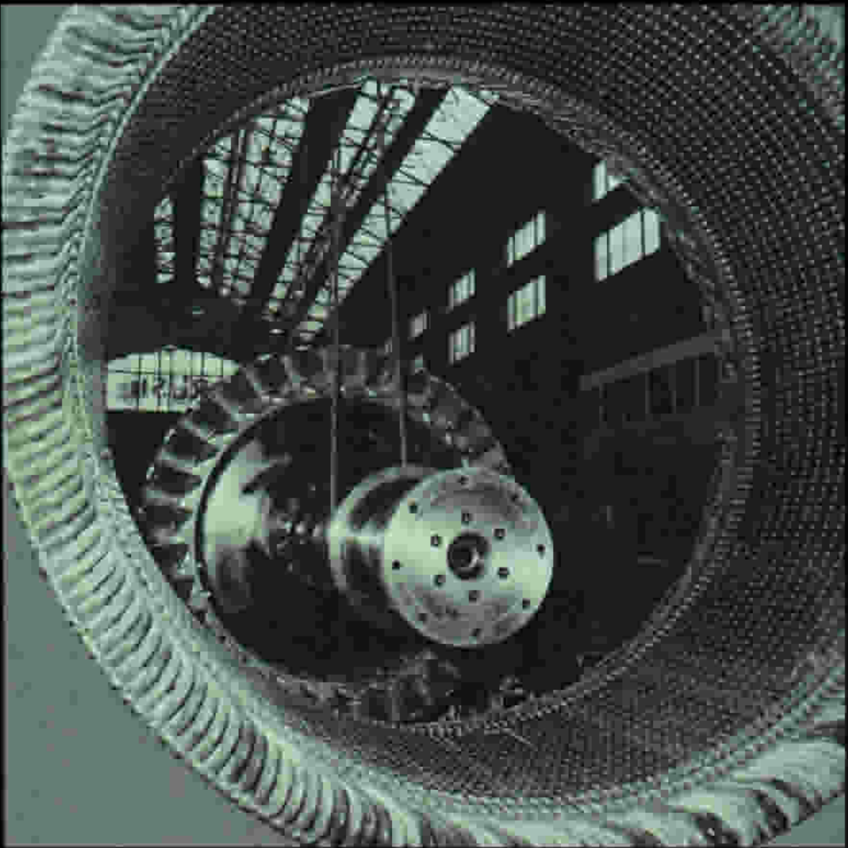
ELECTRIC MOTOR HANDBOOK

Edited by

B. J. Chalmers



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Electric Motor Handbook

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Edited by
B. J. Chalmers
With specialist contributors

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Preface

The opportunity to edit a major new handbook, to stand alongside the long established *J & P Transformer Book* and *J & P Switchgear Book,* is at once an honour and a challenge. The J & P Books have been widely used by generations of engineers all over the world and, in setting out to create this *Electric Motor Handbook,* I was conscious that the target was high.

As with J & P books, the aim has been to compile an authoritative handbook which will be of real practical use to engineers working in a wide range of capacities including plant design, equipment specification, commissioning, operation and maintenance. Such a task would have been over-ambitious for most single authors, and certainly for the present editor. I therefore readily acknowledge the expertise contributed by the individual chapter authors, which represents their combined experience accumulated over many decades. The Chapter contents are essentially theirs and I am greatly indebted to them for the manner in which they responded to the tasks I set them. A world-wide rather than parochial view has been sought by including international authors, considerations and illustrations.

While the chapters are, in the main, independent it may be observed that some points are covered in more than one chapter, e.g. under both commissioning and maintenance. I have taken the view that the reader seeking information in a particular context should not be disappointed. In consequence, a little duplication has been accepted.

I must also acknowledge that the original proposal for this form of handbook was made by K. K. Schwarz, Dr M. R. Lloyd and the late J. C. H. Bone, all then with Laurence, Scott and Electromotors Ltd. It was only after force of circumstances prevented them pursuing the project that I became involved. Their initial outline plans gave me a good starting point.

As one who has for about 30 years been mainly concerned with electromagnetic aspects of electrical machines, I confess I have long believed that the majority of their operational problems, and practically all the manifestations of failure or breakdown, are non-electrical, whether they be of the nature of fracture, abrasion, burning or explosion. Accordingly, a significant proportion of this book relates to the causes, occurrence and avoidance of such problems. It is to be hoped that the availability of this Handbook will help many readers avoid such misfortunes.

The vast majority of textbooks on electrical machines concern methods of performance analysis, while a rather smaller number concern the design process. This book is not primarily directed towards either of these activities. Rather it is intended as a user's handbook, written by engineers for engineers, which may provide a useful source of reference.

Although motor size is strictly dependent upon torque rather than power, the scope of the Handbook has been broadly defined as relating to rotating machines of above about 10 kW output (with normal industrial supply frequency). This somewhat arbitrary lower limit was set in order to restrict the coverage to the more common machines and to eliminate the very diverse range of small motors. It was considered that to attempt to include the great variety of motor types of less than 10 kW output within a reasonable limit of total page number would inevitably have made the treatment somewhat formless and superficial. Similarly, linear motors are excluded.

At a time when many senior engineers are retiring and there is serious concern regarding the shortfall in young recruits into electrical power engineering, it is pleasing to be able to participate in activities which may assist in alleviating this deficiency. The Chapter authors are all, in their own spheres, contributing towards promotion of our profession while it is incumbent upon those of us involved in teaching to stimulate the interest of students in what we believe to be a challenging career area. In Manchester, for so long the home of much heavy electrical engineering, we have taken an initiative and created the 'Manchester Machines Research Group', aiming to provide an increased service in research, training and consultancy. I hope the *Electric Motor Handbook* will serve as a useful adjunct to these endeavours, helping to disseminate technical interests and information to a wide readership. If it approaches the popularity of the J & P books it will provide a valuable service to our industry.

B. J. Chalmers
Manchester, 1987

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1 Characteristics

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1.1 Introduction

Electric motors convert electrical energy, supplied from an a.c. or d.c. source, to mechanical energy at a rotating shaft. There are several different types of electric motor but here we are concerned with the main ones only, namely: (1) the induction motor and its derivatives which are equipped with a commutator, such as the Schrage motor; (2) the synchronous motor; and (3) the d.c. motor.

All electric motors have certain basic features in common. Each has a stationary member, the stator, and a rotating member, the rotor, separated by an airgap. The stator and rotor each have a magnetic core, usually laminated, although on some high-speed a.c. machines the rotor may be of solid steel. The core carries copper or aluminium windings in slots or on salient poles. The windings are insulated except in the case of cage (squirrel cage) rotors. Details of construction and windings are given in Chapters 5 and 6.

The theory of electric motors is described in many textbooks.[1-8] Only the essentials will be summarized here, for balanced polyphase a.c. machines and d.c. machines in the first instance.

Consider first the principles of operation common to all electric motors. The windings carry currents which may be caused to flow by direct conduction or electromagnetic induction. The currents produce m.m.f. waves of magnitude varying approximately sinusoidally around the airgap, circumferentially (very approximately in the case of the d.c. machine). These space-sinusoidally distributed m.m.f. waves can be represented, as shown in Figure 1.1, by space vectors $\mathbf{F}_s$ and $\mathbf{F}_r$ for stator and rotor, respectively. $\mathbf{F}_s$ and $\mathbf{F}_r$ may be stationary or rotating but, for the production of useful torque, must be stationary relative to each other. They add vectorially in accordance with the equation:

$$\mathbf{F}_m = \mathbf{F}_s + \mathbf{F}_r \tag{1.1}$$

to produce the resultant $\mathbf{F}_m$, called the magnetizing m.m.f. The magnetizing m.m.f. produces the magnetizing flux, which can be represented by the space vector $\boldsymbol{\Psi}_m$. Flux and current react to produce torque. It can be shown that the total torque developed is given by the expressions:

$$T_T = K\,\Psi_m\,F_s\,\sin\delta_{ms} \tag{1.2}$$

and

$$T_T = K\,F_r\,\Psi_m\,\sin\delta_{rm} \tag{1.3}$$

where K is a constant and $F_s\sin\,\delta_{ms} = F_r\sin\,\delta_{rm}$, as can be seen from Figure 1.1.

The relationship between flux and m.m.f. is non-linear because of the well-known B–H properties of the magnetic core. However if, for approximation,

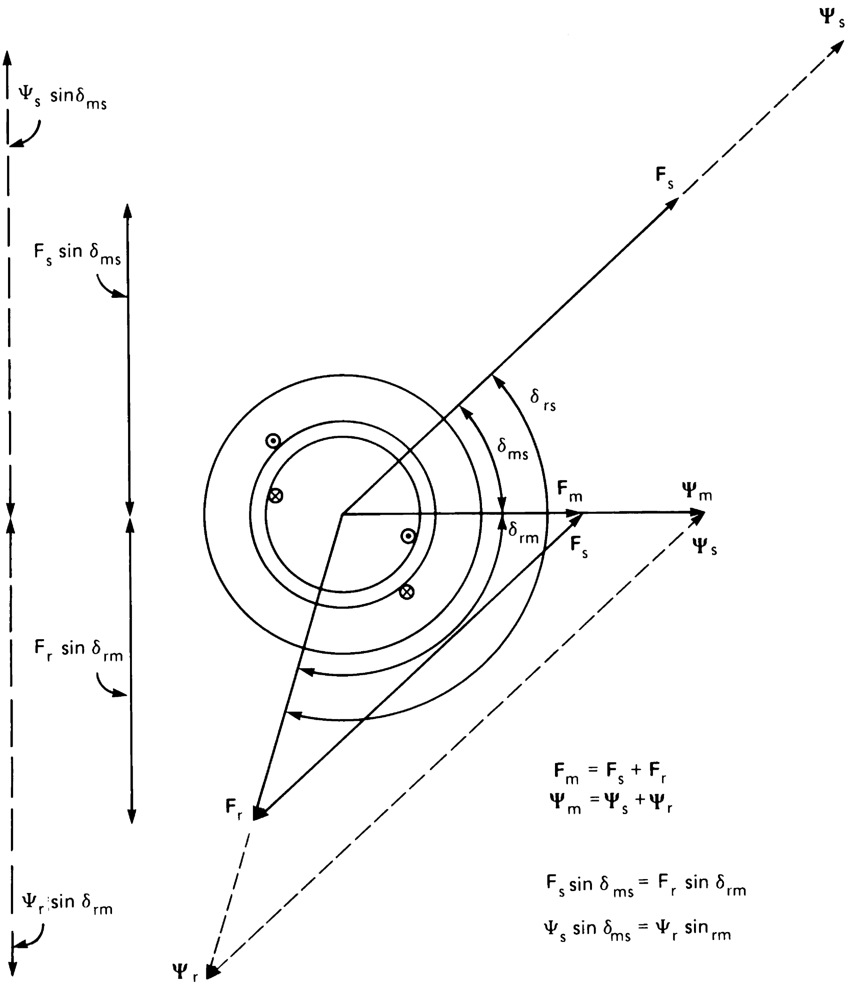


Figure 1.1 The m.m.f. and flux space vectors of an electric motor

magnetic linearity is assumed, i.e. $\mathbf{\Psi}_x \alpha \mathbf{F}_x$, then the m.m.f. space vectors $\mathbf{F}_s$ and $\mathbf{F}_r$ can be regarded as producing separate space vectors $\mathbf{\Psi}_s$ and $\mathbf{\Psi}_r$, respectively, where:

$$\mathbf{\Psi}_m = \mathbf{\Psi}_s + \mathbf{\Psi}_r \tag{1.4}$$

as shown in Figure 1.1.

It follows that, when magnetic linearity is assumed, other expressions for torque can be derived. These can be summarized in the general equation:

$$T_T = K \, F_x \, \Psi_y \sin\delta_{xy} \tag{1.5}$$

where F_x and ψ_y are the magnitudes of m.m.f. and flux space vectors, respectively, and δ_{xy} is the angle between the vectors.

Note that Figure 1.1 includes a representation of a section through the machine in which the directions of current flow in rotor and stator windings are shown. The rotor tends to rotate in the counter-clockwise direction, seeking to maximize the flux linkage by making δ_{rs} equal to zero.

In all polyphase a.c. machines (except the Schrage motor, discussed in Section 1.5.3) the stator m.m.f. wave is developed by alternating currents of supply frequency f_s. It rotates at synchronous speed $N_s = f_s/p$ rev./s, relative to the stator, where p is the number of pole pairs of the winding.

In asynchronous- or induction-type motors, voltages are induced in the rotor windings at frequency $f_r = (N_s - N)p$, where N is the speed of the rotor. These voltages produce currents which develop an m.m.f. wave rotating at speed $N_s - N$, relative to the rotor, i.e. at speed $(N_s - N) + N = N_s$, relative to the stator. Thus, regardless of the speed of the rotor, the m.m.f. space vectors $\mathbf{F}_s$ and $\mathbf{F}_r$ remain stationary relative to each other.

In a synchronous motor, operating in the steady state, the rotor rotates at exactly synchronous speed and therefore no voltage is induced in the rotor windings. The rotor m.m.f. wave is developed usually by d.c. fed to the rotor windings through sliprings. It is therefore stationary relative to the rotor and $\mathbf{F}_s$ and $\mathbf{F}_r$ are again stationary relative to each other.

In a d.c. motor, the field winding on the stator is supplied with d.c. and therefore the stator m.m.f. vector $\mathbf{F}_s$ is stationary. The armature winding, on the rotor, is supplied with d.c. via brushes bearing on a commutator. The action of the commutator simultaneously converts the external d.c. to a.c. within the armature and maintains the rotor m.m.f. vector $\mathbf{F}_r$ stationary and in quadrature with the $\mathbf{F}_s$ vector.

In addition to their contribution to magnetizing flux, the stator currents produce leakage flux which links with the stator windings only, and the rotor currents produce leakage flux which links with the rotor windings only. The effects of leakage flux can be represented by current flowing through leakage inductances.

Equivalent circuits and phasor and vector diagrams, which take account of all the reactions described above, can be used for modelling the performance of the individual machine types. The general forms of the steady state performance characteristics can be deduced from these models.

In deducing the performance characteristics it will be assumed, unless otherwise stated, that the supply voltage and frequency remain constant, regardless of the power taken by the machine.

The steady state performance characteristics of a motor largely determine its suitability for a particular application. However, they give an incomplete picture of the performance and capability of the motor. The complete picture may require an examination of the transient performance of the motor in the system of which it forms a component. Only brief references to transient performance will be made here.

1.2 Modelling of steady state motor performance

The complex interactions in electrical machines can be modelled by relatively simple equivalent circuits when certain simplifying assumptions are adopted. Thus, in the modelling of polyphase a.c. machines, sinusoidal time variation of applied voltage and sinusoidal space variation of airgap m.m.f. will be assumed. Completely balanced operation will also be assumed, enabling performance to be represented on a per-phase basis.

An equivalent-circuit model for a polyphase a.c. machine, operating in the steady state with voltage applied to both stator and rotor windings, is shown in Figure 1.2. This model can be developed in a way which enables the inherent similarities between different machine types to be appreciated. A proper understanding of the model requires a study of one or more of the standard texts mentioned above, but a fair understanding can be gleaned from careful

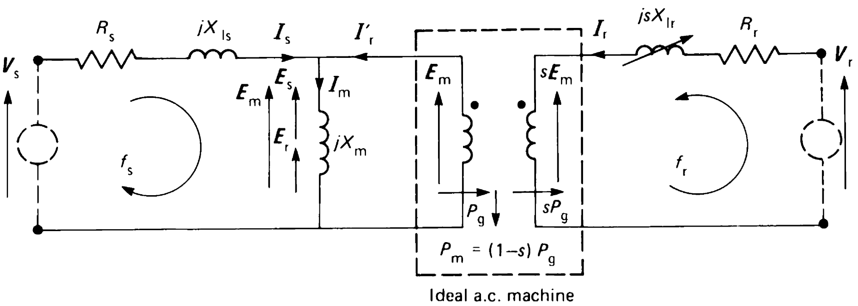


Figure 1.2 Steady state equivalent circuit of a polyphase a.c. machine with voltage applied to both stator and rotor circuits

All stator quantities at stator frequency f_s
All rotor quantities at rotor frequency f_r

V_s	voltage applied to a stator phase
V_r	voltage applied to a rotor phase
I_r	current in a rotor phase
I'_r	component of current in a stator phase (at stator frequency) which produces the same magnetizing effect as I_r
I_m	$= I'_r + I_s$ magnetizing current (the equivalent current in a stator phase required to establish the magnetizing flux)
I_s	total current in a stator phase
jX_m	magnetizing reactance of a stator phase
E_m	$= jX_m I_m$ magnetizing e.m.f. in a stator phase
E_s	$= jX_m I_s$ notional e.m.f. in a stator phase due to stator currents only
E_r	$= jX_m I'_r$ notional e.m.f. in a stator phase due to rotor currents only
jX_{ls}	leakage reactance of a stator phase
jsX_{lr}	leakage reactance of a rotor phase at rotor frequency where jX_{lr} is its value at stator frequency
R_s	resistance of a stator phase
R_r	resistance of a rotor phase
P_g	power transmitted from stator to airgap
sP_g	power transmitted from airgap to rotor circuit
P_m	$= (1-s)P_g$ mechanical power developed

consideration of the circuit parameters and the phasor and vector diagram shown in Figure 1.3.

The first point to note is that all stator parameters are referred to stator frequency f_s and all rotor parameters to rotor frequency f_r where $f_r = sf_s$ and s is the slip $(N_s-N)/N_s$, as already defined. Furthermore, rotor parameters have been referred to the effective number of turns per phase of the stator winding, e.g.:

rotor winding resistance $R_r = n^2 R_{r\ (\text{actual})}$,

rotor voltage $V_r = nV_{r\ (\text{actual})}$

where n is the ratio of effective turns per phase, stator to rotor.

The equivalent circuit incorporates a representation of an ideal polyphase motor. This conceptual device:

(1) Has no winding resistance.
(2) Has perfect coupling between stator and rotor windings and therefore no leakage flux.
(3) Has infinite permeability in its main magnetic path and therefore requires no magnetizing m.m.f.

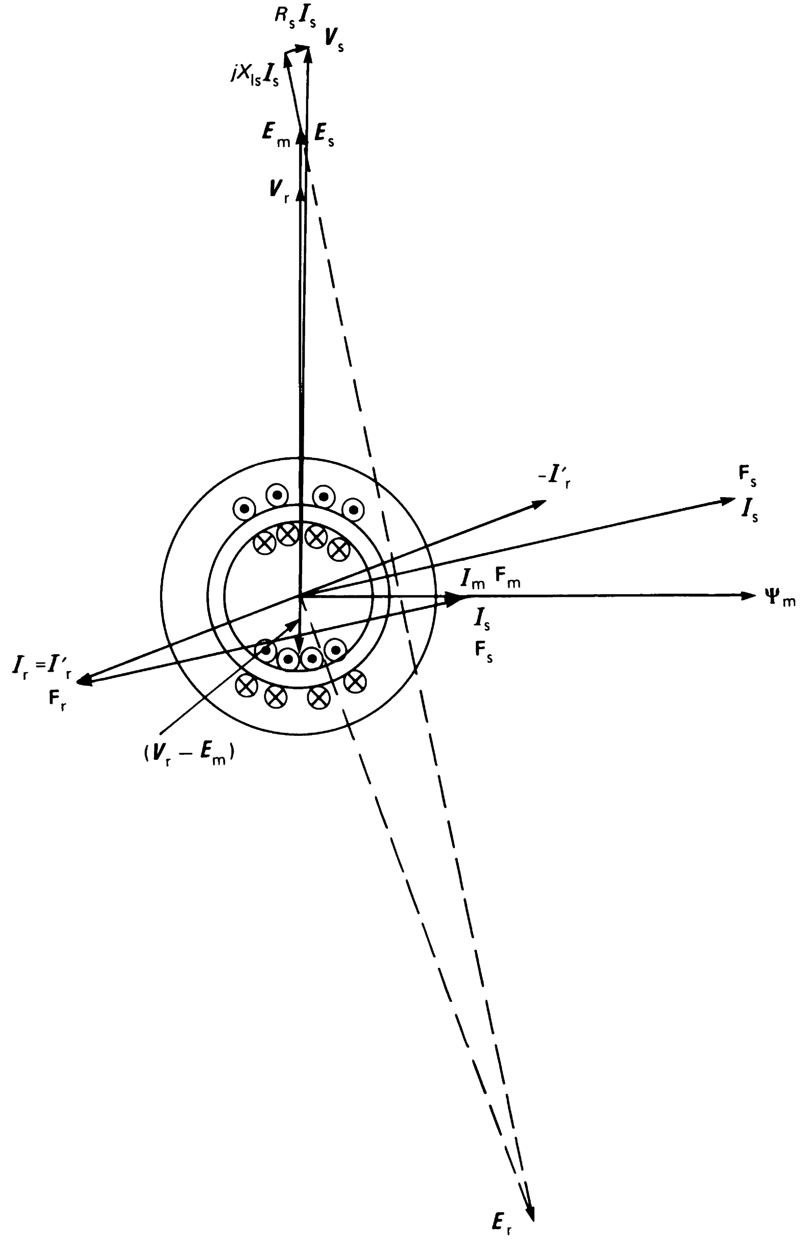


Figure 1.3 Phasor and vector diagram for the equivalent circuit of Figure 1.2 for $s = 1$. (The current direction symbols represent conditions in the reference phases, not resultant current sheets)

(4) Converts voltage, frequency and power from stator to rotor in the ratio 1:s, but converts current in the ratio 1:1.

The effects of:

(1) Winding resistances are accounted for in resistors R_s and R_r.
(2) Leakage flux in leakage reactances jX_{ls} and jsX_{lr}.
(3) Finite permeability in magnetizing reactance jX_m carrying magnetizing current I_m.
(*Note:* Losses in the magnetic core are neglected at this stage.)

Figure 1.3 shows a phasor and vector diagram for a special case in which the rotor is locked with the axes of the stator and rotor reference phases aligned, and in which the voltage V_r is in phase with the e.m.f. E_m. To enable 'time' phasors and 'space' vectors to be meaningfully related, the diagram includes a representation of the reference phases in a transverse section through the machine, assuming two-pole construction. For the locked rotor condition, $s = 1$, $f_r = f_s$, and all phasors and vectors rotate in the counterclockwise direction at the same angular velocity $\omega_s = 2\pi f_s$ rad./s. The diagram is drawn for the instant that the magnetizing current $i_m = I_{m(\max)} \cos \omega_s t$ has its maximum value, i.e. when $\omega_s t = 0$.

The positive direction of current flow is that shown in the conductors of the stator phase. The so-called 'back e.m.f.' convention is adopted for e.m.f.[9] Back e.m.f. $e = d(Li)/dt$, corresponding to $e = d\Psi/dt$, is an e.m.f. in the direction opposing positive current flow.

The voltage driving rotor current in the positive direction is $(V_r - E_m)$. This voltage produces a current I_r, lagging $(V_r - E_m)$ by the angle arctan X_{lr}/R_r. This current is negative at the reference instant, giving the current directions shown in the rotor conductors. A current I_r' in a stator phase would produce the same magnetizing effect as I_r. The stator current I_s must balance this current and in addition establish the magnetizing current I_m. Thus, $I_s = (I_m - I_r')$ or, otherwise expressed:

$$I_m = I_s + I_r' \tag{1.6}$$

Evidently, the stator current is positive at the reference instant.

The resultant polyphase stator and rotor current 'sheets' which are, of course, displaced from the reference phases, produce the m.m.f. space vectors $\mathbf{F}_s$ and $\mathbf{F}_r$ whose axes are cophasal with the time phasors I_s and I_r', respectively. The m.m.f. space vectors produce the resultant magnetizing m.m.f. space vector $\mathbf{F}_m$ in accordance with Equation (1.1).

Alternatively expressed, the polyphase magnetizing currents produce the magnetizing m.m.f. space vector whose axis is cophasal with the time phasor I_m. Corresponding current phasors and m.m.f. space vectors may be taken as equal in per-unit terms.

The magnetizing m.m.f. produces the magnetizing flux, represented by the space vector $\mathbf{\Psi}_m$. The magnetizing flux induces the magnetizing e.m.f.s E_m in a stator phase and, in general, sE_m in a rotor phase.

E_s and E_r are notional e.m.f.s in a stator phase, associated with notional fluxes $\mathbf{\Psi}_s$ and $\mathbf{\Psi}_r$ produced by the m.m.f.s $\mathbf{F}_s$ and $\mathbf{F}_r$, respectively, where

$$E_m = E_s + E_r \tag{1.7}$$

The space vectors $\mathbf{\Psi}_s$ and $\mathbf{\Psi}_r$ are not shown in Figure 1.3 but can be inferred from the corresponding vectors in Figure 1.1. Corresponding flux space vectors and stator e.m.f. phasors may be taken as equal in magnitude in per-unit terms.

Only the magnetizing e.m.f. can be regarded as having a real existence, in the sense that it could be measured in a shadow winding. E_s and E_r can be regarded as existing, as components of E_m, only under linear magnetic conditions. Such conditions have been assumed in Figure 1.3 and therefore the triangle formed by the m.m.f. space vectors and that formed by the e.m.f. phasors are similar. The latter is displaced from the former by a counterclockwise rotation through 90°. The triangle formed by the flux vectors would be identical to that formed by the e.m.f. phasors, except for the 90° displacement.

In the analysis of induction motors, the separate existence of $\boldsymbol{E}_s$ and $\boldsymbol{E}_r$ is not normally considered. However, in the analysis of synchronous motors it is normal practice to consider the equivalents of $\boldsymbol{E}_s$ and $\boldsymbol{E}_r$ as existing separately, as will be shown later.

Finally, it is worth pointing out that there is more to the equation $f_r = sf_s$ than just a simple numerical relationship. For a given value of f_s there are only two possibilities, either f_r is determined by s or s is determined by f_r. The former leads to asynchronous operation, the latter to synchronous operation.[10]

Asynchronous operation occurs when f_r is determined continuously by the rotor speed, as it is when:

(1) V_r is zero, as in the normal induction motor.
(2) V_r is supplied through sliprings at continuously varying frequency f_r.
(3) V_r is supplied through a frequency changer converting supply frequency to f_r, e.g. a commutator converting from f_s externally to f_r internally.

Synchronous operation occurs when the rotor is fed through sliprings at a frequency which is independent of rotor speed. The rotor must then rotate at a speed such that $s = f_r/f_s$. In practice, if f_s is the a.c. supply frequency the only stable operating condition arises when $f_r = 0$, i.e. when d.c. is supplied to the rotor and it rotates at normal synchronous speed so that $s = 0$. Synchronous operation can also occur with alternative forms of rotor which enable similar magnetic field conditions to be established.

1.3 The polyphase induction motor

1.3.1 Note on construction

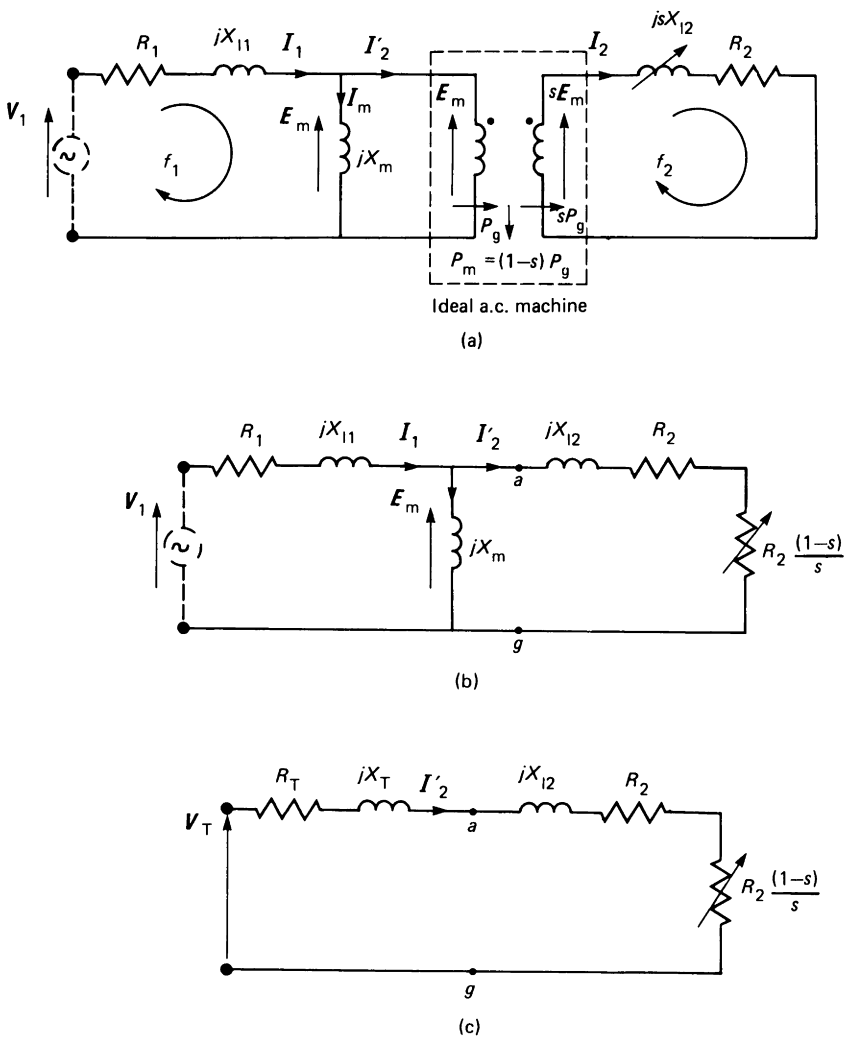
The stator of a polyphase induction motor is wound with a polyphase winding, now almost invariably a three-phase winding. The rotor may be wound with a similar winding but of a different number of turns, star- or delta-connected, with the ends of windings brought out to sliprings. Alternatively, the rotor may have a cage winding consisting of bars through the rotor slots, joined together at each end by endrings. An integral cage winding of aluminium can be formed by the die-casting process.

The airgap is regarded as uniform, in contrast to that resulting from the salient pole construction of d.c. machines and some synchronous machines, as discussed later. However, the fact that the windings are carried in slots punched in the laminations of the magnetic core leads to important second-order effects discussed in Section 1.3.6.

1.3.2 Equivalent circuit and phasor and vector diagram

The reactions in a polyphase induction motor, operating under balanced steady state conditions, can be modelled by an equivalent circuit developed from that shown in Figure 1.2. The modelling of a cage rotor by a circuit applicable to a phase-wound rotor is permissible because, in essence, the rotor manifests itself to the stator as a circuit which produces the m.m.f. vector $\mathbf{F}_r$, rotating at synchronous speed. The stator is unaware not only of the number of phases on the rotor but also of its rotation.

The development of the equivalent circuit to its familiar form is illustrated in Figure 1.4. Figure 1.4(a) is similar to Figure 1.2 except that V_r is taken as zero and,



$$V_T = V_1 \, [jX_m] \, / \, [R_1 + j(X_{l1} + X_m)]$$

$$R_T + jX_T = [jX_m \, (R_1 + jX_{l1})] \, / \, [R_1 + j(X_{l1} + X_m)]$$

Figure 1.4 Alternative forms of steady state equivalent circuit for a polyphase induction motor. (a) Complete form. (b) Normal form. (c) Thévenin-modified form

in accordance with convention, the reference direction of rotor current is reversed and suffixes s and r are replaced by 1 and 2 respectively. In Figure 1.4(b), the circuit is referred to the stator by displacing the representation of an ideal a.c. machine off the diagram to the right. The resulting resistor R_2/s is replaced by resistors R_2 and $R_2(1 - s)/s$, having equivalent total resistance.

Thus, remembering that core losses must be taken into account separately, the electrical losses in the machine are accounted for by the power consumed in the

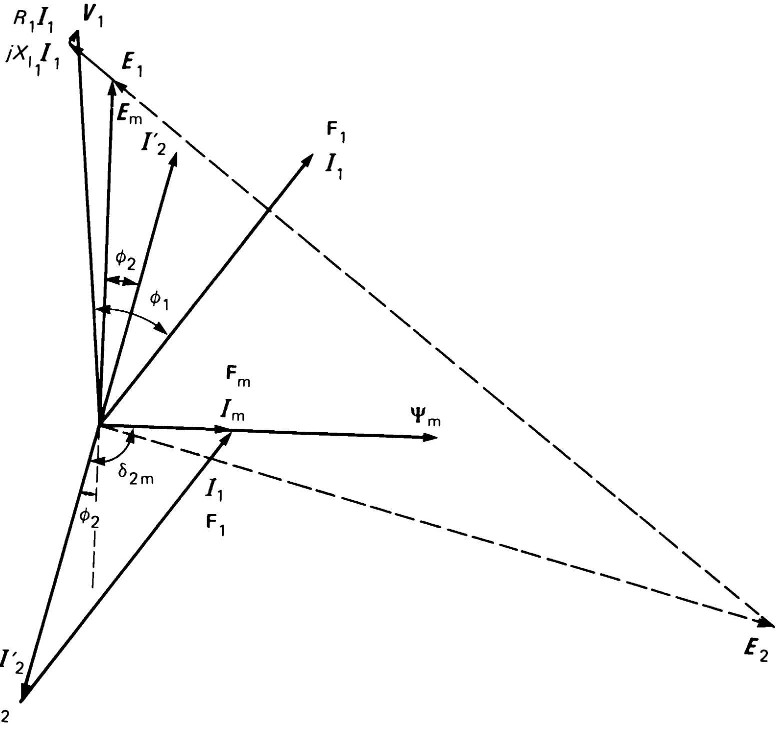


Figure 1.5 Phasor and vector diagram for an induction motor corresponding to the equivalent circuit of Figure 1.4(b)

resistors R_1 and R_2. Therefore, the mechanical power developed is represented by the power consumed in the resistor $R_2(1 - s)/s$.

A typical phasor and vector diagram corresponding to Figure 1.4(b) is shown in Figure 1.5. Note that no rotor-current phasor $\boldsymbol{I}_2$, equivalent to $-\boldsymbol{I}_r$ of Figure 1.3, appears in the diagram, since this phasor would be required here to represent a current of frequency f_2, where $f_2 \neq f_1$. The phasor $-\boldsymbol{I}_2'$, equivalent to $\boldsymbol{I}_r'$ of Figure 1.3, and the space vector $\mathbf{F}_r$ are correctly shown as cophasal. The phasors $\boldsymbol{E}_1 = jX_m\boldsymbol{I}_1$ and $\boldsymbol{E}_2 = -jX_m\boldsymbol{I}_2'$, notional components of $\boldsymbol{E}_m$ for assumed linear magnetic conditions, are included only for later comparison with the corresponding phasors for a synchronous machine (see Figure 1.13, page 21).

It follows from the equivalent circuit and phasor and vector diagram that:

$$\text{Torque per phase} = R_2\frac{(1-s)}{s}\mathrm{I}_2'^2\ \frac{1}{2\pi N}$$

$$= \frac{R_2}{s}\mathrm{I}_2'^2\ \frac{1}{2\pi N_s}$$

$$= \frac{\text{power crossing air gap at a–g}}{2\pi N_s}$$

$$= \frac{\mathrm{E_m\,I_2'}\cos\phi_2}{2\pi N_s} \tag{1.8}$$

where $\mathrm{E_m} \propto \Psi_m \propto \mathrm{F_m}$, $\mathrm{F_2} \propto \mathrm{I_2}$ and $\cos\phi_2 = \sin\delta_{2m}$

Thus, the expression for torque, deduced from essentially power balance considerations, is consistent with the fundamental expression in Equation (1.3).

Given the availability of modern computing aids there can be no justification for introducing the well-known 'approximate' equivalent circuit. However, for our purpose, it is useful to introduce a simplification. By applying Thévenin's theorem to the part of the circuit to the left of a–g in Figure 1.4(b) the simplified circuit of Figure 1.4(c) is obtained. From Figure 1.4(c) the following expressions for the performance characteristics dependent on the current I_2' can readily be deduced:

(1) Stator equivalent of rotor current:

$$I_2' = \frac{V_T}{[(R_T + R_2/s)^2 + (X_T + X_2)^2]^{1/2}} \tag{1.9}$$

(2) Torque developed per phase:

$$T = \frac{R_2}{s} \frac{V_T^2}{(R_T + R_2/s)^2 + (X_T + X_2)^2} \frac{1}{2\pi N_s} \tag{1.10}$$

This has a maximum value at a slip S_{MT} such that:

$$S_{MT} = \frac{R_2}{[R_T^2 + (X_T + X_2)^2]^{1/2}}$$

Evidently, when the other parameters are constant the maximum torque is independent of R_2.

(3) Mechanical power developed per phase:

$$P_m = R_2 \frac{(1 - s)}{s} \frac{V_T^2}{(R_T + R_2/s)^2 + (X_T + X_2)^2} \tag{1.11}$$

This has a maximum value at a slip S_{MP} such that:

$$S_{MP} = \frac{R_2}{R_2 + [(R_T + R_2)^2 + (X_T + X_2)^2]^{1/2}}$$

1.3.3 Performance characteristics of wound-rotor motors

In a wound rotor induction motor, the parameters can be regarded as constants if the effects of saturation of the magnetic paths are neglected. Theoretical performance characteristics can then be calculated from Equations (1.9)–(1.11). However, it is convenient to express values in per-unit of the corresponding values at full load, with slip expressed in its normal per-unit of synchronous speed. For this purpose the following equations can be derived from Equations (1.9)–(1.11):

$$\frac{I_2}{I_{2FL}} = \left\{ \frac{\left[1 + \frac{S_{MT}}{S_{FL}}(1 + Q^2)^{1/2}\right]^2 + Q^2}{\left[1 + \frac{S_{MT}}{s}(1 + Q^2)^{1/2}\right]^2 + Q^2} \right\}^{1/2} \tag{1.12}$$

$$\frac{T}{T_{\mathrm{FL}}} = \frac{1 + \frac{1}{2}(1+Q^2)^{1/2}\left(\frac{S_{\mathrm{FL}}}{S_{\mathrm{MT}}} + \frac{S_{\mathrm{MT}}}{S_{\mathrm{FL}}}\right)}{1 + \frac{1}{2}(1+Q^2)^{1/2}\left(\frac{s}{S_{\mathrm{MT}}} + \frac{S_{\mathrm{MT}}}{s}\right)} \tag{1.13}$$

$$\frac{P}{P_{\mathrm{FL}}} = \frac{T}{T_{\mathrm{FL}}}\,\frac{(1-s)}{(1-S_{\mathrm{FL}})} \tag{1.14}$$

where $Q = (X_{\mathrm{T}} + X_2)/R_{\mathrm{T}}$ and suffix FL denotes full load.

A set of performance characteristics calculated from Equations (1.12)–(1.14) using the values $S_{\mathrm{FL}} = 0.0234$, $S_{\mathrm{MT}} = 0.15$ and $Q = 7$, which are based on the parameters of a particular 75 kW, 50 Hz, three-phase, six-pole motor, is shown in Figure 1.6.

The ratio of maximum torque to full-load torque differs in machines of different rating and/or pole number. A minimum value of 2 is required by American

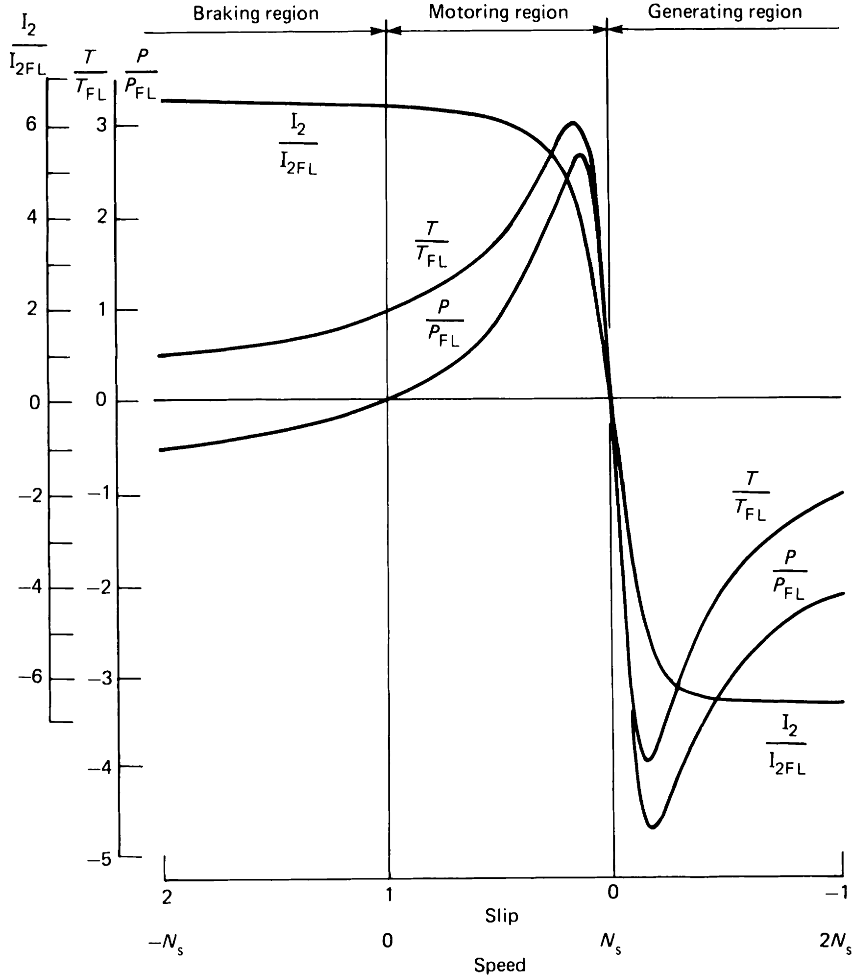


Figure 1.6 Typical performance characteristics of a wound rotor induction motor

Standards, but values of up to 4, for some motors, are claimed in manufacturers' literature. A full-load slip of about 0.025 is typical of a 75 kW motor. A 350 kW machine would have a full-load slip of about 0.01 and a 10 kW machine one of about 0.05. Higher values of full-load slip are associated with higher values of rotor resistance relative to leakage reactance and, consequently, with higher values of slip for maximum torque. However, bearing in mind the variations noted in this paragraph, the characteristics shown in Figure 1.6 can be regarded as typical of those of wound rotor induction motors.

In Figure 1.6 three separate regions can be distinguished, as follows:

(1) The normal motoring region, in which $1 > s > 0$.
(2) A braking region in which $2 > s > 1$. This arises, in so-called 'plugging', when the sequence of the supply voltages to a machine operating at normal speed is deliberately reversed.
(3) A generating region in which s is negative, the machine being driven at super-synchronous speed by external means.

It is evident from an examination of the motoring region that the starting torque is relatively low and the starting current relatively high. However, the characteristics can be modified usefully by the connection of external resistors in series with the rotor circuit, as described in the next section.

1.3.4 Performance characteristics of wound rotor motors with external resistors in rotor circuit

When a balanced polyphase set of resistors is connected in series with the sliprings of a wound rotor induction motor the parameter R_2 in Equations (1.9) and (1.10) is valid for the resistance per-phase of the whole rotor circuit including the external resistance. It is evident from Equation (1.9) that if R_2 and s are changed by the same factor, the current I_2' is unchanged. A similar argument applies to the expression for torque in Equation (1.10). Therefore, the current–slip and torque–slip characteristics for any value of R_2 can easily be deduced from the corresponding characteristics for any other known value of this parameter.

The performance characteristics of the machine of Figure 1.6, for a range of values of $R_2 = R_{2W} + R_{2E}$, where R_{2W} is the resistance of the rotor winding and R_{2E} the resistance added externally, are shown in Figure 1.7. Only the motoring and braking regions need be considered for this aspect of machine performance.

Clearly, the effects of increasing R_2 are to reduce the starting current and increase the starting torque, until a value of R_2 is reached which causes maximum torque to occur at standstill. The use of higher values of resistance then leads to a reduction of starting torque, from the maximum value, and a continued reduction of starting current.

Braking performance over the whole range $2 > s > 1$ can be optimized by suitable choice of R_{2E}. However, in general, good braking performance is associated with relatively high values of torque at standstill and consequently with ideal characteristics for accelerating the machine from standstill in the reverse direction. This is a very unsatisfactory feature of 'plugging', and other methods of braking are generally to be preferred.[11]

1.3.5 Performance characteristics of cage rotor motors

There is no single set of performance characteristics, similar to Figure 1.6, which can be said to be typical of an induction motor with a cage rotor, mainly because a cage rotor does not exhibit constant rotor parameters R_2 and X_2. In virtually all cage motors, to some extent, and with large machines and cages of special design in

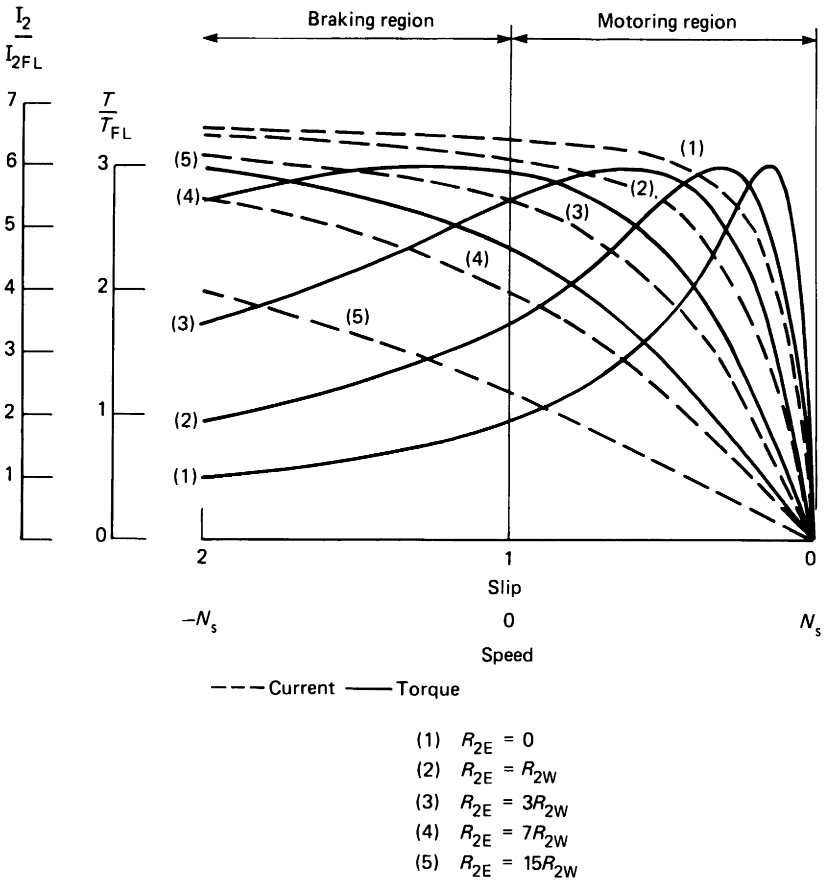


Figure 1.7 Performance characteristics of wound rotor induction motor with external resistors in rotor circuit

particular, these parameters vary with rotor frequency, and therefore speed, because of the so-called ‘current displacement’ or ‘skin’ effect. This effect can be explained by a consideration of the leakage flux paths associated with the cages of special design. For example, Figure 1.8(a) shows a cross-section of a ‘deep’ bar in a rotor slot and the approximate leakage flux paths. The section of conductor at the bottom of the slot is linked by all the leakage flux, that at the top by only a small proportion of this flux. Thus, the leakage inductance of the bottom section is relatively high, that of the top section relatively low. Therefore, when reactance effects are significant, e.g. at standstill where rotor frequency is equal to stator frequency, the impedance of the bottom section of conductor is greater than that of the top section.

Consequently, the current density over the cross-section is non-uniform, current being displaced towards the top. Therefore, the effective resistance of the bar is higher than its resistance to d.c. At rated speed the rotor frequency is very low, the rotor leakage reactance effect is negligible, the current spreads uniformly over the cross-section and the bar exhibits its minimum resistance. This combination of relatively high resistance at standstill and low resistance at rated speed is exactly what is required for good performance, as explained in the discussion related to Figure 1.7 above.

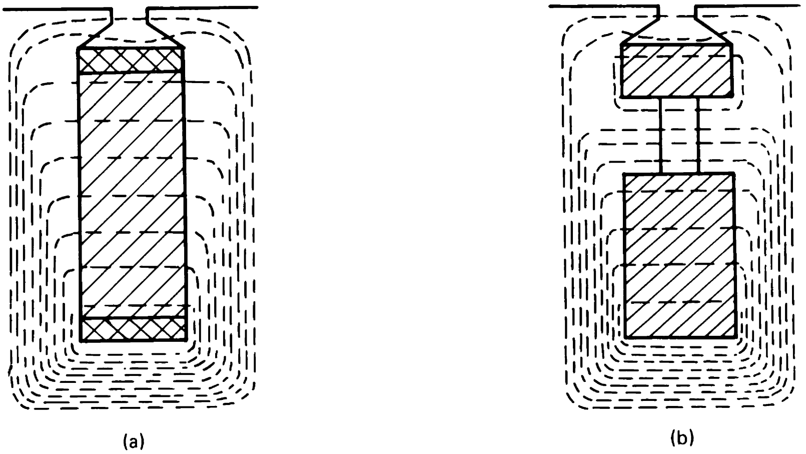


Figure 1.8 Diagrammatic illustration of leakage flux paths of induction motor rotors. (a) Deep-bar. (b) Double cage

The effect can be modified by using bars of various shapes or by using the double-cage construction illustrated in Figure 1.8(b). Evidently, the choice of slot design and bar material, including aluminium in the die-casting process, can lead to a wide range of designs and a wide range of performance characteristics. These are classified in different ways by the various associations responsible for specifying standards.[12,13]

Other factors, affecting the performance characteristics of cage rotor motors more than those of wound rotor motors, should be mentioned. For example, the stator and rotor m.m.f. distributions are not sinusoidal but stepped, because of the concentration of conductors in slots, and the permeance of the gap between stator and rotor is non-uniform because of the slotting and the relative motion of the slotted surfaces. Consequently, the gap flux contains harmonics of the fundamental sinusoid. These harmonics can give rise to both asynchronous and synchronous torques superimposed on that due to the fundamental.[14,15] The design, particularly in the choice of slot combinations, will usually be such as to eliminate the harmonics which lead to synchronous torques, but the asynchronous effects cannot be eliminated completely. The latter can be taken into account in an extended equivalent circuit,[16] but even when this is done there remain other phenomena which can affect performance significantly, as discussed in Section 1.3.6.

Figure 1.9 shows representative characteristics for induction motors with four types of cage rotor. Table 1.1 gives some indication of the range of variation of values at specific points in the characteristics, for a wide range of machines embracing ratings from 1 to 630 kW and pole numbers from two to twelve. The four types correspond to the similarly designated NEMA classifications, but the characteristics, and the information in the table, are based on data from several sources including manufacturers' publications. Reference should be made to the latter for the characteristics of machines of particular type and rating.

1.3.6 Efficiency

The efficiency of an induction motor is best expressed in the form:

Efficiency = 1 − (losses/input)

where the term in brackets can be called the deficiency.

The losses consist of:

(1) Stator and rotor copper losses due to both load and magnetizing currents. These are the only losses taken into account in the simple equivalent circuit of Figure 1.2(a). Strictly, if a.c. values are used for the resistances they include an element for the loss due to pulsation of leakage flux at standstill.

In addition there are:

(2) Stator core losses which are a function of magnetizing flux density and which, at constant supply voltage, are approximately constant.
(3) Rotor core losses which are a function of magnetizing flux density and speed, and which are virtually negligible at rated speed.
(4) Extra stator copper loss associated with the component of stator current required to supply the core losses.
(5) Mechanical losses due to friction and windage.
(6) So-called stray losses, being all the losses additional to the above.

All losses, except stray losses, can be determined with reasonable accuracy from the results of standstill and running-light tests (see Chapter 8).

Table 1.1 Range of variation of particular performance figures for various types of cage induction motor

Designation	*(A)*	*(B)*	*(C)*	*(D)*
Rotor type	Single cage low-resistance	Deep bar or double cage	Double cage	Single cage high-resistance
Full-load slip	$0.01>S_{FL}>0.005$	$0.05>S_{FL}>0.005$	$0.05>S_{FL}>0.005$	$0.15>S_{FL}>0.05$
Breakdown torque	$2.8>T_{BD}>2.4$	$2.0>T_{BD}>1.9$	$2.0>T_{BD}>1.9$	$3.0>T_{BD}>2.6$
Pull-up torque	$1.2>T_{PU}>0.5$	$1.4>T_{PU}>1.0$	$2.0>T_{PU}>1.4$	
Standstill torque	$2.0>T_{SS}>0.8$	$2.0>T_{SS}>1.4$	$2.8>T_{SS}>2.0$	$2.7>T_{SS}>2.5$
Starting current	$5.0<I_{SS}<8.0$	$4.5<I_{SS}<6.0$	$4.5<I_{SS}<6.0$	$4.0<I_{SS}<5.5$

Notes:
(1) All values are in per-unit of corresponding full load value, except slip.
(2) $S_{FL}(A)<S_{FL}(B)<S_{FL}(C)<S_{FL}(D)$ for machines of similar rating.
(3) In all cases, figures on the right apply to machines of higher rating.
(4) Breakdown torque and pull-up torque are designated in Figure 1.9(a).
(5) Breakdown torque may be less than the maximum torque developed by machines of Type C.
(6) Some machines of Type D may not develop a breakdown torque in the region $1>s>0$.

Stray losses, as defined in (6) above, include rotor losses associated with the space harmonics of the magnetic field which induce fundamental, i.e. supply-frequency, voltages in the stator windings. To some extent these may be taken into account in an extended equivalent circuit as already mentioned.[17]

There remain:

(1) Losses produced by magnetic fields which link stator and rotor and which induce voltages in the stator at frequencies other than supply frequency; these voltages are short-circuited by the supply network.
(2) Losses due to pulsations of leakage flux additional to those which occur at standstill.

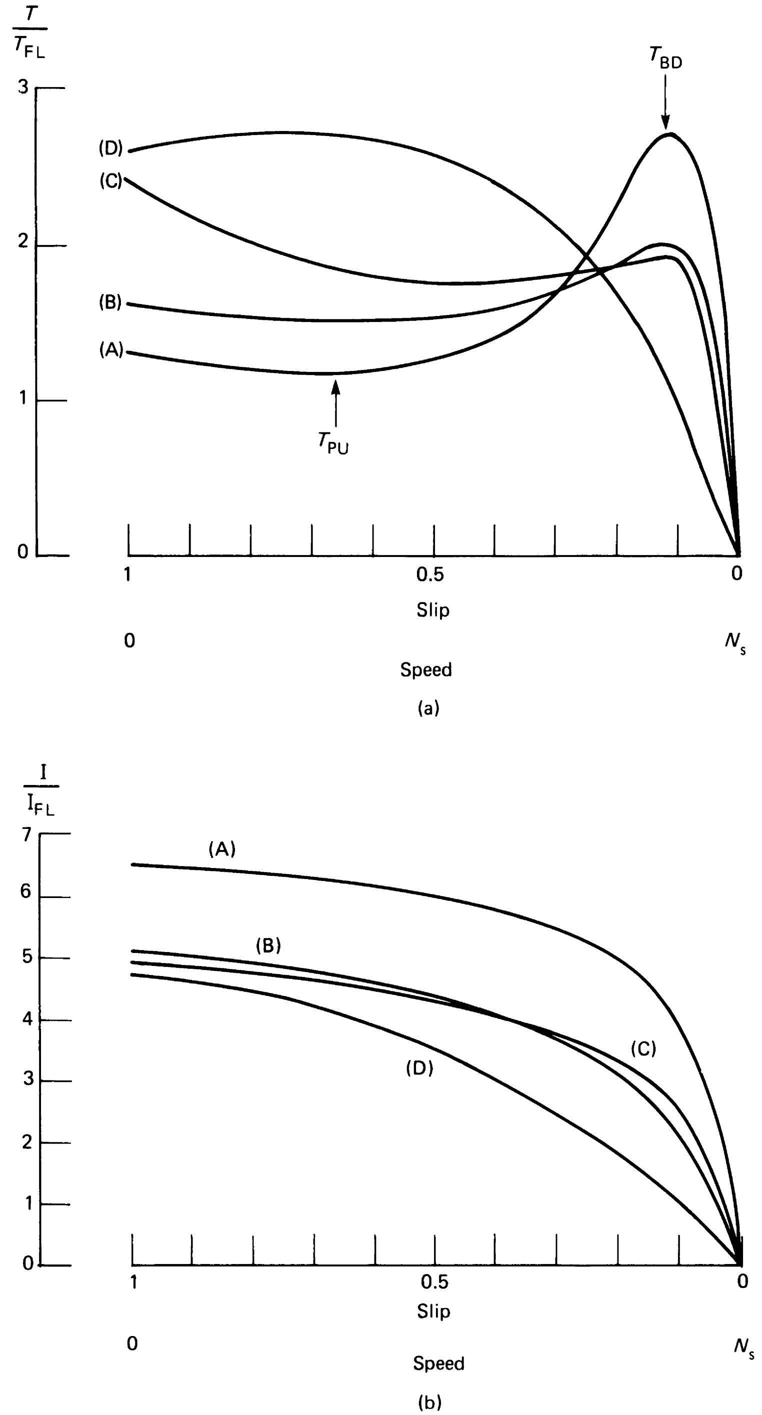


Figure 1.9 Performance characteristics of induction motors with various types of cage rotor. (a) Torque–slip. (b) Current–slip

These arise because of the presence of teeth on stator and rotor, in relative motion, and would exist even with stator windings of notionally perfect sinusoidal distribution.

The interactions causing stray losses are evidently complex.[18,19] These losses can be evaluated at one speed by the reverse-rotation test[20] but this has its limitations.[21] Some designers conclude that, for the purposes of a declared efficiency, it suffices to make an allowance based on experience.[22] However, it must be noted that in the regions $0.9 > s > 0.1$ these losses reduce the motoring torque considerably, and in the region $s > 1.1$ they increase the braking torque considerably.[23]

In an induction motor of full-load efficiency 0.88, the deficiency of 0.12 would be comprised approximately as follows: stator copper loss 0.04, rotor copper loss 0.02, core losses 0.04, mechanical losses 0.01 and stray losses 0.01. The core losses and mechanical losses taken together as rotational losses remain approximately constant; the other losses are load-dependent. Maximum efficiency occurs approximately when these two sets of losses are equal, and usually at a slip less than full-load slip. A typical curve of efficiency against slip is shown in Figure 1.10(b).

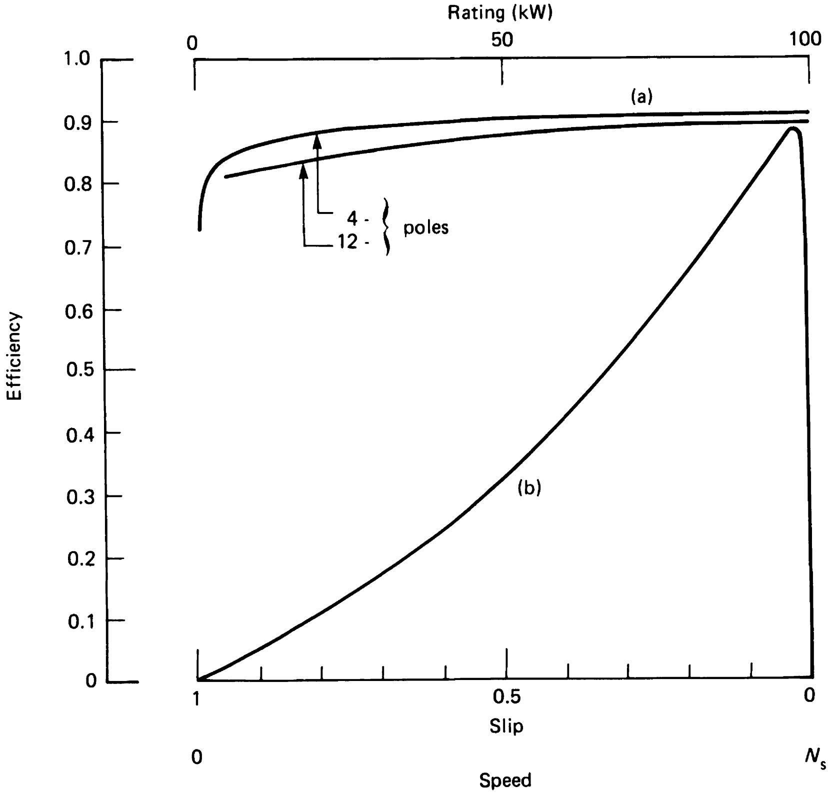


Figure 1.10 Efficiency. (a) Full-load, against rating, for typical cage induction motors. (b) Against slip for a typical motor

Efficiencies of induction motors of a given pole number increase with rating. Those of motors of a given rating decrease with increase in pole number. Curves of full-load efficiency against rating for cage rotor machines of ratings up to 100 kW are included in Figure 1.10. At a rating of 1000 kW, the efficiency would increase by between 0.02 and 0.03.

The efficiencies of wound rotor motors are lower than those of the corresponding cage rotor motors by about 0.02 in the lowest ratings, and by 0.01 or less in the highest ratings.

1.3.7 Power factor

An induction motor operates at a lagging power factor which is a function of slip. Maximum power factor usually occurs at a slip slightly greater than full-load slip. A typical curve of power factor against slip is shown in Figure 1.11(b).

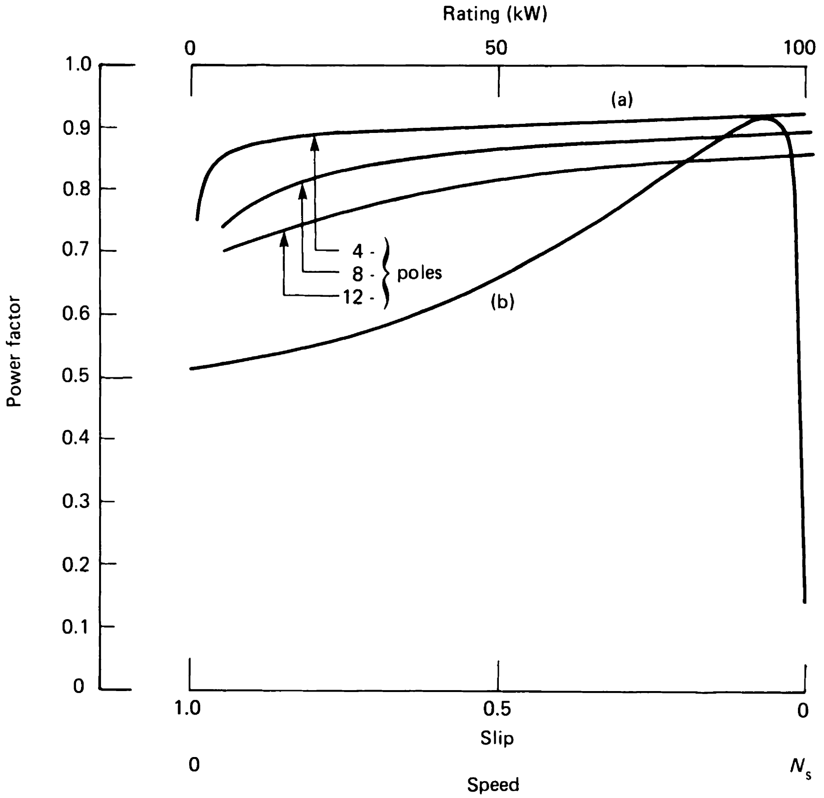


Figure 1.11 Power factor. (a) Full-load, against rating, for typical cage induction motors. (b) Against slip for a typical motor

Power factors of induction motors of a given pole number increase with rating. Those of motors of a given rating decrease with increase in pole number, the decrease being significantly greater than the corresponding decrease in efficiency mentioned in Section 1.3.6. Typical curves of full-load power factor against rating for cage rotor machines of ratings up to 100 kW are included in Figure 1.11(a). At a rating of 1000 kW, the power factor could be expected to rise to 0.91 for a twelve-pole motor and to 0.93 for an eight-pole or four-pole machine.

The power factors of wound rotor motors are lower than those of the corresponding cage rotor motors by about 0.03–0.05 in the lowest ratings, and by about 0.01 or less in the highest ratings.