Jun Mitani

3D Origami Art

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Preface

Paper, one of the ordinary, essential items found around us, is used not only as canvas for drawing and painting but also as a fun material for the creative activity of making shapes through cutting, folding, and gluing. It seems that by cutting and gluing paper, any shape can be made. But if your action is limited to folding, you may feel that only limited shapes can be made. But is that really true? Now take a look into the world of origami, making shapes only through folding. You will find a magical, fascinating space of geometry woven with a variety of representations.

With a centuries-old history in Japan, origami play to fold paper into different shapes is familiar among people regardless of age. Now there are fans of origami all over the world. In its long history, origami techniques of forming a sheet into intended shapes have progressed. The background of this is an accumulation of mathematical knowledge about origami to establish theories on origami design. Furthermore, the advent of computer programs to perform the necessary computations for origami design moved the world of origami forward dramatically.

Many of you may have played by folding paper in your childhood. Origami in the twenty-first century has evolved amazingly from what it was before. In particular, the folding action along a curve and forms having curved surfaces are part of a new origami field realized by the use of computers.

This book introduces three-dimensional creations derived from computation and explains the design method. Many of them have a three-dimensional structure composed of curved surfaces, and some have complicated forms. But the background theory underlying the creations in this book is very simple. Surely you will be surprised to find that totally different-looking origami forms are designed from a common theory. This book contains many photos and design drawings called *crease patterns*. All of these crease patterns are available on my webpage http://mitani.cs.tsukuba.ac.jp/book/3d_origami_art/, so everyone can download and print them.

But I forgot one important thing. The origami creations in this book do not use square paper but use various shapes of sheets such as rectangles and regular polygons. Sometimes gluing may be necessary to stabilize the final shape. A little relaxing of rules greatly widens the range of shapes you can create.

It is my great pleasure to present information in this book so that you can enjoy the mystery and acquire mastery of three-dimensional origami, making lovely, geometric, three-dimensional structures out of a set of lines and curves drawn on a two-dimensional plane.

Prologue: Origami Basics

Origami is one of our familiar playtime activities, but its origins are not at all clear. This introduction covers the historical background of origami, its geometric nature, and the essential information for designing origami.

P.1 Road to Modern Origami

It is thought that origami play of folding cranes and boats began to spread in Japan in the early Edo period (the seventeenth century), although the period depends on how origami is defined. Before that, a decorative origami culture existed in Japan that included wrapping paper (*tatou*) and paper attached to gifts (*noshi*). Origami is deeply rooted in Japanese tradition. Today origami is used worldwide as a term indicating paper-folding play. However, paper folding is an old, established practice in Europe as well. So, the idea that "origami originated from Japan and then spread all over the world" is not entirely correct.

A collection of variations of traditional Japanese origami cranes was published in 1797, more than 200 years ago, entitled *Hiden Senbazuru Orikata* [*Secret Crane Folding Patterns*]. The book focuses on connected cranes and covers as many as 49 types of completion drawings together with information on how to cut a sheet. Many of you may think origami uses a square sheet and should not be cut, but the book tells us that in the past, a more freewheeling style was used to create origami works. *Hiden Senbazuru Orikata* is said to be the world's oldest origami play book. But I learned while writing this book that a text had been discovered in Kuwana, Mie, which dates back several years before *Hiden Senbazuru Orikata*. Origami history has many facts that are yet to be revealed. Paper-folding play has a centuries-old history in Japan and has been loved by people of all generations.

Original modern origami was probably introduced after the middle of the twentieth century. Akira Yoshizawa's contribution to origami is highly valued both at home and abroad. It was he who refined origami into an art and established the method of describing the folding procedure using figures and symbols. Various new folding techniques have been developed since about 1980, when the concept of "design" was introduced to origami. Recent years have seen structurally complex works, such as insects. These are far-removed from classical origami in sophistication. Complicated origami with many folds is called *complex origami*. The First International Meeting of Origami Science and Technology was held in 1989, advancing the study from a scholarly point of view. In 2014, the sixth meeting was held in Tokyo with about 300 participants from 30 countries, and as many as 140 of the latest origami study results were presented.

After 2000, with the spread of computers, many software programs that support origami design and simulate how sheets of paper transform with folds were introduced. Computer-aided origami design and studies are now also known as *computational origami*.

Another new, active movement is the application of origami techniques in engineering fields—the folding of satellite solar panels, the folding of car airbags, and robotics, seeking more use in various industries.

P.2 Origami and Crease Pattern

The first step to getting into origami is to look carefully at the relation between the folded "shape" and the "fold lines" on the unfolded sheet. There are two fold line types—the "mountain" (or "ridge") fold and the "valley" fold. The graphic showing the layout of these fold lines is called a *crease pattern* (Figure P.1). Different from the crease pattern, the *diagram* explains the folding process using figures and symbols.

When you unfold an actual origami work, you may find certain lines left unfolded at the time of completion (but were folded as a mark during the folding process). These lines are called *auxiliary lines* and are generally not included in the crease pattern.

In indicating crease patterns, mountain folds are often colored in red and valley folds in blue. In a monochrome setting, mountain folds are generally shown by dash-dot or solid lines and valley folds by dashed lines (in this book, mountains are shown by solid lines and valleys by dashed lines for visibility).

P.3 Theory of Flat Folding

Most of the traditional origami works are folded flatwise during the folding process. Even an origami crane wing is folded flatwise all the way through the final step. Folding a sheet flatwise is a basic origami action and is called *flat folding*.

Fold lines resulting from flat folding are always straight. So, a flat-folded origami work has a crease pattern with a set of straight lines. Figure P.2 is a finished "bird" from flat folding and its crease pattern. You may see the crease pattern represented by a set of straight lines.

Many studies have been done on the flat-folded state because it is the general origami state. Focusing on the point where fold lines meet, the following two principles

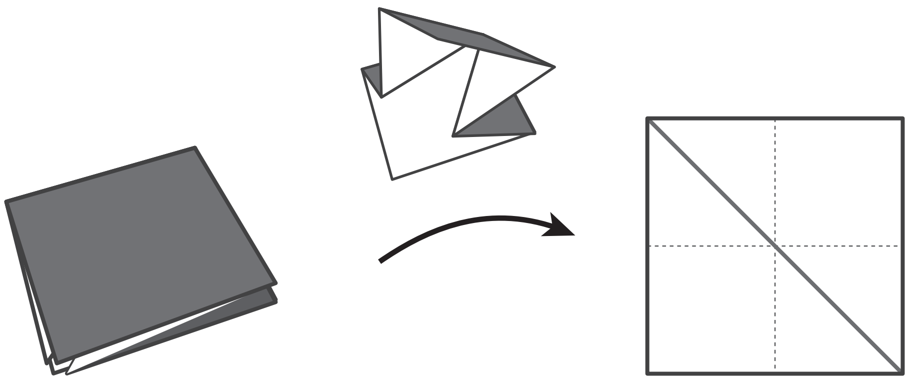

Figure P.1 Unfolded sheet has a crease pattern with mountain and valley lines.

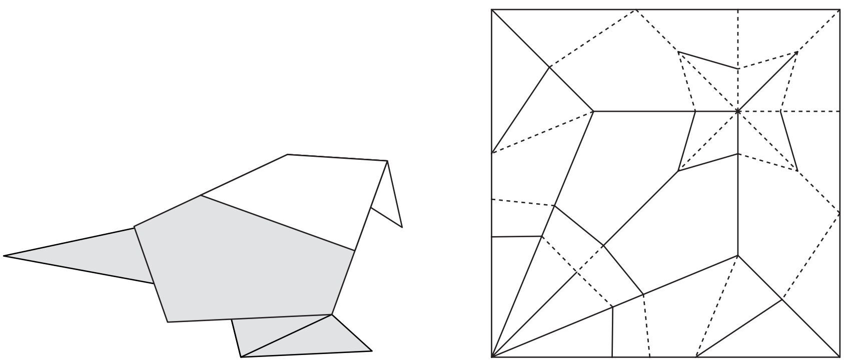

Figure P.2 "Bird" and its crease pattern (viewed from the colored portion).

are always satisfied. These are called *Maekawa's theorem* and *Kawasaki's theorem*.

Maekawa's theorem: The number of mountains and valleys differs by two.

Kawasaki's theorem: The alternating sum of incident angles is 0.

Figure P.3 presents an example of a crease pattern appearing on a flat-folded sheet when it is unfolded. It meets the above two theorems. Let us fold sheets in various ways, and then unfold them to observe that the two theorems are always right. The bird crease pattern in Figure P.2 also meets the theorems at all the intersections.

Note that these theorems are necessary but not a sufficient condition. A flat-foldable crease pattern always meets these two conditions, but meeting these conditions does not always make flat folding possible. There are crease patterns locally but not globally foldable because of a collision. To understand the situation of "partly but not globally foldable," let us look at the simple crease pattern in Figure P.4a. Trying to fold the two valleys flatwise fails because one folded section hits the other. On the contrary, as shown in Figure P.4b, by having the fold lines a bit dislocated from that as shown in Figure P.4a, the paper can be folded without collision. The crease pattern in Figure P.4c may look a little complicated, but it fully meets the above conditions and is foldable globally.

P.4 Tessellation and Twist Folding

Tessellation is one way of creating tiling patterns by folding paper as shown in Figure P.5.

Periodic patterns on a tessellation look like a mosaic. You may enjoy the shades against the light. This folding technique is seen in pleats in clothing and has had many known

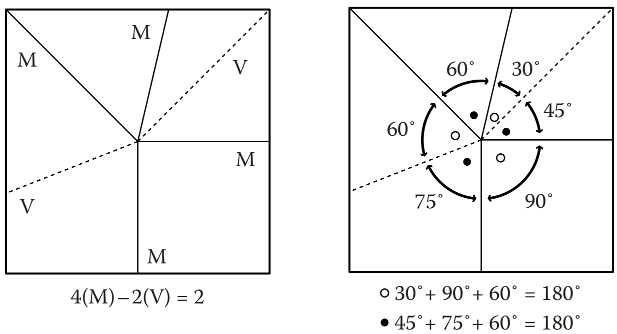

Figure P.3 Explanation of Maekawa's theorem (left) and Kawasaki's theorem (right).

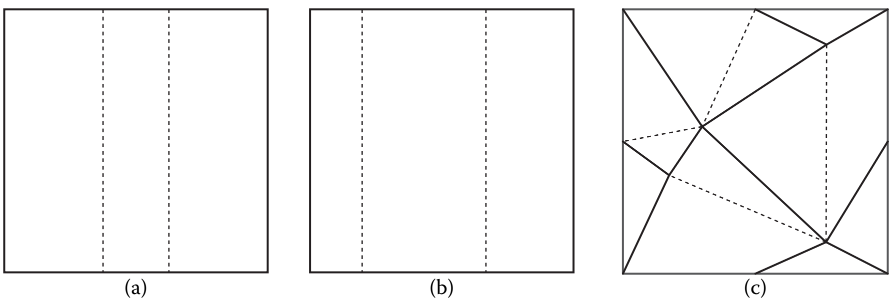

Figure P.4 (a) Flat-foldable crease pattern. (b and c) Flat-foldable crease patterns.

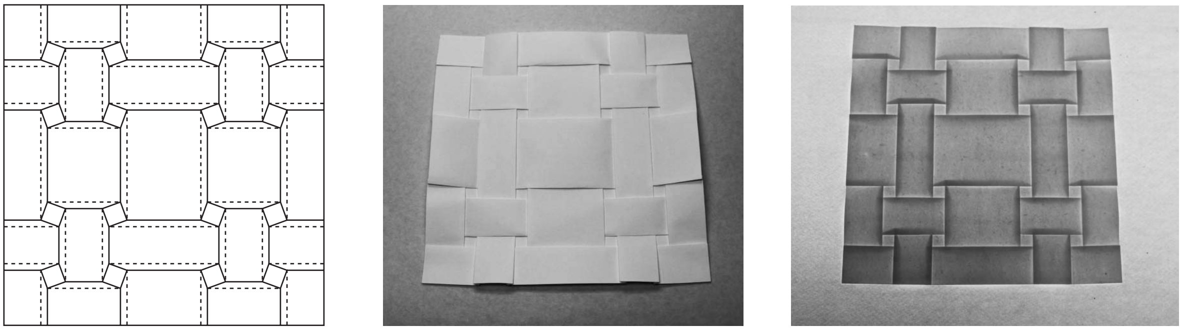

Figure P.5 Tessellation: from the left—crease pattern, photo of the folded paper, and translucent silhouette against the light.

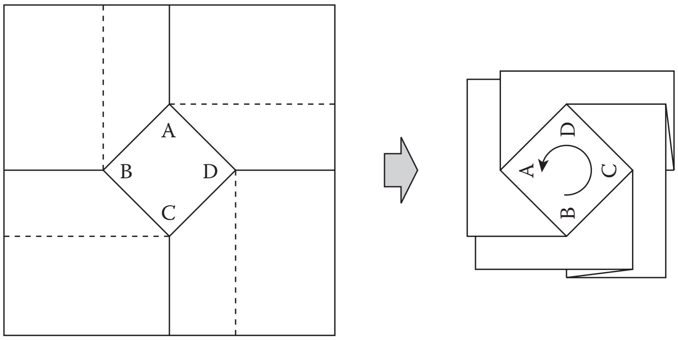

Figure P.6 Square-based twist folding.

patterns since long ago. Shuzo Fujimoto's extensive studies in tessellation conducted in the 1960s are still highly valued.

The concept of "connecting structures into a bigger shape" in tessellation is important for origami design. Chapter 3 of this book explains the making of an origami work having connected multiple three-dimensional shapes on one sheet of paper based on the concept of tessellation. Chapter 6 introduces different forms in an advanced tessellation structure.

Tessellation often uses the "twist folding" shown in Figure P.6. For the crease pattern (left) in Figure P.6 to be folded flatwise, you need to do it as you twist the sheet. At this time, the center square turns 90 degrees. Normally, a sheet is folded straight on and on. Twist folding requires you to move the fold lines all together while twisting at the center.

The twist fold in Figure P.6 can be connected to its mirror inversion side-by-side as in Figure P.7 (top). Then, inverting it vertically gives you the pattern in Figure P.7 (bottom). This has a big closed area in the center but is flat foldable with no problem.

As in Figure P.8, a twist-fold crease pattern can be parallelly shifted and connected to the original pattern. At this time, the fold line's mountain and valley signs must be inverted. As a result, the square area of the twist fold appears on the backside. By carefully looking at the crease pattern and connecting the patterns, you can extend the shape foldable from one sheet of paper.

Placing of square twist folds leads to different tessellation patterns. Besides squares, regular triangles and regular hexagons can also be tiled closely on a plane. So, each of the twist folds in Figure P.9 can also be placed and connected similarly as before.

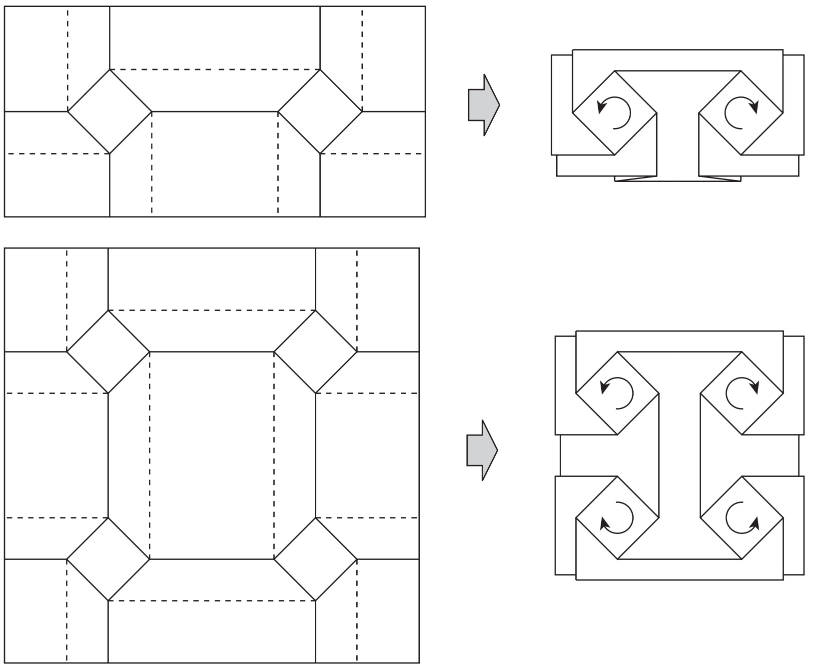

Figure P.7 Connection of twist folds.

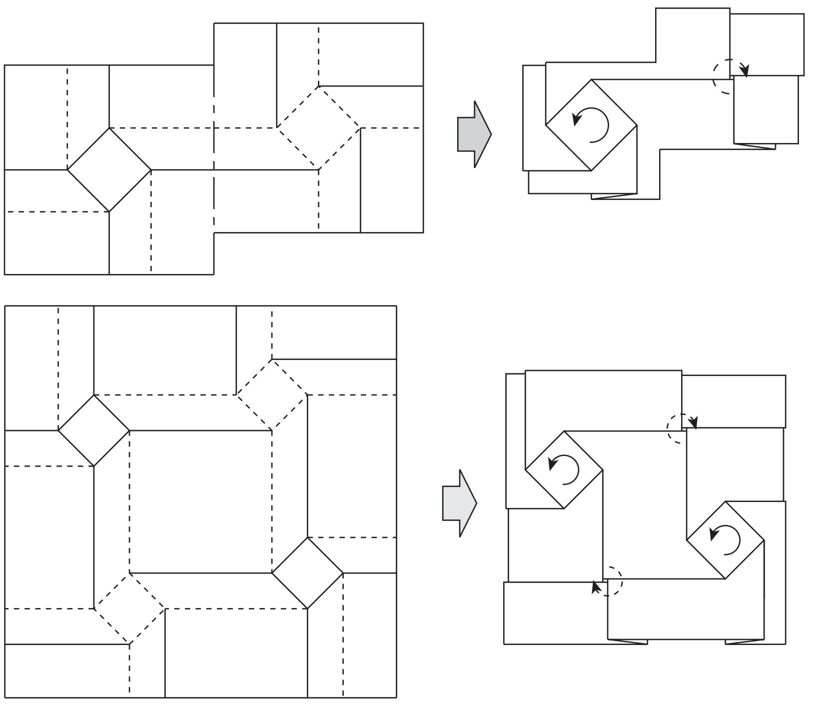

Figure P.8 Connection of twist folds with mountains and valleys inverted.