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On the Cauchy Problem

Sigeru Mizohata

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Sigeru Mizohata

Department of Mathematics
Faculty of Science
Kyoto University

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Preface

Upon the invitation of Prof. Chi, I stayed at the Wuhan University in China during October—November 1983, and gave a series of lectures on evolution equations. The audience was Chinese mathematicians in this field from all over China. At that time Prof. Chi made a wonderful lecture note written in Chinese. This English version is essentially the same as it. However to make the lecture note understandable to every one who attended to the lecture and others, I reorganized the materials, and added to the original version some comments and also appendix at the end of each lecture. It will be my great pleasure if this lecture note could stimulate young mathematicians.

Finally I wish to express my thanks to Mr. Masayoshi Hata (Kyoto University) who took the pain to type my manuscript in this excellent form.

S. Mizohata

August 1984 Kyoto University

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Lecture I. Evolution Equations

The linear partial differential equation of evolution type is defined by

$$\left(\frac{\partial}{\partial t}\right)^m u(x,t) + \sum_{j=1}^{m} a_j(x,t;\frac{\partial}{\partial x}) \left(\frac{\partial}{\partial t}\right)^{m-j} u(x,t) = f(x,t), \qquad (1)$$

where t represents the time variable, $x\ (\in \mathbb{R}^n)$ represents the space variable. $a_j(x,t;\frac{\partial}{\partial x})$ is a differential operator:

$$a_j(x,t;\frac{\partial}{\partial x}) = \sum_{\nu} a_{j\nu}(x,t)\left(\frac{\partial}{\partial x}\right)^{\nu},$$

where the suffix ν runs through a finite set. The Cauchy problem for (1) (with the data at $t = 0$) is expressed by imposing

$$\left(\frac{\partial}{\partial t}\right)^j u(x,t)\Big|_{t=0} = u_j(x) \quad (\ 0 \le j \le m-1\ ). \qquad (2)$$

There are well-known three types of evolution equation:

1) Wave equation

$$\left(\frac{\partial}{\partial t}\right)^2 u - \Delta u = 0 \qquad (3)$$

2) Heat equation

$$\frac{\partial}{\partial t} u - \Delta u = 0 \qquad (4)$$

3) Schrödinger equation

$$i\frac{\partial}{\partial t} u - \Delta u = 0\ . \qquad (5)$$