THOUGHT AND CHOICE IN CHESS

PSYCHOLOGICAL STUDIES 4

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by

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PREFACE

What is so special about the thought processes underlying the skilled chess player's choice of a move? Why do masters find the good moves that patzers overlook? This book is an attempt at answering these questions through systematic description and interpretative analysis of a collection of 'thinking aloud protocols.' The protocols, products of experimental sessions held in the years 1938 to 1943, are verbal reports rendered by subjects performing a simple experimental task: a subject was presented with an unfamiliar position taken from an actual tournament or match game and asked to find and play a move as though he were engaged in a tournament game of his own. The verbal report was to be as full and explicit a rendering of the subject's thoughts as possible, to include his plans, calculations, and other considerations leading to the move decision. The author – at the time a psychology student at the University of Amsterdam and an active international chess player to boot – served as the principal experimenter while the subjects were players of varying strengths, up to masters, grandmasters, and even two world champions.

For the interested reader protocols of the thought processes of such titans as Alexander Alekhine, Max Euwe, Paul Keres, Reuben Fine, Salo Flohr and Savielly Tartakower are appended. Quite apart from the insight the analysis of these protocols has given us into the dynamics of thinking and choosing in chess, these protocols would seem to be of some historic and curiosity value in themselves.

Since the experiments were not in the rigorous laboratory tradition but rather 'free' occasions for goal-directed, productive thinking – so free, indeed, that some might prefer not to call them 'experiments' at all – the analysis of the resulting protocols had to remain largely descriptive and interpretative. Except for a few general expectations about masters doing better – they did – and calculating deeper – they did not – there were no present hypotheses being tested. The idea

was rather to arrive at a generalized description of the structure and the dynamics of the thought process as a basis for theory formation; this to be accomplished through a painstakingly thorough and systematic analysis of the protocols. In other words: The main goal was to describe in general terms what 'happens' (structure) and to infer why it happens as it does (dynamics) in the mental processes preparatory to making a move decision.

As a working basis for this analysis, Otto Selz's conception of thinking as a hierarchically organized linear series of operations was adopted. In a way Selz's theory was tested too – a crucial question being the extent to which the above general conception would suffice for a gapless description and interpretation of the process of *chess* thinking.

So much for the central theme of the investigations – and for the body of the book. All other problems treated in the text – discussions of theory, of method, of an additional series of experiments in chess perception, the concept of mastership, the factors and the development of chess talent, etc. – can be viewed as variations, extensions, or generalizations of the main theme. But at this point we had better refer the reader to the Table of Contents or to Section 5 for a more complete description of the organization of the book.

The present work appeared originally in the Dutch language under the title of *Het Denken van den Schaker* (Amsterdam: North-Holland Publishing Co., Ltd., 1946) as a voluminous doctoral dissertation at the University of Amsterdam. The English edition, although revised, is for the most part a faithful translation of the original text. It is true that terminology was modernized, sections were rearranged, rewritten, or abbreviated, and references and new footnotes added; but in general both the content and composition of the book have remained the same – down to the numbering of sections.

Some changes and revisions should be made mention of, however. First, the English edition contains new material. Tables 8, 9, and 10 were not previously published; nor were the games from which experimental positions A, B, and C were taken (see Appendix I); nor was the collection of protocols (see Appendix II). Second, after twenty-five years the incognito of the grandmaster and master subjects has been lifted; the dead offered no objections while the quick readily consented to having their identity revealed. Third, the list of biographical information of famous masters (Table 15) has been brought up to date. Fourth, along with some other rearrangements in Chapter

II, a brief sketch of the life and work of Otto Selz has been newly written and added to the text (see Section 17).

Finally, the most important changes are in Chapter IX. Although similar problems were treated in the 1946 edition, the text of the original chapter was practically discarded: the whole chapter is a freshly written epilogue, anno 1963. In this ninth chapter too, the old findings of *Het Denken van den Schaker* have been related to new ideas and findings, in particular in the field of machine simulation of cognitive processes.

Apart from the rendering of the translation proper it is hoped that the various revisions and additions have increased the value of the book in the eyes of its prospective audience: psychologists, chess players, computer workers, and librarians.

ADRIAAN D. DE GROOT, *author*
GEORGE W. BAYLOR, *editor*

PREFACE TO THE SECOND EDITION

To the author, the task of writing a few lines by way of introduction to the second edition of his old book – published in Dutch 32 years ago and in English translation more than 12 years ago – is of course most gratifying. To him, the publisher's decision to have the text reissued spells confirmation of his hope that the book has some qualities which enable it to stand up to the test of time. In fact, there appears to be some continuing demand for it in the various quarters for which it was intended: among cognitive psychologists and computer specialists as well as among 'plain' chessplayers of all ranks. Lately, *Thought and Choice in Chess* has been quoted particularly often in publications of artificial intelligence specialists.

The text of this second edition is identical to that of the first apart from a good many relatively minor textual emendations. First, the number of printing errors has been greatly reduced, it is hoped, by correcting whatever mistakes were brought to our attention by the editor of the first edition, Dr. George W. Baylor (Université de Montréal), and by various readers. Second, and more important, in cooperation with the author the whole text has been newly edited, or rather given a linguistic facelift, by Mr. Jop Spiekerman. Awkward translations – a few on almost every page – have been carefully rephrased and closely scrutinized so as to leave meanings intact. Technical terms, as listed in the Index of Subjects, have not been changed, with only one exception: 'considerable moves' has been replaced throughout by 'pertinent moves'.

Thanks are due to all who have cooperated in preparing this second edition. Next to the printer – who did an excellent job – I feel particularly indebted to Jop Spiekerman, who again proved his great specialist competence and fine linguistic acumen by the way he led the refurbishing operation. To the extent that improvements in readability

and style were possible – given the original text and the constraints of time and format – the second edition has become substantially superior to the first.

Schiermonnikoog-Amsterdam, April 1978 ADRIAAN D. DE GROOT

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CHAPTER I

STATEMENT OF THE PROBLEM

A. INTRODUCTION

Section 1: The literature of chess

Although a good many more books have been written about chess than the uninitiated generally realize,[1] the psychological side of the game has remained a largely virgin territory.

The chess literature is for the most part of a purely technical nature. It deals with the play and not with the player and his way of thinking; it treats the problem and not the problem solver. Other than their games, the 'biographies' of famous players contain little more than a record of tournament results, prizes won, and a sketch of their chess careers. Concerning their inner development one finds only scanty accounts; their personalities are scarcely described, if at all; and with questions such as how their powers evolve and what are the bases of their skill, as yet no one has really concerned himself seriously.

Nevertheless, here and there, scattered among books and journals one can find descriptions and discussions which have some value for the psychologist. In some books and articles one finds comparative discussions of the styles of different masters,[2] in others analyses of a partly psychological kind about the game itself;[3] while elsewhere one finds more or less extensive descriptions of individuals.[4]

Further, the psychology of blind simultaneous play has sometimes attracted the attention of chessmasters themselves.[5] Finally, newspaper and magazine accounts of important tournaments and matches sometimes contain psychologically interesting particulars or anecdotes

1 The old chess library of Dr. M. Niemeyer, now part of the *Koninklijke Bibliotheek* in The Hague, one of the most complete in the world, contains about 10,000 items, written in 41 different languages; the oldest books date from the 15th century.

2 E.g. LASKER 1895; TARTAKOWER 1921; RETI 1923 and 1933; BRINCKMANN 1932; EUWE 1938.

3 WALKER 1850; JUNK 1918; VON SCHEVE 1919; TARTAKOWER 1921; KLEIN and PALITZSCH 1924; SPIELMANN 1929; LASKER 1932;

4 ALLEN 1858; MACDONNELL 1894; BUCK 1902; BACHMANN 1910–21; VON GOTTSCHALL 1912; TARRASCH 1912 and 1925; EUWE 1932; VAN HOORN and LIKET 1937.

5 MIESES 1918; ALEKHINE 1932.

on events and reactions during play; sometimes keen character sketches as well.

These references sound more impressive than they are, however. All too often the harvest of psychologically interesting material from the winnowing of thick books is extremely lean; moreover, *thinking* almost never receives attention. Nevertheless, there will be occasional references to the contents of some of these writings in the following pages.

Section 2: Binet's study

Just as the chess literature, written specifically for and by chess players, supplies few connections with psychology; likewise the psychological literature has little bearing on the psychology of chess.

The eminent French pioneer in intelligence testing, Alfred Binet, wrote the first work that treated problems in this field: *Psychologie des Grands Calculateurs et des Joueurs d'Echecs* (BINET 1894). It contains the findings from a study on the memory and imaginative faculty of blindfolded players.

At first Binet considered the fact that many chessmasters are able to play simultaneously a considerable number of games blindfolded, that is, without looking at the chessboard, as an achievement in the realm of *visual memory*.[6] In the older psychological literature this had never been doubted. Hippolyte Taine, misled by the completely atypical case of a blindfold player who was surely no chessmaster, expressly defended this point of view (TAINE 1870, p. 80). It was only natural that Binet, who himself knew the game only superficially, should use this as his starting point. The study of the blindfold player formed for him just another part of a general investigation into *les grandes mémoires*.

In order to collect the opinions of as many experts as possible, Binet first of all conducted a survey. He drew up a list of 14 rather

6 Any player of master strength can play at least four blindfold games simultaneously, if he wants to. Some have specialized in it and have achieved larger numbers. Well known in this area are Morphy (8 games), Zukertort (16), Pillsbury (22), Reti (28), Alekhine (32), Najdorf (40), and Koltanowski (50). A blind simultaneous performance can only be considered a success if the opponents are not too weak (at least Class A or Class B players) and the percentage of wins for the simultaneous player is respectable (say, 75 per cent). Naturally the larger the number of opponents, the longer the duration of the game. Record lengths of time for Alekhine and Koltanowski were more than 12 hours. Needless to say, such performances make enormous demands on one's endurance.

detailed questions concerning the ways and means by which the blindfolded player calls the positions during a game back to mind, concerning the character of the representation, the role of auditory and kinesthetic experiences, etc. Through the assistance of chess organizations in France, England, Germany, and Spain answers were received from many prominent chess players. Furthermore, several Parisian masters (among whom were Rosenthal, Goetz, Arnous de Rivière, Janowski, and Taubenhaus) made themselves available for direct study in the psychological laboratory at the Sorbonne.

In the course of the investigations it became increasingly clear to Binet that his original plan had been too narrow. In the written and verbal communications of the chessmasters a number of points emerged that he had not foreseen at the outset and which consequently led him to revise his concepts about the factors that govern blind play. He came to the conclusion that the ability to play blindfolded rests on three fundamental conditions (BINET 1894, p. 262):

(1) Knowledge and experience in the field of chess *(l'érudition);*
(2) Imagination *(l'imagination);*
(3) Memory *(la mémoire).*

Condition 1 (l'érudition):
It is only because the position is meaningful to the master, is a unity, a well-structured scene of battle, that he is able to keep it in mind. Each position has a character of its own. One of the correspondents wrote: 'Each position that I create or that I see develop speaks to my senses, beyond my reason, it makes on me an impression *sui generis.*'[7] (*Ibid.*, p. 265) This unity of character naturally exists only for the initiated; as a result of his knowledge and experience, as a result of his mastery, the configuration of the position 'integrates itself.'

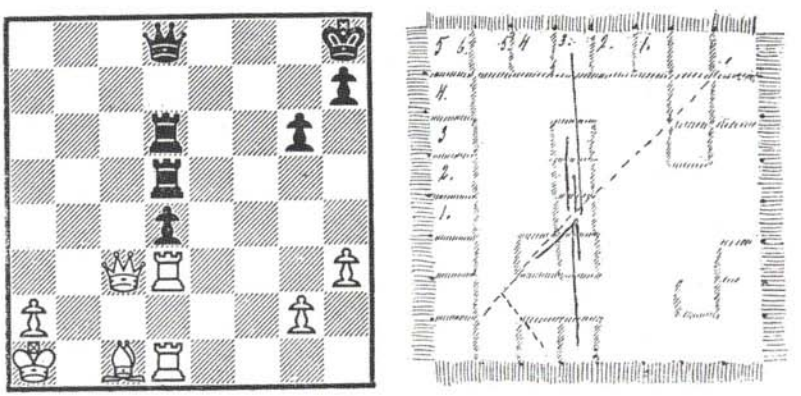
The same holds for the course of an entire game: to the master a game is no mere sequence of independent moves, but a development of a struggle that can be typified by a few characteristic maneuvers and ideas. The feat of memory, which Binet regarded as the most essential, is thus only possible on the basis of the chess mastery of the blindfolded player. Because of the import of the logical, dynamic, and genetic relationships between the elements in this feat of memory, Binet spoke of a *mémoire des idées* that he contrasted with the more mechanically operating memory, *la mémoire des sensations.*

7 'Chaque position que je crée ou que je vois se former devant moi parle au delà de mon raisonnement, à ma sensibilité, elle me fait une impression *sui generis.*'

Condition 2 (l'imagination):
The reports differ about the way in which the blindfolded player pictures the board and the actual situation; in particular, about the localization of the image in space. The descriptions were not all equally clear on this point, probably as a result of a certain vagueness and ambiguity of the localization itself. But this is easily understandable in the light of the highly abstract character of the mental images (see under condition 3). Moreover, Binet concluded from his material that the blind player does not, in general, have a fully *complete* picture of the position before him, but rather only a rough Gestalt which he searches, step by step, or rather: He continually reconstructs (the details of) the position.

Condition 3 (la mémoire):
On the basis of the fact that virtually all of the correspondents laid emphasis on the lack of visual details of color and form, both of the pieces and of the board, Binet came to the conclusion that one must speak here of an *abstract* visual memory that he contrasted with the better known *concrete* visual memory. One of the correspondents wrote very characteristically: 'We know only that it is a Knight or a Pawn without bothering about anything else.' (*Ibid.*, p. 295) Another tried to make a drawing (Figure 1) which represented in a schematic way the nature of the representation of a specific position. Although one cannot show specifically that this drawing is a 'likeness,' it is still very instructive insofar as it shows that indeed everything concrete is lacking. The squares of the board have no color, have only vaguely defined boundaries, and are only incompletely present. Only the

FIGURE 1



numbers resulting from the King's trying to catch the onrushing Pawn are given. The pieces themselves do not appear in the drawing: rather the lines of force that go out from them and that schematically represent their dynamic possibilities.

Apparently one cannot properly speak of a visualization[8] of the position. At the request of the investigator, one of the correspondents attempted to visualize the position occurring in a blind game at the laboratory in a more concrete way. He succeeded quite well but added that he would certainly abandon this method when playing six or eight games simultaneously, since in that case such a procedure 'is not of any help to him but would have no other effect than to tire him.' (*Ibid.*, p. 302)

Finally, in agreement with the remarks of the experts, Binet distinguished in his summary two forms of memory activity in blindfold play: the retention of positions *(mémoire visuelle abstraite)* and the recollection of the course of a game *(mémoire de récapitulation)*, which he did not discuss further.

In studying the book by Binet one cannot escape the impression that the author did not completely overcome the mistake of his starting point. To be sure, he did retract his original opinion concerning the concrete visual character of the chess player's memory, and it is true that he added other factors which are of importance; but with respect to the dynamics of the processes of remembering and recollecting there are but a few negative conclusions and vague words. No psychological analysis of the functioning of the 'abstract visual' memory, *la mémoire des idées*, was offered by his study. Moreover, the difference between the experience *(l'érudition)* of the player and his skill in blind play was not sufficiently recognized, let alone analyzed.

In this connection the few pages that Henri Bergson devotes to Binet's results, in a paper that appeared in 1902, are much more instructive (BERGSON 1902). The author cites the professional memory of the chess player as an example of the activity of a *schéma dynamique*.[9]

8 Binet himself spoke of 'visualization' but he added qualifications like: 'It is most often abstract, that is to say that it abstracts, detaches, and pulls out of the visualized object exclusively those qualities that are necessary for the combinations of the game.' (*Ibid.*, p. 338) ('Elle est le plus souvent abstraite, c'est-à-dire qu'elle abstrait, qu'elle détache, qu'elle arrache de l'objet visualisé les seules qualités nécessaires aux combinaisons du jeu.' We prefer to entirely omit the term 'visualize' (see p. 7).

9 This is the forerunner of the 'schematic anticipation' (anticipatory schema) in the *Denkpsychologie* of Otto Selz (cf. Sections 19 and 44).

By this he means a schematic representation of a complex structure of images and/or ideas that cannot be visualized itself, but can be developed into detailed visual or other sensory representations: 'The schematic representation does not so much contain the images themselves but rather indicates the directions to follow and the operations to perform in order to reconstruct them.'[10] (*Ibid.*, pp. 6–7)

According to Bergson, every mental operation that demands efforts of concentration begins with this kind of dynamic schema of the whole and/or the goal. The experimental work in the later school of *Denkpsychologie* almost completely confirms this notion. The reproduction of the image of a position in chess is, indeed, a telling example. The schema here is approximately that which the chessmasters in Binet's survey called the 'character' of the position. In particular, Dr. Tarrasch's description of the reproductive process is illustrative in this respect. First, the position is identified by its 'character,' that is, the subject must recognize 'which position it is.' In this process some characteristic move or maneuver from that game is often instrumental. Second, from here a more detailed representation of the actual situation on the board is developed; the schema is worked out into actual ('abstract visual') images. Thus according to the reports of the players, 'imagining a position' does not ordinarily occur all at once, but progressively, in successive steps. It is not a matter of simply 'seeing' the position mentally but rather of continual reconstruction which requires some effort: 'the image of the board with its pieces is not present in the memory of the player as it is in a mirror, but at every moment it demands of the player an effort of reconstruction.'[11] (*Ibid.*, pp. 7–8)

Binet never actually arrived at the point where he recognized the process of reproduction as a reconstruction 'by parts' of the position starting out from a dynamic total schema. Nor did he recognize the importance of the distinction between *knowing* 'which position it is' (feeling able to reconstruct it) and *imagining* a position (having a workably complete picture in mind) even though it was expressed rather clearly by several of the correspondents (among whom was Dr. Tarrasch). This would also explain why the distinction

10 'Nous entendons par là que cette représentation contient moins les images elles-mêmes que l'indication des directions à suivre et des opérations à faire pour les réconstituer.'

11 'l'image de l'échiquer avec ses pièces n'est pas présentée à la mémoire du joueur telle quelle, 'comme dans un miroir,' mais... elle exige à tout instant, de la part du joueur, un effort de reconstitution.'

Binet makes between the function of memory and that of imagination is not completely satisfactory. He does not take into account that the development from a schematic to a complete representation of the position is not a purely reproductive process but is in part an inferential construction process and therefore thinking. Where he speaks of 'abstract visual' memory, we are now able to discern two different levels of functioning: that of memory in its narrower sense and that of imaginative inferential reconstruction.

In the above exposition the use of the term 'visual' when speaking of either memory or imagination has purposely been avoided. As to memory, much information concerning positions and games is present in the blindfolded player's mind in the form of a 'knowledge that...' which has hardly any spatial and certainly no visual character. The abstract nature of the imaginative reproduction process, too, makes the term 'visual' somewhat suspect. In the mental representations there are probably many visual as well as tactile and kinesthetic elements that correspond to the observation, picking up, and moving of the pieces, respectively. However, the abstract images with which the player really works can hardly be considered either tactile or visual; rather they are *spatial*, which is more than either tactile or visual.

If the imagery of the chess player were visual, one would have to invent an independent theory for the game without a board for a player such as the late P. A. Koetsheid (class A to B strength) who became (really) blind at the age of seven but did not learn to play chess until he was twelve (compare, too, Géza Révész's exposition on blind mathematicians; RÉVÉSZ 1938, p. 155 ff.).

A precise study of this matter is, of course, outside the scope of the present work. The immediate purpose of the above discussion was merely to draw attention to the problems connected with the use of such terms as 'visual' and 'visualization' in the case of the chess player.

Finally one may note that Binet does not do full justice to blindfolded playing when he places such great emphasis on memory. Alexander Alekhine has rightly remarked that one forgets too easily that the blindfold player has more to do than just remember the positions; he has a second and much greater difficulty to cope with 'namely, to fight blindly, to find in every position, blindly, the best or almost the best move.'[12] (ALEKHINE 1932, p. 19) Because the actual position is missing perceptually, playing is made that much more

12 '...nämlich, blindlings zu kämpfen, blindlings in jeder Position ungefähr den besten Zug zu finden.'

difficult than in front of a board; the position must be continually reconstructed in the imagination.

There are other points in Binet's study that could be criticized. One could on logical grounds make the objection that his three factors are not conceptually on a par. 'Imagination' and 'memory' can best be considered capacities, faculties of the mind, but the player's 'experience' *(l'érudition)* is a concept of a different order. Binet hardly elaborates this point at all. Furthermore, his lack of knowledge of the game sometimes manifests itself in faulty interpretations of passages in the reports made by the chess players as well as in some erroneous inferences of lesser importance. We shall not, however, go into these minor matters now.

All in all, we see that there is quite a bit of criticism that could be made of this book by the French psychologist. However, if one takes into consideration that Binet began his investigation with an erroneous hypothesis (concrete visual memory as a basic factor in chess mastership), carried it out with an insufficient knowledge of the game of chess, and had to draw conclusions that found no support whatever in contemporary psychology, then one can have nothing but admiration for the results he achieved. Psychology in his day knew nothing of 'wordless thought'; it was still stumbling with the theory of sensations and images and their divisions into auditory, visual, and kinesthetic categories, and with other concepts and distinctions that were in no way helpful in understanding the thought process. For that reason Binet had to label many of the reports he received about blindfolded playing as being *fort obscurs*. In fact, they were obscure only insofar as they did not concur with the prevalent conception of thought as a series of concrete images.

The conclusion of Binet's book shows, for that matter, that he himself painfully felt the inadequacies of the academic psychology of his time. In the last paragraphs which radiate the same wholesome common sense which assured his success as a pioneer of mental measurement, he pokes fun at the lifeless, all too abstract way psychologists had so far tried to investigate a function such as memory. Later, when he talks of the role of experience in blindfolded playing he attests to an important gap in theoretical psychology: 'If I am not mistaken, modern psychology has not paid sufficient attention to the role played by old memories in the acquisition of new memories.' (BINET 1894, p. 339) Here he mentions a subject which was not worked out systematically until many years later. With regard to its im-

plications for thought processes Otto Selz was the first to incorporate it into a theoretical framework, namely, in his conception of acquiring 'experience' in some field. This process is described by Selz as a gradual construction and differentiation of a system of both general and more specific reaction dispositions *(Verhaltensweisen)* or, in the case of intellectual skills, a system of immediately available mental operations (solving methods) such as that on which the chessmaster's skill is based.[13]

The problems that were touched upon in this section will be treated later in another context, in Chapter VIII in particular.

Section 3: The question of chess talent

As well as the problem posed by the conspicuous phenomenon of blindfolded playing, the question of chess talent has also, here and there, inspired psychologists to empirical investigations. We shall briefly discuss a few of these investigations.

1. In the year 1925 three Russian professors, Djakow, Rudik, and Petrovsky invited eight participants from the large grandmaster tournament at Moscow to come to the Laboratory of Experimental Psychology and Psychotechnics of the University of Moscow to participate as subjects in a few experiments. The general purpose was to try to unearth the factors that underlie chess talent. The experimental program included, according to the German translation of the booklet the Russians published, the investigation of 'all the essential aspects of the mental activity of a chess player, as they have been sufficiently described in the literature.'[14] (DJAKOW, RUDIK and PETROVSKY 1927, p. 26) The somewhat over-optimistic statement in modern eyes is then followed by a specification of the 'faculties of the mind' that were examined: memory, attention (concentration), higher intellectual processes (combination power, finding of logical regularities, speed of reaction with regard to simple intellectual

13 New in Selz's approach was, as we shall see in some more detail in Chapter II, the emphasis on the operational side of thought and memory. Old memories exert influence on the acquisition of new ones since it is not only content that is stored but also – in computer language – 'programs' for information retrieval and for operational thought which makes use of stored information. Selz's *System von Verhaltensweisen* is essentially a mental program for handling data or solving problems in some particular area.

14 '...alle wesentlichen Seiten der psychischen Arbeit der Schachspieler die ja in der einschlägigen Literatur zur Genüge beschrieben.'

tasks such as checking calculations, and the like), as well as imagination, will power, and psychological type.

Most of the 'faculties' were investigated – nowadays we would say: operationally defined – by means of simple experiments and tests from the early psychotechnical tradition. Judged from a more modern point of view and taking into account what we currently know about 'factors of the mind,' one could say in advance that it is highly improbable that tests of this type – tests for visual memory and combination power, for example – have anything in common with what these terms mean in chess, other than the same name. It is hardly surprising, therefore, that most of the tests did not differentiate between chessmasters and control subjects of comparable intellectual stature. The Russian investigators interpreted their findings, however, as a refutation of commonly held beliefs, namely, that high achievements in chess are based on exceptional visual memory, combinative power, speed of calculation, power of concentration, and logical thinking. They were not aware of the 'faculty fallacy' in their reasoning, which is most obvious with regard to *combination* power and speed of *calculation:* two terms that have a very specific meaning in chess.

Most worthwhile, however, were the results of their experiments in visual memory. The experimenters used different types of stimulus materials in such a way that the resemblance with the customary materials of the chessmaster – board and moving pieces – was experimentally varied. There appeared to be no superiority in the master group where the resemblance to chess was lacking, some superiority where an 8×8 board with moving spots was used, and a definite superiority where actual positions were used. Again, this is hardly surprising. Here we can probably agree with the Russians that a commonly held belief – superiority in visual memory, regardless of the stimulus materials – was refuted. The results appear to be in agreement with our reluctance to speak of 'visual' (memory) in our discussion of Binet's findings.

The faculty of imagination and the two personality traits, will power and psychological type, were operationalized in a rather primitive and highly disputable fashion: by means of Rorschach-indices. The number of responses, R, was used as an index for 'imagination' (power), the number of kinesthetic answers, M, as a negative index for 'will power' *(sic)*. The 'psychological type' of a subject was determined by a somewhat *ad hoc* derived combination of scores. Results indicated that chessmasters had rather high scores on 'will power' and differed very much from one another in 'imagination.'

The latter result was one of the main factors on the basis of which the investigators concluded that two types of chessmasters could be distinguished: the *pedantic* and the *imaginative* types *(Pedanten* and *Phantasten* in the German translation). The latter finding appears to be roughly in agreement with commonly held conceptions on typological differences. From the exposition in the book, however, it is not quite clear to what extent the typological findings were contaminated by a priori ideas of the investigators.

In summary, the methods used were rather primitive and, as a result, the outcomes far from unambiguous. Nonetheless, some of the findings and observations in the book, concerning both the character of the game of chess and chess talent, are of enough interest to be taken up again in Chapter VIII of this study, where the factors underlying chess mastership are discussed.

2. In the year 1920 Franziska Baumgarten had the opportunity to examine psychologically the infant prodigy in chess, the eight year old Samuel Rzeschewski – now U.S. Grandmaster Reshevsky. At that early age he was already quite famous for his simultaneous play against some 20 opponents. Miss Baumgarten published her results, together with findings on other prodigies, in an entertaining but superficial little book, *Wunderkinder* (BAUMGARTEN 1930).

In itself the case of Reshevsky is extremely interesting since in the history of chess there is no other example of a talent that matured so early. Révész is right, it seems, in his statement that a chess talent in general is apt to attain a very high, if not already maximal, level of ability before the 21st year of life (RÉVÉSZ 1921), but to arrive at the master level before the age of 10 is highly exceptional, to say the least.

It was unfortunate that Baumgarten could not examine the child wonder in a more thorough and adequate way than she did. The investigator was careful not to draw any general conclusions on chess talent. She confined herself to relating a number of outcomes from tests and observations and interpreting the case of this particular prodigy – which interpretation is, again, open to much criticism. She found in the little boy Samuel an extremely one-sided mental development. His verbal intelligence was below the average level of five year old Berlin boys. There was only one test where his achievement was exceptional, namely in a memory experiment, where the stimulus material consisted of a matrix of one digit figures: he succeeded in retaining a 4 by 7 schema after a learning period of three

minutes and a 5 by 8 schema in four minutes. On the other hand, his visual memory for less abstract data was rather poor.

Baumgarten concluded that there existed a cleavage between his inborn talent and his actual knowledge due to the rather exceptional environment in which the child grew up. According to her information, Samuel – at his eighth year – had never seen a picture book, had never made a drawing or seen anybody else draw, had not yet been to school, and had so far learned only Talmud and some Hebrew. By contrast, it is quite probable that at that tender age he had already played more games of chess than, for instance, Euwe or many other masters, at the age of fifteen.

We can agree with Baumgarten's statement that the mental development of the young Reshevsky was 'one-sided' in comparison with other children of the same age. We can also agree with her opinion that this one-sidedness can in part be ascribed to the consequences of his position as a child prodigy. As such he had to be 'spoiled' on the one hand and 'exploited' on the other to the detriment of what for normal children would be called a 'more balanced development.' But Baumgarten's interpretation and evaluation of the facts are biased. First, she misinterpreted some of the facts by not sufficiently taking into account the orthodox Jewish educational traditions. Second, the young Reshevsky happened to be extremely – albeit 'one-sidedly' – gifted. Both facts make her comparison with 'normal,' i.e., with non-Jewish, not extremely gifted children of his age, rather futile. She was certainly wrong in blaming the parents for his one-sided development as strongly as she did.

The question of what might have become of the little Samuel if he had been given a more general intellectual and cultural education in his early youth is, of course, unanswerable, but it would seem that Baumgarten's supposition that he could have become an important scientist (*Ibid.*, p. 58) is, again, ill-founded.

3. Besides Reshevsky some other 'cases' have here and there attracted the attention of psychologists and psychiatrists, in particular where there were personality deviations or symptoms of mental illness. One result of such a psychopathologically oriented interest is Ernest Jones's psychoanalytic study of Paul Morphy (Jones 1931). In this work, apart from the case study materials, the author makes a number of general remarks on the nature of the game of chess that are of importance for the analysis of chess talent and chess affinity. Jones's article will be discussed in some detail in Chapter VIII.

4. Finally, there exist a number of psychological theories of a more general scope that in some respects touch upon the problem of chess talent. Révész tackled the problems of chess talent in one article on general and specific forms of giftedness (Révész 1938) and in another on the phenomenon of early maturation of talents in different fields (Révész 1921). Also, in 1938, the present writer published a brief essay on the factors of talent in chess that was, however, not yet based on an experimental investigation (De Groot 1938). In the present work a further analysis of the problems of chess talent will be delayed until after the discussion of the experimental analysis of the thought process. In Chapter VIII a more critical review of some of the above mentioned publications will be found.

B. THE OBJECT OF THIS STUDY

Section 4: The analysis of chess thinking

The purpose of the investigations described in this study is first of all *to carry out an experimentally based psychological analysis of chess thinking.*

Until now there have been no studies that have concerned themselves with a systematic description of the chess player's characteristic attitudes and methods of thinking. Herein, however, lies the heart of the psychological problems involved in chess. Only from a knowledge of the normal thinking of the chessmaster can one understand special arts, such as 'blind' and/or simultaneous play. Only by analyzing the thought process can one arrive at a thorough insight into the demands the game makes on its practitioners. Only along this path can the question of chess aptitude be fully handled. A systematic empirical analysis of the chess player's thinking therefore forms a sound basis for practically every psychological study in the field of chess.

To arrive at such a systematic analysis, a theoretical frame of reference had to be used. The only available conceptual system, aside from obsolete association theory, was the *Denkpsychologie* of Otto Selz. In the second part of his voluminous work, *Zur Psychologie des produktiven Denkens und des Irrtums*, he wrote at the conclusion of a discussion on general methods of problem solving in productive thinking: 'Only through an analysis, as undertaken in the present investigation, of those most general operations that correspond to (mentally set) aims as such ... does a complete (literally: 'gapless') description of the causal connections that govern the total course of

intellectual and/or motor processes come within reach.'[15] (SELZ 1922)

Thus, the task we set ourselves was to attempt such a complete description while using the terminology and applying the laws of Selz's *Denkpsychologie*. In other words, we tried to analyze the structure of the thought process in chess and to search for and pin down the characteristic moments in the chess player's thought dynamics: organization, methods, operations, etc.

The analysis was restricted to thought processes as they occur in playing the game. The field of problem composition or endgame studies was not taken into consideration. As is well known, the field of chess problems is in many respects a domain of its own, quite apart from the playing arena. Interest in playing the game and interest in composing and solving problems do not necessarily go together, nor does skill in one field guarantee skill in the other. Generally, the player's personality differs from that of the problemist, in that the player is primarily prepared to fight and to risk and to play. For that reason we do not come across many names of grandmasters in the problemist's world even though the purely intellectual basis is largely the same in both fields.

The investigation into the player's thinking during the game had to be restricted too. A chess game is a unit; playing it is a single feat, one activity; the concentration span runs from the first to the last move. But this unit is too large for an experimental psychological investigation, as the average duration of a serious competitive game is about four hours. The abundance of material from a protocol of the player's thinking during that entire time would be practically unmanageable. Moreover, it is impossible to have different subjects play the same game.

However, the alternation of moves required by the rules of chess ensures a natural segmentation of the game. During those four hours

15 'Erst durch die in diesen Untersuchungen in Angriff genommene Analyse jener allgemeinsten Operationen, die Zielsetzungen als solchen zugeordnet sind ... wird eine lückenlose Beschreibung des Kausalzusammenhangs determinierter intellektuellen und intellektuell-motorischer Gesamtverläufe möglich.' (P. 609).

16 From a sample of one thousand games that were played in master tournaments and matches in the latter half of the last century the mean number was 42 moves (actually 41.9). Of the 422 wins by White the mean was 41.0 moves; of the 322 wins by Black the mean was 42.0 moves; and of the 256 draws the mean was 43.3. The distribution over the 11–15 moves, 16–20 moves, 21–25 moves, etc., appeared to be clearly skewed as evidenced by the values of the three measures of central tendency:

	mean	*median*	*mode*
	41.9	39.4	34.7

(two hours per person) approximately forty moves are made[16] so that each player thinks an average of three minutes about his next move. Wide fluctuations occur around the three-minute-per-move mean: from part of a second to three quarters of an hour, or more. But in any case, thought processes leading to one move provide more workable units than entire games.

To create the conditions necessary for studying these thought processes a number of positions from actual games were selected and presented to a group of subjects, consisting of grandmasters, masters, experts and less skilled class players. They were not familiar with the positions presented. Each subject was requested to make a move only after proper deliberation, as if he were engaged in his own game, and, in addition, to think out loud as fully as possible so that the protocol would contain a picture of the way by which he had reached his choice of move. This means that the object of the present investigation is not the analysis of the way in which a chess game is built up in its entirety, but rather the analysis of how the chess player (subject) solves the *choice-of-move-problem.*

Section 5: Organization of the book

This study principally consists thus of *an experimentally based, psychological analysis of the chess player's thought processes preceding a move in a serious game.*

Parts C and D of Chapter I contain a closer analysis of the objective nature of the choice-of-move-problem. Chapter II is devoted to the theory of thinking, especially the *Denkpsychologie* of Otto Selz, while Chapter III describes and attempts to justify the experimental method used.

These preparatory chapters lead to the actual analysis contained in Chapters IV, V, and VI. The striking *external phase structure* which appears in chess thinking was made the starting point in the analysis of the protocol material.

Chapter IV, then, contains a detailed description based on one of the protocols as well as a statistical elaboration of a number of characteristics of the external structure. Chapter V deals with the *general problem structure* of chess thought which should be carefully distinguished from the external phase structure. The treatment here is in terms of the hierarchical structure of 'main problem' and 'subproblems' and of 'main goal' and 'subgoals,' respectively. In Chapter VI we leave the description of the external structure still further

behind and proceed to an explanation of the *dynamics of the thought process*. Here the main theme is the psychological development of the thinker's problem. In Chapter VII the most important results from the previous analyses are assembled and worked out theoretically with respect to the general *organization and methodology of the thought process*. With that the systematic analysis may be considered complete.

Having come so far, however, it seemed quite natural to keep going. The results obtained made it possible to reconsider, from a new point of view, other psychological questions with regard to chess and to compare thought in chess to that in other fields. Questions concerning the characteristics of the game and the player, his attitudes in contrast to his aptitudes, could be examined; the factors of talent and its development into mastership of the game could be looked into. It was, of course, impossible to consider these problems as thoroughly as the analysis of chess thought itself. The author had to limit himself to collecting some supplementary data from the literature and to extrapolating from his own experience and some additional experimentation. For that reason Chapter VIII has been written with perhaps less scientific rigor than the preceding chapters. By virtue of this, however, it will probably make the chapter that much more readable, stimulating, and accessible, in particular to a reader not directly interested in the complexities of thought psychology.

Finally, Chapter IX contains a brief, critical discussion of the *Denkpsychologie* of Selz and its further developmental possibilities. Since the attempt to base the analysis of the thought process of the chess player on Selz's ideas can be considered a test of the applicability of his theories and terminology, this subject, too, is a natural outgrowth of the main object of the present study.

In the revised English edition Chapter IX has been enlarged by some methodological remarks and by a discussion of some of the consequences of the findings for the machine simulation of chess thinking and for theories on human thought.

C. THE CHOICE-OF-MOVE-PROBLEM

Section 6: The variability of the problem

It is somewhat misleading to speak of *the* choice-of-move-problem. The character of the problem which confronts the player on move is

extremely variable, both from a psychological and a chess-technical standpoint. It is, of course, highly dependent on the position on the chessboard. It makes a great difference whether the player on move must construct a plan or execute one; whether he must conceive a combination or parry one; whether he must simply exchange pieces, get out of check, look for a safe continuation in a threatening position, or 'lie in wait' for the right opportunity. It makes a difference whether he can follow well-known lines of play – in the opening, the endgame, or a stereotyped middle game position – or must discover and work out new possibilities for himself.

Most 'interesting,' of course, are the turning points in the development of the battle: the moments in which the player discovers and starts to carry out a deep combination or works out a long range plan: those moments that would ordinarily appear in a diagram from the game. It would be most fascinating, for example, to gain some insight into the thought processes of Anderssen during his 'immortal game' against Kiezeritsky, when he decided to sacrifice both of his Rooks for the furthering of his attack.

Mainly, the layman wonders about two things: how the player 'hits upon the idea' and how far he foresees the coming development of the game. How did Euwe hit upon the idea of sacrificing a piece for three Pawns against Alekhine in the famous 26th (Zandvoort) game of his first match for the world championship? And how many of the combinatorial possibilities did Alekhine weigh in his brilliant game against Réti at Baden-Baden in 1925, when he put his Rook *en prise* at K6?

Such questions concerning the tense high points of competitive chess are of most interest to the chess devotee and amateur – next to the psychological explanation of special skills such as simultaneous blindfolded play. It is self-evident, however, that such sensational moments rarely occur in chess and surely are not generally representative of the choice-of-move-problem. Ordinarily positions are not so colorful, and it would certainly have been a serious mistake if brilliant combinations had been at hand in all of the experimental positions. The attempt was to avoid such one-sidedness by selecting a series of experimental positions of a wide and varied character. Only trivial choice-of-move-problems were purposely omitted.

For our purposes one of the important variable characteristics of a chess position is the number of possibilities available, i.e., the player's

freedom of choice. The choice of considerable moves can be large or small, up to the extreme case where there is no choice at all.

The question of the number of possibilities can be examined from three angles: the formal, the chess objective, and the psychological. That is, one can ask:

a. for the number of *possible* moves according to the rules of the game (the *legal* freedom of choice);
b. for the number of *good* moves; and
c. for the number of moves from which the player or subject *really* makes his choice – if there is a question of choice.

To do full justice to point *c*, the most important one, it is first necessary to discuss briefly *a* and *b*.

Section 7: The legal freedom of choice (K)

According to the rules of the game *the number of legally possible moves* in a given chess position (= K) ranges from zero (in a stalemate or checkmate position) to over a hundred. Practically speaking, however, the boundaries lie closer to each other: positions with more than seventy legal possibilities can be constructed but rarely occur in actual play, whereas the lower numbers – under eight – almost only occur when the King is in check. In the latter case the King must be removed from check which can only be done in a limited number of ways.

How great is the legal freedom of choice on the average? If we examine the universe of all positions which have occurred in past tournament and match games, what distribution for K would result? In order to get at least an approximate answer to these questions, the following variables have been tabulated from the 'diagramed positions' of a richly illustrated game collection, TARRASCH's *Dreihundert Schachpartien*:[17]

1. the number of the move to be played at that certain moment;
2. the legal number of possible moves in the given position;
3. the legal number of possible moves for the opponent, after the execution of the actually chosen move.

Variables 2 and 3 were classified according to intervals for variable 1: the 11th to the 15th move, 16th to 20th, 21st to 25th, 26th to

17 The sample is, of course, not random. Both games to be published and positions to be diagramed are chosen for the interest they may arouse in the reader. It does not seem likely, however, that this selective factor and the purely formal variable K are correlated, especially not if the number of legal possibilities for both White and Black are considered in every position.

30th, 31st to 40th, 41st to 50th, 51st and higher. Positions in which check must be answered (check-positions) were excluded from these tabulations. The collection of data was continued until each of the first five classes (starting with the 11th to the 15th) contained 20 positions with White on move and 20 positions with Black on move. Thus, for example, *the 21st to the 25th move class* yielded for 40 positions:

mean move number: 23.4		
mean of K (number of legal possibilities) for White:	38.1	total mean K = 38.0
mean of K (number of legal possibilities) for Black:	37.9	

For the same move interval the following results were obtained for the extreme K-values (lo and hi), the median (Mdn), and the quartiles (Q_1 and Q_3):

lo	Q_1	Mdn	Q_3	hi
21	32.5	38.0	42.5	65

Apparently the median and the total mean coincide in this class. Assuming that the sample is representative, we can now offer the following generalization of the results:

Around the 23rd and 24th move (mean 23.4) the legal freedom of choice (K) lies between 32 and 43 move possibilities in about 50% of all the non-check-positions from master (tournament and match) games; K-values under 21 and above 65 are rare.

The results of all the move intervals are summarized in the graph, Figure 2. The three curves were formed by connecting the values of K of each interval. Approximate data with regard to the value of K for move number i can be found by interpolation. Thus, for instance, the quartiles for K at the i^{th} move, for Black and White averaged together, can be estimated by intersecting the vertical at i with the dotted quartile lines. The extreme values as found in each sample have been indicated by circles on the corresponding vertical.

Not enough of the diagramed positions from Tarrasch's collection were of the opening phase, before the tenth move; consequently, it was necessary to use other material to construct the left side of the graph. This material consisted in all the positions from six frequently played openings, somewhat arbitrarily chosen. On these are based the sample data at i = 3.0; i = 6.0; and i = 9.0. They fit well enough with the rest of the graph. In total the graph is based on a move count of 350 positions.

FIGURE 2. *The relation between legal freedom of choice, K, and move number, Z_i.*

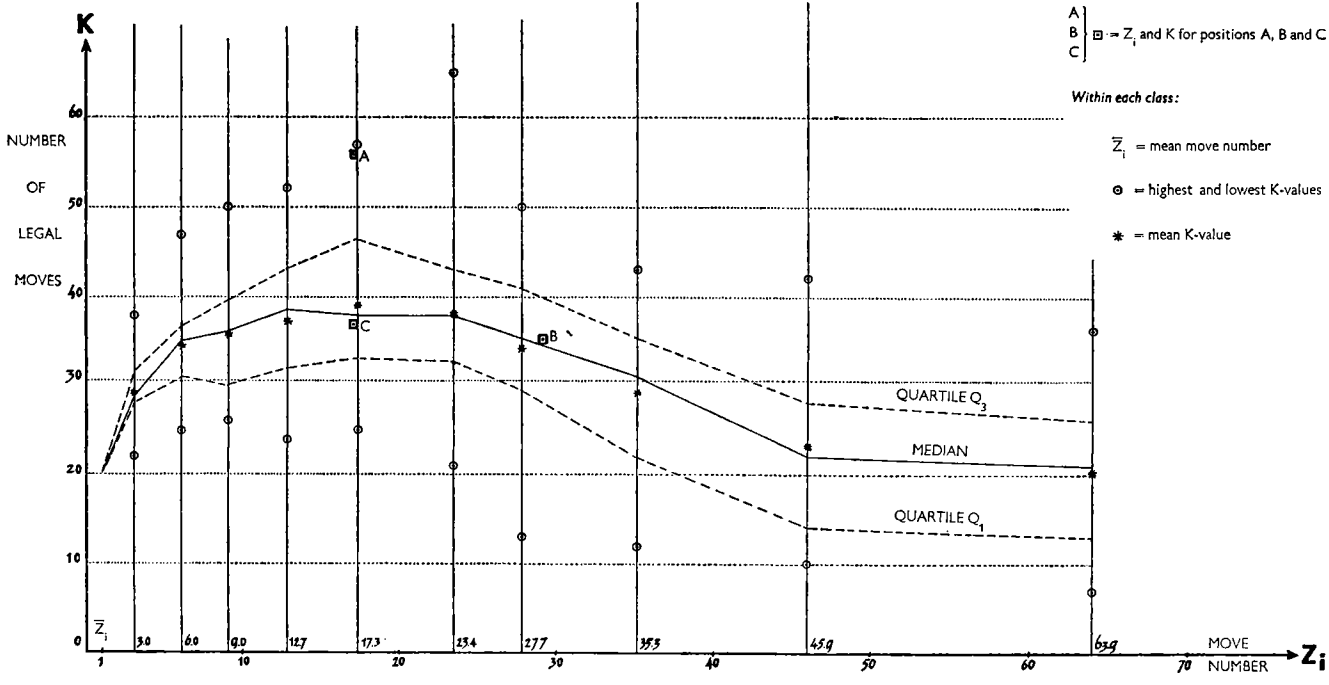


TABLE 1. *The relation between legal freedom of choice, K, and move number, Z_i.*

Move number class (Z_i-class)	Z_i = 2, 3, 4	Z_i = 5, 6, 7	Z_i = 8, 9, 10	Z_i = 11–15	Z_i = 16–20	Z_i = 21–25	Z_i = 26–30	Z_i = 31–40	Z_i = 41–50	Z_i = 51–..
Number of positions investigated (p)	36	34	36	40	40	40	40	40	23	21
Mean move number (Z_i)	3.0	6.0	9.0	12.7	17.3	23.4	27.7	35.5	45.9	63.9
Highest number of legal moves (K_{max})	38	47	50	52	57	65	50	43	42	36
Third quartile of K-values (Q_3)	31.5	37	39.5	43	46.5	42.5	41	35.5	28	26
Median of K-values	29	35	36	38.5	38	38	35.5	31	22	21
Mean number of legal moves (K)	29.1	34.3	35.8	37.1	39.4	38.0	34.0	29.1	23.1	20.3
First quartile of K-values (Q_1)	28	31	30	32	33	32.5	29.5	22	14	13
Lowest number of legal moves (K_{min})	22	25	26	24	25	21	13	12	10	7

The following results can be read from the graph:

1. The mean number of move possibilities on the i^{th} move, K(i), varies from 20 – at the 1st move and around the 65th move in long games – to about 40 around the 17th move. Remarkably enough, less than 20 possibilities rarely occurs before a very advanced stage of the game.
2. As the game 'ages' the variation in the mean number of legal moves remains within modest limits but does show a characteristic curve which clearly reflects, among other things, the development of the pieces during the first 10 to 15 moves.
3. To estimate the mean number of move possibilities from the complete universe of master game positions, it would be necessary to resort to some method of weighting since there exist far more positions at the 12th move, for instance, than at the 74th move: most games contain more than 12 moves but less than 74 moves. Thus, the frequencies in the first 20 moves should count more heavily. Taking that into account, it can be seen from the graph that the total mean lies somewhere between 30 and 35 move possibilities. Indeed, some rough calculations based on the lengths of games as noted in footnote 16, Section 4, yielded 32.3 as a sample estimate for the 'average legal freedom of choice.'
4. From the quartile lines which are close together and from the extreme values, it can be seen that the scatter at any given moment is rather narrow. In all of the material studied, excluding check-positions, there is not a single case before the 26th move (out of 226 positions) in which there are fewer than 20 move possibilities (i.e., where $K < 20$) and before the 51st move not a single case of fewer than 10 possibilities. Even further on, the range is still rather small. The occurrence of 65 possibilities (for $i = 21$ to 25) is quite an exception for the next highest is 57 possibilities (for $i = 16$ to 20).
5. On the graph each game could be represented by a broken line with each position as a point. Positions A, B, and C, with which most of the experimentation was done, are represented as such points. Apparently A is an extremely rich position whereas B and C are normal.

A few words should be said about check-positions in which, of course, the legal necessity of getting out of check greatly delimits the number of move possibilities. In the 45 games from the Margate tournament book, 1938, there were 171 check-positions resulting from a total of

3,640 'half-moves.' Therefore there was one check out of 21.3 half-moves. If we generalize we may roughly set the relative frequency of a check-position at 5 %.

The material on which the graph was based contained 16 check-positions which had to be excluded (4.4 %). The mean of the move possibilities was 3.8, the median was 3.5, the range was from 2 to 9. If a larger number of cases had been taken, these values, especially the extremes, would shift somewhat, but in any case it is clear that this group of positions is exceptional, regardless of the stage of the game. During the progress of a game the giving of check suddenly decapitates the gamut of formal possibilities, leaving three or four replies: a checking move is legally coercive, indeed.

Section 8: Objective freedom of choice

What is more important to us, but more difficult to answer, is the question of *the number of good moves* that can be played in a given position (point *b*).

A 'good move' is, of course, a relative concept, but it can be more or less rigorously defined. For a workable operational definition, however, we shall have to make use of the judgment of experts[18] who must be able to handle the difficult cases in which comparisons of complicated variations and 'deep' evaluations are needed (a so-called *analysis of the position)*. The following definition of a 'good move' may deviate somewhat from the actual usage of the term, but it possesses the advantage of greater rigor and testability:

> A move is good if and only if it is impossible to find another one, after a careful and convincing analysis, that is better.

Or, put another way:

> A move is not good if and only if it is possible by an analysis of the position, to demonstrate convincingly that another one is better.

With this, the relativity of the concept and the necessity of comparison with other moves is clearly shown. As to the meaning of 'analysis,' practical considerations can be taken into account: a master must be able to analyze the position without months of investigation, and

18 Since there are as yet no objective or machine definitons available that are both precise and practically usable, the best basis is still intersubjective agreement among experts.

the results should convince any unbiased skilled player. This definition is, of course, not completely rigid since there is some leeway in the depth of the analysis. It can, however, be tightened and strictly operationalized whenever the situation demands it.

But when move Z_1 is actually better than move Z_2 the question still remains: how can its superiority be proved in an 'analysis'? The answer: When, after considering every possible response, one can show that the minimal results are better with move Z_1 than with move Z_2. This seems like a tautology but in fact gives the means for proving that Z_1 is better than Z_2. 'To achieve results' means to reach positions whose value can be objectively assessed by a skilled player in terms such as 'won by White,' 'very favorable,' 'drawn,' 'equal play,' 'somewhat less,' etc. One could even rate the position (e.g., won = 10, favorable = 7, even = 5, etc.). Naturally such rating is not entirely free from subjectivity; but the assumption that one can actually prove the superiority of Z_1 over Z_2 implies that in practice it must be possible, in each variation, to reach an end position – possibly a 'dead position'[19] – that can be evaluated with the proper degree of objectivity.

A few examples: Move Z_1 is better than move Z_2 if Z_1 leads to a 'won' position against every opposing move while only 'favorable' positions can be reached via move Z_2; if Z_1 yields at least 'even play' against every opposing move and Z_2 no more than a 'less good' position; etc.[20] With this the usage in sufficiently standardized.

We are now able to classify chess positions according to the number of good moves that can be made. But it is not so easy to investigate this statistically – every position would have to undergo analysis. It is certain, however, that the frequencies obtained here are of a different order than those obtained under point *a*. At least nine out

19 A 'dead position' is a position in which an *evaluation of the position*, without further calculations, is in order due to the absence of immediate tactical possibilities. 'Absence of immediate tactical possibilities' can, of course, be defined in different ways, e.g., 'no capturing moves can be made.' The term 'dead position' was introduced into the chess machine terminology by TURING (1953, p. 291).

20 If need be, one can expand the usage of 'Z_1 is better than Z_2' to include cases in which branch end results show no clear differences. Thus Z_1 is also better than Z_2 if the same favorable result (e.g., a won position or even checkmate) can be reached faster by means of Z_1 than Z_2. Furthermore, Z_1 is better than Z_2, even though both moves lead to the same result with the best opposition, if it is much more difficult for the opponent to find the best moves after Z_1 than after Z_2. In practice these cases are included, but here they shall be excluded so as not to unduly complicate the matter.

of ten legal move possibilities do not come in for serious consideration; in general, there will remain only three or four out of the thirty or forty possibilities.

From the analysis of an old tournament game, Halic v.s. De Groot (Munich, 1936) the average number of good moves per position was found to be no more than two. The maximum was five, and in approximately 40% of the positions from this rather ordinary, not particularly brilliant or spirited game there was not more than one good move. The importance of this last category is obvious, for which reason it will be attended to first.

One good move. This is at the same time 'the best move': its superiority over the other moves must be demonstrable, by definition. For example, this case occurs:

a. with a *check* that can be parried in only one way (then there is just one legal possibility, and, a fortiori, but one good move) or which can be parried in only one way without immediate disadvantage;
b. with an *exchange of pieces* where the piece or Pawn must be recovered;
c. with the presence of a *threat* that can be properly parried in only one way (e.g., the opponent threatens checkmate in two moves, or attacks a major piece, or threatens to fork two pieces, etc.);
d. in positions where just *one combination* yields an advantage (e.g., a series of forcing moves which the player calculated beforehand);
e. in positions where a consideration of *timing* necessitates the choice of one move to execute a specific plan (e.g., if one must be ahead of the opponent and forestall his counter measures); etc.

Thus again we have to do with the same sort of curtailment of the gamut of possibilities as in Section 7 with check-positions. In practice, indeed, it is not only the check-positions that have a forcing character. For instance, an error by the opponent, if recognized as such, morally forces upon the player a definite, immediate reaction: namely, the consequent winning continuation.

The degree of difficulty of the choice-of-move-problems of this group can, of course, differ greatly. In all the cases from *a* to *e* one can give both trivial and extremely complicated examples. They have, however, one thing in common: the existence of a single objective solution. One can prove that a specific move is the best: *the choice of move problem is objectively solvable.* The problem for the player (the subject) is then to 'find' the single solution; it is not to 'choose' a move that may be partially based on personal tastes or motives.

Two or more good moves. It might appear from the discussion above that positions in which more than one good move can be played are necessarily not objectively solvable. This is not the case, however. Or rather, it is better not to connect the concept of objective solvability with the 'number of good moves.' If there are two good moves, Z_1 and Z_2, by definition it is not possible to prove their inequality, but it may be possible to prove their *equality. In that case also we must say that the choice-of-move-problem is objectively solvable.* After all, a quadratic equation is no less objectively solvable than a linear equation.

Double and multiple objective solutions are far from rare in chess. They occur, for instance, in:

a. positions in which the sequence of moves of a combination can be varied without changing the end result;
b. check-positions in which the King can move in two or more ways but which does not affect the relevant continuation;
c. positions in which the opponent can be forced into a tight spot by arbitrary 'tempo moves';
d. defensive positions in which one can undertake nothing oneself, but must calmly await the coming developments by moving the available pieces back and forth.

Apart from these, there is another category, namely, *objectively non-solvable positions.* Here two or more good moves are possible but neither the inequality nor the equality of these moves can be practically proved because the position itself cannot be fully analyzed. Such specimens are to be found, for example, in:

a. the opening where one can choose between different variations, none of which has as yet been refuted;
b. the middle game where one can choose, for instance, between different systems (e.g., to open up the position or to keep it closed), whose relative worths cannot be definitely established even through an extensive analysis, etc.

Such objectively non-solvable position problems do not allow of an objectively valid 'right' solution as does a mathematical problem.

21 Notice that the difference is relative. A still playable opening variation can later be refuted by the collective experiences and analyses of chessmasters; a currently objectively non-solvable position can later become solvable. Indeed there is an abundance of cases in the opening where the nineteenth century master still had free choice among various good moves, Z_1, Z_2, and Z_3, while his twentieth century colleague is required to choose move Z_1 because the other two, Z_2 and Z_3, have in the meantime been refuted. The development of opening theory has the consequence that increasingly more 'free' positions are becoming objectively solvable.

At least no one knows the solution or can find it by human calculation[21] so that it is impossible to arrive at a choice without subjective considerations. Such positions have a character of their own, especially if the number of good moves is large (three or more possibilities).

From a psychological standpoint the *objective freedom of choice* – defined as the number of playable, good moves – and especially the objective solvability or non-solvability of a position is of much more importance than the legal freedom of choice, K. The latter, psychologically, is but a purely formal maximum both with respect to the number of possible good moves and the number of moves that the player actually considers. This maximum is seldom attained and then almost only in check-positions. Between the legal freedom of choice and 'objective solvability,' there is little connection: a robust position with 60 or more possible moves may be objectively solvable, whereas an anemic position in which K = 10 may be 'wide-open.'

Just as we can differentiate objectively solvable and objectively non-solvable positions (or to put it more briefly, forcing and wide-open positions) so also can we distinguish two ways in which the chess player can view his problem. If the player regards the situation on the board as objectively solvable, then he must proceed in such a way as to search for the best move, the 'right' continuation. With a 'free' conception of the problem, on the other hand, he asks himself the question, 'What shall I play (choose) now?' He is cognizant of his freedom and of the personal element in his choice; nor does he pretend that his choice will be the 'right' move. Thus we can consider problem situations and play situations as the two limiting cases. It is characteristic of chess that the player's conception of the choice-of move-problem fluctuates between these extremes.

Naturally, the player's conception of the problem does not always correspond with the objective character of the position. Sometimes the player believes he has found and played the only correct move while, in actuality, he could have embarked upon another equally good continuation. In other cases the player chooses a move according to personal preference, presuming nothing; afterwards the move may appear to have been the only right one, or worse, not the only right one. Furthermore, the player's way of thinking and his attitude towards chess in general help to determine his conception of the problem. Typological differences manifest themselves at this point (cf., e.g., Section 59). This, however, in no way detracts from the psychological importance of the criterion of objective solvability.

D. THE PROOF SCHEMA FOR AN OBJECTIVELY SOLVABLE POSITION: MOVE W_1 IS THE BEST

Section 9: The two part structure of the proof schema

Let us assume in a given position with White to play that there is only one 'right' move, W_1. It must be shown then that against any and all replies by Black, White will always achieve better results with W_1 than with W_2, W_3, ..., W_n. By using the ratings introduced in Section 8, it can easily be shown that a complete proof consists of two parts. To prove:

(1) By playing 1.W_1, White's minimum result will be, e.g., 8. (This will constitute the *positive part* of the proof.)
(2) By playing 1.W_2, W_3, ..., W_n, White's maximum result will be, e.g., 6. (This will constitute the *negative part* of the proof.)

In the positive part of the proof every possible Black reprisal must be answerable by one sufficiently strong White move; conversely, in the negative part, every White move must be answerable by one sufficiently strong Black move. The existence of more than one sufficiently strong move is redundant to the proof. In other words: In the positive part branches are only expected when it is Black's move *(counter branches)* and in the negative part only when it is White's move *(own-branches)*.

The positive part ('By playing 1.W_1, White's minimum result will be, e.g., 8.'):

After the move 1.W_1 Black's replies 1...B_{11}, 1...B_{12}, 1...B_{13}, etc., can be answered respectively, by 2.W_{11}, 2.W_{12}, or 2.W_{13}. If after 2.W_{12} Black responds with 2...B_{121} or 2...B_{122}, then the replies 3.W_{121} and 3.W_{122} follow, respectively. Finally, it is assumed that branches will eventually terminate and end positions will be reached where unambiguous, objective values can be assigned. 1.W_1 can then be evaluated by assigning it the minimum terminal value – in this case 8.[22]

22 The evaluation of W_1 may or may not be correct: this depends on whether the subsequently chosen W-moves (W_{11}, W_{12}, W_{121}, W_{122}, etc.) are objectively 'best moves' or at least 'good moves.' In case they are not, the final evaluation may be too *low*. Accordingly the ratings assigned the other possible first moves (W_2, W_3,..., W_n) may be too *high*. It follows that even so the conclusion of the relative superiority of W_1 over other moves remains correct – provided the counter branches in the

The negative part ('By playing 1.W_2, W_3, ..., W_n, White's maximum result will be, e.g., 6'):

For all moves 1.W_2, W_3, ..., W_n, Black can respond 1...B_2, B_3, ..., B_n in such a way that every second move by White 2.W_{i1}, W_{i2}, ..., W_{in} ($i = 2, 3, \ldots, n$) can be correspondingly answered by 2...B_{i1}, B_{i2}, ..., B_{in}, etc. From White's viewpoint no terminus can be assessed at more than 6.

If one succeeds in finding the moves 1.W_i, 2.W_{ij}, 3.W_{ijk} ($i, j, k = 1, 2, \ldots, n$) in the positive part and 1...B_i, 2...B_{ij}, 3...B_{ijk} ($i, j, k = 2, 3, \ldots, n$) in the negative part, in such a way as is required for the proof, then one has indeed proved that 1.W_1 is the strongest move.

We shall use a special case for an illustration. Assume that White is able to checkmate his opponent in three moves. The proof is much simpler here because (1) no final numerical evaluation and comparison of results is needed and (2) the length of no variation can exceed three moves.

To prove:

(1) After 1.W_1 no reply by Black can avert checkmate in three moves, or less.
(2) After any other moves, 1.W_2, W_3, ..., W_n, Black is able to prevent checkmate in three moves.

This can be illustrated by a branching tree making the simplifying assumption of a small number of possible branches (see Figure 3).

Incidentally, it is not necessary that two counters with different subscripts actually be different; e.g., different defenses are sometimes followed by the same sufficiently strong move.

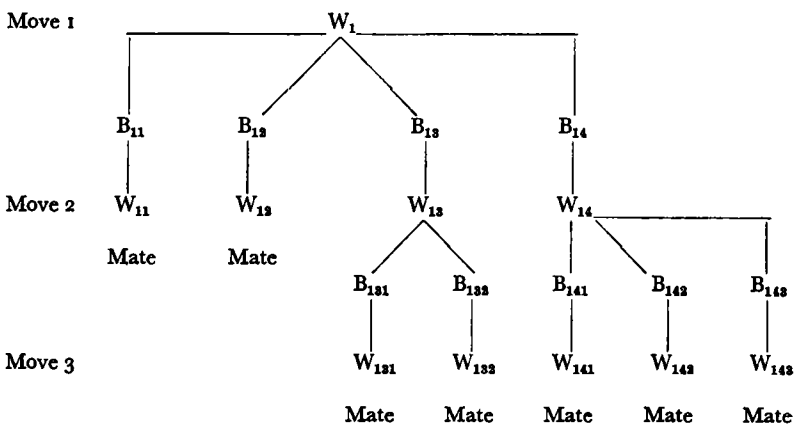
This schema, at least the positive part, may remind the chess player of the way in which chess problems are presented in publications. The problem solver must only find the key move, W_1, and the successive right White move against every possible Black answer. But the problem composer must solve the negative part too! He must avoid 'cooks' or redundant solutions; he must be able to prove that W_1 is the 'only' best move.

positive part and the own-branches in the negative part cover all (relevant) answers.

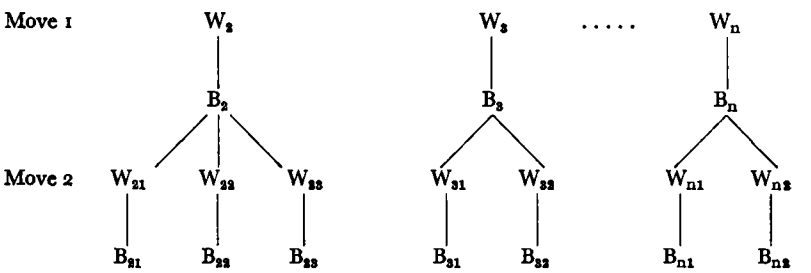
The procedure as described here is, therefore, not completely equivalent to 'blind' *minimaxing* as described in game theory in that it is more parsimonious and less comprehensive. The W-moves in the positive part and the B-moves in the negative part may be intuitively selected; there is no need to consider all possible branches in order to prove the superiority of W_1 over other moves.

FIGURE 3

Positive Part:



Negative Part:



In an actual game the situation is different, of course. A player who sees a chance to force checkmate may not bother to look for other, perhaps even shorter, solutions. Such an indifference towards the negative part can only exist when victory is clearly in sight – immediate winning combinations, threats that cannot be parried, etc. – for in general the negative part must be taken into consideration.

Section 10: The significance of the schema for the thought process

Naturally not every chess player has this proof schema at his continual disposal and certainly not in the form given here. Even in a worked

out *written analysis* this fundamental form is not always easy to disinter, as a result of the fact that groups of variations are considered together or parts of the analysis are omitted. If the opponent has a serious 'threat' in the initial position, all of the moves that do not parry that threat can be omitted from the negative part. With the discovery of a stalemating combination – which legally draws the game – in an apparently hopeless situation, it would be senseless still to ask for the negative proof in order to show that other moves do *not* lead to a draw. If White has a choice between executing a forcing, drastic combination and a number of quiet moves which do not significantly alter the position, the latter group can be treated together in the analysis and rejected at a single glance. Of course, other such simplifying conclusions are also possible.

The *ordinary thought process* follows the lines of such a proof even less. Because of the limited thinking time and the necessity to work everything out in his head, the player at the board often cannot furnish a proof, even where an analysis can. It should be kept in mind, moreover, that in a chess position, what can and must be proved is not given as in a mathematical problem. The player must discover this for himself; the thought process involves both searching and reasoning, both selecting a move and building up an argumentation for the choice. Thus it is comparatively rare for a thought process to constitute a rigorous proof. In the majority of cases a move is played without complete certainty that it actually is the best move.

But, after all, it is a motivated choice, and although the thought process does not consist in a proof it still must involve an argumentation in favor of the move to be chosen. In the present study it was in fact possible to derive from the text of most protocols a reasoning structure comparable to the proof schema set forth above. It was often possible, in particular, to distinguish a positive and negative part in the final argument in spite of the fact that the order of presentation in the protocols was largely unsystematic. Sometimes the many miscarriages of attempted continuations begun with positive intentions collectively form the negative part of the argumentation. In such a case all of the variations previously considered for the opponent become 'dead branches' of the negative analysis. For example: The subject has the choice between the active, combinative move 1.W_1 and the calm continuation 1.W_2. He wants to play W_1 and so calculates four worthwhile retorts for Black: B_{11}, B_{12}, B_{13}, and B_{14}. Against the first three the attack would be driven through, but the fourth possibility, B_{14}, refutes the move W_1. He therefore ends up with 1.W_2. The

negative part now consists of Black's refutation of $1.W_1$, that is, the variant $1.W_1$, B_{14}, etc. The other three Black branches, B_{11}, B_{12}, and B_{13}, are of no further value to the reasoning process; they have become 'dead branches.'[23]

Branchings from either White or Black nodes and the wording with which the subject begins a new variation often tell us whether the calculations are an attempt at a positive proof, at a purely empirical investigation into the best possibilities for both sides, or at a negative proof. The first of these three cases is the most frequent, as we shall see.

In any case a familiarity with the general proof schema for the 'best move' has its utility in the psychological investigation of chess thought (cf., e.g., Chapter V, Section 43; and Chapter VII, Section 54).

23 'Branches' in the proof schema of a chess game can be likened to the metaphorical 'tree of logic' – perhaps even more profitably here than in a mathematical proof. G. Pólya, moreover, speaks of 'dead branches' in an article on this theme (PÓLYA 1939). In the course of this book the metaphorical 'tree' will be frequently employed.