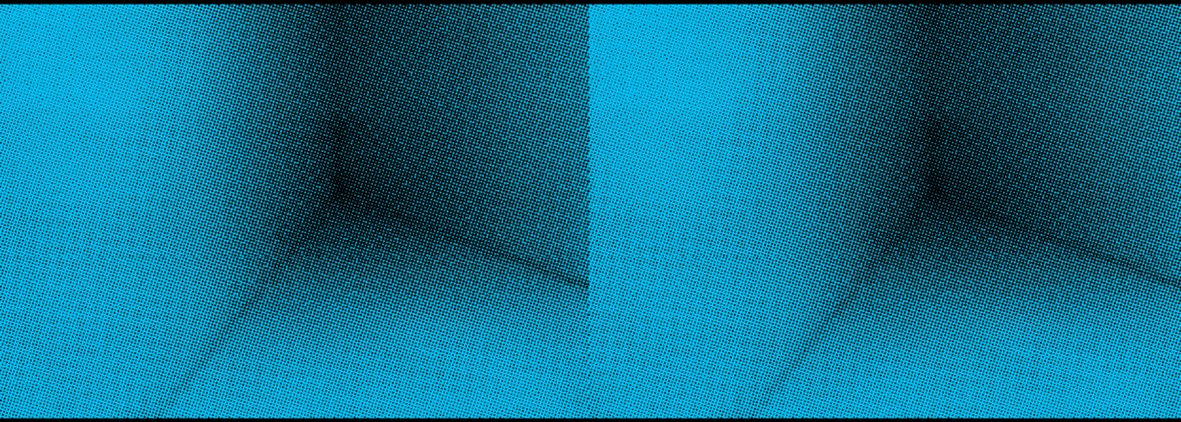
HEINRICH KUTTRUFF



room acoustics

FIFTH EDITION

Spon Press

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Preface to the fifth edition

I am delighted to present a new edition of my book *Room Acoustics*. Preparing a new edition enabled me to take into consideration some new publications in the field of room acoustics. Furthermore, several more conventional subjects which were not incorporated into the earlier editions, although being of great interest with regard to room acoustics, have been included, along with some new figures. The chapter on measuring techniques has been restructured because of the rapid progress in signal processing. Finally, the new edition has provided me with the opportunity to eliminate a number of incomplete, inaccurate or even misleading formulations or expressions and to replace them with more adequate ones.

Despite all these modifications and additions, the original translation by Professor Peter Lord, to whom I owe my sincere gratitude, is still the basis of this text. Once more, I would like to thank the publisher for giving me the opportunity to present a revised version of my work, as well as for their excellent cooperation.

Heinrich Kuttruff
Aachen

Preface to the fourth edition

Almost a decade has elapsed since the third edition and during this period many new ideas and methods have been introduced into room acoustics. I therefore welcome the opportunity to prepare a new edition of this book and to include the more important of those developments, while also introducing new topics which were not dealt with in earlier editions.

In room acoustics, as in many other technical fields, the digital computer has continued its triumphant progress; nowadays hardly any acoustical measurements are carried out without using a computer, allowing previously inconceivable improvements in accuracy and rapidity. Therefore, an update of the chapter on measuring techniques (Chapter 8) was essential. Furthermore, the increased availability of computers has opened new ways for the computation and simulation of sound fields in enclosures. These have led to better and more reliable methods in the practical design of halls; indeed, due to its flexibility and low cost, sound field simulation will probably replace the conventional scale model in the near future. Moreover, by simulation it can be demonstrated what a new theatre or concert hall which is still on the drawing board will sound like when completed ('auralisation'). These developments are described in Chapter 9, which contains a separate section on auralisation.

Also included in the new edition are sections on sound scattering and diffuse reflection, on sound reflection from curved walls, on sound absorption by several special arrangements (freely hanging porous material, Schroeder diffusers) and on the measurement of diffuse reflections from walls.

The preparation of a new edition offered the chance to present some subjects in a more comprehensive and logical way, to improve numerous text passages and formulae and to correct errors and mistakes that inevitably crept into the previous editions. I appreciate the suggestions of many critical readers, who drew my attention to weak or misleading material in the book. Most text passages, however, have been adopted from the previous editions without any changes. Therefore I want to express again my most sincere thanks to Professor Peter Lord of the University of Salford for his competent and sensitive translation. Finally, I want to thank the publishers for their cooperation in preparing this new edition.

Heinrich Kuttruff
Aachen

Preface to the first edition

This book is intended to present the fundamentals of room acoustics in a systematic and comprehensive way so that the information thus provided may be used for the acoustical design of rooms and as a guide to the techniques of associated measurement.

These fundamentals are twofold in nature: the generation and propagation of sound in an enclosure, which are physical processes which can be described without ambiguity in the language of the physicist and engineer; and the physiological and psychological factors, of prime importance but not capable of exact description even within our present state of knowledge. It is the interdependence and the equality of importance of both these aspects of acoustics which are characteristic of room acoustics, whether we are discussing questions of measuring techniques, acoustical design, or the installation of a public address system.

In the earlier part of the book ample space is devoted to the objective description of sound fields in enclosures, but, even at this stage, taking into account, as far as possible, the limitations imposed by the properties of our hearing. Equal weight is given to both the wave and geometrical description of sound fields, the former serving to provide a more basic understanding, the latter lending itself to practical application. In both instances, full use is made of statistical methods; therefore, a separate treatment of what is generally known as 'statistical room acoustics' has been dispensed with.

The treatment of absorption mechanisms is based upon the concept that a thorough understanding of the various absorbers is indispensable for the acoustician. However, in designing a room he will not, in all probability, attempt to calculate the absorptivity of a particular arrangement but instead will rely on collected measurements and data based on experience. It is for this reason that in the chapter on measuring techniques the methods of determining absorption are discussed in some detail.

Some difficulties were encountered in attempting to describe the factors which are important in the perception of sound in rooms, primarily because of the fragmentary nature of the present state of knowledge, which seems to consist of results of isolated experiments which are strongly influenced by the conditions under which they were performed.

We have refrained from giving examples of completed rooms to illustrate how the techniques of room acoustics can be applied. Instead, we have chosen to show how one can progress in designing a room and which parameters need to be considered. Furthermore, because model investigations have proved helpful these are described in detail.

Finally, there is a whole chapter devoted to the design of loudspeaker installations in rooms. This is to take account of the fact that nowadays electroacoustic installations are more than a mere crutch in that they frequently present, even in the most acoustically faultless room, the only means of transmitting the spoken word in an intelligible way. Actually, the installations and their performance play a more important role in determining the acoustical quality of what is heard than certain design details of the room itself.

The book should be understood in its entirety by readers with a reasonable mathematical background and some elementary knowledge of wave propagation. Certain hypotheses may be omitted without detriment by readers with more limited mathematical training.

The literature on room acoustics is so extensive that the author has made no attempt to provide an exhaustive list of references. References have only been given in those cases where the work has been directly mentioned in the text or in order to satisfy possible demand for more detailed information.

The author is greatly indebted to Professor Peter Lord of the University of Salford and Mrs Evelyn Robinson of Prestbury, Cheshire, for their painstaking translation of the German manuscript, and for their efforts to present some ideas expressed in my native language into colloquial English. Furthermore, the author wishes to express his appreciation to the publishers for this carefully prepared edition. Last, but not least, he wishes to thank his wife most sincerely for her patience in the face of numerous evenings and weekends which he has devoted to his manuscript.

Heinrich Kuttruff
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Introduction

We all know that a concert hall, theatre, lecture room or a church may have good or poor 'acoustics'. As far as speech in these rooms is concerned, it is relatively simple to make some sort of judgement on their quality by rating the ease with which the spoken word is understood. However, judging the acoustics of a concert hall or an opera house is generally more difficult, since it requires considerable experience, the opportunity for comparisons and a critical ear. Even so, the inexperienced cannot fail to learn about the acoustical reputation of a certain concert hall should they so desire, for instance by listening to the comments of others, or by reading the critical reviews of concerts in the press.

An everyday experience (although most people are not consciously aware of it) is that living rooms, offices, restaurants and all kinds of rooms for work can be acoustically satisfactory or unsatisfactory. Even rooms which are generally considered insignificant or spaces such as staircases, factories, passenger concourses in railway stations and airports may exhibit different acoustical properties; they may be especially noisy or exceptionally quiet, or they may differ in the ease with which announcements over the public address system can be understood. That is to say, even these spaces have 'acoustics' which may be satisfactory or less than satisfactory.

Despite the fact that people are subconsciously aware of the acoustics to which they are daily subjected, there are only a few who can explain what they really mean by 'good or poor acoustics' and who understand factors which influence or give rise to certain acoustical properties. Even fewer people know that the acoustics of a room is governed by principles which are amenable to scientific treatment. It is frequently thought that the acoustical design of a room is a matter of chance, and that good acoustics cannot be designed into a room with the same precision as a nuclear reactor or space vehicle is designed. This idea is supported by the fact that opinions on the acoustics of a certain room or hall frequently differ as widely as the opinions on the literary qualities of a new book or on the architectural design of a new building. Furthermore, it is well known that sensational failures in this field do occur from time to time. These and similar anomalies add even more weight to the general belief that the acoustics of a room is beyond the scope of calculation or prediction, at least with any reliability, and hence the study of room acoustics is an art rather than an exact science.

In order to shed more light on the nature of room acoustics, let us first compare it to a related field: the design and construction of musical instruments. This comparison is not as senseless as it may appear at first sight, since a concert hall too may be regarded as a large musical instrument, the shape and material of which determine to a considerable extent what the listener will hear. Musical instruments—string instruments for instance—are, as is well known, not designed or built by scientifically trained acousticians but, fortunately, by people who have acquired the necessary experience through long and systematic practical training. Designing or building musical instruments is therefore not a technical or scientific discipline but a sort of craft, or an 'art' in the classical meaning of this word.

Nevertheless, there is no doubt that the way in which a musical instrument functions, i.e. the mechanism of sound generation, the determining of the pitch of the tones generated and their timbre through certain resonances, as well as their radiation into the surrounding air, are all purely physical processes and can therefore be understood rationally, at least in principle. Similarly, there is no mystery in the choice of materials; their mechanical and acoustical properties can be defined by measurements to any required degree of accuracy. (How well these properties can be reproduced is another problem.) Thus, there is nothing intangible nor is there any magic in the construction of a musical instrument: many particular problems which are still unsolved will be understood in the not too distant future. Then one will doubtless be in a position to design a musical instrument according to scientific methods, i.e. not only to predict its timbre but also to give, with scientific accuracy, details for its construction, all of which are necessary to obtain prescribed or desired acoustical qualities.

Room acoustics is in a different position from musical instrument acoustics in that the end product is usually more costly by orders of magnitude. Furthermore, rooms are produced in much smaller numbers and have by no means geometrical shapes which remain unmodified through the centuries. On the contrary, every architect, by the very nature of his profession, strives to create something which is entirely new and original. The materials used are also subject to the rapid development of building technology. Therefore, it is impossible to collect in a purely empirical manner sufficient know-how from which reliable rules for the acoustical design of rooms or halls can be distilled. An acoustical consultant is confronted with quite a new situation with each task, each theatre, concert hall or lecture room to be designed, and it is of little value simply to transfer the experience of former cases to the new project if nothing is known about the conditions under which the transfer may be safely made.

This is in contrast to the making of a musical instrument where the use of unconventional materials as well as the application of new shapes is either firmly rejected as an offence against sacred traditions or dismissed as a whim. As a consequence, time has been sufficient to develop well-established empirical rules. And if their application happens to fail in one case or another, the faulty product is abandoned or withdrawn from service—which is not true for large rooms in an analogous situation.

For the above reasons, the acoustician has been compelled to study sound propagation in closed spaces with increasing thoroughness and to develop the knowledge in this field much further than is the case with musical instruments, even though the acoustical behaviour of a large hall is considerably more complex and involved. Thus, room acoustics has become a science during the past century and those who practise it on a purely empirical basis will fail sooner or later, like a bridge builder who waives calculations and relies on experience or empiricism.

On the other hand, the present level of reliable knowledge in room acoustics is not particularly advanced. Many important factors influencing the acoustical qualities of large rooms are understood only incompletely or even not at all. As will be explained below in more detail, this is due to the complexity of sound fields in closed spaces—or, as may be said equally well—to the large number of 'degrees of freedom' which we have to deal with. Another difficulty is that the acoustical quality of a room ultimately has to be proved by subjective judgements.

In order to gain more understanding about the sort of questions which can be answered eventually by scientific room acoustics, let us look over the procedures for designing the acoustics of a large room. If this room is to be newly built, some ideas will exist as to its intended use. It will have been established, for example, whether it is to be used for the showing of ciné films, for sports events, for concerts or as an open-plan office. One of the first tasks of the consultant is to translate these ideas concerning the practical use into the language of objective sound field parameters and to fix values for them which he thinks will best meet the requirements. During this step he has to keep in mind the limitations and peculiarities of our subjective listening abilities. (It does not make sense, for instance, to fix the duration of sound decay with an accuracy of 1% if no one can subjectively distinguish such small differences.) Ideally, the next step would be to determine the shape of the hall, to choose the materials to be used, to plan the arrangement of the audience, of the orchestra and of other sound sources, and to do all this in such a way that the sound field configuration will develop which has previously been found to be the optimum for the intended purpose. In practice, however, the architect will have worked out already a preliminary design, certain features of which he considers imperative. In this case the acoustical consultant has to examine the objective acoustical properties of the design by calculation, by geometric ray considerations, by model investigations or by computer simulation, and he will eventually have to submit proposals for suitable adjustments. As a general rule there will have to be some compromise in order to obtain a reasonable result.

Frequently the problem is refurbishment of an existing hall, either to remove architectural, acoustical or other technical defects or to adapt it to a new purpose which was not intended when the hall was originally planned. In this case an acoustical diagnosis has to be made first on the basis of appropriate measurement. A reliable measuring technique which yields objective quantities, which are subjectively meaningful at the same time, is an indispensable tool of the acoustician. The subsequent therapeutic step is essentially the same as described above: the acoustical consultant has to propose measures which would result in the intended objective changes in the sound field and consequently in the subjective impressions of the listeners.

In any case, the acoustician is faced with a two-fold problem: on the one hand he has to find and to apply the relations between the structural features of a room—such as shape, materials and so on—with the sound field which will occur in it, and on the other hand he has to take into consideration as far as possible the interrelations between the objective and measurable sound field parameters and the specific subjective hearing impressions effected by them. Whereas the first problem lies completely in the realm of technical reasoning, it is the latter problem which makes room acoustics different from many other technical disciplines in that the success or failure of an acoustical design has finally to be decided by the collective judgement of all ‘consumers’, i.e. by some sort of average, taken over the comments of individuals with widely varying intellectual, educational and aesthetic backgrounds. The measurement of sound field parameters can replace to a certain extent systematic or sporadic questioning of listeners. But, in the final analysis, it is the average opinion of listeners which decides whether the acoustics of a room is favourable or poor. If the majority of the audience (or that part which is vocal) cannot understand what a speaker is saying, or thinks that the sound of an orchestra in a certain hall is too dry, too weak or indistinct, then even though the measured reverberation time is appropriate, or the local or directional distribution of sound is uniform, the listener is always right; the hall does have acoustical deficiencies.

Therefore, acoustical measuring techniques can only be a substitute for the investigation of public opinion on the acoustical qualities of a room and it will serve its purpose better the closer the measured sound field parameters are related to subjective listening categories. Not only must the measuring techniques take into account the hearing response of the listeners but the acoustical theory too will only provide meaningful information if it takes regard of the consumer’s particular listening abilities. It should be mentioned at this point that the sound field in a real room is so complicated that it is not open to exact mathematical treatment. The reason for this is the large number of components which make up the sound field in a closed space regardless of whether we describe it in terms of vibrational modes or, if we prefer, in terms of sound rays which have undergone one or more reflections from boundaries. Each of these components depends on the sound source, the shape of the room and on the materials from which it is made; accordingly, the exact computation of the sound field is usually quite involved. Supposing this procedure were possible with reasonable expenditure, the results would be so confusing that such a treatment would not provide a comprehensive survey and hence would not be of any practical use. For this reason, approximations and simplifications are inevitable; the totality of possible sound field data has to be reduced to averages or average functions which are more tractable and condensed to provide a clearer picture. This is why we have to resort so frequently to statistical methods and models in room acoustics, whichever way we attempt to describe sound fields. The problem is to perform these reductions and simplifications once again in accordance with the properties of human hearing, i.e. in such a way that the remaining average parameters correspond as closely as possible to particular subjective sensations.

From this it follows that essential progress in room acoustics depends to a large extent on the advances in psychological acoustics. As long as the physiological and psychological processes which are involved in hearing are not completely understood, the relevant relations between objective stimuli and subjective sensations must be investigated empirically—and should be taken into account when designing the acoustics of a room.

Many interesting relations of this kind have been detected and successfully investigated during the past few decades. But other questions which are no less important for room acoustics are unanswered so far, and much work remains to be carried out in this field.

It is, of course, the purpose of all efforts in room acoustics to avoid acoustical deficiencies and mistakes. It should be mentioned, on the other hand, that it is neither desirable nor possible to create the 'ideal acoustical environment' for concerts and theatres. It is a fact that the enjoyment when listening to music is a matter not only of the measurable sound waves hitting the ear but also of the listener's personal attitude and his individual taste, and these vary from one person to another. For this reason there will always be varying shades of opinion concerning the acoustics of even the most marvellous concert hall. For the same reason, one can easily imagine a wide variety of concert halls with excellent, but nevertheless different, acoustics. It is this 'lack of uniformity' which is characteristic of the subject of room acoustics, and which is responsible for many of its difficulties, but it also accounts for the continuous power of attraction it exerts on many acousticians.

Chapter 1

Some facts on sound waves, sources and hearing

In principle, any complex sound field can be considered as a superposition of numerous simple sound waves, e.g. plane waves. This is especially true of the very involved sound fields which we have to deal with in room acoustics. So it is useful to describe first the properties of a simple plane or a spherical sound wave or, more basically, the general features of sound propagation. We can, however, restrict our attention to sound propagation in gases, because in room acoustics we are only concerned with air as the medium.

In this chapter we assume the sound propagation to be free of losses and ignore the effect of any obstacles such as walls, i.e. we suppose the medium to be unbounded in all directions. Furthermore, we assume our medium to be homogeneous and at rest. In this case the velocity of sound is constant with reference to space and time. For air, its magnitude is

$$c = (331.4 + 0.6\,\theta) \quad \text{m/s} \tag{1.1}$$

where θ is the temperature in degrees centigrade.

In large halls, variations of temperature and hence of the sound velocity with time and position cannot be entirely avoided. Likewise, because of temperature differences and air conditioning, the air is not completely at rest, and so our assumptions are not fully realised. But the effects which are caused by these inhomogeneities are so small that they can be neglected.

1.1 Basic relations, the wave equation

In any sound wave, the particles of the medium undergo vibrations about their mean positions. Therefore, a wave can be described completely by indicating the instantaneous displacements of these particles. It is more customary, however, to consider the velocity of particle displacement as a basic acoustical quantity rather than the displacement itself.

The vibrations in a sound wave do not take place at all points with the same phase. We can, in fact, find points in a sound field where the particles vibrate in opposite phase. This means that in certain regions the particles are pushed together or compressed and in other regions they are pulled apart or rarefied. Therefore, under the influence of a sound wave, variations of gas density and pressure occur, both of which are functions of time and position. The difference between the instantaneous pressure and the static pressure is called the sound pressure.

The changes of gas pressure caused by a sound wave in general occur so rapidly that heat cannot be exchanged between adjacent volume elements. Consequently, a sound wave causes adiabatic variations of the temperature, and so the temperature too can be considered as a quantity characterising a sound wave.

The various acoustical quantities are connected by some basic laws which enable us to set up a general differential equation governing sound propagation. Firstly, conservation of momentum is expressed by the relation

$$\operatorname{grad} p = -\rho_0 \frac{\partial \mathbf{v}}{\partial t} \tag{1.2}$$

where p denotes the sound pressure, $\mathbf{v}$ the vector particle velocity, t the time and ρ_0 the static value of the gas density. The unit of sound pressure is the pascal: 1 pascal (Pa) $= 1\ \mathrm{N/m^2} = 1\ \mathrm{kg\,m^{-1}\ s^{-2}}$.

Furthermore, conservation of mass leads to

$$\rho_0 \operatorname{div} \mathbf{v} = -\frac{\partial \rho}{\partial t} \tag{1.3}$$

ρ being the total density including its variable part, $\rho = \rho_0 + \delta\rho$. In these equations, it is assumed that the changes of p and ρ are small compared with the static values p_0 and ρ_0 of these quantities; furthermore, the absolute value of the particle velocity $\mathbf{v}$ should be much smaller than the sound velocity c.

Under the further supposition that we are dealing with an ideal gas, the following relations hold between the sound pressure, the density variations and the temperature changes $\delta\theta$:

$$\frac{p}{p_0} = \kappa \frac{\delta\rho}{\rho_0} = \frac{\kappa}{\kappa - 1} \cdot \frac{\delta\theta}{\theta + 273} \tag{1.4}$$

Here κ is the adiabatic or isentropic exponent (for air $\kappa = 1.4$).

The particle velocity $\mathbf{v}$ and the variable part $\delta\rho$ of the density can be eliminated from eqns (1.2) to (1.4). This yields the differential equation

$$c^2 \Delta p = \frac{\partial^2 p}{\partial t^2} \tag{1.5}$$

where

$$c^2 = \kappa \frac{p_0}{\rho_0} \tag{1.6}$$

$\Delta(= \nabla^2 = \text{div grad})$ is the Laplacian operator. This differential equation governs the propagation of sound waves in any lossless fluid and is therefore of central importance for almost all acoustical phenomena. We shall refer to it as the 'wave equation'. It holds not only for sound pressure but also for density and temperature variations.

1.2 Plane waves and spherical waves

Now we assume that the acoustical quantities depend only on the time and on one single direction, which may be chosen as the x-direction of a cartesian coordinate system. Then eqn (1.5) reads

$$c^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{1.7}$$

The general solution of this differential equation is

$$p(x,t) = F(ct - x) + G(ct + x) \tag{1.8a}$$

where F and G are arbitrary functions, the second derivatives of which exist. The first term on the right represents a pressure wave travelling in the positive x-direction with a velocity c, because the value of F remains unaltered if a time increase δt is associated with an increase in the coordinate $\delta x = c\delta t$. For the same reason the second term describes a pressure wave propagated in the negative x-direction. Therefore the constant c is the sound velocity.

Each term of eqn (1.8a) represents a progressive 'plane wave': As shown in Fig. 1.1a, the sound pressure p is constant in any plane perpendicular to the x-axis. These planes of constant sound pressure are called 'wavefronts', and any line perpendicular to them is a 'wave normal'.

According to eqn (1.2), the particle velocity has only one non-vanishing component, which is parallel to the gradient of the sound pressure, i.e. to the x-axis. This means sound waves in fluids are longitudinal waves. The particle velocity may be obtained from applying eqn (1.2) to eqn (1.8a):

$$v(x,t) = \frac{1}{\rho_0 c}\left[F(ct - x) - G(ct + x)\right] \tag{1.8b}$$

As may be seen from eqns (1.8a) and (1.8b) the ratio of sound pressure and particle velocity in a plane wave propagated in the positive direction ($G = 0$)

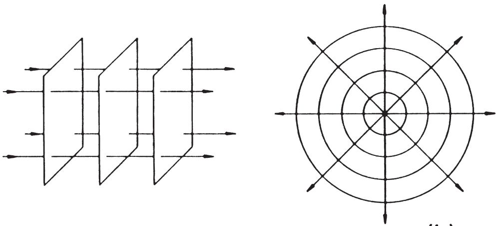

Figure 1.1 Simple types of waves: (a) plane wave; (b) spherical wave.

is frequency independent:

$$\frac{p}{v} = \rho_0 c \tag{1.9}$$

This ratio is called the 'characteristic impedance' of the medium. For air at 20° centigrade its value is

$$\rho_0 c = 416 \frac{\text{Pa}}{\text{m/s}} = 416\ \text{kg m}^{-2}\ \text{s}^{-1} \tag{1.10}$$

If the wave is travelling in the negative x-direction, the ratio of sound pressure and particle velocity is negative.

Of particular importance are harmonic waves in which the time and space dependence of the acoustical quantities, for instance of the sound pressure, follows a sine or cosine function. If we set $G = 0$ and specify F as a cosine function, we obtain an expression for a plane, progressive harmonic wave:

$$p(x,t) = \hat{p}\cos\left[k\,(ct - x)\right] = \hat{p}\cos\left(\omega t - kx)\right) \tag{1.11}$$

with the arbitrary constants $\hat{p}$ and k. Here the angular frequency

$$\omega = kc \tag{1.12}$$

was introduced which is related to the temporal period

$$T = \frac{2\pi}{\omega} \tag{1.13a}$$

of the harmonic vibration represented by eqn (1.11). At the same time this equation describes a spatial harmonic vibration with the period

$$\lambda = \frac{2\pi}{k} \tag{1.13b}$$

This is the 'wavelength' of the harmonic wave. It denotes the distance in the x-direction where equal values of the sound pressure (or any other field quantity) occur. According to eqn (1.12) it is related to the angular frequency by

$$\lambda = \frac{2\pi c}{\omega} = \frac{c}{f} \tag{1.14}$$

where $f = \omega/2\pi = 1/T$ is the frequency of the vibration. It has the dimension second^{-1}; its units are hertz (Hz), kilohertz (1 kHz = 10^3 Hz), megahertz (1 MHz = 10^6 Hz), etc. The quantity $k = \omega/c$ is the propagation constant or the (angular) wave number of the wave, and $\hat{p}$ is its amplitude.

A very useful and efficient representation of harmonic oscillations and waves is obtained by applying the relation (with $\mathrm{i} = \sqrt{-1}$):

$$\exp(\mathrm{i}x) = \cos x + \mathrm{i} \sin x \tag{1.15}$$

This is the complex or symbolic notation of harmonic vibrations and will be employed quite frequently in what follows. Using this relation, eqn (1.11) can be written in the form

$$p(x,t) = \mathrm{Re}\left\{\hat{p} \exp\left[\mathrm{i}\left(\omega t - kx\right)\right]\right\}$$

or, omitting the sign Re:

$$p(x,t) = \hat{p} \exp\left[\mathrm{i}\left(\omega t - kx\right)\right] \tag{1.16}$$

The complex notation has several advantages over the real representation of eqn (1.11). Differentiation or integration with respect to time is equivalent to multiplication or division by i. Furthermore, only the complex notation allows a clear-cut definition of impedances and admittances (see Section 2.1). It fails, however, in all cases where vibrational quantities are to be multiplied or squared. If doubts arise concerning the physical meaning of an expression it is advisable to recall the origin of this notation, i.e. to take the real part of the expression.

As with any complex quantity the complex sound pressure in a plane wave may be represented in a rectangular coordinate system with the horizontal and the vertical axis corresponding to the real and the imaginary part of the pressure, respectively. The quantity is depicted as an arrow, often called 'phasor', pointing from the origin to the point which corresponds to the value of the pressure (see Fig. 1.2). The length of this arrow corresponds to the magnitude of the complex quantity while the angle it includes with the real axis is its phase angle or 'argument' (abbreviated arg p). In the present case the magnitude of the phasor equals the amplitude $\hat{p}$ of the oscillation, the phase angle depends on time t and position x:

$$\arg p = \omega t - kx$$

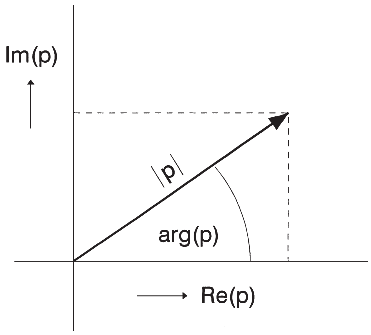

Figure 1.2 Phasor representation of a complex quantity p.

This means, that for a fixed position the arrow or phasor rotates around the origin with an angular velocity ω, which explains the expression 'angular frequency'.

So far it has been assumed that the wave medium is free of losses. If this is not the case, the pressure amplitude does not remain constant in the course of wave propagation but decreases according to an exponential law. Then eqn (1.16) is modified in the following way:

$$p(x,t) = \hat{p}\exp(-mx/2)\exp\left[\mathrm{i}\left(\omega t - kx\right)\right] \tag{1.16a}$$

We can even use the representation of eqn (1.16) if we conceive the wave number k as a complex quantity containing half the attenuation constant m as its imaginary part:

$$k = \frac{\omega}{c} - \mathrm{i}\frac{m}{2} \tag{1.17}$$

Another simple wave type is the spherical wave in which the surfaces of constant pressure, i.e. the wave fronts, are concentric spheres (see Fig. 1.1b). In their common centre we have to imagine some very small source which introduces or withdraws fluid. Such a source is called a 'point source'. The appropriate coordinates for this geometry are polar coordinates with the distance r from the centre as the relevant space coordinate. Transformed into this system, the differential equation (1.5) reads:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{1.18}$$

A simple solution of this equation is

$$p(r,t) = \frac{\rho_0}{4\pi r}\dot{Q}\left(t - \frac{r}{c}\right) \tag{1.19}$$

It represents a spherical wave produced by a point source at $r = 0$ with the 'volume velocity' Q, which is the rate (in m^3/s) at which fluid is expelled by the source. The overdot means partial differentiation with respect to time. Again, the argument $t - r/c$ indicates that any disturbance created by the sound source is propagated outward with velocity c, its strength decreasing as $1/r$. Reversing the sign in the argument of $\dot{Q}$ would result in the unrealistic case of an in-going wave.

The only non-vanishing component of the particle velocity is the radial one, which is calculated by applying eqn (1.2) to eqn (1.19):

$$v_r = \frac{1}{4\pi r^2}\left[Q\left(t - \frac{r}{c}\right) + \frac{r}{c}\dot{Q}\left(t - \frac{r}{c}\right)\right] \tag{1.20}$$

If the volume velocity of the source varies according to $Q(t) = \hat{Q}\exp(\mathrm{i}\omega t)$, we obtain from eqn (1.19)

$$p(r,t) = \frac{\mathrm{i}\omega\rho_0}{4\pi r}\hat{Q}\cdot\exp\left[\,\mathrm{i}\,(\omega t - kr)\right] \tag{1.21}$$

This expression represents—with $k = \omega/\mathrm{c}$—the sound pressure in a harmonic spherical wave. The particle velocity after eqn (1.20) is

$$v_r = \frac{p}{\rho_0 c}\left(1 + \frac{1}{\mathrm{i}kr}\right) \tag{1.22}$$

This formula indicates that the ratio of sound pressure and particle velocity in a spherical sound wave depends on the distance r and the frequency $\omega = kc$. Furthermore, it is complex, i.e. between both quantities there is a phase difference. For $kr \gg 1$, i.e. for distances which are large compared with the wavelength, the ratio p/v_r tends asymptotically to $\rho_0 c$, the characteristic impedance of the medium.

A plane wave is an idealised wave type which does not exist in the real world, at least not in its pure form. However, sound waves travelling in a rigid tube can come very close to a plane wave. Furthermore, a limited region of a spherical wave may also be considered as a good approximation to a plane wave provided the distance r from the centre is large compared with all wavelengths involved, i.e. $kr \gg 1$, see eqn (1.22).

On the other hand, a point source producing a spherical wave can be approximated by any sound source which is small compared with the wavelength and which expels fluid, for instance by a small pulsating sphere or a loudspeaker mounted into one side of an airtight box. Most sound sources, however, do not behave as point sources. In these cases, the sound pressure depends not only on the distance r but also on the direction, which can be characterised by a polar angle ϑ and an azimuth angle φ. For distances exceeding a characteristic range, which depends on the sort of sound source and the frequency, the sound pressure is given by

$$p(r, \vartheta, \varphi, t) = \frac{A}{r}\Gamma(\vartheta, \varphi) \exp\left[\mathrm{i}(\omega t - kr)\right] \tag{1.23}$$

where the 'directional factor' $\Gamma(\vartheta, \varphi)$ is normalised so as to make $\Gamma = 1$ for its absolute maximum. A is a constant.

1.3 Energy density and intensity, radiation

If a sound source, for instance a musical instrument, is to generate a sound wave it has to deliver some energy to a fluid. This energy is carried away by the sound wave. Accordingly we can characterise the amount of energy contained in one unit volume of the wave by the energy density. As with any kind of mechanical energy one has to distinguish between potential and kinetic energy density:

$$w_{\text{pot}} = \frac{p^2}{2\rho_0 c^2}, \qquad w_{\text{kin}} = \frac{\rho_0 |\mathbf{v}|^2}{2} \tag{1.24}$$

where $|\mathbf{v}|$ is the magnitude of the vector $\mathbf{v}$. The total energy density is

$$w = w_{\text{pot}} + w_{\text{kin}} \tag{1.25}$$

Another important quantity is sound intensity, which is a measure of the energy transported in a sound wave. Imagine a window of 1 m^2 perpendicular to the direction of sound propagation. Then the intensity is the energy per second passing this window. Generally the intensity is a vector parallel to the vector $\mathbf{v}$ of the particle velocity and is given by

$$\mathbf{I} = p\mathbf{v} \tag{1.26}$$

The principle of energy conservation requires

$$\frac{\partial w}{\partial t} + \operatorname{div}\mathbf{I} = 0 \tag{1.27}$$

It should be noted that—in contrast to the sound pressure and particle velocity—these energetic quantities do not simply add if two waves are superimposed on each other.

In a plane wave the sound pressure and the longitudinal component of the particle velocity are related by $p = \rho_0 c v$, and the same holds for a spherical wave at a large distance from the centre ($kr \gg 1$, see eqn (1.22)). Hence we can express the particle velocity in terms of the sound pressure. Then the energy density and the intensity are

$$w = \frac{p^2}{\rho_0 c^2} \qquad \text{and} \qquad I = \frac{p^2}{\rho_0 c} \tag{1.28}$$

They are related by

$$I = cw \tag{1.29}$$

Stationary signals which are not limited in time may be characterised by time averages over a sufficiently long time. We introduce the root-mean-square of the sound pressure by

$$p_{rms} = \left(\frac{1}{t_a} \int_0^{t_a} p^2 \mathrm{d}t \right)^{1/2} = \left(\overline{p^2} \right)^{1/2} \tag{1.30}$$

where the overbar is a shorthand notation indicating time averaging. The eqns (1.28) yield

$$\overline{w} = \frac{p_{\mathrm{rms}}^2}{\rho_0 c^2} \qquad \text{and} \qquad I = \frac{p_{\mathrm{rms}}^2}{\rho_0 c} \tag{1.28a}$$

Finally, for a harmonic sound wave with the sound pressure amplitude $\hat{p}$, p_{rms} equals $\hat{p}/\sqrt{2}$, which leads to

$$\overline{w} = \frac{\hat{p}^2}{2\rho_0 c^2} \qquad \text{and} \qquad I = \frac{\hat{p}^2}{2\rho_0 c} \tag{1.28b}$$

The last equation can be used to express the total power output of a point source $P = 4\pi r^2 I(r)$ by its volume velocity. According to eqn (1.21), the sound pressure amplitude in a spherical wave is $\rho_0 \omega \hat{Q}/4\pi r$ and therefore

$$P = \frac{\rho_0}{8\pi c} \hat{Q}^2 \omega^2 \tag{1.31}$$

If, on the other hand, the power output of a point source is given, the root-mean-square of the sound pressure at distance r from the source is

$$p_{\mathrm{rms}} = \frac{1}{r} \sqrt{\frac{\rho_0 c P}{4\pi}} \tag{1.32}$$

1.4 Signals and systems

Any acoustical signal can be unambiguously described by its time function $s(t)$ where s denotes a sound pressure, a component of particle velocity, or the instantaneous density of air, for instance. If this function is a sine or cosine function—or an exponential with imaginary argument—we speak of a harmonic signal, which is closely related to the harmonic waves as introduced in Section 1.2. Harmonic signals and waves play a key role in acoustics although real sound signals are almost never harmonic but show a much more complicated time dependence. The reason for this apparent contradiction is the fact that virtually all signals can be considered as superposition of harmonic signals. This is the fundamental statement of the famous Fourier theorem.

The Fourier theorem can be formulated as follows: let $s(t)$ be a real, non-periodic time function describing, for example, the time dependence of the sound pressure or the volume velocity, this function being sufficiently steady (a requirement which is fulfilled in all practical cases), and the integral $\int_{-\infty}^{\infty}[s(t)]^2 \mathrm{d}t$ having a finite value. Then

$$s(t) = \int_{-\infty}^{\infty} S(f) \exp(2\pi \mathrm{i} f t)\, \mathrm{d}f \tag{1.33a}$$

and

$$S(f) = \int_{-\infty}^{\infty} s(t) \exp(-2\pi \mathrm{i} f t)\, \mathrm{d}t \tag{1.33b}$$

Because of the symmetry of these formulae $S(f)$ is not only the Fourier transform of $s(t)$ but $s(t)$ is the (inverse) Fourier transform of $S(f)$ as well. The complex function $S(f)$ is called the 'spectral function' or the 'complex amplitude spectrum', or simply the 'spectrum' of the signal $s(t)$. It can easily be shown that $S(-f) = S^*(f)$, where the asterisk denotes the transition to the complex conjugate function. $S(f)$ and $s(t)$ are different but equivalent representations of a signal.

According to eqn (1.33a), the signal $s(t)$ is composed of harmonic oscillations with continuously varying frequencies f. The absolute value of the spectral function, which can be written as

$$S(f) = |S(f)| \exp[\mathrm{i}\psi(f)] \tag{1.34}$$

is the complex amplitude of the harmonic oscillation; $\psi(f)$ is its phase angle. The functions $|S(f)|$ and $\psi(f)$ are called the amplitude and the phase spectrum of the signal $s(t)$. A few examples of time function and their amplitude spectra are shown in Fig. 1.8.

The Fourier theorem assumes a slightly different form if $s(t)$ is a periodic function with period T, i.e. if $s(t) = s(t+T)$. Then the integral in eqn (1.33*a*) has to be replaced by a series:

$$s(t) = \sum_{n=-\infty}^{\infty} S_n \exp\left(\frac{2\pi i n t}{T}\right) \tag{1.35a}$$

with the coefficients

$$S_n = \int_{-\infty}^{\infty} s(t) \exp\left(\frac{-2\pi i n t}{T}\right) dt \tag{1.35b}$$

where n is an integer. The steady spectral function has changed now into a set of discrete 'Fourier coefficients', for which $S_{-n} = S_n^*$ as before. Hence a periodic signal consists of discrete harmonic vibrations, the frequencies of which are multiples of a fundamental frequency $1/T$. These components are called 'partial oscillations' or 'harmonics', the first harmonic being identical with the fundamental oscillation.

If the signal is not continuous but consists of a sequence of N discrete, periodically repeated numbers

$$s_0, s_1, s_2, \ldots, s_{N-2}, s_{N-1}, s_0, s_1, \ldots$$

these coefficients are given by

$$S_m = \sum_{n=0}^{N-1} s_n \exp\left(-\frac{2\pi i n m}{N}\right) \qquad (m = 0, 1, 2 \ldots N-1) \tag{1.36a}$$

The sequence of these coefficients, which is also periodic with the period N, is called the discrete fourier transform (DTF) of s_n. The inverse transformation reads

$$s_n = \sum_{n=0}^{N-1} S_n \exp\left(\frac{2\pi i n m}{N}\right) \qquad (m = 0, 1, 2 \ldots N-1) \tag{1.36b}$$

Of course all the formulae above can be written with the angular frequency $\omega = 2\pi f$ instead of the frequency f. A real notation of the formulae above is also possible. For this purpose one just has to separate the real parts from the imaginary parts in eqns (1.33), (1.35) and (1.36), respectively.

Equations (1.33) cannot be applied in this form to stationary non-periodic signals, i.e. to signals which are not limited in time and the statistical properties of which are not time dependent. This holds, for instance, for any kind of random noise. In this case the integrals would not converge. Therefore, firstly a 'window' of width T_0 is cut out of the signal. For this section the spectral function $S_{T_0}(f)$ is well defined and can be evaluated numerically or experimentally. The 'power spectrum' of the whole signal is then given by

$$W(f) = \lim_{T_0 \to \infty} \left[\frac{1}{T_0} S_{T_0}(f) S^*_{T_0}(f) \right] \tag{1.37}$$

The determination of the complex spectrum $S(f)$ or of the power spectrum $W(f)$, known as 'spectral analysis', is of great theoretical and practical importance. Nowadays it is most conveniently achieved by using digital computers. A particularly efficient procedure for computing spectral functions is the 'fast Fourier transform' (FFT) algorithm. To give at least an idea of it we suppose that N is an even number. We note that the sum in eqn (1.36*a*) can be decomposed into one sum containing terms of even order m and a second one comprising the terms with odd m:

$$S_m = \sum_{j=0}^{(N/2)-1} s_{2j} \exp\left(-2\pi\mathrm{i}\frac{jm}{N/2}\right) + \exp\left(-2\pi\mathrm{i}\frac{m}{N}\right) \sum_{j=0}^{(N/2)-1} s_{2j+1} \exp\left(-2\pi\mathrm{i}\frac{2jm}{N/2}\right)$$

Each of these sums represents a discrete Fourier transform of $N/2$ elements. But while the calculation according to eqn (1.36*a*) requires N^2 multiplications, the number of multiplications occurring in the above equation is only $2(N/2)^2 + N$. If N is an integral power of 2, the decomposition of sums can be repeated again and again until the final sums consist just of one term each. It can be shown that the final number of required multiplications is only $(N/2)\log_2(N)$. Thus, the saving of computing time provided by FFT is considerable, particularly if N is large. For a more detailed description of the fast Fourier transform, the reader is referred to the extensive literature on this subject (e.g. Ref. 1). A rough, experimental spectral analysis can also be carried out by applying the signal to a set of bandpass filters.

The power spectrum, which is an even function of the frequency, does not contain all the information on the original signal $s(t)$, because it is based on the absolute value of the spectral function only, whereas all phase information has been eliminated. Inserted into eqn (1.33*a*), it does not restore the original function $s(t)$ but instead yields another important time function, called the 'autocorrelation function' of $s(t)$:

$$\phi_{ss}(\tau) = \int_{-\infty}^{\infty} W(f) \cdot \exp(2\pi \mathrm{i} f \tau) \mathrm{d}f = 2 \int_{0}^{\infty} W(f) \cdot \cos(2\pi f \tau) \mathrm{d}f \qquad (1.38a)$$

The time variable has been denoted by τ in order to indicate that it is not identical with the real time. In the usual definition of the autocorrelation function it occurs as a time shift:

$$\phi_{ss}(\tau) = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s(t)s(t+\tau)\,\mathrm{d}t = \overline{s(t)s(t+\tau)} \qquad (1.38b)$$

The autocorrelation function characterizes the similarity of a signal at time t and the same signal at a different time $t+\tau$.

Since ϕ_{ss} is the Fourier transform of the power spectrum, the latter is also obtained by inverse Fourier transformation of the autocorrelation function:

$$W(f) = \int_{-\infty}^{\infty} \phi_{\mathrm{ss}}(\tau) \cdot \exp(-2\pi \mathrm{i} f \tau) \mathrm{d}\tau = 2 \int_{0}^{\infty} \phi_{\mathrm{ss}}(\tau) \cdot \cos(2\pi f \tau) \mathrm{d}\tau \qquad (1.39)$$

Equations (1.38a) and (1.39) are the mathematical expressions of the theorem of Wiener and Khinchine: power spectrum and autocorrelation function are Fourier transforms of each other.

If $s(t+\tau)$ in eqn (1.38b) is replaced with $s'(t+\tau)$, where s' denotes a time function different from s, one obtains the 'cross-correlation function' of the two signals $s(t)$ and $s'(t)$:

$$\phi_{\mathrm{ss}'}(\tau) = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s(t)s'(t+\tau)\mathrm{d}t = \overline{s(t)s'(t+\tau)} \qquad (1.40)$$

The cross-correlation function provides a measure of the statistical similarity of two functions s and s'. It is closely related to the correlation coefficient

$$\Psi = \frac{\overline{s(t) \cdot s'(t)}}{\sqrt{\overline{s(t)^2} \cdot \overline{s'(t)^2}}} \qquad (1.41)$$

which may vary between $+1$ and -1. If $\Psi = 0$, the functions $s(t)$ and $s'(t)$ are said to be uncorrelated (or incoherent). This is the case when these functions are random or nearly random. As an example, consider the signals

$$s(t)=\sum_n a_n \cos\left(\omega_n t+\varphi_n\right) \qquad \text{and} \qquad s'(t)=\sum_n b_n \cos\left(\omega_n t+\varphi'_n\right)$$

each of them consisting of many sinusoidal components with phase angles which are randomly distributed over the interval from 0 to 2π. First we calculate the averaged square of both sums:

$$\begin{aligned} s^2 &= \sum_n \sum_m a_n a_m \cos\left(\omega_n t+\varphi_n\right) \cos\left(\omega_m t+\varphi_m\right) \\ &= \frac{1}{2} \sum_n \sum_m a_n a_m \left[\cos\left(\omega_n t+\omega_m t+\varphi_n+\varphi_m\right)+\cos\left(\omega_n t-\omega_m t+\varphi_n-\varphi_m\right)\right] \end{aligned}$$

Time averaging eliminates all time-dependent terms, i.e. all terms except those with $n=m$, which are reduced to a_n^2 . Thus we arrive at the important result that under the given conditions the averaged square of the signal $s(t)$ is just

$$\overline{s^2}=\frac{1}{2}\sum_n a_n^2 \tag{1.42}$$

In a similar way it is shown that

$$\overline{s \cdot s'}=\frac{1}{2}\sum_n a_n b_n \cos\left(\varphi_n-\varphi'_n\right)$$

which is zero since the cos terms cancel due to the random distribution of phase angles.

It should be noted that $\Psi=0$ is a necessary but not a sufficient condition for two signals being uncorrelated.

In a certain sense a sine or cosine signal can be considered as an elementary signal; it is unlimited in time and steady in all its derivatives, and its spectrum consists of a single line. The counterpart of it is Dirac's delta function $\delta(t)$: it has one single line in the time domain, so to speak, whereas its amplitude spectrum is constant for all frequencies, i.e. $S(f)=1$ for the delta function. This leads to the following representation:

$$\delta(t)=\lim_{f_0\to\infty}\frac{1}{f_0}\int_{-f_0/2}^{f_0/2}\exp\left(2\pi f t\right)\mathrm{d}f \tag{1.43}$$

The delta function has the following fundamental property:

$$s(t) = \int_{-\infty}^{\infty} s(\tau)\delta(t-\tau)\,\mathrm{d}\tau \tag{1.44}$$

where $s(t)$ is any function of time. Accordingly, any signal can be considered as a close succession of very short pulses, as indicated in Fig. 1.3. Especially, for $s(t) \equiv 1$ we obtain

$$\int_{-\infty}^{\infty} \delta(\tau)\,\mathrm{d}\tau = 1 \tag{1.45}$$

Since the delta function $\delta(t)$ is zero for all $t \neq 0$ it follows from eqn (1.45) that its value at $t = 0$ must be infinite.

Now consider a linear and time-independent but otherwise unspecified transmission system. Examples of acoustical transmission systems are all kinds of ducts (air ducts, mufflers, wind instruments, etc.) and resonators. Likewise, any two points in an enclosure may be considered as the input and output terminal of an acoustic transmission system. Linearity means that multiplying the input signal with a factor results in an output signal which is augmented by the same factor. The properties of such a system are completely characterised by the so-called 'impulse response' $g(t)$, i.e. the output signal which is the response to an impulsive input signal represented by the Dirac function $\delta(t)$ (see Fig. 1.4). Since the response cannot precede the excitation, the impulse response of any causal system must vanish for $t < 0$. If $g(t)$ is known, the output signal $s'(t)$ with respect to any input signal $s(t)$ can be obtained by replacing the Dirac function in eqn (1.44) with its

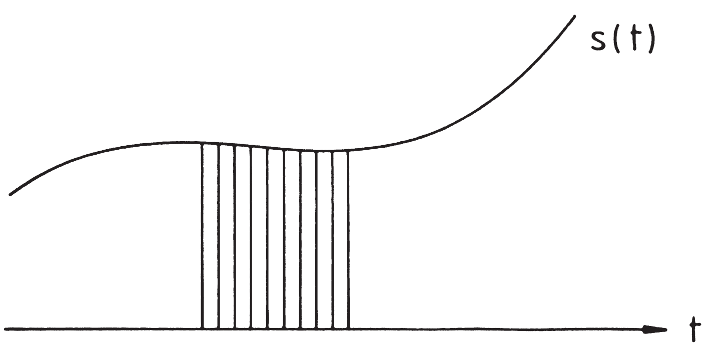

Figure 1.3 Continuous function as the limiting case of a close succession of short impulses.

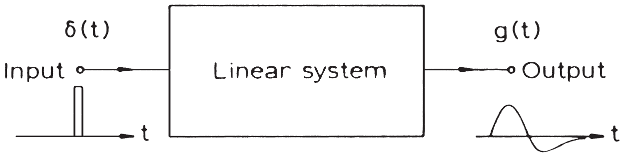

Figure 1.4 Impulse response of a linear system.

response, i.e. with $g(t)$:

$$s'(t) = \int_{-\infty}^{\infty} s(\tau)g(t-\tau)\,\mathrm{d}\tau = \int_{-\infty}^{\infty} g(\tau)s(t-\tau)\,\mathrm{d}\tau \tag{1.46}$$

This operation is known as the convolution of two functions s and g. A common shorthand notation of it is

$$s'(t) = s(t) * g(t) = g(t) * s(t) \tag{1.46a}$$

Equation (1.46) has its analogue in the frequency domain, which looks even simpler. Let $S(f)$ be the complex spectrum of the input signal $s(t)$ of our linear system; then the spectrum of the resulting output signal is

$$S'(f) = G(f) \cdot S(f) \tag{1.47}$$

The complex function $G(f)$ is the 'transmission function' or 'transfer function' of the system; it is related to the impulse response by the Fourier transformation:

$$G(f) = \int_{-\infty}^{\infty} g(t) \exp(-2\pi \mathrm{i} ft)\,\mathrm{d}t \tag{1.48a}$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(2\pi \mathrm{i} ft)\,\mathrm{d}f \tag{1.48b}$$

with $G(-f) = G^*(f)$ since $g(t)$ is a real function. The transfer function $G(f)$ has also a direct meaning: if a harmonic signal with frequency f is applied to a transmission system, its amplitude will be changed by the factor $|G(f)|$ and its phase will be shifted by the phase angle of $G(f)$.

1.5 Sound pressure level and sound power level

In the frequency range in which our hearing is most sensitive (500–5000 Hz) the intensity of the threshold of sensation and of the threshold of pain in hearing differ by about 13 orders of magnitude. For this reason it would be impractical to characterise the strength of a sound signal by its sound pressure or its intensity. Instead, a logarithmic quantity, the so-called 'sound pressure level' is generally used for this purpose, defined by

$$SPL = 20\log_{10}\left(\frac{p_{\mathrm{rms}}}{p_0}\right) \quad \text{decibels} \tag{1.49}$$

In this definition, p_{rms} denotes the 'root-mean-square' pressure, as introduced in Section 1.3. The quantity p_0 is an internationally standardised reference pressure; its value is 2×10^{-5} Pa, which corresponds roughly to the normal hearing threshold at 1000 Hz. The 'decibel' (abbreviated dB) is not a unit in a physical sense but is used rather to recall the above level definition. Strictly speaking, the p_{rms} as well as the SPL are defined only for stationary sound signals since they both imply an averaging process.

According to eqn (1.49), two different sound fields or signals may be compared by their level difference:

$$\Delta SPL = 20\log_{10}\left(\frac{p_{\mathrm{rms1}}}{p_{\mathrm{rms2}}}\right) \quad \text{decibels} \tag{1.49a}$$

It is often convenient to express the sound power delivered by a sound source in terms of the 'sound power level', defined by

$$PL = 10\log_{10}\left(\frac{P}{P_0}\right) \quad \text{decibels} \tag{1.50}$$

where P_0 is a reference power of 10^{-12} W. Using this quantity, the sound pressure level produced by a point source with power P in the free field can be expressed as follows (see eqn (1.32):

$$\mathrm{SPL} = \mathrm{PL} - 20\,\log_{10}\left(\frac{r}{r_0}\right) - 11\,\mathrm{dB} \qquad \text{with } r_0 = 1\text{ m} \tag{1.51}$$

1.6 Some properties of human hearing

Since the ultimate consumer of all room acoustics is the listener, it is important to consider at least a few facts relating to aural perception. More information may be found in Ref. 2, for instance.

One of the most obvious facts of human hearing is that the ear is not equally sensitive to sounds of different frequencies. Generally, the loudness at which a sound is perceived depends, of course, on its objective strength, i.e. on its sound pressure level. Furthermore, it depends in a complicated manner on the spectral composition of the sound signal, on its duration and

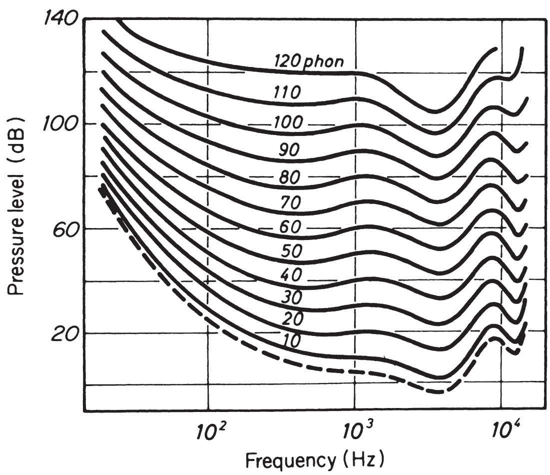

Figure 1.5 Contours of equal loudness level for frontal sound incidence. The dashed curve corresponds to the average hearing threshold.

on several other factors. The loudness is often characterised by the 'loudness level', which is the sound pressure level of a 1000 Hz tone which appears equally loud as the sound to be characterised. The unit of the loudness level is the 'phon'.

Figure 1.5 presents the contours of equal loudness level for sinusoidal sound signals which are presented to a listener in the form of frontally incident plane waves. The numbers next to the curves indicate the loudness level. The lowest, dashed curve which corresponds to a loudness level of 3 phons, marks the threshold of hearing. According to this diagram, a pure tone with a SPL of 40 decibels has by definition a loudness level of 40 phons if its frequency is 1000 Hz. However, at 100 Hz its loudness level would be only 24 phons whereas at 50 Hz it would be almost inaudible.

Using these curves, the loudness level of any pure tone can be determined from its frequency and its sound pressure level. In order to simplify this somewhat tedious procedure, electrical instruments have been constructed which measure the sound pressure level. Basically, they consist of a calibrated microphone which converts the sound into an electrical signal, amplifiers and, most important, a weighting network, the frequency-dependent attenuation of which approximates the contours of equal loudness. Several weighting functions are in use and have been standardised internationally; the most common of them is the A-weighting curve. Consequently, the result of such a measurement is not the loudness level in phons, but the 'A-weighted sound pressure level' in dB(A).