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About the author

Born in 1935, Prof. Sakurai studied Civil Engineering, first at Kobe University (B.E., 1958), then at Kyoto University (M.E., 1960), and finally at Michigan State University USA (Ph.D., 1966), having received his Dr Eng. from Nagoya University in 1975.

Prof. Sakurai worked at Kobe University, where he held the position of Associate Professor (1966–73) and Professor in the Division of Rock Mechanics, Dept. of Civil Engineering (1973–1999), and then worked at the Hiroshima Institute of Technology as President (1999–2003). He is now Professor Emeritus of Kobe University, and also Professor Emeritus of Hiroshima Institute of Technology. Prof. Sakurai worked as President of the Construction Engineering Research Institute Foundation (CERIF) (2003–2011).

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He has given lectures in Brazil, Canada, China, Czech Republic, Germany, Greece, India, Indonesia, Italy, Kazakhstan, Korea, Poland, Taiwan, Thailand, Russia, and many other countries.

In the ISRM, Prof. Sakurai was Vice-President at Large (1988–91), President of the Commission on Communications (1987–91), and Member of the Commissions on Computer Programs (1978–87), on Rock Failure Mechanisms in Underground Openings (1981–91), and on Testing Methods (1983–91). He was also Vice-President of the Japanese Committee for ISRM (the ISRM NG JAPAN) (1995–99).

Professionally, Prof. Sakurai has been involved in various kinds of Rock Mechanics projects (hydropower, nuclear power, pumped storage and compressed air energy storage schemes; highway and railway tunnels; slopes), in Japan and abroad.

His research activities have been principally connected to numerical and analytical methods, back analysis, and field measurements, the aim of these activities being mainly concerned with making a bridge between the theory and practice. Prof. Shunsuke Sakurai is the author or co-author of over 100 publications, and the editor of "Field Measurements in Geomechanics" (Proceedings of the 2nd International Symposium, Kobe, 1987).

Prof. Sakurai received the IUE Award (1974), the JSCE Prize for the Best Paper (1990), and the ICMAG Award for Significant Paper (1994). He also received the Science Award of Hyogo Prefecture (1997).

Chapter 1

Introduction

1.1 AIMS AND SCOPE

This book is dedicated to practising engineers working in rock engineering practice, as well as for graduate students studying and doing research on rock mechanics and rock engineering. The aim of this book is to make the engineers and the students understand how to apply the theory of rock mechanics to engineering practice, in order to achieve the rational design and construction of rock structures such as tunnels, underground caverns, and slopes, and to assess not only the stability of them during/after construction, but also to ensure the safety of the workers.

In order to verify the adequacy of the original design and assess the stability of the rock structures during construction, observational methods are extremely useful. In the method field measurements play a major role, but the measurement data are only numbers unless they are properly interpreted. Back analyses can be used for interpreting the data quantitatively, resulting in the rational design and construction of the structures being achieved.

It is noted that back analysis is a highly non-linear problem, even in the simple case of linear elastic materials. This non-linearity of back analyses may attract the interest of mathematicians in back analysis problems, but only from the mathematical point of view. However, this book is not for mathematicians, but for practising rock engineers so that the back analyses should be used for engineering practice. The contents of this book are mainly based on the original works carried out in the Rock Mechanics Laboratory of Kobe University, Japan.

1.2 FIELD MEASUREMENTS AND BACK ANALYSES

Rock structures such as tunnels, underground caverns, vertical shafts, slopes, etc. are constructed with natural rocks whose geological and mechanical characteristics are extremely complex. This complexity causes difficulty in the evaluation of mechanical characteristics of rock masses, even though various laboratory and *in situ* tests, such as plate bearing tests and direct shear tests, have been developed for determining the mechanical properties of rock masses, such as Young's modulus, strength parameters, underground water condition, permeability, etc. which are important data for design analyses. In addition, the initial stresses of rock masses caused by gravitational and tectonic forces are also important data for the analyses.

It should be noted that the difficulty in the evaluation of the mechanical characteristics and initial stresses of rock masses is a characteristic feature for the design of rock structures, as the mechanical behaviours of the rock structures are extremely complex. This is entirely different from other structures like bridges and buildings, which are built with artificial materials, such as concrete and steel, whose mechanical parameters can be easily determined in laboratory experiments. Moreover, the external forces acting on the structures are also well documented.

In the mechanical behaviours of the rock structures, various uncertainties are involved not only in the mechanical characteristics of rock masses, but also in the design and construction procedures of rock structures. For instance, in tunnel engineering practice the following uncertainties are involved; (1) geological and geomechanical characteristics of rock masses are complex, (2) mechanical modelling of rock masses is extremely difficult, (3) the initial stresses of rock masses are difficult to evaluate, (4) interaction mechanism between tunnel support structures and surrounding rock masses is complex, (5) the mechanical behaviour of tunnels seems to be different for different excavation methods, (6) the mechanical behaviour is also influenced by the skill of tunnel excavation workers, etc.

In rock engineering practice, it is well known that the real behaviour of the rock structures quite often differs from that predicted by numerical analyses carried out at the design stage, even though sophisticated computer programs are used. This discrepancy may be simply because of the various uncertainties described above being involved.

In order to fill in the gap between real and predicted behaviours of rock structures, field measurements are carried out to verify the input data used in the original design, as well as to assess the stability of the rock structures during construction. In addition, it can verify the safety of the workers during the construction. Field measurements are also performed for monitoring long-term stability, for instance the monitoring of landslides. There are many different types of field measurements available, but displacement measurements are most commonly used in rock engineering practice, because they are reliable and easily handled in comparison to others such as stress and strain measurements.

However, it should be noted that the field measurement data are only numbers unless they are properly interpreted. Therefore, the most important aspect of field measurements is the quantitative interpretation of measurement results. For this purpose, back analyses must be a powerful tool.

Chapter 2

Back analysis and forward analysis

2.1 WHAT IS BACK ANALYSIS?

In back analyses, input data are measured values, such as displacements, strains, stresses and pressures, while the output results are the mechanical parameters of rock masses, such as Young's modulus, Poisson's ratio, strength parameters (cohesion and internal friction angle), permeability, and even the initial state of stress. This analysis procedure is entirely a reverse calculation of an ordinary analysis, so that it is called "back analysis", while an ordinary analysis is called "forward analysis" all through this book.

In the design of rock structures, forward analyses (ordinary analyses) are carried out for calculating stresses, strains and displacements of rock masses. The analyses require the input data which are external forces (initial stresses), the mechanical parameters of rock masses, such as Young's modulus, Poisson's ratio, strength parameters (cohesion and internal friction angle), permeability, etc. On the other hand, in the back analyses, the input data are measurement results, such as displacements, strains, stresses, etc., while the output results are the mechanical parameters of rock masses, initial stresses, permeability, etc. It is obvious that the output results of the back analyses correspond to input data of the forward analyses, while the input data for the back analyses are the measurement data. Therefore, the back analyses seem to be entirely a reverse calculation of the forward analyses, as shown in Figure 2.1.

In forward analyses, it is obvious that any sophisticated computer program can be used, no matter how many input data are needed, as long as all the data can be determined by laboratory and *in situ* tests, while in back analyses only a limited number of measurement data (input data for back analyses) are available. This means that all the input data necessary for the forward analyses are hardly identified by back analyses. To overcome this difficulty, a constitutive equation of rock masses used in back analyses should be simple enough to be able to back-calculate all the mechanical parameters of the equation from a limited number of field measurement data.

It should be emphasised that one of the important purposes of field measurements is to monitor whether the present situation of rock structures is stable, or whether an unexpected mechanical behaviour seems to start occurring. To accomplish this purpose, the field measurement results must be properly interpreted during the constructions without delay. To meet this requirement, the back analyses should be capable

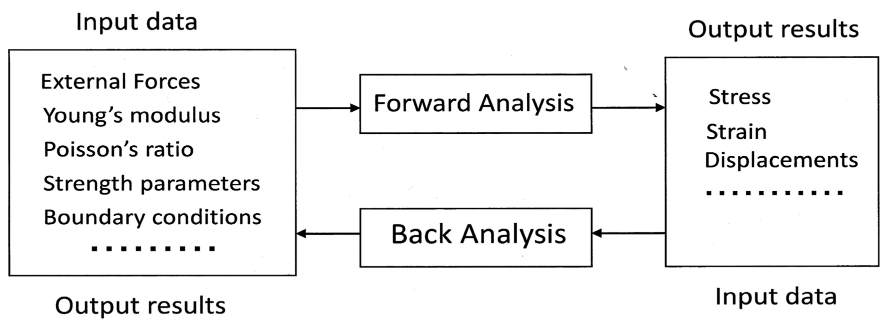


Figure 2.1 Definition of back analysis.

not only of assessing the adequacy of the original design, but also of predicting the catastrophic failure of the structures during the constructions.

2.2 DIFFERENCE BETWEEN BACK ANALYSIS AND FORWARD ANALYSIS

In forward analyses, firstly the mechanical model of rock masses is assumed as to be such as elastic, elasto-plastic, rigid-plastic, visco-elastic, etc., and the mechanical parameters of the model are determined by laboratory and *in situ* tests. Once all the mechanical parameters are determined, we can calculate displacements, strains and stresses of rocks as the outcomes of the forward analyses. This computation procedure provides a one-to-one relationship between the input data and the output results, because modelling (assumption) is done before the determination of input data, as shown in Figure 2.2. This implies that it is extremely important for the forward analyses to assume the most suitable mechanical model for rock masses.

On the other hand, in back analyses we first obtain field measurement data (displacements, strains, stresses, etc.) during constructions. These data are used as input data for back analyses, as seen in Figure 2.2. In order to perform back analyses for determining the mechanical parameters, we must assume a mechanical model. It is obvious that the mechanical parameters determined by the back analysis depend entirely on what mechanical model we assume in the back analyses. For example, if we assume an elastic model, then Young's modulus can be determined, but if a rigid-plastic model is assumed, then Young's modulus cannot be determined. Instead plastic parameters such as cohesion and internal friction angle can be obtained, though the identical input data (measurement results) are used for the both cases. This means that in the back analyses, a one-to-one relationship between the input data and output results cannot be guaranteed, because that mechanical modelling of rock masses is located in-between the input data and output results, as shown in Figure 2.2. In other words, in back analyses a one-to-one relationship between the input data and output results cannot be substantiated.

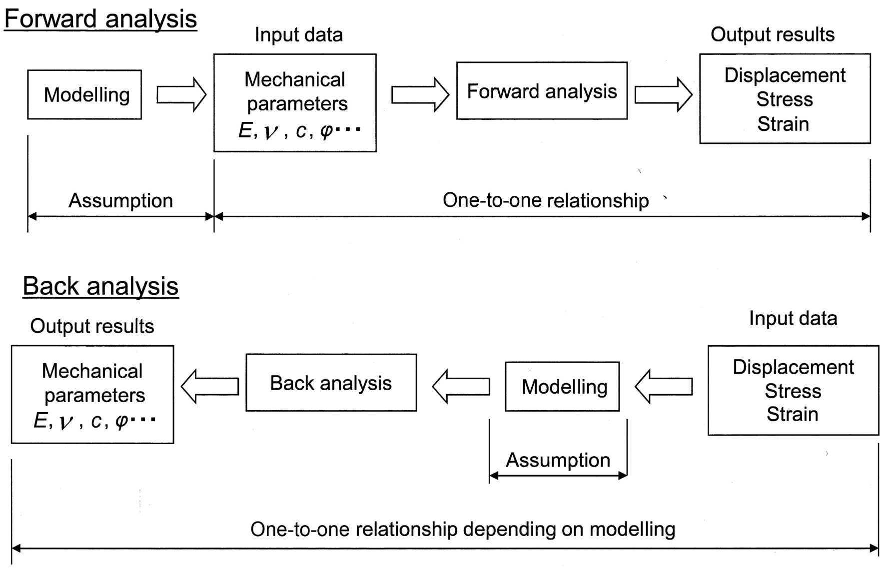


Figure 2.2 Difference between forward analysis and back analysis (Sakurai, 1997a).

We can now conclude that back analysis is not simply a reverse calculation of the forward analysis. Its concept should be different from forward analysis in such a way that back analysis should identify the mechanical model, as well as determine its mechanical parameters from field measurement results.

The mechanical model of rock masses is assumed in the design of structures, and usually only its mechanical parameters are determined by back analyses. In addition, it is noted that the back analyses determining mechanical parameters are a non-linear problem, even for the case of simple linear elastic problems, resulting in that back analyses may attract the interest of mathematicians to solve the non-linear problems. Since the mathematicians are interested in obtaining a stable solution in back analyses only from a mathematical point of view, it does not matter which mechanical model is used in back analyses.

2.3 BACK ANALYSIS PROCEDURES

2.3.1 Introduction

Back analysis problems can be solved by various approaches. Among them, inverse and direct approaches are commonly used in geotechnical engineering practice (Cividini et al., 1981). In the inverse approach the formulation is just the reverse of that in the forward stress analysis, even though the governing equations are identical. On the other hand, the direct approach is based on an iterative optimisation procedure which corrects the trial values of unknown quantities in such a way that the discrepancy between the measured and the computed quantities is minimised. In both inverse and direct methods, the number of measured values should be greater than the number of unknown quantities, otherwise the results cannot be determined uniquely.

However, it is often difficult to determine these values precisely because of the various uncertainties which are usually involved in rock engineering problems. To overcome this difficulty, a probabilistic approach is preferable as it is capable of taking these uncertainties into account. The most advantageous feature of this approach is that the final results are expressed in statistical terms, such as mean and variance.

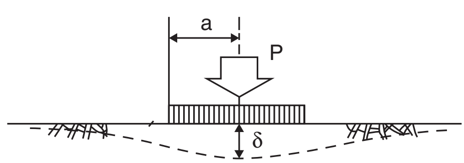
2.3.2 Inverse approach

The inverse approach requires a mathematical formulation in a reverse way to the ordinary stress analysis so that it is only available for the linear elastic materials, whose stress-strain relationship is expressed in a linear form.

A simple example for the inverse approach in rock engineering practice is *in situ* rock tests, such as a plate bearing test, where a displacement δ is measured under a given external force P, as shown in Figure 2.3. Young's modulus E can then be determined by Equation (2.2), which is derived in a reverse formulation of the conventional stress analyses. If the number of the data (measured displacements) is greater than that of back-analysed quantities (Young's modulus, initial stresses, etc.), the least squares method can be used (Sakurai & Takeuchi, 1983).

As another example in the rock engineering field, Kovari and his colleagues (1977) developed an inverse approach called the "integrated measuring technique" for determining the rock pressure acting on tunnel linings from the strain measured on the inner surface of the lining. In this back analysis approach, mathematical equations relating the rock pressure to the axial force and bending moment of the tunnel lining were derived by imposing the equilibrium conditions between the rock pressure and the normal force and bending moment of the lining, which used a fundamental equations to determine the rock pressure acting on the lining.

For more complex engineering problems, an inverse approach can be used on the basis of a Finite Element Method (FEM), which was originally developed for structural engineering problems (Kavanagh & Clough, 1971). In the field of rock mechanics Gioda (1980) modified Kavanagh's algorithm to back-calculate both the bulk and shear moduli by applying static condensation and the least squares method.



$$\delta = \frac{(1-\nu^2)}{2aE} P \tag{2.1}$$

$$E = \frac{(1-\nu^2)}{2a} \frac{P}{\delta} \tag{2.2}$$

Figure 2.3 *In situ* plate bearing test for determining Young's modulus from measure displacements due to applied external force.

The method is defined as "inverse", with respect to the corresponding stress analysis, since it requires the inversion of the equations governing the linear elastic stress analysis problem. If the inversion of the governing equations is possible, this technique is easily applied to engineering practice, because iteration is not necessary in computation, resulting in computation time becoming less compared with the other back analysis approaches.

2.3.3 Direct approach

The direct approach is based on the minimisation of the discrepancy between the field measurement data and the corresponding numerically evaluated quantities, in such a way that an error function δ shown in Equation (2.3) is adopted to define the discrepancy between the measured displacements and those derived from a numerical analysis.

$$\delta = \frac{\sum_{i=1}^{N} (u_i^m - u_i^c)^2}{\sum_{i=1}^{N} u_i^m} \rightarrow \min \tag{2.3}$$

where u_i^m and u_i^c are measured and computed displacements, respectively. N is number of measuring points.

The direct approach has a great advantage in avoiding the inversion of the mathematical equations of stress analyses, resulting in that it can be easily applied to any non-linear problems.

The error function defined by Equation (2.3) is in general a complicated non-linear function of the unknown quantities, and in most cases the analytical expression of its gradient cannot be determined. This is particularly evident for non-linear or elasto-plastic problems. Therefore, the function minimisation algorithm adopted for the problem solution must handle non-linear functions and it should not require the analytical evaluation of the function gradient. The algorithms meeting with these requirements, known in mathematical programming as direct search methods, are based on iterative procedures which perform the minimisation process only by successive evaluations of the error function given in Equation (2.3). Each evaluation requires a stress analysis of the geotechnical problems on the basis of the trial value chosen for the iteration.

In the minimisation algorithm for the error function, any standard algorithms of mathematical programming, such as the Simplex method (Nelder & Mead, 1965), Rosenbrock algorithm (Rosenbrock, 1960), Powell method (Powell, 1964), Conjugate Gradient method (Fletcher & Reeves, 1964), etc. can be used. However, these methods require rather time-consuming computations since a large amount of iteration is usually needed.

Gioda & Maier (1980) demonstrated the applicability of the direct method to back-calculate the non-linear material parameters and the load conditions, using a numerical example of a pressure tunnel test.

2.3.4 Probabilistic approach

Both the inverse and direct methods are based on a deterministic concept, and provide precise values for material constants and load parameters. However, it is often difficult to determine these values quantitatively because of the various uncertainties being included in rock engineering problems. To overcome this difficulty, a probabilistic approach is preferable as it is capable of taking these uncertainties into account. The most advantageous feature of this approach is that the final results are expressed in statistical terms, such as mean and variance.

The field measurement data are in general affected by various errors that depend on the nature of measured quantities, the characteristics of measuring devices, field conditions, etc. In order to evaluate the influence of these errors on the back-calculated mechanical parameters, various methods have been proposed. Among them a simulation technique, such as the Monte Carlo simulation, can be easily applied to engineering practice (Cividini & Gioda, 2003). This simulation technique is an extremely simple implementation, but requires a computational effort which rapidly increases with the increase in number of unknown parameters. In order to overcome this drawback, a probabilistic Bayesian approach is recommended (Cividini et al., 1983).

A typical feature of the Bayesian approach is that *a priori* information on the unknown parameters can be introduced in the back analysis, together with the data deriving from *in situ* measurements. In most cases, the *a priori* information consists of an estimation of the unknown parameters based on the engineer's judgement or on available general information. This leads to a numerical calibration procedure that combines the knowledge deriving from previous similar problems with the results of the *in situ* investigation.

2.3.5 Fuzzy systems, Artificial Intelligence (AI), Neural network, etc.

In a probabilistic approach, the determination of a probability density function for the mechanical parameters of rock masses is extremely difficult. In other words, there is no reliable way to determine the input data for the probabilistic approach. This is entirely different from the case of materials such as steel and concrete, resulting in that the probabilistic approach may be less applicable to rock engineering problems. To overcome this problem, the Fuzzy Set Theory can be used, which can easily provide with all the input data necessary for the back analyses on the basis of engineers' subjective judgements (Zadeh, 1965). This means that the Fuzzy Set Theory goes well with the probabilistic approach of back analyses. It should be noted that the Fuzzy Set Theory must be a potential tool for solving rock mechanics problems in probabilistic approaches (Fairhurst & Lin, 1985; Nguyen & Ashworth, 1985; Sakurai & Shimizu, 1987).

It is obvious that rock masses are extremely complex non-linear systems that include many parameters. In order to solve these complex systems, Feng et al. (2000) proposed a new displacement back analysis approach which is based on a combination of a neural network, an evolutionary technique and numerical analysis methods to identify the mechanical parameters. The method has been successfully applied to the Three Gorges Project permanent shiplock to estimate the mechanical parameters of rock masses.

Feng et al. (2004) also proposed another displacement back analysis method to identify the mechanical parameters based on hybrid intelligent methodology, such as the integration of evolutionary Support Vector Machines (SVMs), numerical analysis and a genetic algorithm.

Considering various uncertainties and complexities involved in rock masses, Khamesi et al. (2015) proposed a novel, intelligent back analysis method for determining the complex and non-linear relation between the displacements and the geomechanical parameters by using a fuzzy system designed by three different methods, i.e. nearest neighbourhood clustering and gradient descent training, particle swarm optimisation, and imperialistic algorithm.

2.4 BRIEF REVIEW OF BACK ANALYSIS

In the early 1970s, identification theories were developed in the field of system engineering (Astrom & Eykhoff, 1971), and applied to various field problems such as structural dynamics (Hart & Yao, 1977). In geomechanics, in earlier times various terms such as identification, characterisation, inverse analysis, etc., were used for identification problems. At that time it was thought that these identifications were mathematical problems, because they are highly non-linear problems, even though a simple elastic model is assumed. Therefore, the main interest of researchers has been on how to solve such non-linear problems so as to obtain a mathematically stable solution with high accuracy. Before the term "back analysis" was being used in rock mechanics field, Sakurai (1974) assumed the ground medium consisting of a visco-elastic material, and proposed a back analysis method to determine the initial stress and mechanical properties of visco-elastic underground media.

The term of "back analysis" appeared for the first time in the rock mechanics field in a paper entitled "Determination of rock mass elastic moduli by back analysis of deformation measurements" (Kirsten, 1976). Ever since that time, various names have been used by different authors. Nevertheless, the term "back analysis" gradually became popular, and it is now commonly used in the rock engineering community. In the rock engineering field, various back analysis procedures have been extensively developed, ranging from simple elastic problems to far more complex non-linear problems, and many papers related to back analysis have been published with particular reference to the interpretation of field measurement results (Gioda & Sakurai, 1987). In rock engineering practice, back analyses are nowadays often used for determining the mechanical properties of rock masses from the data of field measurements carried out during the construction.

Deterministic back analysis procedures are roughly classified into two categories: the inverse approach and the direct approach (Cividini et al., 1981). In the inverse approach, the mathematical formulation is just the reverse of that in an ordinary analysis (forward analysis in this book), although the governing equations are identical.

In the case of a ground represented by a simple mechanical model with simple geomechanical configurations, the closed-form solutions in the theory of elasticity and plasticity may be used. However, for the ground with an arbitrary shape under a more complex geological and geomechanical environment, numerical methods such as FEM, Boundary Element Method (BEM), Discrete Element Method (DEM), etc., seem to be more promising. For example, Kavanagh (1973) proposed a back analysis formulation based on FEM which may make it possible to obtain the material constants not only for isotropic materials, but also for inhomogeneous and anisotropic materials, from both measured displacements and strains.

Gioda (1980) modified Kavanagh's algorithm to back-calculate both the bulk and shear moduli by applying static condensation and the least squares method. In order to obtain the material constants, the displacements alone are sufficient. However, to identify the load conditions in addition to the material constants, the measurements of not only the displacements, but also the values for the loads and pressures are necessary. For this, a numerical procedure of back analysis was proposed for determining the earth pressure acting on tunnel lining on the basis of measured displacements and measured earth pressure at some locations. The optimal earth pressure distribution can be determined by minimising a suitably defined error function (Gioda & Jurina, 1981).

Sakurai & Takeuchi (1983) proposed an inverse method of determining both the initial stress and Young's modulus from measured displacements around a tunnel, assuming homogeneous and isotropic linear elastic media. According to the method, the strain distribution around a tunnel can be determined by the data of a limited number of measured displacements. Since the method is formulated in the stiffness matrix method, the large simultaneous equations have to be solved, resulting in time-consuming numerical computation. To overcome this shortcoming, Sakurai & Shinji (1984) used the flexibility matrix method for solving the identical problem, resulting in drastically reduced computation time. Shimizu & Sakurai (1983) extended the back analysis procedure proposed by Sakurai & Takeuchi (1983) to the three-dimensional case by using BEM to determine both Young's modulus and the *in situ* stress from measured displacements. If the back analyses are carried out with the displacements measured during the excavation of pilot tunnels for underground powerhouse caverns, the back-calculated values are those for the three-dimensional large extent of rock masses, so that they are used for assessing the adequacy of the original design of powerhouse caverns.

Gioda & Maier (1980) demonstrated the applicability of the direct method to back-calculate the non-linear material parameters together with the load conditions by introducing a numerical example of a pressure tunnel test. Cividini et al. (1985) also stated that the direct approach could be employed to determine the time-dependent material constants by applying convergence displacement measurement data taken at various stages of the tunnel construction.

Since various uncertainties are involved in rock engineering problems, it is difficult to determine the mechanical parameters of rock masses quantitatively. To overcome this difficulty, a probabilistic approach is preferable as it is capable of taking these uncertainties into account.

Among various probabilistic procedures, the Monte Carlo simulation can be easily applied to engineering practice (Cividini & Gioda, 2003). This simulation technique is an extremely simple implementation, but requires a computational effort which rapidly increases with the increase in number of unknown parameters. In order to overcome this drawback, the Bayesian approach is promising for back analyses. Cividini et al.