
NOTES AND REPORTS
IN MATHEMATICS
IN SCIENCE AND ENGINEERING

Volume 3

On the Cauchy Problem

Sigeru Mizohata

SCIENCE PRESS

Beijing, China

ACADEMIC PRESS, INC.

Orlando San Diego New York Austin London

Montreal Sydney Tokyo Toronto

On the Cauchy Problem

This is Volume 3 in
NOTES AND REPORTS IN MATHEMATICS
IN SCIENCE AND ENGINEERING
Edited by WILLIAM F. AMES, *Georgia Institute of Technology*

A list of books in this series is available from the publisher on request.

On the Cauchy Problem

Sigeru Mizohata

Department of Mathematics

Faculty of Science

Kyoto University

1985



SCIENCE PRESS

Beijing

The People's Republic of China



ACADEMIC PRESS, INC.

Harcourt Brace Jovanovich, Publishers

Orlando San Diego New York

Austin London Montreal Sydney

Tokyo Toronto

Science Press Rapid Manuscript Reproduction

Responsible Editor Lü Hong

Copyright © 1985 by Science Press and Academic Press, Inc.

Published by Science Press, Beijing, China

Distribution rights throughout the world, excluding the People's Republic of China, granted to Academic Press, Inc., U.S.A.

Printed in Hong Kong

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the copyright owner.

First published 1985

ISBN 0-12-501660-3

Science Press Book No. 5042-76

Library of Congress Card Catalog Number:
85-63843

Preface

Upon the invitation of Prof. Chi, I stayed at the Wuhan University in China during October—November 1983, and gave a series of lectures on evolution equations. The audience was Chinese mathematicians in this field from all over China. At that time Prof. Chi made a wonderful lecture note written in Chinese. This English version is essentially the same as it. However to make the lecture note understandable to every one who attended to the lecture and others, I reorganized the materials, and added to the original version some comments and also appendix at the end of each lecture. It will be my great pleasure if this lecture note could stimulate young mathematicians.

Finally I wish to express my thanks to Mr. Masayoshi Hata (Kyoto University) who took the pain to type my manuscript in this excellent form.

S. Mizohata

August 1984 Kyoto University

This page intentionally left blank

Contents

Lecture I Evolution Equations.....	1
Lecture II H^∞ -wellposedness.....	11
Appendix	20
Lecture III Lax-Mizohata Theorem	
§ 1.....	28
§ 2.....	33
§ 3 Proof of Theorem 3	35
§ 4.....	41
§ 5 Further considerations	45
Appendix	
§ A.1 Preliminaries	49
§ A.2 Proof of (12).....	50
§ A.3 Partition of unity	52
§ A.4 Estimates of $\alpha_n(D)b(x,D)\chi_{n,\pm}(D)$	53
§ A.5 Proof of (9), (10), (11).....	55
§ A.6.....	57
§ A.7.....	58
Lecture IV Cauchy Problems in Gevrey Class	
§ 1 Introduction and results.....	60
§ 2 Fundamental proposition	69
§ 3 Proof of Theorem 4	75
§ 4 Gevrey property in t of solutions.....	81
§ 5 Comments	85

Appendix

§ A.1 Proof of Lemma 4.....	86
§ A.2 Proof of Lemmas 1,2 and 6.....	90
§ A.3 Proof Lemma 3.....	91

Lecture V Micro-local Analysis in Gevrey Class (I)

§ 1 Introduction.....	97
§ 2 Definition of $\{\alpha_n(D),\beta_n(x)\}$	104
§ 3 Criterion of $WF_s(u)$ by S_n	108
§ 4 Some comments on $WF(u)$	111
§ 5 Some comments on $WF_A(u)$	115

Appendix

§ A.1 Partition of unity.....	118
§ A.2 Proof of Theorem 1.....	122
§ A.3 Proof of (18).....	124
§ A.4 Pseudo-local property in $\gamma^{(s)}$	125
§ A.5 Proof of Theorem A.1.....	129

Lecture VI Micro-local Analysis in Gevrey Class (II)

§ 1 Preliminaries.....	135
§ 2 Proof of Theorem 1.....	138
§ 3 Some consequence of Theorem 1.....	145
§ 4 Propagation of singularities in the sense of C^∞	150

Appendix

§ A.1.....	160
§ A.2 Proof of Lemma 2.....	163

Lecture VII Schrödinger Type Equations

§ 1 Introduction (General view-points on evolution equations).....	166
§ 2 Necessity of (C_0)	170
§ 3 Sufficiency for L^2 -wellposedness.....	173

Lecture I. Evolution Equations

The linear partial differential equation of evolution type is defined by

$$\left(\frac{\partial}{\partial t}\right)^m u(x,t) + \sum_{j=1}^m a_j(x,t; \frac{\partial}{\partial x}) \left(\frac{\partial}{\partial t}\right)^{m-j} u(x,t) = f(x,t), \quad (1)$$

where t represents the time variable, $x (\in \mathbb{R}^n)$ represents the space variable. $a_j(x,t; \frac{\partial}{\partial x})$ is a differential operator:

$$a_j(x,t; \frac{\partial}{\partial x}) = \sum_{\nu} a_{j\nu}(x,t) \left(\frac{\partial}{\partial x}\right)^{\nu},$$

where the suffix ν runs through a finite set. The Cauchy problem for (1) (with the data at $t = 0$) is expressed by imposing

$$\left(\frac{\partial}{\partial t}\right)^j u(x,t) \Big|_{t=0} = u_j(x) \quad (0 \leq j \leq m - 1). \quad (2)$$

There are well-known three types of evolution equation:

1) Wave equation

$$\left(\frac{\partial}{\partial t}\right)^2 u - \Delta u = 0 \quad (3)$$

2) Heat equation

$$\frac{\partial}{\partial t} u - \Delta u = 0 \quad (4)$$

3) Schrödinger equation

$$i \frac{\partial}{\partial t} u - \Delta u = 0. \quad (5)$$