

Markus Reiner Eightieth Anniversary Volume

Contributions to Mechanics

Edited by

David Abir

Technion – Israel Institute of Technology, Haifa



CONTRIBUTIONS TO MECHANICS

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PROFESSOR MARKUS REINER, Dr. Tech.

**Gerard Swope Professor of Mechanics,
Technion—Israel Institute of Technology, Haifa.**

**Member of The Israel Academy of the Sciences
and the Humanities.**

Born 5 January 1886.

CONTRIBUTIONS TO MECHANICS

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THE QUEEN'S AWARD
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PREFACE

TO THE numerous friends of Professor Markus Reiner it may come as a surprise to learn that he is in his eighties now (born 5 January 1886). His vitality and activity defy his age. Of the dozens of research projects conducted by Reiner, the two most important were published at the ages of 59 and 62.[†] In 1947, being 61 years old (the retirement age for many people from active work), he opened a new and flourishing chapter in his life as Professor of Mechanics at the Technion—Israel Institute of Technology, Haifa, thus becoming an academic after almost 40 years of work as a practising civil engineer. At the age of 70 Reiner exhibited his centripetal pump, which caused a stir and keen discussion among professional circles.

Reiner continues to display vivid interest and energy in many facets of activity, both intellectual and physical; he still publishes papers and swims almost daily in the sea or pool.

May he carry on his energetic and fruitful activity for many more years!

The prolific and diverse contribution of Professor Reiner in the field of Mechanics in general, and Rheology in particular, is well known. It stands out not only in quality, but also in quantity. Though he spent the first four decades of his adult life as a full-time engineer, doing research as a hobby in his spare time only (apart from two years devoted solely to research), he has published to date an impressive number of publications—172 papers and notes. The more significant of these, sixty-eight in number, are given in the selected list of his works, which follows this Preface. The list also includes Reiner's three fundamental books (one of them written in collaboration with Dr. G.W. Scott Blair), which were translated into several languages.

Reiner's work has been recognized by numerous academic and professional institutions, both in his own country, Israel (where he has been living close to half a century), and in a number of other countries. The list of awards, prizes, honorary memberships, etc., bestowed upon him by many learned societies is both lengthy and noteworthy.

[†] (1) "A mathematical theory of dilatancy", *American Journal of Mathematics* 67, 350–62 (1945). (2) "Elasticity Beyond the Elastic Limit", *ibid.* 70, 433–46 (1948).

The present book, too, is a manifestation of esteem and affection to Professor Reiner, expressed by thirty-three authors from various countries, who eagerly contributed original works. The number of contributors might easily have been much larger; but in order to avoid too voluminous a book, invitations for papers were confined to personal friends of Markus Reiner in the narrow professional field. Even so, it is quite likely that a good number of people who might fall within this category of friends have been left out. To these, sincerest apologies are hereby offered; any such omission was unintentional and a result of unawareness.

The papers in this volume are conveniently grouped in parts, according to their contents. This grouping, however, should not be considered as rigid; neither should it be taken as a measure of relative merit or significance. In each part, the papers are arranged in alphabetical order of the authors.

The list of contributions opens with a biographical survey of Reiner's life, written by Dr. Scott Blair—his close friend for almost four decades.

In editing the volume no attempt was made to unify the style of writing of the various authors; it was thought that the personal way of presentation of the individual contributors should be preserved. Some editorial changes, however, have been introduced in the manuscripts, either for the sake of clarity of the subject-matter or in accordance with English language requirements. By the same token, the spelling of words which are written differently in Great Britain and America has been left intact according to preference of the individual authors; however, in order to avoid misunderstandings and confusion, uniformity has been adopted throughout the book in designating the serial numbers of References by square brackets, and of equations by parentheses.

Finally, thanks are expressed to the authorities of the Technion-Israel Institute of Technology, Haifa, for their encouragement in the preparation of this volume; and sincere gratitude and appreciation are due to Eng. E. Goldberg, who contributed significantly as language editor for part of the material.

DAVID ABIR

SELECTED LIST OF PUBLICATIONS BY MARKUS REINER

BOOKS

- 1943: *Ten Lectures on Theoretical Rheology*, R. Mass, Jerusalem. (Translated into Russian, Moscow, 1947.)
- 1949: *Deformation and Flow*, H. K. Lewis & Co., London. (Translated into Japanese, Tokyo, 1957.)
- 1949: *Twelve Lectures on Theoretical Rheology* (revised and enlarged edition of 1943 book), North Holland Publishing Co., Amsterdam. (Translated into French, Paris, 1955; and Polish, Warsaw, 1958.)
- 1957: *Agricultural Rheology* (in collaboration with G. W. SCOTT BLAIR), Routledge & Kegan Paul, London. (Translated into Japanese, Tokyo, 1960.)
- 1960: *Lectures on Theoretical Rheology* (revised and enlarged edition of 1949 book), North Holland Publishing Co., Amsterdam.
- 1960: *Deformation, Strain and Flow* (revised and enlarged edition of 1949 book), H. K. Lewis & Co., London. (Translated into Russian, Moscow, 1963; and German, Munich, 1967.)

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- 1911: Deformationswiderstand, tangentielle Kohäsion und Reibungswiderstand von Flüssigkeiten, *Zeitschrift des oesterreichischen Ingenieur- u. Architekten Vereins*, **63**, 803–7.
- 1914: Ueber die Torsionsbeanspruchung von Wellen, *Zeitschr. d. oesterr. Ing. u. Arch. Ver.* **18**, 1–6.
- 1915: Ueber das Auftreten von Normalspannungen bei der Torsion prismatischer Stäbe, *Oesterr. Wochenschrift fuer den oeffentlichen Baudienst*.
- 1917: Ueber die Berechnung der Auflagereibung von Tragwerken, *Zeitschr. d. oesterr. Ing. u. Arch. Ver.* **69**, 574–6, 584–7.
- 1924: Structural steelwork reinforced with concrete, *Concrete and Constr. Eng.* **19**, 445.
- 1925: Ueber die Torsion prismatischer Stäbe durch Kräfte, die auf den

- Mantel einwirken. *Zeitschrift f. angewandte Mathematik u. Mechanik*, **5**, 409–17.
- 1925: Safety of retaining walls against sliding, *Concr. and Constr. Eng.* **20**, 183.
- 1926: Ueber die Stroemung einer elastischen Fluessigkeit durch eine Kapillare, *Kolloid-Zeitschrift* **39**, 80–87.
- 1926: Zur Theorie der Strukturtrubulenz, *Kolloid-Zeitschr.* **39**, 314–15.
- 1927: (in collaboration with R. RIWLIN) Ueber die Stroemung einer elastischen Fluessigkeit im Couette Apparat, *Kolloid-Zeitschr.* **43**, 1–5.
- 1929: The general law of flow of matter, *J. Rheology* **1**, 11–20.
- 1930: In search of a general law of flow of matter, *Rheology* **1**, 250–60.
- 1930: Zur Hydrodynamik der Kolloide, *Zeitschr. f. angew. Math. u. Mech.* **10**, 400–13.
- 1931: Slippage in a non-Newtonian liquid, *Rheology* **2**, 337–50.
- 1932: Outline of a systematic survey of rheological theories, *Rheology* **3**, 245–56.
- 1932: Die Berechnung des Einflusses einer festen Wand auf den Aggregatzustand einer Fluessigkeit aus Viskositaetsmessungen, *Physikalische Zeitschrift* **33**, 499–502.
- 1933: Ueber den Gebrauch der Potenzfunktion zur Darstellung einer naturgesetzlichen Beziehung, *Die Naturwissenschaften* **21**, 294–9.
- 1933: A contribution to the theory of the straight beam, *Quarterly J. of Mathematics* (Oxford), **4**, 12–20.
- 1933: (in collaboration with E. C. BINGHAM) The rheological properties of cement and cement mortar stone, *Physics* **4**, 88–96.
- 1933: (in collaboration with R. SCHOENFELD-REINER) Viskometrische Untersuchungen hochmolekularer Naturstoffe: Kautschuk in Toluol, *Kolloid Zeitschr.* **65**, 44–62.
- 1934: The theory of non-Newtonian liquids, *Physics* **5**, 321–41.
- 1934: Viscometric studies of rubber solutions, *Physics* **5**, 342–9.
- 1938: (in collaboration with A. FREUDENTHAL) Failure of a material showing creep (a dynamical theory of strength), *Proc. 5th Intern. Congr. Appl. Mechanics*.
- 1939: (in collaboration with K. WEISSENBERG) A thermodynamical theory of the strength of materials, *Rheology Leaflet* No. 10, pp. 12–20.
- 1945: A mathematical theory of dilatancy, *Am. J. Math.* **67**, 350–62.
- 1945: (in collaboration with A. ARNSTEIN) Creep of cement and cement mortar, *Civil Engineering* **40**, 198–202.

- 1946: The coefficient of viscous traction, *Am. J. Math.* **68**, 672–80.
- 1946: A simple instrument for measuring blast, *J. Scientific Instruments* **23**, No. 12.
- 1948: Research on the workhardening of polycrystalline metals. Palestine Board for Scientific & Industrial Research, Jerusalem.
- 1948: Research on the rheological properties of bitumen and asphalt. Palestine Board for Scientific & Industrial Research, Jerusalem.
- 1948: Elasticity beyond the elastic limit, *Am. J. Math.* **70**, 433–46.
- 1949: On volume or isotropic flow, *Applied Scientific Research A.* **1**, 471–88.
- 1949: (in collaboration with G. W. SCOTT BLAIR and G. MOCQUOT) Sur un aspect de mécanisme de formation des lainures dans le fromage de Gruyère compte, *Le Lait* **29**, 351–7.
- 1949: (in collaboration with G. W. SCOTT BLAIR and H. B. HAWLEY) The Weissenberg effect in sweetened condensed milk, *J. Soc. Chem. Ind.* **68**, 327–8.
- 1950: The volume flow of asphalt, *J. Soc. Chem. Ind.* **69**, 257–60.
- 1950: (in collaboration with A. ARNAN and M. TEINOWITZ) Research on loading tests of reinforced concrete floor structures. Research Council of Israel, Jerusalem.
- 1951: An investigation into the rheological properties of bitumen: Maxwell body and elastic dispersions. *Bull. Res. Council. Israel* **1**, No. 3, 5–25.
- 1952: (in collaboration with I. BRAUN) Problems of cross-viscosity, *Quart. J. Mech. Appl. Math.* **5**, 42–53.
- 1952: (in collaboration with I. BRAUN) Note on dimensions in tensor analysis, *Bull. Res. Council. Israel* **1**, 81–82.
- 1952: A possible cross-viscosity effect in air, *Bull. Res. Council. Israel* **2**, 65.
- 1952: A tea-pot effect, *Bull. Res. Council. Israel* **2**, 265.
- 1953: On volume viscosity, *Proc. 8th Intern. Congr. Appl. Mechanics*, Istanbul, p. 273.
- 1954: Volume rheology, *Proc. 2nd Intern. Congr. Rheology*, 1953, Butterworths, London, pp. 310–15.
- 1955: The complete elasticity law for some metals according to Poynting's observations. *Appl. Sci. Res.*, **A. 5**, 281–95.
- 1956: The tea-pot effect, *Physics Today* **9**, 16–20.
- 1956: (in collaboration with M. HANIN) On isotropic tensor functions and the measure of deformation *ZAMP* **7**, 377–93.
- 1956: Phenomenological macrorheology, *Rheology I* (ed. F. EIRICH), pp. 9–62, Academic Press, New York.

- 1956: (in collaboration with B. POPPER) The application of the centripetal effect in air to the design of a pump, *Brit. J. Appl. Phys.* **7**, 452–3.
- 1957: A centripetal pump effect in air, *Proc. Roy. Soc. A.* **240**, 173–89.
- 1958: (in collaboration with L. RINTEL) The complete stress–strain law in infinitesimal elasticity, *Bull. Res. Council. Israel* **6C**, 113–26.
- 1958: Rheology, *Encyclopaedia of Physics* **6** (ed. S. FLUEGGE), pp. 434–550, Springer-Verlag, Berlin.
- 1958: The centripetal-pump effect in a vacuum pump, *Proc. Roy. Soc. A.* **247**, 152–67.
- 1959: The flow of matter, *Scientific American* **201**, 122–38.
- 1960: The rheology of concrete, *Rheology* **3** (ed. F. EIRICH), pp. 341–64, Academic Press, New York.
- 1960: Cross-stresses in the laminar flow of liquids, *Physics of Fluids* **3**, 427–32.
- 1960: The stress–strain relation of elasticity and the measure of strain, *ZAMM* **40**, 415–20.
- 1960: Plastic yielding in anelasticity, *J. Mech. Phys. Solids* **8**, 255–61.
- 1962: (in collaboration with A. FOUX) Cross-stresses in the flow of air at reduced pressures, IUTAM Intern. Symposium, *Second-order Effects in Elasticity, Plasticity and Fluid Dynamics*, Haifa, 1962 (ed. M. REINER and D. ABIR), Pergamon Press, Oxford, 1964.
- 1962: (in collaboration with Z. KARNI) The general measure of deformation, IUTAM Symposium, Haifa, *ibid.*
- 1964: (in collaboration with A. FOUX) Cross-stresses in the flow of rarefied air, *3rd Intern. Congress in the Aeronautical Sciences*, Stockholm, 1962, Spartan Books, Washington, D.C.
- 1965: (in collaboration with A. FOUX) Cross-stresses in the flow of different gases, *Topics in Applied Mechanics—Schwerin Memorial Volume* (ed. D. ABIR, F. OLLENDORFF and M. REINER), Elsevier Co., Amsterdam.
- 1965: (in collaboration with D. ABIR and H. MANOR) The solid viscosity of mild steel, *ibid.*
- 1965: Anomalous viscosity, *Research Frontiers in Fluid Dynamics* (ed. SEEGER and TEMPLE), pp. 171–91, Interscience Publishers, New York.
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- 1965: Second order stresses in the flow of gases, *Proc. Intern. Congress Rheology*, Providence, 1963, pp. 267–79.

- 1966: Finite simple torsion of a circular cylinder, *Israel J. Technology* **4**, 187–92.
- 1966: Deformation, *Encyclopedia of Polymer Science and Technology* **4**, 620–47, John Wiley & Sons, New York.
- 1967: (in collaboration with Z. KARNI) The complete tensor expressions of the Green and Almansi measures of deformation, *ZAMP* **18**, 131–2.

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PROFESSOR MARKUS REINER: A BIOGRAPHICAL SKETCH

G. W. SCOTT BLAIR

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England*

I FEEL very much honoured to have been invited to write a brief biographical sketch for the Volume commemorating the eightieth birthday of my old friend Markus Reiner. Although we live in different continents, we have worked together on rheology over a period of nearly 40 years and are the closest of friends.

Markus Reiner was born on 5 January 1886, of Jewish parents, in Czernowitz, at that time the capital of the Province of Bukowina which was part of the Austro-Hungarian Empire. The population of Bukowina was mixed: Ukrainians in the north, Rumanians in the south, and about 25% of Jews, who spoke either Yiddish or German, but did not think of themselves as Germans.

His father was, in turn, wine-merchant, restaurateur and proprietor of a delicatessen shop. He and his wife had to work hard to support themselves and their six children. Markus was the youngest child and he was much helped by an older sister, who, among other things, taught him to play the piano, a hobby which he has enjoyed all his life.

He was at Primary and Secondary Schools in Czernowitz and then went to study Civil Engineering at the famous "Technische Hochschule" at Vienna. Among his teachers was J. Finger, whose work covered Second-order Elasticity. Reiner had no idea at this time that he would some day himself work in this field. In 1909 he obtained the degree of "Ingenieur" and, finding better prospects for a good position in Berlin, he moved there as an employee in an office of structural design. (In those days, one was free to move from any one country to another to find a suitable post—excepting only Russia.) He also worked for a time in Essen but later returned to Czernowitz to be near his parents, having found a position in the Structural Design Office of the State Railways Directorate. At this time he was much influenced by the works of Tolstoy and the early writ-

ings of Martin Buber, later to become a close friend. The Jews in Bukovina were mostly in trade, and the young Markus felt the call of the soil, especially in the land of his ancestors, "Erets Israel". He became a Zionist but the First World War started before he could implement his plans to go to Palestine. Just before the war started he was awarded the degree of Doctor of Technology at Vienna. His thesis dealt with problems of torsion and he had already published a paper on Lubrication Theory.

Early in the war the Russians occupied Czernowitz, and Reiner found himself a lieutenant in the Engineering Corps of the Austrian Army, supervising such projects as the reconstruction of blown-up bridges in Serbia and in Italy. After the war he returned for a time to his position in Czernowitz, then part of Rumania; but in 1922 he emigrated to Palestine.

He was greatly attracted by the experiments in communal living embodied in the Kibbutz Movement and he started to work as a labourer at an orange plantation. But, with his age and experience (he was then 36), his comrades decided that he could do better service in a post under the British Mandate Government in the Department of Public Works in Jerusalem, where he stayed for 25 years, until the British left Palestine and the State of Israel came into existence.

As Chief Civil and Structural Engineer of the Department of Public Works, he had many interesting assignments, including structural support for the Church of the Holy Sepulchre and the restoration of irrigation channels in Jericho, originally constructed by Herod the Great.

At this time Reiner frequently co-operated with the Government Chief Architect, A. St. B. Harrison, doing the structural design work for many beautiful buildings designed by him. They have remained lifelong friends.

Until 1926 Reiner had only occasional time to spare for strictly scientific work. At about this time, however, preparations were made for the realization of the old plan for a Hebrew university in Jerusalem. The first Professor to arrive was Professor A. Fodor, a Colloid Chemist. Reiner was then living in the same house with Fodor and they often talked about subjects of common interest. One day Fodor showed him an article in the *Physikalische Zeitschrift* by Freundlich *et al.* on the problem of flow of an elastic liquid through a tube. Though knowing nothing about colloid or physical chemistry, Reiner saw at once that the solution of the problem given was incorrect and worked out his own solution. This Fodor sent to Wo. Ostwald who published it in the *Kolloid-Zeitschrift*. The paper raised great interest. Hatschek wrote from London, congratulating Reiner; and Herschel also wrote from Washington, draw-

ing attention to the fact that Reiner's equation had already been published by Buckingham. It became known as the Buckingham-Reiner equation. Herschel wrote to Eugene C. Bingham at Lafayette College and Bingham invited Reiner to visit him there. This he did for a couple of months in the summer of 1928. Some years previously Bingham had published his famous finding that "paint is not a liquid, but a soft plastic solid" and had become interested in plasticity. He arranged for a Plasticity Symposium at Lafayette College in which both chemists and physicists participated; and he became convinced that it was necessary to find for them a common ground. When Reiner came to Easton, Bingham said to him, "Here am I, a chemist, and you are an engineer working on similar problems. There must be a field in physics to which these problems belong and in which we both can work." To which Reiner replied, "There is such a field; it is called the Mechanics of Continuous Media." "That will not do," said Bingham, "this highbrow designation will frighten away the chemists." He consulted his Professor of Classical Languages who suggested to him "Rheology" in accordance with Heraclitus' saying: *panta rhei*. A Society of Rheology was founded and first met in Washington, D.C., in December 1929, a meeting I well remember: thus rheology was born. This provided Reiner with a field of scientific activities lasting for the subsequent 40 years. However, he continued to work as an engineer and science was merely his hobby until 1947 when he was appointed Professor of Mechanics at the Technion, now the Israel Institute of Technology in Haifa.

He spent 2 years (1932-4) in America with Bingham, acting as an Assistant Editor for the *Journal of Rheology* which Bingham had founded, and doing certain experimental work in which he was assisted by his second wife, Dr. Rebecca Schoenfeld, who was a chemist. Reiner now came into contact with many scientists working in this field, either personally or by correspondence, among the latter, myself. I sent him an inquiry about the date of a paper of his in order to establish priority with a paper of mine. Such questions of priority have in many cases led to antagonisms, the most famous being that between Newton and Leibnitz and their followers about priority in the discovery of the calculus. In this case, however, it led to an intimate friendship which has lasted ever since.

In the years 1929-30 I had the good fortune to hold a Rockefeller Fellowship in America and, during vacations, to visit as many rheologists as I could find, including, of course, Professor Bingham. Wherever I went, I heard of "Dr. Reiner" who had been in America just before me. In view of this and having already corresponded with him, it was a great

pleasure to my wife and myself when the Reiners, passing through England on their way to America in 1931, came and visited us. This was the first time that we met.

The first Mrs. Reiner (*née* Grete Obernik) died while comparatively young, leaving a son and a daughter. His second wife, Rivka (the Hebrew form of her name by which she was always known), was a Colloid Chemist. Her death a few years ago, after a short illness, was a great tragedy for her husband. She had been his constant helper, companion and scientific colleague for some 33 years. She is also greatly missed by her many friends, one of the closest of whom was my wife. Her two daughters, as well as Reiner's two children by his first marriage, are all happily married and have all taken to the Kibbutz way of life, thus fulfilling, in the second generation, their father's own ambitions as a young man. There are also now numerous grandchildren growing up as "sabras" (native-born Israelis) in the four Kibbutzim where their parents work.

Markus Reiner has never been a political party man; but, with his late friends Magnes, Buber, and others, he sponsored a group, in Mandate times, to work for Jewish-Arab unity. He must indeed be pleased that all his four children are members of a movement which stresses the importance of making every attempt, however difficult, to end the enmity with Israel's Arab neighbours (while not ignoring the need for the defence of Israel in the meantime). The Movement also urges that everything should be done to give Israel Arabs their fullest rights as citizens, a cause dear to Markus Reiner's heart.

Markus Reiner is a citizen of the world. Not only has he travelled widely, but he has encouraged his children so far as has been possible to do the same: one daughter was at school for a time in Reading, England, and another recently spent 2 years in London. He himself comes each year to England and, through his initiative, I have been able to work with him on three occasions in Israel, a country which I came to love as much as Reiner loves England; so we have kept continually in touch since the end of the Second World War. Just before its start, in 1938, Markus, Rivka and his two small daughters came for a few months to England and lived at a farm near my home. No one knows how the Hebrew-speaking children so quickly made contact with the farmer's family! It was during this visit that I discovered that Reiner was not only a fine research worker, but an excellent "tutor". Trained as a chemist, I had never learned tensor analysis, which a rheologist should know something about. A little private coaching in the farm-house, by the light of an oil lamp, was a very great help to me.

Reiner has published several books, including a modern version of a course of lectures (*Lectures on Theoretical Rheology*), which he originally gave at the University of Jerusalem and his *Deformation, Strain and Flow* which has been a help to rheologists in many lands. His books have been translated into many languages. One book, which we wrote together, on “Agricultural Rheology” has, apparently, been more appreciated in Japanese than in its original English version!

For many years, he lectured to students on Strength of Materials; now, his work is mainly research, sponsored by the U.S. Navy, Air Force, and Department of Agriculture. Always popular as a lecturer, he is, perhaps, even better as an after-dinner speaker, having, like most Israelis, so many good stories. At a time when he was obviously in great distress of mind (1963), he spoke at Providence, R.I., U.S.A., at the Fourth International Congress on Rheology. At this after-dinner talk he introduced the non-dimensional “Deborah Number” which he based on the saying of the Prophetess Deborah—“The Mountains Flowed Before the Lord”. Unfortunately bad health prevented my being there, but a number of colleagues who were present afterwards told me that they regarded his speech as one of the highlights of the Congress.

It is difficult to summarize a lifetime of research within a few short paragraphs. I can only try to highlight the principal achievements of Reiner’s massive contribution to rheology. He started, as we have seen, with the independent calculation (in 1926) of Buckingham’s equation (1921) for Bingham systems flowing in capillary tubes. Soon afterwards, with the late Miss R. Rivlin (not to be confused with her nephew, Dr. R. S. Rivlin), he derived a similar equation for coaxial cylinder viscometry. He was also considerably influenced at about this time by the work of Wo. Ostwald and wrote several papers on Structural Turbulence and what used to be called “Structural Viscosity” as well as the Newtonian portion of the flow-curve which Ostwald called the “Laminarast”. He also made a critical study of Ostwald’s use of power equations. Around 1930 equations were often sought for non-Newtonian systems not following Bingham’s equation; and Taylor, Maclaurin and other series were tried. Reiner was in the forefront of attempts to find general equations for these complex systems. He also studied slip at the capillary wall and proposed a classification of rheological properties which may be regarded as a precursor to the Tables of Deformations drawn up by the (then) British Rheologists’ Club[†] and by the Dutch and other Societies of Rheology many years later.

[†] Now the British Society of Rheology.

Starting with what are now called Maxwell, Kelvin and Saint Venant Bodies, he was responsible for building and collating many of the model systems used by other workers.

Around 1938–9 he extended his engineering work on strength of materials, publishing several rheological papers on this subject; and immediately after the war, he turned his attention to dilatancy and work-hardening as well as publishing further work on rheological classification.

He was among the first to predict the importance of “secondary normal stresses” and foretold the existence of what are now called “Weissenberg Effects” before anybody had seen such phenomena.

Reiner was always ready to turn his skill as an engineer to unlikely uses. A French colleague (M. Mocquot) and I once consulted him about the causes of some characteristic cracks in Gruyère cheese and he at once found an analogy with cracks which he had often observed in concrete buildings! Later, we were able to show the Weissenberg Effect, which he had predicted, to be present in certain aged condensed milks.

Meanwhile, he had time to help substantially in such diverse fields as his wife’s work on rheology of flour doughs and my own studies on bovine cervical mucus.

But perhaps his most exciting and controversial findings came when he had reached an age when many people have retired. Led by observations in the rheology of bitumen, he started studies on the behaviour of gases and, later, liquids, between two very carefully aligned concentric disks, very close together, one of which is rotated at high speeds. He used this for the construction of a centripetal pump. Some aerodynamic experts maintain that the effect is due simply to the imperfections in the alignment of the apparatus and the whole question still remains *sub judice*.

Meanwhile, Reiner had not forgotten metals and has studied second-order effects in their elastic response; and the changes in length which take place when metal rods are twisted (Poynting Effects).

With Dr. David Abir, he edited the *Proceedings of an International Symposium*, arranged by IUTAM on *Second-order Effects in Elasticity, Plasticity and Fluid Dynamics*, held in Haifa in 1962. It may be mentioned that in his introductory lecture, Professor Truesdell, the well-known authority, said that “direction was given to this field by two classical papers of Reiner, published in 1945 and 1948” in the *American Journal of Mathematics*. Few rheologists—and indeed few research workers in any field—have worked over so wide a range of subjects and materials. Prophets may go without honour in their own countries but with rheologists this is not always so. Markus Reiner’s work has been very fully appreciated in Israel.

He has received many honours, including the Israel Prize, the Rothschild Prize and the Weizmann Prize of the City of Tel-Aviv, and he is a member of the Israel Academy of the Sciences and the Humanities.

Long ago, it was true indeed that “The days of our years are three score years and ten; and if by reason of strength they be fourscore years, yet is their strength labour and sorrow”. Fortunately, today, modern medicine and surgery have extended this span. Markus Reiner has had his share of labour and sorrow, but I am sure that he feels that he has had much happiness too and that he has still many more fruits to contribute to rheology (every time I see him, he passes me *some* useful suggestion). We dare to hope that many more years lie ahead of him, as is indeed very probable in this new age in which we live. So, on his eightieth birthday we may well wish him “Many happy returns of the day”,

יבן ירבו בארץ - ישראל

(And may such men multiply in the Land of Israel.)