The Epistle of the Number
by Ibn al-Aḥdab

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The Epistle of the Number
by Ibn al-Aḥdab

The transmission of Arabic mathematics to Hebrew circles in medieval Sicily

Ilana Wartenberg

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FOREWORD

My book aims to show how Ibn al-Bannā''s famous Arabic text on arithmetic and algebra *Talkhīṣ Aʿmāl al-Ḥisāb* (A Summary of the operations of Calculation) was transmitted into Hebrew by the polymath Isaac Ibn al-Aḥdab, resulting in the extensive text *The Epistle of the Number* (אגרת המספר). My book presents the first edition of *The Epistle of the Number*, which was composed in Syracuse, Sicily, at the end of the 14th century. The Hebrew Epistle plays a pivotal role in the history of medieval Hebrew mathematics because it is the first known Hebrew treatise which includes extensive algebraic theories and procedures. It is also the first (and last) known version of *Talkhīṣ Aʿmāl al-Ḥisāb*. The Hebrew Epistle exposes novel mathematical vocabulary and enhances our understanding of the linguistic mechanisms which helped create scientific vocabulary in medieval Hebrew. My book also depicts the fascinating figure of Isaac Ibn al-Aḥdab: astronomer, mathematician, poet and exegete – shedding new light on his persona and intellectual activity.

Isaac probably left his homeland before the 1391 persecutions in Castile. According to the colophon of *The Epistle of the Number*, he also spent some time in Muslim land, probably in North Africa, where he studied *Talkhīṣ Aʿmāl al-Ḥisāb*. Later, during a perilous voyage to the Holy Land, Isaac was ship-wrecked in Syracuse. Some of his friends, members of the Jewish community there, asked him to compose a book on mathematics. In response to their request, Isaac decided to translate the Arabic text into Hebrew and adapt it for the Jewish community. This is how *The Epistle of the Number* came to be.

I have endeavoured to present a broad picture of Isaac's life within the historical and intellectual context of his time. I describe Isaac's exchanges of knowledge with medieval Jewish scholars such as Samuel Ibn Ṣarṣa, the RiBaSH and Judah bar Asher the Second, who was Isaac's astronomy teacher. Isaac was mostly known for his astronomical treatises and their high scientific level. He even improved existing astronomical tools, as described in his treatise *The Intermediate Instrument* (כלי ממוצע). Isaac also wrote about the Jewish calendar and conversion algorithms between the Muslim and Jewish calendars. In addition, he wrote numerous poems, an exegesis of the Passover *Haggadah* and a book on weights and measures in the Bible.

The Epistle of the Number includes an impeccable translation of the *Talkhīṣ Aʿmāl al-Ḥisāb* as well as lengthy commentaries and numerous examples as well as theoretical additions and rectifications by Isaac. He explicitly expressed his wish to be understood by his readers and he indeed succeeded in being highly methodical and clear. Not only did he explain the meaning of new mathematical terms, but he

may have been the one who coined some of them. He also discussed the etymology of several terms, thereby demonstrating his linguistic sensitivity and creativity. In addition to the abundant philological discussions within, *The Epistle of the Number* embarks upon interesting analyses – for example, the analogue between the arithmetician, who is expected to apply mathematical operations upon numbers rather than study their nature and the doctor, who is expected to heal, i.e. apply practical medical knowledge, rather than study the quiddity of Man.

The Hebrew Epistle bears witness to the multidisciplinary nature of Hebrew scientific works in the Middle Ages as well as the broad expertise of their authors. *The Epistle of the Number* is a rich text which introduces a wide variety of mathematical themes, both in arithmetic and algebra. In complete congruence with its Arabic source, it is divided into two books. The first book includes the presentation of the place-value decimal system. It elaborates on the basic arithmetical operations: addition, subtraction, multiplication, division and the extraction of the square root applied on integers, fractions, as well as expressible and inexpressible numbers.[1] Numerous algorithms are provided and carefully explained, including the famous ancient Greek method to identify prime numbers, known as *Eratosthenes' sieve*.

The second book in the Hebrew Epistle presents rules and procedures which "allow us to find the value of the sought unknown from the given known": the rule of three, the rule of double false position and algebraic procedures, such as the solution of equations of the first and second degree. The domain of definition of the arithmetical operations defined in the first book is extended in the second book from numbers to various algebraic species: roots, squares and cubes, thus creating *algebraic expressions*, the very remote "ancestors" of polynomials. After a rigorous analysis of the algebraic theories, we find a series of problems of verbal reasoning: distance-time-velocity calculations, charity distribution and the purchase of a horse by several associates. I have composed a commentary on the mathematical themes in *The Epistle of the Number* focussing on Book II. I have cautiously used modern mathematical notation to describe the various theories, which were originally written in prose.

To the best of our knowledge, there is only one surviving copy of *The Epistle of the Number*, a mid-16th century copy from Constantinople. The manuscript is kept at Cambridge University Library, and its shelf-mark is MS Heb. Add. 492.1 (folios 1v–38v).[2] I present the palaeographical and codicological features of the unicum and the codex in which it is found, followed by an edition of *The Epistle of the Number*, with notes and a translation into English.

To summarize: my book aspires to show the various intellectual facets in Isaac Ibn al-Aḥdab's work, in terms of text and context, focussing on *The Epistle of the*

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[1] In modern mathematical language: rational and irrational numbers, respectively.

[2] See Stefan C. Reif, *Hebrew Manuscripts at Cambridge University Library*, *A description and introduction*, (Cambridge, 1997), p. 590.

Number. Apart from being a tract on a novel subject on the medieval Hebrew mathematical bookshelf, the Epistle is manifold testimony of the transmission of Arabic science to Jewish circles in Christian Syracuse at the end of the 14th century and reflects the scientific activity of the Jews there, of which still relatively little is known. My book also aims to fill a gap in the exposure of medieval Hebrew mathematical texts[3] as only a small part of Hebrew mathematical materials from the Middle Ages have been edited so far, with an even smaller number of tracts which were also translated or commentated upon.

This book will be of interest to scholars in the fields of Jewish Studies, Hebrew philology, medieval studies, mathematics, history of science, and, in particular, history of medieval Hebrew and Arabic mathematics.

[3] Furthermore, my book comprises the only extant study of *The Epistle of the Number*, preceded only by one article on selected aspects of the text by Tony Lévy as well as five short articles which I have written, which include materials from my doctoral dissertation and thus, from this book. See Lévy, Tony, 'L'algèbre arabe dans les textes hébraïques (I), Un Ouvrage Inédit D'Isaac ben Solomon al-Aḥdab (XIV siècle)', in *Arabic Sciences and Philosophy*, 13, (2003), 269–301 and Ilana Wartenberg, 'The Epistle of the Number: An Episode of Algebra in Hebrew', *Zutot: Perspective on Jewish Culture* 5 (2008), 95–101; ead., 'Iggeret ha-Mispar: by Isaac ben Solomon Ibn al-Ahdab (Sicily, 14th century) (Part I: The Author)', *Judaica* 64 (2008), Heft 1, 18–36; ead., 'Iggeret ha-Mispar: by Isaac ben Solomon Ibn al-Ahdab (Sicily, 14th century) (Part II: The Text)', *Judaica* 64, Heft 2/3 (2008), 149–161; ead., 'The Epistle of the Number: The Diffusion of Arabic Mathematics in Medieval Europe', *Circolazione Dei Saperi Nel Mediterraneo: Filosofia E Scienze (Secoli IX–XVII), Circulation des Savoirs Autour de la Méditerranée: Philosophie Et Sciences (IXe–XVIIe Siècle): Atti Del VII Colloquio Internazionale Della Société Internationale d'Histoire des Sciences Et de la Philosophie Arabes Et Islamiques: Firenze, 16–28 Febbraio 2006*, Ed. Graziella Federici-Vescovini and Ahmad Hasnaoui, pp. 101–110 (Edizioni Cadmo, 2013) and ead., 'The Naissance of the Medieval Hebrew Mathematical Language as manifest in Ibn al-Ahdab's *Iggeret ha-Mispar*', in *A Universal Art: Hebrew grammar across disciplines and faiths*, Ed. Nadia Vidro, Irene Zwiep, Judith Olszowy-Schlanger, (Brill, 2014), pp. 117-131.

HEBREW ABBREVIATIONS

רשימת ראשי תיבות וקיצורים:

א. ראשי תיבות:

א"א = אי אפשר

אח"כ = אחר כך

א"כ = אחרי כן

אע"פ = אע"ף = אף על פי

בג"ה = בגזירת השם

בה"י = בעזרת השם יתברך

בס"ד בנל"ך ואע"י אס"ה = בסייעתא דשמייא ברוך נתן ליעף כח ולאין אונים עצמה ירבה[1] אין סוף הוא(?)

בע"הו = בעזרת השם וישועתו

ג"כ = גם כן

ד"מ = דרך משל

ועלא"ם עי"א =?[2]

ע"ד = על דרך

עד"מ = על דרך משל

ע"י = על ידי

ע"כ = על כן

עכ"ז = עם\על כל זה\זאת

ר"ל = רוצה לומר

ת"ו = תבנה ותיכון

ב. קיצורים:[3]

אמ' = אמר

יר' = ירצה

פי' = פירוש

[1] "And to him that hath no might He increaseth strength" [Isaiah 40:29].

[2] I have not been able to decipher this abbreviation, which appears on folio 17r.

[3] Often words at the end of a line were shortened, i.e. their end was omitted, for justification purposes, and they are easily decipherable for Hebrew readers.

HEBREW AND ARABIC TRANSLITERATION

I. Hebrew has been transliterated in accordance with the following system:[1]

Consonants

r	ר	y	י	ʾ	א
š	שׁ	k	כ	b	בּ
s	שׂ	kh	כ	v	ב
t	ת	l	ל	g	ג
		m	מ	d	ד
		n	נ	h	ה
		s	ס	w	ו
		ʿ	ע	u	וּ
		p	פ	o	וֹ
		f	פ	z	ז
		ṣ	צ	ḥ	ח
		q	ק	ṭ	ט

Vowels

Vowel length is not represented. Vocalic šewa is not transliterated

The transcription of *ḥaṭef* vowels is identical with that of their full counterparts.

qamaṣ, paṭaḥ = a
ṣere, segol = e
ḥiriq = i
ḥolam, qamaṣ qaṭan = o
shuruq, qubbuṣ = u

[1] However, for *standardized* English spellings of terms or names (e.g. RiBaSH) I have not altered their transliteration, even if they do not correspond to the system I use elsewhere. The same goes for spelling used by other authors in the bibliography and for Arabic terms and names.

II. Arabic has been transliterated in accordance with the following system:

ḍ	ض	ʾ	ا
ṭ	ط	b	ب
ẓ	ظ	t	ت
ʿ	ع	ṯ	ث
ġ	غ	j	ج
f	ف	ḥ	ح
q	ق	kh	خ
k	ك	d	د
l	ل	ḏ	ذ
m	م	r	ر
n	ن	z	ز
h	ه	s	س
w	و	š	ش
y	ي	ṣ	ص

Long Vowels

ا = ā

و = ū

ي = ī

Short Vowels

fatḥa = a

kasra = i

ḍamma = u

CHAPTER 1: IBN AL-AḤDAB

INTRODUCTION

In 1899, the father of modern Jewish bibliography and one of the founders of modern Jewish scholarship (Wissenschaft des Judentums), Moritz Steinschneider (1816–1907), wrote in his seminal work *Mathematik bei den Juden*, that Isaac ben Solomon Ibn al-Aḥdab "has not won the interest of scholars of Hebrew literature in the way that he deserves. They only mention him every now and then".[1] This statement is no longer true today, as research has revealed new facets of Isaac's scientific, literary and exegetical works. However, to date no thorough study of his mathematical work has ever been made and this books aims to fill this gap.

This chapter presents the highlights of Isaac's life and *ensemble d'oeuvres*, as well as his reading sources and intellectual and familial milieu, briefly describing the historical circumstances in Castile and Sicily during Isaac's lifetime and Isaac's whereabouts during the persecutions of 1391 – a key traumatic event to the Jewish communities in the Iberian Peninsula and Italy. Finally, the relevant contextual elements behind the composition of *The Epistle of the Number* are examined, shedding light on the cultural and religious setting in which Isaac was active.

ISAAC'S LIFE AND TIMES

Isaac ben Solomon Ibn al-Aḥdab ben Ṣaddiq ha-Sefardi (i.e. the Spaniard, Sephardi) was born in Castile between 1340 and 1350 and passed away sometime between 1429 and 1433. In a poem of admiration dating from 1377, which Isaac wrote to the Samuel Ibn Ṣarṣa, Isaac refers to himself as "young".[2] 1429 is the last known year in which it is certain that Isaac was alive. It was in this year that his student, Nissim ben Sabetai Abū al-Faraj, copied one of Isaac's texts in which he sought God's blessing to prolong his teacher's life. But in a treatise copied in 1433 by the same scribe, Isaac is referred to "in blessed memory" (z"l).[3]

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[1] "Isak Alchadib hat bisher nicht das verdiente Interesse der Forscher auf dem Gebiete der jüdischen Literatur gefunden, die nur obenhin oder gelegentlich seiner erwähnen…". See Moritz Steinschneider, *Mathematik bei den Juden*, (Hildesheim, 1964), p. 167.

[2] Dov Schwartz, *The religious philosophy of Samuel Ibn Ṣarṣa*, (in Hebrew), (Ph.D. dissertation, Bar-Ilan University, Ramat-Gan, 1989), pp. 5–7.

[3] *The Intermediate Instrument*, Munich, Bayerische Staatsbibliothek, MS Cod. Hebr. 246/9, fol. 67v (IMHM 1102), *A Poem on Esther*, Munich, Bayerische Staatsbibliothek, MS Cod.

Isaac was trained in Castile by the astronomer Judah ben Asher the Second, the great-grandson of Rabbi Asher ben Yeḥiel (the RoSH).[4] At some point, probably before the 1391 persecutions, Isaac left Castile. As we read in the colophon of *The Epistle of the Number*, Isaac also lived in a Muslim land, either in the Muslim part of the Iberian Peninsula (Granada) or, more likely, in North Africa. In his astronomical treatise *A Precious Instrument* (כלי חמדה) Isaac tells us that in 1396 he was in Syracuse on the island of Sicily.

> In the year [5]156 to the Creation[5] I am in Syracuse, which is in the Island of cold Sicily, and God is in front of me. I have invented an instrument, which is very easy to use; also, with it one can easily establish the positions of the planets…[6]

In the introduction to *The Epistle of the Number* we read of Isaac's arrival in Syracuse:

> Later, while I was going through the sea to the Holy Land, may it be built and existent in our days, the waves of the sea were threatening us, deep calleth unto deep,[7] our soul is bowed down to dust.[8] Observing us from His Seat, The Observer of His world in general and in particular, may His name be praised, exalted and raised, calmed the stormy waves of the sea and brought us all safe to the glorious town of Syracuse in Sicily.
>
> [*The Epistle of the Number*, Cambridge, University Library, MS Heb. Add. 492.1, fol. 1v]

Isaac recounts his maritime journey to the Holy Land, a journey interrupted by a vicious storm, leaving him stranded in Syracuse, Sicily. However, this account should be taken with a pinch of salt since this type of story, involving elements of a

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Hebr. 246/19, fol. 254r (IMHM 01102) and *An article on the definition of things*, Munich, Bayerische Staatsbibliothek, MS Cod. Hebr. 246/8, fol. 67r (IMHM 01102). The number in the round brackets refers to the microfilm or microfiche number (the latter is explicitly marked as 'Fiche') at the Institute of Microfilmed Hebrew Manuscripts (=IMHM) at the National Library of Israel in Jerusalem.

[4] Leopold Zunz, *Zur Geschichte und Literatur*, (Berlin, 1874); reprint (Berlin, 1919), p. 423, Alfred Freimann, 'Die Ascheriden 1267–1391', in *Jahrbuch der jüdisch-literarischen Gesellschaft*, 13, (Berlin, 1919), pp. 155–156.

[5] I.e. 1395/6 CE.

[6] *A Precious Instrument*, London, British Library, MS Or. 2806, fol. 11r (IMHM 06385). According to the entry on top of the folio, we learn that the owner of the manuscript is not other than (the young) Mordekhai Comtino, 15th century Turkish Talmudist, astronomer and mathematician. He is not the scribe of the text, since his entry is in a substantially different handwriting than the one we see in the text.

[7] Psalm 42:7.

[8] In Psalm 44:25 we find the semantically equivalent שחה לעפר נפשינו "our soul is bowed down to the dust…" Isaac slightly adapted the verse according to rules known as *meliṣa* מליצה.

voyage to the Holy Land, a storm and a shipwreck,[9] belongs to a literary topos typical of Hebrew literature in the Middle Ages and is not necessarily a reflection of reality.[10] Isaac first stayed in Syracuse, where he composed *The Epistle of the Number* and the astronomical tract *A Precious Instrument*. The last evidence of Isaac's location can be found in his astronomical tract *The Intermediate Instrument*, from which we learn that Isaac was in Palermo in 1426.[11]

THE HISTORICAL CIRCUMSTANCES

A full description of the historical events which directly influenced the Jews in Castile and Aragon in the fourteenth century is outside the scope of this book. I shall only point to the highlights of the complex circumstances which belong to Isaac's lifetime – a tumultuous period for Jews both in Castile and Aragon.[12]

Castile

The fourteenth century marked the beginning of the destruction of Castilian Jewry. Between 1348 and 1350, Jews were accused of causing the Black Plague, and as a consequence, they were persecuted. Furthermore, the middle of the fourteenth century was characterized by a bloody civil war in Castile, which bore tragic consequences for the Jews as well as the wider population. Pedro, the legitimate son of Alfonso the Eleventh, ruled Castile with the help of Jews, the most famous of whom was Don Samuel ha-Levi. In 1366, the son of Alfonso's mistress, Enrico the Second, rebelled against his brother's reign, denouncing Pedro as the son of a Jew. In 1369, Pedro was killed and Enrico the Second became the supreme authority in Castile. Many Jews were massacred and numerous Talmudic academies were destroyed, as one can read in a letter by Rabbi ben Jacob Lattes. The Jews also had to pay heavy forfeits to the new king, creating an even greater burden. The

[9] Although no boat or ship is explicitly mentioned by Isaac and in the absence of medieval submarines, it is hard to think of other means to travel around the Mediterranean Sea back then.

[10] Tova Rosen, 'The Hebrew Mariner and the Beast', *Mediterranean Historical Review*, 238–244.

[11] Giuseppe Mandalà has made an important discovery; he identified Isaac Ibn al-Aḥdab as Gaudio Alachadeb, a known arbitrary (*borer*), *notaries Iudeorum,* and probably a spiritual judge (*dayan*) in Palermo. Mandalà researched documents showing that Isaac/Gaudio was already living in Palermo in 1418. I wish to thank Giuseppe Mandalà for sending me his article, which also sheds new light on the life of Isaac, his family, the Sicilian Jewish community as well as the historical context in Sicily. See Giuseppe Mandalà, Da Toledo a Palermo: Yiṣḥaq ben Šelomoh ibn al-Aḥdab in Sicilia (ca. 1395/96 – 1431), *Flavio Mitridate mediatore fra culture nel contesto dell'ebraismo siciliano del XV secolo, Atti del convegno internazionale di studi, Caltabellotta (Agrigento), 30 giugno-1 luglio 2008 - Palermo*, ed. Mauro Perani and Giacomo Corazzol. (Palermo, 2012), pp. 1–16.

[12] This section presents succinct highlights from Yitzhak Baer, *Toldot ha-Yehudim bi Sefarad ha-Notsrit*, (Tel-Aviv, 1959), pp. 179–363.

humiliation of the Jews was further extended when they were forced to wear a distinctive Jewish mark.

Enrico's son, John the First, ruled between 1379 and 1390. During his reign, various anti-Jewish laws were enforced, while at the same time several Castilian Jews held high rank in the Royal Courts. John the First of Castile died in 1390, before his heir was mature enough to inherit the crown. The custodians appointed by the late king were unable to curb the rebelling forces in Castile. On June 4th, 1391, these forces set fire to Jewish homes in Seville, massacring part of the population, coercing others to convert to Christianity and selling some of them to the Muslims. This was to become the pivotal and most traumatic event for Iberian Jews in the fourteenth century. Many Jews chose martyrdom, as related in Ḥasdai Crescas' letter to Avignon's rabbis, in which he describes the martyrdom of Judah ben Asher the Second, Isaac's teacher of astronomy. Synagogues were converted into churches and Jewish neighbourhoods were occupied by Christians, leading to the complete destruction of Jewish communities, such as the one in Valencia.

Aragon

Historians deem the fate of Jews in Aragon in the fourteenth century to have been better than that of the Jews in Castile, although it was not completely devoid of violent acts against Jews. Sicily, which was part of Aragon's domain, enjoyed an even greater peace in comparison to the rest of Aragon. Pere the Third ruled Aragon between 1336 and 1387. His reign was relatively calm and was characterized by the reconstruction of the Aragonese Jewish community. In general, Pere the Third protected the rights of the Jews and allowed their commercial life to flourish. In particular, he gave them permission to import fabrics from England and Flanders. Both Pere II and his son John the First, were interested in astronomy and had Jewish scholars in their court.[13]

It is very difficult to establish a realistic picture of the conditions of the Jews in Aragon in spite of the existing documents written in Hebrew by Jewish merchants and doctors, some preserved in archives in Pamplona. These documents indicate that Jews were secure both financially and socially. Pere the Third seems to have made an effort to be an enlightened king. Despite sporadic instances of malicious suspicion of the Jews, he was served by Jewish doctors, astronomers, financial advisors and translators. He is also known to have given religious autonomy to the Jews in regard to capital cases.

Profound hatred against the Jews was rooted both in Castile and Aragon, but its manifestation was more extensive and overt in Castile. Baer explains that this phenomenon was related to the popular belief that Jews had desecrated the host and

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[13] Chabás, José and Goldstein, Bernard R., 'Isaac Ibn al-Ḥadib and Flavius Mithridates: The Diffusion of an Iberian Astronomical Tradition in the Late Middle Ages', *Journal for the History of Astronomy*, xxxvii, (2006), 147.

that Jewish religious writings, such as Maimonides' – the RaMBaM's – *Mishneh Torah*, contained anti-Christian sayings.

In 1387, Pedro's son, John the First, assumed power in Aragon until 1396. Once becoming king, he dismissed all libellous stories against the Jews, even though he had himself promulgated similar allegations earlier. Like his father, he maintained good relationships with the Jews of Aragon, in particular with Ḥasdai Crescas. However, Pedro was a weak king. In July 1391, the first rumours of persecutions in Castile had reached Aragon and caused uproars. John the First published orders stating that the Jews were considered property of the King and that the clerks must protect them. He even sent chevaliers and soldiers to protect the Jews in various cities in Aragon but at the same time did not hesitate to steal the property of the Jews whose lives had been lost in the persecutions. Violence persisted for several years. In 1392, Jews in Sicily were confined to their quarters. There were also persecutions in the summer of that year, which ended shortly after Martin the Fifth of Aragon ordered the punishment of the perpetrators, thus protecting the Jewish community in Sicily.

The Jewish community in Sicily

During the fourteenth century, Sicily was under Aragonese Christian rule. Jews and Muslims living in Christian Spain were grouped in local *Aljamas* ruled by lay as well as religious leadership. Long after the end of Muslim rule in Sicily in 1072, we know that Sicily still preserved strong linguistic and cultural Arabic imprints. Arabic was still in widespread use among Sicilian Jews in the fourteenth century. Manuscripts as well as commercial and legal certificates indicate the usage of Hebrew, Arabic, Catalan and Aragonese, although the quality of the Arabic was at times poor.[14] Arabic culture still influenced Jews in Aragon, as manifest in poetry and artistic work, songs and even Jewish food and language of the time. For example, Jews referred to Sunday *al-aḥad*[15] and not *domingo*. Commercial documents suggest that transactions between Muslims and Jews were common. In the thirteenth century, legislation against Jews converting to Islam was enacted both in Aragon and Castile, suggesting that conversion to Islam was a common phenomenon among the Jews there. In general, the Jews living in Castile and Aragon bore a strong affinity to Muslim culture and religion.[16]

This affinity is corroborated in Isaac's writings: the knowledge of Arabic, at least by some members of the Jewish community in Syracuse, is alluded to in *The Epistle of the Number*. In the introduction to the Epistle, Isaac clearly expresses his concern that some Sicilian Jews who understand Arabic might find out that some passages from the Arabic source are missing from his Hebrew translation.

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[14] Henri Bresc, *Arabes de langue, Juifs de religion*, (Paris, 2001), pp. 46–50.

[15] Or *alhad*, which is still used in Judaeo-Spanish nowadays.

[16] Eleazar Gutwirth, 'Hispano-Jewish Attitudes to the Moors in the fifteenth Century', *Sefarad*, (1989), 245–250.

> I saw fit to omit a few parts from the Epistle, since, in my view, they are of no use. I shall point out to those places as I reach them so that someone who will read the Epistle in the Arabic language will not be able to catch me *in flagrante delicto*.
>
> [fol. 1v]

Sicilian Jews were successfully involved in the local economy. They practiced various professions: tax-collectors, financiers and merchants. Surviving certificates indicate that many Sicilian Jews in the fourteenth century were in possession of small plots of land. There was a Jewish Sicilian bourgeoisie mind-set in regard to housing and concern with domestic comfort – Jews contributed to the commercial developments of Sicilian towns and enjoyed a great freedom of movement.[17]

It is difficult to say much about the interest of fourteenth-century Sicilian Jews in mathematics *per se*. Isaac's Epistle is the only known Hebrew mathematical book to have been written in Sicily at the time. A Hebrew commentary on Menelaus' *De Sphaera*, attributed to Jeremiah ha-Kohen of Palermo, is in actuality a treatise on the sundial with little mathematical value.[18]

Furthermore, the Sicilian Jewish communities did not enjoy a high reputation for Talmudic knowledge. The casuistic literature of the medieval Rabbinic Responsa includes few references to Sicilian Jewry.[19] In the introduction to *The Epistle of the Number* Isaac describes some members of the Jewish community in Syracuse who were interested both in Torah and in mathematics.[20]

> There, I found many honourable men of high rank, who busied themselves with the Torah and the precepts. Amongst them, their seed is established in their sight

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[17] Idem, 'Widows, Artisans and the Issues of Life: Hispano-Jewish Bourgeois Ideology', *Iberia and Beyond, Hispanic Jews between Cultures*, (1992), pp. 151–152.

[18] The attribution derives from a confusion in the description of Vatican MS Ebr. 379 in S. Assemanus and J. Assemanus, *Bibliothecae Apostolicae Vaticanae…I: Codices ebraicos et samaritanos* (Rome, 1756), p. 354, where several passages on folios 67–74 are described as a *Commentarius de Tractatum de Sphaera* by Jeremiah ha-Kohen of Palermo. This mistake is repeated by C. Roth, "Jewish Intellectual Life in Medieval Sicily," *The Jewish Quarterly Review* 47 (1957), p. 324, who relies on L. Zunz, *Zur Geschichte*. M. Steinschneider, Die hebraeischen Übersetzungen des Mittelalters (Berlin, 1893), p. 542, explained the source of the confusion and showed that the text by Jeremiah, written in 1486, is actually on the sundial. I have examined this text and it is of little value for the history of Hebrew mathematics in Sicily. In any case, it is known that astronomical activity was more abundant than mathematical one in Sicily (and elsewhere), as can be testified also by the number of astronomical works by Isaac. I wish to thank Steven Harvey for having pointed this confusion out to me.

[19] Cecil Roth, 'Jewish Intellectual Life in Medieval Sicily', *The Jewish Quarterly Review*, XLVII, (April 1957), 323–334.

[20] Astronomical activity was more abundant than mathematics in Sicily, as can be testified from al-Aḥdab's known works in the field. This is discussed later in this chapter.

> with them.[21] They were educated, lovely young men, whose friendship I enjoyed. I chose their company; some of them studied with me the teachings of our *Holy* Torah. In their free time, some of them were studying the Science of the Number. One day, they asked me to compose for them a short book in this science, as comprehensive in all matters as possible.
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> [fol. 1v]

PEOPLE IN DIRECT CONTACT WITH ISAAC

The RiBaSH (Valencia 1326–Algiers 1408)

Rabbi Isaac bar Sheshet Perfet, or Barfat, the RiBaSH, was a prominent Jewish scholar of noble descent. He was born in Barcelona, and in 1372, he moved to Saragossa, where he was leading the local Jewish community from 1385 until 1391. Regarding foreign sciences he stated:

> One must not discuss the laws of the Bible and its precepts using the wisdom of Nature and medicine, because if we were to believe their sayings, then the Bible would not come from Heaven, God forbid… we rely upon our wise men, of blessed memory, even if they tell us that right is left… We should not believe the Greek scholars and the Muslims, who only talk out of their own assumptions and experience.[22]

During the persecutions of 1391, the RiBaSH was one of the few who were able to escape from Valencia. Regarding the relationship between the RiBash and Isaac, we only know that in a responsum addressed to Judah ben Asher the Second, the RiBaSH stated that he had met Isaac in Castile before the persecutions. He refers to "three quires, which your student, al-Aḥdab, laid here, may God save him; he wrote them for me to send to you". The content of these notebooks is unknown.[23]

The RoSH's family: Judah ben Asher the Second

Judah ben Asher the Second was the great-grandson of the RoSH, i.e. Rabbi Asher ben Yeḥiel, the chief Rabbi of Toledo. At the beginning of the fourteenth century, the RoSH was invited to Toledo by the local Jewish community. Despite the fact that the RoSH originated from an Ashkenazi milieu (Germany), he was highly influential in the life of the Sephardi community of Toledo and his religious rulings were fully respected. After his death, one of his grandsons, Judah the First, succeeded him. Then, after Judah's death in 1349, the RoSH's great-grandson, Judah ben Asher the Second from Burgos, became Rabbi in Toledo. The latter wrote a book on astronomy and was Isaac's teacher on the subject. During the persecutions

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[21] Job 21:8.

[22] Baer, *Toldot ha-Yehudim*, p. 271.

[23] The Responsa project, (in Hebrew), Bar-Ilan University, (Ramat-Gan, 1972–2002), Responsa 240 (CD ROM).

of 1391, Judah chose martyrdom in Toledo, as did many of his friends and family members.[24]

Samuel Ibn Ṣarṣa (Valencia, second half of the 14th century)

Samuel Ibn Ṣarṣa was active in Castile between 1360 and 1380. In his treatises on philosophy he wrote about the theory of divine attributes, the Creation of the World, morality (based on Maimonidean theories), divine Providence, and the source of the soul. Ibn Ṣarṣa wrote two exegetical works: *The Source of Life* and *A Perfect Beauty*. *The Source of Life* comments on Ibn Ezra's biblical exegesis. *A Perfect Beauty* is a commentary on tales of our Sages of Blessed Memory and their *Midrashim*, i.e. homiletic interpretations. Ibn Ṣarṣa's period is dominated by Averroistic rationalistic theories, but in many respects Ibn Ṣarṣa's doctrine is neo-Platonic. Ibn Ṣarṣa was trying to create a synthesis between rationalism and Neo-Platonism, interwoven with strong elements of astrology and astral magic, which endows human beings the capacity to change nature.[25]

Around 1377, after having studied Ibn Ṣarṣa's works, Isaac sent the author a letter of admiration written in verse, at the end of which he refers to himself as "Ibn al-Aḥdab the young". This letter is the only known correspondence between Isaac and Ibn Ṣarṣa. The latter was also known by the name Ibn/Ben Senne, but its non-vocalised form in Hebrew SNH (סנה) has misled historians into believing that Isaac may have studied medical or philosophical works of Ibn Sinā (Avicenna). As a matter of fact, the Jewish name Ibn Seneh derives from the translation of the Spanish name Ṣarṣa, a *thorn bush*, into the Hebrew Seneh. This letter sheds some light on Isaac's philosophical and exegetical background since in it he expresses his profound respect for Ben Senne, whose wisdom and splendour had superseded all men of his generation. Isaac prayed for the welfare of Ibn Ṣarṣa and expressed his hope that God would prolong his days. Isaac was familiar with medical knowledge, at least on some level. In *The Epistle of the Number*, he mentions Ibn Sinā's(?) *Canon of Medicine* (ספר הרפואה) in Book I, when he discusses ratios of a series of 4 numbers.[26]

Zaraḥia Ibn Danon

Zaraḥia Ibn Danon was a poet with whom Isaac had a polemic exchange of poems, the last of which Isaac wrote on Ibn Danon's tomb, immediately after the latter's death.[27]

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[24] Baer, *Toldot ha-Yehudim*, p. 285.

[25] Schwartz, *The religious philosophy*, pp. 13–18.

[26] *The Epistle*, fol. 3r. In 1481, the five books of the *Canon of Medicine* in Hebrew translation were sold together with other books to Jews in Polizzi by Isaac's grandson Gaudio the Younger – these books had probably been in the possession of the grandfather Isaac/Gaudio. See Mandalà, 'Da Toledo a Palermo', 12.

[27] Ora Raanan, *Poems of Rabbi Isḥak ben Shlomo ben Ẓaddik al-Aḥdab*, (Lod, 1988), p. 26.

Isaac's family

Ibn al-Aḥdab literally means in Arabic 'the son of the hunchback'. Isaac's surname appears in the manuscripts under several forms, for example: al-Aḥdab, al-Ḥadib, al-Ḥadab, and al-Khadib. Several members of the al-Aḥdab family are known to have lived in Toledo: Solomon bar Samuel Ibn al-Aḥdab, as we learn from an anonymous inscription of the names on Jewish tombs. According to the memorial stone in the cemetery of Toledo, Solomon Ibn al-Aḥdab died in 1349, during the Black Plague.[28] What we know for certain is that Isaac had (at least) three sons: Abraham, Ṣaddiq, and Jacob. The latter had a son named Gaudio (a clear case of papponomy), who was a doctor. Isaac and some of his descendants played an important public, intellectual as well as economical role in the Sicilian Jewish community during the fifteenth century.[29]

Jacob

Jacob was Isaac's youngest son. He wrote: "I am the youngest at my father's home".[30] Jacob copied some of his father's poems and wrote commentaries on *The Paved Way* (אורח סלולה). He also elaborated on his father's astronomical tables, in response to a friend's request.[31] Jacob is also known to have predicted the solar eclipse of 1463. In a commentary on *The Paved Way*, Jacob writes:

> In our times our brain is meagre and meek, and we have no strength to enter the ample ascensions established for us by the first scholars in the Temples of Wisdom many years ago, the wisdom of astronomers and astrologers. Due to the work, labour and toil of our time, our knowledge has diminished such that when we find commentated books accompanied by commentated exegesis with the paucity of our mind we are not capable of understanding these matters, in particular, the books on astronomical tables which involve calculations and numbers, unless there are numerical examples and commentaries. Therefore, I, the youngest at my father's home, have consented to the request of a beloved

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[28] Yosef S. Spiegel, *Haggadah shel pessach, pessach dorot, dinei leil ha-seder u-perush 'al ha-haggadah le-rabeinu Yiẓḥaq ben Shlomoh al-Aḥdab mi-ḥakhmei Sefarad ve-Sizilia ba-me'a ha-shniya la-elef ha-ḥamishi*, (The Passover *Haggadah* commentated by al-Aḥdab), (Jerusalem, 2000), p. 1. Could the mentioned Solomon have been Isaac's father? If so, then Isaac was born in 1350 at the latest. Another possible family connection: in his commentary on the Passover *Haggadah*, Isaac refers to a certain Judah Krishef as his grandfather. But we know of a Judah Krishef who was one the RoSH's sons-in-law. In his commentary on the Passover *Haggadah*, Isaac refers to a certain Judah Krishef as his grandfather. If both Judahs are the same person, then Isaac must have been the RoSH's great-grandson. See further idem, *Haggadah shel pessach*, pp. 9–10.

[29] For details see Mandalà, 'Da Toledo a Palermo', 9–16.

[30] *The Paved Way,* London, British Library, MS Or. 2806, fol. 20v (IMHM 06385).

[31] *The Paved Way,* Paris, Biblothèque Nationale, MS héb. 1047/18, fol. 172r (IMHM 14650).

> friend, who wishes to learn the tables of *The Paved Way*, composed by my master, my father, of blessed memory. My friend has asked me to give him an example from the gentiles' tables, which will facilitate the calculation[32] of an eclipse and the opposition of the luminaries.[33] For the sake of his friendship and in order to fulfil his will and respond to his request, I have written this example for him and all those who wish to study gentile tables with ease. I ask the reader to forgive me if he finds any mistakes, may God save us from errors. I shall start, with God's help, may His name be blessed and exalted…[34]

Abraham and Ṣaddiq

Abraham and Ṣaddiq were the elder brothers of Jacob, for whom Isaac wrote affectionate poems on their wedding day. He advises them to behave morally and how to choose the right path in life. As can be read in his poems, despite Isaac's extensive scientific activities, like all medieval Jewish scholars he attached foremost importance to religious studies. According to Isaac's poems, science is of secondary importance to, in particular, foreign sciences. One should distance oneself from it if it contradicts the Jewish sources.

Isaac's descendants

Some scholars are identified as Isaac's descendants, such as Rabbi Abraham bar Solomon al-Ḥadab, who was a religious judge in Corfu in 1530. Other possible descendants up to the eighteenth century are to be found in Fez, Venice, and different towns of today's Greece as well as in Tiberias.[35]

WHERE WAS ISAAC IN 1391?

It is not clear where Isaac was during the persecutions of 1391, as none of his writings mention these tragic events. The strongest term he uses against Christians is 'Christian exile' (גלות אדום), which appears in his commentary on the Passover *Haggadah*; an expression which seems to be a rather general term designating the Reconquista than the persecutions themselves. However, Shirman is strongly

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[32] The text says הוצאת, lit. 'the extraction of'.

[33] The text says לקות וניגוד המאורים, but one would expect to find 'conjunction and opposition of the luminaries', which are found in the astronomical tables. An eclipse can be either solar (at conjunction, in the beginning of the lunar month) or lunar (at opposition, in the middle of the lunar month) and thus, there seems to be a mismatch of the semantic categories of the used terms. Perhaps this was an imprecise, popular manner to describe eclipses.

[34] *The Paved Way*, London, British Library, MS Heb. Add. 26921, fol. 20v (IMHM 05469).

[35] Zunz, *Zur Geschichte*, pp. 423–424 and Spiegel, *Haggadah shel pessach*, p. 9.

convinced that Isaac fled the persecutions in Castile.[36] It is surprising that no one has ever considered the possibility that Isaac was spared these horrible events, especially since no counter evidence for this option exists.

A careful reading of Isaac's poems does not reveal any hint of the persecutions, but perhaps the poems that reached us date from before 1391. However, none of his other works – whether exegetical or astronomical – written after 1391, bear any indication of these events and in particular of the atrocious death of Isaac's teacher, Judah ben Asher the Second. Several hypotheses are possible: perhaps the works in which these events were mentioned were lost. It is also possible that the scribe was confined by censorship and hence, could not copy the account of such horrible events, but could this have been the case of all the surviving copies of Isaac's post-1391 work?

Unfortunately, at the moment I am not able to provide any details regarding Isaac's whereabouts in 1391. He may have still been in Castile and miraculously escaped the persecutions. He may have already been in Muslim lands learning mathematics with Muslim scholars or already in Sicily. In any case, I have reason to believe that in 1391, Isaac had long left Castile. Assuming no censorship considerations are valid, I would argue that had Isaac left Castile after the persecutions, one would have expected the introduction of *The Epistle of the Number*, probably written after 1391, to include a reference to these horrors, in particular because Isaac's own teacher chose martyrdom in those events. It seems strange that he would tell us about his stay in Muslim lands and the voyage to the Holy Land which was interrupted by a vicious storm, whereas the latter event pales when compared to the persecution.

Hence, although I cannot provide a definite answer to Isaac's location in 1391, I hope to raise the readers' attention to the *possibility* that Isaac may have not present, or even knowledgeable of the 1391 persecutions. New evidence may however emerge in the future, which will shed light on this matter.

Isaac's reading sources

The following list incorporates Isaac's identifiable reading sources:

- Ibn al-Bannā''s *Talkhīṣ Aʿmāl al-Ḥisāb* (Compendium on the operations of Calculation), its author's commentary *Rafʿ al-Ḥijāb* (Unveiler of the Veil) and possibly other commentaries.[37]
- Within *The Epistle of the Number*, in his discussion that one is not a number, Isaac also mentions Ibn Rushd's commentary. His formulation, however, is very different from other Hebrew versions we know of.[38]

[36] Shirman, Ḥayim, *ha-Shirah ha-ʿIvrit bi-Sefarad uve-Provans: mivḥar shirim ve-sipurim meḥorazim,* (The Hebrew Poetry in Spain and in Provence), (Jerusalem, 1954), p. 582.

[37] See details in chapter 2, pp. 25–28.

[38] *The Epistle*, fol. 1v.

- Euclid's *Elements*: Isaac must have been familiar with the contents of this book, either with one of its translations or commentaries either in Arabic or Hebrew, since in *The Epistle of the Number* he cites two common notions from *The Elements*.[39] However, the linguistic discrepancy between the Hebrew version given in *The Epistle of the Number* and the other Hebrew translations of Euclid which are known to us, strongly hints that Isaac may have not read the Hebrew translations of *The Elements*. He probably studied one of its Arabic translations during his stay in a Muslim country.
- Abraham Ibn Ezra's[40] (Tudela, c. 1089 – c. 1167) work(s): Isaac refers to Ibn Ezra's biblical exegesis in his *Wedge of Gold* (לשון הזהב), a book on weights and measures in the Bible (described further on). It is possible that Isaac also knew Ibn Ezra's mathematical works such as *Book of the One* (ספר האחד) and *Book of the Number* (ספר המספר) since these books were in wide circulation at the time. In the Epistle Isaac mentions the well-known definition of multiplication in the Holy language, by which he probably refers to Ibn Ezra's definition.[41]
- Ibn Sinā's *Canon of Medicine* (ספר הרפואה). Isaac explicitly mentions it when he compares the ratio between any four numbers and the four humours.[42]
- Aristotle's *Physics*, or a commentary thereof: Isaac explicitly mentions the four causes.[43] Also, Isaac probably read the *Organon* or any of its commentaries since he discussed the division of *discrete* and *continuous* quantities.[44]

The work of the following scholars is referred to in Isaac's commentary to the Passover *Hagaddah*:[45]

- Maimonides' *Guide for the Perplexed* (מורה נבוכים).
- RaSHi's exegesis to the Bible.

The following medieval astronomers' work influenced Isaac's writing:

- Immanuel Bonfils ben Jacob (c. 1350), active in Tarascon: In 1365 Bonfils wrote an astronomical work *Six Wings* (שש כנפים), which became popular and was translated into Latin in 1406. This book is believed to have motivated Isaac to improve the quadrant.[46]

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[39] Ibid, fol. 28r.

[40] Although עזרא should have been transliterated as 'Ezra, Ezra has become the Standard English transliteration.

[41] *The Epistle*, fol. 10r.

[42] Ibid, fol. 3r and also see footnote 26 above

[43] Ibid, fol. 6v.

[44] Ibid, fol. 13v.

[45] Spiegel, *Haggadah shel pessach*, pp. 24–25.

[46] See section on *The Intermediate Instrument*.

- Ibn ar-Raqqām (c. 1250 – c. 1315), who was the authoritative astronomer in Tunis during Isaac's time, was a source for Isaac's astronomical works. Other sources include the works of al-Battānī (c. 858 – c. 929) from Harran who was active in ar-Raqqah and Ibn al-Kammād (twelfth century, al-Andalus).[47]

ISAAC'S WORKS

Altogether, there are over eighty manuscripts which refer to Isaac's works, scattered in tens of libraries around the world. The richness of the domains covered designates Isaac as a polymath in various fields, including astronomy, mathematics, poetry, philosophy, and exegesis.

Mathematics

The Epistle of the Number, the kernel of this book, is Isaac's only known purely mathematical text. As mentioned earlier, it is the first (and only) known version of Ibn al-Bannā''s renowned *Talḫīṣ Aʿmāl al-Ḥisāb*. It includes a perfect translation of the Arabic source as well as long elaborations. *The Epistle of the Number* plays a special role in the history of medieval Hebrew mathematics because not only is it the first known Hebrew treatise to include extensive algebraic theories and procedures, but it also exposes a rich novel mathematical vocabulary.[48]

The Wedge of Gold (לשון הזהב) is an exegetical work of mathematical nature in which Isaac comments on the measures and weights mentioned in the Bible. In the course of time, this book was erroneously considered to be lost and then it was mistakenly believed to have been published in Venice in 1552. In this tract, Isaac builds a model of the tabernacle and explores numismatics.[49] This treatise involves many area calculations and hence, apart from being an exegetical work, *The Wedge of Gold* can be considered of some, albeit marginal, mathematical nature as well. The

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[47] Spiegel, *Haggadah shel pessach*, p. 14 and José Chabás and Bernard R. Goldstein, Bernard R., *Flavius Mithridates*, 148. An explicit reference to Ibn ar-Raqqām and Al-Battānī can be found in *Tables of The Paved Way*, London, British Library, MS Or. 2806, fol. 33v.

[48] Mathematical elements (mainly calculations) can be found in his *The Wedge of Gold*, an exegetical work, the conversion algorithm between Jewish and Muslim months mentioned above and the various astronomical tables, but in none of these texts is mathematics the main theme, it is only a tool for calculation. In the past, a mathematical treatise by the name *A Procedure of Calculation* (מעשה חושב) was erroneously attributed to Isaac. See Lévy, Tony, 'L'algèbre arabe dans les textes hébraïques (I), Un Ouvrage Inédit D'Isaac ben Solomon al-Aḥdab (XIV siècle)', in *Arabic Sciences and Philosophy*, 13, (2003), p. 301.

[49] Isaac ben Solomon Ibn al-Aḥdab, *Leshon ha-Zahav*, (A Wedge of Gold), London, British Library, MS Or. 10660, fol. 113v (IMHM 07975). The Hebrew edition of the text with commentary was published by Yaaqov Spiegel, Leshon ha-Zahav le-R. Isaac ben Solomon al-Aḥdab, *B.D.D.*, Volume 12 (Bar Ilan University), Winter 2000, pp. 5–34. I wish to thank Jesús del Prado Plumed for this reference.

type of mathematics used here involves basic arithmetic, mainly multiplication.[50] The types of numbers present in the text are integers and fractions.[51] The numeration system is either rhetorical or alphanumerical, i.e. according to the *ABJAD* system, in which the numerical value of a letter (1, 2, … 9, 10, 20, … 90, 100, …) corresponds to its position in the alphabet.

Astronomy

The study of astronomy was legitimized and widely encouraged in medieval Jewish circles because of its importance for the reckoning of the Jewish calendar. Indeed, one finds original Jewish astronomical writing in the Middle Ages. It is not surprising that the majority of the known copies of Isaac's manuscripts belong to this domain. Isaac composed astronomical tables and wrote about astronomical instruments such as the astrolabe, the equatorium and the quadrant as well as some of his inventions or adaptations of existing tools.

The Paved Way (אורח סלולה).[52] This tract was composed in Syracuse in 1396 and about 25 copies of it survived, an impressive number for a medieval Hebrew text on science, indicating its status as a medieval "best-seller". *The Paved Way* elaborates on the motion of the luminaries and it includes a set of user friendly tables on conjunctions and oppositions[53] as well as solar and lunar eclipses. This work was part of a rich astronomical tradition in the Iberian Peninsula and Provence in Isaac's time, which consists of works by Immanuel Bonfils of Tarascon, Abraham Zacut and Judah ben Verga. This tradition derives from the Ptolemaic astronomy, elaborated in al-Andalus, then diffused to Jewish and Christian communities in Europe. This tradition gave birth to a separate branch of tables, which is independent of the Toledan or Parisian Alfonsine Tables.[54]

A Precious Instrument (כלי חמדה). Divided into two sections, consisting of four and thirteen chapters respectively, we find records of the construction and use of the equatorium, an astronomical instrument which was devised by Isaac in Syracuse in 1396 for the determination of the position of the planets with greater precision. It only requires the turning of dials, instead of cumbersome calculations with

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[50] To designate multiplication, Isaac uses only the term כפל, whereas הכאה, a calque of the Arabic ضرب, does appear in *The Epistle of the Number*. This may indicate that this exegetical work had been composed before Isaac came in contact with Arabic mathematics.

[51] Geometric terms include היקף (circumference), עובי (thickness), שטח (area), רוחב (width), אורך (length). The arithmetical terms used are identical to those in Book I of *The Epistle of the Number*. These terms are: בקירוב (approximately), קיבוץ (addition) and the fractions חומש (one fifth) and שתות (one sixth).

[52] The title derives from a biblical expression [Proverbs 15:19].

[53] See the section on Jacob above.

[54] José Chabás and Bernard R. Goldstein, *Flavius Mithridates*, 147–148, 170 (n. 4).

astronomical tables, which are prone to arithmetical error by the (human) calculator.[55]

In the colophon we read:

> Epistles on the precious instrument, composed by the wise man, Rabbi Isaac Ibn al-Aḥdab, be his name in blessed memory, who says: the precious instrument is precious to you above all instruments of marvels. Its name is dear and pleasant. Isaac ben Solomon is like a father to you, and you are to him like a beloved child (נטע שעשועים).[56]

The Intermediate Instrument (כלי ממוצע). In this treatise, which includes 26 chapters, Isaac describes an instrument which is a combination of an astrolabe and the new quadrant invented by Jacob ben Makhir around the year 1300.[57] This new quadrant, which became very popular in Christian Europe, derives from the astrolabe by folding the stereographic projection of the heavens in half, twice. Isaac composed two epistles on the subject, one about the construction of the instrument and the other about its application. Isaac followed the doctrine of ar-Raqqām and used tables calculated by both al-Battānī and ar-Raqqām. This device was designed for latitude 36°, even though Syracuse lies in latitude 37°. It is possible that latitude 36° was chosen for the convenience of calculation.[58]

The Epistle of the Number (אגרת המספר). In this text, Isaac seems to allude to Jacob ben Makhir. In the chapter on multiplication of numbers he comments on vertical multiplication on paper, where intermediate results cannot be erased (as is the case of wax boards), Isaac refers to the table representation of one type of vertical multiplication as *The Owner of Wings* (בעל כנפים),[59] alluding to the title of Jacob ben Makhir's important astronomical work *Six Wings* (שש כנפים). The numerical example given in folio 11r in *The Epistle of the Number* is the multiplication of 255,225 by 879. With a slight stretch of imagination, the shape of the table could perhaps remind us of a bird spreading its wings:

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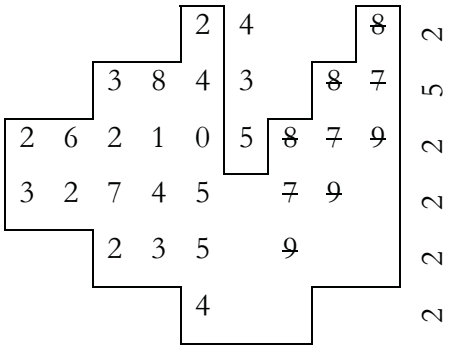
[55] Bernard R. Goldstein, Astronomy in the medieval Jewish Spanish community, *Between Demonstration and Imagination: Essays in the History of Science and Philosophy presented to John D. North*, ed. L. Nauta and A. Vanderjagt. (Leiden, 1999), pp. 233–234.

[56] *A Precious Instrument*, London, British Library, Or. 2806, fol. 11r (IMHM 06385). נטע שעשועים literally means 'a plant of delight', in the sense of 'a beloved child'. See Goldstein, Bernard R., 'Descriptions of Astronomical Instruments in Hebrew', *From deferent to equant: a volume of studies in the history of science in the ancient and medieval Near East in honor of E.S. Kennedy*, ed. David A King and George Saliba, (New York, 1987), pp. 124–128.

[57] See Goldstein, 'Instruments', pp. 121–123.

[58] Spiegel, *Haggadah shel pessach*, pp. 16–17.

[59] Lit. 'a bird'.



The Jewish calendar

A fragmentary text composed by Isaac includes a calendrical table of the Muslim months and their (various) equivalents in the Jewish calendar.[60] A preliminary study of this fragment shows that this text was composed around the year 1387, possibly while Isaac was still in Castile. It was copied in Sicily in by Nissim Abū al-Faraj. In the margins we find references to the years 1430 and 1445, which fit the period when Al-Faraj was involved in the copying of Isaac's texts. The aim of the devised algorithm is to enable one to find the Muslim month in which any 1 Tishri (the first day of the month in the fixed Jewish calendar) will fall. No algorithm can provide the exact date (i.e. the actual day in the Muslim month) because the Muslim calendar was and still is (for example in Saudi Arabia) based on lunar sighting. However, the lunar component both in the Muslim and Jewish calendars ensures that the algorithm will yield pretty accurate results. My initial study of this fragment indicates that Isaac provided a mathematization of conversion tables already found in Isaac Israeli's *Yesod Olam* (The Foundation if the World). The latter is a lengthy calendrical scientific treatise composed in Toledo in 1310, which discusses astronomical and mathematical issues at length – laying solid foundations for understanding all aspects of the Jewish calendar. It is important to note that *Yesod Olam* was dedicated to no other than the RoSH, and as we know, Isaac was taught astronomy by the RoSH's great-grandson Judah ben Asher the Second, and he may have even been part of the RoSH's family, as mentioned earlier in the chapter. Regardless of the exact nature of the connection between Isaac and the RoSH, it is highly probable that Isaac had direct access to Israeli's treatise. In fact, we find calendrical notes by

[60] Raanan, *Poems*, p. 17. This algorithm has survived in one copy only as far as we know, see Rome, Biblioteca Casanatense, MS Ebr. 3082, folios 28v (table) and 43r (instructions and examples), with unrelated astronomical materials inserted in between. As far as I know, this calendrical fragment has never been published and I intend to publish it in the future.

Isaac Ibn al-Aḥdab which follow *Yesod Olam* in MS Poc. 368 in the Bodleian Library.[61]

'A table to [find] the unknown *molad*'[62] (לוח למולד הנעלם). The *molad* (literally 'birth') refers to the new moon. The calculation of the *molad*, in particular the *molad* of the month of Tishri, the Jewish New Year, is of extreme importance in the Jewish tradition. The determination of the time of the *molad* is based on the calculation of the length of all lunations which elapsed from Creation until the desired *molad*. The value of an average lunation used is 29 days 12 hours and 793 parts of 1080 parts of the hour. Isaac devises methods that render the calculation of the *molad* easier.[63]

Commentaries and exegeses

A discourse on the definition of the sayings (מאמר בגדרי הדברים): in this tract Isaac writes about the meaning of the mind, the soul, angels and stars.[64]

A comment on the seven precepts for the sons of Noah (הערה על ז' מצוות בני נח): these seven precepts (six negative and one positive) are perceived by Jews as the basis of the ethical code for non-Jews.[65]

A commentary on the Azharot[66] composed by Solomon Ibn Gabirol. In it, Isaac declares that he "has only come to comment on the purity of the [author's] language and the beauty of its subject".[67]

Exegesis of the Passover *Hagaddah* (פסח דורות). This book includes rules for the *Seder* dinner (דינים לליל הסדר), followed by an ample commentary on the *Hagaddah*. In the introduction, Isaac says that the purpose of his writing is "to explain some of the terms in the Passover *Hagaddah*, as requested by some of his friends". As mentioned above, in the marginalia we find Isaac's reference to the

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[61] This is most probably a Sephardi manuscript from the fifteenth century. The exact connection between the writing of the two Isaacs needs to be carefully investigated. One has to wonder whether the calendrical notes may even be in al-Aḥdab's own handwriting. See also the next footnote.

[62] Oxford, Bodleian Library, MS Poc. 368, fols. 218r–219r (IMHM 19329).

[63] 1 Tishri may need to be postponed by a day or two from when the molad of Tishri falls, as a result of the religious prohibition that 1 Tishri fall on a Sunday, Wednesday or a Friday. Also, if the *molad* of Tishri occurs at noon or later (old molad) then 1 Tishri needs to be postponed to the following day and possibly postponed by another day if the previous restriction applies.

[64] Spiegel, *Haggadah shel pessach*, p. 18.

[65] Raanan, *Poems*, p. 18.

[66] *Azharot* are didactic liturgical poems which treat the 613 precepts of the Jewish Law, 365 negative ones and 248, the number of parts in the human body (ר'מ'ח' איברים), positive ones. In the Middle Ages it was common to dedicate a poem to one precept or precepts which belong to the same subject. The famous Jewish poet Solomon Ibn Gabirol wrote *Azharot*.

[67] Spiegel, *Haggadah shel pessach*, pp. 18–19.

Reconquista (גלות אדום) *sans plus*. His language is clear and witty. He elaborates on every deed of the Passover Eve. This exegesis was probably written in Sicily, since the local habit to have a lengthy *Kiddush* is mentioned.[68] Isaac also discusses the Sicilian habit of adding dates, grains and seedless raisins, apples, almonds, hazelnuts and spices to the Passover horseradish (חזרת), instead of the habitual vinegar, in order "to render the *Matzah* tasty".[69] In this tract, Isaac focuses on the interpretation of terms in the *Hagaddah* and unlike other exegeses of this kind, he does not collect previous commentaries; Isaac's writing is characterized by a strong personal and independent interpretative style.[70] Only one copy of this exegesis is known to have survived and it dates from the end of the fifteenth century, now in a private collection.[71] Originally, this copy belonged to descendants of Isaac, as can be read from the signature of the owner, Ḥabib Ibn Isaac, and the signatures of other family members on the cover. There are very few Passover exegeses which survived the persecutions on the Iberian Peninsula. This fact enhances even more the value of Isaac's exegesis and the unicum that survived.

Poetry

Isaac belongs to the generation of Jewish poets which seal the period of Sephardi Hebrew poetry and fight for its survival. He laments the decline of Hebrew poetry. In some of his poems, he expresses admiration to the early great poets such as Moses Ibn Ezra, Solomon Ibn Gabirol and Judah ha-Levi. He emphasizes the importance of poetry but insists that a poem must be novel and not a repetition of old ideas and writing styles. Unlike the earlier poets in al-Andalus (Muslim Spain), whose poems were highly rhetorical, full of puns and metaphors, poets in Christian Spain wrote in a didactic manner, conveying scientific, moral and social messages.[72] This literary trend comes as no surprise given that this was the prevalent poetical style in the Christian Castilian milieu, which clearly influenced Jewish poets.

There are about ninety known poems written by Isaac, all of which were studied and categorized.[73] These poems have various natures: most of them are of didactic-ethical or religious. A few are humorous-satirical. In the poems dedicated to his sons on their wedding days, Isaac includes Maimonides' *Thirteen articles of the Creed* and guides his sons to adhere to religious piety. He encourages his son Abraham to learn astronomy, study the rules of the Jewish calendar, which include the

[68] Ibid, p. 23.

[69] Ibid, p. 29.

[70] Ibid, p. 23.

[71] At the private collection of the aDMoR from Karlin-Stolin in Jerusalem.

[72] On this influence and the comparison between Jews poetry in Muslim and Christian Spain see Doron, Aviva, 'Ha-Shir Bokhe 'al mapalato, Itshaq al-aḥdab – meshorer wivri be-shalhey tif'eret sefarad' (The poem laments its defeat – Isaac al-Aḥdab – a Hebrew poet at the end of the Glory of Sefarad) *Leot Zikkaron*, in Memorial volume dedicated to the memory of Aharon Mirsky, (Jerusalem 2006), pp. 333–345 (in Hebrew).

[73] Raanan, *Poems*.

calculation of the *molad* (new moon), *ʿibbur* (intercalation) and *tequfot* (i.e. seasonal turning points: spring equinox, summer solstice, autumn equinox and winter solstice), but he warns his sons to stay away from foreign sciences which are not in unison with Jewish sources.[74]

Another poem concerns the thirty-two hermeneutic principles interpreting the Torah according to the Jewish tradition. Isaac also wrote poems as prefaces to books, proverbs, polemics and rhymed prose. He is believed to have written a humorous prayer in which all words start with the Hebrew letter nun נ.[75] No trace of this poem exists, but it is known that this type of poetry was common in the Iberian Jewish milieu, mainly as a brain teaser, a sort of medieval Sudoku.

In one poem, Isaac complains about the religious interdiction to kill fleas on Sabbath. Two poems are addressed to the poet Zaraḥia Ibn Danon, as part of a polemic exchange between the two poets regarding who was a better poet, as mentioned above. Another category of poems includes strophic poems, i.e. poems in which the same rhyme scheme is repeated from one stanza to the other, such as in *Poem on Esther* and *A diligent man*. The latter is a well-known poem written with much wit, sarcasm and humour. In its composition, Isaac was probably influenced by Immanuel the Roman, who himself may have been influenced by the *serventese* and *vanti* poems, filled with bragging and vanity, a style which was common in Christian Italy but not in Christian Castile. In this poem, Isaac joyfully enumerates tens of social skills or trades which he himself practised. There are thirty-six stanzas, the first twenty-two of which open with the letters of the Hebrew alphabet in sequence and the last stanzas form the acronym חזק ואמץ (be strong and of good courage). The narrator tells us about his frequent change of professions. This long list of occupations, which ranges from medicine to wafer making, can shed light on urban life styles among the Jews at that time. We also learn about the Jewish perceptions of those professions, such as in the case of doctors, who are characterized as cupid and womanizers.[76] Furthermore, Isaac is also attributed with a book on the art of poetry, of which all traces are lost.[77]

Medicine

Isaac's name was erroneously linked with the illustrious Arab medical writer Ibn Sinā (c. 980 – c. 1037), known under the Latin name Avicenna, since one of Isaac's letters was addressed to Ibn Senne, wrongly identified as Ibn Sinā, as described above.[78]

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[74] Ibid, pp. 334, 337–339, 341–343.

[75] Raanan, *Poems*, p. 17.

[76] Gutwirth, Eleazar, 'Widows', pp. 154–156.

[77] Raanan, *Poems*, p.19.

[78] Nevertheless, in Book I in the Epistle, Isaac compares ratios and numbers with temperaments, possibly referring to Ibn Sinā's *Canon of Medicine*. See footnote 26 above.

ISAAC'S LINGUISTIC SKILLS AS MANIFEST IN THE EPISTLE

Isaac's language in the Epistle is generally very clear and precise. Reading through the Epistle demonstrates Isaac's great value as a pedagogue, since he often expresses his wish to be understood by his students and he finds alternative explanations and numerical examples. Whether the students were solely readers (including his friends who asked him to compose a mathematical treatise upon his arrival in Syracuse), or whether he was teaching these materials in groups, is not clear.

Isaac's sensitivity to language and capability to play with the language and create puns are manifest in his explanations of the etymology of several mathematical terms. For example, when explaining the meaning of prime numbers, he says that "They are also called *deaf* numbers, since they do not listen to the voice of divisors."[79] Isaac's central linguistic contribution to the Hebrew Epistle is the probable creation of novel mathematical vocabulary, mainly in the field of algebra. Given the phonological and lexical proximity between Arabic and Hebrew, Isaac often coined a Hebrew term that sounded close to its Arabic origin, while preserving the process of semantic extension. For example, in coining the algebraic term הקבלה, opposition[80] Isaac was phonetically and semantically inspired by مقابلة.

Regarding Isaac's command of languages other than Hebrew, the impeccable translation of *Talkhīṣ Aʿmāl al-Ḥisāb* into Hebrew demonstrates his proficiency of the Arabic language. We also find textual proof that Isaac possessed at least partial knowledge of some Romance languages (spoken both in his native Castile and in Aragonese Sicily), perhaps he even knew Latin. Towards the end of folio 16v Isaac says that some languages allow for the form 'one twentieth', i.e. the name of a fraction with denominator greater than ten, which directly derives from the name of the integer in the denominator (this is the case of English). Neither Arabic nor Hebrew allow for a fraction corresponding to a number greater than ten, X, to be expressed as Xth, only as *one [part] over X*. The Romance languages, on the other hand, do permit to construct a fraction in one word which directly derives from its corresponding integer, in the way English does. Given Isaac's geographical whereabouts, it is probable that by referring to "other languages" he meant either Latin, Medieval Castilian, Catalan or a certain Sicilian dialect. One twentieth in Latin is v*icesimus*, which derives from *viginti*, *vigèsim* in Catalan and *vigesimo* in Castilian.[81]

THE ECHOES OF ISAAC'S WORK

In some manuscripts, we find names of people and places relating to Isaac's work: students, copyists and readers. The leading figures are described below:

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[79] *The Epistle*, fol. 2r. The Hebrew expression creates a pun and means that the prime numbers do not "obey" their divisors.

[80] See lexicon.

[81] Also see Isaac's remark in fol. 17v.

Nissim ben Sabetai Abū (or ben) al-Faraj, mentioned above, is known to have been Isaac's student. He copied a number of Isaac's manuscripts in astronomy, philosophy, exegesis and poetry, leaving notes such as:

> I, the youngest of his students, Faraj ben al-Faraj, am writing…[82]

We have an interesting family connection here, which seems to have contributed to the distribution of Isaac's astronomical tables into Christian Italy: Nissim's son, Samuel, a rather shadowy figure, was born in Caltabellota, Sicily. He converted to Christianity in the 1460s, changed his name to William Raymond of Moncada, the name of his first patron, the Count of Adrano, but then became known as Flavius Mithridates. He was a student at the University of Naples in 1473 and four years later, he arrived in Rome and met the Duke of Urbino, to whom he dedicated some astronomical tables in Latin. These tables derived almost entirely from Isaac's *The Paved Way* copied by his father Nissim and depend on Isaac's zīj, but Mithridates never mentioned the source of his tables. Around 1486, Mithridates translated kabbalistic tracts into Latin for Giovanni Pico della Mirandola, for whom he served as advisor. A rather controversial personality, he nevertheless evoked admiration thanks to his apparent genius and linguistic proficiency in Hebrew, Aramaic, Arabic and Syriac.[83]

Muscato Bar Menaḥem completed copying *An Epistle on A Precious Instrument* (אגרת כלי חמדה) in 1482.[84]

Abraham Zacut (fifteenth century) refers to Isaac's *The Paved Way* in his astronomical treatise *The Almagest* (החיבור הגדול).[85]

Abraham ben Ḥayim Gascon wrote a commentary on Isaac's astronomical tables in 1542.[86]

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[82] *A Table of the Muslim calendar and its analogue in the Jewish Calendar*, Rome, Biblioteca Casanatense, MS Ebr. 3082, fol. 43r (IMHM 00072), probably composed in or around 1428.

[83] Chabás, José and Goldstein, Bernard R. 'Isaac Ibn al-Ḥadib and Flavius Mithridates: The Diffusion of an Iberian Astronomical Tradition in the Late Middle Ages', *Journal for the History of Astronomy*, xxxvii, (2006), pp. 147–148, 169 and Scandaliato, Angela, 'Le radici familiari culturali di Guglielmo Raimondo Moncada, ebreo convertito del rinascimento, nella Sicilia del sec. XV', *Una manna buona per Mantova, Man Tov le-Man Tovah, Studi in onore di Vittore Colorni*, ed. Perani Mauro (Florence, 2004), 204–205.

[84] *A Precious Instrument,* Paris, Bibliothèque Nationale, MS Héb. 1051, fol. 138v (IMHM 14656).

[85] See, for example, Berthold Cohn, *Der Almanach perpetuum des Abraham Zacuto* (Strassburg 1918: Schriften der Wissenschaftlichen Gesellschaft in Strassburg, 32. Heft): p. 12 (German translation of Zacut's introduction) and p. 46 (Hebrew introduction to Zacut's treatise). Zacut was very well informed of the astronomical contribution by his Jewish predecessors, many of whom he mentions. For details, see José Chabás and Bernard R. Goldstein, *Astronomy in the Iberian Peninsula: Abraham Zacut and the Transition from Manuscript to Print.* Transactions of the American Philosophical Society, New Series, Vol. 90, No. 2. (2000), p. 49.

Rabbi Mordekhai ben Ḥayim completed a copy of *An Epistle on A Precious Instrument* in 1422 i.e. during Isaac's lifetime.[87]

Rabbi Samuel bar Yoav from Modena wrote a commentary on *The Paved Way*.[88]

Joseph ben Suleyman copied Isaac's poem to his son Abraham in Baghdad in 1680.[89]

Isaac's astronomical tables and *A Precious Instrument* were used in Naples in 1492,[90] as well as in Baghdad, Jerusalem, Damascus and Palestine in 1738.[91]

Mordekhai Finzi, the fifteenth-century Hebrew scholar, mentions Isaac's astronomical works.[92] However, it is not known whether Finzi had read *The Epistle of the Number* or any of Isaac's other scientific works.

SUMMARY

Isaac lived during a tumultuous time for the Jewish people; yet, it is far from clear whether he was directly affected by the events, in particular the persecutions of 1391. From Castile to Sicily, passing through Muslim land and the Mediterranean Sea, Isaac transmitted science and poetry to the Sicilian Jewish world. His mathematical training in Muslim lands enabled him to present a rich tract on arithmetic and algebra to the Jewish community in Syracuse which derived from Ibn al-Bannā''s *Talkhīṣ Aʿmāl al-Ḥisāb*. At least some of the members of this community apparently expressed interest in mathematics.

Isaac was an astronomer, a poet, an exegete, a mathematician, a 'calendar man' and a coiner of mathematical terminology. *The Wedge of Gold* presents mathematical calculations within a religious context (biblical exegesis) whereas *The Epistle of the Number* exposes Arabic arithmetic and algebra in the Hebrew language outside a

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[86] Examples following the Tables in *The Paved Way,* New-York, Jewish Theological Seminary, MS 2571, fol. 1r (IMHM 28824). See also, Bernard R. Goldstein, 'The Hebrew Astronomical Tradition: New Sources', *Isis* 72. (1981), 240, 244 (reprint: *Theory and Observation in Ancient and Medieval Astronomy*, A collection of 24 essays, revised with a new preface. London: Variorum, 1985).

[87] Paris, Bibliothèque Nationale, MS héb. 1065, fol. 70r (IMHM 31301).

[88] *Commentary on The Paved Way*, Budapest, Magyar Tudomanyos Akademia, Kaufmann Coll. MS A13/14, fol. 242r (IMHM Fiche 029).

[89] *A Poem to the Wedding day of Abraham,* Jerusalem, National Library of Israel, MS Sassoon 778, fol. 105r (IMHM 09556).

[90] *A Precious Instrument,* Naples, Biblioteca Nazionale Vittorio Emanuele III, MS F 12, fols. 171r–172v (IMHM 11526).

[91] *Astronomical and Astrological Tables,* Jerusalem, National Library of Israel, MS Sassoon 52, fol. 139r, (IMHM 08941).

[92] Y. Tzvi Langermann, 'The Scientific Writings of Mordekhai Finzi', *Italia*, VII. Nr. 1–2, (1988), 17, 20.

religious context. Isaac's numerous astronomical works survived in abundance, in strong contrast to the paucity of traces of his purely mathematical writing.[93]

[93] This fact may corroborate Freudenthal's conjecture concerning the marginality of mathematics among medieval Jewish communities. Freudenthal claims that Jews in twelfth-century Provence could not have a-priori been interested in algebra because it was too foreign a science and irrelevant to religious practices. See 'Science in the Medieval Jewish Culture of Southern France', *History of Science, xxxiii* (1995), 36-37. The existence of *The Epistle of the Number* with very weak traces of algebra preceding it does not necessarily contradict Freudenthal's general line of argumentation but in general, one should avoid *argumentum ex silentio*. For the discussion of previous traces of algebra in Hebrew see chapter 3, pp. 88–93.

CHAPTER 2: THE HEBREW AND ARABIC EPISTLES

THE ARABIC SOURCE

Identification of the Arabic source of *The Epistle of the Number* was complicated by the fact that neither author nor source title are explicitly evoked in the text. However, in the exordium to the Hebrew Epistle, Isaac recounts the story of a certain epistle in Arabic, which he later translated into Hebrew for the Jews of Syracuse. The only clue in the Hebrew text to its Arabic source is a reference to the later commentary to this unnamed Epistle by the same Arab author, entitled *The Unveiler of the Veil* (מסיר המסוה), which translates into *Rafʿ al-Ḥijāb* in Arabic. The latter is the title of a known commentary by a thirteenth-century mathematician, Ibn al-Bannāʾ on one of his mathematical tracts, *Talkhīṣ Aʿmāl al-Ḥisāb*.[1] This key detail enabled Tony Lévy to trace *The Epistle of the Number* back to its Arabic source, *Talkhīṣ Aʿmāl al-Ḥisāb*.[2]

> Isaac ben Solomon ben Ṣaddiq Ibn al-Aḥdab the Spaniard says: A Muslim scholar was once asked by some of his friends to compose a short treatise for them which would encompass all matters of the Science of the Number in a concise manner. He complied with their request and composed a very short epistle. He was doing marvels[3] in presenting its methods and in abbreviating it according to its matters and sent it to them. When the Epistle reached his friends, it was not accessible to them and it seemed to have transcended their comprehension.[4] They asked him to explain it. Sensing the limitation of their intellect, the author also reacted with cunning, by composing for them a commentary of the Epistle, which was so unusual and profound, that only a logician and a person who understands the giving of the causes of things could understand it. He named it

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[1] The complete title of Ibn al-Bannāʾ's commentary is *Rafʿ al-Ḥijāb ʿan Wujūh Aʿmāl al-Ḥisāb*, i.e. The Unveiling of the Veil on the Methods of the operations of Calculation.

[2] Lévy, 'L'algèbre Arabe', p. 288.

[3] The Hebrew expression הגדיל לעשות is found in Joel 2:20, where it is translated, for example, in the King James Bible version, as "he has done great things." I translated this expression in my own words in order to suit the context. I usually do not follow any standard translation of the biblical expressions in the introduction to *The Epistle of the Number*, but rather, I chose what I saw fit in the context of our text.

[4] Lit. 'the eyes of their wisdom'.

> *The Unveiler of the Veil*.[5] The readers replied by the same coin saying that its name is *The one who returns the Veil*.[6] In return, he also wrote to them the following: "I am committed to making the effort to expose the subject matters from the source of their fundaments. However, I am not obliged to try to make the wild beasts[7] understand." Later on, the Epistle spread among the wise,[8] for whom it became an epistle of utmost beauty. Its nature became known amongst them and they wrote on it many commentaries of many types, all very long. When I reached their countries I was dwelling in the *Tents of Kedar*[9] and that Epistle reached me. I studied it with of one of their sages. I also saw its author's commentary and the commentary of others until I reached its inner treasures and revealed all its mysteries.
>
> [fol. 1v]

Ibn al-Bannāʾ

Abū-l ʿAbbas Aḥmad Ibn Muḥammad Ibn ʿUtman al-ʾAzdi Ibn al-Bannāʾ (1256–1321) was born in Marrakesh, Morocco, where he was educated in a wide range of subjects: religion, literature, grammar, mathematics and astronomy. He was renowned for his excellent pedagogical skills. He made original contributions to the study of mathematics, in particular to the field of combinatorics. *Talkẖīṣ Aʿmāl al-Ḥisāb* is his most well-known mathematical tract. It became the standard textbook in mathematical education between the fourteenth and seventeenth centuries in North Africa. Many commentaries were written to *Talkẖīṣ Aʿmāl al-Ḥisāb*, including one by Ibn al-Bannāʾ himself, which is discussed in the next section. Many of his disciples promulgated his school of thought,[10] however much criticism was directed at Ibn al-Bannāʾ for being too concise in his writing. One of Ibn al-Bannāʾ's disciples, Ibn al-Qādī, recounts an anecdote supposedly based on a quote by Ibn al-Bannāʾ, which sheds light on Ibn al-Bannāʾ's elitist attitude, similarly to that described in the introduction to the Hebrew Epistle.[11]

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[5] Exodus 34:33.

[6] Exodus 34:34.

[7] Lit. 'the beasts of the forests'.

[8] Lit. 'those with wise hearts'.

[9] I.e. in a Muslim land. In the biblical context, Kedar was the second son of Ishmael. See Psalm 120:5.

[10] For more details on Ibn al-Bannāʾ's life, work and legacy, see Djebbar, *L'algèbre arabe genèse d'un art*, (Paris, 2005), p. 130 and *Talkẖīṣ*, pp. 7–35.

[11] One of the critiques was al-ʾAbili, who was Ibn Khaldūn's teacher of mathematics. For details on Ibn al-Bannāʾ's school see *Talkẖīṣ*, pp. 15–19 (or pp. 17–21 in the French part). Unless otherwise stated, the references to the *Talkẖīṣ* are in the Arabic part.

Naming the Hebrew Epistle

The text we now know as *The Epistle of the Number* bears no title and provides no hint as to what it was called at the time of its composition. Its modern name was given by Tony Lévy who "baptized" the text with this title because Isaac tells us about his decision to translate an Arabic Epistle on the Science of the Number into Hebrew, thus creating a Hebrew Epistle (albeit a very long one!).[12]

The Hebrew and Arabic texts compared

Talkhīṣ Aʿmāl al-Ḥisāb is a succinct book on the rules of arithmetic with a chapter dedicated to algebra. Isaac, as he promises in the introduction to *The Epistle of the Number*, presented a lengthy Hebrew text, which goes beyond a mere translation of the Arabic source. As we have seen above, Isaac also cautions his readers that he planned to omit a few sections from the Arabic source. However, a meticulous comparison between *The Epistle of the Number* and *Talkhīṣ Aʿmāl al-Ḥisāb* reveals no material lacunae in the Hebrew text.[13] In fact *The Epistle of the Number* is an expansion of its main Arabic source: numerical examples in abundance, verbal problems and commentaries and a wide range of algorithms and various elaborations of religious, linguistic and philosophical nature.

The readership and propagation of the Hebrew and Arabic texts

The aim and the readership of the Arabic and Hebrew texts diverge significantly, as does their mode of transmission. The core differences lie in the length and the format of the texts. Ibn al-Bannāʾ wrote a concise textbook for a wide group of potential students, probably as part of an organized curriculum in schools of administration in North Africa. His text inspired many commentaries, and it enjoyed a vast diffusion over a period of several centuries. Isaac, on the other hand, wrote an isolated treatise for the Jewish community in Syracuse after being requested by friends to compose a book on mathematics. With the exception of Nissim Abū al-Faraj, who also copied many of Isaac's treatises,[14] Isaac's student readership is largely unknown. It is not clear whether the Hebrew Epistle was written for self-study or whether it was studied in a group. It is also not known how many copies of the Hebrew Epistle were actually made throughout the centuries. The only surviving fragmentary copy from the sixteenth century attests to some interest in the text, but anything more is speculation.

As for the applicability of the mathematics within the Hebrew Epistle to the daily lives of Jews in Syracuse or elsewhere in Sicily, one can easily see how the basic arithmetical rules in the Epistle could have been applied in daily life, for example, the addition of numbers or division of a sum into unequal parts. On the other hand,

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[12] Lévy, 'L'algèbre Arabe', p. 286.

[13] The missing materials are a few tricks of divisibility that Isaac explicitly says that he would omit in his Hebrew version. See *The Epistle*, fol. 16.

[14] See chapter 1.

algebra does not distinguish itself as a particularly useful branch of 'daily mathematics'. As for the linguistic and philosophical discussions in *The Epistle of the Number*, they seem to be of even lesser practical value, but without doubt they are intellectually enriching. Bearing in mind Isaac's description of some of his Sicilian coreligionists as people who enjoyed learning mathematics in their spare time, it comes as no surprise that Isaac aimed to cater for his readers' intellectual curiosity without, perhaps, being overly concerned with practicalities.

TABLE OF CONTENTS OF *THE EPISTLE OF THE NUMBER*[15]

Book I: Arithmetical operations on integers, fractions and roots

Part I: Operations on Integers

Chapter I: Definition of a number, its different categories and ranks

- The categorization of numbers into integers and fractions
- The categorization of integers into even and odd numbers
- The categorization of even numbers: even-times even, even-times odd and even-times even-times odd numbers
- The categorization of odd numbers: prime and odd-times odd numbers
- The three ranks of numbers: units, tens, hundreds
- The building blocks of numbers
- The place-value decimal system with its graphical presentation in Hindu-Arabic numerals*
- The correspondence between the rank of a number and its name
- Calculating the rank of the extended number

Chapter II: The five types of addition

- The addition of arbitrary numbers and the test of addition by subtraction, the geometric sequence with a ratio of 2 (chequered squares)
- The sequence of natural numbers
- The sequence of the squares of natural numbers
- The sequence of cubes
- The sequence of odd numbers
- The sequence of the squares of odd numbers
- The sequence of the cubes of odd numbers
- The sequence of even numbers
- The sequence of the squares of even numbers
- The sequence of the cubes of even numbers

Chapter III: Subtraction

- Subtracting one number from another
- Casting by 7, 8, 9 as a means of testing

[15] A star indicates the themes in the Hebrew text which are absent from *Talkhīṣ Aʿmāl al-Ḥisāb*.

- Test by addition

Chapter IV: Multiplication
- Multiplication by translation
- Multiplication by half-translation
- Multiplication without translation
- Table of multiplication

Chapter V: Division
- Division of a large number by a smaller number (division of different categories: continuous and discrete*)
- Denomination- division of a small number by a larger one
- Division into unequal parts
- Division of sums
- Prime Numbers
- Divisibility tests (classifying prime number greater than ten as partials*)
- Eratosthenes' sieve

Chapter VI: Restoration and Reduction
- Restoration
- Reduction

Part II: Operations upon Fractions

Chapter I:
- The naming of fractions and the determination of their numerator

Chapter II:
- The addition and subtraction of fractions

Chapter III:
- The multiplication of fractions

Chapter IV:
- The division of one fraction by another and the naming of the result

Chapter V:
- The restoration and reduction of fractions

Chapter VI:
- The conversion of fractions

Part III: Operations upon the roots of expressible and inexpressible quantities[16]

Chapter I:
- The roots of integers and roots of fractions, approximation algorithms.

[16] I.e. rational and irrational numbers.

Chapter II:
- The addition and subtraction of binomials and apotomes

Chapter III:
- The multiplication of roots

Chapter IV:
- The division of roots

Book II: The rules which enable to obtain the unknown from the given known

Part I: Proportions and scales

Chapter I:
- Proportions: the rule of three
- The method of scales: the rule of double false position

Part II: Restoration and opposition

Chapter I:
- The basic algebraic operations: restoration, opposition and equalization
- The algebraic species: numbers, roots, squares, cubes, square squares etc.
- Classes of numbers*

Chapter II: The procedure for the six types of equalization
- Abbreviated notation for roots, squares and cubes.*
- Solving the six equations

Chapter III: Algebraic expressions
- The construction of algebraic expressions
- The particle of addition *and*
- The particle of subtraction *less*
- Permissible values for algebraic expressions*

Chapter IV: Multiplication of algebraic expressions
- The degrees of the different species
- The names of the degrees
- The multiplication of species
- The multiplication of an algebraic species by a number
- Equations of third and fourth degrees which contain no number
- The multiplication of an additive, a subtractive by a subtractive and an additive by a subtractive
- The method for multiplying algebraic expressions with the help of tables

Chapter V: Division
- The division of monomials
- The division of monomials with the same degrees
- The division of a monomial by a number
- The division of a subtractive algebraic expression by another one containing only one term

- The impossibility of dividing a lower degree by a higher one
- The impossibility of dividing by a subtractive

*Part III: Problems of Practical Nature**

Chapter I: Theory and practice for the Solution of practical problems

- Six problems illustrating the six equations
- A linear problem with one variable – the distribution of money to charity
- Two linear problem of encounter
- Five linear problems with many variables – the joint purchase of a horse (truncated immediately after the fifth problem)