Mathematics as a Cultural System

RAYMOND L. WILDER
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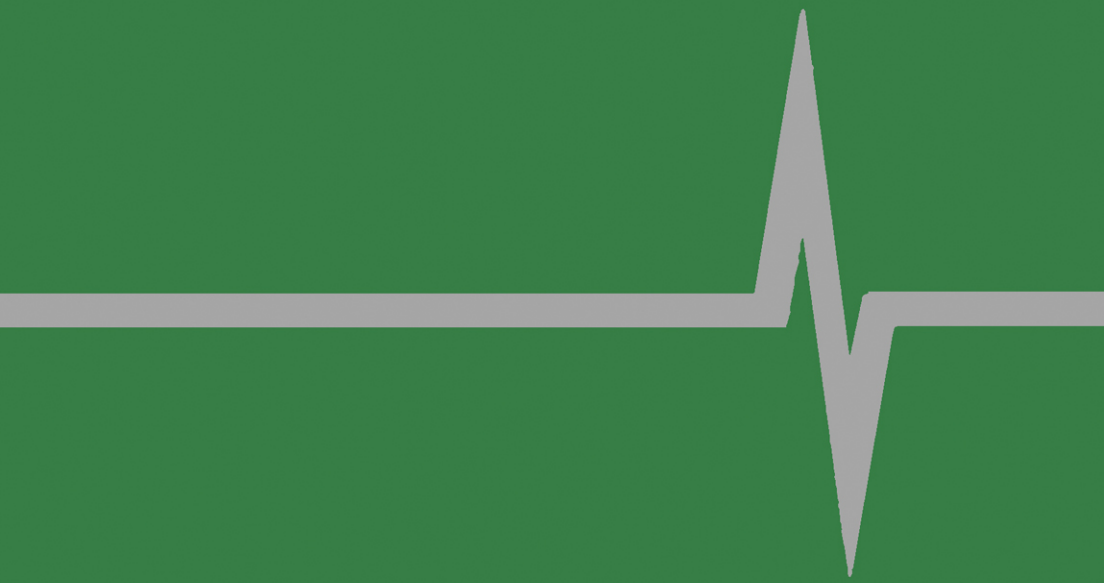

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by

RAYMOND L. WILDER

University of California, Santa Barbara

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To my daughter
Beth Dillingham

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Introductory Note

There is presented here *a way of looking* at mathematics and its history. Justification for this may be found in any aspect of modern science that has proceeded from the empirical to the theoretical. In dealing with empirical activities, the word "truth" may be aptly used. It is true that certain species of birds fly south in winter and north in summer. But theories explaining such behavior may or may not be true in the same sense. If the behavior seems to conform to a theory T, then T may be accepted as an explanation of the behavior; but to call T "true" in the same sense in which we state observed properties of the behavior would be unjustified.

Similar remarks apply even more forcefully to physical theories — the "big bang" theory, for instance. The classical Newtonian theory of the universe is a good case in point; long considered "true", we know now that it is only a theory, even though a fine instrument with which to work within suitable limitations.

Similarly, the way of looking at mathematics presented herein is not asserted to be the "true" state of affairs. I do assert, however, that to conceive of mathematics as a cultural system does offer a way of explaining many anomalies that, in my opinion, have not been heretofore satisfactorily explained by philosophical or psychological means. Moreover, to consider the evolution of mathematics from a culturological point of view is no more demeaning to the individual mathematician than is the biological theory of evolution demeaning to the individual *Homo sapiens.* Whether it presents a *true* picture or not is not for me — or anyone else — to affirm. That is does constitute a logical and satisfying explanation of mathematical behavior and its history seems to me true — as I hope that some, at least, of my readers will agree.

Also I hope that this work will not be considered as a *history,* such as was the fate of my earlier work *Evolution of Mathematical Concepts* ("EMC"). "Evolution" and "History" are not, I believe, synonymous, although

many seem to think so. Historical events are cited in the present work, but only to exemplify or justify the theory; sometimes the same historical event will be cited more than once when its various aspects furnish evidence for more than one theory. It will not be expected, then, that I shall always go to prime sources such as archives, since I shall usually cite the place in the literature where the event will most easily be found by the general reader.

Familiarity with EMC is not assumed in the present work, although sometimes references to it will be made. While EMC may be adumbrative of the present work, the latter is intended as a more mature treatment in that the citations to mathematical theory are not restricted to number and geometry, as in EMC, and concepts (e.g. consolidation, hereditrary stress) which were introduced somewhat superficially in EMC are here analyzed and more explicitly related to mathematical developments. It is not intended, however, that this book is designed only for mathematical readers. Although a particular mathematical concept here and there may be unfamiliar to the general reader, usually its omission will not cause misunderstanding of the general context. Certainly social scientists and philosophers should be able to read and understand. Despite the general misunderstanding of the intent of EMC, it was gratifying that many philosophers and social scientists seem to have discovered and understood the book.

Regarding details of presentation. The Bibliography is organized by author and date; thus Kroeber, 1917:168 refers to the work of Kroeber cited in the Bibliography under the date 1917, and specifically to p. 168 thereof. Since there have been two editions of EMC — a hard cover (Wilder, 1968) and a paperback (Wilder, 1974, 1978) — I shall use "EMC_1, to denote the paperback edition, while "EMC" generally denotes the hardback edition. However, when the reference to EMC and EMC_1 are the same (thus "EMC:Chap 2" and "EMC_1:Chap 2"), the one reference EMC will be used.

Reference to chapters will by by roman numerals. Thus, III-6 refers to section 6 of Chapter III.

In conclusion, I wish to express my thanks to my daughter Professor Beth Dillingham, a professional anthropologist, who encouraged me to write this book and has read the manuscript with a critical eye. Naturally, any errors that I have made are my responsibility, not hers.

R. L. Wilder

University of California
Santa Barbara
June 22, 1980

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