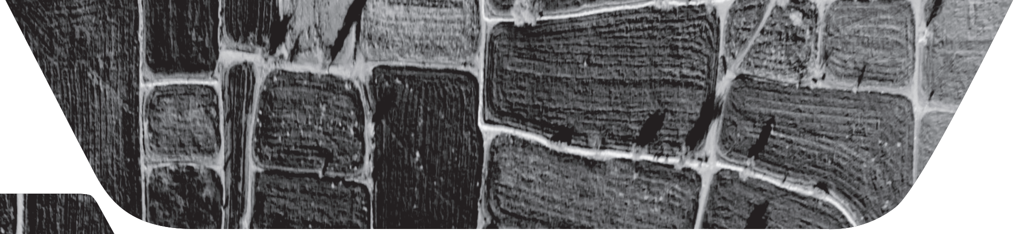

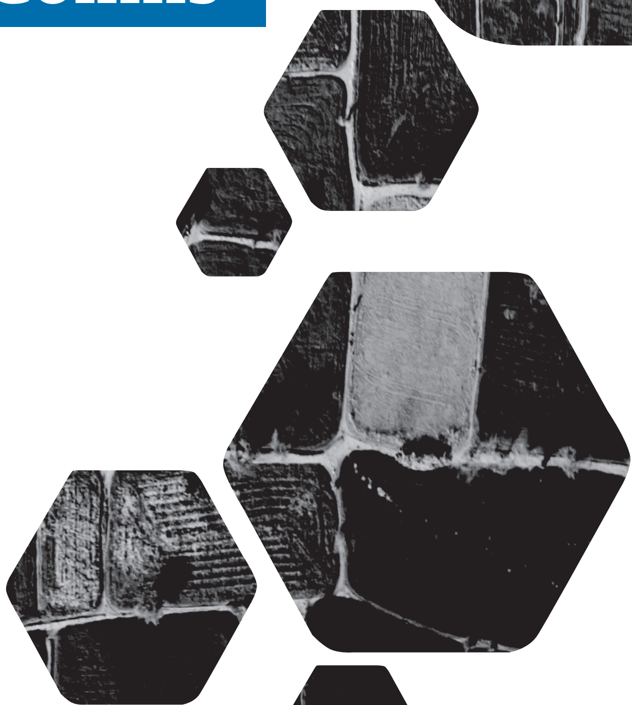

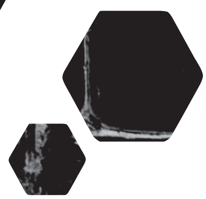

Maths Now

8

Author
Mona Jhunjhunwala

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Preface

Maths Now is a series of eight books for classes 1 to 8 based on the NCERT syllabus. The series follows an activity-oriented approach to make mathematics engaging for students through emphasizing connections between mathematics and day-to-day experiences. This series also features the balanced use of manipulatives, virtual manipulatives, abstract ideas and other interesting features to improve inherent mathematical skills of students by creating foundational interest in the subject.

This series has been created with a view to enhance the students' understanding of the key concepts of mathematical problem-solving and to increase practical learning by bringing in contexts from outside the classroom. The main aim of the series is to eradicate maths phobia among students, make mathematical concepts crystal clear so that students appreciate the beauty of the subject and the role it plays in one's life.

Key Features

Let's Get Started Chapter starter in the form of a picture-based exercise

Mental Maths Objective-type questions to develop quick-thinking skills

Go Easy! Additional tips helping students to calculate quickly

Do You Know? Nuggets of information to add real-world context to abstract mathematical concepts

Common Errors Pointers highlighting common mistakes and misconceptions

Solved Examples Exercises with step-by-step solutions

Word Problems Textual questions based on real-life situations

Exercises In-text objective-type questions for quick review

Crossword and Puzzles Mathematical problems to stimulate the students' engagement

Maths Lab Activity Hands-on activity to connect concepts with their practical uses in real-life situations

Concept Map Graphic organizer to logically represent relationships between concepts under one topic

Key Concepts List to concisely give an overview of concepts in each topic

Chapter Revision Exercises at the end of each chapter for a comprehensive review

Skill Up!

- **Project** Practical activities to enhance real-world application of concepts
- **Life Skills** Questions to inculcate positive behaviour and add a layer of ethical thinking while solving practical mathematical questions
- **Mind Buzzer** Questions to provide challenging questions relating to real-life examples

Teacher's Notes Important tips related to concepts for the teacher

Worksheets Exercises that covers financial literacy, inferential and experiential learning

Reasoning Worksheet Questions to stimulate rational thinking using mathematical skills

Eminent Mathematicians Brief write-up on eminent mathematicians and their contributions

Poster Important points and formulae in the form of a pull-out page

Review Corner Variety of questions at the end of the book for additional practice

I would like to take this opportunity to thank all the teachers and educationist, especially Dr. C.B. Mishra, Prudence Group of School, who reviewed the books and provided their feedback, which helped in improving the quality of the content.

I would like to dedicate this series to my father, late Mr. S.P. Gupta.

Feedback, invaluable comments and suggestions from users are welcome.

Author
Mona Jhunjhunwala

Key Features

Different features interspersed within the book aim to provide active learning tools and techniques. These tools and techniques have been designed keeping in mind the 5E principle based on the constructivist approach to learning. These features can be used as learning strategies to enhance the understanding of key concepts of mathematics and increase practical learning and problem-solving by bringing in real-life contexts from outside the classroom.

Engage: To capture the attention of students, become mentally engaged and make connections between past and present knowledge

- **Let's Get Started**
- **Common Errors**
- **Go Easy!**
- **Vedic Maths (classes 6 to 8)**
- **Eminent Mathematicians**
- **Poster**

Explore: To enable the students to explore the concepts through hands-on activities, investigations, etc.

- **Maths Lab Activity**
- **Crosswords**
- **Puzzles**
- **Mental Maths**

Explain: To enable the students to understand the main ideas of the topics

- **Learning Objectives**
- **Note**
- **Solved Examples**
- **Concept Map**
- **Key Concepts**

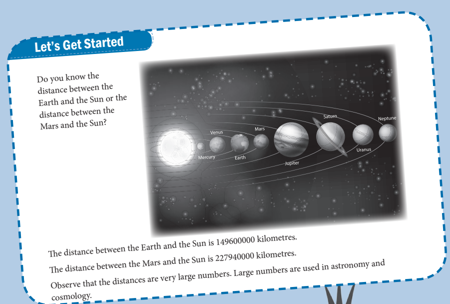
Extend: To enable the students to stimulate thinking and apply prior and current knowledge to real-life situations

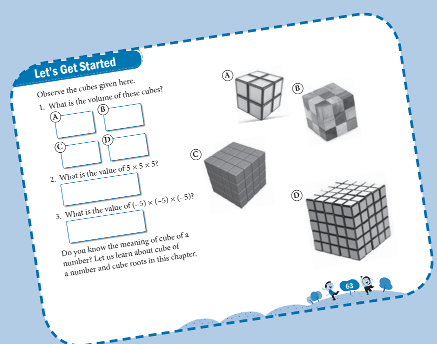
- **Do You Know?**
- **Word Problems**
- **Skill Up!**
  - **Project**
  - **Life Skills**
  - **Mind Buzzer**

Evaluate: To review, assess and evaluate what students have learnt

- **Exercises**
- **Chapter Revision**
- **Reasoning Worksheet**
- **Worksheet**
- **Review Corner**

Balanced use of Manipulatives, Virtual Manipulatives and Abstract Ideas!





About the Features

Engage

Let's Get Started
Picture-based chapter-opening exercise

Common Errors
Pointers highlighting common mistakes and misconceptions

Eminent Mathematicians
Brief write-up on eminent mathematicians and their contributions

Go Easy!
Shortcuts for easy and fast calculations

VEDIC MATHS
Shortcut techniques to solve mathematical calculations

Poster
Important points and formulae in the form of a pull-out page

Explore

Maths Lab Activity
Hands-on activity to help in improving investigation, reinforcement and extension of concepts just learnt

Mental Maths
Objective-type questions to develop quick-thinking skills

PUZZLE!
Questions in the form of a puzzle to stimulate engagement with the concepts

CROSSWORD
Questions in the form of a crossword to stimulate engagement with the concepts

Explain

Note
Pointers for better understanding of concepts

SOLVED EXAMPLES
Example exercises with step-by-step solutions

Concept Map
Logical representation of relationships between concepts in a chapter

Key Concepts
• Definition of important terms and formulae given as bulleted list

Extend

Do you know?
Nuggets of information to add real-world context to abstract mathematical concepts

WORD PROBLEMS
Textual questions based on real-life situations

SKILL UP!

Project
Practical activities to enhance real-world application of concepts

Life Skills
Thought-provoking questions to help develop sensible life skills such as better awareness, empathy and attitude towards self and others

Mind Buzzer
Questions to arouse intellectual curiosity and encourage the students to think beyond the classroom

Evaluate

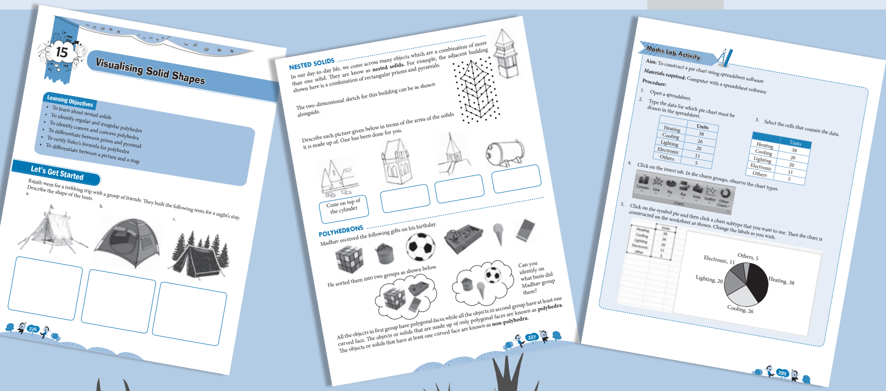
EXERCISE
In-text objective-type questions for quick review, assessment and evaluation

Chapter Revision
Chapter-end questions with varieties of objective and subjective-type questions

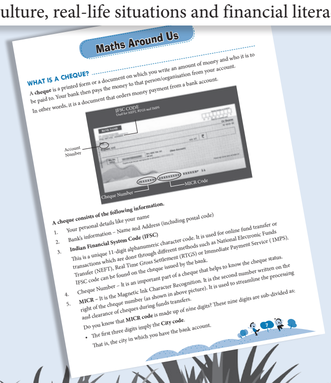
REASONING WORKSHEET
Questions to stimulate rational thinking using mathematical skills

WORKSHEET
Picture-based, fun problem-solving questions to reinforce concepts

REVIEW CORNER
Variety of questions at the end of the book for additional practice



Maths Around Us
A two-page feature providing a variety of interesting information and activities that connect maths and Indian history, art, culture, real-life situations and financial literacy



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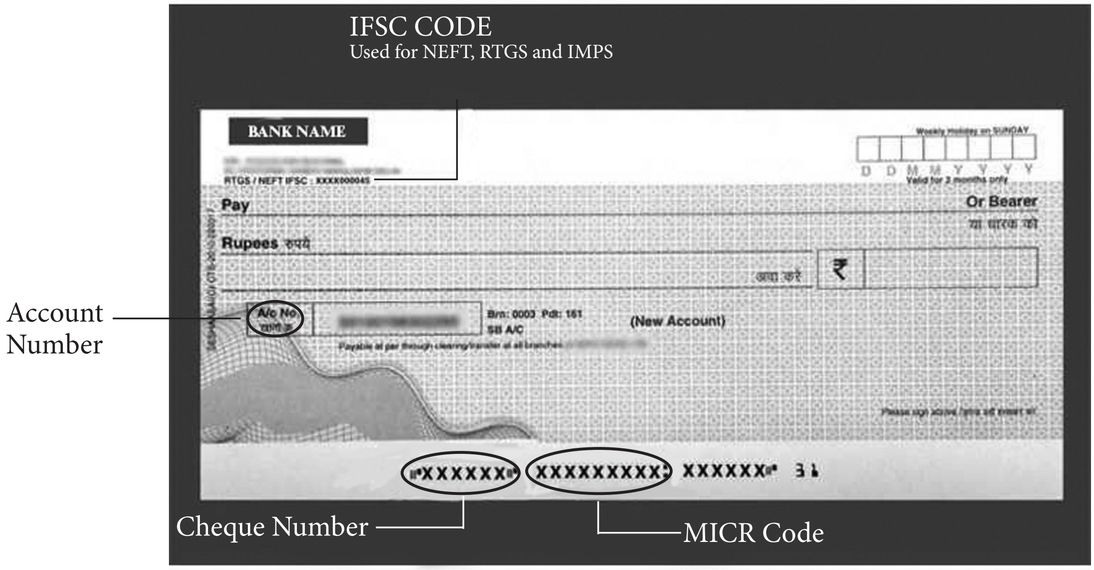
Maths Around Us

WHAT IS A CHEQUE?

A **cheque** is a printed form or a document on which you write an amount of money and who it is to be paid to. Your bank then pays the money to that person/organisation from your account.

In other words, it is a document that orders money payment from a bank account.



A cheque consists of the following information.

1. Your personal details like your name
2. Bank's information – Name and Address (including postal code)
3. **Indian Financial System Code (IFSC)**

   This is a unique 11-digit alphanumeric character code. It is used for online fund transfer or transactions which are done through different methods such as National Electronic Funds Transfer (NEFT), Real Time Gross Settlement (RTGS) or Immediate Payment Service (IMPS). IFSC code can be found on the cheque issued by the bank.
4. Cheque Number – It is an important part of a cheque that helps to know the cheque status.
5. **MICR –** It is the Magnetic Ink Character Recognition. It is the second number written on the right of the cheque number (as shown in above picture). It is used to streamline the processing and clearance of cheques during funds transfers.

   Do you know that **MICR code** is made up of nine digits? These nine digits are sub-divided as:

   - The first three digits imply the **City code**.

     That is, the city in which you have the bank account.

- The next three digits imply the **Bank code**. That is, the bank in in that city.
- The last three digits imply the **Branch code**.

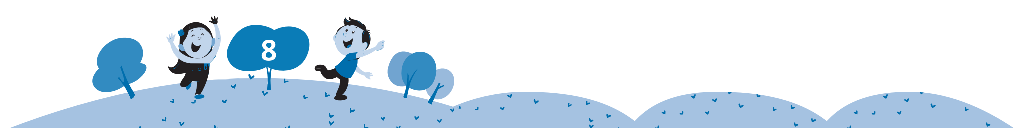
  That is, the particular branch of that particular bank. This is also called the Transaction code.

6. The third number on the right side of the MICR code is your bank account number as per the Reserve Bank of India.
7. The last number signifies whether the account is a savings account or a current account.

   It helps in processing outstation cheques faster.

Your parents must be having a bank account. Ask them to show you what a cheque looks like. Then list down the following details.

Name of the person	
Bank's Name and Address	
Cheque Number	
MICR Code	
Bank Account Number as per RBI	
Savings account or Current Account	
IFSC code	
City Code	
Bank Code	
Branch Code	



1 Rational Numbers

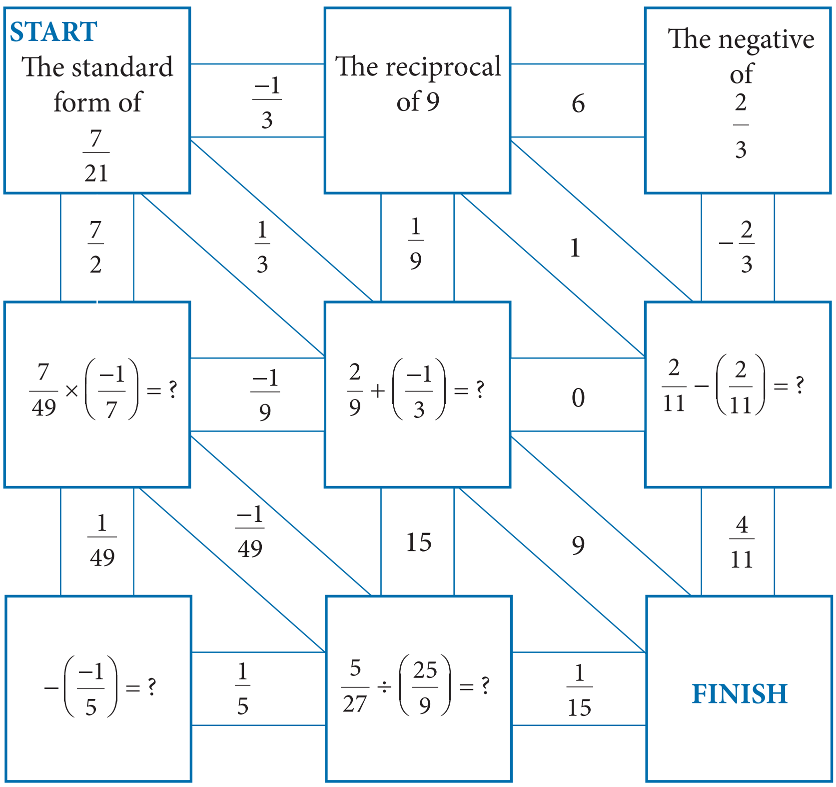
Learning Objectives

- To learn the properties of rational numbers
- To understand the distributive property
- To represent rational numbers on the number line
- To find rational numbers between two rational numbers

Let's Get Started

Make your way from START to FINISH by answering the question in the box and following the path with the correct answer.

Do you remember what these numbers are called?



Easy Recall of the Rule of Signs

1. $(+a) \times (+b) = (+ab)$ 2. $(+a) \times (-b) = (-ab)$ 3. $(-a) \times (+b) = (-ab)$ 4. $(-a) \times (-b) = (+ab)$

Examples: 1. $(+5) \times (+4) = (+20)$ 2. $(+5) \times (-4) = (-20)$
3. $(-5) \times (+4) = (-20)$ 4. $(-5) \times (-4) = (+20)$

PROPERTIES OF RATIONAL NUMBERS

A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is known as a rational number.

> **Note**
>
> 0 can be written as
>
> $0 = \frac{0}{q}$, where $q \neq 0$

For example: $\frac{-5}{4}, \frac{2}{3}, \frac{0}{1}, \frac{4}{7}$ etc. are rational numbers.

Rational numbers include natural numbers (N), whole numbers (W), integers (Z) and all negative and positive fractions (Q).

In the rational number $\frac{p}{q}$, $q \neq 0$ because division by zero is not defined.

Any natural number can be expressed as a rational number. For example, $2 = \frac{2}{1}$, which is a rational number. You have learned about properties of different operations for whole numbers and integers. Similarly, there are some properties of rational numbers.

Closure Property

Let 'a' and 'b' be any two rational numbers, then $a * b$ is also a rational number, where $*$ represents the basic arithmetic operation.

Table 1.1 Closure property of rational numbers

Closure property	Explanation and examples
for addition	To check if the set of rational numbers is closed under addition. Let us take two rational numbers, say, $\frac{-7}{12}$ and $\frac{3}{7}$. $\frac{-7}{12} + \frac{3}{7} = \frac{-7 \times 7 + 3 \times 12}{84} = \frac{-49 + 36}{84} = -\frac{13}{84}$ The resulting number after addition of these two rational numbers is also a rational number. Thus, closure property holds true for addition of rational numbers. For any two rational numbers 'a' and 'b', $a + b$ is also a rational number.
for subtraction	To check if the set of rational numbers is closed under subtraction. Let us take two rational numbers, say, $\frac{4}{5}$ and $\frac{2}{5}$. $\frac{4}{5} - \frac{2}{5} = \frac{4-2}{5} = \frac{2}{5}$ The resulting number after subtraction of these numbers is also a rational number. Thus, closure property holds true for subtraction of rational numbers. For any two rational numbers 'a' and 'b', $a - b$ is also a rational number.

for multiplication	To check if the set of rational numbers is closed under multiplication. Let us take two rational numbers, say, $\frac{1}{7}$ and $\frac{3}{2}$. $\frac{1}{7} \times \frac{3}{2} = \frac{1 \times 3}{7 \times 2} = \frac{3}{14}$ The resulting number after the multiplication of these numbers is also a rational number. Thus, closure property holds true for multiplication of rational numbers. For any two rational numbers '*a*' and '*b*', $a \times b$ is also a rational number.
for division	To check if the set of rational numbers is not closed under division. Closure law does not hold good for division of all rational numbers. Let us take two rational numbers, say, $\frac{4}{5}$ and 0. $\frac{4}{5} \div 0 = \frac{4}{5} \times \frac{1}{0} =$ Not defined Thus, closure property does not hold true for division of all rational numbers. For any two rational numbers '*a*' and '*b*', $\frac{a}{b}$ is not a rational number.

Note

When we divide a number by zero, then division is not defined. So, closure property does not hold true for division by zero. That means, except zero, all numbers are closed under division.

Complete the following table.

Table 1.2 Verification of closure property of rational numbers

a	$\frac{2}{3}$	$\frac{2}{5}$	**0**	**Is it a rational number in all the cases?**
b	$\frac{-5}{9}$	**0**	$\frac{1}{2}$	
$a + b$				YES
$a - b$				
$a \times b$				
$a \div b$				

From the above table, it can be observed that rational numbers are closed under addition, subtraction and multiplication whereas they are not closed under division.

SOLVED EXAMPLES

Example 1: Is $\frac{2}{3} + \frac{-2}{5}$ a rational number?

Solution:

$$\frac{2}{3} + \frac{-2}{5} = \frac{10}{15} + \frac{-6}{15} = \frac{4}{15}$$

Yes, $\frac{2}{3} + \frac{-2}{5}$ gives a rational number.

Commutative Property

Let 'a' and 'b' be any two rational numbers, then $a * b = b * a$, where $*$ represents the basic arithmetic operation.

Table 1.3 Commutative property of rational numbers

Commutative property	Explanation and examples
for addition	The set of rational numbers is commutative under addition. Let us take two rational numbers, say $\frac{7}{8}$ and $\frac{-5}{12}$. $\frac{7}{8}+\left(\frac{-5}{12}\right)=\frac{7\times 3+(-5)\times 2}{24}=\frac{21-10}{24}=\frac{11}{24}$ $\frac{-5}{12}+\frac{7}{8}=\frac{(-5)\times 2+7\times 3}{24}=\frac{-10+21}{24}=\frac{11}{24}$ Thus, $\frac{7}{8}+\left(\frac{-5}{12}\right)=\frac{-5}{12}+\frac{7}{8}$. Therefore, commutative property holds true for addition of rational numbers. For any two rational numbers 'a' and 'b', $a + b = b + a$.
for subtraction	To check if the set of rational numbers is commutative under subtraction. Let us take two rational numbers, say $\frac{4}{5}$ and $\frac{2}{3}$. $\frac{4}{5}-\frac{2}{3}=\frac{4\times 3-2\times 5}{15}=\frac{12-10}{15}=\frac{2}{15}$ $\frac{2}{3}-\frac{4}{5}=\frac{2\times 5-4\times 3}{15}=\frac{10-12}{15}=\frac{-2}{15}$ Thus, $\frac{4}{5}-\frac{2}{3}\neq\frac{2}{3}-\frac{4}{5}$. Therefore, commutative property does not hold true for subtraction of rational numbers. For any two rational numbers 'a' and 'b', $a - b$ **is not equal** to $b - a$.
for multiplication	To check if the set of rational numbers is commutative under multiplication. Let us take two rational numbers, say $\frac{-3}{2}$ and $\frac{7}{5}$. $\frac{-3}{2}\times\frac{7}{5}=\frac{-3\times 7}{2\times 5}=-\frac{21}{10}$ and $\frac{7}{5}\times\frac{-3}{2}=\frac{7\times -3}{5\times 2}=-\frac{21}{10}$ Thus, $\frac{-3}{2}\times\frac{7}{5}=\frac{7}{5}\times\frac{-3}{2}$. Therefore, commutative property holds true for multiplication of rational numbers. For any two rational numbers 'a' and 'b', $a \times b = b \times a$.

for division	To check if the set of rational numbers is commutative under division. Let us take two rational numbers, say $\frac{-3}{2}$ and $\frac{6}{5}$. $\frac{-3}{2} \div \frac{6}{5} = \frac{-3}{2} \times \frac{5}{6} = -\frac{5}{4}$ $\frac{6}{5} \div \frac{-3}{2} = \frac{6}{5} \times \left(-\frac{2}{3}\right) = -\frac{4}{5}$ Thus, $\frac{-3}{2} \div \frac{6}{5} \neq \frac{6}{5} \div \frac{-3}{2}$. Therefore, commutative property does not hold true for division of rational numbers. For any two rational numbers 'a' and 'b', $\frac{a}{b}$ is not equal to $\frac{b}{a}$.

Complete the following table.

Table 1.4 Verification of commutative property of rational numbers

a	$\frac{1}{6}$	$\frac{1}{7}$	**0**	
b	$\frac{-1}{5}$	**0**	$\frac{-2}{7}$	
$a + b$			Is $a + b = b + a$?	
$b + a$				
$a - b$			Is $a - b = b - a$?	
$b - a$				
$a \times b$			Is $a \times b = b \times a$?	
$b \times a$				
$a \div b$			Is $a \div b = b \div a$?	
$b \div a$				

From the above table, it can be observed that

- Sum of two rational numbers is same irrespective of the order in which they are added.
- Product of two rational numbers is same irrespective of the order in which they are multiplied.
- Difference between the two rational numbers is not same if the order of the numbers is changed.
- Division of two rational numbers is not same if the order of the numbers is changed.

Thus, for rational numbers, addition and multiplication are commutative whereas subtraction and division are not commutative.

SOLVED EXAMPLES

Example 2: Is $\frac{1}{2}-\frac{2}{3}=\frac{3}{6}-\frac{4}{6}$?

Solution:

LHS

$$\frac{1}{2}-\frac{2}{3}=\frac{3}{6}-\frac{4}{6}$$
$$=-\frac{1}{6}$$

RHS

$$\frac{2}{3}-\frac{1}{2}=\frac{4}{6}-\frac{3}{6}$$
$$=\frac{1}{6}$$

LHS ≠ RHS

Therefore, $\frac{1}{2}-\frac{2}{3}\neq\frac{2}{3}-\frac{1}{2}$.

Example 3: Is $\frac{1}{4}\times\frac{-8}{3}=\frac{-8}{3}\times\frac{1}{4}$?

Solution:

LHS

$$\frac{1}{\cancel{4}}\times\frac{-\cancel{8}^{\,2}}{3}$$
$$=\frac{-2}{3}$$

RHS

$$\frac{-\overset{2}{\cancel{8}}}{3}\times\frac{1}{\cancel{4}}$$
$$=\frac{-2}{3}$$

LHS = RHS

Therefore, $\frac{1}{4}\times\frac{-8}{3}=\frac{-8}{3}\times\frac{1}{4}$

Associative Property

Let 'a', 'b' and 'c' be any three rational numbers, then $a * (b * c) = (a * b) * c$, where $*$ represents the basic arithmetic operation.

Table 1.5 Associative property of rational numbers

Associative property	Explanation and examples
for addition	To check if the set of rational numbers is associative under addition. Let us take three rational numbers, say $\frac{3}{24}, \frac{7}{8}$ and $\frac{-5}{12}$ $\frac{3}{24}+\left\{\frac{7}{8}+\left(\frac{-5}{12}\right)\right\}=\frac{3}{24}+\left(\frac{21-10}{24}\right)$ $=\frac{3}{24}+\frac{11}{24}$ $=\frac{3+11}{24}$ $=\frac{14}{24}\quad=\frac{7}{12}$ $\left(\frac{3}{24}+\frac{7}{8}\right)+\left(\frac{-5}{12}\right)=\left(\frac{3+21}{24}\right)+\left(\frac{-5}{12}\right)$ $=\frac{24}{24}+\left(\frac{-5}{12}\right)$ $=1+\left(\frac{-5}{12}\right)$ $=\frac{12-5}{12}=\frac{7}{12}$ Thus, $\frac{3}{24}+\left\{\frac{7}{8}+\left(\frac{-5}{12}\right)\right\}=\left(\frac{3}{24}+\frac{7}{8}\right)+\left(\frac{-5}{12}\right)$. Therefore, associative property holds true for addition of rational numbers. If 'a', 'b' and 'c' are three rational numbers, then $a + (b + c) = (a + b) + c$

for subtraction	To check if the set of rational numbers is associative under subtraction. Let us take three rational numbers, say $\frac{3}{24}, \frac{7}{8}$ and $\frac{5}{12}$. $\frac{3}{24} - \left(\frac{7}{8} - \frac{5}{12}\right) = \frac{3}{24} - \left(\frac{21-10}{24}\right) = \frac{3}{24} - \frac{11}{24} = \frac{3-11}{24} = -\frac{8}{24}$ $\left(\frac{3}{24} - \frac{7}{8}\right) - \frac{5}{12} = \left(\frac{3-21}{24}\right) - \frac{5}{12} = -\frac{18}{24} - \frac{5}{12} = \frac{-18-10}{24} = -\frac{28}{24}$ Thus, $\frac{3}{24} - \left(\frac{7}{8} - \frac{5}{12}\right) \neq \left(\frac{3}{24} - \frac{7}{8}\right) - \frac{5}{12}$. Therefore, associative property does not hold true for subtraction of rational numbers. If 'a', 'b' and 'c' are three rational numbers, then $a - (b - c)$ **is not equal to** $(a - b) - c$
for multiplication	To check if the set of rational numbers is associative under multiplication. Let us take three rational numbers, say $\frac{2}{7}, \frac{-3}{2}$ and $\frac{7}{5}$. $\frac{2}{7} \times \left(\frac{-3}{2} \times \frac{7}{5}\right) = \frac{\cancel{2}}{\cancel{7}} \times \left(-\frac{\overset{3}{\cancel{21}}}{\underset{5}{\cancel{10}}}\right) = -\frac{3}{5}$ $\left(\frac{2}{7} \times \frac{-3}{2}\right) \times \frac{7}{5} = -\frac{3}{\cancel{7}} \times \frac{\cancel{7}}{5} = -\frac{3}{5}$ Thus, $\frac{2}{7} \times \left(\frac{-3}{2} \times \frac{7}{5}\right) = \left(\frac{2}{7} \times \frac{-3}{2}\right) \times \frac{7}{5}$. Therefore, associative property holds true for multiplication of rational numbers. If 'a', 'b' and 'c' are three rational numbers, then $a \times (b \times c) = (a \times b) \times c$
for division	To check if the set of rational numbers is associative under division. Let us take three rational numbers, say, $\frac{2}{5}, \frac{-3}{2}$ and $\frac{6}{5}$ $\frac{2}{5} \div \left(\frac{-3}{2} \div \frac{6}{5}\right) = \frac{2}{5} \div \left(\frac{-3}{2} \times \frac{5}{6}\right) = \frac{2}{5} \div \left(-\frac{5}{4}\right) = \frac{2}{5} \times \left(-\frac{4}{5}\right) = -\frac{8}{25}$ $\left(\frac{2}{5} \div \frac{-3}{2}\right) \div \frac{6}{5} = \left(\frac{2}{5} \times \frac{-2}{3}\right) \div \frac{6}{5} = -\frac{4}{15} \div \frac{6}{5} = -\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{15}}} \times \frac{\cancel{5}}{\underset{3}{\cancel{6}}} = -\frac{2}{9}$ Thus, $\frac{2}{5} \div \left(\frac{-3}{2} \div \frac{6}{5}\right) \neq \left(\frac{2}{5} \div \frac{-3}{2}\right) \div \frac{6}{5}$. Therefore, associative property does not hold true for division of rational numbers.

Complete the following table.

Table 1.6 Verification of associative property of rational numbers

a	$\frac{1}{6}$	$\frac{2}{13}$	$\frac{-1}{8}$	
b	$\frac{3}{11}$	**0**	$\frac{-3}{8}$	
c	$\frac{-1}{6}$	$\frac{1}{13}$	**0**	
$a + (b + c)$			Is $a + (b + c) = (a + b) + c$?	
$(a + b) + c$				
$a - (b - c)$			Is $a - (b - c) = (a - b) - c$?	
$(a - b) - c$				
$a \times (b \times c)$			Is $a \times (b \times c) = (a \times b) \times c$?	
$(a \times b) \times c$				
$a \div (b \div c)$			Is $a \div (b \div c) = (a \div b) \div c$?	
$(a \div b) \div c$				

From the above table, it can be observed that for rational numbers, addition and multiplication are associative whereas subtraction and division are not associative.

SOLVED EXAMPLES

Example 4: Simplify $\frac{1}{2} \times \left(\frac{2}{5} \times \frac{5}{8}\right)$.

Solution:

$$\frac{1}{2} \times \left(\frac{2}{5} \times \frac{5}{8}\right) = \frac{1}{\cancel{2}} \times \frac{\cancel{10}^{\,5}}{\cancel{40}_{\,8}}$$

$$= \frac{1}{8}$$

Therefore, the value of $\frac{1}{2} \times \left(\frac{2}{5} \times \frac{5}{8}\right)$ is $\frac{1}{8}$.

Example 5: Prove that $\frac{1}{9}+\left(\frac{2}{7}+\frac{3}{5}\right)=\left(\frac{1}{9}+\frac{2}{7}\right)+\frac{3}{5}$.

Solution:

LHS

$$\frac{1}{9}+\left(\frac{2}{7}+\frac{3}{5}\right)=\frac{1}{9}+\left(\frac{10}{35}+\frac{21}{35}\right)$$
$$=\frac{1}{9}+\frac{31}{35}$$
$$=\frac{35}{315}+\frac{279}{315}$$
$$=\frac{314}{315}$$

RHS

$$\left(\frac{1}{9}+\frac{2}{7}\right)+\frac{3}{5}=\left(\frac{7}{63}+\frac{18}{63}\right)+\frac{3}{5}$$
$$=\frac{25}{63}+\frac{3}{5}$$
$$=\frac{125}{315}+\frac{189}{315}$$
$$=\frac{314}{315}$$

Hence, LHS = RHS.

Therefore, $\frac{1}{9}+\left(\frac{2}{7}+\frac{3}{5}\right)=\left(\frac{1}{9}+\frac{2}{7}\right)+\frac{3}{5}$

EXERCISE 1A

1. Write T for true and F for false.

a. Rational numbers are not closed under multiplication.

b. Commutative property does not hold true for subtraction of rational numbers.

c. Associative property holds true for addition of rational numbers.

d. $\frac{1}{9}+\frac{2}{7}=\frac{2}{7}+\frac{1}{9}$

e. $\frac{1}{5}\times\left(\frac{3}{4}\times\frac{7}{9}\right)=\left(\frac{1}{5}+\frac{3}{4}\right)\times\frac{7}{9}$

2. Fill in the blanks.

a. If a and b are rational numbers, then $a + b$ is a ______________ number.

b. $a \times (b \times c) = (\_\_\_\_ \times b) \times \_\_\_\_$ represents associative property of multiplication.

c. $\frac{1}{2}+\frac{2}{3}=$ ________ $+\ \frac{1}{2}$

d. $\frac{1}{7}\times\left(\frac{3}{7}\times\frac{14}{15}\right)=\left(\frac{1}{7}\times\frac{3}{7}\right)\times$ ______.

e. $\frac{3}{8}\div\left(\frac{-51}{24}\div\frac{17}{12}\right)$ is equal to _______.

3. Solve the following.

a. $\frac{1}{5}+\frac{-2}{9}$ b. $\frac{12}{26}\times\frac{13}{48}$ c. $\frac{1}{6}-\frac{2}{5}$

d. $\frac{7}{24}\div\frac{21}{8}$ e. $\frac{11}{55}\times\frac{-5}{4}$ f. $\frac{5}{8}\div\frac{15}{16}$

4. Simplify the following using associative property of rational numbers.

a. $\frac{1}{5}+\left(\frac{2}{5}+\frac{3}{5}\right)$ b. $\frac{1}{2}+\left(\frac{2}{3}+\frac{4}{3}\right)$ c. $\frac{1}{3}\times\left(\frac{-12}{15}\times\frac{5}{-8}\right)$

d. $\left(\frac{2}{11}\times\frac{11}{8}\right)\times\frac{4}{7}$ e. $\left(\frac{-2}{9}+\frac{2}{9}\right)+\frac{5}{9}$ f. $\left(\frac{3}{13}+\frac{8}{13}\right)+\frac{1}{3}$

5. Verify the following.

a. $\frac{1}{3}+\left(\frac{2}{4}+\frac{3}{5}\right)=\left(\frac{1}{3}+\frac{2}{4}\right)+\frac{3}{5}$ b. $\frac{2}{11}\times\left(\frac{11}{2}\times\frac{4}{5}\right)=\left(\frac{2}{11}\times\frac{11}{2}\right)\times\frac{4}{5}$

c. $\left(\frac{7}{5}+\frac{2}{5}\right)+\frac{1}{5}=\frac{7}{5}+\left(\frac{2}{5}+\frac{1}{5}\right)$ d. $\frac{5}{9}\times\left(\frac{1}{2}\times\frac{3}{5}\right)=\left(\frac{5}{9}\times\frac{1}{2}\right)\times\frac{3}{5}$

6. Answer the following questions.

a. Subtract the sum of $\frac{-8}{7}$ and $\frac{5}{4}$ from the sum of $\frac{3}{2}$ and $\frac{-31}{28}$.

b. The sum of two numbers is $\frac{7}{20}$. If one of them is $\frac{1}{4}$, find the other.

c. What number should be subtracted from $\frac{5}{9}$ to get $\frac{1}{3}$?

d. The area of a rectangular field is $7\frac{1}{3}$ sq. m. Also, the breadth of the field is $2\frac{3}{4}$ m. Find the length of the field.

SOME MORE PROPERTIES

The Property of Zero (0)

What do you observe when 0 is added to an integer, and to a whole number?

$\boxed{2 + 0 = 2}$ $\boxed{-2 + 0 = -2}$

We get the same integer or the whole number. Similarly, we get the same rational number when zero is added to a rational number. This is known as zero property.

Note

0 is known as the additive identity or the identity element for the addition of rational numbers.

Look at the table given below and complete it.

Table 1.7 Verification of the zero property of rational numbers

Rational numbers	Zero property	Rational number
0	$0 + 0 =$	0
$\frac{1}{2}$	$\frac{1}{2} + 0 =$	
$\frac{a}{b}$	$\frac{a}{b} + 0 =$	

In general, let $\frac{a}{b}$ be any rational number. Then, $\frac{a}{b} + 0 = \frac{a}{b} = 0 + \frac{a}{b}$.

The Property of One (1)

When an integer or a whole number is multiplied with 1 , the product is the same integer or whole number.

$\boxed{2 \times 1 = 2}$ $\boxed{-2 \times 1 = -2}$

Similarly, what do you get when a rational number is multiplied with 1? $\frac{1}{2} \times 1 = \frac{1}{2}$

This is known as multiplicative property.

Look at the table given below and complete it.

Table 1.8 Verification of the multiplicative property of rational numbers

Rational numbers	Multiplicative property	Rational number
0	$0 \times 1 =$	0
1	$1 \times 1 =$	
$\frac{1}{2}$	$\frac{1}{2} \times 1 =$	
$\frac{-2}{5}$	$\frac{-2}{5} \times 1 =$	
$\frac{a}{b}$	$\frac{a}{b} \times 1 =$	

In general, let $\frac{a}{b}$ be any rational number, then $\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b}$.

Additive Inverse or Negative of a Rational Number

Additive inverse of a positive integer is obtained by just writing a negative sign before it. For example, additive inverse of 3 is –3.

In general, let $\frac{a}{b}$ be any rational number, then $-\frac{a}{b}$ is its additive inverse and is a rational number. Then, $\frac{a}{b} + \left(-\frac{a}{b}\right) = \left(-\frac{a}{b}\right) + \frac{a}{b} = 0$.

Note

$\frac{-a}{b}$ is known as negative of $\frac{a}{b}$.

It is the additive inverse of $\frac{a}{b}$.

Reciprocal of a Rational Number

The reciprocal of a number is 1 divided by that number. A fraction's numerator and denominator are interchanged to get its reciprocal. It is same for a rational number too.

For example, the reciprocal of $\frac{4}{9} = 1 \div \frac{4}{9} = \frac{9}{4}$.

In general, the reciprocal of a number $\frac{a}{b}$ is given by $\frac{b}{a}$ and it is denoted by $\left(\frac{a}{b}\right)^{-1}$. In other words, $\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^{-1} = \frac{\boldsymbol{b}}{\boldsymbol{a}}$.

Note

- 0 has no reciprocal.
- The reciprocal of 1 is 1 and the reciprocal of (−1) is (−1).

DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION AND SUBTRACTION

Let $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ be any three rational numbers. Then, $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) + \left(\frac{a}{b} \times \frac{e}{f}\right)$ and $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) - \left(\frac{a}{b} \times \frac{e}{f}\right)$.

Prove that: $\frac{1}{5} \times \left(\frac{3}{2} + \frac{2}{3}\right) = \left(\frac{1}{5} \times \frac{3}{2}\right) + \left(\frac{1}{5} \times \frac{2}{3}\right)$

LHS:

$$\frac{1}{5} \times \left(\frac{3}{2} + \frac{2}{3}\right) = \frac{1}{5} \times \left(\frac{3\times3+2\times2}{6}\right) = \frac{1}{5} \times \left(\frac{9+4}{6}\right) = \frac{1}{5} \times \frac{13}{6} = \frac{13}{30}$$

(By the distributive property of multiplication over addition)

RHS: $\left(\frac{1}{5} \times \frac{3}{2}\right) + \left(\frac{1}{5} \times \frac{2}{3}\right) = \frac{1\times3}{5\times2} + \frac{1\times2}{5\times3} = \frac{3}{10} + \frac{2}{15} = \frac{(3\times3)+(2\times2)}{30} = \frac{9+4}{30} = \frac{13}{30}$

Both the approaches give the same result. This shows that $\frac{1}{5} \times \left(\frac{3}{2} + \frac{2}{3}\right) = \left(\frac{1}{5} \times \frac{3}{2}\right) + \left(\frac{1}{5} \times \frac{2}{3}\right)$.

Prove that: $\boldsymbol{\frac{1}{5} \times \left(\frac{3}{2} - \frac{2}{3}\right) = \left(\frac{1}{5} \times \frac{3}{2}\right) - \left(\frac{1}{5} \times \frac{2}{3}\right)}$

LHS: $\frac{1}{5} \times \left(\frac{3}{2} - \frac{2}{3}\right) = \frac{1}{5} \times \left(\frac{3\times3-2\times2}{6}\right) = \frac{1}{5} \times \left(\frac{9-4}{6}\right) = \frac{1}{5} \times \frac{5}{6} = \frac{1}{6}$

(By the distributive property of multiplication over subtraction)

$$\left(\frac{1}{5} \times \frac{3}{2}\right) - \left(\frac{1}{5} \times \frac{2}{3}\right) = \frac{1\times3}{5\times2} - \frac{1\times2}{5\times3} = \frac{3}{10} - \frac{2}{15} = \frac{(3\times3)-(2\times2)}{30} = \frac{9-4}{30} = \frac{5}{30} = \frac{1}{6}$$

Both the approaches give the same result.

This shows that $\frac{1}{5} \times \left(\frac{3}{2} - \frac{2}{3}\right) = \left(\frac{1}{5} \times \frac{3}{2}\right) - \left(\frac{1}{5} \times \frac{2}{3}\right)$.

Mental Maths

1. Find a rational number which is an additive inverse of its own.
2. What is the reciprocal of the additive inverse of $\frac{1}{3}$?

Complete the table given below.

Table 1.9 Verification of distributive property of multiplication over addition and subtraction

a	$\frac{1}{8}$	0	$\frac{-1}{7}$	
b	$\frac{1}{6}$	$\frac{2}{5}$	$\frac{1}{7}$	
c	$\frac{-1}{4}$	$\frac{3}{5}$	0	
$a \times (b + c)$			Is $a \times (b + c) = (a \times b) + (a \times c)$?	
$(a \times b) + (a \times c)$				
$a \times (b - c)$			Is $a \times (b - c) = (a \times b) - (a \times c)$?	
$(a \times b) - (a \times c)$				

SOLVED EXAMPLES

Example 6: Write the additive inverse of the following rational numbers.

a. $\frac{1}{9}$

b. $\frac{-5}{6}$

Solution:

a. Given: $\frac{1}{9}$

$-\frac{1}{9}$ is the additive inverse of $\frac{1}{9}$

since $\frac{1}{9}+\left(-\frac{1}{9}\right)=0$.

b. Given: $\frac{-5}{6}$

$\frac{5}{6}$ is the additive inverse of $\frac{-5}{6}$

since $\frac{5}{6}+\left(\frac{-5}{6}\right)=0$.

Example 7: Write the reciprocal of the following rational numbers.

a. $\frac{1}{8}$

b. $\frac{-3}{5}$

Solution:

a. Given: $\frac{1}{8}$

The reciprocal of $\frac{1}{8} = \frac{8}{1} = 8$

b. Given: $\frac{-3}{5}$

$\frac{5}{-3}$ is the reciprocal of $\frac{-3}{5}$ since $\frac{-3}{5} \times \frac{5}{-3} = 1$

We write $\frac{5}{-3}$ as $\frac{-5}{3}$.

Example 8: Simplify $\frac{4}{5}\left(\frac{5}{8}+\frac{1}{2}\right)$ using distributive property of rational numbers.

Solution: Given: $\frac{4}{5}\left(\frac{5}{8}+\frac{1}{2}\right)$

Use distributive property, $\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right)$

$$\begin{aligned}\frac{4}{5}\left(\frac{5}{8}+\frac{1}{2}\right)&=\left(\frac{4}{5}\times\frac{5}{8}\right)+\left(\frac{4}{5}\times\frac{1}{2}\right)\\&=\left(\frac{1}{2}\right)+\left(\frac{2}{5}\right)\\&=\frac{1\times5+2\times2}{10}\\&=\frac{9}{10}\end{aligned}$$

Therefore, $\frac{4}{5}\left(\frac{5}{8}+\frac{1}{2}\right)=\frac{9}{10}$.

Example 9: Verify that $\frac{1}{2}\left(\frac{3}{4}+\frac{5}{6}\right)=\left(\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{5}{6}\right)$

Solution:

LHS

$$\begin{aligned}\frac{1}{2}\left(\frac{3}{4}+\frac{5}{6}\right)&=\frac{1}{2}\left(\frac{3\times3+5\times2}{12}\right)\\&=\frac{1}{2}\times\frac{19}{12}\\&=\frac{19}{24}\end{aligned}$$

RHS

$$\begin{aligned}\left(\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{5}{6}\right)&=\frac{3}{8}+\frac{5}{12}\\&=\frac{3\times3+5\times2}{24}\\&=\frac{19}{24}\end{aligned}$$

LHS = RHS

Since, $\frac{1}{2}\left(\frac{3}{4}+\frac{5}{6}\right)=\left(\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{5}{6}\right)$.

Hence, verified.

EXERCISE 1B

1. Fill in the blanks.

a. $\frac{1}{9} \times 0 =$ _____.

b. The additive inverse of $\frac{-2}{15}$ is _____.

c. $\frac{6}{11} \times$ __________ $= 1$.

d. $\frac{-11}{13} + 0 =$ _____.

e. The reciprocal of $\frac{7}{13}$ is _____.

f. $-\frac{5}{23} \times 1 =$ _____.

2. Match the following.

a. The multiplicative inverse of 9 — i. 9

b. Multiplicative identity — ii. $-\frac{1}{9}$

c. Additive identity — iii. $\frac{1}{9}$

d. The reciprocal of $\frac{1}{9}$ — iv. 0

e. The additive inverse of $\frac{1}{9}$ — v. 1

3. Write the additive inverse of the following rational numbers.

a. 1 b. 0 c. $\frac{-2}{7}$ d. $\frac{11}{17}$ e. $\frac{-1}{10}$ f. $\frac{1}{19}$

4. Write the reciprocal of the following rational numbers.

a. $\frac{-3}{11}$ b. $\frac{5}{12}$ c. $\frac{9}{15}$ d. $\frac{12}{-17}$ e. -8 f. 5

5. Simplify the following using distributive property of rational numbers.

a. $\frac{2}{5} \times \left(-\frac{3}{4} + \frac{1}{4}\right)$

b. $\frac{3}{5} \times \left(\frac{27}{21} + \frac{8}{1}\right)$

c. $\frac{3}{7} \times \left(\frac{7}{16} - \frac{21}{4}\right)$

d. $\left(\frac{3}{4} \times \frac{1}{4}\right) + \left(\frac{3}{4} \times \frac{5}{4}\right)$

e. $\left(\frac{5}{9} \times \frac{2}{3}\right) + \left(\frac{4}{3} \times \frac{5}{9}\right)$

f. $\left(\frac{9}{13} \times \frac{2}{5}\right) - \left(\frac{1}{5} \times \frac{9}{13}\right)$

6. Verify the following calculations.

a. $\frac{4}{-5} \times \left(\frac{8}{11} - \frac{7}{13}\right) = \left(\frac{4}{-5} \times \frac{8}{11}\right) - \left(\frac{4}{-5} \times \frac{7}{13}\right)$

b. $\frac{2}{5}\left(\frac{3}{4} + \frac{5}{7}\right) = \left(\frac{2}{5} \times \frac{3}{4}\right) + \left(\frac{2}{5} \times \frac{5}{7}\right)$

c. $\frac{3}{7}\left(\frac{2}{5} + \frac{1}{3}\right) = \left(\frac{3}{7} \times \frac{2}{5}\right) + \left(\frac{3}{7} \times \frac{1}{3}\right)$

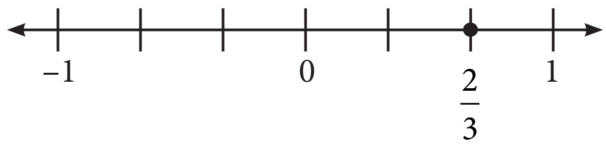
d. $\frac{1}{5} \times \left(\frac{3}{2} - \frac{2}{5}\right) = \left(\frac{1}{5} \times \frac{3}{2}\right) - \left(\frac{1}{5} \times \frac{2}{5}\right)$

REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

We have learnt to represent integers and fractions on the number line. Representing rational numbers is the same as representing fractions.

Rational numbers are represented on the number line by dividing a unit into as many divisions as the denominator of the rational number. Number of extra markings made is given by subtracting 1 from the denominator.

For example, $\frac{2}{3}$ is represented on the number line by dividing each unit into 3 equal parts as shown below. Number of extra markings made is $3 - 1 = 2$.

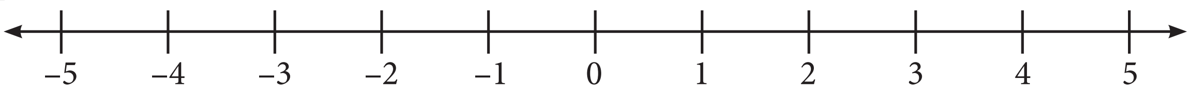


SOLVED EXAMPLES

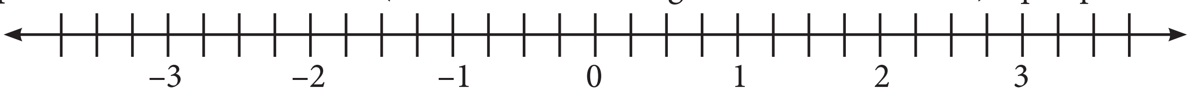
Example 10: Represent $\frac{-7}{4}$ on number line.

Solution:

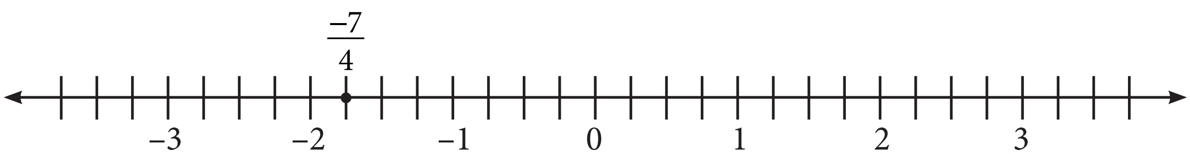
Step 1: Draw a number line such that the distance between two consecutive numbers is 1 unit.



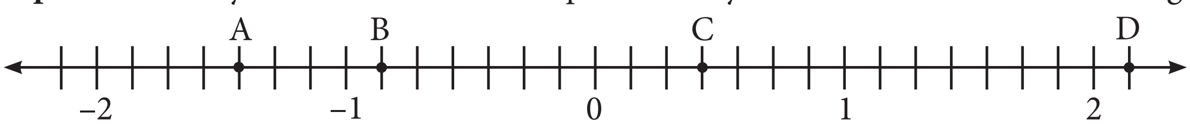
Step 2: Divide each unit into 4 (denominator of the given rational number) equal parts.



Step 3: To represent $\frac{-7}{4}$, move 7 units to the left from 0 since it is a negative rational number and place a dot over the 7th division.



Example 11: Identify the rational number represented by the letters on the number line given below.



Solution:

Each unit is divided into 7 equal parts.

A is on the 10th mark to the left of 0. So, A represents $\frac{-10}{7}$.

B is on the 6th mark to the left of 0. So, B represents $\frac{-6}{7}$.

C is on the 3rd mark to the right of 0. So, C represents $\frac{3}{7}$.

D is on the 15th mark to the right of 0. So, D represents $\frac{15}{7}$.

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

As we know that there are infinite rational numbers between any two given rational numbers, there are different methods to find rational numbers between any two given rational numbers.

Method 1: LCM method

The following steps should be followed when the rational numbers are with different denominators.

Step 1: First, find the LCM of the denominators.

Step 2: Convert the rational numbers to the equivalent rational numbers with the LCM as the common denominator.

Step 3: Keeping the LCM as the denominator, choose integers in-between the numerators of the equivalent rational numbers as numerators.

> **Note**
>
> LCM method is very useful for finding many rational numbers between given rational numbers. The calculations are much simpler and can be done faster.

Method 2: Mean method

Consider the given two rational numbers.

$\frac{3}{5}$ and $\frac{4}{5}$

↓

Find the mean of the two rational numbers.

$$\frac{\frac{3}{5}+\frac{4}{5}}{2}=\frac{7}{10}$$

↓

Find the mean of the first rational number and the mean obtained in the previous step.

$$\frac{\frac{3}{5}+\frac{7}{10}}{2}=\frac{\frac{3\times2+7}{10}}{2}=\frac{13}{20}$$

Find the mean of the second rational number and the mean obtained in the previous step.

$$\frac{\frac{4}{5}+\frac{7}{10}}{2}=\frac{\frac{4\times2+7}{10}}{2}=\frac{15}{20}$$

↓

Continue the process till the desired number of rational numbers are found.

↓

Write the rational numbers found in the previous steps.

SOLVED EXAMPLES

Example 12: Find any five rational numbers between $\frac{-2}{5}$ and $\frac{3}{8}$.

Solution:

Make the denominators same by taking the LCM of the denominators.

LCM of 5 and 8 is 40.

So, $\frac{-2}{5}=\frac{-2\times8}{5\times8}=\frac{-16}{40}$. Similarly, $\frac{3}{8}=\frac{3\times5}{8\times5}=\frac{15}{40}$.

Thus, the rational numbers are $\frac{-15}{40},\frac{-14}{40},\frac{-13}{40},\ldots\frac{13}{40},\frac{14}{40},$ between the two given rational numbers. We can take any 5 of them.

Example 13: Find a rational number between $\frac{1}{5}$ and $\frac{2}{5}$.

Solution:

In the mean method, we go on finding the average of the two given numbers to obtain new rational numbers.

So, the mean of $\frac{1}{5}$ and $\frac{2}{5}$ is $= \dfrac{\frac{1}{5}+\frac{2}{5}}{2} = \dfrac{\frac{3}{5}}{2} = \frac{3}{10}$.

Therefore, $\frac{3}{10}$ is a rational number between $\frac{1}{5}$ and $\frac{2}{5}$.

Note

Again, by finding the mean of $\frac{1}{5}$ and $\frac{3}{10}$, another rational number in between the two rational numbers can be found.

Example 14: Find three rational numbers between $\frac{2}{3}$ and $\frac{7}{2}$.

Solution:

In the mean method, we go on finding the average of the two given numbers to obtain new rational numbers.

The mean of $\frac{2}{3}$ and $\frac{7}{2}$ is $= \dfrac{\frac{2}{3}+\frac{7}{2}}{2} = \dfrac{\frac{4+21}{6}}{2} = \frac{25}{12}$.

Therefore, $\frac{25}{12}$ is a rational number between $\frac{2}{3}$ and $\frac{7}{2}$.

To find the second rational number, find the mean of $\frac{2}{3}$ and $\frac{25}{12}$.

The mean of $\frac{2}{3}$ and $\frac{25}{12}$ is $= \dfrac{\frac{2}{3}+\frac{25}{12}}{2} = \dfrac{\frac{8+25}{12}}{2} = \frac{33}{24}$.

Therefore, $\frac{33}{24}$ is another rational number between $\frac{2}{3}$ and $\frac{7}{2}$.

To find the third rational number, find the mean of $\frac{33}{24}$ and $\frac{7}{2}$.

The mean of $\frac{33}{24}$ and $\frac{7}{2}$ is $= \dfrac{\frac{33}{24}+\frac{7}{2}}{2} = \dfrac{\frac{33+84}{24}}{2} = \frac{117}{48}$.

Therefore, $\frac{117}{48}$ is another rational number between $\frac{2}{3}$ and $\frac{7}{2}$.

Hence, the three rational numbers between $\frac{2}{3}$ and $\frac{7}{2}$ are $\frac{25}{12}, \frac{33}{24}$, and $\frac{117}{48}$.

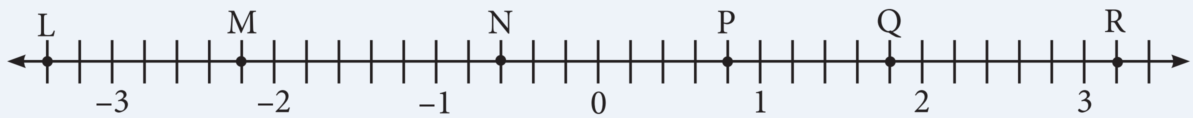
EXERCISE 1C

1. **Represent the following rational numbers on the number line.**

a. $\frac{-4}{5}$ b. $\frac{3}{7}$ c. $\frac{5}{3}$

d. $\frac{-6}{4}$ e. $\frac{7}{2}$ f. $\frac{-2}{4}$

2. **Write the rational numbers represented by the letters given on the number line.**



L: ________, M: ________, N: ________, P: ________, Q: ________, R: ________.

3. **Write five rational numbers between the given rational numbers.**

a. $\frac{-1}{5}$ and $\frac{1}{5}$ b. $\frac{-1}{10}$ and $\frac{3}{10}$ c. $\frac{2}{7}$ and $\frac{3}{5}$

d. $\frac{-1}{3}$ and $\frac{2}{11}$ e. 0 and –1 f. 1 and 2

4. **Find a rational number between the given rational numbers using the method of means.**

a. 2 and 3 b. –2 and –1 c. $\frac{1}{3}$ and $\frac{2}{3}$

d. $\frac{-1}{4}$ and $\frac{-3}{4}$ e. $\frac{1}{7}$ and $\frac{2}{7}$ f. 0 and 1

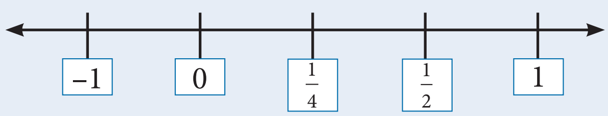
Maths Lab Activity

Aim: To find '*n*' number of rational numbers between two rational numbers.

Materials required: The whole class and the blackboard/whiteboard

Procedure:

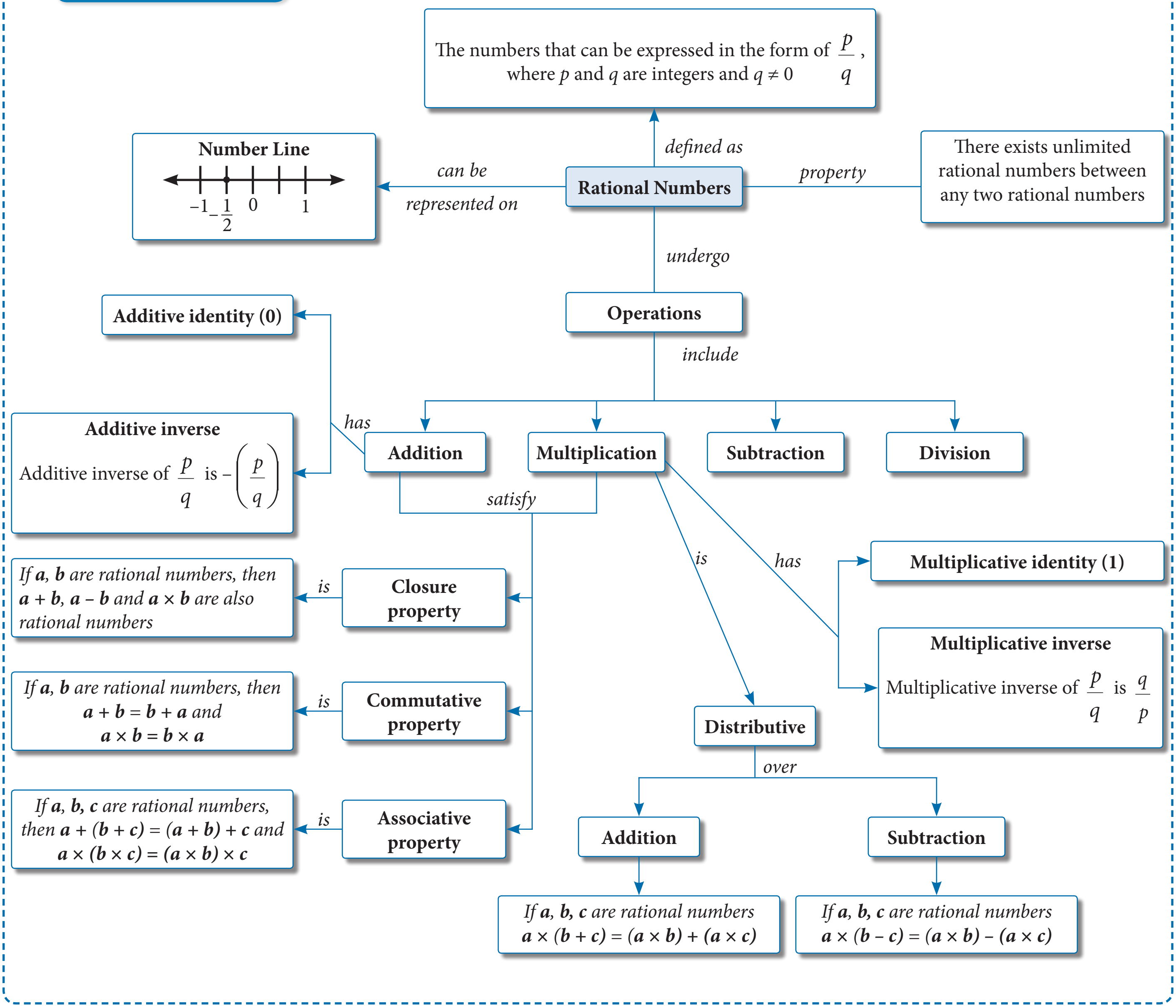
1. Divide the whole class into three groups, name the groups as A, B and C.
2. Draw a number line on the blackboard /whiteboard and plot $\frac{1}{4}$ and $\frac{1}{2}$ on it as shown.



3. Ask a student from group A to mark a rational number between $\frac{1}{4}$ and $\frac{1}{2}$.
4. Ask a student from group B to mark a rational number between $\frac{1}{4}$ and the new rational number.
5. Ask a student from group C to mark a rational number between $\frac{1}{2}$ and the new rational number.
6. Continue this process for at least three rounds.
7. Discuss the different methods through which each group found a rational number between two rational numbers.
8. Provide some more pairs of rational numbers and ask the students to find a rational number between those pairs.

Conclusion: There are infinite rational numbers between two rational numbers.

Concept Map



Key Concepts

- **Rational Number:** A number which is in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- **Additive identity:** It is the number which when added to any number gives the same number as the result. 0 is the additive identity for rational numbers.
- **Additive inverse:** A number which is added to a given number to get zero as the result.
- **Reciprocal:** A number that is obtained by interchanging the numerator and the denominator of a fraction. This is also known as multiplicative inverse.
- **Additive inverse or negative** of a positive integer is obtained by just writing a negative sign before it.

CHAPTER REVISION

A. Match the following.

1. $\frac{2}{9}+0$ — a. 1
2. $-\frac{3}{8}\times-\frac{8}{3}$ — b. 0
3. Additive inverse of $-\frac{3}{13}$ — c. $0+\frac{2}{9}$
4. $\frac{1}{8}\times 0$ — d. $\frac{1}{7}$
5. Reciprocal of 7 — e. $\frac{3}{13}$

B. Write T for true and F for false.

1. Additive inverse of a rational number is same as the given rational number.
2. Reciprocal of a rational number is obtained by interchanging the numerator with the denominator of the given rational number.
3. 0 is the additive identity for rational numbers.
4. –1 is the multiplicative identity for rational numbers.
5. There are infinite rational numbers between any two rational numbers.

C. Fill in the blanks.

1. $\frac{2}{5}-\frac{3}{10}=\frac{[\quad]}{[\quad]}$
2. $\frac{-12}{25}+\_\_\_\_=\frac{-12}{25}$
3. $\frac{[\quad]}{[\quad]}\times\frac{112}{215}=1$
4. $\frac{11}{21}+\frac{22}{35}=\frac{[\quad]}{[\quad]}+\frac{11}{21}$
5. $\left(\frac{5}{12}\times\frac{3}{8}\right)\times\frac{6}{7}=\frac{[\quad]}{[\quad]}\times\left(\frac{3}{8}\times\frac{6}{7}\right)$
6. $\frac{2}{3}\left(\frac{5}{7}+\frac{3}{4}\right)=\left(\frac{2}{3}\times-\right)+\left(\frac{2}{3}\times-\right)$

D. Find the following.

1. Additive inverse of $\frac{-1}{5}$.
2. Multiplicative inverse of $\frac{3}{8}$.
3. Negative of $\frac{23}{108}$.
4. Additive inverse of $\frac{3}{17}$.
5. Additive inverse of reciprocal of $\frac{1}{17}$.
6. Reciprocal of additive inverse of $\frac{13}{28}$.

E. Name the property used in the following.

1. $\frac{2}{6} \times \frac{7}{8} = \frac{7}{8} \times \frac{2}{6}$

2. $-\frac{4}{5} \times 1 = -\frac{4}{5} = 1 \times -\frac{4}{5}$

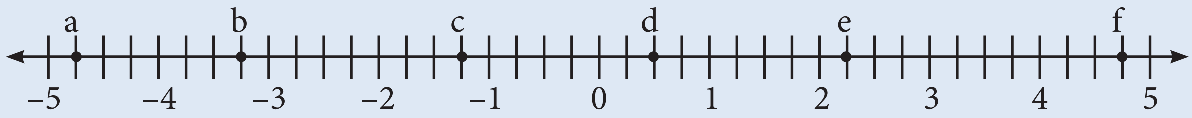
3. $\frac{3}{5} \times \left(\frac{9}{12} + \frac{8}{16}\right) = \left(\frac{3}{5} \times \frac{9}{12}\right) + \left(\frac{3}{5} \times \frac{8}{16}\right)$

4. $-\frac{5}{21} \times \left(-\frac{21}{5}\right) = 1$

5. $\frac{2}{6} \times \left(\frac{7}{8} \times \frac{4}{3}\right) = \left(\frac{2}{6} \times \frac{7}{8}\right) \times \frac{4}{3}$

6. $\frac{1}{11} + 0 = \frac{1}{11} = 0 + \frac{1}{11}$

F. Identify the rational numbers represented by the letters given on the number line.



a: ________, b: ________, c: ________, d: ________, e: ________, f: ________.

G. Find a rational number between the two given rational numbers.

1. 3 and 4
2. $\frac{3}{7}$ and $\frac{4}{7}$
3. $\frac{1}{3}$ and $\frac{2}{3}$
4. $\frac{-11}{3}$ and $\frac{-12}{3}$
5. $\frac{-2}{4}$ and $\frac{-3}{4}$
6. –1 and 0

H. Answer the following questions.

1. The product of two rational numbers is –22. If one of the numbers is –22, find the other.
2. By what rational number should $\frac{-25}{49}$ be divided to get $\frac{-5}{7}$?
3. Divide the sum of $\frac{3}{8}$ and $\frac{-5}{3}$ by $\frac{4}{3}$.
4. The total area of Mansi's land is 300 square feet. She wants to construct a house in $\frac{1}{3}$ of the land and design the rest of the land as a garden. What part of the land can be designed as a garden?
5. John has 5 acres of land. He wishes to donate $\frac{2}{9}$ of the land to an NGO (Non-Governmental Organisation) to build an animal shelter. If he distributes the remaining land between his three children, how much land will each of his children get?

SKILL UP!

Project

1. Find out the blood group of people in your building/locality/street (about 10–20 people). Recreate the table as given below. Find the fraction of people who are universal donors and those who are universal recipients.

Blood Type	Number of people	Fraction of people
A+		
O+		

Universal donors: ______________Universal recipients: ______________

2. You must have learnt about elements in your chapters of chemistry. Represent the number of metals, non-metals and metalloids as rational numbers in the simplest form.

Life Skills

You are the team leader of your group that has to plan and organise the school annual day celebration. You are allowed to choose 20 students out of 90 students to help you. These students have different qualities (A few examples are given in the table below. Add a few more).

Choose 20 students from the table to help you giving reasons for your choice.

Qualities	Fraction of students	Number of students	Number of students chosen	Reason
Good team player	$\frac{1}{5}$			
Creative	$\frac{1}{15}$			
Good communication skills	$\frac{1}{15}$			

Mind Buzzer

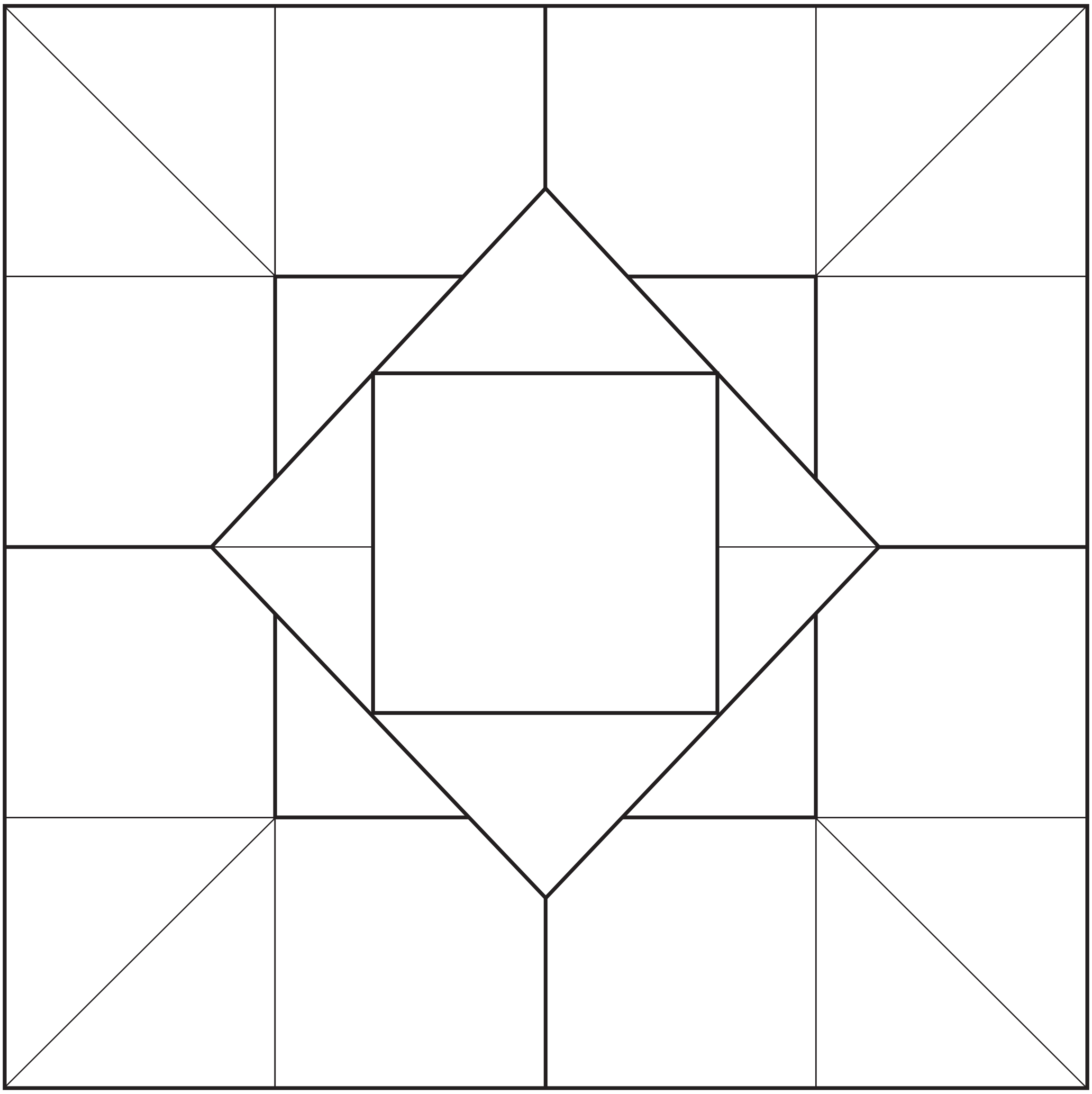
In the list of languages spoken all over the world, Hindi and Bengali find a place. Hindi is spoken by approximately 260 million people and Bengali is spoken by approximately 242 million people. If the population of the world is about 7.7 billion, then what fraction of the world speaks in Hindi and Bengali?

Teacher's Notes

- Before discussing the properties of rational numbers, recall operations on rational numbers and then proceed with the properties.
- While discussing the properties of rational numbers, compare the same properties with whole numbers and integers. It helps students to compare and understand the properties better.

WORKSHEET

Look at the squares and the triangles in the figure given below and answer the following questions.



a. Count the squares and colour $\frac{1}{7}$ of the number of squares.

b. Count the triangles and colour $\frac{3}{8}$ of the number of triangles.

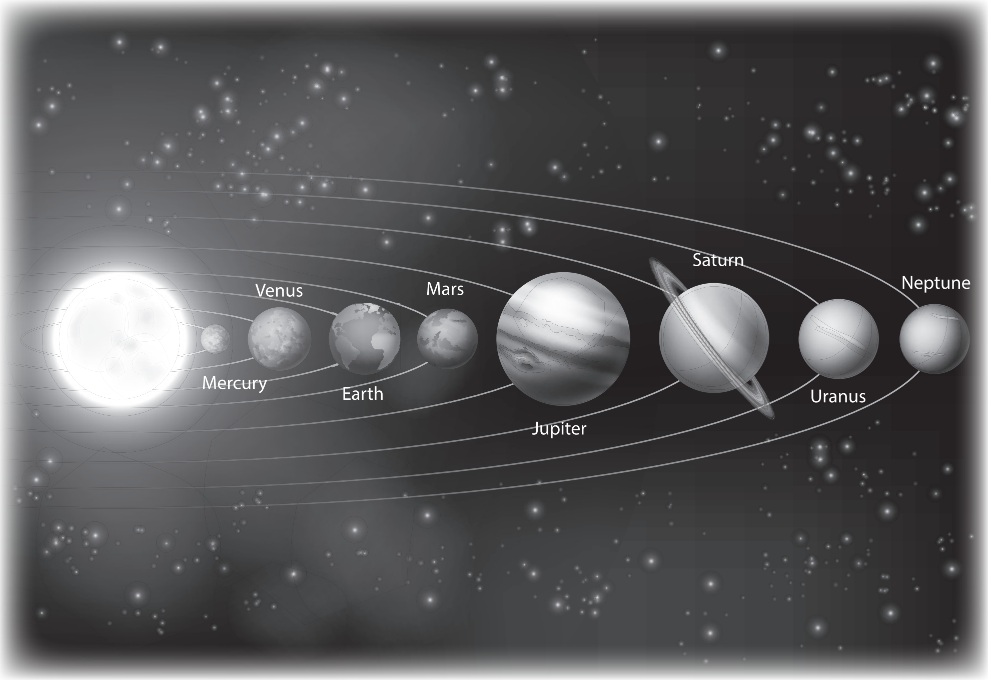
2 Exponents and Powers

Learning Objectives

- To express integers as exponents
- To simplify a given expression using the laws of exponents
- To express a number in standard notation

Let's Get Started

Do you know the distance between the Earth and the Sun or the distance between the Mars and the Sun?



The distance between the Earth and the Sun is 149600000 kilometres.

The distance between the Mars and the Sun is 227940000 kilometres.

Observe that the distances are very large numbers. Large numbers are used in astronomy and cosmology.

How can we easily express such numbers?

We use exponents to easily express such numbers.

EXPONENTS AND POWERS

A number which is multiplied many times by itself can be written in index/exponent/power form.

base → 2^4 ← exponent

↑ exponential notation

$a^n = a \times a \times a \times \ldots\ldots$ n **times**

In a^n, the base is a and the exponent is n. a^n is read as nth power of a or a raised to the power n.

For example,

- 5^4, where 5 is the base and 4 is the exponent. You will read 5^4 as 5 raised to the power 4.
- $(-4) \times (-4) \times (-4) = (-4)^3$ Base $= -4$ and exponent $= 3$. You will read $(-4)^3$ as -4 raised to the power 3.

Note

- When a positive number is raised to any power, the answer is positive. $(1)^{\text{natural number}} = 1$
- When a negative number is raised to the power of an odd natural number, the answer is negative. $(-1)^{\text{odd natural number}} = -1$
- When a negative number is raised to the power of an even natural number, the answer is positive. $(-1)^{\text{even natural number}} = 1$
- If $\frac{p}{q}$ is any rational number and n any integer, then $\left(\frac{p}{q}\right)^n = \frac{p^n}{q^n}$.

SOLVED EXAMPLES

Example 1: Express each of the following in exponential form.

a. $8 \times 8 \times 8$

b. $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$

c. $\left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right)$

d. $(-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$

Solution:

a. $8 \times 8 \times 8 = 8^3$

b. $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \left(\frac{3}{7}\right)^4$

c. $\left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) = \left(-\frac{1}{5}\right)^6$

d. $(-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = (-2)^6$

Example 2: Express each of the following in exponential form.

a. 144 b. -216 c. $\frac{-125}{512}$ d. $\frac{729}{15625}$

Solution: For expressing the numbers in exponential form, factorise the numbers first.

On factorizing the numbers, we get

a. $144 = 12 \times 12 = 12^2$

b. $-216 = (-6) \times (-6) \times (-6) = (-6)^3$

c. $-\frac{125}{512} = \left(-\frac{5}{8}\right) \times \left(-\frac{5}{8}\right) \times \left(-\frac{5}{8}\right) = \left(-\frac{5}{8}\right)^3$

d. $\frac{729}{15625} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{5 \times 5 \times 5 \times 5 \times 5 \times 5} = \frac{3^6}{5^6} = \left(\frac{3}{5}\right)^6$