Introduction to Equilibrium Analysis

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INTRODUCTION TO EQUILIBRIUM ANALYSIS
Variations on themes by Edgeworth and Walras
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Variations on themes by Edgeworth and Walras

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Preface

This book is a simple but formal account of work done to date in that part of economics which we have chosen to call equilibrium analysis. This last term is a little pretentious since we confine ourselves to a discussion of the oldest problem in economics, that of the exchange of commodities among individuals. We show how two different approaches to the solution of the exchange problem lead, in large economies, to essentially the same result. The two approaches derive from ideas developed by Edgeworth and Walras. The first approach is based upon improvements on suggested allocations by cooperation amongst individuals. The second is concerned with the decentralization of choices by prices. The tools we employ are now standard in modern mathematical economics. In sum, our book might be described more appropriately by a fuller version of our subtitle: 'Variations on themes by Edgeworth and Walras scored for modern instruments, convex sets and fixed points.'

We should say at the outset that the reader who hopes to find an elegant formalization of all that is traditionally thought of as the domain of economics will be sadly disillusioned. We can only offer two remarks to mitigate this disappointment. Firstly, one has to start somewhere; and secondly perhaps one of our readers will be inspired to make an important step towards applying rigorous analysis to more ‘relevant’ economic problems.

The book is intended to be used as a basis for a course for senior undergraduates or graduate students. It has been used by the authors in this way at Berkeley, Bonn, Brussels and Warwick, and we are indebted to our students at these universities for their comments and corrections. Throughout the book we have endeavoured to keep the mathematics employed elementary. Of course, this involves a loss of generality in the results, but means that the mathematical prerequisites for reading the book are
negligible. Indeed, any reader who has a logical mind and is reasonably persistent should not have great difficulty.

To help in the preparation of a course it may be of some use to know which parts of the book are more or less essential. Chs. 2 and 4 are the most central. Ch. 2 develops our model and ch. 4 gives our basic results on the relationship between the two solution concepts, the core and the competitive equilibrium, associated with our two themes. Ch. 5 shows how the two themes become similar in large enough economies. Chs. 3 and 6 are concerned respectively with proving that the two solutions, the core and the set of competitive equilibria, are not empty. Ch. 1 gives an elementary geometric account of the contents of the book for a special case. This chapter will, at least in part, be redundant for those who have a sound basic training in economic theory. However, it should give those approaching this sort of material for the first time an intuitive feel for what is done here.

We should next make explicit acknowledgement of our intellectual indebtedness. Our debt to Edgeworth and Walras is obvious, but throughout this book the influence of more recent mathematical economists is felt. In particular, the names of Aumann, Debreu, Scarf and Shapley occur again and again.

Finally, we must thank Liz Thompson for her painstaking and accurate typing of the various versions of the manuscript, Jill Pothecary for drawing the diagrams and Franz Tillmann for his careful proof reading and for preparing the index.
Contents

Preface ................................................................. v

Chapter 1. An introductory outline ................................. 1
1. Introduction ....................................................... 1
2. Basic example .................................................... 1
3. Improving by cooperation ....................................... 5
4. Decentralization by prices ....................................... 9
5. Coincidence of two themes ...................................... 18
6. Conjuring trick .................................................... 23
7. Limit economies and insignificance .............................. 25
8. From limits to limiting results ................................... 26
9. Back to Walras ...................................................... 28
10. Less well behaved individuals ................................. 31
11. Forewarned is forearmed ......................................... 33

Chapter 2. Exchange economies ........................................ 35
1. Introduction ....................................................... 35
2. Commodity space ................................................ 36
3. Agents' characteristics .......................................... 37
4. Economies ........................................................ 48
5. Appendix .......................................................... 53

Chapter 3. The core of a game ......................................... 65
1. Introduction ....................................................... 65
2. Balancedness ...................................................... 69
3. Non-classical exchange economies ............................... 76
4. Appendix .......................................................... 86

Chapter 4. Large economies ........................................... 93
1. Introduction ....................................................... 93
2. Prices and demand ............................................... 94
# Contents

3. Walras equilibrium ................................................. 97  
4. Economies with a continuum of agents .................... 105

## Chapter 5. Limit theorems for the core ...................... 115
1. Limit theorem for replica economies ....................... 115  
2. Limit theorems for type economies ......................... 119  
3. Uniform boundedness of core allocations ................. 126  
4. Asymptotic equal treatment for most ..................... 130  
5. Asymptotic equal treatment for all ....................... 135  
6. Proof of limit theorem 5.2 ................................. 140  
7. Emergence of prices ........................................... 141  
8. Alternative proof for section 6 ........................... 145

## Chapter 6. Existence of competitive equilibria ............... 147
1. Introduction .................................................. 147  
2. Individual demand .......................................... 149  
3. Total demand ................................................. 153  
4. Equilibria for strongly convex preferences ............. 154  
5. Equilibria for economies with convex preferences .... 162  
6. Equilibria without convexity of preferences ............ 165  
7. Conclusions ................................................. 169  
8. Appendix .................................................... 170

## Mathematical appendix I: Topological concepts in $\mathbb{R}^1$ .... 175
1. Convergence of $\mathbb{R}^1$ .................................. 176  
2. Closed and open sets ....................................... 176

## Mathematical appendix II: Separation theorems .......... 183

## Mathematical appendix III: Continuous correspondences ....... 187
1. Introduction .................................................. 187  
2. Upper hemi-continuous correspondences .................. 188  
3. Lower hemi-continuous correspondences ................. 195  
4. Continuous correspondences ................................ 198

## Mathematical appendix IV: Fixed point theorems .......... 201

## List of mathematical symbols .................................. 205

## References ...................................................... 209

## Author index ................................................... 213

## Subject index .................................................. 215