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19

Buckling of Structures

Theory and Experiment

edited by

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Buckling of Structures

Theory and Experiment

The Josef Singer Anniversary Volume

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Presented to

Josef Singer

by
Friends, Colleagues and Former Students
on the occasion of his sixty-fifth birthday
on August 24, 1988.



DEDICATION TO PROFESSOR JOSEF SINGER

Josef Singer was born on 24th August 1923 in Vienna. He was the younger of the two sons of Zvi and Etel Singer. In 1929 the family moved to Berlin, where he attended elementary school, till his parents sensed that it was time to leave Europe and early in 1933 the family settled in Haifa. There he grew up attending the Reali Secondary School, where the basis was laid for his scientific and cultural interests. He also engaged in sports, mainly long-distance running, swimming and sailing.

Already at the age of 14, Singer clearly knew that he wanted to become an aeronautical engineer and started to plan accordingly. At that time it was not considered wise in Israel to strive towards such a goal, as there was no sign of an aeronautical industry anywhere on the horizon and hardly any flying activity. Singer joined the Aero Club, built models, started gliding, instructed in aerodynamics, then started flying and at the age of 18 was among the first few who obtained their private pilot license at the only flying school in the country. He reasoned that as a future aeronautical engineer, flying was essential for better understanding of aeroplanes.

He then started his aeronautical engineering studies by correspondence (in Israel there was no place to study this profession, and World War II prevented going abroad), and passed the Royal Aeronautical Society Associate Fellowship examination in 1943. At the same time he headed the Tel Aviv branch of the Aero Club, building up its activities and instructing many of the first generation of Israel's aviation professionals.

In 1943 Singer joined the Royal Air Force as a flying cadet, but as he was about to be sent to Flying School in Rhodesia, courses there were curtailed. Disappointed, he worked as a mechanic and later in the design office of RAF 107 MU in Egypt. He also continued his studies in the evenings. In 1946, upon his release from the RAF, Singer went to Imperial College, University of London, and obtained his B.Sc. (Eng.) with 1st class honors in 1948 and later his D.I.C. in Aeronautics, studying under Sir Arnold Hall.

Upon his return to Israel he joined the Israel Air Force and served for 6 years as an Engineering Officer in the emerging Engineering Department, where he worked on the design, installation and testing of weapon systems, making fighting machines out of the multitude of second-hand planes the fledgling Air Force had managed to buy. Later he was head of the Test and Development Section with the rank of Major. He soon discovered that in these airplanes structural problems were more frequent and more challenging than those in aerodynamics, which he originally aimed at. Thus he turned to structural mechanics, and in 1952 was sent by the Air Force to obtain his Master degree at the Polytechnic Institute of Brooklyn, under Professor N.J. Hoff. In 1954 Singer married Shoshana Praeger, his devoted companion; they have three children, Gidon, Tamar and Uri.

In 1955 Josef Singer joined the Technion, Israel Institute of Technology, where an Aeronautical Engineering Department was being established as a result of the foresight of Professor Sydney Goldstein. Singer initiated the aerostructures activities

and the structures laboratory of the department. At the end of 1955 he went again to the Polytechnic Institute of Brooklyn for two years, to obtain his Doctorate under Professor Hoff, while simultaneously planning the future Structures courses and research program for the Technion Department.

Upon his return to Haifa, he taught the Structures courses to the first class of Aeronautical Engineering students at the Technion, initiated the Aircraft Structures research work, set up and equipped the laboratory and organized the Aerostructures group. Soon the Technion Aircraft Structures Laboratory became internationally known, its research was funded by the U.S. Air Force and other agencies abroad and in Israel, and cooperation with centers around the world flourished.

Twice, in 1958-60 and 1965-67, Singer served as Head of the Department of Aeronautical Engineering. He greatly contributed to its growth in all its branches. He became full Professor in 1965.

Professor Singer always kept close contacts with the developing aeronautical industries in Israel and served as consultant to Israel Aircraft Industries, and other organizations. In 1971 he was called by the President of the Israel Aircraft Industries, to serve as Senior Vice President and Head of the Engineering Division. Not wishing to leave research and education at the Technion, Singer agreed to serve for two-and-a-half years. This was the period when the first Israeli built civil airplanes, the Westwind and the Arava, were certified, and when the first fighter, the Kfir, moved into production and its entirely new version, the Canard Kfir C-2, was conceived and developed. During those years the IAI Engineering Division matured and grew and its capability of avionic integration, essential for modern airplane design, was initiated. He also introduced long-range planning to the Division.

At the end of 1973 Singer resumed his academic duties at the Technion. In 1982 he was elected President of the Institute and served a four-year term. Though these were financially difficult years for Israeli universities, the Technion continued to grow and advance academically and emerged financially stronger, with its endowment funds nearly doubled. Among the many new academic activities initiated during his term of office, was also the Multidisciplinary Space Research Institute.

Professor Singer's scientific and technical work focused primarily on the stability of shells and in particular on experimental studies. Singer and his students and co-workers have made the Technion Aircraft Structures Laboratory into one of the internationally recognized centers in this field, and the methods developed there are being used by researchers and industry all over the world, not only in the aerospace industry, but also in off-shore structures and other marine and civil engineering applications. His work on conical shells, stiffened cylindrical shells, correlation between vibration and buckling, and influence and measurement of imperfections is universally quoted. But as is evident from his list of publications, he contributed to many other fields of structural mechanics, such as thermal stresses and thermal buckling of wings, creep rupture, buckling under impact, durability of shear panels, dynamic buckling and

durability of composite structures and plastic buckling. He is presently working on a book "Experimental Methods in Buckling of Thin-Walled Structures" (with J. Arbocz as co-author). Singer's work is characterized by a search for better physical understanding, by thoroughness of execution and by a balance between theory and experiment and between basic studies and practical applicability. He imparted this balanced approach to his many M.Sc. and D.Sc. students.

In the course of his sabbatical leaves, Josef Singer was visiting professor at Stanford University 1963-4, and California Institute of Technology (Caltech) 1968-9, Vinton Hayes Senior Fellow at Harvard University 1976-7 and is currently a Sherman Fairchild Distinguished Scholar at Caltech. His international research activities involved many joint projects with colleagues in Israel, the USA, West Germany and the Netherlands.

Professor Singer is very active in international organizations, in particular in the International Council of Aeronautical Sciences (ICAS), where he served as Member of the Program Committee and then its Chairman and Member of the Executive Committee. Subsequently he was elected as President of ICAS for two terms 1982-6. He was instrumental in making ICAS the major international forum for aeronautical engineering and sciences. He has also been active in the International Astronautical Federation (IAF), as session chairman and member of the program committee for some years. He is also active in IUTAM (the International Union of Theoretical and Applied Mechanics), where since 1976 he has served on the Congress Committee.

In Israel Professor Singer has been a member of many professional committees: Israel Council for Higher Education (1975-81), Advisory Committee of the Chief Scientist, Ministry of Defense (1968-76), Chairman, Advisory Committee on Structures of the Research and Development Unit (1976-77). For a decade he has been a member of the Board of Directors of Israel Aircraft Industries, serving as Chairman of its Research and Development Committee (1979-82 and 1986-7) and in 1986-7 served as the Chairman of its Board of Directors.

Professor Singer was a founding member of the Israel Society of Aeronautics and Astronautics and many times its President. He was active in the initiation and organization of the Israel Annual Conference on Aviation and Astronautics, being several times chairman of the Organizing Committee. He was also Chairman of the Organizing Committee of the 9th ICAS Congress held in Haifa (1974) and is presently Chairman of the Organizing Committee of the 16th ICAS Congress (Jerusalem, 1988).

Josef Singer is a Foreign Associate of the U.S. National Academy of Engineering, Member of the International Academy of Astronautics, Foreign Associate of the Académie Nationale de l'Air et de l'Espace, France, Fellow of the American Institute of Aeronautics and Astronautics (AIAA), Fellow of the Royal Aeronautical Society, Fellow of the Institution of Mechanical Engineers (U.K.), Member of the Society for Experimental Stress Analysis (U.S.A.).

The honors he has been awarded include: named to the L. Shirley Tark Chair in Aircraft Structures at the Technion, 1973, Dr. Sc.

h.c. Polytechnic University of New York 1983. Fellow, City and Guilds of London Institute 1986, Dr. h.c. Université D'Aix Marseilles II, 1986, Officier dans l'Ordre des Palmes Académiques 1986. Honorary Member, Deutsche Gesellschaft für Luft- und Raumfahrt (DGLR) (1987).

Professor Singer is one of the pioneers of aeronautical engineering in Israel, has been deeply involved in the development of its aeronautical research, education and industry, and has fostered international cooperation in the field. He has made lasting contributions to our knowledge in aerospace structures. However, his most important contribution in the Aeronautical Engineering profession in Israel is, undoubtedly, the large number of students who lead Israeli aeronautical activities today.

On behalf of all authors of this Volume, including those friends who were unable to contribute, we wish Josef Singer ! כה לחי .

The Editors

Haifa and Delft
February, 1988

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THE BUCKLING OF AXIALLY COMPRESSED IMPERFECT SHELLS WITH ELASTIC EDGE SUPPORTS

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SUMMARY

A rigorous solution is presented for the case of axially compressed stiffened cylindrical shells with general imperfections, where the edge supports are provided by symmetrical or unsymmetrical elastic rings. The circumferential dependence is eliminated by a truncated Fourier series. The resulting nonlinear 2-point boundary value problem is solved numerically via the 'Shooting Method'. The changing deformation patterns resulting from the different degrees of interaction between the given initial imperfections and the specified end rings are displayed. Recommendations are made as to the minimum stiffnesses required for optimal load carrying configurations.

INTRODUCTION

Since thinwalled structures exhibit very favorable strength over weight ratios the design of stiffened or unstiffened shells continue to play an important role in modern engineering. Unfortunately, thinwalled shells are prone to buckling instabilities.

In the last decades initial geometric imperfections [1]-[2] and general elastic supports [3] have been widely accepted as the explanation for the wide experimental scatter and the poor correlation between the predictions based on a linearized small deflection theory with SS-3 ($N_x = v = w = M_x = 0$) boundary conditions and the experimental values.

The effect of different combinations of in-plane boundary conditions on the stability of axially compressed perfect shells or shells with axisymmetric imperfections have been studied analytically and numerically by Hoff [4] and Almroth [5]. Recently Singer and his coworkers [3] have developed an experimental technique which makes it possible to estimate the degree of elastic support present in a particular test set-up.

Despite all these theoretical and experimental results the shell design manuals in use at the present time adhere to the so-called 'Lower Bound Design Philosophy', which involves the use of a so-called 'knockdown factor'. The empirical 'knockdown factor γ ' is so chosen that when it is multiplied with the

buckling load of the perfect structure P_c a lower bound to all available experimental data is obtained.

It has been hoped that with the large scale introduction of computer codes with advanced nonlinear capabilities an alternate design procedure could be developed which would no longer penalize innovative shell design because of the poor experimental results obtained elsewhere.

As a step towards this goal Arbocz [6] in 1984 published the results of an extensive numerical study of the well characterized stringer stiffened shell AS-2, which has been tested at Caltech in 1970 [7].

Using an early finite difference version of the well known nonlinear shell code STAGS [8] the complete shell was modeled. The measured initial imperfections were fitted by a bivariate cubic spline fit. This model was then used to compute the first derivatives of the measured initial imperfections with respect to x and θ at all nodal points. Employing C-4 ($u = v = w = w_{,x} = 0$) boundary conditions an iterative step-by-step procedure then located the limit point of the prebuckling states. The calculated collapse load of $\rho_s = 0.8563$ has been normalized by -320.8 N/cm, the buckling load of the perfect shell using membrane prebuckling and the same C-4 boundary conditions. The calculated collapse load is unexpectedly high since the shell AS-2 buckled at $\rho_{exp} = 0.715$.

In looking for an explanation, a comparison of the calculated prebuckling deformation for C-4 boundary conditions (see Fig. 1) with the experimentally measured prebuckling deformation (see Fig. 2) is helpful.

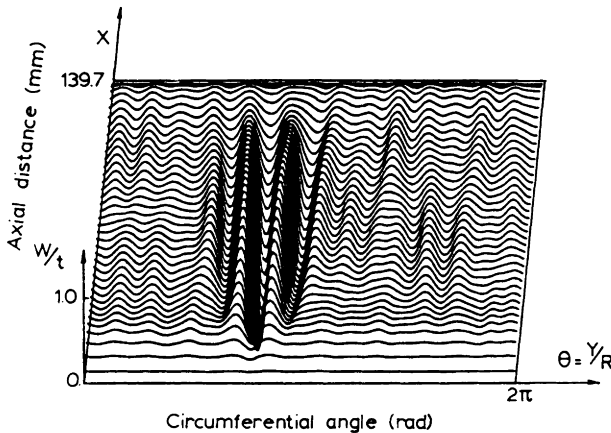


Fig. 1. Calculated prebuckling growth of the stringer stiffened shell AS-2 at $\rho_s = 0.8563$ ($41 \times 161 = 6601$ mesh points).
(Boundary conditions: $u = v = w = w_{,x} = 0$).

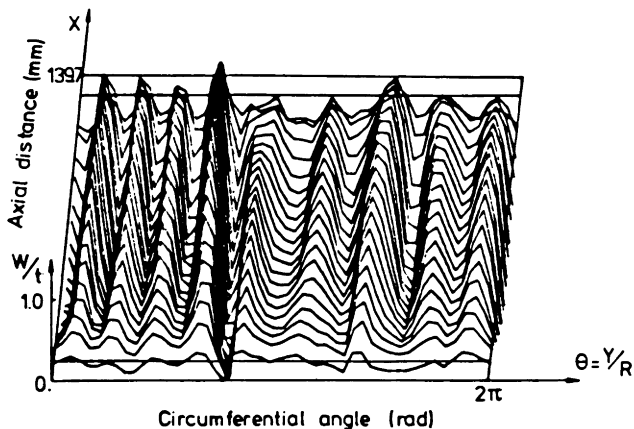


Fig. 2. Measured prebuckling growth of the stringer stiffened shell AS-2 at $\rho=0.629$ ($21 \times 49 = 1029$ data points).

After looking at these figures it is obvious that the two deformation patterns are strikingly different. Since the measured initial imperfections are modeled quite accurately by the bivariate cubic spline fit used, therefore the answer must be sought in a possible difference between the C-4 boundary conditions used with the numerical calculations and the actual elastic boundary conditions present at the experimental set-up.

This statement is reinforced by the results shown in Figures 3 and 4 of rerunning the current discrete model using the same spline fitted initial imperfections as input but changing the boundary conditions successively to C-3 ($N_x = v = w = w_x = 0$) and to SS-3 ($N_x = v = w = M_x = 0$).

It must be mentioned here that for the C-3 boundary conditions the limit load $\rho_s=0.8153$ is normalized by -256.9 N/cm, whereas for the SS-3 boundary conditions the limit load $\rho_s=0.8095$ is normalized by -229.8 N/cm. These normalizing factors are the bifurcation buckling loads of the perfect AS-2 shell using membrane prebuckling and the indicated boundary conditions.

From a comparison of the calculated prebuckling deformations using the same initial imperfections but different boundary conditions with the experimentally measured prebuckling growth it appears that the best agreement occurs for the SS-3 boundary conditions.