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# **Intensive Science and Virtual Philosophy**

**Manuel DeLanda**

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To **Julieta Pereda Pineda**,  
who taught so much about the world

# Preface: Ten Years After

It has been a decade since the original publication of this book and the trend towards a renewal of realism with which it is concerned has become ever more intense. Young continental philosophers today acknowledge that it was in 2002 that a realist ontological stance was openly proclaimed, “the first time this has been done with a straight face in the recent continental tradition.”<sup>1</sup> In other words, the first time that the autonomy of material entities from the human mind was asserted without any sense of postmodern irony. In the years that followed several forms of continental realism have flourished, grouped together under the term *speculative* to stress the fact that this autonomous world is not the world of everyday perception and common sense, but that a great deal of philosophical work must be performed to reconstruct it from the impoverished version given to direct experience. The different realist positions that have emerged differ precisely in what they postulate must be speculatively added to what is given to yield a richer picture of things and events. Given how recent this “speculative turn” is, the variety of proposals is a welcome antidote to the premature freezing of a single ontological solution to the problem.

In the present text the necessary enrichment of the world-view is performed by adding to the properties that define an object’s identity, its *tendencies and capacities*. Whereas properties are always actual, tendencies and capacities can be real without being actual, if they are not currently manifested or exercised. Thus, the mind-independent identity of a given body of water can be established by determining its actual properties (its volume, purity of composition, temperature, speed of flow) but that determination does not exhaust its reality. Such a body of water may exist presently in the liquid state, but it is part of its reality that at a certain temperature it can become steam or ice, that is, that it

has a real tendency to boil or freeze under certain conditions. The fact that it is not in a gaseous or crystalline state at this moment does not make its tendency to become gas or crystal any less real. Similarly, the identity of a body of water is partly determined by its capacity to affect other substances, such as its capacity to dissolve them. The exercise of this capacity demands an actual interaction with acids, alkalis, or salts, but the absence of interactions does not make it any less real. Like tendencies, capacities become actual as events, but in this case the events are always double, to dissolve/to be dissolved. The reason is that a capacity to affect must always be coupled to a capacity to be affected: water is a solvent but only when interacting with substances that are soluble in it.

In the analytical tradition, where the trend towards realism began in the 1980's, the need to supplement the properties of a mind-independent world with tendencies and capacities is now generally acknowledged.<sup>2</sup> Moreover, the effect that this ontological supplementation may have on *epistemology* has also been explored. In particular, gaining knowledge about objective entities involves creating representations of them as possessors of a set of properties, as well as performing *interventions on them* with the aim of forcing them to manifest their tendencies, or of getting them to interact with a variety of other entities so that they exhibit their full repertoire of capacities.<sup>3</sup> This way, epistemology is liberated from passive observation. Nowhere is this epistemological change more evident than in realist theories of causality. Whereas for idealists causality can be reduced to a concept (transcendental or conventional), and for empiricists it can be reduced to the observed constant conjunction of a cause and its effect, for realists causality is a form of *objective synthesis*, the production of one event by another event.<sup>4</sup> Human interventions in the realm of science share this productive or synthetic aspect of causality.

Postulating objective tendencies and capacities may not seem overly speculative, but this book goes one step beyond that. It offers a new philosophical concept to ground the reality of the non-actual determinants of identity, the concept of *the structure of a possibility space*. This structure is easier to determine for tendencies than for capacities, since for the former it can be defined by the mathematics of critical points, like boiling or freezing points. For capacities we need a way of thinking about spaces with a changing number of dimensions,

given that what an entity can affect depends on a possibly unlimited number of other entities that can be differently affected by it.<sup>5</sup> Moreover, the present text dares to speculate not only about the structure of the possibility spaces associated with actual entities, but also about the relations that these spaces (or more exactly, their structure) may have with each other. Do they form another dimension of reality, one defying not only observation and representation but also intervention?. Do they form an immanent plane, an ideally continuous space from which actual beings derive as so many discontinuous segments?. The task of making this ideal plane an object of thought goes beyond mathematics and science. It is a defining task for any future realist philosophy.

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- 4 Mario Bunge. *Causality and Modern Science*. (New York: Dover, 1979). p. 47.
- 5 My most ambitious attempt to define the concept of the structure of possibility spaces, for both tendencies and capacities, is in:



# Introduction: Deleuze's World

There are always dangers in writing a book with a specific audience in mind. The most obvious one is the danger of missing the target audience completely, either because the subject matter fails to grab its attention or because the style of presentation does not meet its standards or expectations. Then there is the associated danger of losing readers who, had not that particular target been chosen, would have formed the real audience of the book. A book may end up this way without any readership at all. In the world of Western philosophy, for example, history and geography have conspired to divide this world into two almost mutually exclusive camps, the Anglo-American and the Continental camps, each with its own style, research priorities and long traditions to defend. A philosophical book which refuses to take sides, attempting, for example, to present the work of a philosopher of one camp in the terms and style of the other, may end up being a book without an audience: too Anglo-American for the Continentals, and too Continental for the Anglo-Americans.

Such a danger is evident in a book like this, which attempts to present the work of the philosopher Gilles Deleuze to an audience of analytical philosophers of science, and of scientists interested in philosophical questions. When confronted with Deleuze's original texts this audience is bound to be puzzled, and may even be repelled by the superficial similarity of these texts with books belonging to what has come to be known as the "post-modern" tradition. Although as I argue in these pages Deleuze has absolutely nothing in common with that tradition, his experimental style is bound to create that impression. Another source

of difficulty is the philosophical *resources* which Deleuze brings to his project. Despite the fact that authors like Spinoza and Leibniz, Nietzsche and Bergson, have much to offer to philosophy today, they are not generally perceived by scientists or analytical philosophers of science as a legitimate resource. For this reason what I offer here is not a direct interpretation of Deleuze texts but a *reconstruction* of his philosophy, using entirely different theoretical resources and lines of argument. The point of this reconstruction is not just to make his ideas seem legitimate to my intended audience, but also to show that his conclusions do not depend on his particular choice of resources, or the particular lines of argument he uses, but that they are *robust to changes* in theoretical assumptions and strategies. Clearly, if the same conclusions can be reached from entirely different points of departure and following entirely different paths, the validity of those conclusions is thereby strengthened.

I must qualify this statement, however, because what I attempt here is far from a comprehensive reconstruction of all of Deleuze's philosophical ideas. Instead, I focus on a particular yet fundamental aspect of his work: *his ontology*. A philosopher's ontology is the set of entities he or she assumes to exist in reality, the types of entities he or she is committed to assert actually exist. Although in the history of philosophy there are a great variety of ontological commitments, we can very roughly classify these into three main groups. For some philosophers reality has no existence independently from the human mind that perceives it, so their ontology consists mostly of mental entities, whether these are thought as transcendent objects or, on the contrary, as linguistic representations or social conventions. Other philosophers grant to the objects of everyday experience a mind-independent existence, but remain unconvinced that theoretical entities, whether unobservable relations such as physical causes, or unobservable entities such as electrons, possess such an ontological autonomy. Finally, there are philosophers who grant reality full autonomy from the human mind, disregarding the difference between the observable and the unobservable, and the anthropocentrism this distinction implies. These philosophers are said to have a *realist ontology*. Deleuze is such a realist philosopher, a fact that by itself should distinguish him from most post-modern philosophies which remain basically non-realist.

Realist philosophers, on the other hand, need not agree about the contents of this mind-independent reality. In particular, Deleuze

rejects several of the entities taken for granted in ordinary forms of realism. To take the most obvious example, in some realist approaches the world is thought to be composed of fully formed objects whose identity is guaranteed by their possession of an *essence*, a core set of properties that defines what these objects are. Deleuze is *not* a realist about essences, or any other *transcendent* entity, so in his philosophy something else is needed to explain what gives objects their identity and what preserves this identity through time. Briefly, this something else is *dynamical processes*. Some of these processes are material and energetic, some are not, but even the latter remain *immanent* to the world of matter and energy. Thus, Deleuze's process ontology breaks with the essentialism that characterizes naive realism and, simultaneously, removes one of the main objections which non-realists make against the postulation of an autonomous reality. The extent to which he indeed deprives non-realists from this easy way out depends, on the other hand, on the details of his account of how the entities that populate reality are produced without the need for anything transcendent. For this reason I will not be concerned in this reconstruction with the textual source of Deleuze's ideas, nor with his style of argumentation or his use of language. In short, I will not be concerned with Deleuze's *words* only with Deleuze's *world*.

The basic plan of the book is as follows. Chapter 1 introduces the formal ideas needed to think about the abstract (or rather *virtual*) structure of dynamical processes. I draw on the same mathematical resources as Deleuze (differential geometry, group theory) but, unlike him, I do not assume the reader is already familiar with these fields. Deleuze's grasp of the technical details involved is, I hope to show, completely adequate (by analytical philosophy standards), but his discussion of technical details is so compressed, and assumes so much on the part of the reader, that it is bound to be misinterpreted. Chapter 1 is written as an alternative to his own presentation of the subject, guiding the reader step by step through the different mathematical ideas involved (manifolds, transformation groups, vector fields) and giving examples of the application of these abstract ideas to the task of modelling concrete physical processes. Despite my efforts at unpacking as much as possible the contents of Deleuze's highly compressed descriptions, however, the subject matter remains technical and some readers may still find it hard to follow. I recommend that such readers skip this first

chapter and, if need be, come back to it once the point of the formal resources becomes clear in its applications to less abstract matters in the following chapters.

Chapters 2 and 3 deal with the production of the different entities that populate Deleuze's world. The basic theme is that, within a realist perspective, one does not get rid of essences until one replaces them with something else. This is a burden which affects only the realist philosopher given that a non-realist can simply declare essences mental entities or reduce them to social conventions. One way to think about essentialism is as a theory of the genesis of form, that is, as a theory of *morphogenesis*, in which physical entities are viewed as more or less faithful realizations of ideal forms. The details of the process of realization are typically never given. Essences are thought to act as *models*, eternally maintaining their identity, while particular entities are conceived as mere *copies* of these models, resembling them with a higher or lower degree of perfection. Deleuze replaces the false genesis implied by these pre-existing forms which remain *the same* for all time, with a theory of morphogenesis based on the notion of *the different*. He conceives difference not negatively, as lack of resemblance, but positively or productively, as that which drives a dynamical process. The best examples are *intensive differences*, the differences in temperature, pressure, speed, chemical concentration, which are key to the scientific explanation of the genesis of the form of inorganic crystals, or of the forms of organic plants and animals. Chapter 2 is concerned with the *spatial* aspects of this intensive genesis while Chapter 3 deals with its *temporal* aspects.

After reconstructing Deleuze's ontology I move on in Chapter 4 to give a brief account of his *epistemology*. For any realist philosopher these two areas must be, in fact, intimately related. This may be most clearly seen in the case of naive realism, where truth is conceived as a relation of *correspondence* between, on one hand, a series of facts about the classes of entities populating reality and, on the other, a series of sentences expressing those facts. If one assumes that a class of entities is defined by the essence which its members share in common, it becomes relatively simple to conclude that these classes are basically given, and that they exhaust all there is to know about the world. The ontological assumption that the world is basically closed, that entirely novel classes of entities cannot emerge spontaneously, may now be

coupled with the epistemological one, and the correspondence between true sentences and real facts can be made absolute. It is unclear to what extent any realist philosopher actually subscribes to this extremely naive view, but it is clear that a reconstruction of Deleuze's realism must reject each one of these assumptions and replace them with different ones.

While in the first three chapters I attempt to eliminate the erroneous assumption of a closed world, in Chapter 4 I try to replace not only the idea of a simple correspondence but, beyond that, *to devalue the very idea of truth*. In other words, I will argue that even if one accepts that there are true sentences expressing real facts it can still be maintained that most of these factual sentences are *trivial*. The role of the thinker is not so much to utter truths or establish facts, but to distinguish among the large population of true facts those that are important and relevant from those that are not. *Importance and relevance*, not truth, are the key concepts in Deleuze's epistemology, the task of realism being to ground these concepts preventing them from being reduced to subjective evaluations or social conventions. This point can be made clearer if we contrast Deleuze's position not with the linguistic version of correspondence theory but with the mathematical one. In this case a relation of correspondence is postulated to exist between the *states* of a physical object and the *solutions* to mathematical models capturing the essence of that object. By contrast, Deleuze stresses the role of correctly posed *problems*, rather than their true solutions, a problem being well posed if it captures an objective distribution of the important and the unimportant, or more mathematically, of *the singular and the ordinary*.

Chapter 4 explores this *problematic epistemology* and compares it with the more familiar axiomatic or theorematic versions which predominate in the physical sciences. To anticipate the main conclusion of the chapter, while in an axiomatic epistemology one stresses the role of *general laws*, in a problematic one laws as such disappear but without sacrificing the objectivity of physical knowledge, an objectivity now captured by distributions of the singular and the ordinary. If such a conclusion can indeed be made plausible, it follows that despite the fact that I reconstruct Deleuze to cater to an audience of scientists and analytical philosophers of science, nothing is yielded to the orthodox positions held by these two groups of thinkers. On the contrary, both

physical science and analytical philosophy emerge transformed from this encounter with Deleuze, the former retaining its objectivity but losing the laws it holds so dear, the latter maintaining its rigour and clarity but losing its exclusive focus on facts and solutions. And more importantly, *the world itself* emerges transformed: the very idea that there can be a set of true sentences which give us the facts once and for all, an idea presupposing a closed and finished world, gives way to an open world full of divergent processes yielding novel and unexpected entities, the kind of world that would not sit still long enough for us to take a snapshot of it and present it as the final truth.

To conclude this introduction I must say a few words concerning that other audience which my reconstruction may seem to overlook: Deleuzian philosophers, as well as thinkers and artists of different kinds who are interested in the philosophy of Deleuze. First of all, there is much more to Deleuze's books than just an ontology of processes and an epistemology of problems. He made contributions to such diverse subjects as the nature of cinema, painting and literature, and he held very specific views on the nature and genesis of subjectivity and language. For better or for worse, these are the subjects that have captured the attention of most readers of Deleuze, so it will come as a surprise that I will have nothing to say about them. Nevertheless, if I manage to reconstruct Deleuze's world these other subjects should be illuminated as well, at least indirectly: once we understand Deleuze's world we will be in a better position to understand what could cinema, language or subjectivity be in that world.

On the other hand, if this reconstruction is to be faithful to Deleuze's world it is clear that I must rely on an adequate interpretation of his words. There is a certain violence which Deleuze's texts must endure in order to be reconstructed for an audience they were not intended for, so whenever I break with his own way of presenting an idea I explain in detail the degree of rupture and the reason for it in a footnote. A different kind of violence is involved in wrenching his ideas from his collaboration with Félix Guattari. In this reconstruction I use Deleuze's ontology and epistemology as exposed in his early texts, and use only those parts of his collaborative work which can be directly traced to those early texts. For this reason I always ascribe the source of those ideas to him, using the pronoun "he" instead of "they" even when quoting from their joint texts. Finally, there is the violence done to Deleuze's fluid style, to the

way he fights the premature solidification of a terminology by always keeping it in a state of flux. Fixing his terminology will seem to some akin to pinning down a live butterfly. As an antidote I offer an appendix where I relate the terms used in my reconstruction to all the different terminologies he uses in his own texts and in his collaborative work, setting his words free once again after they have served their purpose of giving us his world. The hope is that this world will retain all its openness and divergence, so that the intense expressivity and even madness so often attributed to Deleuze's words may be seen as integral properties of the world itself.



# Chapter 1

## The Mathematics of the Virtual: Manifolds, Vector Fields and Transformation Groups

Of all the concepts which populate the work of Gilles Deleuze there is one that stands out for its longevity: the concept of *multiplicity*. This concept makes its appearance in his early books and remains one of central importance, with almost unchanged meaning and function, until his final work.<sup>1</sup> Its formal definition is highly technical, including elements from several different branches of mathematics: differential geometry, group theory and dynamical systems theory. In this chapter I will discuss the technical background needed to define this important concept but some preliminary informal remarks will prove helpful in setting the stage for the formal discussion. In the first place, one may ask what role the concept of a multiplicity is supposed to play and the answer would be a replacement for the much older philosophical concept of an *essence*. The essence of a thing is that which explains its *identity*, that is, those fundamental traits without which an object would not be what it is. If such an essence is shared by many objects, then possession of a common essence would also explain the fact that these objects *resemble* each other and, indeed, that they form a distinct *natural kind* of things.

Let's take one of the most traditional illustrations of an essence. When one asks what makes someone a member of the human species the answer may be, for example, being a "rational animal". The exact definition of the human essence is not what is at issue here (if rationality and animality are not considered to be essential human properties some other set will do). The important point is that there be some set of defining characteristics, and that this set explain both the identity of the human species and the fact that particular members of the species resemble each other. In a Deleuzian ontology, on the other hand, a species (or any other natural kind) is not defined by its essential traits but rather by the *morphogenetic process* that gave rise to it. Rather than representing timeless categories, species are historically constituted entities, the resemblance of their members explained by having undergone common processes of natural selection, and the enduring identity of the species itself guaranteed by the fact that it has become reproductively isolated from other species. In short, while an essentialist account of species is basically static, a morphogenetic account is inherently dynamic. And while an essentialist account may rely on factors that transcend the realm of matter and energy (eternal archetypes, for instance), a morphogenetic account gets rid of all *transcendent* factors using exclusively form-generating resources which are *immanent* to the material world.

Animal and plant species are not, of course, the only natural kinds traditionally defined by essences. Many other natural kinds, the chemical elements or the set of elementary particles, for example, are also typically so defined. In each of these cases we would need to replace timeless categories by historical processes. Yet, even if successful this replacement would take us only half-way towards our goal. The reason is that even if the details of a given process account for the resemblance among its products, the similarities which make us classify them as members of the same kind, there may be *similarities of process* which still demand an explanation. And when accounting for these common features we may be tempted to reintroduce essences through the back door. These would not be essences of objects or kinds of objects, but essences of processes, yet essences nevertheless. It is in order to break this vicious circle that multiplicities are introduced. And it is because of the tenacity of this circle that the concept of multiplicity must be so carefully constructed, justifying each step in the construction by

the way it avoids the pitfalls of essentialism. To anticipate the conclusion I will reach after a long and technical definitional journey: multiplicities specify *the structure of spaces of possibilities*, spaces which, in turn, explain the regularities exhibited by morphogenetic processes. I will begin by defining an appropriate notion of “space”, a notion which must not be purely geometrical but also capable of being linked to questions of process.

The term “multiplicity” is closely related to that of “manifold”, a term which designates a geometrical space with certain characteristic properties. To grasp what is special about manifolds (and what resources this concept can offer to avoid essentialism) it will be useful to give a brief account of its historical origins. Although the use of geometrical procedures for the solution of problems is an ancient practice inherited from the Greeks, the extensive use of curves and trajectories in the formulation of a variety of physical problems from the sixteenth century on made it necessary to develop new problem-solving resources. With this in mind, René; Descartes and Pierre de Fermat invented the now familiar method of embedding curves into a two-dimensional space on which arbitrary axes could be fixed. Once so embedded, the fixed axes allowed the assignment of a pair of numbers, or coordinates, to every point of the curve, so that the geometric relations between points could now be expressed as relations between numbers, a task for which the newly developed algebra was perfectly suited. This translation scheme, in short, allowed the combinatorial resources of algebra to be brought to bear on the solution of geometrical problems.

The term “manifold” does not belong to the analytical geometry of Descartes and Fermat, but to the *differential geometry* of Friedrich Gauss and Bernhard Riemann, but the basic idea was the same: tapping into a new reservoir of problem-solving resources, the reservoir in this case being the differential and integral calculus. In its original application the calculus was used to solve problems involving relations between the changes of two or more quantities. In particular, if these relations were expressed as a *rate of change* of one quantity relative to another, the calculus allowed finding the instantaneous value for that rate. For example, if the changing quantities were spatial position and time, one could find instantaneous values for the rate of change of one relative to the other, that is, for velocity. Using this idea as a resource in geometry involved the realization that a geometrical object, a curved

line or surface, for instance, could also be characterized by the rate at which some of its properties changed, for example, the rate at which *its curvature* changed between different points. Using the tools of the calculus mathematicians could now find “instantaneous” values for this rate of change, that is, the value of the curvature at a given infinitesimally small point.

In the early nineteenth century, when Gauss began to tap into these differential resources, a curved two-dimensional surface was studied using the old Cartesian method: the surface was embedded in a three-dimensional space complete with its own fixed set of axes; then, using those axes, coordinates would be assigned to every point of the surface; finally, the geometric links between points determining the form of the surface would be expressed as algebraic relations between the numbers. But Gauss realized that the calculus, focusing as it does on infinitesimal points on the surface itself (that is, operating entirely with local information), allowed the study of the surface *without any reference to a global embedding space*. Basically, Gauss developed a method to implant the coordinate axes on the surface itself (that is, a method of “coordinatizing” the surface) and, once points had been so translated into numbers, to use differential (not algebraic) equations to characterize their relations. As the mathematician and historian Morris Kline observes, by getting rid of the global embedding space and dealing with the surface through its own local properties “Gauss advanced the totally new concept that *a surface is a space in itself*”.<sup>2</sup>

The idea of studying a surface as a space in itself was further developed by Riemann. Gauss had tackled the two-dimensional case, so one would have expected his disciple to treat the next case, three-dimensional curved surfaces. Instead, Riemann went on to successfully attack a much more general problem: that of  $N$ -dimensional surfaces or spaces. It is these  $N$ -dimensional curved structures, defined exclusively through their intrinsic features, that were originally referred to by the term “manifold”. Riemann’s was a very bold move, one that took him into a realm of abstract spaces with a variable number of dimensions, spaces which could be studied without the need to embed them into a higher-dimensional  $(N+1)$  space. As Morris Kline puts it: “The geometry of space offered by Riemann was not just an extension of Gauss’s differential geometry. It reconsidered the whole approach to the study of space.”<sup>3</sup> And we could add that this new way of *posing*

*spatial problems* would, a few decades later in the hands of Einstein and others, completely alter the way physicists approached the question of space (or more exactly, of spacetime).

A Deleuzian multiplicity takes as its first defining feature these two traits of a manifold: its variable number of dimensions and, more importantly, the absence of a supplementary (higher) dimension imposing an extrinsic coordinatization, and hence, *an extrinsically defined unity*. As Deleuze writes: “Multiplicity must not designate a combination of the many and the one, but rather an organization belonging to the many as such, which has no need whatsoever of unity in order to form a system.”<sup>4</sup> Essences, on the other hand, do possess a defining unity (e.g. the unity of rationality and animality defining the human essence) and, moreover, are taken to exist in a transcendent space which serves as a container for them or in which they are embedded. A multiplicity, on the other hand, “however many dimensions it may have, . . . never has a supplementary dimension to that which transpires upon it. This alone makes it natural and immanent.”<sup>5</sup> It may be objected that these are purely *formal* differences between concepts, and that as such, they do not necessarily point to a deeper ontological difference. If we are to replace essences as the explanation of the identity of material objects and natural kinds we need to specify the way in which multiplicities relate to the physical processes which generate those material objects and kinds.

Achieving this goal implies establishing a more intimate relation between the geometric properties of manifolds and the properties which define morphogenetic processes. The resources in this case come from the theory of dynamical systems where the dimensions of a manifold are used to represent properties of a particular physical process or system, while the manifold itself becomes *the space of possible states* which the physical system can have.<sup>6</sup> In other words, in this theory manifolds are connected to material reality by their use as *models* of physical processes. When one attempts to model the dynamical behaviour of a particular physical object (say, the dynamical behaviour of a pendulum or a bicycle, to stick to relatively simple cases) the first step is to determine the number of relevant ways in which such an object can *change* (these are known as an object’s *degrees of freedom*), and then to relate those changes to one another using the differential calculus. A pendulum, for instance, can change only in its position and momentum, so it has

two degrees of freedom. (A pendulum can, of course, be melted at high temperatures, or be exploded by dynamite. These are, indeed, other ways in which this object can change, they simply are not relevant ways from the point of view of dynamics.) A bicycle, if we consider all its moving parts (handlebars, front wheels, crank-chain-rear-wheel assembly and the two pedals) has ten degrees of freedom (each of the five parts can change in both position and momentum).<sup>7</sup>

Next, one maps each degree of freedom into one of the dimensions of a manifold. A pendulum's space of possibilities will need a two-dimensional plane, but the bicycle will involve a ten-dimensional space. After this mapping operation, the state of the object at any given instant of time becomes a *single point* in the manifold, which is now called a *state space*. In addition, we can capture in this model an object's *changes of state* if we allow the representative point to move in this abstract space, one tick of the clock at a time, describing a curve or trajectory. A physicist can then study the changing behaviour of an object by studying the behaviour of these representative trajectories. It is important to notice that even though my example involves two objects, what their state space captures is not their static properties but the way these properties change, that is, *it captures a process*. As with any model, there is a trade-off here: we exchange the complexity of the object's changes of state for the complexity of the modelling space. In other words, an object's instantaneous state, no matter how complex, becomes a single point, a great simplification, but the space in which the object's state is embedded becomes more complex (e.g. the three-dimensional space of the bicycle becomes a ten-dimensional state space).

Besides the great simplification achieved by modelling complex dynamical processes as trajectories in a space of possible states, there is the added advantage that mathematicians can bring new resources to bear to the study and solution of the physical problems involved. In particular, *topological resources* may be used to analyse certain features of these spaces, features which determine *recurrent or typical behaviour* common to many different models, and by extension, common to many physical processes. The main pioneer of this approach was another great nineteenth-century mathematician, Henri Poincaré. Poincaré began his study not with a differential equation modelling a real physical system, but with a very simple equation, so simple it had no physical