

***COMPUTATIONAL  
MECHANICS IN  
STRUCTURAL  
ENGINEERING***

**RECENT DEVELOPMENTS  
AND FUTURE TRENDS**

EDITED BY  
**FRANKLIN Y. CHENG  
FU ZIZHI**



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**COMPUTATIONAL MECHANICS IN  
STRUCTURAL ENGINEERING**

**Recent Developments and Future Trends**

**Proceedings of Sino-US Joint Symposium/Workshop on Recent Developments and Future Trends of Computational Mechanics in Structural Engineering, Beijing, China, September 24-28, 1991.**

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# COMPUTATIONAL MECHANICS IN STRUCTURAL ENGINEERING

**Recent Developments and Future Trends**

*Edited by*

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## PREFACE

It is generally acknowledged that the field of computational structural mechanics has enriched the rational application of mechanics and numerical methods in the practice of modern structural analysis, design and construction of building systems and other structures. In recent years, both the US and China have been actively engaged in research on computational structural mechanics for various engineering disciplines, and have sponsored cooperative research projects in areas of mutually beneficial interest. Researchers from both sides have shared a strong interest in exchanging state-of-the-art information in research and practice.

The Sino-US Symposium/Workshop on Recent Developments and Future Trends of Computational Mechanics in Structural Engineering was held during September 24–28, 1991, in Beijing, China. Its emphasis was to review the current state of knowledge and practice in computational structural engineering, to identify frontal research and development needs, and to stimulate cooperative research efforts among investigators in the US and China as well as other countries.

The technical program of the symposium included the following sessions: failure analysis and finite elements; shells and walls; structural control; computational strategies for buildings; nonlinear analysis; structural optimization; structural reliability; supercomputers, CAD and expert systems.

US delegates were:

- Dr. Charles A. Anderson (in collaboration with Dr. Charles R. Farrar)—  
Los Alamos National Laboratory, New Mexico
- Dr. A. H-S. Ang—University of California at Irvine
- Dr. James E. Beavers—Center for Natural Phenomena Engineering, Martin  
Marietta Energy Systems, Inc.
- Dr. Ted Belytschko—Northwestern University
- Dr. Franklin Y. Cheng (Chairman)—University of Missouri-Rolla
- Dr. K. P. Chong—National Science Foundation
- Dr. S. C. Liu—National Science Foundation
- Dr. Graham H. Powell—University of California-Berkeley
- Dr. T. T. (Larry) Soong—State University of New York at Buffalo
- Dr. Gerald A. Wempner—Georgia Institute of Technology

Chinese delegates were:

Professor Bao Shihua—Tsinghua University  
 Professor Ying-Jun Chen—Northern Jiao Tong University  
 Professor Deng Dahuan—Chinese Academy of Agricultural Mechanization Sciences  
 Professor Fu Zizhi (Chairman)—Petroleum Design Institute  
 Professor Huang Yih—Xian Institute of Metallurgy and Construction Engineering  
 Professor Li Guiqing—Institute of Aseismic Engineering Structures  
 Professor Li Mingrui—Beijing Agricultural Engineering University  
 Professor Li Xingsi—Dalian University of Technology  
 Professor Liang Qizhi—South China University of Technology  
 Professor Lin Shaopei—Shanghai Jiao Tong University  
 Professor Liu Guang-dong—Hunan University  
 Professor Lu Wenda—Tongji University  
 Professor Wang Guangyuan and Professor Ou Jinping—Harbin Architectural and Civil Engineering Institute  
 Assc Prof Wang Qizheng—Tsinghua University  
 Professor Xu Bohou—Zhejiang University  
 Professor Zhong Wanxie—Dalian University of Technology  
 Professor Zhou Fu-Lin—Guangzhou Urban Construction Institute

Guest observers and paper contributors for the symposium included: Professor Teizo Fujiwara and Professor Yoshikazu Yamada of Kyoto University, Dr. M. Shoji and Mr. Takashi Matsumoto of Kajima Corporation, Dr. S. C. Fan of Nanyang Technological University, Dr. D. S. Juang of National Central University, Dr. T. W. Lin of National Taiwan University, Dr. J. P. Pu of Feng Chia University, Dr. P. H. Wang of Chung-Yung Christian University, and Dr. Y. B. Yang of National Taiwan University.

The joint symposium/workshop was jointly sponsored by the US National Science Foundation, the Chinese Association of Computational Mechanics and the National Natural Science Foundation of China. As chairmen of the symposium/workshop and editors of the proceedings, we gratefully acknowledge their support.

FRANKLIN Y. CHENG  
 FU ZIZHI

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Group picture after opening ceremony



Visit to the National Science Foundation of China (NSFC)



Welcoming remarks at reception by Dr S. C. Liu,  
Program Director, US NSF



Speech at banquet by Dr K.P. Chong, Program Director, US NSF



Welcoming remarks at banquet by Prof. Hu Zhaosen,  
exec. vice-chairman, NSFC



Delegation friendship toast at banquet by Dr Franklin Y. Cheng  
and Prof. Fu Zizhi



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## Opening Speech

Prof. Fu Zizhi, Vice-Chairman, CACM

Dear Colleagues and Friends,

The Sino-U.S. Joint Symposium / Workshop on Recent Development and Future Trends of Computational Mechanics in Structural Engineering is now declared to be opened. On behalf of the Chinese Association for Computational Mechanics (CACM) under CSTAM, let me express our warm welcome to all our Scholars present here today.

In this Symposium / Workshop, we have delegates from China, the United States of America, Japan and Singapore. They have been working in the foremost fronts of various domains of Computational Mechanics and are of the first class level. The purpose of this meeting is to promote mutual understanding through academic exchanges and discuss the future trends of Computational Mechanics as well as the contents and forms of cooperation. We sincerely hope that this meeting will serve as a starting point to promote a well planned long term exchange and promotion of Sino-U.S. cooperation in Computational Mechanics.

This Symposium / Workshop was first proposed by Professor K.P. Chong in 1989, sponsored by CACM and the National Science Foundation of the United States of America and also supported by CSTAM and National Natural Science Foundation of China. It is the result of two years of preparation. Professor K.P. Chong and Professor S.C. Liu, Program Directors, Engineering Directorate, NSF, Prof. Wu Jiexiang, Director of the Bureau of International Cooperation of the NSFC, Prof. Jin Zhengmo, Head of the Mechanics Division of the same Foundation have lent us great support. We would like to mention especially Dr. Franklin Y. Cheng, Curators' Professor of the University of Missouri-Rolla who has spent great amount of energy and time in the preparation. Our cooperation in this respect has been very sincere and pleasant. To all those mentioned above, I hereby express our heartfelt thanks.

The meeting is short but it will produce deep effects in promoting the substantial cooperations in the future. I firmly believe that we can expect more cooperations between the Science Foundations of the two countries and the academic exchanges between our scholars.

I send my greeting to the success of this meeting and my best wishes to the happiness of all present here today.

Thanks for your kind attention.

**OPENING REMARKS**

*Franklin Y. Cheng, Curators' Professor of Civil Engineering  
University of Missouri-Rolla, USA*

Good morning, distinguished participants, ladies and gentlemen.

On behalf of the US delegates, I am pleased that the Sino-U.S. Joint Symposium/Workshop on Recent Developments and Future Trends of Computational Mechanics in Structural Engineering has now formally opened. We welcome our guest observers wholeheartedly.

In recent years, researchers in the US and China have actively engaged in development and practice of computational structural mechanics. They share a strong interest in exchanging research information. The objectives of the symposium/workshop are to (1) review the current state of knowledge and practice in computational structural engineering; (2) identify the frontal joint research opportunities necessary to advance the current state of knowledge; and (3) stimulate future cooperative research and development in the areas of common needs in the US and China as well as other countries.

The symposium is jointly sponsored by the US National Science Foundation and the Chinese Association for Computational Mechanics with support of CSTAM and the National Natural Science Foundation of China. I gratefully acknowledge their sponsorship and support. We deem it a great honor that Dr. S.C. Liu and Dr. Ken P. Chong, program directors of the National Science Foundation, are here today. Dr. Henry J. Lagorio of NSF is unable to be with us, but he sends his best wishes for a successful meeting. We are thankful for the tireless efforts of Professor Fu Zizhi in arranging all local details.

Please permit me to remind you that the major task now is for us to diligently exchange research information and to successfully develop a long-term cooperative program. I think the resolutions to be established during the workshop this Friday will have a great impact on the success of this symposium. Let us work together closely this week to achieve our ultimate goal.

Thank you.

Welcome by Ken P. Chong, Director of Structural Systems and Construction Processes, National Science Foundation, USA

On behalf of the National Science Foundation I would like to welcome all of you to this prestigious workshop. As you know mechanics and structural engineering are the "backbones" of civil engineering, and computational mechanics play a key role in these areas. In the United States computational resources seem plentiful. However some of our super-computer centers are being strained. In some cases only 50 % of computer requests can be accommodated. On the other hand, researchers and engineers in China have always been very careful in minimizing computational labor and maximizing its efficiency. This is one of the areas where US - Chinese interaction would be beneficial.

I wish you all the success, and I would like to take this opportunity to thank the workshop co-chairmen, Professors Cheng and Fu, for their hard work over the last two years. Without their tireless effort we will not be here today.

**Address to the China-US workshop on Structural  
Engineering and Computation**

Prof. Hu Zhaosen  
Exec. Vice Chairman, NSFC

Mr. Chairman, Ladies, Gentlemen and friends:

I feel great honoured to have this opportunity to meet so many distinguished experts from home and abroad working in the structural Engineering and Computation field.

On behalf of the National Natural Science Foundation of China to extend my sincere congratulations on the opening of the workshop on structural Engineering and Computation and express my warmest welcome to all the participants to come to Beijing.

The National Natural Science Foundation of China is an organization with both management and academic nature. In order to bring prosperity to science, economy and society of China, our foundation devotes all efforts to coordinate, support and grant basic research and part of applied research including research in the fields of Structural engineering and computation.

One of the tasks of our foundation is to support international scientific exchange and cooperation. These includes the joint research attended by Chinese and foreign scientists, supporting international conferences held in China and the attendance of Chinese scientists at international conferences held abroad, and other cooperative activities.

Therefore I am very pleased to see so many Chinese and foreign experts getting together to exchange experience and to discuss scientific problems of common interest. I hope this will lead to the further cooperation between Chinese scientists and our foreign friends.

Finally, I sincerely wish the symposium a great success and all the delegates good health.

Thank you!

Ladies and Gentlemen,

The purpose of the "Sino-U.S. joint symposium/workshop on recent developments and future trends of computational mechanics in structural engineering" is to provide a forum for the scientists of both sides, to increase mutual understanding and to promote exchanges and cooperations. The contents of the symposium will be related to structural control, structural reliability, structural nonlinear properties, structural optimizations, CAD, expert systems and the application of supercomputers etc. All these reflect the latest developments of structural computational mechanics with high theoretical and practical values. We are confident that this symposium/workshop is a high level academic meeting.

The technical program consists of a symposium with three days presentations of research papers and a one day simultaneous workshop. We hope that this symposium/workshop will serve not only for academic exchange on the three days but also a starting point of cooperation between the colleagues of the two countries.

Now let us have a coffee break and then begin with the first general session.

A Talk by Prof. Zhong Wanxie, chairman, organizing committee

**RESOLUTIONS OF SINO-US SYMPOSIUM WORKSHOP  
ON RECENT DEVELOPMENTS AND FUTURE TRENDS  
OF COMPUTATIONAL MECHANICS IN STRUCTURAL ENGINEERING**

The Sino-U.S. Joint Symposium/Workshop on Recent Developments and Future Trends of Computational Mechanics in Structural Engineering was held during September 24-28, 1991, in Beijing, China. Participants conducted fruitful scientific exchanges in the areas of Buildings, Reliability, Structural Optimization and Control, Supercomputers, Shell Structures, Intelligent CAD and Expert Systems, and Nonlinear and Failure Analysis.

At the conclusion of the symposium/workshop, participants unanimously passed the following resolutions:

- (A) Collaborative research between Chinese and U.S. researchers requires cross-flow of research results as published in journals and conference proceedings. While Chinese investigators have relatively easy access to international publications, Chinese research papers are often inaccessible to U.S. researchers because they are published either in Chinese or in journals not widely circulated in the West. Therefore: (1) Chinese researchers should be encouraged to publish in English and in international journals; (2) More Chinese-sponsored journals and conference proceedings should be published in the West; (3) Effective measures should be devised to disseminate research results to users; (4) Chinese researchers are encouraged to serve on editorial boards and as editors of international journals.
- (B) Based on the success of this event as experienced by all participants, it is resolved that a joint symposium/workshop be continued at intervals of two or three years.
- (C) Topics of mutual interest for further investigation, preferably as joint research activities, are listed in the following groups.

**I. STRUCTURES, RELIABILITY, STRUCTURAL OPTIMIZATION AND CONTROL**

The consensus of the group chaired by T.T. Soong and Li Guiqing was that reliability, optimization and control are closely related topics. A number of the papers presented at the workshop showed promise for collaborative research in this area. Specific topics discussed are noted below.

(a) Control - Although active control research is currently going on in China, passive control is considered more practical at this time. In the structural control area, the following topics are recommended for possible joint research.

1. More in-depth studies of merits as well as limitations of active control from the standpoint of reliability. Life safety enhancement and cost effectiveness through damage avoidance need to be carefully examined.
2. Computational issues in relation to the development of new control algorithms.
3. More emphasis on control of wind-induced vibrations. Structures that may benefit from such studies include radio towers in Shanghai and Nanjing and a number of cable-stayed or long-span suspension bridges.
4. Active control with other applications, such as construction processes.

(b) Reliability - The general feeling was that reliability research and applications have not been well developed in China. As an example, reliability-based codes for bridges are yet to be implemented there. The following were pointed out as potential joint research topics.

1. Implementational issues related to reliability-based codes, particularly those dealing with characterization of uncertainties.
2. Reliability as an integral part of the "full-range" analysis which includes design, optimization, analysis, identification and control.

(c) Optimization - Generally speaking, a strong base of optimization research exists in China. Joint research in the following directions could be particularly fruitful.

1. Basic work in nonlinear structural optimization and sensitivity analysis.
2. Engineering issues in optimization, e.g., reliability-based optimization and optimization with multiple objectives and multiple constraints.

## II. SUPERCOMPUTERS, SHELL STRUCTURES, INTELLIGENT CAD AND EXPERT SYSTEMS

The group chaired by Graham Powell and Ou Jinping considered the above three topics. For each topic, key points from discussion are summarized below.

(a) Supercomputers - The use of supercomputers is driven by the need to perform extensive computations, most likely for large finite element models. Supercomputers are not currently available in China. As a result, Chinese researchers restrict themselves to problems which can be solved with modest computational effort. This has led to some interesting and innovative solution techniques for

such structures as tall buildings. Parallel processing could be a viable alternative in China. US researchers in the group have access to supercomputers through their laboratories or NSF programs. A joint research possibility is in-depth studies of merits as well as limitations of analytical models used by Chinese and US researchers on aforementioned structures by comparing numerical results and computational efforts.

(b) Shell Structures - Chinese researchers in this area are interested mainly in reinforced concrete shells, and have attained a high level of expertise in modeling and analyzing shells. While US researchers and engineers tend to rely on finite element analyses, Chinese researchers pursue more traditional analytic methods. Their level of expertise in the analysis of cooling towers, for example, is very advanced. US researchers may not be aware of Chinese research. A possible joint effort is to develop analytical models and numerical algorithms with emphasis on computational efficiency.

(c) Intelligent CAD, Expert Systems - This is a broad area, ranging from conceptual design to routine draft. Several Chinese researchers are interested in different aspects of the broad problem, and are performing innovative research, such as intelligent draft, optimum design for seismic loading, structural design with hierarchical decomposition and multilevel optimization, design decision-making with application of the principles of fuzzy logic. US researchers are engaged in research on data models, data base design, and the structure of the design process. Chinese and US researchers share an interest in topics of intelligent CAD and optimum structural design. These could be fruitful areas for increased cooperative research.

### III. NONLINEAR AND FAILURE ANALYSIS

This group chaired by T. Belytschko and Li Mingrui discussed areas of mutual interest in computational mechanics for nonlinear and failure analysis. In these discussions, an effort was made to identify areas for joint projects as follows.

(a) Nonlinear and Dynamic Stability Problems: Continuation Methods and Other Algorithms; Thin-Walled Structures - This topic seemed to be of most interest to Chinese participants. There was considerable interest in the development of more accurate and efficient algorithms for nonlinear problems, reflected in the subtopics under nonlinear dynamics of buildings and bridges.

(b) Adaptive Methods: Methods for Crack Growth and Localization; Error Criteria - US delegates felt that the Chinese research community, with its strong theoretical background, is in a position to make contributions to

developing error criteria. Adaptive methods offer one of the few possibilities for solving important problems in failure such as crack propagation and strain localization.

(c) Nonlinear Dynamics of Buildings and Bridges: Efficient, Accurate Algorithms; Methodologies for Seismic Loading - There was considerable interest in this topic because of its practical impact. Chinese delegates also felt that significant improvements in algorithms are necessary to enable practical nonlinear transient analysis of buildings and bridges.

(d) Interaction and Coupled Problems: Fluid-Structure Interaction; Structure-Media Interaction - The focus of these problem areas is the safety of dams. Assessing the safety of dams involves many coupled field problems; for example, the seepage of fluid below a given dam foundation is a coupled problem of importance to safety.

(e) Boundary Elements for Nonlinear Problems: Software Development; Object-Oriented Programming - The boundary element method was felt to be useful in nonlinear problems such as crack propagation, where it can reduce the amount of remeshing. Contact problems were also mentioned as a viable application.

**NSF PROGRAMS IN COMPUTATIONAL AND STRUCTURAL MECHANICS**

K.P. CHONG, S.C. LIU and H.J. LAGORIO  
Program Directors, Directorate of Engineering  
National Science Foundation  
Washington, DC 20550 USA

**ABSTRACT**

The U.S. National Science Foundation (NSF) spends over U.S. \$40 million annually for basic research in civil engineering, covering areas in structures, construction processes, earthquake engineering, building systems, solid mechanics, geomechanics and environmental engineering. Significant amounts of basic knowledge and technology have been advanced through these research efforts. Computational and structural mechanics are important components of civil engineering research. In this paper, examples of current and recent research projects and findings are presented. Some significant developments, not necessarily funded by NSF, are also discussed.

**INTRODUCTION**

In a recent book by Boresi and Chong [1], it was indicated that finite element methods provide perhaps the most versatile tools available in numerical analysis of engineering problems in solid mechanics, fluid mechanics, flow in porous media, heat transfer, and so on. Finite element methods are constantly being developed and applied to ever more complex problems. New developments and applications of finite element methods include refined finite elements, advanced computing methods for stability and dynamics analyses, shell analysis, design sensitivity and optimization of nonlinear systems, viscoelasticity, and fracture. An extensive list of references is given.

Finite elements are widely used also in computer-aided engineering design [2] (CAE,CAD) and benchmark programs have been designed to evaluate finite element codes [3,4]. In finite element methods, the solution of simultaneous equations is very computationally intensive. The traditional Gaussian elimination method [5,6,7] is not very efficient compared with for example, iterative gradient

methods in vectorized finite elements [8,9]. The availability of super computers and parallel computers [10,11,12] allows for vectorized processing and parallel processing, opening up powerful computational capabilities [13,14]. Substructuring or partitioning of matrices (for example, the stiffness matrix) are adaptable to parallel processing [15]. Efficient substructuring techniques have been developed for static, dynamic and stability analyses [9,13,14,16-24]. Unfortunately, nearly all finite element codes are written in sequential form which differ greatly from parallel procedures. Storaasli and Bergan [12] indicated that it is more effective to write new parallel algorithms than to modify existing algorithms. Corresponding developments in parallel processing hardware are also available. They include the NASA finite element machine [10], local memory multiprocessors [26] such as transputers which are widely used in Europe and in hypercube networks [9]. Local memory multiprocessors (as opposed to shared ones) possess individual memory for each processor. On the other hand, shared memory multiprocessors, which access a common memory, are more powerful and increasingly used in finite element analyses [27].

In most finite element formulations, the stiffness or displacement method is used in which the unknowns are nodal displacements. In the displacement method, stresses and strains are derived by differentiation which reduces accuracy. An alternative method is the force method in which member forces including moments are the primary variables (unknowns) and displacements are derived. Patnaik [28] developed an integrated force method (IFM) which uses all member forces including redundants as the unknowns. This method increases the accuracy of stress predictions with less computational labor [29,30]. A dual IFM is being developed to enable IFM elements to be introduced into existing displacement-based finite element codes such as NASTRAN, with minimal modification.

Based on Hamilton's principle, Marjadi [31] formulated an alternative force method for static, dynamic and buckling analyses, using isoparametric three-dimensional beam elements. Optimization and sensitivity derivatives have been investigated.

#### **STABILITY, DYNAMICS, AND NONLINEAR ANALYSES**

The following discussions in this section is taken from the book by Boresi and Chong [1]. Wempner [32] presented a comprehensive review on the mechanics and finite element analysis of shell structures, including developments in instability, inelasticity, nonlinearity, and excessive stiffness (locking) in modeling. About two hundred references were quoted. Shear-locking is also discussed by Ravichandran and Sridharan [33]. To investigate localized plastic collapse and post-collapse behavior of rotational shells, a mixture of general shell elements has been used (in localized regions with severe nonlinearities), with rotational

shell elements elsewhere (Zienkiewicz and Taylor, [6]), and transitional shell elements in between [34,35].

In nonlinear and dynamic analyses, commercial programs such as ADINA (Automated Dynamic Incremental Nonlinear Analysis; Bathe [36]) and ABAQUS [37] have been used to model the impact responses and energy dissipation of metallic tubes [38,39], as well as the post-buckling of domes [40] and creep behavior. For thermoelastic-plastic and/or creep analyses, Snyder and Bathe [41] developed a finite element solution procedure, the so-called  $\alpha$ -method which utilizes Euler forward and backward methods. The one-parameter integration method, which is stable, uses one time-step size for the computation of nodal displacements and another (smaller) time-step size for the integration of creep strains. Other effective time integration schemes have been presented by Zienkiewicz and Corneau [42] and Bushnell [43].

Optimal structural design and design sensitivity of linear responses of structural systems, subject to static and dynamic loads, are well developed (see, for example, Haftka and Adelman [44], Haug and Arora [45], Lahey [46]). However, sensitivity analysis of nonlinear responses is still in its infancy [47]. Extensive computational efforts are needed for these sensitivity analyses.

The three-dimensional nonlinear finite element package for microcomputers, DIANA, (Borst et al., [48]) has been used to model crack propagation and fracture in concrete (Rots et al., [49]). Shear transfer and tension softening in the crack region (Chong et al. [50]) have been considered. Fracture parameters have also been investigated with boundary elements (Brebbia and Dominguez [51], Aliabadi [52], Blandford et al. [53]).

A finite element reliability method (Liu and Der Kiureghian [54]) has been developed, which combines the finite element method with the reliability method for reliability assessment of structural safety. Analyses of uncertainties and probabilities (Der Kiureghian and Ke, [55]) are included for geometrically nonlinear, elastic structures. Other investigators (Spanos and Ghanem [56]; Liu et al., [57]; Mahadevan and Haldar [58]) have contributed to stochastic finite element methods for structural dynamics, optimization and reliability.

One common denominator of numerical analyses is that elasticity and constitutive relationships always play a key role [59] in computation and modeling.

#### COMPUTATIONAL AND STRUCTURAL MECHANICS

The following is a list of examples of NSF-supported research projects in computational mechanics and structural mechanics with computational elements.

**Finite Element Methods****Adaptive Finite Element Methods for Variational Inequalities with Application to Contact Problems and Plasticity**

Oden, J.; University of Texas-Austin

To develop reliable a-posteriori error estimates of problems described by variational inequalities, development of adaptive strategies based on these error estimates and finally an algorithm development based on the developed strategies.

**Mixed/Hybrid Plate and Shell Elements**

Taylor, Robert; University of California-Berkeley

To develop new shell finite elements which can be used in a wider variety of problems than existing ones, to avoid artificial locking mechanisms, incorporate nonlinear elasticity, permit large rotations and some types of inelastic material behavior.

**Computer Modeling and Examination of 3D Inelastic Beam-Column Joint Behavior in Steel Structures**

Deierlein, Gregory; Cornell University

A finite element analysis procedure for including joint flexibility in the analysis and design of 3D frame structures will be developed. This procedure will be capable of geometric and/or material nonlinear analysis.

**Large Deformation Analysis of Nonlinear Homogeneous and Heterogeneous Media Using an Adaptive Arbitrary Lagrangian Eulerian Finite Element Method with Applications in Industrial Forming**

Ghosh, Somnath; University of Alabama, Tuscaloosa

To develop stable, accurate and efficient computational models for large deformations of metals and composites. The finite element method with arbitrary Lagrangian-Eulerian kinematic description will be implemented for industrial forming processes.

**A Spectral Approach For Analyzing Structures With Random Parameters**

Spanos, Pol D.; Rice University

Research involves structures with uncertain (random) properties and loads, approximate representation of random processes describing the material properties or the structural loads by a finite number of random variables, using the classical concepts of the Karhunen-Loeve and the Polynomials Chaos expansions, to be incorporated into spatial discretizations of structural elements

such as beams and plates, in accordance with standard finite element procedures.

**International Workshop on Finite Element Analysis of Reinforced Concrete**

Meyer, Christian; Columbia University

An international workshop on finite element analysis and modeling of reinforced concrete structures, organized by the ACI/ASCE Committee 447 and its Japanese counter-part, the JCI Committee, were held in June, 1991. Objectives include critical review of various material models, assessing the state-of-the-art and identifying research needs.

**Finite Element Reliability Methods**

Der Kiureghian, Armen; University of California-Berkeley

To combine finite elements and structural reliability into a comprehensive methodology for safety and reliability assessment of non-linear structures with uncertain characteristics and subjected to random loads with spatial and/or temporal stochasticity.

**Local-Global Analysis for Plastic Collapse of Shells**

Gould, Phillip; Washington University, St. Louis

To develop an analytical model which can be used for the study of localized collapse of shells. General quadrilateral degenerate shell elements incorporating large deflection and plasticity effects are to be used to capture the localization of deformation.

**Finite Element Analysis of Transverse Shear Failures in Reinforced Concrete Plates and Shells**

Harmon, Thomas; Washington University, St. Louis

Study of the punching shear resistance of thick plates and shells made of reinforced concrete are being conducted.

**Stochastic Finite Element Methods**

Veneziano, Daniele; MIT

The objective was to develop new uncertainty-propagation, reliability-analysis, and variance-decomposition procedures for general-purpose FE codes. The methods were to be implemented on the NONSAP-C and NIKE2D programs and applied to a variety of linear and nonlinear, static and dynamic, time invariant and time-dependent problems. Project completed in 1987.

**A Three-Dimensional Mixed Finite Element Method to Integrate the**

**Dynamic Crack Growth and Failure Mechanisms in a Composite Laminate**  
Vaicaitis, Rimas; Columbia University

To model the three-dimensional dynamic crack growth and failure mechanism in a composite laminate subjected to low velocity impact, incorporating variationally consistent fracture mechanics elements, multi-failure criteria and parallel processing algorithms.

**Destructive Field Testing of a RC Bridge as a Benchmark To Validate FE Modeling and Analysis of RC Structures**  
Aktan, Ahmet; University of Cincinnati

To exploit tests for the advancement of analytical modeling and nonlinear analysis of reinforced concrete structures.

**Dynamic Buckle Propagation in Submarine Pipelines**  
Carney, John; Vanderbilt University, Nashville

Pipeline buckle propagation and arrest are to be studied by means of a three-dimensional finite element model which will account for the energies dissipated by both circumferential deformations and longitudinal bending and shearing deformations.

**Mixed Mode Fracture of Concrete**  
Hawkins, Neil; University of Washington

Shear transfer mechanism will be studied using beam specimens with short starter cracks and various loading histories which are crack-length dependent. Moire and loading data will be used interactively with a finite element code to deduce the crack closure and shear forces in the fracture process zone under mixed mode loading.

**Development of Space-Time Finite Elements for Nonlinear Transient Response of Flexible Structures**  
Hodges, Dewey; Georgia Tech

Focuses on the computational difficulties associated with steep gradients in the velocity and displacement fields in structures subjected to impulsive loadings. The approach adopted herein, finite elements formulated in the space-time domain, offers the possibility of overcoming these computational problems.

**Nonlinear and Probabilistic Theory for Concrete Creep**  
Bazant, Zdenek; Northwestern University

A numerical algorithm for step-by-step integration in time in finite element programs will be developed, in addition to creep tests.

**Impact Damage in Curved Laminated Composites**  
Chang, Fu-Kuo; Stanford University

Stresses and strains in laminates will be calculated by a proposed finite element method which is based on transient dynamic analysis with the consideration of material and geometric nonlinearities.

**Influence of Concrete Creep on the Load Capacity of Reinforced Concrete Shell Structures**  
Schnobrich, William C.; University of Illinois

With finite elements, it is possible to evaluate a failure load for concrete shell roofs, including creep, nonlinear geometry, and nonlinear material responses.

**Computational Methods**

**Finite Rotations and Finite Elements of Thin Elastic Shells**  
Wempner, Gerald; Georgia Tech

The purpose of the proposal is to seek the answers about accuracy and convergence of numerical procedures by scrutinizing the theoretical bases, the alternative functionals stationary and/or external criteria. Complementary functionals provide means to that end. Computational programs for numerical and symbolic investigations are other means for exploring and for testing the practicality of the results.

**Multi-Scale Methods for Structural Dynamics**  
Liu, Wing K.; Northwestern University

Multi-scale Galerkin methods will be developed for the study of structural/building systems in which the response is characterized by multiple-spatial and multiple-time scales, often dominated by the middle part of the frequency spectrum. New temporal and spatial discretization methods will be developed for the multi-time scale structural dynamic problems.

**Post-Buckling Behavior And Imperfection Sensitivity of Space Trusses With Multiple Eigenmodes**  
Peek, Ralf; University of Michigan

To determine the worst shape of imperfection for any lattice type structure, by means of the Lyapunov-Schmidt-Koiter decomposition

followed by asymptotic results, as well as by an iterative numerical technique.

**Foundation and Special Reference of Inelasticity**

Simo, Juan Carlos; Stanford University

Concerned with geometric and computational aspects of modern nonlinear continuum mechanics, with special emphasis on inelasticity, and suitable for large scale inelastic computation.

**Hypersingular Boundary Integral Equations in Mechanics**

Rudolph, Thomas; Iowa State University

To evaluate the computational basis for evaluating hypersingular equations. The first application areas will be in calculating stresses near cracks in elastic bodies and the scattering of elastic waves from thin bodies.

**An Automated Environment for Engineering Software Development**

Chen, Wai-Fah; Purdue University

To develop prototype domain-specific software environment using structural engineering domain as a test case. This environment will be centered around the concept of software reuse and will be composed of Computer Aided Software Engineering (CASE) tools for managing and integrating reusable software components into applications.

**Reliability and Uncertainty**

**Applications of ARMA and Related Random Process Models in Structural Engineering**

Shinozuka, M.; Columbia University

Auto-regressive moving-average (ARMA) and related models were developed for stationary Gaussian multivariate random processes, constructed for multi-dimensional random processes with prescribed correlation functions, adapted to structural engineering applications.

**Workshop on Reliability in Computational Mechanics**

Oden, J.; University of Texas-Austin

This workshop sensitizes researchers to current methodologies for evaluating reliability as it applies to computational mechanics problems.

**Performance-Optimized Reliability Analysis of Structural Systems**  
Corotis, Ross; Johns Hopkins University, Baltimore, MD

To develop practical procedures whereby characterization of limit state consequences can be incorporated into an analysis for reliability-constrained structural system optimization. Multiobjective mathematical programming will be used to identify the limit state surface for multiple, independently varying loads. Efficient methods of combining Monte Carlo directional simulation with mathematical programming, and especially linear programming, will be utilized to permit a comprehensive assessment of structural system reliability.

**A New Approach - Convexity Modelling - to an Old Research Topic: Buckling of Viscoelastic and Nonlinear Elastic Structures with Uncertain Imperfections**  
Elishakoff, Isaac; Florida Atlantic University

To study the uncertainty in imperfections in viscoelastic plates and their effects on buckling. The main objective is to investigate the feasibility of convex modeling - a new concept of uncertainty in imperfection sensitivity analysis.

**Sensitivity Analysis and Failure Cost Design of Random Structural Systems in Load Space**  
Soltani, Mehrdad; University of Miami

A nonlinear structural analysis program is utilized to derive ultimate and serviceability limit states in load space. System probability of failure is computed by integration of the joint density function of applied loads in the failure region in a three-dimensional load space.

**Structural Engineering and Reliability Analysis**  
Tallin, Andrew; Polytechnic University, Brooklyn, NY

Research includes computer-based reliability analysis with special emphasis on systems with nonlinear dynamic response and hierarchical damage states, development of a hybrid reliability approach applied to seismic response and fatigue crack growth.

**The First International Symposium on Uncertainty Modeling and Analysis: Fuzzy Reasoning, Probabilistic Methods, and Risk Management**  
Ayyub, Bilal; University of Maryland

Probabilistic methods, Bayesian approaches, statistical techniques as well as fuzzy logic and reasoning are currently being used by researchers to deal with uncertainty. In other countries,

especially Japan, researchers and engineers are currently using all these methods more effectively in many engineering applications. The Symposium provides the opportunity for U.S. researchers and engineers to interact with counterparts from other countries.

**Reliability of Existing Framed Structures Using Approximate Reasoning**

Chou, Karen; Syracuse University

Research emphasizes the reliability and safety of existing and newly constructed plane frame structures. Classical reliability measure will be updated utilizing the fuzzy set logic to include the quality of connections and structural members through inspections.

**Energy-Based Damage Model and Procedure for Safety Analysis of Structural Systems In Steel Buildings**

Sohal, Iqbal Singh; Rutgers University

An energy-based damage model will be developed for computation of residual strength and stiffness of damaged steel structural members. For safety evaluation of structural systems, a damage analysis computer code is to be developed.

**Integration of System Reliability in the Structural Design Process  
System Reliability-Based Design of Transmission Structures**

Dagher, H. Joseph; University of Maine

To develop a computer program that designers and utilities can use to incorporate system reliability in the design of transmission structures.

**Structural Optimization**

**Design Sensitivity Analysis and Optimization of Nonlinear Structural Systems**

Arora, Jasbir; University of Iowa

The variables to be considered are: structural dynamics, nonlinear buckling loads, general nonlinearity, nonconservative aspects, elastic-plastic, viscoelastic, viscoplastic and related topics.

**Design Sensitivities and Shape Optimization for Nonlinear Solid Mechanics Problems by Boundary Element Methods**

Mukherjee, Subrata; Cornell University

To calculate design sensitivity coefficients for nonlinear solid mechanics problems and their use in design.

**Optimal Design of Large-Scaled Structures with Multianalysis-Type Constraints**

McGee, Oliver; Ohio State University

To develop efficient and reliable iterative procedures for minimum-weight design of large structures subject to multianalysis-type constraints (e.g., displacements, stresses, natural frequencies, and elastic stability).

**A Decomposition Strategy for Optimization of Reinforced Concrete Systems**

Balling, Richard; Brigham Young University

Decomposing the design into a system level and a component level. Continuous and discrete optimization techniques are applied at both level. Heuristic configuration search algorithms are used at the component level.

**Chaos**

**Chaotic Dynamics of Elasto-plastic Structures: Experiments and Novel Computational Methods**

Moon, Francis, C.; Cornell University

Attempt to develop radically new computational tools for analysis of nonlinear vibrations by inventing a way to put the computation on a chip using nonlinear cellular networks, as well as using both existing supercomputers.

**Plastic Instability Phenomena with Aspects of Chaos of Fixed-Edge Structures Subjected to Short Pulse Loading**

Symonds, Paul S.; Brown University

Anomalous phenomena have been studied in uniform beams by finite element analysis and indications of deterministic chaos appear. Their novelty and interest lie in their appearance with workless constraints and in the absence of harmonic or other sustained excitation. Research aims to confirm and amplify the preliminary results already obtained to gain deeper understanding by further analysis and extensions, and to provide methods for the estimation of parameter ranges in which they are likely to occur.

**Chaos and Dimensionality in the Dynamics of Elastic Solids: Application of the Theory of Chaotic Vibrations to the Nonlinear**

**Modelling of Flexible Structures**

Cusumano, Joseph; Pennsylvania State University

To develop better techniques for obtaining finite degree of freedom nonlinear models of flexible structures. Research includes testing, signal analysis and modeling.

**Computational Modeling and Others****Intersection of Shells of Revolution**

Steele, Charles; Stanford University

To develop more efficient methods of analysis of stress and deflections in intersecting shell structures. Finite element codes are intrinsically inefficient. The project is a combination of analytical and numerical methods.

**Studies of Non-Linear Theory of Shear Deformable Elastic Shells**

Reissner, Eric; University of California-San Diego

Based on the status of the existing nonlinear theory of shear-deformable elastic shells with independent descriptions for translational and rotational displacement states, it was proposed to undertake research on the form of two-dimensional constitutive relations for finite deformations of shells, and on possible new solutions of specific problems for shallow shells of revolutions and helicoidal shells. Completed in 1985.

**Initiation and Growth of Tearing in Ductile Sheet Metal**

Wierzbicki, Tomasz; MIT

To study plastic folding and tearing mechanisms of sheet metal and to develop models of fracture initiation and growth.

**Application of Advanced Computer-Based Technologies in Civil Engineering**

Rehak, Daniel; Carnegie Mellon University

Specific research includes finite element systems, database management systems, and integrated computer-aided engineering systems. To create a friendly, responsive, and powerful computer-based engineering environment.

**A New Approach for the Analysis of Piece-Wise Periodic Structures of Finite Length**

Lyrintzis, Constantinos; San Diego State University

A combination of finite elements, transfer matrices and wave

propagation, the computations to proceed in the direction of wave propagation only, so that the results are always numerically stable.

#### **Studies in Structural Mechanics**

Miller, Gregory; University of Washington, Seattle

Emphasis on the application of advanced analytical and numerical techniques to the study of problems in materials characterization and structural response, including fracture mechanics, plasticity, fatigue analysis, micro-mechanics, and nonlinear dynamics.

#### **SUMMARY AND CONCLUSION**

An overview of the state of the art and NSF projects in computational and structural mechanics is presented. The authors hope that this paper will act as a catalyst, sparking interest and further research in these areas. This paper reflects the personal views of the authors, not necessarily those of the National Science Foundation.

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**Funding Programs of the Computational  
Mechanics by National Natural Science  
Foundation of China (NSFC)**

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Department of Mathematical and Physical Sciences, NSFC.

I. General Survey of NSFC

• **Duty:** The basic duty of NSFC is to support basic research and parts of applied basic research. The areas of these researches are mathematics, physics, chemistry, astronomy, geology, biology, information science, engineering and material science, and so on.

The ratio of funds for basic and applied basic research is three to seven. This ratio is naturally formed.

• **Organization:** NSFC consists of one office, four bureaux and six science departments.

NSFC consists of about 200 staff, among whom there are about 80 persons who once worked in the field of science and technology.

• **Sources of Funds:** The funds of NSFC come from the governmental financial allocation. It has increased from eighty million yuan in 1986 to one hundred and seventy five million yuan in 1991. The annual increasing rate is seventy percent.

• **Three Levels of Programs Supported:**

The programs supported by NSFC are divided into three kinds: general programs, key programs and major programs. Their funds percentages are 75%, 15% and 10% respectively.

• **Principles and steps of program evaluation:** Applying programs are evaluated under the principle of "relying on experts, promoting democracy, supporting best programs, and being in a fair and reasonable way". After initial evaluation done by the academic committee of the unit where the applicant works, and peer-reviews, the subject evaluation panels give funding opinions to each program, and finally the Foundation ratifies the

execution. The evaluation of the programs focuses on creativity, originality, frontiers and intersections of the disciplines.

Every year NSFC receives a great deal of applications of which about a quarter is usually supported. Table one is the survey of the general programs applied and supported in 1990. As an example, it shows the outlook of the funds work.

- The international cooperation and academic exchange: Since the founding of NSFC, it has signed agreements or memorandum of understanding with fourteen countries and regions. In 1990, NSFC has supported 505 programs which spent about 3.3 million yuan.

## II. Brief Introduction of Mechanics Discipline

Mechanics is a basic as well as a technical discipline. It plays an important role in the development of science and technology in China.

- The main areas of research supported by Mechanics Section are solid mechanics, fluid mechanics, general mechanics, experimental mechanics, computational mechanics, rock and soil mechanics, biomechanics, explosion mechanics, physical mechanics, rational mechanics and so on.

- The areas of research encouraged are mechanical properties of materials including their constitutive relations; study of the nonlinear behavior in large deformation under various conditions of some typical materials with combined macroscopic, mesoscopic and microscopic methods; turbulence and its control; vortex-dominated flows and separated flows, boundary - layer theory and drag reduction, nonlinear waves and coupled fluid-solid problems; dynamic response and optimization theory of complex, nonlinear, large-scale structures; environmental mechanics and so on.

- General programs are evaluated once a year. Table two records the datum about application and support of mechanical programs in 1990. Table three gives us the distribution of the applied and supported programs in the disciplines in 1990.

- The key programs were not taken out from the general programs until 1990.

During 1986 - 1990, three major programs were supported:

- Mechanical Problems in Ocean Engineering;
- Complicated Flows Dominated by Vortex, Shock and Nonequilibrium Phenomenon;
- Fracture and Constitutive Relations of Metallic Materials. ,

### III. Some Achievements in the Work of Funding for Computational Mechanics

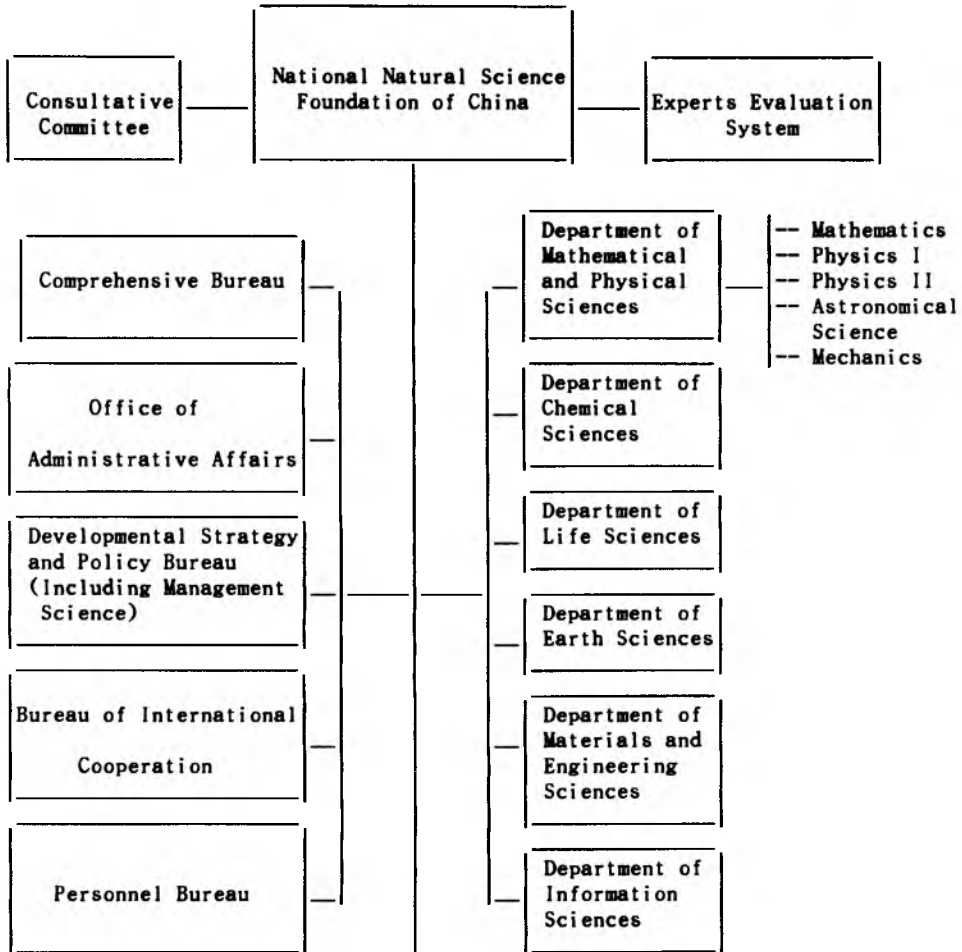
In recent years, NSFC has supported computational mechanics in three directions. They are research in numerical methods, structure optimization design, and developing computer software and standardization. Chinese scholars have done a lot of work in these directions and got some achievements.

### IV. Developing International Cooperation and Promoting the Development of Science

As basic research is generally characterized by international and opening features, NSFC pays a great deal of attention to international exchange and co-operation, encourages and supports academic exchanges. Of course, this co-operation should be sincere, equal, mutual beneficial and mutual respect. The results of the co-operative research should be shared by both sides. In recent years, we have six joint programs with England and Bulgaria which achieved satisfactory results.

With these results, we are confident that this symposium will expand and deepen international cooperation and exchanges, and we will make more contributions to the development of science and technology.

## NSFC Organization Chart



**Table 1** An Overview of the Amount of Support and Application for General Projects in 1990

Unit : 1,000 yuan (RMB)

	Project	Amount	Intensity	Rate of Funding
Application	14438	759273		
Support	3478	119792	34.4	24.1%

**Table 2** An Overview of the Amount of Support and Application for General Projects in Mechanics Discipline in 1990

Unit : 1,000 yuan (RMB)

	Project	Amount	Intensity	Rate of Funding
Application	422	24070		
Support	114	3650	32	27.0%

**Table 3** The Distribution of the Applying and Support Disciplines (Percentage)

Branches of Mechanics	Applying	Supporting
Solid Mechanics	29.3	30.7
Fluid Mechanics	21.8	25.3
General Mechanics	10.7	12.0
Experimental Mechanics	10.0	9.3
Computational Mechanics	12.8	12.0
Others	15.4	10.7

## Partial Differential Equations and Hamiltonian System

Zhong Wanxie

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Dalian Univ. of Tech., China)

(Higher Education Press  
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### Abstract

In theory of mechanics and/or mathematical physics problems in prismatic domain, the method of separation of variables usually leads to the Sturm-Liouville type self-adjoint eigen-problems, and then the eigenfunction expansion method can be applied in equation solving. However, a number of important application problems cannot lead to self-adjoint operator for the transverse coordinate. From the minimum potential energy variational principle, by selecting the state and its dual variables, the generalized variational principle is derived, and then based on the analogy between the theory of structural mechanics and optimal control, the present paper derives the problem to Hamiltonian system. The finite dimensional theory for Hamiltonian system is extended to the corresponding theory of Hamiltonian operator matrix and adjoint symplectic spaces. The adjoint symplectic ortho-normality relation is proved for the whole state eigenfunction vectors, and then the expansion of an arbitrary whole state function vector by the eigenfunction vectors is established. Thus the classical method of separation of variables is greatly extended.

**Keywords:** *Separation of variables; Hamiltonian system; Symplectic; Eigenfunction Expansion; Plate bending; Whole state space.*

### 1. Introduction

The analogy theory between computational structural mechanics and optimal control had been given in some detail<sup>[1-5]</sup>. However, all the problems given in these papers are for finite dimensional state vector. Considered from the analogy between the semi-analytical method of elliptic partial differential equation (PDE) and the continuous-time optimal control, it is recognized that the dimension of the PDE is originally infinite, for the establishment of the analogy relation to finite dimensional control system, semi-analytical method is invoked. In creatises on semi-analytical method<sup>[6,7]</sup>, the derivations are almost all based on the displacement method.

By using the finite dimensional subspace in the transverse coordinate, the semi-analytical method approximates the infinite dimensional space of the original PDE. Hence there is the problem of appropriate selection of subspace for better approximations. For the PDEs, for which the

variables can be separated, it is natural to select the eigen-solutions of its Sturm-Liouville problem<sup>[8]</sup> as the basis functions of subspace, such that the ordinary differential equations derived are ultimately de-coupled. Unfortunately, the Sturm-Liouville problem can only be applied to self-adjoint operators after the separation of variables, which brings strict limitation to its extent of application. Thus semi-analytical method was proposed as the extension. However, the appropriate selection of the subspace remains a tremendous problem to be solved.

In the investigation of eigen-problems of large scale system theory, the various reduced state space methods<sup>[9]</sup> cannot ensure the eigenvalues of the original problem unchanged. For the invariance of eigen-solutions of the original system, it is necessary to select subspace from the whole state space, which corresponds to the theory of contact transformation of the Hamiltonian dynamic system, i.e. to select the adjoint symplectic subspace<sup>[10,11]</sup>. The main eigen-solutions dominate the long term property, so that in the numerical method for PDE, this factor must be considered.

The minimum potential energy variational principle corresponds to the displacement method<sup>[12]</sup>. From the analogy between structural mechanics and optimal control, the displacement corresponds to the state vector in control theory. Hence the minimum potential energy based semi-analytical method corresponds to the reduced state space method in large scale system theory. Unfortunately, this method corresponds only to the extended point transformation in Hamiltonian dynamic system theory, but not the contact transformation<sup>[13]</sup>. Generally speaking, the main eigen-solutions will be disturbed in the subspace for using only extended point transformations.

From Hamiltonian system theory, the general contact transformation should be executed in whole state space, which corresponds to the combination of spaces of displacement and internal force in structural mechanics. In PDE theory, the whole state space should be the combination of variables composed of the field function and its dual. From the viewpoint of variational methods<sup>[12]</sup> in theory of elasticity, the generalized variational principle<sup>[12]</sup> should be used.

Applying the theory of contact transformation to the transformed PDE and introducing the eigen-problem of the Hamiltonian system, the eigenfunction expansion method can then be used to solve the PDE. The classical Sturm-Liouville problem and its eigen-function expansion for PDE is thus a special case of the present method.

## 2. The Hamiltonian operator matrix and variational principles

The principle described in the introduction is quite general, but the theory is better described with some typical problems, in which the principle given in the previous section should be reviewed frequently, so as to capture the essence of the present method.

For simplicity, the second order elliptic PDE is selected as the first problem, then the plate bending problem in a strip domain is solved by the eigen-function expansion method in the latter text. By using these two examples to describe the general method, it is conceivable that all the elastic analysis problem in a prismatic domain can be solved by the method of Hamiltonian operator matrix for whole state space, the generalized variational principle, the eigen-solutions in the adjoint symplectic space and the eigen-function expansion solution.

As the first problem, the second order elliptic PDE is considered

$$a\delta^2 u/\delta x^2 + b\delta^2 u/\delta x\delta y + c\delta^2 u/\delta y^2 - eu = 0 \quad (2.1)$$

For simplicity, constant coefficients  $a, b, c, e$  are assumed, and  $b^2 - 4ac < 0$ . The domain  $\mathcal{D}$  is taken to be a strip of

$$\mathcal{D}: \quad 0 < x < h, \quad 0 < y < 1, \quad h \gg 1 \quad (2.2)$$

$x$  is the longitudinal direction. The boundary conditions are

$$u = 0, \quad \text{when } y=0 \text{ or } y=1; \quad (2.3)$$

$$u = \text{given functions } u_{0*}(y) \text{ or } u_{h*}(y), \quad \text{when } x=0 \text{ or } x=h \quad (2.4)$$

The corresponding variational equation can be given as

$$\min_u \iint_{\mathcal{D}} \left[ \frac{a}{2} \left( \frac{\partial u}{\partial x} \right)^2 + \frac{c}{2} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{b}{2} \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{e}{2} u^2 \right] dx dy \quad (2.5)$$

which is the minimum potential energy principle, where only the field function  $u(x,y)$  is in variation. The finite terms semi-analytical method based on the above variational equation<sup>[2,6]</sup> corresponds to the reduction from transverse infinite dimensional space of the PDE to finite dimensional subspace by the reduced state space method, for which the main eigen-solutions will be disturbed, which is not preferred. Hence, the dual function should be introduced.

By partially differentiating the integrand of (2.5) with respect to  $(\partial u/\partial x)$  and taking negative, the 'internal force' dual function  $n(x,y)$  is introduced as

$$n(x,y) = -(a\delta u/\delta x + b/2 \cdot \delta u/\delta y), \quad \text{or } \delta u/\delta x = -(n - b/2 \cdot \delta u/\delta y)/a \quad (2.6)$$

Substituting the above equation into (2.1), it gives

$$\delta n/\delta x = - \left[ e - (c-b^2/4a)\delta^2/\delta y^2 \right] u - (b/2a) \cdot \delta n/\delta y \quad (2.7)$$

Now the dual equations (2.6) and (2.7) can be written in operator form

$$\left. \begin{aligned} \delta u/\delta x &= -[(b/2a)\delta/\delta y]u - (1/a)n = \underline{F} u - \underline{G} n \\ \delta n/\delta x &= -[e - (c-b^2/4a)\delta^2/\delta y^2]u - [(b/2a)\delta/\delta y]n = -\underline{Q} u - \underline{F}^T n \end{aligned} \right\} \quad (2.8)$$

where the operators  $\underline{F}, \underline{G}, \underline{Q}$  are

$$\underline{F} = -(b/2a)\delta/\delta y, \quad \underline{F}^T = (b/2a)\delta/\delta y \quad (2.9)$$

$$\underline{G} = 1/a, \quad \underline{Q} = e - (c-b^2/4a)\delta^2/\delta y^2 \quad (2.10)$$

The generalized variational principle of the above dual equations is necessary for clarifying the meaning of the operators. By integration by parts and taking account of the boundary conditions it can be verified that the variational equation of

$$\delta \int_0^h \int_0^1 \left[ -n \frac{\partial u}{\partial x} - \frac{b}{2a} n \frac{\partial u}{\partial y} - \frac{n^2}{2a} + \frac{e}{2} u^2 - \frac{1}{2} \left( c - \frac{b^2}{4a} \right) u \frac{\partial^2 u}{\partial y^2} \right] dy dx = 0,$$

where  $u$  and  $n$  are considered as independent functions in the variational operation, can be used to derive the dual equation set (2.8). Based on the analogy of PDE with optimal control theory, the longitudinal coordinate  $x$  corresponds to the time coordinate in control. The above variational equation can be expressed in the operator form