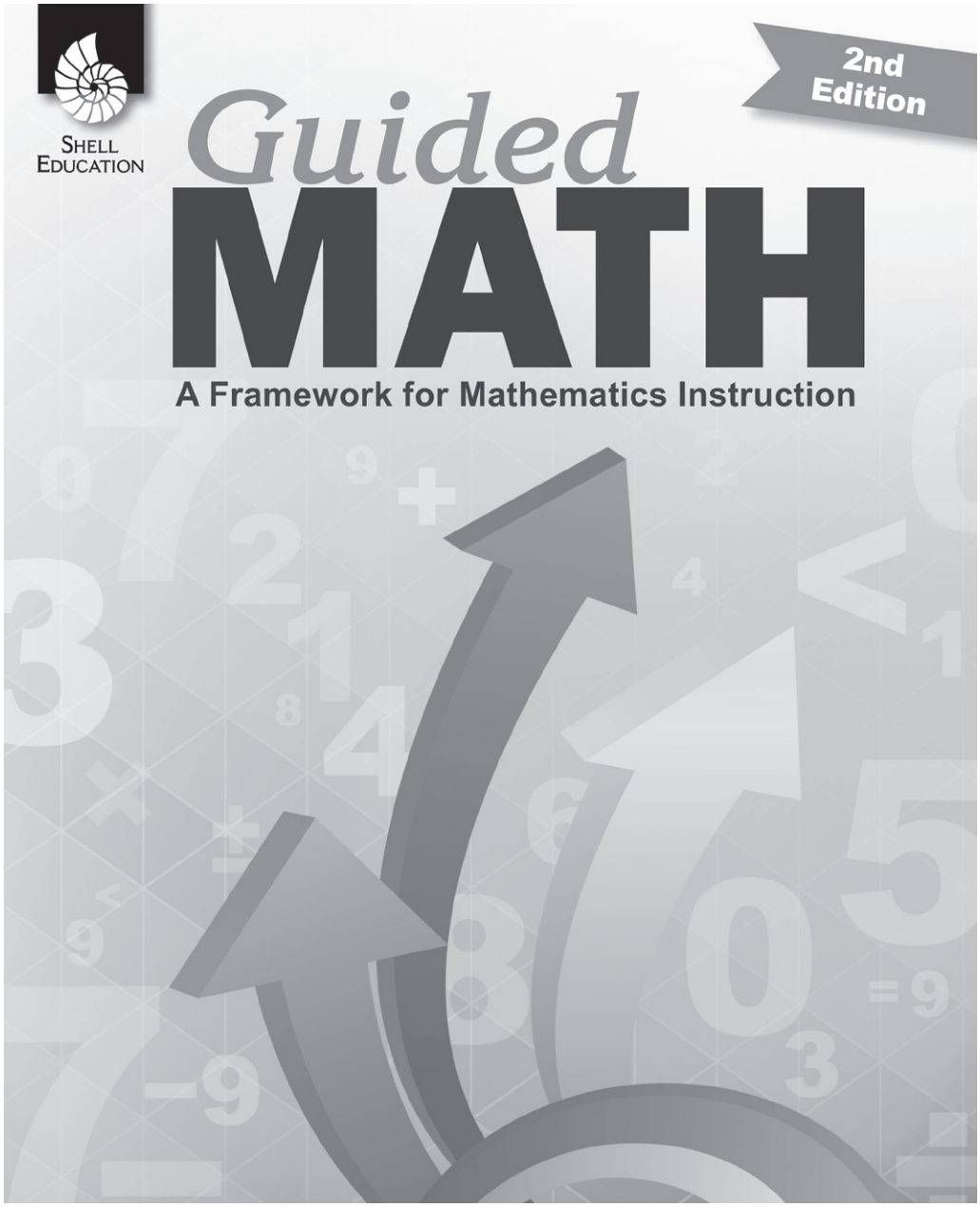
SHELL EDUCATION

2nd Edition

Guided MATH

A Framework for Mathematics Instruction

Laney Sammons
Foreword by Donna Boucher



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Foreword

In 2005, I was teaching 5th grade math at a school in the suburbs of Houston. I had taught 3rd and 4th grade before moving to 5th grade, so I was familiar with gaps in the mathematical understanding of my students at these grade levels. After several years teaching 5th grade, I realized that with each successive year, the achievement gaps I saw in my students were growing deeper and wider. Once students fell behind, the gaps rarely closed with the traditional, whole-class instructional setting. I was remediating 2nd grade mathematical skills while trying to teach 5th grade concepts and standards. Without a strong foundation, the new concepts made no sense to students. By 5th grade, students who struggle in math have likely been struggling for most of their elementary careers. I experienced the heartbreaking consequences of that failure on a daily basis. Some students tried to mask their failure by acting out, while others just withdrew and accepted defeat.

I realized that I needed to change the way I taught to meet the needs of my students. Clearly, a "one-size-fits-all" type of instruction was not getting students where they needed to be. In fact, this approach was likely to blame for them being in the predicament they were currently in. I searched for books to help me restructure my class to better meet the needs of my students, but to be honest, there just wasn't much available. So, I experimented. While I primarily still used whole-group instruction, I began using small-group lessons as part of my math block. Books on differentiation helped me create tiered tasks for my math centers. Over the next five years, through trial and error, I cobbled together an effective classroom structure. My students flourished. For the first time in years, they were experiencing success!

Five years later, I stepped onto a campus in an instructional coach role. I thought I had dealt with mathematical gaps at my previous school, but the new campus was an eye-opening experience. Kindergarten students had vastly different experiences prior to entering school, and the range of readiness was dramatic. With a large percentage of students functioning below grade level, we had our work cut out for us. The philosophy of our administrators was that

students learn best by doing, and hands-on experiences were valued over worksheets and textbooks. Our teachers used Lucy Calkins's Reading and Writing Workshop approach for literacy, so I began working with the teachers to implement a similar approach to math while incorporating what I had done previously. Moving toward small-group lessons was a huge paradigm shift. As a coach, I once again found myself looking for resources to help support their growth and understanding.

Thankfully, that was the year Laney Sammons's original *Guided Math* book was published. My first reaction was a selfish one, wondering where this book had been when I needed it! I immediately recognized how helpful the book would be for our teachers as they made the transition from only using whole-group instruction in mathematics. I kicked off a campus book study, and our journey toward becoming a Guided Math campus began. Each year, more teachers and teaching teams embraced the framework as they witnessed the great math instruction that was taking place in these Guided Math classrooms. It was an exciting and rewarding experience.

The changes in this new edition are exciting! For each of the components of the Guided Math framework, Laney has included new and practical details for implementation. Some of these suggestions include games to allow students to interact with the math word wall, strategies for facilitating math discussions during math stretches, options for different math workshop models, and a 15-day plan for implementing Guided Math. Teachers will find this new edition provides practical information to make the transition to Guided Math feel less daunting. User-friendly features, such as chapter snapshots and reflection opportunities, support teacher learning and are useful for book study groups.

If I had to choose a new favorite chapter, it would be the chapter on small-group lessons. The planning and implementation of small-group lessons is often the biggest hurdle teachers face when moving away from whole-class instruction. This new chapter not only provides additional resources for planning and implementing small-group lessons, but it also reflects a fundamental shift in thinking about the best way to use small-group lessons within the Guided

Math framework. When the first edition was published, small-group lessons were primarily used for differentiation following whole-group instruction. Experience has shown, however, that the most effective use of small-group lessons is to present new concepts. Step-by-step directions for planning small-group lessons are included along with samples of planning tools and examples of full lessons. This is sure to take much of the mystery out of teaching in small groups.

I began blogging in 2012, often citing Laney's work and creating resources to support the Guided Math framework. Each additional book in the Guided Math suite found a place on my bookshelf. Laney's body of work has had a profound impact on me personally and on teachers and students across the country and internationally. I crossed paths with Laney at the National Council of Teachers of Mathematics Annual Conference in 2014. That was the beginning of our personal and professional relationship. Over the past four years, we have collaborated on conferences, presented Guided Math institutes and workshops together, and co-authored a book called *Guided Math Workshop*. Since the publication of the original book, Laney has worked extensively with teachers, schools, and districts implementing Guided Math. The knowledge she gained from those boots-on-the-ground experiences makes this second edition an even more valuable resource for teachers trying to navigate the changing landscape of mathematics instruction. Whether you are replacing your dog-eared original copy or just starting out on your Guided Math journey, this book is a must have!

Donna Boucher
Co-author of *Guided Math Workshop*

Preface to the Second Edition

Without a doubt, most people join the teaching profession because they care deeply about young people and place great value on education. I am no exception. In 1989, when my younger child entered first grade, I tentatively entered a kindergarten classroom as a rookie teacher. I don't know who was more nervous—the 18 youngsters who were beginning school for the first time or me.

The vast differences among my students became apparent to me very quickly. One of my students did not know what crayons were. She had never held a pencil before. Several children had attended a preschool focused on traditional kindergarten skills. They knew their numbers through 10 and they could write their names, as well as several sight words, beautifully. Another child, before he had even put his jacket in his cubby, curled up on the floor in deep slumber. Some children had a strong sense of story and loved listening to read-alouds. Others were unable to follow and comprehend simple stories. A few students had difficulty getting along with their peers. Almost all of them struggled initially with the idea of doing things together as a class rather than following their individual interests. As I led lessons and tried to establish smooth transitions, it often felt as if I were trying to carry a large load of laundry. One sock would fall to the floor, and as I picked it up, another would drop. And so it was with my students. Just as one student was coaxed back into the group, another would head off to pursue something that had just attracted their attention.

The differences I observed in my kindergarten class that year are mirrored in classrooms throughout the country in elementary, middle, and high schools. Fortunately for me, there was a very talented and experienced full-time teaching assistant in my classroom. She was able to provide support and scaffolding for me as a rookie teacher, and she and I were able to do the same for my "rookie" students. Because we worked with these students in small groups and differentiated instruction for them, they were successful and ready to move on to the first grade.

I've often heard the expression, "Everything I need to know for life I learned in kindergarten." I might change that slightly to say, "Everything I need to know to teach I learned in kindergarten." My years of teaching kindergarten taught me how important it is to find out all you possibly can about your students and to adjust your instruction to meet their ever-changing needs. Rather than becoming frustrated teaching one-size-fits-all lessons, my teaching assistant and I spent lots of time supporting below-level learners, prodding reluctant learners, piquing the curiosity of adventurous learners, and praising the accomplishments of all learners as we guided them toward the next steps of their learning.

In the subsequent years, I moved to other schools and grade levels. Although the needs of the students in each grade I taught were as diverse as those in my first kindergarten class, the support of a full-time teaching assistant was no longer available. I faced the daunting task of trying to help my students achieve academic success while recognizing that they each had unique needs. Whole-class instruction made this extremely difficult. I knew how students' needs could be met with a teaching assistant in the classroom. How could I adapt that knowledge to make it feasible to implement in my classroom without any assistance? My colleagues were facing the same challenges. Together, we devised plans for working with small groups of students while the rest of the class was engaged in independent work. Unit pretesting gave us an idea of our students' proficiencies so we could create need-based groups. Generally, the three small groups we created each worked with the teacher for 20 minutes of the 60-minute class period. The lessons we planned were designed to respond to what we had identified as their needs. We began to feel a little better about addressing the needs of our students. They responded by showing much greater interest and by making gains in their achievement. But we were still novices at this game.

Eventually, my school implemented Guided Reading based on the model developed by Irene Fountas and Gay Su Pinnell (1996, 2001). As we applied this teaching framework to language arts instruction, it became apparent to me that this model was an effective method for differentiating instruction to meet the needs of my learners. And, with

some tweaking, I began to apply it to my math instruction, too. This method allowed me to teach each group at an instructional level that maximized its learning, and by using the approach with both language arts and mathematics, it was easy to establish and teach consistent routines and procedures to students. Soon, some of my colleagues and I began using what we now call Guided Math.

Later, as an instructional coach, I worked with our entire staff to make the transition to teaching math using the Guided Math framework. From my communication with teachers across the country, it is obvious that many of them are beginning the process of implementing their own versions of Guided Math. It is my hope that this book will provide some encouragement and guidance to those teachers. There is no one "right" way to use Guided Math. By sharing our ideas, we can help each other implement Guided Math in ways that work best with our teaching styles and in our classrooms.

In the almost 10 years since *Guided Math: A Framework for Mathematics Instruction* was first published, it has been gratifying to see teachers throughout the United States and Canada enthusiastically embrace the framework. Over the years, I have been privileged to work closely with many teachers as they implemented Guided Math in their classrooms and adapted it to make it their own. I am fortunate to have had these grassroots experiences. From them, I have learned much. I have also learned from the many excellent math educators who generously shared their ideas and experiences in articles, books, conference presentations, and on social media. Not surprisingly, my vision for Guided Math has evolved over the years. The time has come to share these ideas with others in a revised second edition of *Guided Math*.

This chart describes some of the revisions you will find in this book:

An expanded grade-level recommendation

As the framework has been implemented in elementary and middle schools, many high school teachers are now discovering it. They recognize its practicality in meeting the learning needs of all students and providing a format that enables greater student engagement. This edition is written for all those who teach mathematics, not just kindergarten through eighth grade teachers.

The inclusion of updated research

Much has been written about mathematics education in the past 10 years. Many new ideas have arisen from continued research on the most effective ways to teach math. Some of those ideas have been incorporated into this book. For those of you who, like me, enjoy keeping abreast of the latest in mathematical education articles and books, look to the References Cited section of this book (pages 366–373) for my most recent research sources.

A variety of ideas for creating an environment of numeracy in the classroom and for planning engaging and instructionally valuable math warm-ups

Additional suggestions for these components of the Guided Math framework are described based on real-life classroom experiences.

An expanded and more in-depth chapter on planning and teaching small-group lessons

Small-group instruction is at the heart of Guided Math. You will find strategies for identifying the gaps in knowledge and skills that students need to be successful in learning new content. While identifying gaps is important, it is even more important for teachers to take effective steps to fill those gaps. The small-group lesson structure allows teachers to plan a single lesson with options for differentiation that address the needs of students who lack the prerequisite knowledge and skills as well as those of students who may require additional challenges.

Specific guidance in establishing a manageable math workshop including designing workstations

An effective math workshop structure is a hallmark of the Guided Math framework. It allows the teacher instructional flexibility as their students work independently. Students may review previously mastered material for greater understanding and retention, work on computational fluency, or engage in mathematical investigations. Working independently in heterogeneous groups, learners explore mathematics together. Chapter 6 on math workshop describes workshop models and workstation tasks.

Additional insights on one-on-one math conferences with students

These student/teacher conversations offer valuable glimpses into students' mathematical thinking and help teachers target specific learning needs. This second edition offers strong guidance in implementing this component of Guided Math.

Assessment options that support student learning

To make the most of the Guided Math framework, assessment data is essential before, during, and after new mathematical content is taught. Chapter 8 focuses on multiple methods of assessment to identify learning needs.

A First Fifteen Days plan for introducing Guided Math in the classroom

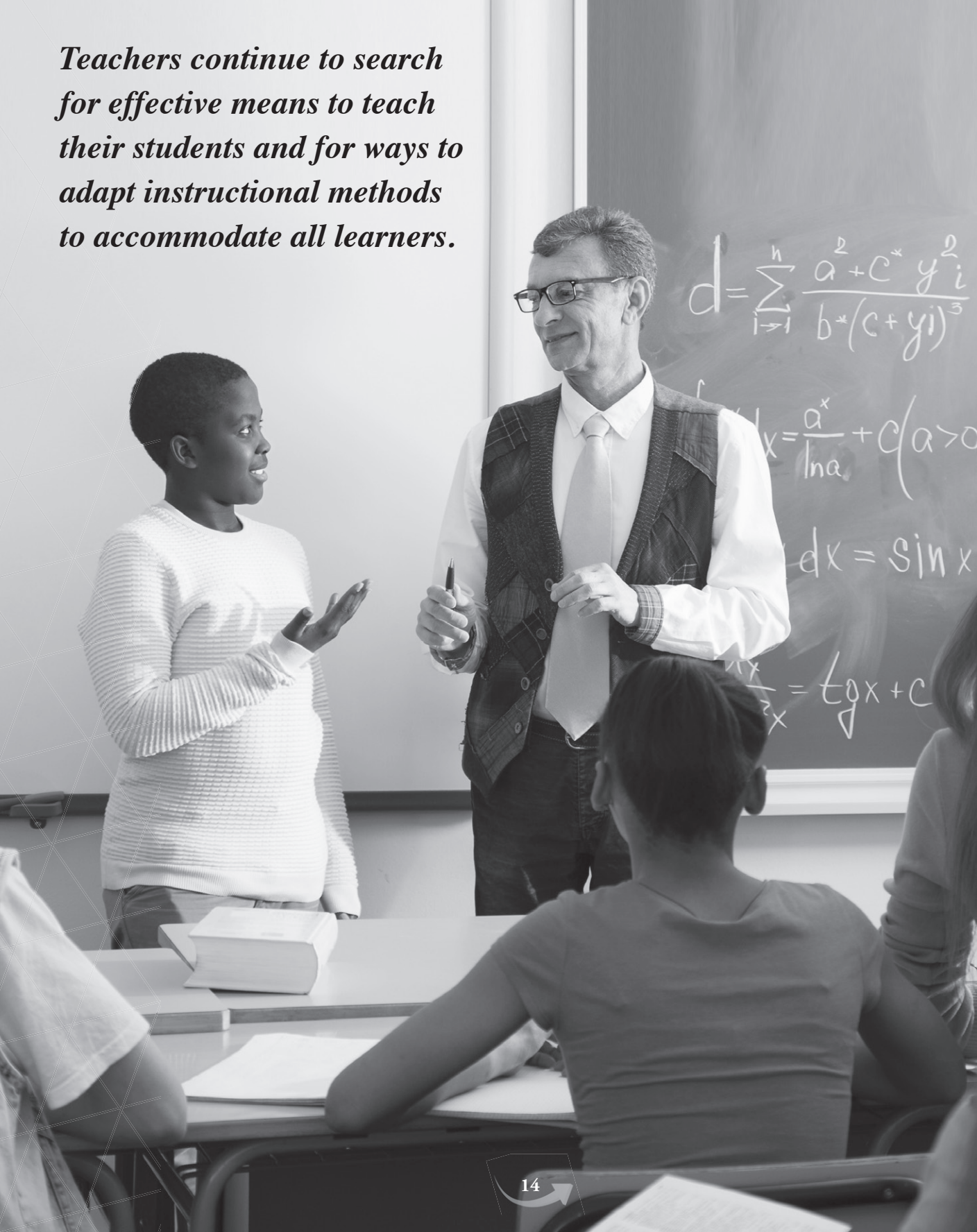
Practical routines and procedures must not only be established but also taught to students prior to implementing Guided Math if its implementation is to be successful. Included in this edition is a 15-day plan that walks teachers through this vital step.

As always, I am eternally grateful to my entire family for their ongoing and enthusiastic support of my work. Although it has been a great inconvenience to him at times, my husband Jack has been understanding and has gone way beyond what might reasonably be expected in assuming additional household responsibilities. My thanks and love to him and my whole family.

For the new ideas in this book, I owe a debt of gratitude to the many math educators from whom I continue to learn so much. To those of you who are in classrooms working with young mathematicians daily, those of you who are coaches who continue to educate yourselves and share new ideas with teachers, and those of you who are administrators who provide support to those teachers who are willing to try new ideas to make their instruction the best it can be: thank you. I am also grateful for the researchers who are studying how to make mathematical education for students challenging, rigorous, and effective. Thank you for sharing your knowledge through articles, books, podcasts, conference presentations, Twitter posts, and any other ways you use to spread the word. Speaking for the many of us who follow your inspiring work: thank you.

—Laney Sammons, 2019

Teachers continue to search for effective means to teach their students and for ways to adapt instructional methods to accommodate all learners.



Chapter 1

Guided Math: A Framework for Mathematics Instruction

Think back to your school days. Picture your math classes. What do you remember? Many of us recall instructions to get out our math books and open to a specified page. The teacher explains the lesson using the chalkboard or overhead projector. One or two students may be called on to solve problems at the board as the rest of the students practice at their desks. Some of us may remember using manipulatives in our early grades but probably not beyond second grade. Then finally, the teacher assigns problems from the book for classwork and homework. These assignments are later turned in, checked, and graded. Periodically, quizzes are given to check understanding. At the end of the chapter, a test is given. The teacher then moves on to the next chapter.

Was this method successful? For many of us, the answer is yes. The teacher-centered approach provided the instruction we needed. We applied this instruction to problems to be completed, and our understanding increased. If it didn't, we comforted ourselves with the knowledge that some people just don't have mathematical minds. We decided to make the most of our other skills. Many of us simply opted out of math classes as soon as we could. All too often, this was considered good enough. Students either "got it" or they didn't. Their grades indicated how well they "got it." Unfortunately, too many of us didn't "get it."

Mathematical literacy has been, and continues to be, a serious problem in the United States (U.S. Department of Education 2008). In 2007, research indicated that 78 percent of adults could not explain how to compute the interest paid on a loan, 71 percent could not calculate miles per gallon on a trip, and 58 percent could not calculate a 10 percent tip for a lunch bill (Phillips 2007). According to the U.S. Department of Education's National Mathematics Advisory Panel, "there are persistent disparities in mathematics achievement related to race and income—disparities that are not only devastating for the individuals and families but also project poorly for the nation's future, given the youthfulness and high growth rates of the largest minority populations" (2008, 12).

Recent results from the National Assessment of Educational Progress (NAEP) show that only 40 percent of fourth grade students rated as proficient or advanced in 2017, while 20 percent ranked in the lowest level, below basic. The scores for eighth grade students are similar with 34 percent scoring as proficient or advanced and 30 percent scoring in the lowest performance level (National Center for Education Statistics 2004). Additionally, according to the 2015 results of the Programme for International Student Assessment, the United States placed 38 out of 71 countries in math. This ranking is behind countries such as Estonia, Vietnam, and Latvia, and below the average of the 35 members of the Organization of Economic Cooperation and Development that sponsors the test (Pew Research Center 2017).

We must change our mathematics instruction because too many of our students are falling behind. Unfortunately, many teachers are still using the traditional, whole-class instructional method in classrooms. Some teachers recognize the need for change from traditional instructional methods and are making those changes. However, the teacher-centered, large-group instruction model of teaching is still prominent in mathematics classrooms across the nation.

Because of the limitations of this method of instruction, students are often presented with the message that there is a particular way in which mathematics must be done—that there is only one right answer

and only one right way to find that answer. The emphasis is on learning a set procedure rather than on conceptual understanding. In his book *The Math Instinct*, Keith Devlin states, "The problem many people have with school arithmetic is that they never get to the meaning stage; it remains forever an abstract game of formal symbols" (2005, 248). As Arthur Hyde (2006) points out, by fourth or fifth grade, students seem to have lost the problem-solving skills they had when they began kindergarten. Lack of conceptual understanding handicaps many students as they face more difficult math challenges in the upper grades.

Rather than inspiring students to understand and make sense of math, current instructional methods too frequently focus on memorization and formalized procedures. This focus on memorization squelches the natural curiosity learners have about mathematics. To improve the quality of mathematics education, Jo Boaler urges educators to equip their students with a mathematical mindset so that they "approach math with the desire to understand it and to think about it, and with the confidence that they can make sense of it" (2016, 34). But unless traditional instructional methods change, teachers will continue to struggle to teach mathematics as a "flexible conceptual subject that is all about thinking and sense-making" (35).

Furthermore, the traditional methods for teaching math offer few options for effectively addressing the diverse learning needs of students. As Jennifer Taylor-Cox so aptly describes: "We aim for the middle and pray for ricochet. We hope the knowledge we impart to the center will bounce around until everyone gets it" (2008, 1). Students who don't "get it" fall further and further behind as teachers struggle to find the time to help them. Teachers are frustrated trying to meet the needs of those students while continuing to challenge others who master the concepts quickly. Some students complain of being bored while others fail miserably in understanding the content being taught. It is easy for teachers to feel caught in the middle of a tug-of-war game when trying to balance the needs of all learners. With the ever-increasing diversity of students in classrooms today, it has become evident that students' mathematical success hinges on teachers' ability to differentiate instruction so that all learners are both supported and challenged as they work to master the required curricular standards (Sammons 2013).

The frustrations felt by educators are only increased by the demands for accountability enacted by state and federal governments. School systems are struggling to eliminate the gaps in achievement between minority and majority students, between special education and general education students, and between students receiving free and reduced lunches and the rest of the student population. It is no longer acceptable to have a portion of our students underachieve in mathematics. Disappointingly, after several years of gaps in the scores of these groups slowly narrowing (according to NAEP assessments), the 2017 results show no further narrowing of these gaps, and they even show slight increases in the gaps since the 2015 assessments (Nation's Report Card n.d.).

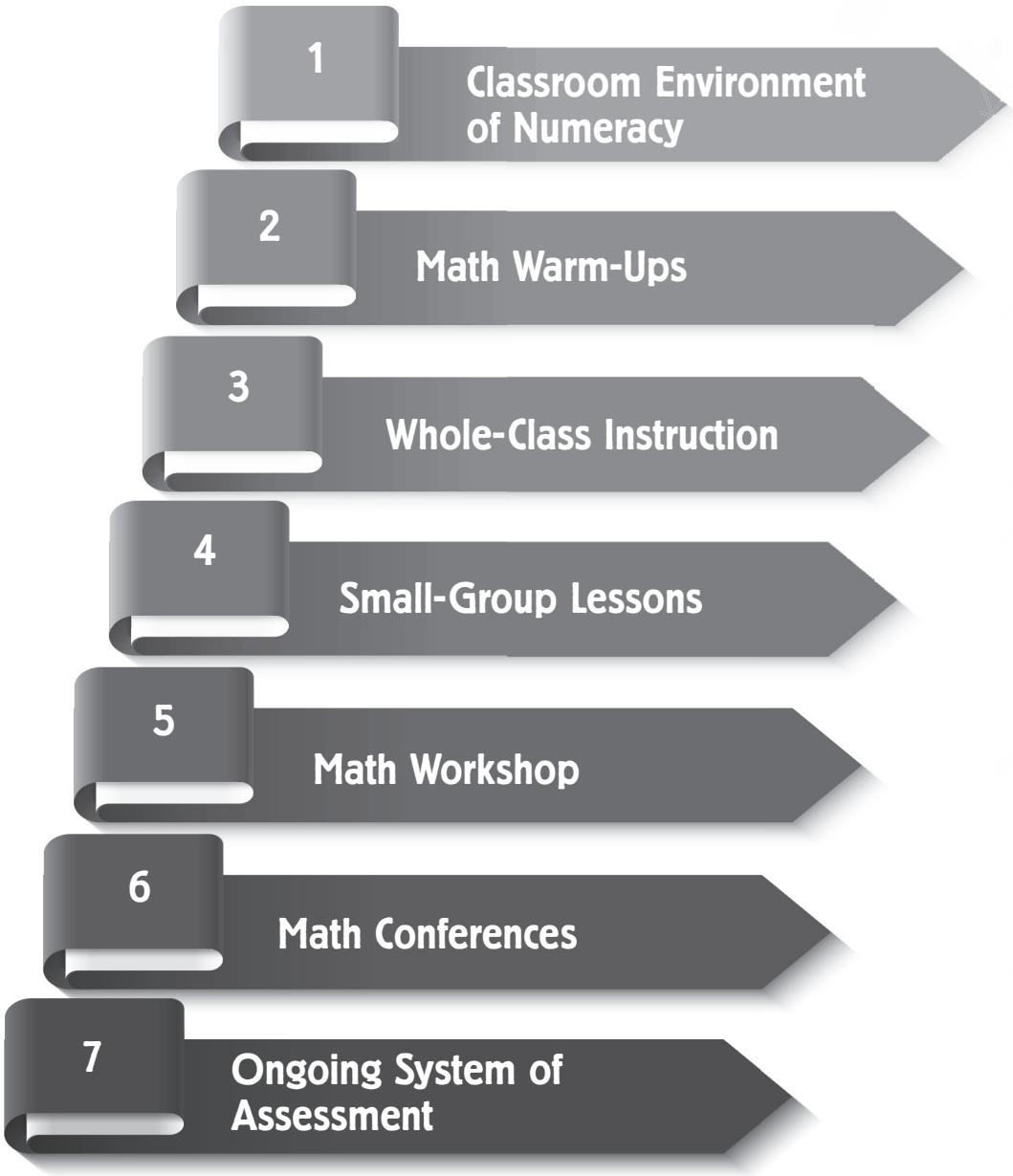
Driven by these pressures and their own professional desires to provide quality mathematical education, teachers continue to search for effective means to teach their students and for ways to adapt instructional methods to accommodate all learners. Making this task even more complicated is the fact that students who are slower learners for one concept in mathematics may very well be accelerated learners with other concepts.

As states have upped the ante with the adoption of more demanding math standards based on the standards developed by the National Council of Teachers of Mathematics (2000) and the Common Core State Standards (Common Core State Standards Initiative 2015), teachers have discovered that methods they have used successfully in the past are no longer effective. The demands of the more rigorous curriculum standards call for new ways of teaching.

As I grappled with these frustrations in my own classroom, I gradually developed a model that offers all students opportunities to develop their mathematical skills at increasingly challenging levels of difficulty with the ultimate goal of helping them gain the ability to function independently in the world of mathematics. I learned the importance of establishing and maintaining a classroom framework that is organized to support numeracy, just as teachers have done for literacy for many years.

Instructional Components of Guided Math

These are the instructional components of the framework:



Together, these components allow teachers to support each student's efforts at varying levels according to their immediate learning needs, to promote the development of mathematical mindsets, and to provide rigorous instruction.

Classroom Environment of Numeracy

Environments rich in mathematical opportunities for children are essential if we want children to develop a thorough understanding of mathematics. When students begin to recognize how numbers and problem solving affect their everyday lives, mathematics becomes more meaningful. Because learning is both a social and constructive process, students learn best through active engagement in authentic opportunities to use and extend their number sense.

The creation of a classroom environment supporting numeracy enables students to build on their previously acquired understanding of numbers. An organized mathematical support system for students requires that we encourage learners to use manipulatives, compute, compare, categorize, question, estimate, solve problems, converse, and write about their thinking processes. Ideally, a math-rich classroom environment and engaging activities will help students become increasingly aware of mathematical and problem-solving opportunities throughout their everyday lives, thus putting a "math curse" on students as authors Jon Scieszka and Lane Smith describe facetiously in their children's book of that title (1995).

The establishment of a community of learners is inherent within a classroom supporting the learning of mathematics. Students who feel respected and supported are willing to take risks in problem solving and in exploring mathematical ideas. They openly share their thinking with others and learn that mistakes are valuable learning experiences.

Through their engagement in mathematical dialogue, students construct the meaning of mathematics and develop enduring understandings of the big ideas or concepts while they acquire procedural and computational fluency. To establish a strong and flourishing learning community, teachers need to understand both their students and the mathematical "landscape of learning" through

which they hope to guide them (Fosnot and Dolk 2001). With careful planning and ongoing reflection, teachers are able to foster the strong social aspects that are integral for learning, to teach behaviors that promote constructive conversation, to organize the physical aspects of their classrooms for immersing students in an environment of numeracy, and to provide classroom procedures that encourage student participation in all components of the Guided Math framework. Chapter 2 focuses on ways to establish numeracy-rich environments within the classroom.

Math Warm-Ups

Math warm-ups are designed to take place first thing in the morning in self-contained classrooms or, alternatively, at the beginning of math class in departmentalized settings. These brief, focused math tasks set the tone for students' mathematical learning. Upon their arrival or as math class begins, students are asked to answer a question, consider a mathematical problem, or reflect on something they have previously learned.

Math stretches are one type of warm-up that teachers can easily implement. For example, students may be asked to add to a Number of the Day chart in which the teacher selects and writes a number at the top of a sheet of chart paper. Each student is expected to record the number in a different way on the chart. Ways of expressing the number will vary depending on grade level. Since the ways of writing the number will vary depending on the student, close observation of the chart will provide the teacher with valuable information about the number sense of individual students as well as an overall picture of the level of understanding of the entire class. More ideas regarding math stretches can be found in Chapter 3.

Number of the Day
4

K–2 Students

$3 + 1$ or $6 - 2$

3–6 Students

$(5 \times 4) - (2 \times 8)$ or 2^2

Secondary Students

$32 \div (-8 \times 2 \div -4^0) \times 2$

A calendar board may also be incorporated in this component of Guided Math provided that the tasks are not overly repetitious and are completed in a timely manner. These activities usually begin as students observe the calendar and date. From there, the teacher briefly reviews previously covered mathematical skills, previews upcoming skills, provides practice in rote counting skills or math facts, encourages mental math skills, and engages students in problem-solving activities. Often, students sit on the floor around the calendar board, but sometimes, students remain at their desks to participate in these oral or written activities. This daily warm-up encourages students to nimbly move from one area of mathematics to another in a nonthreatening, fast-paced way. The predominately oral nature of these activities promotes conversations about mathematical concepts. This fosters a deeper, more enduring understanding in students. Chapter 3 further explores instructional ideas for mathematical warm-ups.

Whole-Class Instruction

Many educators today are moving away from the traditional, teacher-directed method of instruction. However, this type of instruction can have a place in today's classroom, providing it isn't the only, or even primary, method of instruction.

Certainly, whole-class instruction requires the least amount of teacher preparation. In its most common form, the teacher introduces the lesson, teaches it as students listen and are questioned, provides a practice activity for students, and either summarizes the lesson or has students summarize it. Traditionally, students remain in their desks and face the teacher, who is at the front of the classroom. Most of us are familiar with this traditional method of teaching from our own time as students. Whole-class instruction remains an option within the Guided Math framework to be used when there is a compelling reason for it. The Guided Math framework offers teachers a variety of instructional methods to use instead of relying solely on whole-class instruction.

Whole-class instruction can be an excellent method for presenting activating strategies, for making literature connections at the beginning of lessons, or for reviewing mastered concepts. With this format, teachers may choose to present mini lessons or model problem-solving

strategies, thinking aloud as they do so. Moreover, this component can be used as a time for a "math congress" (Fosnot and Dolk 2001) or for a math huddle, as it is called in the Guided Math framework. In a congress or math huddle, students come together following mathematical investigations to share their discoveries or to discuss their mathematical thinking.

Teaching to the whole class is a very straightforward method of instruction, but it requires a remarkable amount of teacher skill to be done well. Although it often appears that discourse during this kind of instruction is "off the cuff," to be effective, teachers must juggle what they know about their students and the mathematical concepts on their "horizons" to guide conversations with meaningful questions. Even a skillful teacher may be unable to reach some students because of students' lack of attention, boredom, inability to understand the instruction, or their often incorrect confidence that they already know how to do the activity and therefore don't need to listen. Chapter 4 offers suggestions on how to use this instructional component of Guided Math most effectively and when it should be avoided.

Small-Group Lessons

Small-group lessons designed to teach new mathematical content while also targeting specific student mathematical learning needs are at the heart of the Guided Math framework. Teachers should first assess their students formally or informally and then group them according to their proficiencies in a given skill for small-group lessons. The groups are homogeneous yet fluid. As individual students' levels of understanding change and gaps in prerequisite knowledge and skills vary, so do the groups. This method of mathematics instruction is analogous to Guided Reading instruction as espoused by Irene Fountas and Gay Su Pinnell in their books *Guided Reading: Good First Teaching for All Children* (1996) and *Guiding Readers and Writers Grades 3–6* (2001).

Using Guided Math instruction, teachers can work with small groups whose composition is determined specifically by students' instructional needs. This allows teachers to closely observe student work, monitor student attention, provide strong support for below-level learners, and extend extra challenges for proficient learners.

With small-group lessons, teachers strive to move all students forward in the appropriate grade-level curricular progression while at the same time addressing gaps in knowledge and skills. For students with significant gaps, teachers briefly address those specifically and then move them into the current lesson with scaffolding that challenges them to successfully experience the new content being taught.

Using this model for small-group instruction, teachers have the option of varying the amount of time they spend with each group according to the specific needs of the students. For example, when a teacher is introducing a new concept, one group may quickly grasp the concept or skill and be able to move on to independent practice. Another group may need significantly more time working directly with the teacher in the small-group setting. Rather than boring those students who have already mastered the concept with continued whole-class instruction, this model encourages those students to move on to independent work quickly, freeing time for more intensive instruction with students who struggle with specific concepts.

Not only can the amount of instructional time differ but so can the content of the material addressed and the amount and level of difficulty of the practice work assigned. Guided Math groups offer teachers an efficient way to provide differentiated instruction to meet the needs of diverse learners. Chapter 5 examines, in greater depth, how to establish and effectively use small-group lessons for Guided Math.

Math Workshop

So, when the teacher meets with small groups or conducts one-on-one conferences with students, what are the other students doing? For small-group instruction and conferencing to be effective, it must be uninterrupted. Students who are not engaged directly with the teacher should have meaningful work to do and know how to follow established and practiced procedures for independent or group work. These students work independently in a math workshop.

As the school year begins, students are taught how to work independently. The teacher establishes expectations and routines for this work during the first few weeks of school. Students learn how to access materials they may need, how to follow rules for working with

manipulatives, how to handle any questions they may have, and what to do if they complete their assigned work. Periodically, the teacher may need to revisit these expectations.

Because each instructional minute of the day is so important, it is essential that each student is engaged in meaningful work. Providing something beyond busy work also helps prevent discipline problems because students who are working on challenging tasks are less likely to disrupt the class. At the same time, it is essential that the work be something that students can successfully complete independently. Math workstation tasks might consist of practice working with previously mastered content to deepen students' understanding, number sense activities that will help them improve their computational fluency, or math inquiries or investigations. In Chapter 6, math workshop is described in more detail with suggestions for establishing procedures and routines, planning workstation tasks, and effectively managing this essential component of the Guided Math framework.

Power Phrase

math workstation: *a task students complete independently that provides practice, promotes computational fluency, or encourages mathematical curiosity*

Math Conferences

The Guided Math framework offers teachers valuable opportunities to interact with students and observe communication between students as they work during small-group lessons. Sometimes one-on-one time with students is needed to aid the teacher in assessing student understanding of mathematical skills or concepts, clarifying or correcting student misunderstandings and errors, or extending and refining a student's understanding.

At any time, teachers can confer with individual students. In very much the same way that teachers have used one-on-one reading and writing conferences, these math conferences encourage learners to further their understandings of mathematical concepts. Because most students enjoy supportive individual attention from their teachers, math conferences also strengthen the relationships between students and teachers.

When one-on-one conferences are used effectively, teachers encourage students to reflect on their own understanding. Additionally, these conversations can generate rich information about how to best work with students. Teachers are able to identify specific teaching points for individual learners and for the class as a whole.

In Chapter 7, individual math conferences are covered in greater depth. A basic structure for individual conferences is presented, strategies for successful conferring are described, and methods for recording anecdotal notes are suggested.

Ongoing System of Assessment

How do teachers determine the mathematical learning needs of their students? How do they know how to group their students for effective small-group lessons? How do they determine individual mathematical learning needs so that they can differentiate instruction in a productive way?

Ongoing and accurate assessment provides teachers with timely information about individual and class needs. In mathematics instruction, a student's level of proficiency can vary drastically from concept to concept. This makes assessing mathematical knowledge and thinking skills more challenging than assessing reading ability, where periodic running records and comprehension questions provide a strong indication of reading level.

Teaching Guided Math entails much more than simply following a textbook chapter by chapter and assigning the same set of problems to the entire class. With instructional time being so valuable, it is important not to waste time teaching what students already know. It is also important to refrain from moving ahead page by page if students are struggling. So, how do Guided Math teachers determine student needs?

A balanced system of assessment offers teachers a complete picture of each child's understanding rather than just a glimpse from the results of a single test. Formative and summative assessments—including observations of students' work, discussions with students, and assessment of their finished products—all give valuable perspectives

on students' capabilities and needs. In addition, to maximize student learning, students themselves should be involved in assessing their own work based on established criteria, rubrics, or exemplars. Balanced assessment involves far more than just grades on tests and report cards. Chapter 8 examines the kinds of individual and class assessments that help teachers refine and extend their instruction to meet the needs of each student.

Guided Math in Practice

What does Guided Math look like in a real classroom? What would others see if they dropped in for a visit? Chapter 9 provides an overview of how the components of this approach come together and are applied. A single classroom teacher can implement Guided Math, but collaboration among teachers makes the process easier and richer. Teachers should consider the various components of Guided Math using the Menu of Instruction (see Figure 1.1 on page 28) as they begin implementation. It may be easier for some teachers to focus first on a few of the components and then gradually add more. Others may prefer to carefully plan and then implement the entire framework at once. There is no one right way to approach implementation.

The very nature of this approach to teaching mathematics allows it to be incorporated flexibly into daily schedules. The constant daily features are the environment of numeracy, math warm-ups, conferencing, and assessment. Teachers in self-contained classrooms may incorporate them throughout the day. During the specific block of time allotted for mathematics instruction, teachers may choose from whole-group instruction, small-group lessons, math workshop, or a combination based on the level of support needed by the class for the mathematical content being taught.

Figure 1.1—Guided Math Menu of Instruction

Daily: Classroom Environment of Numeracy—A classroom should be a community where students are surrounded by mathematics. This includes real-life math tasks, data analysis, math word walls, measurement tools, mathematical communication, class-created math anchor charts, graphic organizers, calendars, and evidence of problem solving.

Daily: Math Warm-Ups—These daily tasks set the tone for math instruction. They may include calendar activities, math stretches, problems of the week, data work, incredible equations, classroom responsibilities, reviews of skills to be maintained, and previews of math content to come.

Your Choice: Whole-Class Instruction—This component is effective when students are working at the same level of achievement or when introducing new content with an activating strategy. It may also be used for teacher modeling and think-alouds, read-alouds of math-related literature, math huddles, whole-class math games, reviews of previously mastered skills, setting the stage for math workshop, and administering written assessments.

Your Choice: Small-Group Lessons—Students are instructed in small groups whose composition varies based on learning needs. These lessons are excellent ways to introduce new concepts, practice new skills, work with manipulatives, provide intensive and targeted instruction to learners who have gaps in background knowledge or skills, introduce activities that will later become part of math workshop, conduct informal assessments, and reteach based on student needs. During this component, students can be actively engaged in the mathematical practices, working and communicating with each other as the teacher facilitates their mathematical construction of meaning.

Your Choice: Math Workshop—Students are provided with independent workstation tasks to complete individually, in pairs, or in cooperative groups. The work may consist of practicing previously mastered skills for greater understanding and retention, tasks to promote computational fluency, mathematical investigations, math games, math journals or other mathematical communication, or interdisciplinary work.

Daily: Conferencing—To enhance learning, teachers confer individually with students, informally assess their understandings, provide opportunities for one-on-one mathematical communications, and determine and deliver teaching points for individual students. Conferences can also help teachers identify previously unknown instructional needs that may become apparent during discussions with students.

Daily: Assessment—A balanced assessment system includes ample assessment for learning to inform instruction along with appropriate assessment of learning for each unit.

Levels of Teacher Support

Levels of teacher support vary according to the instructional approach chosen by the teacher. Figure 1.2 provides an outline of instructional approaches, teacher activities, and support levels that complement the approaches and activities.

Figure 1.2—Levels of Support for Guided Math Components

Instructional Approach	Teacher Activity	Level of Support
Whole Group	Activating strategies Modeling Think-alouds Mini lessons (only when there is a compelling reason for them) Leading students in formulating conjectures or math congresses Directed review	Full support for all students by the teacher specifying the approach to problem solving or guiding the conversation; responsibility on the teacher
Small Group	Facilitate student exploration of new concepts/skills Extend understanding of previously introduced concepts	Moderate level of support and targeted instruction based on student needs; more responsibility is released to students as the teacher facilitates and provides scaffolding for students who may have gaps in prerequisite knowledge and skills
Math Workshop	Prepare tasks and materials for workstations	Low level of support; tasks should be those which students can complete without teacher assistance; responsibility is shifted to students

Whole-Group Instruction: Full Level of Teacher Support

The ultimate goal of mathematics instruction is to lead students to a comprehensive understanding of mathematical concepts that they can then apply independently throughout their lives. The flexible nature of the Guided Math model allows for the gradual release of responsibility from teacher to students as described by P. David Pearson and Margaret C. Gallagher (1983). With this model, teachers are intentional about providing students with varying levels of instructional support. Ultimately, teachers enable their students to assume full responsibility for acting independently as mathematicians. This model is not the "I, We, You" model. In the "I, We, You" model, teachers simply show students a new procedure (I), lead the class in working together to apply the procedure (We), and then ask students to try it independently (You).

While the Guided Math "gradual release of responsibility" model begins with teachers doing most of the work during the whole-class instruction component of the framework, the focus at this time should not be on showing a set procedure to be replicated later by learners. Instead, teachers might lead students through an activating strategy to help them tap into their background knowledge and stimulate interest in a mathematical concept. Or, through modeling and think-alouds, teachers might share their thought processes as they engage in the mathematical practices. For example, in demonstrating how mathematicians approach problem solving, teachers might opt to model how they identify and clarify the information they have, determine whether or not they need to know more, consider what mathematical knowledge they already have that may help them solve the problem, visualize the problem, or review a variety of strategies to determine which might be most efficient to use.

This kind of modeling is vastly different from demonstrating a particular procedure that students will be expected to use without an understanding of the mathematical concepts. Rather than teachers using their time of maximum support to provide "students with a pre-packaged solution...right away and then...a bunch of (often contrived) problems which that solution can be used to solve" (Bledsoe 2014, n.p.), the time is used to develop their students' mathematical thinking and practices.

For this phase of the model to be most effective, it must be teacher-centered. Students listen, answer questions, and turn and talk with partners when requested. As a result, the teacher has minimal opportunity to monitor comprehension or communicate with most of the class. Therefore, it should be used sparingly and only with specific purposes in mind.

Small-Group Lessons: Moderate Level of Teacher Support

The next stage of the "gradual release of responsibility" model for Guided Math calls for students to take responsibility for learning concepts and skills as they work in small groups. After determining the prerequisite knowledge and skills that learners need for success with each lesson, teachers identify which, if any, students have gaps in those areas. When introducing new content, teachers can then provide the appropriate scaffolding to fill those gaps. Students receive just enough scaffolding to challenge them to move beyond their current independent capabilities while still holding them responsible for working diligently to construct new mathematical meaning from their learning experiences.

When teaching small-group lessons, it is especially important for teachers to understand the positive impact of productive struggle. Students must be allowed to grapple with the complexities of math. James Hiebert and Douglas Grouws describe mathematical tasks that lead to productive struggle as those that are "within reach but that present enough challenge, so there is something new to figure out" (2007, 388). To make small-group lessons as effective as possible, teachers must recognize the very fine distinction between productive struggle and unproductive frustration when they scaffold the learning of their students. In general, many teachers find it difficult to avoid stepping in to "help" before students really have a chance to wrestle with challenging problems. Too much scaffolding deprives students of valuable learning opportunities. As teachers, we must consider how we can "provide scaffolds, such as purposeful questioning, that honor and build on the thinking of students without removing the demands of the task or doing the thinking for them" (Huinker and Bill 2017, 213).

Small-group lessons are ideal for challenging students with rich, open-ended tasks using what is sometimes referred to as a "You, Y'all, We" structure (Green 2014; Bledsoe 2014; Rawding 2018). With this instructional model (see Figure 1.3), students are first asked to try to make sense of a given task on their own. This is the "You" phase. After students have had time to explore and grapple with the task independently, they work collaboratively on the task with a partner. In this "Y'all" phase, working in pairs, students share their thinking and continue their efforts to make sense of the task. Finally, in the "We" phase, the entire small group comes together to discuss their thinking, the connections they made, and their findings in a math conversation facilitated by the teacher.

Figure 1.3—Responsibilities in the You, Y'all, We Structure

	Teacher Responsibilities	Student Responsibilities
You	• Provide a rich, open-ended task. • Observe students closely as they work. • Confer with students to learn more about their mathematical reasoning. • Ask purposeful questions to spur deeper thinking.	• Individually explore the task. • Confirm understanding of task. • Make math connections. • Consider strategies to solve problems or to construct mathematical meaning.
Y'all	• Observe pairs of students as they work. • Monitor the working relationships of the student partners to encourage productive collaboration. • Confer with pairs of students to learn about their reasoning. • Ask purposeful questions to spur deeper thinking.	• Collaborate with a partner to further explore the task. • Share mathematical thinking about the task with a partner. • Brainstorm ideas for a solution or understanding. • Jointly devise a way to express how to solve the problem or explain new mathematical insights.

	Teacher Responsibilities	Student Responsibilities
We	• Maintain an environment in which students are comfortable sharing their mathematical thinking with others. • Facilitate a math conversation among students regarding the task. • Encourage student reflection. • Ensure the participation of all students. • Ask purposeful questions to spur deeper thinking. • Support students in generating a summary of the ideas shared or mathematical conclusions reached.	• Exhibit active listening. • Show respect for everyone and for the ideas shared. • Share the ideas generated individually and with a partner. • Consider carefully the ideas shared by others. • Express agreement or disagreement with ideas shared by others along with supporting evidence. • Actively collaborate with the group to arrive at a summary or conclusion.

The small-group format is a setting in which students are comfortable sharing their thinking and taking moderate risks with teacher support. Teachers are able to closely observe and listen to students talk—a valuable informal assessment opportunity that allows for immediate feedback, the identification of misconceptions, the delivery of brief teaching points, and the type of questioning that provokes deeper and more critical thinking by fledgling mathematicians.

Math Workshop: Low Level of Teacher Support

In the final stage of the Guided Math "gradual release of responsibility" model, students assume complete responsibility for their learning during math workshop. They work independently, individually, or in small groups on math workstation tasks. Before participating in this independent work, they should be very familiar with the procedures and expectations of the teacher and should also be able to carry out the assigned work with no additional guidance. It is during this time that they draw upon the mathematical concepts and skills they have already mastered and engage in tasks to improve computational fluency. Students might be involved in problem solving, investigations, games, math-related reading, mathematical writing, or other tasks to increase their independent mathematical competency.

The "gradual release of responsibility" model challenges students to assume more and more responsibility for their own conceptual understandings and problem-solving skills. They also become adept with mathematical practices and computational fluency. Since learning is rarely a completely linear process, the level of teacher support required for each student may vary from day to day and from lesson to lesson. Guided Math offers teachers an instructional framework that encourages students to gradually assume increasing responsibility as they learn, while at the same time providing scaffolding and support when needed.

Scheduling Guided Math Components

Most teachers who implement Guided Math in their classrooms feel that it is important to have small-group lessons and math workshop every day. Maintaining a regular routine is beneficial for all students. They know what to expect each day. Usually, three to four small-group lessons are conducted daily as other students work independently at math workstations. It is with this kind of consistency that the Guided Math framework is most effective.

For teachers just beginning implementation or who desire more options for their mathematics instruction, the framework offers flexibility. Figure 1.4 (page 36) shows how the components of Guided Math may be woven together for instruction during the week.

On Monday, the entire mathematics period is taught as a whole group as the class begins with an activating strategy. A problem is then presented, which the teacher solves by thinking aloud to explain the thought process—not a procedure—to students. Following the problem-solving activity, the independent work for the week is introduced with a mini lesson.

On Tuesday, the class begins with a read-aloud of mathematics-related literature as the whole class gathers to listen and then to discuss the mathematical connections. Following the read-aloud, students begin independent work in math workshop. The tasks were explained on Monday, so students should be ready to begin with little

additional direction. For the first 15 minutes of math workshop, the teacher circulates around the classroom, conferring with individual students. For the last 30 minutes, the teacher meets with Guided Math group 1. The lesson is tailored specifically to the needs of these students who have been grouped together because of their similar skills. They may be students who have already mastered what most of the students are currently working on, and they can be given more challenging instruction. Or they may be students who the teacher noticed have a particular problem that can be addressed easily through small-group instruction. Sometimes, they are students who need additional scaffolding and support with the concepts on which the class is working.

On Wednesday, the structure is similar to that of Tuesday. The class begins with a mini lesson, but this time, a problem is posed for the class to solve and discuss. Math workshop begins with the teacher initially engaged in conferencing. The teacher meets with Guided Math group 2 after conferencing, which is a group with different needs.

On Thursday, there is no mini lesson so that the teacher can meet with two groups. Students begin math workshop immediately. The teacher spends 15 minutes conferencing, followed by 30 minutes with Guided Math group 3. For the last 30 minutes of the class, the teacher meets again with group 1 from Tuesday for additional instruction.

On Friday, the schedule is quite different. Students meet for whole-class instruction as they participate in a math huddle. During this time, they share and discuss their observations, problem-solving strategies, conjectures, or representations with the class. The ideas shared are recorded in a chart or graphic organizer that is posted in the room for future reference.

Figure 1.4—Sample Flexible Guided Math Schedule

	Activity	Guided Math Component
Monday	• activating strategy • problem-solving think-aloud by teacher • explanation of independent work for the week (investigation, paper/pencil practice, games)	Whole Class
Tuesday	• mini lesson (read-aloud) • independent math work with teacher conferencing for the first 15 minutes • Guided Math with group 1	Whole Class Workshop Conferencing Small Group
Wednesday	• problem challenge mini lesson • independent math work with teacher conferencing for the first 15 minutes • Guided Math with group 2	Whole Class Workshop Conferencing Small Group
Thursday	• independent math work with teacher conferencing for the first 15 minutes • Guided Math with group 3 • Guided Math with group 1	Workshop Conferencing Small Group
Friday	• math huddle (students share their observations, problem-solving strategies, conjectures, representations) • create a chart or graphic organizer to post in the classroom for reference	Whole Class

This weekly plan is just one example of the kind of flexibility the framework offers teachers. When planning, it is important to consider the curriculum and the students to determine which of the components work best for each day of instruction.

The U.S. Department of Education's National Mathematics Advisory Panel (2008) suggests that research does not support the contention that mathematics instruction should be completely teacher-centered or student-centered. Instead, it should be "informed by high-quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers" (xiv). While small-group lessons are at the heart of Guided Math and are an essential characteristic of the framework, teachers are encouraged to use their professional judgment to structure mathematics instruction to meet the diverse needs of the students in their classes, curricular demands, and even their own teaching styles. Guided Math frees teachers from the one-size-fits-all model and empowers them to determine the best instructional strategies for each student, for the class, and for the concepts being taught each day.

Review and Reflect

Think of the way you currently teach mathematics.

1. What aspects of your instruction are the most successful?

2. What aspects of your instruction trouble you? Why? In what ways might you wish to change your instruction?

3. Does your math instruction lead your students to a deep conceptual understanding of the math standards that they are learning? If so, what are you doing that contributes to that? If not, how do you think you would like to change your teaching?