BLACKBODY RADIATION

A HISTORY OF THERMAL RADIATION COMPUTATIONAL AIDS AND NUMERICAL METHODS

Seán M. Stewart • R. Barry Johnson

CRC Press
Taylor & Francis Group

BLACKBODY RADIATION

A HISTORY OF THERMAL RADIATION COMPUTATIONAL AIDS AND NUMERICAL METHODS

Optical Sciences and Applications of Light

Series Editor
James C. Wyant
University of Arizona

Nanofabrication: Principles to Laboratory Practice, *Andrew Sarangan*

Blackbody Radiation: A History of Thermal Radiation Computational Aids and Numerical Methods, *Sean M. Stewart and R. Barry Johnson*

High-Speed 3D Imaging with Digital Fringe Projection Techniques, *Song Zhang*

Introduction to Optical Metrology, *Rajpal S. Sirohi*

Charged Particle Optics Theory: An Introduction, *Timothy R. Groves*

Nonlinear Optics: Principles and Applications, *Karsten Rottwitt and Peter Tidemand-Lichtenberg*

Photonics Modelling and Design, *Slawomir Sujecki*

Numerical Methods in Photonics, *Andrei V. Lavrinenko, Jesper Lægsgaard, Niels Gregersen, Frank Schmidt, and Thomas Søndergaard*

BLACKBODY RADIATION

A HISTORY OF THERMAL RADIATION COMPUTATIONAL AIDS AND NUMERICAL METHODS

Seán M. Stewart • R. Barry Johnson

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed on acid-free paper
Version Date: 20160714

International Standard Book Number-13: 978-1-4822-6312-1 (Hardback)

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

Dedication

To those scientists and engineers, immortalized in this work, whose tireless investigations into blackbody radiation created the beginnings of quantum physics;
and
to Donald and Cathy Stewart, loving parents,
and Marianne F. Johnson, loving wife,
for their unceasing support and encouragement.

A computer who must make many difficult calculations usually
has a book of tables... or a slide rule, close at hand.
Maurice L. Hartung writing in 1960 in the preface
to "How to Use Log Log Slide Rules."

The spectral density of black body radiation ... represents
something absolute, and since the search for the absolutes
has always appeared to me to be the highest form of research,
I applied myself vigorously to its solution.
-Max Planck
In Michael Dudley Sturge, *Statistical and Thermal Physics* (2003), 201.

Contents

Preface ... xiii
List of Figures ... xxiii
List of Tables ... xxvii
Authors ... xxix

SECTION I The blackbody problem

Chapter 1 Thermal radiation and the blackbody problem ... 3

1.1 Towards a solution to the blackbody problem ... 4
1.2 Planck and the blackbody problem ... 9
1.3 The work of the experimentalists ... 12
1.4 Thermal laws from dimensional analysis ... 16
1.5 Transition and new beginnings ... 26

SECTION II Theoretical and numerical matters

Chapter 2 Theoretical developments ... 33

2.1 Spectral representations ... 33
2.2 Two important special functions ... 36
2.2.1 Polylogarithms ... 36
2.2.2 The Lambert W function ... 39
2.3 Two common spectral scales used to represent blackbody radiation ... 41
2.4 Other spectral scale representations ... 46
2.5 Ephemeral spectral peaks ... 52
2.6 Logarithmic spectral scales ... 57
2.7 The radiometric and actinometric cases ... 58
2.8 Normalized spectral exitance ... 61
2.9 The Stefan–Boltzmann law ... 62
2.9.1 The traditional approach ... 63
2.9.2 A polylogarithmic approach ... 66
2.10 Fractional functions of the first kind ... 68
2.11 Fractional functions of other kinds ... 73
2.12 Centroid and median wavelengths ... 77

2.13 The standard probability distribution and cumulative probability distribution functions for blackbody radiation 83
2.14 Infrared, visible, and ultraviolet components in the spectral distribution of blackbody radiation 86

Chapter 3 Computational and numerical developments 97

3.1 Approximations to the spectral exitance 97
3.1.1 The laws of Wien and Rayleigh–Jeans 97
3.1.2 Extended Wien and Rayleigh–Jeans approximations 100
3.1.3 Polynomial interpolation and logarithmic correction factors 101
3.1.4 Laurent polynomials and non-rational approximations of Erminy 103
3.2 Computation of the fractional function of the first kind 107
3.2.1 Series expansion methods 108
3.2.1.1 Large arguments 108
3.2.1.2 Small arguments 114
3.2.1.3 Division point 116
3.2.2 Approximation of the integrand first 118
3.2.3 Gauss–Laguerre and generalised Gauss–Laguerre quadrature 119
3.2.4 Asymptotic expansion 123
3.2.5 Other methods 126

Chapter 4 Blackbody sources and basic radiometry 131

4.1 Blackbody sources 131
4.2 Goniometric characteristics of surfaces 133
4.3 Inverse square law 137
4.4 Extended source radiometry 139
4.5 Radiometry of images 143
4.6 Example problem 152

SECTION III Computational aids

Chapter 5 Nomograms and graphs used for thermal radiation calculations 157

5.1 Nomograms 157
5.2 Graphs 170
5.3 The legacy of graphical aids 180

Chapter 6 Slide rules used for thermal radiation calculations 183

6.1 Three early slide rules from Germany, England, and the United States 184
6.1.1 The System Czerny rule 184
6.1.2 General Electric Radiation Calculators 191
6.1.3 The Admiralty rule 202
6.2 Two mysterious rules 210
6.3 The DENEM Nuclear Radiation Calculator 212
6.4 A circular slide rule 215
6.5 "Do-it-yourself" slide rules and charts 220
6.6 Radiation slide charts from the 1960s 222
6.6.1 The Block rule 222
6.6.2 The Infrared Slide Rule 226
6.7 The last of the real slide rules 229

Chapter 7 Tables used for thermal radiation calculations 243

7.1 Notational conventions used for tables 246
7.2 From the earliest tables to 1939 247
7.3 Tables from 1940 to 1954 260
7.4 Tables from 1955 onwards 270
7.5 Tables then and now 296

Chapter 8 Beyond the analogue aids of old 301

Appendix A Miscellaneous mathematical results 307

A.1 Series expansion for the Lambert W function 307
A.2 Bernoulli numbers and the Riemann zeta function 308
A.3 Number of real roots to an equation 312

Appendix B Computer program for the fractional function of the first kind 317

Appendix C Chronological table of events 321

References 329

Reference Author Index 367

Index 375

Preface

It is often said mathematics is the queen of all the sciences. Computation, then, is its most loyal and humble servant. Computation has always been an integral part of science and until quite recently was the sole domain of the human computer. As anyone who has ever been required to do so can confirm, computation by hand is a slow and error prone task. Not surprisingly, much effort went into the creation of a number of computational strategies and mechanical aids designed to relieve the burden imposed by such work. Before the advent of the digital computer the speed and ability with which calculations could be made often placed very real human limits on the types of problems one could realistically hope to solve. Nowadays, the swiftness and ease with which calculations can be made makes it all too easy to forget a time, not that long ago, when even relatively simple calculations were a minor undertaking. Who today pays much attention to how the calculations we seek are made, as long as they can be made? Fast is all that matters.

Until computers became affordable and widely available in the mid-1970s, escape from the computational mire for many scientists and engineers came in the form of a number of surprisingly simple and unassuming aids. The oldest and perhaps most well-known examples are those of tables. One conservative estimate puts tables as having been with us for some 4500 years, and having been used as the main aid to calculation for at least the last two millennia. They were, however, not the only aids developed and used. In addition to tables, by the late nineteenth century graphical aids such as nomograms and graphs, and mechanical aids such as slide rules and calculating machines, had become available and were beginning to play an important role in aiding computation. Collectively their importance lay in the substantial time savings that were to be had from their use. No longer burdened by having to perform many tedious and time consuming calculations, it allowed scientists and engineers to focus their time and attention on the real problem at hand.

At its heart, this book is about computation. It focuses on those methods used for one very specific, yet historically important, problem — that of *blackbody* radiation. In doing so, we examine the often overlooked and largely forgotten computational aspects relating to an important area of science and engineering before and after the coming of the digital age. We develop and discuss many of the mathematical techniques and methods that have been developed and continue to be applied to evaluating quantities associated with blackbody radiation. The book also concerns itself with the many aids, such as nomograms and graphs, slide rules, and tables, that were produced, together with their more recent digital arrivals that continue to be developed to aid calculations encountered in this field. The story of these aids, from their initial inception until what for many was the time of their final demise at the hands of the impending digital age, forms a central part of the narrative we attempt to tell.

The genesis of this book lies in an article the first author wrote in 2011 entitled “Blackbody radiation functions and polylogarithms.” The paper concerned itself with a way of writing many of the functions associated with blackbody radiation in terms of a special function known as the polylogarithm function. In setting the stage for such work, its short introduction attempted to place the importance of these functions within their proper historical context. In so doing he was to discover that in the not too distant past, many slide rules and extensive sets of tables had been developed and used for finding values to these and other closely related functions. Intrigued, he resolved to further explore these “analogue” aids of old once time permitted. His searches with requests for information led him to the second author who, back in the day, had been a user of such aids. Still in possession of many examples, a fruitful correspondence ensued with the end result being the book you now have before you.

The second author began using radiation slide rules in the late 1960s for the development of infrared systems for medical, industrial, and military applications. In the early 1970s he was part of a group studying the dynamic behavior of materials of interest in the ballistic missile defence field. As this required untold millions of evaluations of Planck’s equation, it led him to investigate the possibility of using Gauss–Laguerre quadrature to significantly reduce the amount of computational time required in order to integrate Planck’s equation over any specified spectral band. It worked, and resulted in significant savings in time per each run. Four decades later, he still keeps a radiation slide rule nearby that he continues to use for quick estimates when a computer is not immediately available but an answer is needed.

Historically, thermal radiation, the radiation emitted from an object due to its temperature alone, not only played a pivotal role in the development of modern physics, particularly in what later became known as quantum mechanics, but technologically its importance over time grew as more and more applications begun to emerge. Reflecting the growing paramountcy of the technology, many mathematical attempts were made to find efficient means for the evaluation of the most common quantities associated with thermal radiation, while in time, a large array of aids would eventually become associated with the field. The latter were designed to help assist in the growing computational demands being placed on the working scientist and engineer. In their day, they were recognized as an important labor saving device and were widely used. While they may only represent a small aspect of the scientific enterprise, in most accounts until now, they are either relegated to a mere footnote, or entirely ignored altogether.

By the last decade of the nineteenth century, driven largely by growth in industries such as electrical lighting and photography, the study of radiation emitted from an object due to its temperature alone had become a subject of intense scientific investigation. The occurrence of very hot objects changing color as their temperature increases is familiar to most. For example, a solid iron rod glows a dull red color at a much lower temperature than when it is glowing a bright, yellowish-white color. The technological problem faced by the electric lighting industry at the turn of the twentieth century was one of efficiency. How was the maximum amount of visible light emitted from an object to be achieved using the least amount of energy? It was a question whose answer ultimately depended on a sound theoretical understanding of the laws of thermal radiation. Knowing the body that emits the greatest amount of radiation compared to all other bodies at the same temperature is of considerable importance. Knowledge of such a body's characteristics allows it to serve as the ideal theoretical standard against which all real radiating bodies can be compared. Known as a blackbody, it was first introduced as a theoretical concept by the German physicist Gustav Robert Kirchhoff in 1860. A further forty years would pass before a correct mathematical description of the radiation emitted by a blackbody was given final form by Max Planck in 1900. Embodied in his now famous law for thermal radiation, calculations made using Planck's equation is a slow process owing to its particularly cumbersome mathematical form. Planck's equation, and those functions closely related to it, thus readily lend themselves to being given a graphical, mechanical, or numerically tabulated form for their evaluation.

This book attempts to give a comprehensive account of the development of Planck's equation, and other closely related functions, together with the many methods used in their evaluation. Due importance is paid to computational techniques and the various aids developed to facilitate such calculations. To help achieve this goal, the book has been divided into three parts. In the first, which consists of Chapter 1, thermal radiation and the blackbody problem is introduced and discussed. It examines early developments made by the experimentalists and the theoreticians as they strove to understand the problem of a blackbody. Introduced here are all the well-known early "classical" laws discovered before 1900 which one today associates with thermal radiation such as the Stefan–Boltzmann law, Wien's homologous law, and Wien's displacement law. Though each law antedates Planck's law, each is a consequence of the latter's law. In attempting to demonstrate their essentially classical character, techniques from dimensional analysis are introduced and used to show how these laws follow directly from properties of their basic physical dimensions.

The second part of the book concerns itself with a number of theoretical developments that stem directly from Planck's law, and the various computational matters that arise when numerical evaluation is required. Some basic elements of radiometry that tie together and use many of the theoretical and computational ideas developed are also considered. These matters are taken up and developed in Chapters 2 through 4.

The different spectral representations that can be used to represent Planck's law are often not fully appreciated. The selection of a particular spectral representation has an important affect not only on the shape of the spectral distribution curve, but also on its corresponding peak location. Chapter 2 begins with a general discussion on spectral representations for continuous radiation sources such as a blackbody and the consequences this has on the form of Planck's law. The development of Planck's equation is also considered in this chapters as are the all important fractional functions of the first and second kinds which result when Planck's equation is integrated between finite limits. For the latter functions, they are expressed in closed form in terms of a special function known as the polylogarithm function and is done using the recently introduced polylogarithmic reformulation for the blackbody problem.

Chapter 3 focuses on the various computational methods developed to obtain numerical values from the various quantities associated with thermal radiation. Here all the well-known approximations, together with many of the not so well-known approximations to Planck's law, are introduced and considered while various methods used to evaluate the fractional functions such as rapidly converging infinite series, Gauss–Laguerre quadrature, integrand approximations, and asymptotic expansions methods are developed and analyzed. Concepts as they pertain to basic radiometry and its measurement methods are introduced in Chapter 4. The latter makes use of many of the ideas developed in the preceding two chapters and is used as an example to show how the forgoing theoretical and computational developments can be used in practice.

Shortly after Planck introduced his law, early users of his equation were quick to discover how overwhelming the task of having to perform many calculations by hand can be. Unsurprisingly, it was not long before the first computational aids designed to alleviate this burden first started to appear. These quickly grew in importance and number as their general utility was realized. Widely used, they would go on to relieve generations of scientists and engineers alike from the onerous task of human computation in an age before digital computers existed, and even after they did, before digital computers became widely available. These aids are the subject of the third part of the book which are explored in Chapters 5 through 8.

These individuals or groups of individuals who embarked on creating the various computational aids used for thermal radiation calculations and what drove them on is often just as interesting as the aids they produced. Much effort, first human then computer, went into the creation of these aids. It was laborious work, and for their respective creators, lacked the prospect of any new physical or technological insight being gained from their production. And yet aids of the highest quality continued to be made for the better part of the twentieth century. An absence of any reputation to be gained as a result of their production usually meant their creation was undertaken on an ad hoc basis rather than as a result of any general desire to produce such aids. Many tables, for example, were created in this way as the by-product of other work, while the first of the radiation slide rules only came about when a need to quickly estimate the integral of Planck's equation arose due to the pressures of World War Two. Occasionally the task of creating specific aids was carefully planned in advance. Most notably, tables appearing in book form were usually produced in this way. To their credit, many saw the creation of aids as important work, and if it had not been for their collective efforts, we would have not been left with as rich a computational legacy as the one we now have.

The greatest need for computational aids designed to assist in calculations involving thermal radiation would not come until the technological importance of the infrared spectrum was fully realized. At first there seems to have been little interest in infrared systems. After Theodore Case developed the first infrared system in 1917 for ship-to-ship communications, his work remained largely ignored and forgotten until the mid-1930s when the drums of war began to distantly play once more. As part of his postgraduate work in Germany, Edgar Kutzscher discovered that several classes of lead salt materials could be used for infrared detection. His discoveries lead to a variety of German military applications. At about the same time, researchers in both England and the United States were pursuing similar developments. The coming of the Second World War, however changed everything. With their obvious military applications, by the war's end the importance of utilizing detection devices capable of operating in the infrared portion of the electromagnetic spectrum had been firmly established. Mirroring these developments, the number of new aids produced in the years during and immediately following the war steadily grew.

While several different types of slide rules and a number of nomograms and graphs appeared, it was in the production of tables where the greatest number of aids for thermal radiation were to be found. Chapter 7 is devoted to a discussion of these aids. They appeared surprisingly quickly, just a few years after Planck first introduced his equation. For the most part, these early tables consisted of direct tabulations of Planck's equation as a function of wavelength and temperature. The production of these early tables was driven largely by the needs of those working in the field of illumination, such as photometrists and colorimetrists and it would be several decades before the first tables produced specifically for those working in the infrared emerged.

During the early part of the twentieth century, values for the fundamental constants of nature were still very much in a state of flux. Early tables quickly became out of date with each new revision in the values for the fundamental constants. To overcome the problem of frequent and continual changes in the values of these constants, tables in dimensionless form started to appear. Since their values no longer depended on the values used for the fundamental constants, tables of this type had a considerable advantage over previously produced tables that were not dimensionless. While the longevity of tables produced in this manner was assured, compared to tables which gave the quantity of interest directly, a number of additional calculations from the dimensionless tabulated value had to be made before the quantity of final interest could be found. Despite the uncertainties in the values for the fundamental constants, by the late 1920s the importance of blackbody radiation had been firmly established. Demand for tabulations of these functions increased and remained high for many decades to come. From the earliest of times, tables for these functions were included in a number of the more widely available handbooks and general texts on tables. Later, tables relating to thermal radiation were included in many discipline-specific texts and they remain one of the few places today where tabulations for these functions continue to be found. Of all the aids, tables dominated the computational aid landscape for thermal radiation in terms of total number produced from the time of their inception at the beginning of the twentieth century until the late 1970s when they were finally displaced by the arrival of affordable digital computers.

The advent of high-speed digital computers in the 1950s changed the task of table making but did not, at least initially, remove the need for tables altogether. While the digital computer may have shifted the computational effort from slow and error prone human computers to fast and accurate digital machines, they remained extraordinarily rare, and access to them was limited to the very few. A decade later, as mainframe computers and minicomputers slowly began their inextricable march through the universities, government laboratories, and scientific agencies of the industrialized world, turnaround time remained long, with a day to three days not being uncommon. Coinciding with this period, infrared technology was rapidly evolving and drove the need for increasingly accurate values for many blackbody radiation functions.

As the breadth of infrared system applications continued to grow throughout the 1960s, a marked increase in the number of new tables produced during this period is found. From the mid-1950s until the late 1970s many tables, and often for very specific purposes, appeared. On account of being computer rather than human generated, tables produced during this period also differed from their predecessors in two important ways. The first was in an increase in the number of significant figures used. Values tabulated between seven to nine significant figures were not uncommon. The second was in their sheer physical size. Large extensions in the range of values used for the independent variables coupled with smaller interval sizes resulted in some tables running to unprecedented numbers of pages. Finding tables produced during this period with anywhere from a hundred to several hundred pages is not unusual. During this time, the vast majority of the tables produced were published as either technical reports or in book form. Unlike many of the tables which appeared as technical reports, those published as books were available to a far wider and more general audience. Books of tables prepared by Marianus Czerny and Alwin Walther, and by Mark Pivovonsky and Max R. Nagel, proved to be incredibly popular. Each had their origin in Germany, appeared in 1961, were highly cited in the literature, and went on to have a lasting influence in those fields where they were put to widest use.

Beginning in the mid-1920s the very earliest of the nomograms start to appear and that is the subject of Chapter 5. The nomogram was a relatively recent addition to the class of graphical methods used to aid calculation, having only been developed in the 1880s. Compared to early tables, where values for Planck's equation were typically given to three or four significant figures, nomograms were only suitable for order-of-magnitude estimates. Their advantage lay in their ability to provide rapid estimates without the need of having to interpolate between adjacent values found in tables. A number were produced, and the inventiveness displayed in some is quite remarkable. Graphs from which values for various quantities relating to thermal radiation could be estimated also start to appear around this time and would go on to be produced for many years to come. Often found in texts, like nomograms, they were again only suitable for order-of-magnitude estimates. Many more graphs appeared, but as we shall see, are far less versatile as a graphical aid compared to those of nomograms.

The final years of the Second World War saw the development of the first of the radiation slide rules and is the subject of Chapter 6. By this time the slide rule was already a very old invention. The English mathematician and clergyman William Oughtred is generally credited as its inventor in 1622, however, it would not be until the 1850s that usage in countries such as England, France, and Germany started to become common. By the turn of the twentieth century, many general and special purpose slide rules were in widespread use across the industrialized world. Motivated by military needs the German experimental physicist Marianus Czerny designed and constructed the first of the radiation slide rules in 1944. In the immediate decades after its introduction, the radiation slide rule was widely adopted by those working in the infrared and recognized as a useful and important tool for engineers and scientists alike. The accuracy of the slide rules varied considerably, but it was not uncommon to achieve estimates of one per cent or better from such devices. More accurate than a nomogram and far quicker to use than tables meant that in the early decades of infrared technology, the radiation slide rule was the definitive "workhorse" for a generation of those involved in infrared systems design and engineering. By the latter half of the 1970s, with the appearance of affordable, hand-held programmable calculators and digital desktop computers, the radiation slide rule went into steep decline and all but disappeared within a few short years.

Collectively, computational aids such as nomograms and graphs, slide rules, and tables made more rapid development possible at a time when work requiring excessive computation could severely limit the rate of progress. They had a profound impact on the field of illumination from the early 1920s onwards, and on the entire infrared industry during its early growth years, beginning in the 1950s. Beyond radiometry, photometry, colorimetry, and optical pyrometry, they also played a more modest role in other related fields where a need for such things occasionally arose. These were mostly in the form of tables produced for and by those working in the fields of astrophysics or meteorology.

Astrophysical work in the early 1930s was one area where a need for suitable computational aids arose. Stemming from his work on planetary nebulae and stellar temperatures, tables for the integral of Planck's equation over a finite spectral band were devised by the Dutch astronomer Herman Zanstra in 1931. Often, early aids produced in one field went for many years unnoticed by those working in other fields as was the case with the tables prepared by Zanstra. Tables for the astrophysical and meteorological communities also tended to differ from many of the tables produced for work in infrared or illumination work. Tables intended for astrophysical use were prepared to very-high values for the temperature while exceptionally narrow terrestrial temperature ranges with far smaller interval sizes were the norm for tables intended for meteorological use.

By the late 1970s the growing computational abilities offered by relatively cheap and widely available digital computers meant that there no longer existed a need for most of the computational aids of old. Calculations formerly performed by the use of graphical, mechanical, or numerically tabulated means were rapidly replaced by computations that could now be performed on demand by the user using their own programmable calculator or personal computer. Their use rapidly declined and within the space of a few years most analogue aids had became antediluvian. A number of digital aid replacements did however emerge. Most were in the form of either online calculators, or programs for particular programming languages or certain electronic devices, with the best of these frequently inspired by their analogue predecessors. It is this recent and continually unfolding story we take up and tell in Chapter 8.

Assembling the many historical documents and artefacts considered in parts of this book would not have been possible without the gracious help of others. Special thanks are extended to Prof. William L. Wolfe, who kindly supplied photographs of the now very rare Aristo Nr. 10048 slide rule and its prototype, to Ms Nina Senger-Mertens from Arithmeum in Bonn who provided the photograph of the equally rare Aristo Nr. 922 rule, and to Prof. John E. Greivenkamp from the University of Arizona for providing photographs of the Vahlo slide chart. We are also grateful to Barbara Grant from *Lines and Lights Technology* for providing copies of the EG&G Judson and the Engineering Summer Conferences Infrared Radiation Calculators, to Chari Gallen from Teledyne Judson Technologies and Gary Wilson from BAE Systems for providing copies of their company's latest Infrared Radiation Calculators gratis, to Brent Lindstrom, Sales and Marketing Director at Electro Optical Industries in Santa Barbara, for providing a number of beautiful specimens of their commemorative Blackbody Radiation Slide Rule, and to Steven Peters who not only drew our attention to the recent Infrared Radiation Calculator from BAE Systems but also answered a number of questions regarding his Blackbody Radiation Calculator app for Android.

Locating copies of some of the more obscure documents cited in this work was a particular challenge. Many individuals met requests for copies of tables either they or their institutions had produced. They were, in no particular order: Prof. Stephen M. Robinson from the University of Wisconsin-Madison, Prof. Yalçın A. Göğüş from the Middle East Technical University in Turkey, Perry Abramowitz from RAND Corporation, Julie Niesen from the Nelson Institute Center for Climatic Research at the University of Wisconsin-Madison, Sharon Wilson from the National Physical Laboratory in the UK, Steven D. Vanstone from Redstone Arsenal in the United States, and Sugiura Yoshio from the National Institute of Advanced Industrial Science and Technology in Japan. All are gratefully acknowledged. We thank Shingo Ushida from the Japan Meteorological Agency for providing high-quality copies of two nomograms reprinted in this book. We would also like to thank the following individuals for their recollections of using radiation slide rules: Prof. William L. Wolfe, Prof. Hans J. Queisser, Prof. Raymond J. Chandos, Mr Max J. Riedl, Mr Jack R. White, and Mr Arthur Cussen and Mrs Marjorie Cussen. These have been invaluable. Finally, we are both indebted to the Deputy Editor-in-Chief of the *Journal of Infrared and Millimetre Waves*, Dr Hong Shen, who drew our attention to relevant work published in the Chinese literature.

The first author would like to extend his gratitude to the College of Engineering, Technology and Physical Sciences at Alabama A&M University in Huntsville who hosted him for two weeks during the summer of 2015 as the final draft of the manuscript for the book was being completed. Support from the first author's home institution in the form of a small grant for such an undertaking is also gratefully acknowledged.

A book of this nature readily leads itself to the inclusion of many figures, particularly figures from the original sources. We have made every effort to determine original sources and obtain permissions for the use of these illustrations where necessary. Credit is to be found in the figure captions. Figures and photographs created by the authors are typically not given any particular credit. While all reasonable effort has been made to contact the holders of copyright material reproduced in the book, any omissions will be rectified in future printings if notice is given to the publisher.

Seán M. Stewart
Astana

R. Barry Johnson
Huntsville, Alabama
January, 2016

List of Figures

2.1 Lambert W function for real arguments 40
2.2 Spectral radiant exitance in the linear wavelength representation as a function of wavelength 44
2.3 Transformation procedure used by Langley to obtain the "normal" spectrum 50
2.4 Thermal spectra drawn by Langley in the 1880s using four different spectral representations 51
2.5 Wien peaks as a function of spectral scale parameter 54
2.6 Normalized spectral radiant exitance for a blackbody as a function of the wavelength–temperature product 62
2.7 Normalized spectral radiant exitance for a blackbody as a function of the frequency–temperature quotient 63
2.8 Factional functions of the first kind as a function of the wavelength--temperature product 72
2.9 Fractional functions of the second kind as a function of the wavelength–temperature product 76
2.10 Normalized fractional centroid peak as a function of the wavelength–temperature product 80
2.11 Standard probability distribution function for blackbody radiation 86
2.12 Fractional spectral components radiated by a blackbody in the visible, infrared, and ultraviolet regions of the spectrum 89
2.13 Optimal temperature maximising the amount of radiation radiated by a blackbody into a finite waveband 90

3.1 Rayleigh–Jeans and Wien approximations and extended approximations 102
3.2 Various approximations to Planck's equation 107
3.3 Fractional error that results when Gauss–Laguerre quadrature is used to estimate the fractional amount. 121

4.1 Representative cavity geometries for blackbody sources 132
4.2 Geometric definition of solid angle 134
4.3 Parameters to determine solid angle 134
4.4 Radiation pattern for a point source 135
4.5 Illustration of Lambert's Law 135
4.6 Determination of total power radiated into a hemisphere 136
4.7 Irradiance at a plane due to a point source 138
4.8 Irradiance at an observational plane due to a finite size source 138
4.9 Effect of a lens on the inverse square law 139

4.10 Apparent radiant intensity for an extended source ... 140
4.11 Irradiance at a receptor ... 140
4.12 Extended source radiometry ... 141
4.13 Irradiance at receptor due to a circular source ... 142
4.14 Irradiance at receptor due to an extended source where the solid angle is determined by a circular hole. ... 143
4.15 Irradiance at receptor when a lens is located between extended source and circular hole. ... 143
4.16 Irradiance at receptor when a lens is located between a circular hole and receptor. ... 144
4.17 Numerical aperture of a lens. ... 144
4.18 Radiometry of an aplanatic lens ... 145
4.19 Off-axis irradiance for imaging systems ... 146
4.20 Extended source imaging larger than the field-of-view of the detector ... 147
4.21 Visual radiometry. ... 148
4.22 Radiometry of a telescope. ... 149
4.23 Radiometry of a lens for any axial observation point. ... 150
4.24 Radiometry of a beacon for any axial observation point ... 151
4.25 Relationship of image height and object angular height. ... 151
4.26 Radiometry of a telecentric lens. ... 152
4.27 Determination of irradiance at a detector due to a Lambert source . 153

5.1 A very early nomogram devised in 1926 by Walsh for estimating the spectral radiant exitance of a blackbody ... 160
5.2 A simple early nomogram based on the calculation of the Stefan–Boltzmann law given by Lax and Pirani in 1929 ... 161
5.3 An extension of Miduno's nomogram made by Terada in 1939 to estimate the spectral radiant exitance of a blackbody ... 162
5.4 A second nomogram devised by Terada to estimate the ratio of the spectral radiant exitance of a blackbody at the spectral peak and the integrated fractional amount ... 163
5.5 A simple alignment chart devised by Béhar in 1940 to estimate the spectral radiant exitance of a blackbody ... 165
5.6 A nomogram devised by Chernoch in 1957 for the purpose of simplifying many thermal radiation calculations ... 166
5.7 A very simple nomogram given by Hackforth in 1960 ... 168
5.8 A late and unusual ladder nomogram devised by Zanker in 1978 ... 169
5.9 Spectral radiant exitance isotherms for blackbody radiation depicted using a double logarithmic scale ... 171
5.10 An example of a common combined graph for the normalized spectral radiant exitance and the fractional amount for blackbody radiation ... 172
5.11 An unusual looking graph devised by Omoto in 1936 for estimating the spectral radiant exitance of a blackbody ... 173

5.12 A modification on Omoto's multiscale design given by Sala in 1986 for estimating the spectral radiant exitance of a blackbody 174
5.13 An uncommon graph devised by Rutgers in 1958 for estimating the total radiant exitance of a blackbody 175
5.14 An alignment chart devised by Gill in 1957 for estimating the temperature derivative of the spectral radiant exitance of a blackbody.. 176
5.15 An uncommon graph devised by Kiele in 1941 for estimating the absolute gradient of a blackbody as a function of wavelength and temperature 177
5.16 A three-dimensional plot for the spectral radiant exitance of a blackbody as a function of both the wavelength and temperature.... 179
5.17 The first attempt at a three-dimensional plot for the spectral radiant exitance of a blackbody as a function of the wavelength and temperature made by Crova in 1880 180

6.1 Czerny's first hand-built radiation slide rule 185
6.2 Front face of the Aristo Nr. 10048 – Rechenschieber für Temperaturstrahlung from the late 1940s 186
6.3 Front face of a rule believed to be a late prototype for the *System Czerny* radiation slide rule 188
6.4 Front face of the Aristo Nr. 922 – Rechenstab für Temperaturstrahlung from the late 1950s 190
6.5 Front and back faces of the General Electric slide chart GEN-15 192
6.6 Front and back faces of the General Electric slide chart GEN-15C .. 198
6.7 Front faces for four different Infrared Radiation Calculators made by Perrygraf for Sensors, Inc., Engineering Summer Conferences, EG&G Judson, and IRIA between 1975 and 1986 201
6.8 Front faces of two recent Infrared Radiation Calculators made by Perrygraf for Teledyne Judson Technologies, LLC, and BAE Systems 202
6.9 Front face of the F5100 radiation slide rule made by A. G. Thornton Ltd for the Admiralty Research Laboratory at Teddington, England 206
6.10 Front and back faces of the Photon Detector Slide Rule made for Vahlo 212
6.11 Front and back faces of the DENEM Nuclear Radiation Calculator . 213
6.12 Front face of the Autonetics Photon Calculator made for North American Aviation, Inc. in the late 1960s 218
6.13 Illustration of Reid's do-it-yourself sliding template devised in 1960 222
6.14 Front and back faces of the Radiation Calculator made for Block Associates, Inc. in 1962 225
6.15 Front faces of two different Infrared Slide Rules made for Infrared Industries, Inc. in the mid-1960s 229
6.16 Front and back faces of the Pickett Model 17 radiation slide rule made for Electro Optical Industries, Inc. in 1971 233

6.17 Front and back faces of the commemorative rule made for Electro Optical Industries, Inc. in 2014 as part of their fiftieth anniversary celebrations since the founding of the company 237

7.1 A typical example of a set of tables produced during the early table making era 262

List of Tables

1.1 Physical variables and fundamental constants for the problem of finding the form for the total radiant exitance. 19
1.2 Exponents for the fundamental dimensions corresponding to the five dimensional quantities related to finding the form for the total radiant exitance. 20
1.3 Physical variables and fundamental constants for the problem of finding the form for the spectral radiant exitance. 21
1.4 Exponents for the fundamental dimensions corresponding to the six dimensional quantities related to finding the form for the spectral radiant exitance. 22

2.1 The first few Eulerian numbers 38
2.2 Eight common spectral scales used to represent blackbody radiation. 48
2.3 Wien peaks for several common spectral representations. 55
2.4 Wien and centroid peaks for six common spectral representations 79
2.5 Equivalent spectral scales corresponding to eleven different percentile amounts 83
2.6 Fraction of the total radiant exitance found at the Wien peak 83
2.7 Portion of radiation radiated by a blackbody into the infrared, visible, and ultraviolet portions of the spectrum for temperatures ranging from 500 K to 15 000 K. 88

3.1 Number of terms needed to be summed for nine significance figure accuracy in the fractional function $\Phi l_s(z)$ (exponential form) 113
3.2 Number of terms needed to be summed for nine significance figure accuracy in the fractional function $\Phi l_s(z)$ (Bernoulli form). 117
3.3 Table of values for the radiometric fractional function comparing those obtained exactly with those obtained using the first two terms from its asymptotic expansion 127

Authors

Seán M. Stewart is an Associate Professor at Nazarbayev University and previously was at The Petroleum Institute for over a decade. Dr. Stewart has written numerous technical articles in the infrared field and developed the polylogarithmic formulation to the blackbody problem described in this book. He is a Member of the Institute of Physics, the Australian Institute of Physics, the Optical Society of America (OSA), and SPIE – The International Society for Optics and Photonics.

R. Barry Johnson, a Senior Research Professor at Alabama A&M University, has been involved for over 40 years in infrared technology. Dr. Johnson developed the Gauss–Laguerre quadrature method of integrating Planck's equation. He is an SPIE Fellow and Life Member, OSA Fellow, an SPIE past president (1987), has been awarded many patents, published numerous technical articles, and was awarded the 2012 OSA/SPIE Joseph W. Goodman Book Writing Award for *Lens Design Fundamentals, Second Edition.*

Section I

The blackbody problem

1 Thermal radiation and the blackbody problem

All objects emit radiation as a result of their temperature. At room temperature an object does not appear self-luminous, as almost all the radiation emitted is in the infrared portion of the electromagnetic spectrum.[1] As the temperature increases, the emitted radiation becomes visible, first appearing as a faint, almost colorless grey glow[2] before becoming a deep, dark, blood red in color. Continuing to increase the temperature further, the glow gradually changes color from a dull red to a bright red, before appearing yellowish and finally white. At still higher temperatures its color would change from a white glow to one which appears bluish-white in color.

Known as thermal radiation, the body which radiates the greatest amount of energy compared to all other bodies at the same temperature is referred to as a blackbody. The German experimental physicist Gustav Robert Kirchhoff (1824–1887) was the first to recognize the important theoretical role blackbodies played in understanding the emission of radiation from real bodies. An understanding of the nature of blackbody radiation was one of nineteenth-century physics' crowning achievements and paved the way to the quantum revolution that followed during the early part of the twentieth century.

A blackbody is defined as a body which absorbs all thermal radiation incident upon it. The name given to the body is appropriate.[3] Provided a body is not hot enough so as to appear self-luminous, as no radiation is reflected by the body, it appears completely black. The complete absorption of radiation by a blackbody holds true for radiation at all wavelengths and for all angles of incident upon the body. For a blackbody in thermal equilibrium with its surroundings it means that all radiation received through absorption must be emitted if its temperature is to remain constant as there is no other mechanism available to the body to lose energy without a corresponding increase in its temperature. A blackbody therefore radiates more energy per unit time in any given wavelength interval and more total energy per unit time over all wavelengths than any other body at the same temperature and is independent of the nature (its size, shape, or material from which it is made) of the radiating body. The radiation from a blackbody is referred to as *blackbody radiation*. By the late nineteenth century, the problem of the blackbody had become one of finding a mathematical expression that could describe the amount of energy emitted within a given spectral range as a function of both wavelength and temperature.

Not surprisingly it was Kirchhoff, having first proposed the blackbody problem, who made the first serious attempt at its solution. In the winter of

1859–60, following a simple yet brilliant line of physical reasoning based on the second law of thermodynamics, he postulated that the radiation emitted from a blackbody within a given wavelength interval is a universal function depending only on the temperature of the body [354, 355]. It was an important first step, and constitutes what today is known as *Kirchhoff's law of thermal radiation*, but there was still a long way to go. Kirchhoff did not give the mathematical form for the universal function associated with blackbody radiation but understood its importance and suspected it must be relatively simple.[4] Its universal character did however, ensure that the problem remained an important one in need of a solution. Kirchhoff left the job of determining the final form to others. The task they inherited, however, proved far more difficult than even Kirchhoff himself could have anticipated. Its solution, when finally found, shook the very foundations of physics and ultimately led to the development of a completely new branch of physics which today we know as quantum mechanics.

In this chapter we consider a number of events that historically were important in understanding the nature of the blackbody problem. The chapter sets the scene for understanding the wider problem of blackbody radiation by introducing quantities central to its description and the laws it obeys. These, in turn, will be discussed in greater detail in the chapters to follow. The important interplay to be found between theory and experiment resulting from the various attempts made towards arriving at a final solution to the problem are traced out and considered. Along the way many of the well known names associated with the early history of the field are introduced. The chapter closes with an examination of two important laws for thermal radiation, which while initially found using arguments based on classical physics and thermodynamics, will be arrived at using arguments based on a powerful though often overlooked technique known as dimensional analysis.

1.1 TOWARDS A SOLUTION TO THE BLACKBODY PROBLEM

By the close of the nineteenth century the exact mathematical form for the universal function of blackbody radiation continued to allude scientists. But it was not from a want of trying. By the late 1880s a number of theoretical attempts to derive the form for Kirchhoff's universal function had been made. The earliest of these was perhaps that of the German physicist Eugen Cornelius Joseph von Lommel (1837–1899) in 1878, then holder of the chair in experimental physics at the Friedrich-Alexander-Universität Erlangen-Nürnberg [394]. Lommel sought to understand, using mechanical principles, absorption; a process he saw as an interaction between light waves and the atoms of a body; and was convinced Kirchhoff's law of absorption followed from his theory. After Lommel, other attempts quickly followed [173] but advanced little in the search for the ultimate form for Kirchhoff's universal function.

Like Kirchhoff, all who approached the problem theoretically were convinced of the simplicity in form the universal function should take.

Reiterating Kirchhoff's words some thirty years later in 1889, the great nineteenth century English physicist Lord Rayleigh (1842–1919) [545] wrote "... the function ... being independent of the properties of any particular kind of matter, is likely to be simple in form," before quickly adding its expected simplicity had led to "... speculations [that had] naturally not been wanting."[5]

In the opinion of Lord Rayleigh, speculations that had been wanting were two attempts that came in the late 1880s. These were the theoretical attempt made by the Russian physicist Vladimir Aleksandrovich Michelson (1860–1927)[6] in 1887 [444, 445] and the empirical law advanced by Heinrich Friedrich Weber (1843–1912), a Professor of Theoretical and Technical Physics at the Polytechnikum, Zürich, in 1888 [677]. While neither adequately described the solution to the blackbody problem, each made a number of predictions which, at least in the case of Weber, proved correct.[7]

While these early attempts to find a correct form for Kirchhoff's universal function were quickly found to be inadequate, beyond trying to theoretically understand the spectral distribution of radiation from a blackbody with temperature, two important results relating to the radiation emitted by a blackbody would be established before the close of the century. The first of these related to the total amount of energy emitted from the surface of a blackbody at a given temperature and is today embodied in what we call the *Stefan–Boltzmann law*. It states the total energy emitted from the surface of a blackbody at temperature T per unit time per unit area in all directions into the half space above a surface is proportional to the fourth power of the temperature. Mathematically

$$M_{\mathrm{e}}^{\mathrm{b}}(T) = \sigma T^4. \tag{1.1}$$

Here $M_{\mathrm{e}}^{\mathrm{b}}$ is the total radiant exitance and is measured in units of watts per square meter [$\mathrm{W{\cdot}m^{-2}}$] while σ is the Stefan–Boltzmann constant.[8] The law is named in honor of Jožef Štefan (1835–1893) who deduced it empirically in 1879 [619][9] and Ludwig Boltzmann (1844–1906) who derived the result theoretically some five years later using arguments based on thermodynamics [92]. While the Stefan–Boltzmann law allowed the total amount of energy radiated from the surface of a blackbody at a given temperature to be found, as it gave no information about the energy radiated from a blackbody within a specific wavelength interval, it meant the form of Kirchhoff's universal function remained as elusive as ever.

The second important result to be found in the last decade of the nineteenth century related to the particular functional form Kirchhoff's universal function could take. In addition to knowing the total energy radiated by a blackbody at a given temperature, understanding how this energy is distributed throughout the spectrum as a function of wavelength is equally important. By the late nineteenth century it was experimentally known that the radiation emitted from a blackbody was spread continuously over a singly peaked spectrum consisting of all wavelengths. However, finding the mathematical form that described this spectral distribution became one of the great unsolved problems facing physics at the close of the nineteenth century.

In 1893 the recently licensed docent at the Universität zu Berlin, Wilhelm Carl Werner Otto Fritz Franz Wien (1864–1928), was able to extend the work of Boltzmann in an important way. The universal function of Kirchhoff's, for a given wavelength λ, can be identified with what is today known as the *spectral radiant exitance* $M^{\mathrm{b}}_{\mathrm{e},\lambda}(\lambda, T)$. The spectral radiant exitance will be discussed in greater detail in Chapter 2, where the sub- and superscripts appearing here will be clearly explained. Physically, the spectral radiant exitance corresponds to the amount of energy emitted by a body into a hemispherical envelope in space per unit time per unit area within the unit wavelength interval λ to $\lambda + d\lambda$. Using thermodynamic arguments together with a principle related to the change in wavelength a wave experiences as it moves relative to a source known as the Doppler effect, Wien was able to deduce theoretically [684] the important result equivalent[10] to

$$M^{\mathrm{b}}_{\mathrm{e},\lambda}(\lambda, T) = \frac{1}{\lambda^5} F(\lambda T). \tag{1.2}$$

Equation (1.2) shows that Kirchhoff's universal function, which depends on both wavelength and temperature, can be reduced to an unknown universal function F of a single variable equal to a product between the wavelength and temperature only. At first sight this may not seem very significant. After all, the final form for the unknown function F remains to be determined and would appear to bring one no closer to a solution of the blackbody problem than before. Superficially at least, it appears one unknown function has simply been replaced by another unknown function. And indeed it has, but to see it only in this light is to completely miss the point.

To understand the true significance of Eq. (1.2), in this result we see that if the spectral radiant exitance is known at one temperature its form at any other temperature can be found. Importantly, any temperature will do. So once the spectral distribution of a blackbody is known at a single temperature, Wien's result ensures the spectral distribution for a blackbody at any other temperature can be found. Graphically, if $\lambda^5 M^{\mathrm{b}}_{\mathrm{e},\lambda}(\lambda, T)$ were to be plotted against the wavelength–temperature product λT, a single spectral curve for the radiation from a blackbody results — regardless of its temperature. The result of this homologous relationship is generally known as *Wien's law* or sometimes as *Wien's general displacement law*. The suitability of either of these terms to describe the law is however, far from ideal. In order to avoid confusion with a number of other laws due to Wien that are to follow, neither of these terms will be used. Instead, the homologous relationship for the law suggests a more appropriate name would be *Wien's homologous law* and that is the name we intend to use.[11]

While Wien's homologous law was a tremendous step forward, it remained to determine the spectral radiant exitance for a blackbody at a single temperature. Of course, it could be obtained experimentally but intellectually is far less satisfying compared to a proper theoretical treatment. Throughout the late 1890s the experimentalists quickly provided many curves for the spectral radiant exitance at various temperatures to great accuracy out to wavelengths extending deep into the infrared. A correct theoretical description however, remained illusive and would have to wait until the close of the nineteenth century and the work of the great German theoretical physicist Max Karl Ernst Ludwig Planck (1858–1947).

Despite the ultimate shortcomings in Wien's homologous law, surprisingly it can still be used to deduce the Stefan–Boltzmann law. The total radiant exitance is found by summing the spectral radiant exitance over all wavelengths. Mathematically this corresponds to integrating $M^{\rm b}_{{\rm e},\lambda}(\lambda, T)$ from zero to infinity, namely

$$M^{\rm b}_{\rm e}(T) = \int_0^\infty M^{\rm b}_{{\rm e},\lambda}(\lambda, T) d\lambda. \tag{1.3}$$

Substituting Wien's homologous law into Eq. (1.3) gives

$$M^{\rm b}_{\rm e}(T) = \int_0^\infty \frac{F(\lambda T)}{\lambda^5} d\lambda = T^4 \int_0^\infty \frac{F(x)}{x^5} dx, \tag{1.4}$$

after the change of variable $x = \lambda T$ is made. The second of the improper integrals, provided it exists, is just a number since it is dimensionless and the Stefan–Boltzmann law follows with the proportionally to the fourth power of the temperature.

The special case of Wien's homologous law is often cited. If $\lambda_{\rm max}$ is the wavelength at which $M^{\rm b}_{{\rm e},\lambda}(\lambda, T)$ attains its maximum value, the peak in the spectral curve will be determined by a fixed value of λT, that is, by

$$\lambda_{\rm max} T = b. \tag{1.5}$$

Here b is a constant referred to as Wien's displacement law constant[12] and it is in this special form the law is known simply as *Wien's displacement law*.[13] The law derives its name from the fact that as the temperature increases, the peak in the curve for the spectral radiant exitance, when plotted as a function of wavelength, becomes "displaced" towards shorter wavelengths.

Wien himself in his original publication of 1893 [684] and subsequent publications of 1893 [685] and 1894 [686] on the same topic did not give either of the two laws we have associated with his name in the form given by Eqs (1.2) and (1.5). For his displacement law Wien wrote

$$\lambda \vartheta = \lambda_0 \vartheta_0, \tag{1.6}$$

where λ_0 and ϑ_0 were some fixed wavelength and temperature (here Wien used the symbol ϑ for absolute temperature in place of the modern day T) respectively and wrote:

> In the normal emission spectrum of a blackbody each wavelength shifts as the temperature changes in such a way that the product of temperature and wavelength remains constant.[14]

Wien did not come to any conclusion about the displacement of the maximum in the spectral curve which today his law has become most widely associated with, instead preferring to give a wider interpretation to the form he presents. It is likely he knew of the more limited form of the spectral maximum to which his displacement law could be applied, since at the end of his first paper of 1893 he wrote that the result found agreed with the shift in the spectral maximum correctly deduced by Weber five years earlier. Weber had used a form for the spectral distribution function of a blackbody he had deduced empirically but which, as we have already noted, later proved to be incorrect. The writing of Wien's displacement law chiefly in terms of the maximum is due largely to the extensive experimental work performed by the German experimental physicist Louis Karl Heinrich Friedrich Paschen (1865–1947) between the years 1895 and 1901 [479, 480, 481, 483, 484, 487].

On his homologous law, as presented in Eq. (1.2), Wien again did not give it in the form we have given. Instead the form Wien gives towards the very end of his first 1893 paper on the subject, in terms of the spectral energy density ϕ_λ; a quantity proportional to the spectral radiant exitance; was

$$\phi_\lambda = \phi_{\lambda,0} \frac{\vartheta^5}{\vartheta_0^5}. \tag{1.7}$$

Here $\phi_{\lambda,0}$ and ϑ_0 are fixed values for the spectral energy density and absolute temperature respectively. If Eqs (1.6) and (1.7) are combined, one obtains

$$\phi_\lambda \lambda^5 = \phi_{\lambda,0} \lambda_0^5, \tag{1.8}$$

and is constant. Obviously, as seen in Eq. (1.7), the spectral energy density for a blackbody is a function of the absolute temperature. But as Wien had already shown, the wavelength–temperature product is constant, so one may rewrite Eq. (1.8) as

$$\phi_\lambda = C\lambda^{-5} f(\lambda T), \tag{1.9}$$

for some constant C and some unknown universal function f. Note Eq. (1.9) reduces to Eq. (1.2) on setting $Cf(\lambda T) = F(\lambda T)$.

In terms of priority it was the Northern Irish physicist Sir Joseph Larmor (1857–1942) who, at the Seventieth Meeting of the British Association for the Advancement of Science in 1900, first gave the form for Wien's homologous law we have used here [375, 384, 376]. We will refer to it as the Larmor form for the homologous law. However, unbeknownst to Larmor, two years earlier an equivalent form

$$M_{e,\lambda}^{b} = T^5 \psi(\lambda T), \tag{1.10}$$

had been given by Lord Rayleigh [547], but attracted little attention, so much so it was independently given for a second time two years later by the German theoretical physicist Max Ferdinand Thiesen (1849–1936) [644]. In this form we will refer to it as the Rayleigh–Thiesen form for the homologous law. The two forms can be seen to be equivalent if we write

$$M^{\mathrm{b}}_{\mathrm{e},\lambda} = \lambda^{-5} F(\lambda T) = T^5 \left[\frac{F(\lambda T)}{(\lambda T)^5}\right] = T^5 \psi(\lambda T). \tag{1.11}$$

Wien's result, as would finally become apparent only a few years later, represented the limit of what could be determined for a blackbody using classical physics and thermodynamics alone. Wien had managed to bring one tantalizingly close to the solution of the blackbody problem, closer than anyone before him had managed to do, yet its final solution continued to remain as elusive as ever. It appeared the complete solution to the problem of blackbody radiation would only come from a radical new way of thinking about the problem. One would not have to wait long and it was to come from a theoretical physicist who also found himself working at the epicenter of research into the nature of the blackbody problem in the late 1890s. It, of course, was Planck himself.

1.2 PLANCK AND THE BLACKBODY PROBLEM

Despite what had been achieved by the close of the nineteenth century, the final form for the universal function $F(\lambda T)$ deduced by Wien remained unknown. This is not to say explicit forms for it had not been proposed. They had, but under closer experimental scrutiny all were found wanting in one way or another in their description of the radiation emitted from a blackbody over all wavelengths.

In 1896 Wien himself proposed a radiation function which initially appeared to provide a solution to the blackbody problem [687, 688] and for a time it seemed to be supported by the available experimental evidence [481]. Further theoretical support to Wien's proposed form for the distribution law of a blackbody would come from Planck himself. Using arguments quite different from those used by Wien, in a series of several long papers commencing in 1897 and culminating in 1900, Planck also managed to arrive at the same result as Wien [503, 504, 505, 506, 507, 508, 509]. *Wien's displacement law*, as it become known, took the form

$$M^{\mathrm{b}}_{\mathrm{e},\lambda}(\lambda, T) = c_1 \lambda^{-5} \exp\left(-\frac{c_2}{\lambda T}\right). \tag{1.12}$$

Here c_1 and c_2 were two unknown radiation constants to be determined.

Wien had proposed his law in June of 1896. In the same month, and quite independently of Wien, Paschen, based on his own extensive series of measurements, proposed a similar looking empirical law for the spectral distribution function of a blackbody [481]

$$M^{\mathrm{b}}_{\mathrm{e},\lambda}(\lambda, T) = c_1^* \lambda^{-\alpha} \exp\left(-\frac{c_2^*}{\lambda T}\right). \tag{1.13}$$

Here c_1^* and c_2^* were again two radiation constants to be determined while α was an adjustable parameter. Paschen found the best fit to his data was obtained when $\alpha = 5.660$.

The closeness between the two forms was convincing evidence in itself that Wien's law was correct while the later theoretical work of Planck only added to the conviction of correctness. Only with newly improved experiments for radiation at wavelengths out to an incredible 51 μm, which put them deep in the infrared, were experimentalists able to show for the first time inconsistencies between those predictions based on Wien's law and experiment [410, 411, 572, 574]. The final solution to the blackbody problem, which for a time was thought to be solved, once more found itself at the beginning of the new century the object of intense theoretical and experimental investigation.

As inconsistencies between predictions based on Wien's law and the work of the experimentalists emerged, others quickly turned their attention back to one of the nineteenth century's greatest unsolved problems. The first to do so was the German theoretical physicist Max Thiesen, who working in Berlin, was a colleague of Planck's. He made his attack on the problem in March of 1900 [644]. On the basis that any distribution function proposed must satisfy Wien's homologous law,[15] Thiesen gave the following family of solutions for the distribution law

$$\phi[x] = \phi_m \left[\frac{x_m}{x} \exp\left(1 - \frac{x_m}{x}\right)\right]^\alpha. \tag{1.14}$$

Here ϕ_m and x_m were two universal constants while the index α was a number between two and five. Setting $\alpha = 5$ Thiesen's law immediately reduces to Wien's law with the two constants appearing in Wien's formula being equal to

$$c_1 = \phi_m x_m^5 \mathrm{e}^5 \quad \text{and} \quad c_2 = 5x_m. \tag{1.15}$$

But in order to provide a better fit with the most recent experimental data, Thiesen found he had to set the value for the index α equal to 4.5. For this value his distribution law takes the form

$$M_{\mathrm{e},\lambda}^{\mathrm{b}}(\lambda, T) = c_1' \sqrt{\lambda T}\, \lambda^{-5} \exp\left(-\frac{c_2'}{\lambda T}\right), \tag{1.16}$$

where c_1' and c_2' are once again two radiation constants to be determined.

Next into the fray was Lord Rayleigh. In June of 1900, using reasoning which appeared to him to be more probable a priori, being based, as it was, on an analogy between the behavior of radiation whose wavelengths are large with the theory of sound, suggested Wien's law could be modified as follows

$$M_{\mathrm{e},\lambda}^{\mathrm{b}}(\lambda, T) = \bar{c}_1 \lambda^{-4} T \exp\left(-\frac{\bar{c}_2}{\lambda T}\right). \tag{1.17}$$

Once more $\bar{c}_1$ and $\bar{c}_2$ were two radiation constants to be determined.

Based on his analogy with sound, Lord Rayleigh had in fact arrived at the following form for the distribution law

$$M_{\mathrm{e},\lambda}^{\mathrm{b}}(\lambda, T) = \bar{c}_1 \lambda^{-4} T, \tag{1.18}$$

which he stated was valid in the long wavelength, high temperature limit. The additional exponential factor was added afterwards to ensure the final distribution function tended to zero in the short wavelength, low temperature regime.

Whether his proposed distribution law more closely represented the experimental facts Lord Rayleigh was not in a position to say but hoped "...the question may soon receive an answer at the hands of the distinguished experimenters who have been occupied with this subject" [548].[16] The final solution however, when it came in late 1900, would require a complete conceptual break in how physics up until that time had conceived energy to be. And it was to come from the man who would go on to become one of the greatest theoretical physicists of all time — Max Planck.

Planck, as we saw, first turned his attention to the problem of blackbody radiation in 1897. He had initially been drawn to the problem by the universal character of the law required by Kirchhoff's law. At the time, Planck was almost forty years old and it would have been tempting to think his best work lay behind him. Up until this point he had spent his entire scientific career investigating the second law of thermodynamics and its consequences. The prospect of him solving one of the late nineteenth century's great unsolved problems seemed remote. After three years of solid effort working on the problem, by the end of 1900, by considering the energy emitted from a blackbody is not continuous but rather discrete and only comes in amounts made up of multiples of some fundamental "quanta," Planck was finally able to give what later proved to be the correct mathematical description for the spectral radiant exitance of a blackbody valid for all wavelengths.

Planck initially achieved his correct form for Kirchhoff's universal function using what has undoubtedly become one of the most famous and productive interpolations ever made in the history of theoretical physics. By taking Wien's radiation law, a result known to be valid in the short wavelength, low temperature limit, together with Lord Rayleigh's classical result from June 1900 and known to be valid in the long wavelength, high temperature limit, and interpolating between the two, by October 1900 [510] Planck was able to arrive at, at least mathematically to begin with, a form for the radiation law that now bears his name [511, 518]. While Planck had an equation which seemed to fit all known experimental data better than any of the previously proposed distributions laws, particularly at the high temperature, long wavelength limit, it lacked physical justification. After what he later described as the most intense period of work in his life, by December 1900 he had a physically plausible theory to back his earlier result obtained through interpolation [512, 519]. Finessing his ideas the following year [513, 514, 515] it is now known as *Planck's law* for thermal radiation and has held up to all subsequent experimental scrutiny [58, 570, 672, 575, 576, 621, 151]. In modern