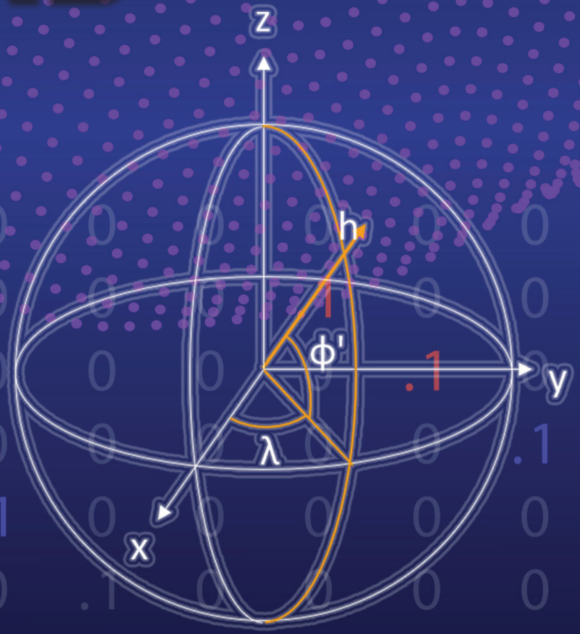
Practical Geolocation for Electronic Warfare Using MATLAB®



Nicholas A. O'Donoughue

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10 9 8 7 6 5 4 3 2 1

To my eternally patient wife

Contents

Preface

This text explores many (but not all!) of the practical realities that arise from the employment of geolocation for electronic warfare in real-world systems. These realities include the impact of a priori knowledge about the position of the target, errors in sensor position, orientation, or velocity, and the impact of repeated measurements over time.

We build upon the derivations and source code from *Emitter Detection and Geolocation for Electronic Warfare*, published in 2019. Where it is necessary, material from that text is included or summarized here, but it is recommended that readers first familiarize themselves with that text.

The companion software for both books can be downloaded from the publicly available Git repository.

The intended audience for this book includes engineers and electronic warfare practitioners who are interested in the development or employment of geolocation algorithms. As with my last text, the MATLAB® code is intended to provide a starting point for the development of novel algorithms in a way that allows for comparison to classical approaches.

Chapter 1

Introduction

Control of the electromagnetic spectrum (EMS) is critical for success on the modern battlefield. In 2020, then-Secretary of Defense, Hon. Mark Esper, wrote "These challenges [from peer and near-peer adversaries] have exposed the cross-cutting reliance of U.S. forces on the EMS" [1]. Later in that document, he introduced a significant organizational change in how the United States will approach EMS Operations (EMSO). These doctrinal and policy changes are not the focus of this text, but they do set the tone for the importance of EW, as a principal component of modern military operations.

In this text, we will focus on a subset of EW: the geolocation of sources of electromagnetic energy. What sets this apart from previous texts, such as *Emitter Detection and Geolocation for Electronic Warfare* [2], is our inclusion of realistic constraints and sources of uncertainty, which we will briefly describe in this chapter.

1.1 RECEIVER PROCESSING

We will not discuss in detail the reception of signals or their processing to detect transmissions of interest and to estimate key parameters. For clarity, this process is briefly reviewed here.

A sample receiver block diagram is shown in Figure 1.1. Note that there are many receiver architectures, with more or fewer components, than are shown here. This diagram is not meant to be definitive or exhaustive, but illustrative. It begins at the left edge with an aperture, followed by a series of filters and mixers that isolate the band of interest and convert it to a frequency that can be sampled. After digitization, the signal is fed through a detector to identify the presence of signals of interest, and a pulse parameter estimation block, to estimate critical pulse parameters, such as center frequency, chirp rate, and modulation type. At this point, most EW systems generate a *pulse descriptor word* (PDW) to describe the intercepted pulse.

The next step, typically, is to associate PDWs over time, in order to keep track of an emitter from pulse to pulse, and to estimate additional parameters, such as the

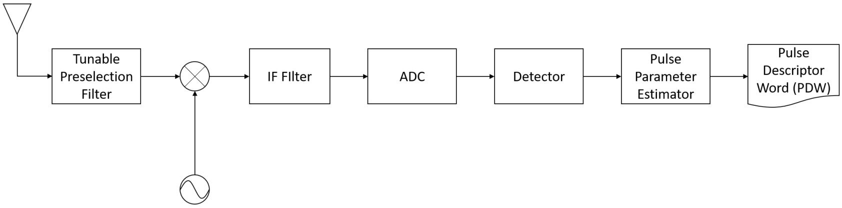

Figure 1.1 Block diagram of a notional receiver.

pulse repetition interval (PRI), duty cycle, or frequency hopping characteristics. At this point, a classifier may attempt to interpret the type of signal (communications, weather sensing, surveillance radar, etc.).

We will assume in this text that all of these steps have occurred already and that we are presented with some set of measurements (ζ) of underlying signal parameters ($\mathbf{z}$); our task is to perform geolocation of the source of those emissions to determine the coordinates ($\mathbf{x}$) from which they were transmitted.

For an in-depth discussion of receivers, including specific hardware choices and signal conditioning, interested readers are referred to Poisel's *Electronic Warfare Receivers and Receiving Systems* [3]. For a discussion of the detection and parameter estimation steps, see Parts I and II of *Emitter Detection and Geolocation for Electronic Warfare* [2], as well as Chapter 6 of [4], and Chapters 2 (for detection of standard communications signals) and 4 (for signals with low probability of intercept features, such as frequency hopping) of [5]. For a more theoretical and general discussion of detection and estimation theory, see [6], [7].

1.2 GEOLOCATION

Geolocation can be simplified to the process of estimation of a source's position $\mathbf{x}$, given some set of observable parameters $\mathbf{z}$, and a equation $f()$ that relates them:[1]

$$\mathbf{z} = f\left(\mathbf{x}\right) \tag{1.1}$$

This process is more difficult than a simple inverse problem because the parameters $\mathbf{z}$ are not sampled directly, rather we are presented with a stochastic variable ζ that varies with the parameters $\mathbf{z}$ as well as random fluctuations, such as those due to measurement noise. In many cases, we can approximate ζ with a Gaussian random variable distributed:

$$\zeta \sim \mathcal{N}\left(\mathbf{z}, \mathbf{C}_{\mathbf{z}}\right) \tag{1.2}$$

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[1] This is a simplified form of the measurement model $f()$, which must by necessity also consider the position of the sensors ($\mathbf{x}_n, n = 0, \dots, N-1$), and possibly additional parameters, such as their velocities (for FDOA).

where the expectation of ζ is $\mathbf{z}$, but the covariance matrix $\mathbf{C_z}$ represents the fluctuations.[2] These statistical underpinnings of target geolocation are reviewed in Chapter 2.

In Part III of *Emitter Detection and Geolocation for Electronic Warfare* [2], we reviewed three common phenomenologies for geolocation: angle of arrival (or triangulation), time difference of arrival (TDOA), and frequency difference of arrival (FDOA), illustrated in Figure 1.2.

Figure 1.2(a) illustrates AOA geolocation via the crossing of three lines of bearing that intersect near the source of a signal. In this case, the measurements are the direction of arrival of the signal at each sensor. Two sensors are required to form a solution in a 2-D space, and three sensors are required for a 3-D solution. Algorithms to solve for geolocation estimates via AOA can be found in the `+triang` folder of the provided MATLAB® source code.

Figure 1.2(b) illustrates TDOA geolocation via the use of time of arrival estimates for a signal at three stations, which are compared to generate two *isochrones*, that represent lines of constant time *difference* of arrival between two pairs of stations. These isochrones intersect at the estimated location of the signal's source. In this manner, three stations are required for a 2-D solution, but four are required for 3-D. Algorithms to solve for geolocation estimates via TDOA can be found in the `+tdoa` folder of the provided MATLAB® source code.

Figure 1.2(c) illustrates FDOA geolocation via the use of frequency estimates for a signal at three stations, which are compared to generate two *isodoppler contours* that represent lines of constant frequency *difference* of arrival between two pairs of stations. Since FDOA is reliant not just on where the source and sensors are, but also on their relative velocities, the contours are much more complex than in AOA or TDOA. Algorithms to solve for geolocation estimates via FDOA can be found in the `+fdoa` folder of the provided MATLAB® source code.

Any of these techniques can be combined for improved stability, or reliable 3-D geolocation with fewer sensors. Algorithms to solve for geolocation estimates via hybrid AOA/TDOA/FDOA can be found in the `+hybrid` folder of the provided MATLAB® source code.

1.2.1 Practical Considerations of Geolocation

While the algorithms presented in [2] provide working solutions for AOA, TDOA, and FDOA geolocation, they rely on a set of simplifying assumptions for clarity. These assumptions are: (1) that all target positions are equally likely, (2) that all sensors have perfect knowledge of their position, orientation, and velocity, and (3) that all measurements are simultaneous (or near enough to simultaneous that the target is

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[2] It may be there is also a *bias* term $\mathbf{b}$ in our measurements, in which case the expectation would be given $\mathbf{z} + \mathbf{b}$. It's also possible that the expectation could be a nonlinear function of the true parameter. We will assume in this text that the measurement vector ζ has been properly processed such that any bias or nonlinearities are removed and its expectation is $\mathbf{z}$.

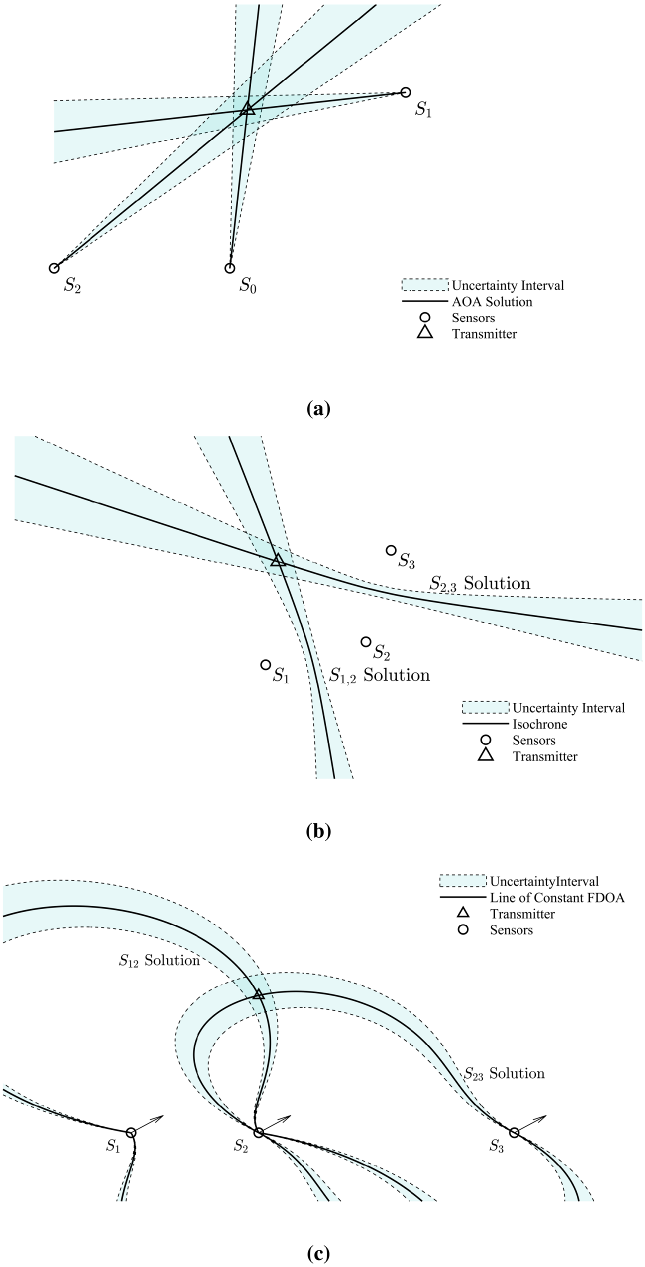

Figure 1.2 Common methods for solving geolocation, discussed in [2]. (a) AOA, (b) TDOA, and (c) FDOA.

assumed to be stationary). We now briefly discuss each, and how they will be relaxed in this text.

1.2.1.1 Target Constraints

In our previous text, we assumed that all possible geolocation solutions were equally likely. In other words, there was no a priori information on target location. While this assumption is reasonable algorithmically, it is often not operationally reasonable. This can be violated in two ways: hard constraints and soft constraints.

Hard constraints take the form of fixed equality or inequality conditions, such as knowledge that the target is a maritime vessel and must therefore be on the Earth's surface, or that an aircraft is traveling at a known altitude. Hard constraints are expressed in the form

$$\begin{array}{ll} \mathbf{a}^T\mathbf{x} = 0 & \text{Equality constraint} \\ \mathbf{b}^T\mathbf{x} \leq 0 & \text{Inequality constraint} \end{array} \tag{1.3}$$

If there are multiple constraints, we can express them concisely in matrix/vector notation:

$$\begin{array}{ll} \mathbf{A}\mathbf{x} = \mathbf{0} & \text{Equality constraints} \\ \mathbf{B}\mathbf{x} \leq \mathbf{0} & \text{Inequality constraints} \end{array} \tag{1.4}$$

If the constraints are nonlinear, then we write them:

$$\begin{array}{ll} \mathbf{a}(\mathbf{x}) = \mathbf{0} & \text{Nonlinear equality constraints} \\ \mathbf{b}(\mathbf{x}) \leq \mathbf{0} & \text{Nonlinear inequality constraints} \end{array} \tag{1.5}$$

It is straightforward to implement a constrained solver that considers this information, particularly with linear constraints, which are convex [8].[3] Predicting estimation performance is slightly more difficult; we will primarily use the *constrained* Cramér-Rao lower bound (CRLB) to predict performance [9]–[11].

Soft constraints present statistical information about where the target is or is not likely to be located. Soft constraints can come from external measurements, such as a separate system's estimated target position, or prior estimates of a target's position and velocity (e.g., from a tracker). Soft constraints are represented as a prior probability distribution function $f_{\mathbf{X}}(\mathbf{x})$. In this manner, the *posterior* likelihood function for any set of measurements $\boldsymbol{\zeta}$ is given:

$$\ell(\boldsymbol{\zeta}, \mathbf{x}) = \ell(\boldsymbol{\zeta}|\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}). \tag{1.6}$$

where $\ell(\boldsymbol{\zeta}|\mathbf{x})$ is the *likelihood function*, which computes how likely it is that the measurement $\boldsymbol{\zeta}$ would occur if the true target location were $\mathbf{x}$. The *posterior* likelihood

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[3] Note that for convex optimization methods to be used, both the constraints and the *cost function* (in this case, the likelihood function or a simplified form of it) must also be convex. This is generally not true with geolocation problems, but they can often be assumed to be locally convex in a region around the true location $\mathbf{x}$.

function $\ell(\zeta, \mathbf{x})$ reflects the combination of the measurement likelihood function and the *a priori* probability distribution function $f_{\mathbf{X}}(\mathbf{x})$.

In Chapter 5, we will discuss constraints, solving them, and their impact on performance.

1.2.1.2 Sensor Uncertainties

Traditionally, sensors that engage in geolocation of adversary transmitters have been placed on fixed (permanent, or semimobile) towers, satellites with well-known trajectories, or even wide-bodied aircraft (such as the RC-135 Rivet Joint used by the U.S. Air Force, shown in Figure 1.3(a)). In these scenarios, it is often reasonable to assume that the sensor's position, orientation, and velocity are known accurately.

However, these sensors are increasingly miniaturized, and the modern battlefield may include their use on smaller platforms that operate in a heavily contested electromagnetic spectrum, such as the Multi-Function Electronic Warfare–Air Large (MFEW-AL) Army program, to be fielded on the MQ-1C Gray Eagle (an artist's rendition is shown in Figure 1.3(b)), or the backpack sensor being fielded by the Army for use by dismounted soldiers: the Versatile Radio Observation and Direction (VROD) system (shown in Figure 1.3(c)). The end result is that ESM sensors are (a) more susceptible to turbulent airflow (and, in the case of ground vehicles or dismounted soldiers, uneven terrain), (b) constrained in the quality of their navigational sensors (due to limited size, weight, and power, and lower cost constraints), and (c) potentially unable to utilize precision navigation and timing services, such as GPS (due to proximity to hostile forces), resulting in increased sensor position errors.

Thus, it is necessary to determine the impact of errors in not only a sensor's position, but also its orientation (for AOA), velocity (for FDOA), and drift in both the clock (for TDOA) and local oscillator (for FDOA). In Chapter 6, we will discuss these errors, and formulate their impact to the geolocation problem via their impact to the received signal vector ζ.

1.2.1.3 Repeated Measurements

For fixed or stationary targets, the use of repeated measurements over time results in high accuracy, even if the individual sensor updates have poor accuracy, as long as they are *unbiased*. This process is illustrated in the iterative estimation block diagram of Figure 1.4.

In Chapter 7, we will examine an optimal solution, in which all prior measurements are stored, and the estimate is recomputed at each update interval using all available information. This will then be compared with an iterative approach that matches Figure 1.4, wherein an estimate of the transmitter's position (along with expected error covariance for that estimate) is propagated forward and combined with the new sensor measurements. We will show the sufficiency of iteratively updating measurements.

(a) **(b)**

(c)

Figure 1.3 Example ESM sensors. (a) RC-135U Combat Sent and RC-135U/V Rivet Joint (the aircraft designation depends on mission configuration). © 2012 U.S. Air Force[12]. (b) Artist rendering of MQ-1C Gray Eagle UAV with a proposed MFEW-Air Large sensor pod. © 2019 Lockheed Martin[13]. (c) Soldier with VROD backpack sensor. © 2019 U.S. Army[14].

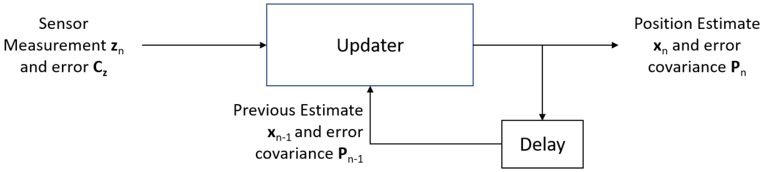

Figure 1.4 Block diagram of iterative estimation.

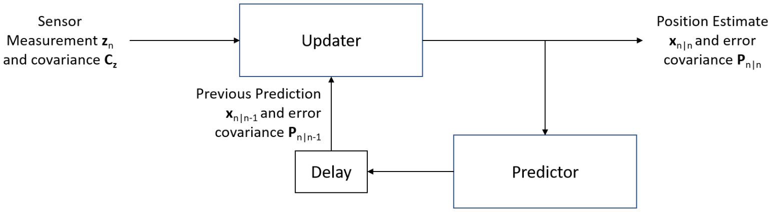

Figure 1.5 Block diagram of tracker predict-update cycle.

1.2.1.4 Tracking

For mobile targets, iterative estimation is replaced with tracking, since the underlying position $\mathbf{x}$ is no longer constant (replaced with $\mathbf{x}(t)$). A basic tracker loop consists of an iterative process of first updating the estimate of the underlying target state $\mathbf{x}_{n|n}$ (in this case, position and, potentially, velocity and acceleration terms) to consider the latest sensor measurement (ζ_n), and then prediction of what that state will be when the next sensor measurement is taken ($\mathbf{x}_{n+1|n}$).[4.] When the next measurement is available, the process repeats, as shown in Figure 1.5.

Much can be written about tracking; indeed countless textbooks have been, and so we will only scratch the surface here.

In Chapter 8, we will introduce trackers with a discussion of the basic $\alpha - \beta$ and $\alpha - \beta - \gamma$ filters and the Kalman Filter, along with common target motion models, and relevant measurement models. We will briefly discuss issues around convergence of the tracker and prediction of steady-state performance. Deployed trackers must contend with a complex problem of data association (assigning measurements to new or existing tracks), track management (removing, or *pruning*, tracks that have gone stale), complex models (e.g., motion models that represent different target states, such as level flight vs. turning), bias reduction, accuracy prediction, and confidence scoring. Most of these topics are beyond the scope of this report, but interested readers are referred to [15] for a good detailed text on the formulation and execution of trackers, including detailed discussion of data association, as well as [16], which focuses on challenges associated

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4. The subscript $\cdot_{n+1|n}$ indicates that the prediction is for time index $n+1$ but only considers information available at time index n.

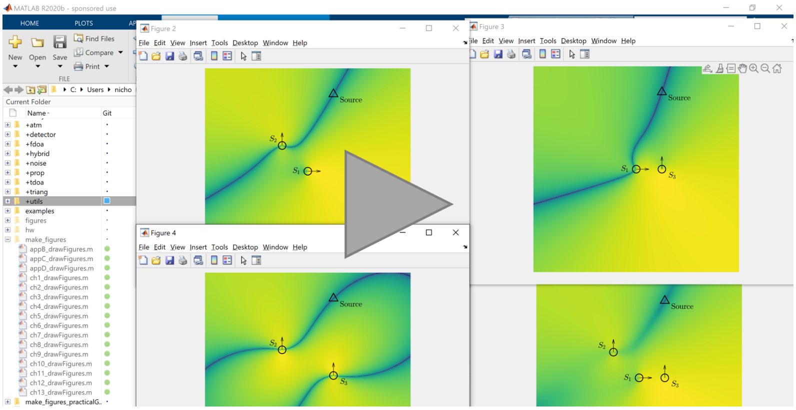

Figure 1.6 Video walkthrough of MATLAB® usage.

with tracking multiple targets, as well as Section 7.3 of [17] for a discussion of Kalman filters within a single sensor (radar) tracking context, and Section 7.3 and Chapter 9 of [18]. A formulation similar to that necessary for passive sensors can be found in [19], which contains a discussion of tracking in sensor coordinates (Chapter 7) and in target coordinates (Chapter 9).

1.3 ASSOCIATED SOFTWARE

This book is written with companion MATLAB® software to illustrate the concepts discussed, and to provide a functional starting point for readers to conduct their own research. The software is repeatedly referenced, and readers are strongly encouraged to run the examples and figure generation scripts as they read, to reinforce understanding.

1.3.1 MATLAB®

We use MATLAB® as our primary development environment, since it is widely employed as a modeling and simulation tool across the Department of Defense research enterprise, and is widely taught in engineering programs. We tested our code on MATLAB® R2020b, but do not expect significant errors with releases as old as R2015a.

Figure 1.6 is a quick introduction to the MATLAB® environment. The code was developed to be largely free of dependencies on special toolboxes, but users may want to consider the *Statistics & Machine Learning Toolbox*.[5].

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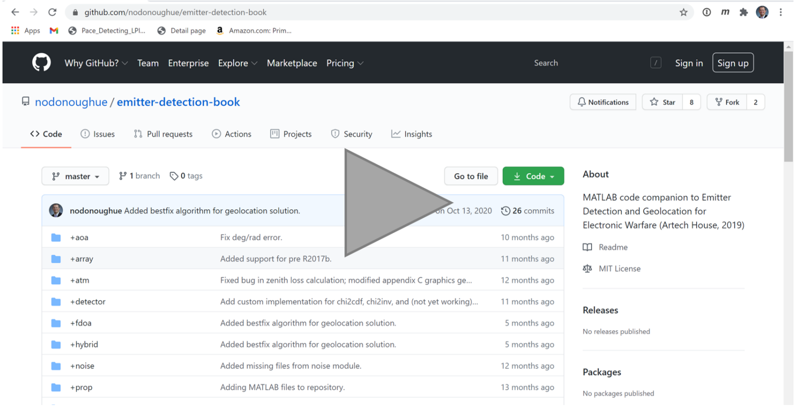
[5] There are workarounds in the +utils folder for some, but not all, functions referenced in the *Statistics & Machine Learning Toolbox*.



nodonoughue / emitter-detection-book

nodonoughue Added bestfix algorithm for geolocation solution. on Oct 13, 2020 26 commits

+aoa	Fix deg/rad error.	10 months ago
+array	Added support for pre R2017b.	11 months ago
+atm	Fixed bug in zenith loss calculation; modified appendix C graphics ge...	12 months ago
+detector	Add custom implementation for chi2cdf, chi2inv, and (not yet working)...	11 months ago
+fdoa	Added bestfix algorithm for geolocation solution.	5 months ago
+hybrid	Added bestfix algorithm for geolocation solution.	5 months ago
+noise	Added missing files from noise module.	12 months ago
+prop	Adding MATLAB files to repository.	13 months ago

About

MATLAB code companion to Emitter Detection and Geolocation for Electronic Warfare (Artech House, 2019)

Readme

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Releases

No releases published

Packages

No packages published

Figure 1.7 Video walkthrough of textbook software installation.

1.3.2 Textbook Software

The software for this text can be found in the git repository, which contains a combined package of source code for both this text and [2]. To use the code, simply download the repository as a .zip file and extract it to your computer, or clone the repository (if you are familiar with git). Then open MATLAB® in the extracted folder. For ease of use, you can run the command `setup`; this will add the folder to your MATLAB® path, so that you can use the tools every time you open MATLAB®.

```
>>setup
```

Figure 1.7 is a link to a video walkthrough of how to download, install, and run the provided source code from the git repository.

1.3.3 Python Software

At the time of publication, a port to Python was in process, but not yet complete. None of the examples, figures, or videos in this text reference the Python implementation, but users may access the latest version of the source code from the git repository.

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