Barsotti Symposium in Algebraic Geometry

Edited by

Valentino Cristante

William Messing

PERSPECTIVES IN MATHEMATICS

S. HELGASON, EDITOR

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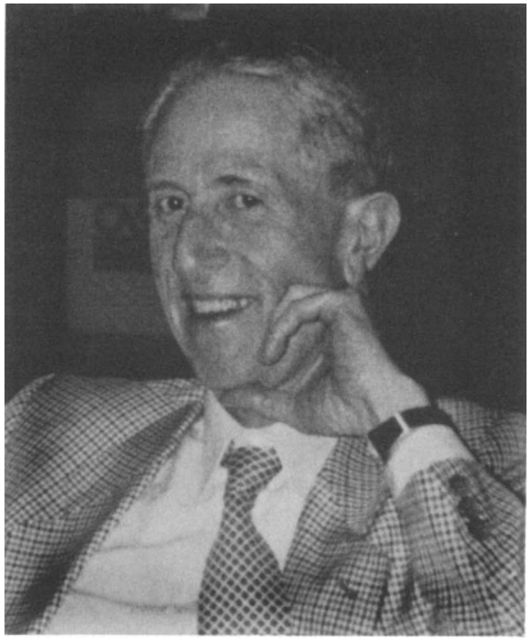

IN MEMORIAM: Iacopo Barsotti (1921–1987)

PERSPECTIVES IN MATHEMATICS, Vol. 15

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Introduction

During the period June 24-27, 1991 a memorial meeting was held in Abano Terme in honor of Iacopo Barsotti. This volume contains submitted papers corresponding to the lectures of some of the invited participants.

Almost all who knew him can attest to Barsotti's levelheadedness, his rather impish sense of humor and his rather strong inclination to puncture the balloons of pomposity. Hence, even though we do not propose to enter into details, one might wonder as to what sort of wry comment the following brief synopsis of Barsotti's work would have elicited from him. In 1970 he himself produced a manuscript in which he commented on his own work up to that point. He neither published nor circulated this manuscript and it was only found among his papers after his death. While we have decided, perhaps unwisely, not to publish this manuscript, we must indicate that a small portion of the following reflects a reading of it.

Barsotti was born in Turin in 1921 and died, following a long illness, in Padua in October 1987. He came of mathematical age following the War, his first paper appearing in 1946. These early papers in what he labelled "pure algebra" were regarded by him as "such as might be expected from someone who's pretty sharp and well versed in the subject". Nevertheless he did prove, subsequent to but independently of H. Cartan, the result which was Cartan's portion of the Cartan-Brauer-Hua theorem.

Barsotti chose an independent path, whose intrinsic difficulties, rather than hindering him, served as a stimulus. He put himself in competition with the most distinguished algebraic geometers of his day. Initially his feeling of creative tension was very strong, but, as might be expected, it evolved over time, although to say he mellowed would be a crude approximation to a much more subtle reality.

It was during the period in the early fifties while he was working on foundational questions in algebraic geometry that these situations initially arose. The period 1945-65 was one of intense activity in this area, initially led by Weil and Zariski who each utilized his own particular language to treat these questions. Barsotti chose to emphasize the role of valuation theory, thus adopting the approach of Krull and Zariski. In the first two of the three papers he dedicated to the foundations of algebraic geometry, he develops a purely algebraic theory of correspondences between projective varieties. The principal tools he introduces are the so–called "associate form" of a cycle (apparently the first treatment valid in any characteristic), and the theory of correspondences between a field and a variety. These two together give a "specialization" theory (and also a "reduction" theory) for cycles and varieties. This was subsequentely generalized to the case of unequal characteristic by Shimura. In the third paper, as an application of the first two, he established an intersection theory for cycles of a projective variety. It is worth noticing that Weil and Chevalley took the reverse path since they obtained the theory of correspondences as a consequence of the theory of intersections. In his approach Barsotti was closer to the ideas of classical Italian algebraic geometers. At a minimum it can be said that Barsotti felt that this foundational work was not appropriately recognized. In any case his independent (of any school) credentials as an algebraic geometer were firmly established by it.

During the next period of his career he turned to the study of algebraic groups and abelian varieties; the latter proving to be the love of his mathematical life. In his first paper on algebraic groups, Barsotti gives a proof of what is now called the projective embeddability of abelian varieties. More precisely, he proves that a projective variety (actually in all his papers he never uses varieties that are not projective) with a normal law of composition is birationally equivalent to an algebraic group (with the law of composition induced by the normal law). This was intended to be a fundamental step in building a theory of the Picard Variety associated to a projective variety without enlarging the realm of algebraic varieties. He then proved that any algebraic group is an extension of an abelian variety by a linear group (usually referred to as Chevalley's theorem), calculated the groups of extension classes of an abelian variety by the additive (resp. multiplicative) group and the relations of these with differential forms (the result regarding extensions by the multiplicative group is frequently called the Weil-Barsotti theorem), proved that the Néron-Severi group of an abelian variety is torsion free, ... This was a period of intense activity on abelian varieties and there was a mutual overlap between the work of Barsotti and that of Cartier, Chevalley, Chow, Matsusaka, Rosenlicht, Serre, Weil,....

Already in 1955 Barsotti posed the questions that would occupy his attention for the rest of his career. He saw that the theory of abelian varieties over a field of characteristic p could be further developed by considering the formal group obtained by expanding the group law at the origin. Like Dieudonné, slightly earlier, Barsotti was struck by the fact that the Taylor series coefficients could not be expressed in terms of iterated derivations and thus the Lie algebra was a very weak tool in characteristic p. Unlike Dieudonné, for Barsotti abelian varieties were always the prime focus of his interest and even while he developed extensively the theory of p-divisible groups, baptized by Grothendieck Barsotti-Tate groups, the source never wavered. Already in 1956 he indicated that endomorphisms of an abelian variety could be represented by "p-adic matrices" and explicitly constructed a representation associated to the multiplicative part of the p-divisible group, analogously to the l-adic representations introduced and studied by Weil. This idea was independently developed by Serre.

Barsotti's language indicates that his underlying intuition was to try to develop tools and techniques that would provide a characteristic p analogue of the transcendental description of an abelian variety. It was the Dieudonné module, Barsotti's canonical module, associated to the p-divisible group of the abelian variety which was to play the role of the complex vector space of differentials of the second kind modulo the exact ones. He arrived at the concept of a p-divisible group only gradually. At first he considered only the connected ones (commutative formal groups of finite height) but already by 1961 he had introduced the étale part and referred to the entire p-divisible group or rather it's coordinate ring as an equi-dimensional hyperdomain. Perhaps because his interests were primarily geometric, for Barsotti these objects lived over an algebraically closed field and only much later did he consider such groups over more general rings. To study these groups

Barsotti introduced the Witt covectors and bivectors as well as the module (resp. vector space) of canonical covectors (resp. bivectors). These enabled him to study p-divisible groups up to isomorphism or up to isogeny as well as to interpret in a more conceptual manner the duality theorem for abelian varieties which had been independently proven by Nishi and Cartier. Barsotti with characteristic frankness referred to Cartier's paper on duality as the unique paper on "structure of group varieties" that he regretted not having written. Barsotti also introduced the characteristic p analogue of the Riemann bilinear form associated to a polarization and proved the necessity of the symmetry condition for a p-divisible group to arise (up to isogeny) as the p-divisible group associated to an abelian variety.

Continuing along his analytic path, he turned his attention to the theory of theta functions. In the period 1968-70 he developed an algebraic theory valid in characteristic zero; this theory, although free of periods, gives back the classical theory over the complex numbers. We suggest that the reader interested in Barsotti's point of view on complex theta functions consult section 4 of his paper ***Considerazioni sulle funzioni theta.*** During the next ten years the theory was extended to work over (perfect) fields of positive characteristic. As opposed to the characteristic zero case where the theta function associated to a divisor lies in the completion of the local ring at the origin, in characteristic p, it lies in the completion of the perfect closure of this ring. The theory utilized most of the tools (covectors, bivectors, hyperfields, ...) that Barsotti had developed over the preceeding twenty-five years. We can do no better than suggest that the interested reader consult Barsotti's Rennes conference paper on theta functions, not only for its mathematical content but also for a somewhat uninhibited expression of his views concerning notation, terminology and mathematical concepts which he preferred to use or, on the other hand, to avoid.

In the last part of his mathematical career, which continued until his last days, he worked on the theory of the differentials equations satisfied by "his" thetas, and succeeded in obtaining a differential description of almost all commutative group varieties. For an exposition which gives an idea of his last results cf. *A New Look for Thetas*, which appeared posthumously.

Barsotti was fiercely independent, certainly an iconoclast. Starting in the mid nineteen fifties he developed his own terminology and notational style. The fact that comparatively few others followed his conventions seemed to have delighted him. Cohomology he regarded with suspicion, although he was among the first to calculate some important H^1's, other concepts he referred to as "complicated simplifications". It is clear that had he adopted a different style his papers would have been more widely read. This was his choice. In any event he saw early and he saw deeply the profound p-adic aspects of the theory of abelian varieties and Barsotti-Tate groups in non-zero characteristic. Barsotti has left us not only his published papers, but also the school of algebraic geometry in Padua which he founded and led. His legacy is rich and extends the deep tradition of Italian algebraic geometry.

Valentino Cristante

William Messing

Biographical Notices about Iacopo BARSOTTI *

Born in Torino, Italy, 1921
Married (Clare A.), 1951
Daughter (Adriana), 1961
Internal Fellow at Scuola Normale Superiore, 1938–1942 (in absentia during military service)
Doctorate in Mathematical Sciences, cum laude, University of Pisa, 1942 (military service 1940–1945)
Postdoctoral Fellow, Scuola Normale Superiore, 1945–46
Assistant, University of Roma, 1946–1948
Postdoctoral Fellow, Princeton University, 1948–49
Associate Professor, and Professor after 3 years, University of Pittsburgh, 1949–1960
Professor, Brown University, 1960-61
Professor, University of Pisa, 1961–1968
Professor, University of Padova, since 1968

Temporary positions while holding another permanent position:

Visiting Associate professor, and then Visiting Professor, University of Southern California, 1954–1956
Fullbright Scholar, University of Pisa, 1957–1959
Professor, University of Massachussets at Boston, 1966–67
Visiting Professor, University of Southern California, second semester 1968
Senior Foreign Scholar, Yale University, 1970–71
Member, Institute for Advanced Study (Princeton), first semester 1981–82

Offices:

Chairman, Istituto di Matematica, University of Pisa, for several years
Chairman, Istituto di Algebra e Geometria, University of Padova, for several years
Director, Seminario Matematico (= collection of all mathematical institutes), University of Padova, again for several years
Member Board of Trustees, University of Padova, for two years
Vice–President, Istituto Nazionale Alta Matematica Francesco Severi, since 1979

Membership in Academies:

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Accademia Patavina Scienze, Lettere e Arti
Accademia Nazionale dei Lincei
Accademia Nazionale delle Scienze detta dei XL

Nationality (and citizenship): Italian

* Barsotti himself wrote these notices for his Curriculum Vitae

Barsotti's Publications

1. *Studi sopra le algebre senza base finita*, Rend. Acc. Naz. Lincei, 1, 1946, pp.1187–1189
2. *Algebre senza base finita* (I), Ann. Matem. Pura e Appl., 26, 1947, pp.57–66
3. *Algebre senza base finita* (II), ibid., 27, 1948, pp. 243–260
   Errata ..., ibid., 28, 1949, p. 346
4. *A proof of two fundamental theorems on linear transformations in Hilbert space, without use of the axiom of choice*, Bull. Amer. Math. Soc., 53, 1947, pp.943–949
5. *Ricerche sopra le algebre divisorie di tipo 1, e sopra le algebre divisorie non algebriche*, Rend. Matem. e sue Appl., 7, 1948, pp.1–30
6. *Il gruppo di Brauer delle algebre semplici di tipo 1*, Rend. Acc. Naz. Lincei, 3, 1947, pp. 188–192
7. *Sopra alcune proprietà delle sub–algebre normali di un'algebra di tipo 1*, Rend. Matem. e sue Appl., 7, 1948, pp. 184–193
8. *Elementi algebrici di algebre divisorie non algebriche*, Ann. Scuola Norm. Sup. 14, 1945, pp. 31–45
9. *Osservazioni elementari intorno al differente di un corpo o di un'algebra sopra un corpo algebrico o sopra un corpo di funzioni algebriche di una variabile*, Boll. Unione Matem. Ital., 3, 1948, pp. 3–7
10. *Struttura delle algebre semplici di genere basso su corpi di funzioni algebriche di una variabile*, Rend. Matem. e sue Appl., 8, 1949, pp. 134–167
11. *Valutazioni nelle algebre divisorie senza base finita*, ibid., pp. 168–185
12. *Algebraic Correspondences between algebraic varieties*, Ann. of Math., 52, 1950, pp. 427–464
    Errata..., ibid., 53, 1951, p. 587
13. *Local properties of algebraic correspondences*, Trans. Amer. Math. Soc., 71, 1951, pp. 349–378
14. *Intersection theory for cycles of an algebraic variety*, Pac. Journ. of Math., 2, 1952, pp. 473–521
15. *A note on abelian varieties*, Rend. Circ. Matem. Palermo, 2, 1954, pp. 236–257
16. *Structure theorems for group–varieties*, Ann. Matem. Pura e Appl., 38, 1955, pp. 77–119
17. *Il teorema di dualità per le varietà abeliane ed altri risultati*, Rend. Matem. e sue Appl., 13, 1954, pp. 98–114
18. *Un teorema di struttura per le varietà gruppali*, Rend. Acc. Naz. Lincei, 18, 1955, pp. 43–50
19. *Factor sets and differentials on abelian varieties*, Trans. Amer. Math. Soc., 84, 1957, pp. 85–108
20. *Algebraic group–varieties*, Bull. Amer. Math. Soc., 62, 1956, pp. 519–530
21. *Abelian varieties over fields of positive characteristic*, Rend. Circ. Matem. Palermo, 5, 1956, pp. 145–169
22. *Gli endomorfismi delle varietà abeliane su corpi di caratteristica positiva*, Ann. Sc. Norm. Sup., 10, 1956, pp. 1–24
23. *Repartitions on abelian varieties*, Ill. Journ. of Math., 2, 1958, pp. 43–70