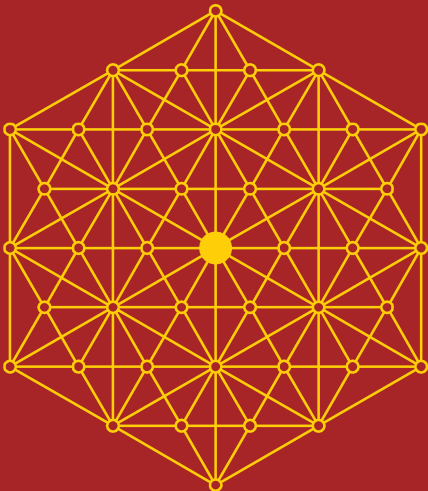
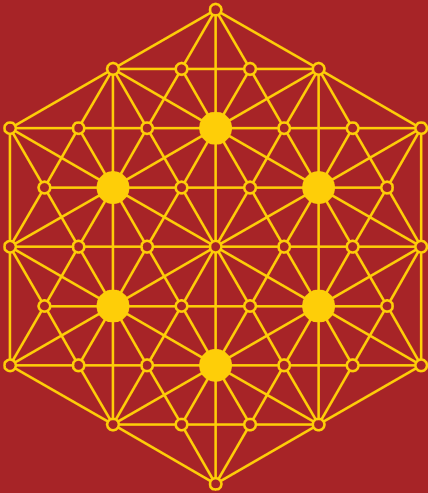
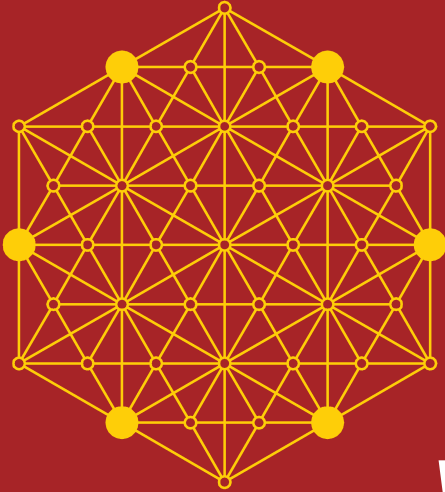


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A SPON BOOK



Vibration Analysis and Structural Dynamics for Civil Engineers

Essentials and
Group-Theoretic
Formulations

Alphose Zingoni

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Essentials and Group-Theoretic
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Preface

It is not an easy task to write a book that covers the fundamentals of a subject, at the same time going reasonably far enough on a particular aspect of the subject to interest the more specialist reader. This book attempts to do that for the subject of the vibration of engineering systems, later focusing on how symmetry (common in engineering systems) affects vibration behaviour, a topic that, interestingly, has been the subject of research for many years in a number of areas of physics and chemistry, such as crystallography, quantum mechanics and molecular symmetry. After covering the fundamentals (which every student must know), the same approaches that have yielded fruitful insights in studying various problems of symmetry in physics and chemistry are harnessed here for studying vibration problems in engineering.

The book is divided into two parts. The first part covers the essentials of vibration analysis, and is intended for all civil, structural and mechanical engineering students taking a basic course on structural dynamics, as well as practicing engineers wishing to acquaint themselves with the fundamentals of vibration modelling. The treatment is concise yet thorough, with several fully worked out examples. Tutorial questions are featured at the end of the first three chapters to provide further practice for the student. Considerations are extended to continuous systems and the use of the finite-element method in vibration analysis.

The second part introduces a novel approach to the vibration analysis of symmetric systems. The mathematical concepts of group theory are used to simplify a range of vibration problems in structural mechanics, demonstrating the computational benefits of the approach, and also providing valuable insights into the vibration behaviour of symmetric systems as well as logical explanations of certain observed phenomena (e.g. coincident frequencies, similarity of modes and stationary points). This feature sets the book apart from other books on vibration and makes it of interest not only to students and teachers of vibration analysis, but also to researchers involved with the development of computational procedures and the study of new phenomena. A more detailed description follows.

Part I begins (in Chapter 1) with a general introduction to vibration and its modelling. After these preliminaries, treatment then focuses on the vibration of discrete systems or lumped parameter models. Chapter 2 considers the free vibration response of both damped and undamped single degree-of-freedom systems on the basis of a rectilinear spring–mass model, and extends the ideas to a wider range of problems in structural and mechanical engineering through the concept of an equivalent spring stiffness. This is followed by treatment of the forced response to harmonic excitation of one degree-of-freedom systems.

Chapter 3 is concerned with the vibration of systems with two or more degrees of freedom. The equations of motion are formulated on the basis of both the stiffness matrix and the flexibility matrix, and the derivations of these matrices are amply illustrated by reference to both structural and mechanical systems. Formulation of the eigenvalue problem is followed by evaluation of natural frequencies of vibration and mode shapes. The concept of a generalized mass and a generalized stiffness is then introduced, after which the mode superposition method (or modal analysis) is described, and applied to the evaluation of the forced response of multi-degree-of-freedom systems.

The theoretical formulation of the problems of the vibration of continuous systems (distributed parameter modelling) is dealt with in Chapter 4, by reference to the transverse vibration of strings, the axial vibration of rods and the flexural vibration of beams. Closed-form solutions for free vibration response are obtained. The orthogonality of natural modes is demonstrated, leading to the development of modal analysis as a technique for evaluating the dynamic response of a system to a time-varying excitation force.

The essence of the finite-element formulation is presented in Chapter 5. After an outline of the basic theory, stiffness and consistent mass matrices for truss, beam and rectangular plane–stress finite elements are derived. A simple example is used to illustrate the process of assembly of the system eigenvalue problem and solution of the ensuing equations.

Part II begins (in Chapter 6) with an outline of group theory, a description of symmetry groups and a summary of the most important results of associated representation theory. The chapters that follow illustrate how group theory is implemented in simplifying the vibration analysis of various structural systems exhibiting symmetry properties.

By reference to rectilinear shaft–disc torsional and spring–mass extensional systems (Chapter 7), plane structural grids (Chapter 8) and high-tension cable nets (Chapter 9), the computation of natural frequencies and mode shapes for discrete systems is shown to be considerably simplified through the presented group-theoretic formulations. It is seen how group theory predicts all the symmetry types associated with a particular configuration of the structure, where stationary points and nodal lines occur, and why certain frequencies have the same values. These are important insights that help us to better understand the nature of vibration phenomena in structures.

Considerations are then extended to the vibration analysis of continuous systems by numerical methods. A group-theoretic formulation of the finite-difference method is presented in Chapter 10, and applied to the vibration analysis of square and rectangular plates. It is seen how group-theoretic decomposition of the vector space of the problem permits the evaluation of system frequencies within decoupled subspaces, thus avoiding numerical problems associated with the computation of clustered frequencies. In the last chapter of the book (Chapter 11), a group-theoretic formulation for finite elements is developed, and illustrated by reference to solid rectangular elements.

The second part of the book, which is essentially a brief exposition of a relatively new computational approach for special vibration problems, will appeal to all students of vibration analysis keen to learn new methods, irrespective of engineering discipline. This part will also be of considerable interest to researchers concerned with the development of improved computational methods for vibration analysis, and with the understanding of complex vibration phenomena associated with symmetry groups of high order.

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About the author

Alphose Zingoni, PhD, is professor of structural engineering and mechanics in the Department of Civil Engineering at the University of Cape Town. He holds an MSc in structural engineering and a PhD in shell structures, both earned at Imperial College London. He served as dean of the Faculty of Engineering at the University of Zimbabwe from 1996 to 1999, before moving to the University of Cape Town in 1999, where he served as head of the Department of Civil Engineering from 2008 to 2012.

A past recipient of a Royal Commission for the Exhibition of 1851 Postdoctoral Fellowship of the UK (1992–1994), Dr. Zingoni has research interests encompassing shell structures, space structures, vibration analysis and applications of group theory to problems in computational structural mechanics. He has written more than 100 scientific papers on these topics, which have been published in leading international journals and presented at various international conferences worldwide. He has authored two other books: *Shell Structures in Civil and Mechanical Engineering*, published by Thomas Telford (London, 1997), and *Theory and Analysis of Structures*, published by UNESCO (Nairobi, 2000).

In 2001, he founded the Structural Engineering, Mechanics and Computation (SEMC) series of international conferences, now held in Cape Town (South Africa) every 3 years. The proceedings of these conferences, which he edits, feature peer-reviewed research papers on a wide variety of topics, including vibration analysis and structural dynamics, and the related topics of vibration control, seismic response and earthquake-resistant design. Currently Dr. Zingoni serves on the editorial boards of seven international journals. Over the past 10 years, he has served on the scientific committees of up to 50 international conferences in fields related to structural engineering and mechanics.

Throughout his academic career, he has taught various courses at both the undergraduate and postgraduate levels, which have included courses on structural analysis, structural design, plates and shells, vibration analysis

and structural dynamics. He has also supervised a large number of post-graduate students undertaking research in these areas.

Dr. Zingoni is a fellow of the Institution of Structural Engineers (London), a fellow of the International Association for Bridge and Structural Engineering (Zurich), a fellow of the South African Academy of Engineering, a member of the Academy of Science of South Africa, a registered professional engineer with the Engineering Council of South Africa and a registered chartered engineer with the Engineering Council of the UK.

Part I

Essentials
