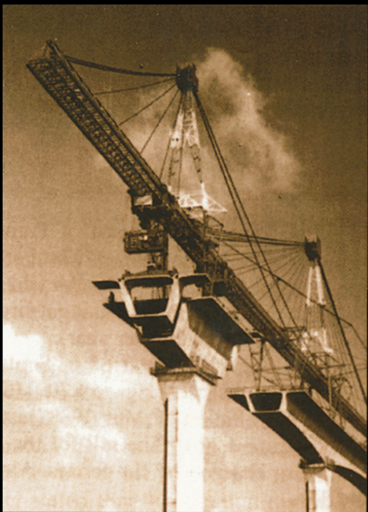
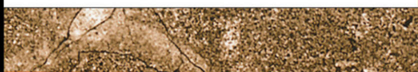
E & FN SPON
An Imprint of Routledge





CONCRETE MIXTURE PROPORTIONING

A SCIENTIFIC APPROACH

F. de LARRARD

MODERN CONCRETE TECHNOLOGY 9

Concrete Mixture Proportioning

Modern Concrete Technology Series
Series Editors

Arnon Bentur
National Building Research Institute
Technion–Israel Institute of Technology
Technion City
Haifa 32 000
Israel

Sydney Mindess
Department of Civil Engineering
University of British Columbia
2324 Main Mall
Vancouver
British Columbia
Canada V6T 1W5

1. **Fibre Reinforced Cementitious Composites**
   A. Bentur and S. Mindess

2. **Concrete in the Marine Environment**
   P.K. Mehta

3. **Concrete in Hot Environments**
   I. Soroka

4. **Durability of Concrete in Cold Environments**
   M. Pigeon and R. Pleau

5. **High-Performance Concrete**
   P.-C. Aïtcin

6. **Steel Corrosion in Concrete**
   Fundamentals and civil engineering practice
   A. Bentur, S. Diamond and N. Berke

7. **Optimization Methods for Material Design of Cement-based Composites**
   Edited by A. Brandt

8. **Special Inorganic Cements**
   I. Odler (forthcoming)

9. **Concrete Mixture Proportioning**
   F. de Larrard

10. **Permeability and Poresize in Cement-based Materials**
    K. Aligizaki and P.D. Cady (forthcoming)

11. **Sulfate Attack on Concrete**
    J.P. Skalny, P.W. Brown, V. Johansen and J. Marchand (forthcoming)

12. **Fundamentals of Durable Reinforced Concrete**
    M.G. Richardson (forthcoming)

Concrete Mixture Proportioning
A scientific approach

François de Larrard

E & FN SPON
An Imprint of Routledge
London and New York

First published 1999
by E & FN Spon, an imprint of Routledge
11 New Fetter Lane, London EC4P 4EE

Simultaneously published in the USA and Canada
by Routledge
29 West 35th Street, New York, NY 10001

© 1999 François de Larrard

All rights reserved. No part of this book may be reprinted or reproduced or utilized in any form or by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying and recording, or in any information storage or retrieval system, without permission in writing from the publishers.

British Library Cataloguing in Publication Data
A catalogue record for this book is available
from the British library

Library of Congress Cataloging in Publication Data
Larrard, François de
Concrete mixture proportioning : a scientific approach / François de Larrard.
p. cm. -- (Modern concrete technology)
Includes bibliographical references and index.
ISBN 0-419-23500-0
1. Concrete--Mixing. I. Title. II. Series: Modern concrete technology series (E. & F.N. Spon)
TA439.L32 1999
666'.893--dc21 98-39234
CIP

ISBN 0 419 23500 0

Contents

Foreword

To write a foreword to this book is a pleasure but the task also presents some difficulties. The pleasure arises from the great importance of the work, but I feel I should first dispose of my 'difficulties'.

As François de Larrard acknowledges in his letter inviting me to write this foreword, my approach to concrete is pragmatic, coupled with doubts about ready-to-use mathematical models for mix design. This is true; in my book *Properties of Concrete* (4th edition), published in 1995, I say: 'an exact determination of mix proportions by means of tables or computer data is generally not possible: the materials used are essentially variable and many of their properties cannot be assessed truly quantitatively. For example, aggregate grading, shape and texture cannot be defined in a fully satisfactory manner.'

My views arose from consideration of commercial models, developed in Australia, the United Kingdom and the United States, each based on a limited range of experimental data, and yet each claiming universal validity. The critical feature of those models is that they are more-or-less statistical fits to experimental data, without the necessary explanation in terms of actual physical phenomena. Correlation of variables should not necessarily be taken to imply causation. For example, over a period of ten years, in a given state, the consumption of alcohol increased and, at the same time, the salaries of teachers increased too. It was concluded that the more the teachers are paid the more alcohol is drunk. This may be an apocryphal story, but *si non è vero è ben trovato.*

The fact is that some engineers fall into that kind of trap. Not so, de Larrard. His models are physical models of which the parameters, in so far as possible, correspond to measured quantities. Herein lies one considerable virtue of his work. There is no denying that, if observed phenomena cannot be expressed in mathematical terms, generalized use of the observed relations is not possible. At the same time, if the mathematical terms are not related to physical phenomena in a manner that stands up to a logical interpretation, there is a great risk of using a model whose limits of validity are not known.

Having read de Larrard's book, I fully accept that he is on the right path. This path must be followed in order to bring the use of concrete and, above all, the mix design, into line with the use of other materials, such as polymers and metals. In other words, de Larrard's approach will make concrete a truly purpose-designed manmade material, instead of just a reconstituted rock.

My use of the future tense might give the impression that the problem has not been fully solved. This is no criticism of de Larrard's work: nobody could achieve more today, if only because we are still unable to quantify all the properties of aggregate, as I mentioned earlier. And this is a formidable task. The physical means to achieve it are available without further basic research; the difficulty lies in the fact that no commercial part of the 'concrete system' has an interest in financing the necessary development.

As for de Larrard's models, I am impressed by the way in which he has validated them by independent assemblies of coherent data, without ever contradicting practical experience. His approach is the best way to progress from today's most common building material, that is, bad concrete, to concrete which, in every case, will be purpose-designed and tailored to the expected use.

There are a few, more general, comments which I would like to make. François de Larrard, a Frenchman in the great tradition of École Polytechnique, has chosen to write this book, not in what the French love to call *la langue de Molière*, which is also *la langue de Descartes*, quoted by de Larrard, but in what has indubitably become the worldwide language of technology, science, and much more. Yet he has retained the Cartesian approach and, for bringing this rigour to concrete, we should be very grateful to him.

Indeed, up to now, there has been very little interaction between the French, very mathematical, approach to concrete on the one hand and, on the other, the British and American pragmatic approach. French books on concrete are rarely translated into English; likewise, it is only in 1998 that, for example, my book *Properties of Concrete* appeared in a French translation (having, over the years, been translated into twelve other languages). An yet, it is the same well-designed concrete mix that is needed on both sides of the Channel and on both sides of the Atlantic, and the marrying of the French and Anglo-American approaches cannot but be highly beneficial for all concrete users. So, this 'French' book in English is very welcome. Its title includes the words 'a scientific approach'. It is a scientific approach by a civil engineer, and this is the way forward.

Adam Neville CBE, FEng
London
15 April 1998

Preface

...diviser chacune des difficultés que j'examinerais en autant de parcelles qu'il se pourrait, et qu'il serait requis pour les mieux résoudre...
...conduire par ordre mes pensées, en commençant par les objets les plus simples et les plus aisés à connaître, pour remonter peu à peu, comme par degrés, jusques à la connaissance des plus composés....
...faire partour des dénombrements si entiers et des revues si générales, que je fusse assuré de ne rien omettre.

...to divide every difficulty that I should examine into as many parts as possible and as necessary to better solve it...
...to drive my thoughts in order, starting by the most simple objects, the easiest to know, so as to rise gradually, step by step, up to the knowledge of the most compound ones...
...to make such complete accounts and so general reviews, that I might be assured of missing no thing.

(René Descartes, *Discours de la méthode*, 1637)

The aim of this book is to build a consistent, rational and scientifically based approach to designing concrete mixtures for most civil engineering applications. An attempt has been made to consider the following facts, which change the nature of the problem, as compared with the situation faced by our predecessors (from René Féret and Duff Abrams to more recent contributors):

- Concretes are no longer made of aggregate, Portland cement and water only. Often, if not always, they incorporate at least one of the following products: organic admixtures, supplementary cementitious materials, fibres.
- Nowadays concretes must meet a comprehensive list of requirements, which are not limited to the final compressive strength, but include rheological properties, early-age characteristics, deformability properties and durability aspects.

- As for the desired properties, the range of attainable values has displayed a dramatic increase in the past few years. For instance, no-slump to self-compacting concrete can be used. The compressive strength at 28 days may be as low as 10 MPa for some dam mass concretes, or as high as 200 MPa or more for some special precast products.
- A purely experimental and empirical optimization is less and less likely to succeed, because of the high number of parameters involved (both input and output), the high labour expenses of such studies, and the economic and time constraints that are characteristics of the industrial world.

But besides these negative aspects (with regard to the ease of optimizing a concrete), there are, fortunately, positive ones:

- Concrete technology is no longer a young technology; since the beginning of this century a huge amount of experimental data has been published, which can be exploited and synthesized. The latest edition of Adam Neville's book provides a unique survey of this literature (Neville, 1995).
- The development of computers helps the researcher to manage large amounts of experimental data. From these data, he may discover the physical laws underlying the behaviour of the concrete system, and translate them into semi-empirical mathematical models, linked with databases (Kaëtzel and Clifton, 1995).
- Once those models are incorporated in user-friendly software, practitioners may readily use them, even with limited knowledge of the vast complexity of the system. Thus optimal responses to industrial problems may be given quickly. At the present state of development, experiments are still necessary, but the software allows the formulator to cut the number of tests drastically, to focus the experimental programmes better and take maximum advantage of the new data generated.

On the basis of these statements, the objectives of this book are twofold: (i) to develop simplified models linking concrete composition and properties, so that an analytical solution of optimization problems becomes possible, allowing one to demonstrate the main rules and trends observed, and therefore understand the concrete system better; (ii) to build more comprehensive and complex models, which may be easily implemented in software and be used in practice for designing concretes with real materials for real applications.

Designing a concrete is first of all a packing problem. All existing methods implicitly recognize this statement, either by suggesting measurement of the packing parameters of some components (ACI

211), or by approximating an 'ideal' grading curve that is assumed to lead to maximum compactness with real materials. It is noteworthy that all authors propose different curves (or families of curves), which raises doubts as the soundness of this concept. Chapter 1 of this book presents in detail a theory developed by the author and his colleagues after a 12-year research effort. It is the first one, to the author's knowledge, to solve the question of the packing density of dry mixtures in all its general extent, with sufficient accuracy for practical application. The theory agrees with most classical results, and shows that the ideal proportions of a given set of aggregate fractions depend not only on the grading curves, but also on the packing abilities of each grain fraction, including the fine ones.

Chapter 2 is devoted to an analysis of the relationships between the composition of concrete and its properties. The models developed often refer to packing concepts, considering either the whole range of solid materials in fresh concrete or the aggregate skeleton in hardened concrete. Most engineering properties are dealt with, but with less emphasis on durability. This is not because the subject is not important, but simply because at present few data are available that both cover a wide range of concretes and relate to true durability-related material properties.

Chapter 3 lists the constituent properties that control concrete properties in the previously developed models. It is noteworthy that this list appears limited compared with the huge collections of parameters that encumber some concrete studies.

Chapter 4 shows how to use the models presented in Chapter 2. It is the core of the book. With the help of simplified models, optimization problems are first solved analytically. This approach qualitatively highlights the most important features of the concrete system. Then a more refined solution is worked out by using the complete models together with a spreadsheet package containing an optimization module (*solver*). General questions dealing with mixture proportioning are discussed in the light of numerical simulations, and some existing empirical methods are critically reviewed.

Chapter 5 presents a number of applications of the proposed approach to a series of industrial problems. It is shown here that the book provides a conceptual framework that applies to any cementitious granular material, including mortars, roller-compacted concrete and sprayed concrete. A special emphasis is put on high-performance concrete, but it is demonstrated that the approach applies just as well to very common 'commodity' concrete. Real concrete formulae, with measured properties, are presented, and are compared with the model predictions obtained for the same specifications.

In the conclusion, research needs for improving the models are stressed. Clearly, the area of durability is one that deserves the highest effort. Engineers must eventually be able to design a concrete for a given structure life span, in a given environment. Today we are lagging far behind this objective. It is also recognized that some models are too empirical, because the scientific basis for some problems is still lacking, making extrapolations sometimes hazardous. However, the author's defence is that an engineer must solve a problem with the most appropriate means that are available to him. Nevertheless, future research should aim to decrease as much as possible the part that empiricism plays.

In regard to the practical use of this book, a flowchart is provided in Appendix 1, which makes it possible to implement the various models in software. Also, it is briefly shown that the concepts presented may be used not only for designing new mixtures but also for quality control purposes. In an industrial production process, as raw materials are continuously changing, the concrete system should remain alive, adapting itself to give an output that is as constant as possible (Day, 1995). This is probably one of the greatest challenges of today's concrete industry, equal to the development of new and 'exotic' formulae.

This book is not a state of the art: that is, a compilation of existing and published knowledge about mixture proportioning. It contains essentially original findings, acquired by the author and his colleagues at the Laboratoire Central des Ponts et Chaussées (Central Laboratory for Roads and Bridges), Paris. Here the author had the chance to spend 12 years in an exceptional scientific environment, while being in contact with the 'real concrete world' through the network of the regional 'Ponts et Chausseés' laboratories. The approach tries to be ahead of the current technology. However, the views presented herein are often personal, and it is the author's hope that this book will help in sharing them with the international scientific and technical community.

A last remark addressed to civil engineers: this book contains very little (if any) chemistry.[1] This is partly because the author is not a chemist, but also because the level at which mix design of concrete is envisaged is that of assembling the components. Facts related to the phases are taken as hypotheses; only their consequences in concrete are studied in this book. However, the author is conscious that more chemistry would be required, if a comparable approach were to be extended to all durability aspects.

F. de Larrard
Bouguenais, January 1998

[1] This remark may be considered either as negative or positive, depending on the reader's background.

Acknowledgements

My deepest gratitude goes to various people without whom this work would not have been possible:

- to the two institutions in which I had the fortune to serve as a researcher (Laboratoire Central des Ponts et Chaussées, Paris/Nantes, and National Institute of Standards and Technology, Gaithersburg, Maryland, who kindly welcomed me as a guest researcher from September 1996 to June 1997);
- to all the people who supported me in the project of writing such a book by their interest regarding our research: our industrial partners, our students and the practitioners who attended the numerous concrete training sessions to which I have contributed for about 10 years;
- to all the friends who participated as co-researchers in the scientific adventure related in this book: Paul Acker, Daniel Angot, Chiara F. Ferraris, Chong Hu, Ali Kheirbeck, Pierre Laplante, Robert Le Roy, Jacques Marchand, Thierry Sedran, Tannie Stovall, Patrick Tondat, Vincent Waller;
- to all my colleagues who were kind enough to communicate precious unpublished data for model calibration/validation: François Cussigh, Pierre Laplante, Gilbert Peiffer, Paul Poitevin, Thierry Sedran, Erik Sellevold;
- to all my colleagues who spent time correcting some parts of the manuscript: Mony Ben-Bassat, Dale Bentz, Nicholas Carino, Gilles Chanvillard, Jim Clifton, Clarissa Ferraris, Edward Garboczi, Geneviève Girouy, Robert Le Roy, Thierry Sedran, Vincent Waller;
- and last but not least to my wife and my children, who suffered to see their husband/father glued to his computer for months instead of doing everything they could normally expect from him.

Finally, this book is dedicated to Albert Belloc, Senior Technician, who spent 39 years at LCPC managing the concrete production team. By his courage and rigour he was a unique example for generations of concrete technologists, and played a major role in many original experiments related in this book. I am also extending this dedication to all experimentalists in LCPC and elsewhere, without whom no models or theories can be conceived.

Note to readers: when using the proportioning method readers are advised to check material properties on trial batches.

Taylor & Francis
Taylor & Francis Group
http://taylorandfrancis.com

1 Packing density and homogeneity of granular mixes

The packing density of a granular mix is defined as the solid volume Φ in a unit total volume. Alternatively, the compaction may be described by the porosity:

$$\pi = 1 - \Phi \qquad (1.1)$$

or by the voids index:

$$e = \frac{\pi}{\Phi} = \frac{1}{\Phi} - 1 \qquad (1.2)$$

The prediction of the packing density of a granular mixture is a problem of extreme relevance for concrete (Johansen and Andersen, 1991), but it is also relevant in many other industrial sectors (Guyon and Troadec, 1994). Many composite materials, like concrete, are made up of granular inclusions embedded in a binding matrix. The aim is often to combine grains in order to minimize the porosity, which allows the use of the least possible amount of binder.

The packing density of a polydisperse grain mixture depends on three main parameters:

- the size of the grains considered (described by the grading curves);
- the shape of the grains;
- the method of processing the packing.

In the past, the design strategy has generally been to proportion the different grains to obtain a grading curve close to an 'ideal' grading curve, which is supposed to produce the maximum packing density. The final optimization is achieved by trial and error. Empirical models exist to describe the variation of the packing density of a given mixture with some parameters describing the compaction process. But few models were available (to the author's knowledge) that provided sufficient accuracy to design granular mixtures (accounting for the three parameters listed above). In particular, most available models (Johansen and Andersen, 1991; Dewar, 1993) either deal with a limited number of aggregate fractions, or assume a simplified grading distribution of each individual granular class. By contrast, the *compressible packing model* (CPM), the theoretical content of which is described in this chapter, covers combinations of any number of individual aggregate fractions, having any type of size distribution.

The CPM is a refined version of a previous model, the linear packing density model for grain mixtures (Stovall *et al.*, 1986; de Larrard, 1988), which was later transformed into the solid suspension model (de Larrard *et al.*, 1994a,b). This work has been carried out independently of another model, similar to the linear model, which has been proposed by Lee (1970). In section 1.1, the fundamentals of the linear model are presented. Here the virtual packing density is dealt with, defined as the maximum packing density attainable with a given grain mixture (with maximum compaction). From this virtual packing density the actual packing density is deduced, by reference to a value of a compaction index. The calculation of the actual packing density, together with a series of experimental validations, is given in section 1.2. Then the question of the influence of the boundary conditions is examined and modelled in section 1.3 (namely the wall effect exerted by the container, and the perturbation due to inclusion of fibres). In section 1.4 the model developed is used for the research of optimal grain size distributions. Finally, the limited knowledge of segregation is briefly reviewed in section 1.5, and the definition of two new concepts, namely the filling diagram and segregation potential, is proposed, with a view to characterizing mixtures that are likely to segregate.

1.1 VIRTUAL PACKING DENSITY OF A GRANULAR MIX

The *virtual packing density* is defined as the maximum packing density achievable with a given mixture, each particle keeping its original shape and being placed one by one. For instance, the virtual packing density of a mix of monosize spheres is equal to 0.74 (or $\pi/3\sqrt{2}$), the packing density of a face-centred cubic lattice of touching spheres, while the physical packing density that can be measured in a *random* mix is close to 0.60/0.64 (see e.g. Cumberland and Crawford, 1987), depending on the compaction. Throughout this section, the packing density dealt with is the virtual one.

1.1.1 Binary mix without interaction

Let us consider a mixture of grains 1 and 2, the diameters of which are d_1

and d_2. The two grain classes are said to be *without interaction* if

$$d_1 \gg d_2 \quad (1.3)$$

'Without interaction' means that the local arrangement of an assembly of grains of one size is not perturbed by the vicinity of a grain of the second size.

Let us calculate the (virtual) packing density of a mixture of these grains. First, we must know the result of the packing for each class on its own. The *residual* packing density of each class is denoted by β_1 and β_2 for grains 1 and 2 respectively. Second, the mutual volume fractions are y_1 and y_2, with

$$y_1 + y_2 = 1 \quad (1.4)$$

The *partial volumes* Φ_1 and Φ_2 are the volumes occupied by each class in a unit bulk volume of the granular mix. We have

$$y_i = \frac{\Phi_i}{\Phi_1 + \Phi_2} \quad (1.5)$$

and the packing density is

$$\gamma = \Phi_1 + \Phi_2 \quad (1.6)$$

We now make a distinction between two situations: *dominant coarse grains* and *dominant fine grains.*

When the coarse grains are dominant, they fill the available volume as if no fine grains were present (Fig. 1.1) so that

$$\Phi_1 = \beta_1 \quad (1.7)$$

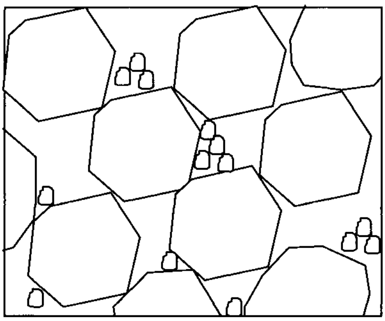


Figure 1.1 Binary mixture without interaction. Coarse grains dominant.

Therefore the packing density may be calculated as follows:

$$\begin{aligned}\gamma &= \Phi_1 + \Phi_2 = \beta_1 + \Phi_2 \\ &= \beta_1 + (\Phi_1 + \Phi_2)\,y_2 \\ &= \beta_1 + \gamma\,y_2\end{aligned}$$

then

$$\gamma = \gamma_1 = \frac{\beta_1}{1 - y_2} \tag{1.8}$$

When the small grains are dominant, they are fully packed in the porosity of the coarse grains (Fig. 1.2), so that

$$\Phi_2 = \beta_2(1 - \Phi_1) \tag{1.9}$$

With a similar approach, it is deduced that

$$\gamma = \gamma_2 = \frac{\beta_2}{1 - (1 - \beta_2)y_1} \tag{1.10}$$

Note that γ_1 and γ_2 may be calculated, whatever the dominant class. Hence in any case we can state

$$\begin{aligned}&\gamma \leqslant \gamma_1 \\ &\Leftrightarrow \Phi_1 + \Phi_2 \leqslant \frac{\beta_1}{1 - \dfrac{\Phi_2}{\Phi_1 + \Phi_2}} \\ &\Leftrightarrow \Phi_1 \leqslant \beta_1\end{aligned} \tag{1.11}$$

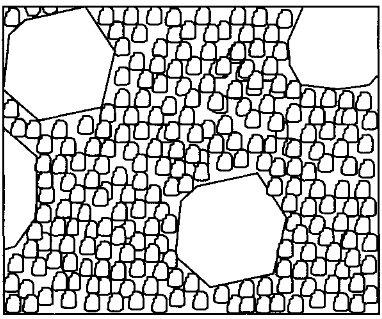


Figure 1.2 Binary mixture without interaction. Fine grains dominant.

in which we recognize equation (1.7). This last inequality is called the *impenetrability constraint* relative to class 1. Similarly, we always have

$$\begin{aligned}
&\gamma \leqslant \gamma_2 \\
&\Leftrightarrow \Phi_1 + \Phi_2 \leqslant \frac{\beta_2}{1-(1-\beta_2)\Phi_1/(\Phi_1+\Phi_2)} \\
&\Leftrightarrow \Phi_2 \leqslant \beta_2(1-\Phi_1) \qquad (1.12)
\end{aligned}$$

because of the validity of the impenetrability constraint relative to class 2. As either class 1 or 2 is dominant,[1] we can write, with no more concern about which is the dominant class:

$$\gamma = \text{Min}(\gamma_1, \gamma_2) \qquad (1.13)$$

In Fig. 1.3 we can see the evolution of the packing density, from pure coarse-grain packing on the left-hand side to pure fine-grain packing on the right. Going from $y_2 = 0$ towards the peak, the packing density increases since a part of the coarse grain interstices is filled by fine grains. At the peak, the fine grains just fill all the space that is made available by the coarse grains. For larger values of y_2, coarse grains are replaced by an equivalent bulk volume of fine grains, decreasing the overall solid volume.

1.1.2 Binary mix with total interaction

Two grain populations are said to have total interaction when

$$d_1 = d_2 \qquad (1.14)$$

while the β_i are generally different. For calculating the packing density of such a mix we state that total segregation does not change the mean

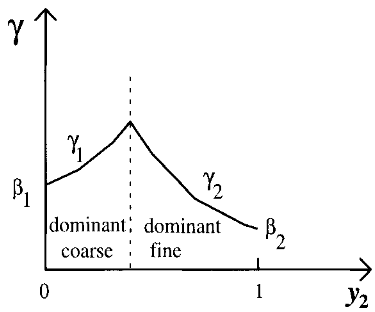


Figure 1.3 Evolution of the packing density vs. fine grain proportion, for a binary mix without interaction.

compactness, so that it is possible to consider that part of the container is filled with only class 1 grains, while the rest is filled with class 2 grains (Fig. 1.4). Therefore

$$\frac{\Phi_1}{\beta_1} + \frac{\Phi_2}{\beta_2} = 1 \tag{1.15}$$

The packing density is calculated by replacing in equation (1.6) one of the partial volumes by its expression as a function of the other one, taken from equation (1.15). In order to keep the same formalism as in section 1.1.1 we may write

$$\begin{aligned} \gamma_1 &= \frac{\beta_1}{1 - (1 - \beta_1/\beta_2) y_2} \\ \gamma_2 &= \frac{\beta_2}{1 - (1 - \beta_2/\beta_1) y_1} \end{aligned} \tag{1.16}$$

so that equation (1.13) still applies. By using the relation $y_1 + y_2 = 1$, it is easy to see that in this particular case $\gamma = \gamma_1 = \gamma_2$ (Fig. 1.5).

1.1.3 Binary mix with partial interaction

We now consider a partial interaction between the classes, defined by the following inequality:

$$d_1 \geqslant d_2 \tag{1.17}$$

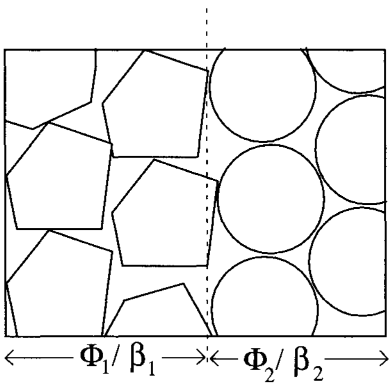


Figure 1.4 Calculation of the packing density in the case of total interaction.

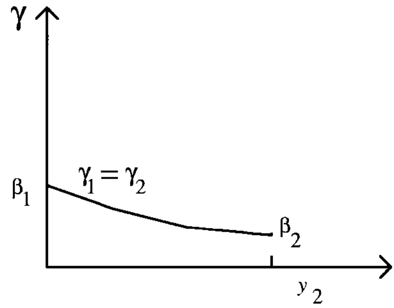


Figure 1.5 Evolution of the packing density vs. fine grain proportion, for a binary mix with total interaction.

We shall begin by describing two physical effects that can be found in binary mixes. Then we shall build general equations, incorporating these effects consistently with the two previous and ideal cases.

If a class 2 grain is inserted in the porosity of a coarse-grain packing (coarse grains dominant), and if it is no longer able to fit in a void, there is locally a decrease of volume of class 1 grains (loosening effect, Fig. 1.6). If each fine grain is sufficiently far from the next, this effect can be considered as a linear function of the volume of class 2 grains, so that

$$\begin{aligned}\gamma &= \Phi_1 + \Phi_2 = \beta_1(1 - \lambda_{2\to 1}\Phi_2) + \Phi_2 \\ &= \beta_1 + (\Phi_1 + \Phi_2)(1 - \beta_1\lambda_{2\to 1})y_2 \\ &= \beta_1 + \gamma(1 - \beta_1\lambda_{2\to 1})y_2\end{aligned}$$

where $\lambda_{2\to 1}$ is a constant that depends on the characteristics of both

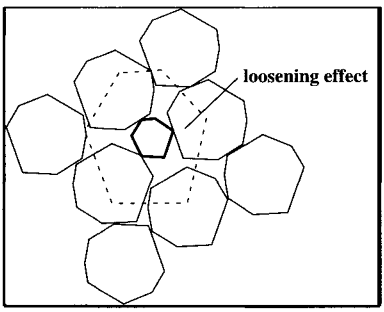


Figure 1.6 Loosening effect exerted by a fine grain in a coarse grain packing.

grain populations. The packing density is then

$$\gamma = \gamma_1 = \frac{\beta_1}{1 - (1 - \beta_1 \lambda_{2 \to 1}) y_2} \tag{1.18}$$

However, to adopt a continuous model that covers all the cases, we prefer the following form:

$$\gamma_1 = \frac{\beta_1}{1 - (1 - a_{12}\beta_1/\beta_2) y_2} \tag{1.19}$$

where a_{12} is the *loosening effect coefficient*. When $d_1 \geqslant d_2$ (no interaction), $a_{12} = 0$, while when $d_1 = d_2$ (total interaction), $a_{12} = 1$.

When some isolated coarse grains are immersed in a sea of fine grains (which are dominant), there is a further amount of voids in the packing of class 2 grains located in the interface vicinity (wall effect, Fig. 1.7). If the coarse grains are sufficiently far from each other this loss of solid volume can be considered as proportional to $\Phi_1/(1 - \Phi_1)$, so that we can write:

$$\begin{aligned} \gamma &= \Phi_1 + \Phi_2 \\ &= \Phi_1 + \beta_2 \left(1 - \frac{\Phi_1}{1 - \Phi_1} \lambda_{1 \to 2} \right) (1 - \Phi_1) \\ &= \beta_2 + \gamma [1 - \beta_2 (1 + \lambda_{1 \to 2})] y_1 \end{aligned}$$

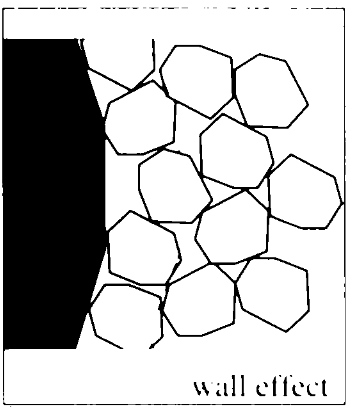


Figure 1.7 Wall effect exerted by a coarse grain on a fine grain packing.

where $\lambda_{1\rightarrow 2}$ is another constant, depending on the characteristics of both grain populations. The packing density is then

$$\gamma = \gamma_2 = \frac{\beta_2}{1 - [1 - \beta_2(1 + \lambda_{1\rightarrow 2})]y_1} \tag{1.20}$$

which we prefer to write as follows:

$$\gamma_2 = \frac{\beta_2}{1 - [1 - \beta_2 + b_{21}\beta_2(1 - 1/\beta_1)]y_1} \tag{1.21}$$

b_{21} is the *wall effect coefficient*. When $d_1 \gg d_2$ (no interaction), $b_{21} = 0$, while when $d_1 = d_2$ (total interaction), $b_{21} = 1$.

As for the binary mix without interaction, it is easy to demonstrate that for any set of proportions y_i we have

$$\gamma = \mathrm{Min}(\gamma_1, \gamma_2)$$

1.1.4 Polydisperse mix without interaction

Let us now consider a mix with n classes of grains ($n > 2$), with

$$d_1 \gg d_2 \ldots \gg d_n \tag{1.22}$$

Class i grains are dominant if

$$\Phi_i = \beta_i(1 - \Phi_1 - \Phi_2 - \ldots - \Phi_{i-1}) \tag{1.23}$$

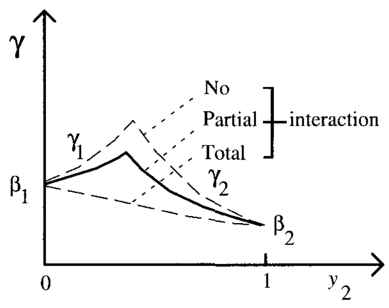


Figure 1.8 Evolution of the packing density of a binary mix vs. fine grain proportion. General case.

In that case, the packing density is calculated as follows:

$$\begin{aligned}
\gamma &= \sum_{j=1}^{n} \Phi_j \\
&= \sum_{\substack{j=1 \\ j \neq i}}^{n} \Phi_j + \beta_i \left(1 - \sum_{j=1}^{i-1} \Phi_j \right) \\
&= \beta_i + (1 - \beta_i) \sum_{j=1}^{i-1} \Phi_j + \sum_{j=i+1}^{n} \Phi_j \\
&= \beta_i + \gamma \left[(1 - \beta_i) \sum_{j=1}^{i-1} y_j + \sum_{j=i+1}^{n} y_j \right]
\end{aligned}$$

Then

$$\gamma = \gamma_i = \frac{\beta_i}{1 - (1 - \beta_i) \sum_{j=1}^{i-1} y_j - \sum_{j=i+1}^{n} y_j} \tag{1.24}$$

Now, let us demonstrate that there is always at least one dominant class. If class 1 is not dominant, then we have

$$\Phi_1 < \beta_1 \tag{1.25}$$

Focusing on the interstitial medium of class 1 grains (that is, the packing of finer particles and the voids volume), if class 2 is not dominant, we can state

$$\Phi_2 < \beta_2(1 - \Phi_1) \tag{1.26}$$

By considering smaller and smaller scales, and still assuming that no class is dominant, we have finally:

$$\Phi_n < \beta_n(1 - \Phi_1 - \ldots \Phi_{n-1}) \tag{1.27}$$

When these n inequalities are strictly and simultaneously verified, each class of grain has a certain clearance as regards the volume available (Fig. 1.9). Therefore the mix is no longer a packing, but rather a suspension. We conclude that at least one equation of the type (1.24) is verified. Otherwise, as for the case dealt with in the previous section dealing with binary mixes, the impenetrability constraint relative to the

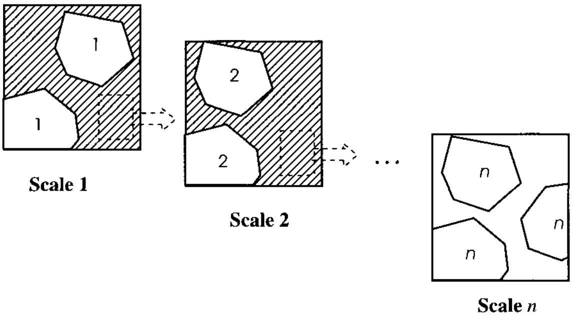


Figure 1.9 A polydisperse mix examined at various scales, where no class is dominant.

ith class is equivalent to

$$\gamma \leqslant \gamma_i \tag{1.28}$$

so that

$$\gamma = \min_{1 \leqslant i \leqslant n} \gamma_i \tag{1.29}$$

1.1.5 Polydisperse mix: general case

Let us first consider the case of a ternary mix, in which

$$d_1 \geqslant d_2 \geqslant d_3 \tag{1.30}$$

Let us assume that class 2 is dominant. Here, class 2 grains are submitted to a loosening effect exerted by class 3 grains, plus a wall effect exerted by class 1 grains (Fig. 1.10). Therefore the packing density

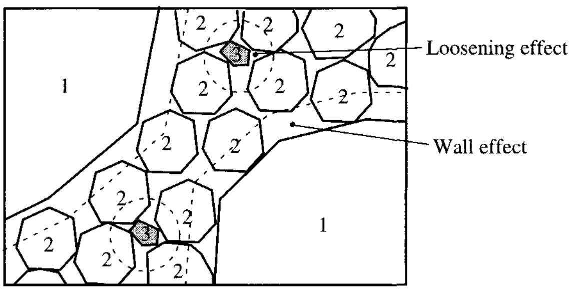


Figure 1.10 Perturbations exerted on the intermediate class by coarse and fine grains in a ternary mix.

of the mix is

$$\begin{aligned}\gamma &= \Phi_1 + \Phi_2 + \Phi_3 \\ &= \Phi_1 + \beta_2\left(1 - \lambda_{3\to 2}\frac{\Phi_3}{1-\Phi_1} - \lambda_{1\to 2}\frac{\Phi_1}{1-\Phi_1}\right)(1-\Phi_1) + \Phi_3 \\ &= \beta_2 + \gamma[1 - \beta_2(1+\lambda_{1\to 2})]y_1 + \gamma(1-\lambda_{3\to 2})y_3 \end{aligned} \tag{1.31}$$

and

$$\begin{aligned}\gamma &= \gamma_2 \\ &= \frac{\beta_2}{1 - [1-\beta_2(1+\lambda_{1\to 2})]y_1 - (1-\lambda_{3\to 2})y_3} \\ &= \frac{\beta_2}{1 - [1-\beta_2 + b_{21}\beta_2(1-1/\beta_1)]y_1 - (1-a_{23}\beta_2/\beta_3)y_3} \end{aligned} \tag{1.32}$$

The linear formulation ensures the additivity of all interactions suffered by one class. This derivation can be easily generalized for n classes of grains with interactions. Also, equation (1.29) still applies, and the equation defining the packing density when class i is dominant is

$$\gamma_i = \frac{\beta_i}{1 - \sum_{j=1}^{i-1}[1-\beta_i + b_{ij}\beta_i(1-1/\beta_j)]y_j - \sum_{j=i+1}^{n}[1-a_{ij}\beta_i/\beta_j]y_j} \tag{1.33}$$

giving the most general formulation for the virtual packing density of a granular mix.

As the virtual packing density is, by definition, non-accessible by experiments, we are going first to continue with the theory to show how we can calculate the actual packing density. Then we shall be able to calibrate and validate the model. The calibration of the model consists essentially in the determination of the a_{ij} and b_{ij} coefficients.

1.2 ACTUAL PACKING DENSITY: THE COMPRESSIBLE PACKING MODEL

We now consider an actual packing of particles, which has been physically built by a certain process. Let Φ be the total solid content, with $\Phi < \gamma$.

1.2.1 Compaction index and actual packing density

We are looking for a scalar index K, which would take a value that depends only on the process of building the packing. By analogy with some viscosity models (Mooney, 1951), we assume that this index is of the following form:

$$K = \sum_{i=1}^{n} K_i \tag{1.34}$$

with $K_i = H\left(\dfrac{\Phi_i}{\Phi_i^*}\right)$

where Φ_i is the actual solid volume of class i, while Φ_i^* is the maximum volume that particles i may occupy, given the presence of the other particles. In other words, the n classes of grain with partial volumes equal to Φ_0, Φ_1, ..., Φ_{i-1}, Φ_i^*, Φ_{i+1}, ..., Φ_n would form a virtual packing.

Function H can be calculated by considering only the self-consistency of the model. Let us deal with a binary mix, the two classes of which are identical (that is, $d_1 = d_2$; $\beta_1 = \beta_2 = \beta$). The only impenetrability constraint is

$$\Phi_1 + \Phi_2 \leqslant \beta \tag{1.35}$$

For calculating the compaction index of the mix, we can write

$$K = H\left(\frac{\Phi_1}{\beta - \Phi_2}\right) + H\left(\frac{\Phi_2}{\beta - \Phi_1}\right) = H\left(\frac{\Phi_1 + \Phi_2}{\beta}\right) \tag{1.36}$$

which corresponds to the following functional equation:

$$H\left(\frac{x}{1-y}\right) + H\left(\frac{y}{1-x}\right) = H(x+y) \tag{1.37}$$

with $x = \dfrac{\Phi_1}{\beta}$; $y = \dfrac{\Phi_2}{\beta}$

Let us show that the only functions for which equation (1.37) is verified are of the form

$$H(u) = k\,\frac{u}{1-u} \tag{1.38}$$

Define the function $k(u)$ such that

$$H(u) = k(u)\left(\frac{u}{1-u}\right) \tag{1.39}$$

Replacing H in the functional equation (1.37), it becomes

$$(x+y)\cdot k(x+y) = x\cdot k\left(\frac{x}{1-y}\right) + y\cdot k\left(\frac{y}{1-x}\right) \tag{1.40}$$

If $x = y = u/2$ then

$$k(u) = k\left(\frac{u/2}{1-u/2}\right) = k\left[\frac{u/4}{1-\left(\frac{1}{2}+\frac{1}{4}\right)u}\right]$$

$$= \ldots k\left[\frac{u/2^n}{1-\left(1-\frac{1}{2^n}\right)u}\right] = \ldots k(0) \tag{1.41}$$

Therefore $k(u)$ is a constant for $u \in [0, 1[$, which is the domain of variation of Φ_i/Φ_i^*. For the sake of simplicity, we take

$$k = 1 \tag{1.42}$$

Thus the compaction index becomes

$$K = \sum_{i=1}^{n}\left(\frac{\Phi_i/\Phi_i^*}{1-\Phi_i/\Phi_i^*}\right) \tag{1.43}$$

Φ_i^* is equal to Φ_i when class i is dominant, so that we can use the same approach as in equation (1.31), replacing Φ_i^* by the expression

(generalized to n classes, i is dominant)

$$\Phi_i^* = \beta_i \left[1 - \sum_{j=1}^{i-1} \left(1 - b_{ij} \left[1 - \frac{1}{\beta_j} \right] \right) \Phi_j - \sum_{j=i+1}^{n} \frac{a_{ij}}{\beta_j} \Phi_j \right] \tag{1.44}$$

which gives for K:

$$K = \sum_{i=1}^{n} \frac{y_i/\beta_i}{\dfrac{1}{\Phi} - \left\{ \sum_{j=1}^{i-1} \left[1 - b_{ij} \left(1 - \dfrac{1}{\beta_j} \right) \right] y_j - \sum_{j=i+1}^{n} \dfrac{a_{ij}}{\beta_j} y_j + \dfrac{y_i}{\beta_i} \right\}} \tag{1.45}$$

However, replacing 1 by $\sum_{i=1}^{n} y_i$ in equation (1.33), we recognize that

$$\sum_{j=1}^{i-1} \left[1 - b_{ij} \left(1 - \frac{1}{\beta_j} \right) \right] y_j - \sum_{j=i+1}^{n} \frac{a_{ij}}{\beta_j} y_j + \frac{y_i}{\beta_i} = \frac{1}{\gamma_i} \tag{1.46}$$

so we reach the final expression for the compaction index

$$K = \sum_{i=1}^{n} K_i = \sum_{i=1}^{n} \frac{y_i/\beta_i}{1/\Phi - 1/\gamma_i} \tag{1.47}$$

As K is a characteristic of the packing process, the packing density is then the value of Φ defined implicitly by equation (1.47). Actually, K is a strictly increasing function of Φ, as the sum of such functions, so that there is a unique value of Φ satisfying this equation for any positive K value (Fig. 1.11).

The y_i are the control parameters of the experiment, β_i are characteristics of the grain classes, the γ_i are given by equation (1.33), and the value of K depends on the process of making the mixture. For a

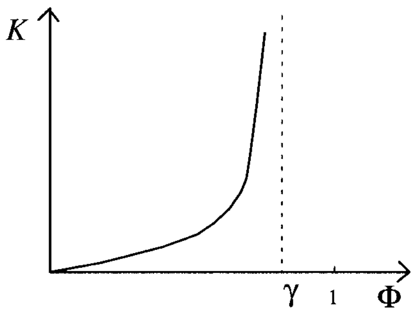


Figure 1.11 Variation of K vs. Φ.

monodisperse packing, it follows that

$$K = \frac{1}{\beta/\Phi - 1} \tag{1.48}$$

1.2.2 Calibration of the model with binary data

Tests related in this section have been carried out at the Laboratoire Régional des Ponts et Chaussées de Blois (de Larrard *et al.*, 1994b). The aim was first to prepare a series of binary mixes, for calibration of the interaction coefficients. The individual fractions were selected in order to be as monodisperse as possible.

Materials

Two families of aggregate were selected and sieved:

- rounded aggregate from the Loire (Decize quarry), with nearly spherical shapes;
- crushed angular aggregate from the Pont de Colonne quarry at Arnay le Duc.

These materials were expected to cover the range of civil engineering materials, from smooth quasi-spherical grains (such as those of fly ash) to angular, flat and elongated ones (such as certain crushed aggregate).

For each family, five monosize classes were prepared, limited by two adjacent sieves in the Renard series. This French standard series has diameters in geometrical progression, with a ratio of $\sqrt[10]{10}$ ($\approx$1.26). The mean sizes of the classes were chosen for obtaining size ratios of 1/2, 1/4, 1/8 and 1/16. Extreme diameters were limited on the high side, for container dimension purposes, and on the low side, to avoid materials that were too humidity sensitive. For polydisperse mixtures (see section 1.2.3) a sixth class was added, ranging between 0.08 and 0.5 mm, for increasing both the grading span and the maximum packing values obtained. Photographs of the particles used are given in the original publication (de Larrard *et al.*, 1994b), and their characteristics appear in Tables 1.1 and 1.2.

Packing process used

After weighing the granular classes, selected to obtain a 7 kg sample, the mixes were homogenized. When the size ratio did not exceed 4, the grains were poured in a Deval machine (following French standard P 18 577), equipped with a modified cylindrical container. This container was then rotated around an oblique axis (with regard to the symmetry

Table 1.1 Characteristics of rounded aggregates used in packing experiments. As the same grains were used for a series of experiments, a certain attrition took place, increasing the β_i values. Corrected packing density values have been obtained by linear regressions in the binary mixes.

Names	d_{min} *(mm)*	d_{max} *(mm)*	*Packing density*	*Corrected packing density*
R < 05	0.08	0.5	0.593	–
R05	0.5	0.63	0.592	0.594
R1	1	1.25	0.609	0.613
R2	2	2.5	0.616	0.620
R4	4	5	0.6195	0.629
R8	8	10	0.628	0.632

Table 1.2 Characteristics of crushed aggregate used in the packing experiments.

Names	d_{min} *(mm)*	d_{max} *(mm)*	*Packing density*	*Corrected packing density*
C < 05	0.08	0.5	0.630	–
C05	0.5	0.63	0.516	0.523
C1	1	1.25	0.507	0.528
C2	2	2.5	0.529	0.525
C4	4	5	0.537	0.557
C8	8	10	0.572	0.585

axis), for 2 min or 66 revolutions. The cylinder has a diameter of 160 mm and a height of 320 mm, and supported another 160 × 160 mm cylinder, used for pouring the uncompacted mixture. After removal, the container served for measuring the packing density.

For the other binary mixes that were prone to segregation, a manual homogenization was carried out: aggregates were poured after mixing in horizontal layers, and then removed vertically with a shovel and cast by successive layers in the cylindrical mould. Both techniques of preparation were used for polydisperse mixtures.

Then the cylinder containing aggregates was closed with a 20 kg steel piston, applying a mean compression of 10 kPa on the top of the sample. The whole set was put on a vibrating table and submitted to the following vibration sequence: 2 min at a 0.4 mm amplitude, 40 s at 0.2 mm and 1 min at 0.08 mm. The height of the sample was recorded continuously by an ultrasound telemeter having a precision of 0.001 mm. The vibration process was fixed for having a comparable response with all granular classes, while keeping the total process within a reasonable time. Hence no definite stabilization of the height appears in such a process, which confirms that actual packing density is not a material property, but rather depends on the mixture *and* the process. The packing density of the mixture was calculated by dividing the mass of the sample by the mean density of the aggregate and by the total volume of the specimen. Each experimental value obtained for the model was the mean of two successive measurements carried out on the same aggregate sample. Between the two measurements, the cylinder was emptied and reconstructed using the entire process.

Packing density of monosize classes

Actual packing densities are given in Tables 1.1 and 1.2. For the two families, they increase with the diameter of the particles (Fig. 1.12). This could be due to differences in the shape of the grains, but is mostly due to the fact that in spite of our efforts, vibration is more efficient in compacting coarse grains than fine ones, because fewer contacts are present per unit volume of the mix. At equal size, rounded aggregates are more compact than crushed ones. Incidentally, no general law seem to govern the relationship between size and packing density of aggregates.

Calibration of the model

The interaction coefficients, a_{ij} and b_{ij}, were expected to depend mainly on the ratio between the particle diameters d_i and d_j. However, to assess the soundness of this hypothesis, an attempt has been made to duplicate, when possible, the binary series of a given size ratio, for both coarse grain and fine grain. For the smallest ratio (1/16), only a single series could be produced. For each class combination, the variation of packing density was expected to be steeper on the dominant coarse grain side than on the dominant fine grain side (Fig. 1.8). This is why the fine grain proportion was incremented by 5% steps between 0 and 30%, and by 10% steps afterwards. Obtained packing density values are summarized in Tables 1.3 and 1.4, and Figs 1.17 and 1.18. Here we note that, while the size ratio

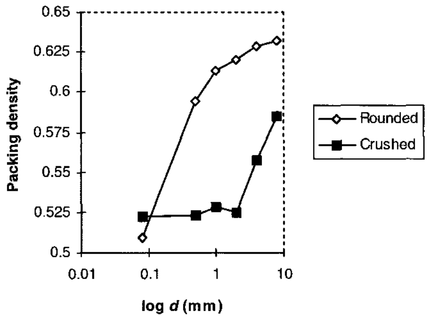


Figure 1.12 Experimental packing density of monodisperse classes vs. size of particles.

Table 1.3 Packing densities of binary mixtures: rounded grains.

	R8R05		*R8R1*		*R4R05*		*R8R2*		*R2R05*		*R8R4*		*R1R05*	
% fine	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*
0	0.628	0.632	0.628	0.632	0.6195	0.629	0.628	0.632	0.616	0.62	0.628	0.632	0.609	0.613
5	0.657	0.6557	0.6545	0.6526	0.645	0.6492	0.653	0.6481	0.635	0.6354	0.6375	0.6416	0.624	0.6217
10	0.6865	0.6808	0.6795	0.6743	0.6715	0.6703	0.682	0.6646	0.663	0.6512	0.643	0.6511	0.633	0.6304
15	0.71	0.7069	0.707	0.6966	0.689	0.692	0.697	0.6813	0.678	0.6671	0.654	0.6603	0.64	0.6386
20	0.729	0.733	0.724	0.7189	0.706	0.7136	0.714	0.6976	0.692	0.6826	0.66	0.6689	0.656	0.6462
25	0.754	0.7558	0.742	0.7391	0.7265	0.7326	0.7235	0.7122	0.708	0.6964	0.663	0.6762	0.666	0.6526
30	0.758	0.7677	0.748	0.7528	0.7485	0.7446	0.728	0.7229	0.718	0.7063	0.6595	0.6817	0.6705	0.6572
40	0.753	0.7544	0.7285	0.7496	0.736	0.7379	0.723	0.7251	0.708	0.7067	0.6565	0.6853	0.6635	0.659
50	0.7385	0.7256	0.7095	0.7277	0.725	0.7138	0.705	0.7111	0.693	0.6909	0.6535	0.6806	0.6545	0.6525
60	0.7165	0.6959	0.6965	0.7029	0.7	0.6875	0.689	0.6927	0.67	0.6709	0.649	0.6719	0.644	0.6421
70	0.68	0.6677	0.677	0.6786	0.6745	0.662	0.671	0.6737	0.656	0.6506	0.6445	0.6616	0.636	0.6303
80	0.652	0.6414	0.6585	0.6554	0.648	0.6379	0.646	0.6551	0.633	0.6308	0.638	0.6508	0.6215	0.6181
90	0.6195	0.6168	0.635	0.6336	0.614	0.6152	0.632	0.6371	0.613	0.6119	0.629	0.6398	0.61	0.6059
100	0.592	0.594	0.609	0.613	0.592	0.594	0.616	0.62	0.592	0.594	0.6195	0.629	0.592	0.594

Table 1.4 Packing densities of binary mixtures: crushed grains.

	C8C05		*C8C1*		*C4C05*		*C8C2*		*C2C05*		*C8C4*		*C1C05*	
% fine	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*	*exp*	*theo*
0	0.572	0.585	0.572	0.585	0.537	0.557	0.572	0.585	0.529	0.525	0.572	0.585	0.507	0.528
5	0.62	0.6066	0.613	0.6034	0.591	0.575	0.597	0.5986	0.54	0.5388	0.5825	0.5931	0.527	0.5362
10	0.642	0.6295	0.646	0.6226	0.6185	0.594	0.611	0.6125	0.552	0.5531	0.5875	0.6011	0.532	0.5444
15	0.676	0.6535	0.6755	0.6425	0.638	0.6137	0.625	0.6264	0.5515	0.5679	0.588	0.6087	0.545	0.5524
20	0.705	0.6779	0.699	0.6624	0.669	0.634	0.634	0.6398	0.566	0.583	0.592	0.6158	0.552	0.5602
25	0.731	0.7001	0.7215	0.6806	0.693	0.6536	0.643	0.6516	0.573	0.5979	0.5955	0.6217	0.5485	0.5673
30	0.7365	0.7137	0.7245	0.693	0.711	0.67	0.651	0.6594	0.594	0.6115	0.594	0.6259	0.555	0.5733
40	0.723	0.6998	0.7025	0.6861	0.691	0.6741	0.643	0.6554	0.588	0.626	0.5875	0.6271	0.556	0.5792
50	0.6941	0.6666	0.6705	0.6586	0.667	0.6502	0.6335	0.6349	0.582	0.617	0.587	0.6198	0.549	0.576
60	0.6585	0.6331	0.638	0.629	0.64	0.6219	0.6245	0.6111	0.579	0.5987	0.587	0.6084	0.546	0.5674
70	0.616	0.6019	0.611	0.6008	0.603	0.5945	0.5975	0.5877	0.568	0.5788	0.572	0.5956	0.5425	0.5567
80	0.583	0.5732	0.5965	0.5746	0.571	0.5688	0.5695	0.5654	0.5555	0.5593	0.564	0.5825	0.537	0.5455
90	0.5655	0.547	0.5435	0.5504	0.545	0.545	0.5435	0.5445	0.534	0.5406	0.553	0.5696	0.53	0.5341
100	0.516	0.523	0.507	0.528	0.516	0.523	0.529	0.525	0.516	0.523	0.537	0.557	0.516	0.523

appears to be the main parameter controlling the behaviour of the binary mix, significant differences appear for pairs of equal size ratio.

Each binary mixture series displays one experimental point for the two coefficients a and b. The relationship between the voids index and the fine grain proportions appears to be two straight lines linked by a curved part. The slope of these lines directly expresses the granular interaction between classes (Powers, 1968) (Fig. 1.13).

In the case of an infinite value for the compaction index (that is, when dealing with virtual packing density), one can easily show using equation (1.33) that

$$a_{12} = \beta_2 \left(\left| \frac{\partial e}{\partial y_2} \right|_{y_2 = 0} + \frac{1}{\beta_1} \right)$$
$$b_{21} = \frac{1/\beta_2 - 1 - \left| \dfrac{\partial e}{\partial y_2} \right|_{y_2 = 1}}{1/\beta_1 - 1} \tag{1.49}$$

Actually, for sufficiently high values of K, the real packing density curves have approximately the same tangent as the virtual one (Fig. 1.14). One may therefore apply the previous equation to the present data, replacing the β_i by the α_i (actual residual packing densities of monodisperse classes). Obtained values for a and b are given in Table 1.5.

In Fig. 1.16, the experimental points for a and b have been plotted against the size ratios. These coefficients increase with the ratio d_2/d_1, which matches the theory. At first sight, other parameters should also play a role. However, no systematic trend appears between rounded and crushed aggregate, or between coarse and fine pairs (as shown by the

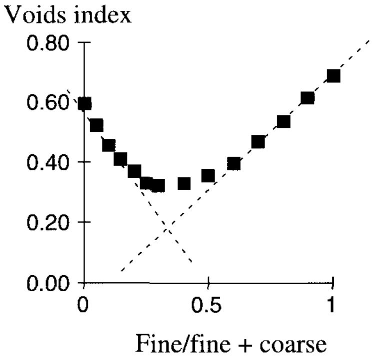


Figure 1.13 Voids index vs. fine grain proportion, for the R8/R01 mixture.

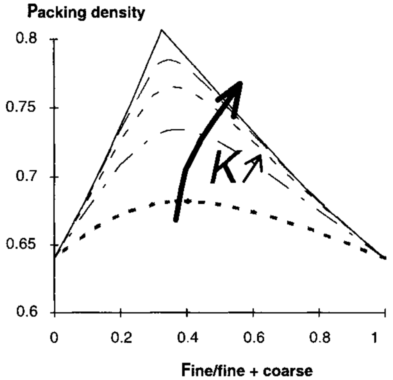


Figure 1.14 Packing density of binary mix of grains with a size ratio of 1/8 after the compressible packing model. Actual residual packing densities of the two classes are assumed to be equal to 0.64, and the different curves stand for low to high **K** values.

Table 1.5 Experimental values for the interaction functions deducted from binary mix experiments.

	d_1/d_2						
	16	*8*	*8*	*4*	*4*	*2*	*2*
Rounded	R8R05	R8R1	R4R05	R8R2	R2R05	R8R4	R1R05
a	0.26	0.31	0.50	0.63	0.46	0.71	0.66
b	−0.05	0.30	0.08	0.33	0.33	0.65	0.56
Crushed	C8C05	C8C1	C4C05	C8C2	C2C05	C8C4	C1C05
a	0.21	0.14	0.31	0.50	0.67	0.77	0.72
b	−0.03	0.28	0.05	0.11	0.47	0.68	0.70

comparison of the R8R4 and R1R05, or C8C4 and C1C05 mix series. However, the sieves used in civil engineering often suffer some flaws (either initial ones, or those that result from the wear exerted by aggregates). Thus the actual ratio of a fraction couple with a theoretical ratio of 1/2 may probably range between 0.4 and 0.6. This would correspond to a horizontal shifting of the corresponding points in the figures. One may conclude that the scattering of the points is not necessarily the sign of another influence, related to the grains' shape, for example.

This hypothesis of inaccurate sieve size does not explain the two *negative* values of the *b* coefficient, obtained for the R8R05 and C8C05 mix series. For those couples, on the fine dominant grain side, the packing density evolves faster than in the case without interaction. In other terms, the fine grains suffered an *anti-wall effect*. This observation is related to the effect of vibration on compaction. Coarse grains probably act as internal vibrators in the mixture of fine grains (Aïtcin and Albinger, 1989). In smoothing the results, this phenomenon has been neglected, as the model is expected to apply in general, including non-vibrated mixtures (see section 1.2.3).

The regression function used to fit the experimental values has to satisfy the following conditions:

- Continuity with the case of binary mixture without interaction ($d_2/d_1 = 0$): $a = b = 0$.
- Continuity with the case of binary mixture with total interaction ($d_2/d_1 = 1$): $a = b = 1$.
- Moreover, if one considers the case of a binary mixture in which y_1 is small, d_2 is fixed, and d_1 varies around d_2 (Fig. 1.15), one should have

  when $d_1 \geqslant d_2$:

$$\gamma = \frac{\beta_2}{1 - (1 - \beta_2 + b_{21}\beta_2(1 - 1/\beta_1)y_1)} \tag{1.50}$$

  when $d_1 \leqslant d_2$:

$$\gamma = \frac{\beta_2}{1 - (1 - a_{21}\beta_2/\beta_1)y_1} \tag{1.51}$$

  and the derived function should be also continuous in $d_2 = d_1 = d$,

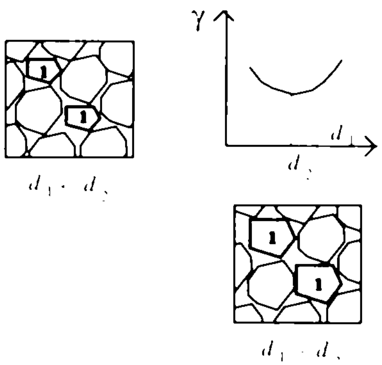


Figure 1.15 Continuity between dominant coarse grain and dominant fine grain cases.

which gives

$$-\left(1-\frac{1}{\beta_1}\right)\left[\frac{\partial b}{\partial x}\right]_{x=1}+\frac{1}{\beta_1}\left[\frac{\partial a}{\partial x}\right]_{x=1}=0 \tag{1.52}$$

where x is the ratio of fine to coarse grain diameters.

As the two factors in this equation are positive, this implies for the derived values of a and b:

$$\left[\frac{\partial a}{\partial x}\right]_{x=1}=\left[\frac{\partial b}{\partial x}\right]_{x=1}=0 \tag{1.53}$$

The following functions verify the three listed conditions, while giving a reasonable approximation of the experimental points:

$$\begin{aligned} a_{ij} &= \sqrt{1-(1-d_j/d_i)^{1.02}} \\ \mathrm{b}_{ij} &= 1-(1-d_i/d_j)^{1.50} \end{aligned} \tag{1.54}$$

For the calibration of the model, the compaction index remains to be fixed. As shown in Fig. 1.14, the higher the value of K, the sharper the binary curves. With the above interaction functions and a K value of 9, the experimental values of packing density obtained are best smoothed with a mean error in absolute value equal to 0.77% for the rounded grains and 1.71% for the crushed grains (Figs 1.17, 1.18). By comparison, the means of within-test standard deviations, describing the repeatability of the measurements, are 0.0026 and 0.0078 for rounded and crushed aggregates respectively. Therefore one may estimate that the better predictions

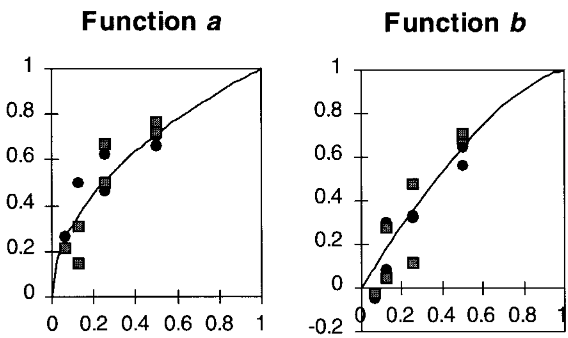


Figure 1.16 Fitting of interaction functions a or b vs. size ratio. Squares and circles stand for crushed and rounded aggregates respectively.

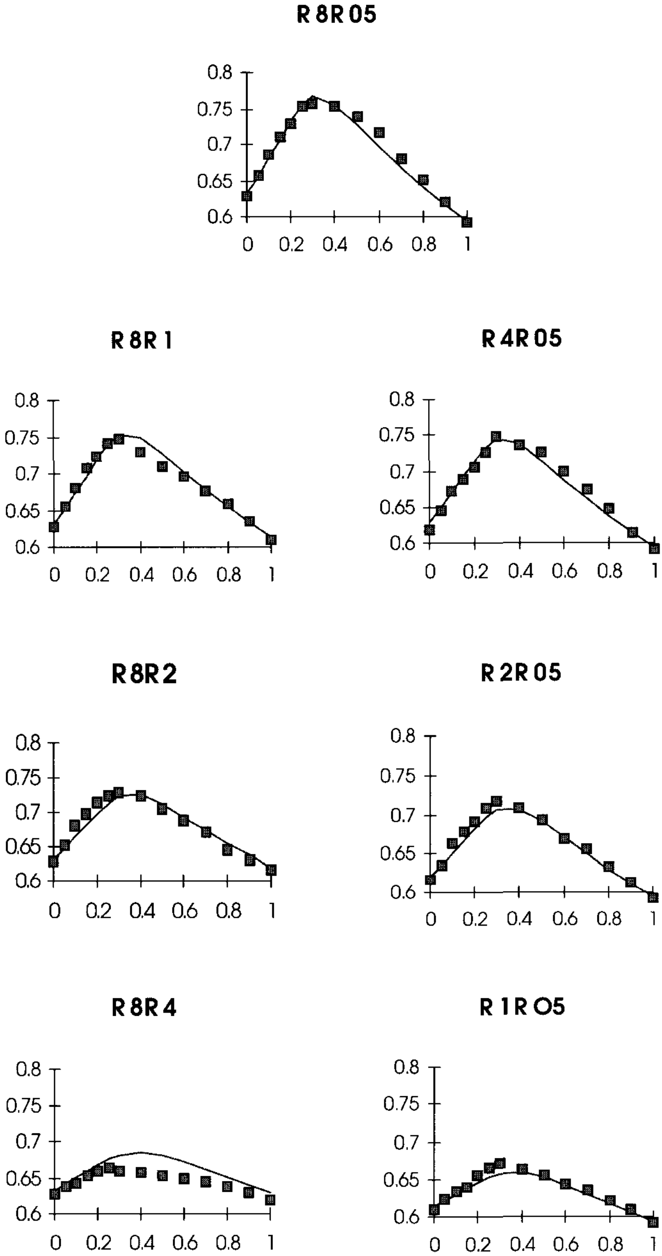


Figure 1.17 Binary mixes of rounded grains. Packing density vs. fine grain proportion. The dots stand for experimental points, while the curves are the model smoothing.