PAUL HINDEMITH

Paul Hindemith
The Craft of musical composition

Book 1: Theory

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Book I

Theory

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HINDEMITH

THE CRAFT OF MUSICAL COMPOSITION

Theoretical Part

CHAPTER I

Introductory

> "Perhaps some will wonder at my undertaking to write about music, when there are at hand the opinions of so many excellent men who have written learnedly and sufficiently about it, and particularly at my doing so at a time when Music has become an almost arbitrary matter, and composers will no longer be bound by laws and rules, but avoid the names of School and Law as they would Death itself . . ."

Thus wrote Johann Joseph Fux in the foreword to his *Gradus ad Parnassum* (1725),* the textbook of counterpoint according to whose basic principles the student to this day still learns his craft. Now for us the first decades of the 18th century represent the fullest flowering of the technique of composition. When Fux's book was published, J. S. Bach was forty years old, and at the summit of his skill and creative power; and the minor masters who were to be found all over Europe exhibited even in works which were by no means heaven-storming a complete technical mastery. But Fux, the strict contrapuntist, whose field is vocal music, cannot reconcile himself to the shift of the center of the composer's work to the instrumental domain, with all that that shift implies for the style of writing. The step from the noble but narrowly limited art of writing for voices, in which instruments must always play a secondary rôle, toward freer and livelier tone-progressions

* *"Mirabuntur fortassis nonnulli, cùm tot præstantissimorum Virorum exstent monumenta, qui de Musica perquàm doctè, & abundanter scripserunt, cur ego ad hoc scribendi genus me contulerim, hoc maximè tempore, quo, Musicâ ferè arbitrariâ factâ, Compositores nullis præceptis, nullisque institutis obstringi volentes, Legum, ac Scholæ nomen ad mortis instar exhorrescunt; . . ."*

such as naturally occur to the gifted instrumentalist, appears to him not as the beginning of a path into a new land, but as a descent which must be halted. How in both word and deed he opposed what he considered the barbarization of music may be seen both in his compositions and—with explicit reference to the master of the purest and most perfect style of writing, Palestrina—in the *Gradus*.

Perhaps the craft of composition would really have fallen into decline if a genius like Bach had not fought his way through to the highest and most complete mastery of his material, and if Fux's *Gradus* had not put a brake upon caprice and exaggeration, and set up a standard of excellence in writing. For this was the first real textbook of composition in a time which had known on the one hand only the passing on from master to pupil of specific devices and tricks of the trade, or, on the other, deep-searching theoretical works that were of little help in learning the practical art of composition.

A musician who feels called upon in these times to contribute to the preservation and transmission of the craft of composition is, like Fux, on the defensive. He is, in fact, even more so than Fux, for in no other field of artistic activity has a period of overdevelopment of materials and of their application been followed by such confusion as reigns in this one. We are constantly brought face to face with this confusion by a manner of writing which puts tones together according to no system except that dictated by pure whim, or that into which facile and misleading fingers draw the writer as they glide over the keys. Now something that cannot be understood by the analysis of a musician, making every conceivable allowance for individual characteristics, cannot possibly be more convincing to the naive listener. In *Die Meistersinger* one reads, it is true, that the composer must make his own rules and then follow them. But this privilege is granted only to a *master*—one, moreover, who knows, or at least feels, the bases of his work provided by Nature.

It is not surprising that things have developed as they have. The discovery, in the last century, of the extreme limits of power and

subtlety in the effect of musical tone extended the boundaries of the tonal domain at the disposal of the composer into hitherto undreamed-of distances. New combinations of tones came to be recognized, and new ways of bending a melodic line were discovered. It seemed as if the sun had risen upon a new, glowing, iridescent land, into which our musician-discoverers rushed headlong. Blinded by the immense store of materials never used before, deafened by the fantastic novelty of sound, everyone seized without reflection at whatever he felt he could use. At this point instruction failed. Either it fell into the same frenzy as practice, and devoted itself to flimsy speculation, instead of adapting its systems of teaching to the new material, or it lapsed into inactivity, and what had never been a very strong urge towards novelty turned into a barren clinging to the past. Confidence in inherited methods vanished; they seemed barely adequate now to guide the beginner's first steps. Whoever wished to make any progress gave himself unreservedly to the New, neither helped nor hindered by theoretical instruction, which had simply become inadequate to the occasion.

2

A considerable portion of the responsibility for the failure of instruction belongs to the instructors themselves. Is it not strange that since Bach hardly any of the great composers have been outstanding teachers? One would expect every musician to have the desire to pass on to others what he had labored to acquire himself. Yet in the last century the teaching of composition was looked on as drudgery, as an obstacle in the way of creative activity. Only rarely did a composer integrate it as a component part of himself; the feeling of responsibility for future generations of musicians seemed to have become a thing of the past. Not until the last few decades do we again find composers who feel it their duty to educate pupils. These men act in the spirit of the old handicraftsmen, who aimed to hand on their skills intact. In times that boast of an enviable flowering of the craft of composition, great masters can afford to devote themselves exclusively to their own creations, paying no attention

to those who are to come after them. It is then the task of the teachers, who follow at a distance, to mint into current coin the wealth which the composers have mined. But today, when there is a general lack of skill in the technique of composition, no composer should withdraw from teaching.

There are two types of theorists: the teaching composer, and the avowed specialist in teaching musical theory. A gifted composer is not always a good teacher. But his instruction is bound to have a certain creative warmth, even when the composer is of modest gifts, because he is passing on directly what he himself has experienced. This is not true of the usual theory instruction, such as is given in most schools. The specialist who gives such instruction without himself being gifted for composition is in a difficult position. In the painful early stages of trying to bring dry series of figured basses and sets of rules to life, he cannot fall back upon his own creative activity. Thus he is likely to turn this most interesting of fields, which lies directly adjacent to that of free composition itself, into a morass of disappointment, instead of exploiting the many stimuli it offers for the better understanding of past and present styles of composition. Not every theory teacher can reach that high estate of knowledge and ability attained by the teachers (and textbook writers) of the last century, for they owed the richness of their harvest to the fact that composers had left the field of instruction to them. But he can at least prevent the theory lesson from becoming what students too often consider it: a boring, incomprehensible, and useless burden. And is it really any more than that when it consists in handing out "music" in the form of dead chord-progressions and monotonous, meaningless melodic lines?

The teacher must not base his instruction simply on the rules of textbooks. He must continually refresh and complete his knowledge from the practice of singing and playing. What he teaches must have been developed out of his own exercises in writing. For it is his task not only to teach the pupil a correct technique, but also to help him obtain a comprehensive musical education, seeing to it that his work in the practical fields is supplemented by an intelligent understanding of the theoretical side. It is up to him to pass

on the most personal and most painfully arrived at secrets of the great composers, so that he may call forth in the student at least a small reflection of their light. At the same time, he must exercise a guiding and calming influence on the young musician who is in the throes of experiment; he must steer him between the Scylla of blind worship of the past and the Charybdis of idolatry of the present. Anyone who follows this profession for the sole purpose of earning his living is just as unworthy of it as is the (fortunately not too common) composer who submits unwillingly to what he considers the slavery of teaching, and poisons the student with his inevitable musical bad temper.

One thing that makes instruction in this field more difficult is the unfortunate fact of its division into two separate parts. Of course the material must be presented to the student in easily graspable form; exercises for the development of melodic invention must alternate with those for the acquisition of a cogent harmonic style, just as the student of an instrumental technique employs both finger exercises and pieces for performance in learning to master his instrument. But complete separation of the harmonic material, followed after a whole year of work by melodic studies which are themselves insufficient, is as thoroughly wrong as would be a method of skating in which one had to practise exercises for working each leg separately before one learned to move on the ice.

In Fux's time, it was possible to get along with the material he worked out. But when the technique of composition developed further, particularly on the harmonic side, the teaching of harmonic phenomena and their treatment was set off in a separate field known as Harmony. This took place about the beginning of the 19th century (although it was based upon much research of earlier times). In this procedure, the new progress in composition seemed to have found the educational method appropriate to it. But that method soon turned out to be inadequate, and after barely a hundred years of an apparently brilliant existence, the fabric, which from the beginning has needed patch after patch, is now worn threadbare. On the other hand, the Fux system has lasted two hundred years, and is still passed on from teacher to student almost in its original form

—a grotesque state of affairs when one realizes that the practice of composition has long since forsaken the bases of this system. This fabric was never patched; it was made of more durable stuff. In fact, the stuff was so durable that it might well have been ripped out and worked over again. But no successors came to adapt its basic principles to new needs. Some were for greater strictness, and stripped Fux of an ornament here and there; others added new spangles to the old garment. But the truth is that no matter what is done to make it more presentable, it no longer fits our needs, and the want of something more suited to our own problems has long been felt.

If, then, every music student must go through these two courses of study, adapting himself painfully at first to the one, only to be torn away from it before he is really used to it and then have to begin all over again on the other, finally to realize that even when he has mastered the new discipline he has acquired no real mastery over his tonal materials—is it any wonder that the idea should arise that a composer should not let himself be disturbed by what he has learned in his theory lessons?

It is in the nature of such a teaching procedure that in the case of an unusually gifted student there comes a day when the teacher can no longer follow the activities of his pupil. He does not understand what the student is aiming at—although in technical matters there can be no secrets—and, as is touchingly described in many biographies, he lets him depart with his blessing, since there is nothing more he can teach him. Unfortunately for the teacher, in most cases the pupil comes to this conclusion on his own account, and does not wait to be dismissed.

3

Because of the situation described above, it is a particularly difficult task today to give a student instruction in composition. One teacher sticks close to what he has had handed down to him. For him what Riemann or Prout said is iron-bound law. His pupil learns the old styles of writing: he can modulate, write counterpoint almost in his sleep, from the first species to the florid fifth, and con-

struct fugues to order, made according to the text-book rules, and containing anything but music. If the precocious student seeks to know more, mentioning that in the music he plays and hears there is more to be discovered, he is hushed up, or given excuses, false explanations, or denials; or the teacher becomes angry; or relations between pupil and teacher are broken off altogether. Or, on the other hand, the teacher may let the pupil flounder around in a field in which both are lost. But none of these ways brings the pupil any nearer to his goal.

Among the younger teachers—who have themselves experienced the impact of the new music on their own work, and now wish to spare the pupil the things that once caused them pain, anger, and disappointment, without arming them for later struggles—many allow the pupil considerable freedom from the start. But freedom is a bad thing at the beginning, since it does not provide the student with the necessary support. A conscientious teacher, who can hardly justify to himself the process of continuing to dispense outworn materials, is in a perpetual state of uncertainty, since no usable new method is at his disposal. How, in particular, shall he treat the more advanced student? He can settle technical problems with him only by relying on nothing more substantial than his taste, citing his own opinion and that of other honest seekers, and exploring the situation with the student. None of these solutions will do except the last, and this can be fruitful only if the teacher and pupil are sympathetically attuned, and both of exceptional gifts. But no general system of instruction can be based on such a happy combination of circumstances.

If confusion in the technique of composition is not to increase and spread, if the conflicting results of an outworn system of instruction are not to bring disaster in the wake of uncertainty, a new and firm foundation must be constructed.

4

I propose to attempt the construction of such a foundation. I am not animated by any desire to freeze into permanent shape what I

have been teaching for years, either to get it out of my system or to be rid of the burden of continually improvising new forms of the material which I have often handed out. Anyone who has for years taught students who wish to know why the masters are free to do what is denied to them, why one theme is good and another poor, why harmonic progressions may be satisfactory or irritating, why sense and order must prevail even in the wildest turmoil of sounds, and why such order cannot be arrived at with the traditional tools; anyone who has not sidestepped this unending struggle with the Why of things, and, at the risk of laying himself bare before his pupils, has taken each new question as a stimulus to deeper and more searching study—anyone who has faced these issues, I say, will understand why I feel called upon to devote to the writing of a theoretical work the time and trouble which I would rather spend in composing living music.

I have experienced the needs of the teacher as well as the strivings of the composer. I have lived through the transition from conservative training to a new freedom perhaps more intensely than anyone else. The new land had to be explored if it was to be conquered, and everyone who took part in this process knows that it was not without danger. The path to knowledge was neither straight nor smooth. Yet today I feel that the new domain lies clearly spread out before our eyes, that we have penetrated the secrets of its organization. This was not accomplished by the stubbornness of those who simply put up a pretense of strength by persisting in their accustomed disorder, or by those who were so self-righteous that they never experienced temptation. Anyone who is familiar with the development of music after the first World War will find step by step in these pages, which are intended to afford entrance to the newly won territory, traces of struggle with external circumstances as well as of that inner strife whose aim is the perfection of one's own work. But even a wider circle of readers will understand, at this first stopping-place on the road to complete clarification of both contemplation and action, that an attempt to explain the music of the present day had to be undertaken, if only to satisfy a personal need to pass on to new

learners what had been acquired by learning, and to shorten for them the paths which until now have been inevitably roundabout.

I address myself above all to the teacher. True, I cannot (as is understandable in the case of a composer whose theorizing is only incidental and enforced) offer him a book of rules, polished down to the last detail, in which he can simply assign to his pupils three pages per lesson. Perfection cannot be attained at the outset of an innovation such as the present one, and the comprehensive working out of the material presented here will require the efforts and the experience of many musicians. The teacher will find in this book basic principles of composition, derived from the natural characteristics of tones, and consequently valid for all periods. To the harmony and counterpoint he has already learned—which have been purely studies in the history of style: the one based on the vocal style of the 16th and 17th centuries, the other on the instrumental style of the 18th—he must now add a new technique, which, proceeding from the firm foundation of the laws of nature, will enable him to make expeditions into domains of composition which have not hitherto been open to orderly penetration.

To the composer, as well as the teacher, the book offers new perspectives on his materials, and makes clear that for a well-intentioned but arbitrary arrangement of sounds he must substitute an order which only to the uninitiated will seem a restriction of the creative process. In reality, wisely and sensibly directed work will result in greater variety than a profusion of over-seasoned or over-sweet progressions, the formula for which is soon transparent and thus available even to those who have no inner musical vocation.

The reader who lightly turns these pages in the hope of a stimulating general discussion will not be well rewarded. He will find the subject matter remote and dry, the more so as he is used to meeting the materials of music in living and flowering form, rather than on the dissecting table. Moreover, he finds more pleasure in the actual sound of music than in reading about it, and thus he may well leave the present accumulation of descriptions of abstract tone-successions, practical rules, and musical examples to those who can feel

the pulse of music beneath the monotonous consideration of its materials.

Those industrious ones, too, who think that by memorizing and working hard at rules and precepts they will come by a recipe for producing convincing music had better give up the search. Finally, this book would represent a disappointment to the beginner who expected to find in it a reliable guide for self-instruction. It presupposes a considerable body of knowledge, and will therefore be valuable only to those who are already somewhat familiar with the technique of composition.

In the present Theoretical Part of this work, the underlying principles of the new theory are first established and then developed. I have limited myself to what is really new, and to that older material to which I give a new interpretation. Such elements of older theories as have remained intact throughout all periods and styles, because they are independent of period and style, I have left untouched, except where it has been more convenient to rename or reclassify them. Nothing is said here about those things which remain unchanged, or about the actual writing of music, with its rules for voice-leading, spacing, and so on. Book II, devoted to actual writing in two parts, contains all this material, both old and new, presented in pedagogic order.

In Fux's foreword, the following additional passage occurs:

> "The practice of medicine is intended for the sick, not for the well—although my work does not aim, and I do not lay claim to the power, to control the current of a gushing stream that has overflowed its banks, or to reconvert composers from their heretical way of writing. For aught I care, everyone may follow his own fancy." *

But despite Fux's modest estimate of his powers, he actually did do away with heresy. His "gushing stream" seems to us, compared

* *Ægrotantibus etenim, ac non bonâ valetudine gaudentibus medicina paratur. Quanquam labor meus non eò tendit, nec tantum mihi roboris arrogo, ut quasi torrenti extra limites præcipitanter erranti cursum inhibere, Compositoresque de licentiosa scribendi hæresi ad resipiscentiam revocare me posse confidam. Per me liceat cuique sequi suum consilium.*

with the torrential flood of today, a mere overflowing mountain brook. Perhaps a single man's strength will not suffice today to dam the flood; perhaps what he attempts will not even be understood, much less valued. Yet the success of Fux's work shall be a good omen for mine.

5

The reader who thinks that the views here expressed amount to a deification of materials, an undue exaggeration of the importance of mere craft, should remember that there was once a time when he had to absorb the rules of harmony, which, despite the limitations of the chordal material dealt with, were numerous enough. There is no denying the fact that to learn a new system takes time and trouble. But if one gains both a wider outlook and a more complete mastery, it is worth it. Technical skill can never be great enough. No one is too able or too accomplished to learn more than he knows. Technique must be learned as a child learns to move his limbs: what was difficult at first must become easy; it must be at one's instantaneous disposal; it must function so perfectly that its action is no longer noticed; it must sink to the level of subconscious activity.

Although the creative process in its highest stages may always remain hidden from human comprehension, as may the mysterious source of artistic work in general, yet the dividing point between conscious and unconscious work can be raised to an extraordinary degree. If this were not true, everyone in whom this point lies at a very low level could assert that he is creating the greatest works of art. There would be no difference between Beethoven and any other composer, who had with difficulty achieved a mere quarter, say, of the height of artistic achievement that men may attain, and knew nothing of the other three quarters that still lay above him. Such a little man would not care to speak of technical matters, but would instead refer to his impulse, his feeling, his heart, which had prescribed the way for him. But must not this impulse be tiny and this feeling negligible if they can express themselves with so little knowledge? Is not an immense mastery of the medium needed to

translate into tones what the heart dictates? Can the inner vision of the music that the composer has glimpsed make itself at all clear to another if the resistance of the tones and the refractoriness of tonal progressions is continually coming between the impulse and its expression in sound?

The road from the head to the hand is a long one while one is still conscious of it. The man who does not so control his hand as to maintain it in unbroken contact with his thought does not know what composition is. (Nor does he whose well-routined hand runs along without any impulse or feeling behind it.) The goal must always be such mastery that technique does not obtrude itself, and a free path is prepared for thought and feeling. The man to whom the tones are a necessary evil with which he must wrestle; or who sees in them a perfectly tractable medium in which he can express himself without any restraint; or who climbs up on them as on a ladder, or wallows in them as in a bog—such a man is simply adding to the infinity of pieces that are written every year without moving a human ear or spirit. The initiated know that most of the music that is produced every day represents everything except the composer: memory, cheap compilation, mental indolence, habit, imitation, and above all the obstinacy of the tones themselves. Our principal task is to overcome the latter. To do this we need precise knowledge of the tones and of the forces that reside in them, free from aesthetic dogma and stylistic exercises such as have characterized previous methods of instruction, but leading the composer rather according to natural laws and technical experience.

In this attitude toward the technical side of composition I am in agreement with views which were held long before the classic masters. We find such views in early antiquity, and far-sighted composers of the Middle Ages and of modern times hold firmly to them and pass them on. What did tonal materials mean to the ancients? Intervals spoke to them of the first days of the creation of the world: mysterious as Number, of the same stuff as the basic concepts of time and space, the very dimensions of the audible as of the visible world, building stones of the universe, which, in their minds, was constructed in the same proportions as the overtone se-

ries, so that measure, music, and the cosmos inseparably merged. And the art of composition itself? To pious musicians it was a means of praising God, and of enabling the community of listeners to take part in that praise. That the work has been created to the glory of the Highest Being, of whose support it is thus assured, we can feel in the music of many composers—above all in that of Bach, whose "Jesu iuva" in his scores was for him no empty formula.

We cannot conjure up past times, although every man must come to some agreement with himself about the bases of his work. But that our consideration of tonal materials and its application by all who may concern themselves with it may catch a kindling spark from the spirit of the old masters is the hope from which this work springs.

CHAPTER II

The Medium

1

General Considerations

If we were to ask an intelligent musician, who knew his *métier* and who had a certain theoretical knowledge, what tones he would choose from among the audible range, what series he would consider the most natural, the simplest, and the most practical raw material for composition, he would undoubtedly reply after a moment's thought that we must mean a scale, for without a scale no ordered music would be conceivable. He would be thinking of the major and minor scales, which provide an inexhaustible supply of tones for all possible harmonic combinations, and according to which all melodies known to him can be classified. He would be forgetting, however, that our ancestors made use of other scales, and that even today peoples of other cultures use scales that often have little similarity to ours.

Even the simplest musical activity, uninfluenced by education or experience—the song of the savage, or the first attempts to draw tones out of a hollow bone or a reed pipe—must make use of some interval-progressions which are based fundamentally on a series of adjacent tones. The primitive musician, giving direct expression to his mood, will at first not be interested in the exact distance of one tone from another. Not until considerable experience has broadened his knowledge and raised the level of his desires will he feel the need of bringing order into the luxuriant tonal wilderness.

It will then develop that certain intervals make similar impres-

sions upon all men. When even the man of the lowest level of civilization hears the interval of an octave, he will feel that the upper note is the higher image of the lower. Accordingly, in all known tonal systems, the basic scale-patterns, with few exceptions, fill in the space between two tones an octave apart.

After the octave, the next fixed point to be felt is the fifth. But the conception of this interval as something fixed and unchangeable is for the untrained ear a more difficult matter. The two tones do not merge completely into one sound, as do those of the octave. The upper tone is not felt, as is the upper tone of the octave, to be the mere higher duplication of the lower. Nevertheless, the interval of the pure fifth is so unambiguous and independent that it is to be found in almost every scale system. Other intervals (thirds, sixths, sevenths, and seconds) are less easily determined. The distance between the two tones of a major sixth, for example, can be diminished or augmented to a certain extent without destroying the impression of a major sixth. The slightest alteration in the size of an octave or a fifth, on the other hand, changes these intervals completely, so that the ear perceives them only as greatly expanded sevenths and fourths or greatly contracted ninths and sixths.

2

Overtones

We find the intervals embedded in the tonal raw material which Nature has made ready for musical use, consisting of an infinite number of tones, from the deepest barely perceptible drone to the whistle that lies at the other limit of audibility. Into this inchoate tonal mass we can introduce a certain order by the use of the immutable measures of the octave and the fifth. Nature, in fact, has herself introduced this order, and put at our disposal a whole series of other intervals as well.

The eye perceives in light which has been split up by a prism a natural series of vibration frequencies. The light of the sun always produces the same immutable series of colors, familiar to us in the rainbow. Now, just as light consists of graduated colors of the

spectrum, so a tone consists of many partial tones. The spectrum of the world of sound is the harmonic or overtone series. A tone produced by a voice or instrument carries with it a greater or lesser number of barely audible overtones. Their order is not arbitrary: it is determined by a strict law, and is as immutable as the color series of the rainbow. The series extends theoretically to infinite heights, but in practice a sounding tone is supported by only a limited number of overtones. It is well that this is so, for a tone accompanied by all the possible overtones, to the upper limit of audibility, would be obscured by their profusion, would lose its character, and would suffocate. Bells of poor quality, with their great number of prominent overtones, give us an idea of what such an overloaded tone would be like. For theirs is a chaos of tones, rather than a single tone, and as such it is of almost no use for musical purposes.

A tone completely devoid of overtones, on the other hand, is characterless. It has no profile; no expression. It cannot be produced on our musical instruments. A completely pure tone of this sort can be produced only electrically by means of an oscillator or similar apparatus. It is of virtually no musical value. Even tones which are relatively poor in overtones are not used in practical music, as the almost pure tones of tuning forks show. The soft, insipid tones of recorders of medium register, for example, or of the similar labial pipes of the organ, are of good effect only in combination with sharper tone-colors (*i. e.*, those richer in overtones). Our musical instruments, of which the larynx is one, produce their tone by the joint action of vibrating solid bodies, which in turn cause the air to vibrate. All the vibrating parts of the instrument possess one or more tones proper to them, as we can observe by tapping the wood of the violin or the brass of the trumpet. Such tones are inseparably connected with the principal tone produced by the instrument. Even if the latter were free of overtones, the tones proper to the material of which the instrument is made would be heard along with it.

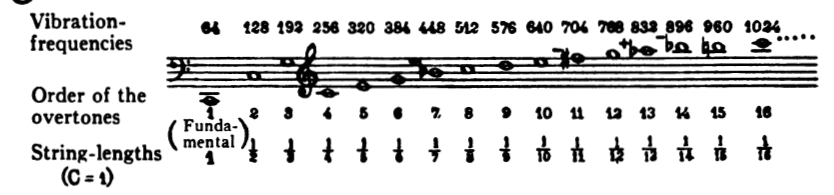
The overtones accordingly have an important relation to tone-color. The latter depends not only on the nature of the material and

construction of the instrument producing it, but also on the manner in which its vibrations are excited: the articulation, the manner of drawing the bow, the touch, all have an important influence on the distribution of the overtones. Every tone-color corresponds to a certain grouping of the overtones. The ear hardly hears them separately; it only perceives the disappearance of some or the addition of others as changes in tone-color.

3

Nature of the Overtone Series

In order to study the nature and construction of the overtone series, we shall take the structure based on the fundamental tone C as the basis of our observations:



We know that this fundamental tone supports a series consisting of the octave, the fifth of the octave, a second octave, the major third of the latter, the octave of the earlier fifth, and so on, as illustrated by Fig. 1. The spaces between the individual overtones thus grow progressively smaller, in arithmetical series. If we assume a frequency of 64 vibrations per second for the fundamental tone C (a convenient and usual basis for reckoning, even though the C actually used in our music nowadays is somewhat higher—*i. e.*, has a somewhat faster vibration rate)—then the second tone of the series will vibrate 128 times per second, the third 192, and similarly each tone will have 64 more vibrations per second than its predecessor in the series. The octave has twice as many vibrations as the fundamental, the twelfth three times as many, the double octave four times.