MATHEMATICS EXPLAINED for HEALTHCARE PRACTITIONERS

DEREK HAYLOCK
& PAUL WARBURTON

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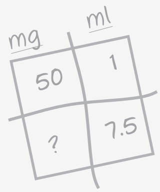

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Derek Haylock is an education consultant and author. He is a Senior Fellow at the University of East Anglia, Norwich, where he worked for over 30 years in teacher education. He is internationally known for his research and writing in the field of mathematics education; his work in mathematics education has taken him to Germany, Belgium, Lesotho, Kenya, Brunei and India. Derek is a major author in mathematics education, with a large number of published books and chapters in books. His best-selling book *Mathematics Explained for Primary Teachers* (Sage Publications) has been the leader in the field for many years, with a fourth edition published in 2010. His work is characterized by a commitment to explain mathematics in simple and accessible ways and to enable children and adults to learn the subject with understanding and confidence.

Paul Warburton is a registered nurse, senior lecturer and the coordinator of the Non-Medical Prescribing (NMP) programme at Edge Hill University, Ormskirk, Lancashire. Paul was also the project leader for the development and implementation of the NHS North-West Strategic Health Authority's on-line Numeracy Assessment Tool. Paul developed his interest in the numeracy skills of healthcare practitioners when he noted the incidence of qualified and even senior healthcare practitioners failing the NMP programme because of poor numeracy. He has extensive experience of writing and conference presentation about numeracy, non-medical prescribing and patient safety.

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INTRODUCTION

Welcome to this book on mathematics in healthcare practice and, assuming you have, well done in deciding to read the introduction! It is likely that you have picked up this book because you are working or training in some area of healthcare and have some concern about your personal confidence in the aspects of numeracy that are required in your professional work or training. You may be keen to learn more about the applications of mathematical understanding and skills in healthcare practice; or you may just be looking to revise and improve your own skills. Whatever the reason, we hope that your engagement with this book will prove to be an important step on the journey to improved mathematical skills, knowledge and understanding.

Why is this book needed?

The book is written for both qualified and pre-registration healthcare practitioners, whatever their chosen profession, and contains information, explanation, exercises and advice that will enable you, the reader, to improve your numeracy skills in practice. Our aim is also that you will develop an *understanding* of the underpinning mathematical processes and concepts involved. We appreciate that there is a risk that your enthusiasm to continue reading may now start to decline, but we really do want to help the reader to learn with understanding, not just to learn rules and recipes that make little sense to them. We recognize that many people are challenged by the numerical demands that occur in professional practice and become anxious when people start talking about 'mathematics', 'numeracy' or 'calculation'. Our aim is to replace that anxiety with confidence, the kind of confidence that is based on some degree of understanding of what is going on when we use and manipulate numbers. We hope that even those practitioners who start out with deeply-embedded feelings of anxiety and insecurity about mathematics will find the content and style of the book to be accessible, relevant, interesting, stimulating, and even enjoyable.

The material that we cover here is selected to enable practitioners to deal confidently and competently with the calculations and numerical situations that they will encounter on a daily basis in their work. Recognizing the importance of understanding, we encourage the reader to develop mental strategies for handling calculations, to draw upon their existing knowledge, to check that answers make sense and to develop a feeling for numbers and the relationships they have with each other. Throughout the book examples of calculations are taken, where appropriate, from a healthcare context to explain the mathematical ideas and to illustrate how the calculations are performed in practice.

Accuracy in numeracy skills and understanding of numbers are important life skills that are needed by everyone to cope confidently with a range of everyday situations such as shopping, working out the cost of utility bills, how much money is left in a bank account and dividing a pizza fairly. In a healthcare setting there are specific numeracy skills that are essential to enable practitioners to deal accurately and confidently with such things as drug doses, infusion rates, body mass indices and patient fluid balances. In everyday life mediocre or low numeracy skills may cause occasional embarrassment or even leave a person open to exploitation by others. But in a healthcare setting the consequences of poor numeracy skills can be serious not just for the practitioner but also for their patients. Patients have the right to expect that healthcare professionals are competent in the calculations that relate to their care and treatment. However, there is evidence from studies in the United Kingdom and elsewhere that inadequate levels of numeracy amongst healthcare practitioners is not uncommon, and that some healthcare staff demonstrate such a low level of competence in numeracy that this can lead to error and put patients at risk.

Problems with numeracy amongst the general population in the United Kingdom are well recognized and acknowledged as a cause of concern for successive governments. Despite recurrent investment in projects supporting adult numeracy in the UK, up to half of the working population in England has been found to have mathematics skills that are no better than those expected of a primary school pupil (www.nationalnumeracy.org.uk, 2012). Since it is from this population that the majority of UK healthcare professionals are recruited, it is reasonable to conclude that numeracy skills may be inadequate within a significant proportion of the large workforce employed in the healthcare professions. This is now a recognized cause of concern for healthcare employers, educators and professional regulators.

Healthcare practitioners enter their chosen profession to improve people's lives and to perform their role safely, competently and to the best of their ability. All healthcare practitioners are educated to ensure they are competent in their roles and it is reassuring that most healthcare is delivered to patients safely and appropriately. However, inaccurate dose calculations by healthcare staff continue to occur, leading to medication errors. Drug dosage errors in the prescribing and administration of medications are a common cause of the patient safety incidents reported to the National Patient Safety Agency (NPSA) in the United Kingdom, and a leading cause of patient harm. In children, where the calculations can be more complex than in adults and where the patient's tolerance of error is lower, medication errors due to incorrect dosage or strength of medication continue to be significant and common causes of reported harm to patients. Calculation errors by a factor of 10, for example, are a common cause of such errors in paediatric settings.

All qualified healthcare practitioners have a duty to remain competent in their practice. Professional regulators such as the General Pharmaceutical Council, the Health Professions Council and the Nursing and Midwifery Council acknowledge the importance of numeracy skills in the role of their registrants. Each of these regulators sets minimum mathematics qualifications or standards of numeracy for entry to pre-registration educational programmes, progression within these programmes and final qualification. In writing this book we have ensured that the mathematical skills needed to meet such standards are covered in depth.

How is the book structured?

Although there are many books on the subject of numeracy we hope that this one will prove to be distinctive. First, it is structured around the mathematical ideas, drawing on healthcare contexts to illustrate and explain these. Second, rather than merely teaching rules for specific healthcare tasks, we aim to improve the reader's numeracy skills in a range of contexts through their understanding of mathematical concepts and principles and by building confidence in handling all kinds of numbers: whole numbers, decimals, fractions, percentages, and especially numbers in the context of measurement, such as dosages, rates and concentrations. Third, we do not assume that there is only one approved way of doing any particular kind of mathematical calculation, or that formal, written methods are superior to mental and informal strategies. We encourage the reader to be alert to how the relationships between numbers can be exploited and to use intuitive approaches to calculations. If after working through this book you still feel you need to use 'long multiplication' or a calculator to multiply 0.25 grams by 12, for example, then we have failed!

A set of specific numeracy objectives is set out at the start of each chapter. These objectives indicate what the practitioner should be able to do in a practical healthcare context when they have mastered the material in that chapter.

These objectives are followed by a feature called *Spot the errors* (except in Chapter 15). The reader is challenged to read quickly through a number of statements and to identify any mathematical errors. This section is intended to simulate the way that practitioners often have to deal with mathematical calculations and instructions under pressure without much time to stop and reflect. The errors are then identified and explained and, from Chapter 4 onwards, the significance of the errors in a healthcare context is discussed. If you spot all our deliberate errors, then you have our authority to give yourself a hearty pat on the back!

Each chapter then provides detailed explanation and illustration of the mathematical ideas and processes specified in the objectives, with examples of their application in practice and with key points highlighted in boxes throughout the text. At the end of each chapter there is a section called *Have a check-up*. This consists of some exercises for you to work through to check your learning of the content of that chapter; annotated answers to these check-up questions are given at the end of the book.

We have set out to write an accessible book on a subject that we know many readers find challenging. The book combines Derek's mathematical knowledge and understanding of the teaching and learning of the subject with Paul's clinical experience and understanding of the importance of numeracy in the healthcare environment. We hope that this combination will enable the reader to develop an understanding and confidence in the use of numbers in both their daily life and their professional practice.

How to use this book

The first thing to say is that to engage with the material in each chapter of this book you will need to work through the mathematics, with a pencil and paper; have a go at the *Spot the errors* section; work through the examples; do the check-up questions, don't just look up the answers! We recommend that you check our working and scribble your own mathematical jottings all over the book; this will help you to engage with the material (and help us by undermining the second-hand book market!).

For most readers we would recommend that you work through the book chapter by chapter. As we have explained above, the book is written to improve the reader's understanding of numeracy and their skills and confidence in using numbers. It takes the reader through the various mathematical processes and concepts, building their knowledge and understanding of these chapter by chapter. So it is best to work through the chapters in the order they are written, ensuring that you have a thorough understanding of the content of one chapter before moving on to the next. This approach will enable the gradual development of a broad understanding of the different processes and concepts that are likely to be encountered in healthcare practice and daily life. We hope that you will then want to use the book in the future as a reference book, to revise and reinforce particular skills and to check up on things you have forgotten.

A DISCLAIMER

Readers should be aware that the healthcare examples used in this book are chosen to illustrate and explain specific mathematical ideas. While we have taken care to make these examples as genuine as possible and to use realistic or recommended doses of particular drugs, it must be understood that this book is not a manual for healthcare practice or a guide to drug dosages.

UNDERSTANDING THE NUMBER SYSTEM

1

OBJECTIVES

The practitioner should be able to:

- understand the place-value principle in our number system for both whole numbers and decimal numbers
- use and interpret powers of 10
- recognize the positions of whole numbers or decimal numbers on a section of a number line
- appreciate where zeros are needed in decimal numbers
- express a decimal number as tenths, hundredths or thousandths

Our mission throughout this book is to increase your understanding of mathematics, because we know that this will help to make you a more confident and competent healthcare practitioner. So, we begin with refreshing your understanding of numbers! The purpose of this chapter is to remind you of some fundamental ideas about whole numbers and decimals and to ensure that you have a clearer grasp of how our base-ten place-value number system works. The practitioner will need to be able to apply this understanding of the number system in the context of their healthcare practice, especially when the numbers are used in measurement. Various aspects of measurement are not explained until Chapters 4–7 of this book, so it is from those chapters on that we will illustrate and apply mathematical ideas more specifically in healthcare contexts.

SPOT THE ERRORS

Identify any obvious mathematical errors in the following ten statements.

1. The digit 7 in the number 87 654 represents seven thousand.
2. Counting in ones, the next number after three thousand and ninety-nine is four thousand.
3. On a number line 8050 lies halfway between 8000 and 8100.
4. To calculate the cost per day of a 28-day course of medicine costing £75.60, a pharmacist enters 75.60 ÷ 28 on a calculator and gets the result 2.7; this means the cost per day is two pounds and seven pence.
5. The number 0.0008 is ‘eight ten-thousandths’.
6. The number halfway between 0.007 and 0.008 is 0.075.
7. The number 4567 is equal to $(4 \times 10^3) + (5 \times 10^2) + (6 \times 10^1) + (7 \times 10^0)$.
8. The number 0.067 is equal to $(6 \times 10^{-2}) + (7 \times 10^{-3})$.
9. In this set {0.09, 0.8, 0.084, 0.18, 0.48}, the greatest number is 0.8 and the smallest number is 0.084.
10. 1.275 is equal to 1 unit and 275 thousandths, which is 1275 thousandths.

(errors identified on page 4)

How does place value work?

Over history there have been many systems for representing numbers. The system used internationally today is essentially a Hindu-Arabic system that has ten as its base and uses the principle of *place value*. This principle means that, for example, the symbol '5' occurring in a numeral might sometimes mean 'five' (as in 65), but it could mean 'fifty' (as in 56) or 'five thousand', as in (5678), or 'five hundredths' (as in 2.75), and so on, depending on the *place* in which it is written. Most ancient number systems did not use this principle. For example, in Roman numerals the symbol 'V' wherever it is written, for example in XV or in CLXVIII, means 'five'.

Numeral and number

A numeral is a symbol used to represent a number. For example, the number of teeth that an adult should have is represented by the numeral 32.

The place-value system for numbers is based on *powers* of ten. These are: one, ten, a hundred, a thousand, ten thousand, a hundred thousand, a million, and so on, getting ten times bigger each time. These powers of ten can be expressed symbolically and in words in a number of ways as shown in the table in Figure 1.1.

one	1	10^0	ten to the power 0	1
ten	10	10^1	ten to the power 1	1 × 10
a hundred	100	10^2	ten to the power 2	1 × 10 × 10
a thousand	1000	10^3	ten to the power 3	1 × 10 × 10 × 10
ten thousand	10 000	10^4	ten to the power 4	1 × 10 × 10 × 10 × 10
a hundred thousand	100 000	10^5	ten to the power 5	1 × 10 × 10 × 10 × 10 × 10
a million	1 000 000	10^6	ten to the power 6	1 × 10 × 10 × 10 × 10 × 10 × 10

Figure 1.1 A table showing increasing powers of 10

There are a few things to note here. In general, ten to the power n (10^n) means 1 multiplied by 10, n times, and this is equal to a number written as 1 followed by n zeros. For example, 10^9 would be 1 multiplied by 9 tens: $1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$; written out in full as a numeral this number (a billion) is 1 followed by 9 zeros.

The expression 10^2 (ten to the power 2) is also read as 'ten squared'; and 10^3 (ten to the power 3) is also read as 'ten cubed'.

We have included 'ten to the power zero' at the top of the table above, as a way of writing the number 1. Although this might seem a bit weird, you should at least be able to see how it fits into the pattern of the table. This use of 'to the power zero' is a mathematical convention adopted for the sake of completeness; it also provides a bridge for extending the place-value system from whole numbers to decimal numbers (see below). Think of it as meaning '1 not multiplied by any tens' or '1 followed by no zeros'.

This table could go on for ever, continuing with ten million, a hundred million, a billion, ten billion, and so on. But the larger the power of ten the less useful becomes the actual name. For example, 'a trillion' might not convey much to us, until someone tells

ERRORS IDENTIFIED

The obvious errors are in Statements 2, 4 and 6.

Statement 2

The next number after three thousand and ninety-nine (3099) is three thousand one hundred (3100). We hope this is obvious now you see the numbers written as numerals.

Statement 4

The cost is two pounds and seventy pence (£2.70) per day. The numbers 2.7, 2.70, 2.700, 2.7000, and so on, all represent the same quantity, since the zeros simply tell us that there are no tenths, no hundredths, no thousandths, and so on. Calculators usually do not display these ‘trailing zeros’ and will give any of these results as 2.7. The ‘7’ in this context means ‘7 tenths of a pound’, which is seven ten-pences, or seventy pence. For sums of money the convention (which calculators ignore) is always to write them with two digits after the point, in order to avoid this kind of confusion.

Statement 6

The number halfway between 0.007 and 0.008 is actually 0.0075. This is more obvious if we write all the numbers with four digits after the decimal point: 0.0070, 0.0075, 0.0080. They are shown on a number-line diagram in Figure 1.2, along with a few other numbers in their vicinity.

(now continue reading from page 3)

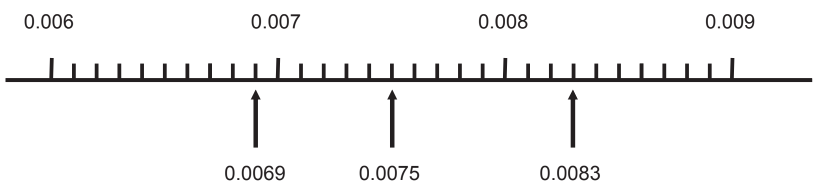

Figure 1.2 A number-line diagram from 0.006 to 0.009

us that it actually stands for 10^{12} (1 followed by 12 zeros). An important (and very large) number in science that we will mention in Chapter 14 involves 10^{23}; written like this, as a power of ten, this is much easier to grasp than referring to it as 'a hundred sextillion'!

Note also that we use the recognized convention for writing numbers with more than four digits: the digits are grouped in threes from the right, and then written with spaces between the groups of three. So, a million is correctly written 1 000 000, not 1,000,000. The reason for this is that some countries use a comma where we use a decimal point.

Having got these powers of ten, we can now see how our base-ten place-value number system works. First, we need only ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and with these we can (theoretically) count any number in the universe! We count beyond 9 by using the place-value principle: the value of a digit in a numeral depends on where it is written. As we work from right to left (for whole numbers) the first place we might write a digit represents 'ones' (sometimes called units), the next place 'tens', the next 'hundreds', the next 'thousands', and so on. So, the numeral, 2345, for example, represents '5 ones, 4 tens, 3 hundreds and 2 thousands'.

Digits

A *digit* is one of the ten symbols used to construct numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

But, of course, we actually read these numbers from left to right – and we use certain language conventions when expressing the numbers in words. So we read 2345 as: 'two thousand, three hundred and forty-five'. 'Forty' here is shorthand for 'four tens', of course.

This system means that the digit at the front of the number (on the left) is the most significant, because it always represents the greatest quantity – and the digits become less significant in terms of what they represent as we move to the right. We discuss the idea of 'significant digits' further in Chapter 8.

So, the numeral 2345 is a condensed way of representing a number that could also be written in full as: 2000 + 300 + 40 + 5.

Or, using powers of ten: $(2 \times 10^3) + (3 \times 10^2) + (4 \times 10^1) + (5 \times 10^0)$.

EXAMPLE 1.1

What are the values represented by the digits 3 and 0 in 53 028? How do you read this number out loud? What is it expressed in powers of 10?

(a) The digit 3 represents 3 thousands, or 3000, or 3×10^3.

The digit 0 (zero) represents 'no hundreds', or 0×10^2.

(b) This number is read as: 'fifty-three thousand and twenty-eight'.

(c) $53\,028 = (5 \times 10^4) + (3 \times 10^3) + (0 \times 10^2) + (2 \times 10^1) + (8 \times 10^0)$.

The digit zero in a numeral like 53 028 is called a 'place-holder', occupying an otherwise empty place (there are no hundreds) and ensuring that the preceding digits are in their correct place. In the history of our number system the invention of a symbol for zero for this purpose was a highly significant breakthrough. This aspect of zero is particularly important in some decimal numbers, as we shall see below.

How are numbers positioned on a number line or numerical scale?

Number lines, particularly in the form of various measuring scales, are important in healthcare practice. They are all derived from the simple idea of representing numbers as points on a line. This enables us to recognize visually the positional relationship between numbers. We start with simple number lines using only whole numbers.

Figure 1.3(a) shows a section of a number line from 2000 to 5000. Each step on this line represents a thousand, as we progress from 2000 to 3000, to 4000, to 5000. The number 3758 must lie on this line, somewhere between 3000 and 4000, because it is greater than 3000 and less than 4000. In Figure 1.3(b) the section between 3000 and 4000 is shown enlarged and divided into ten equal sections. Each of these sections must represent a step of a hundred along the line (because ten hundreds equal a thousand). So the points marked on the number line in (b) are 3000, 3100, 3200, 3300, and so on,

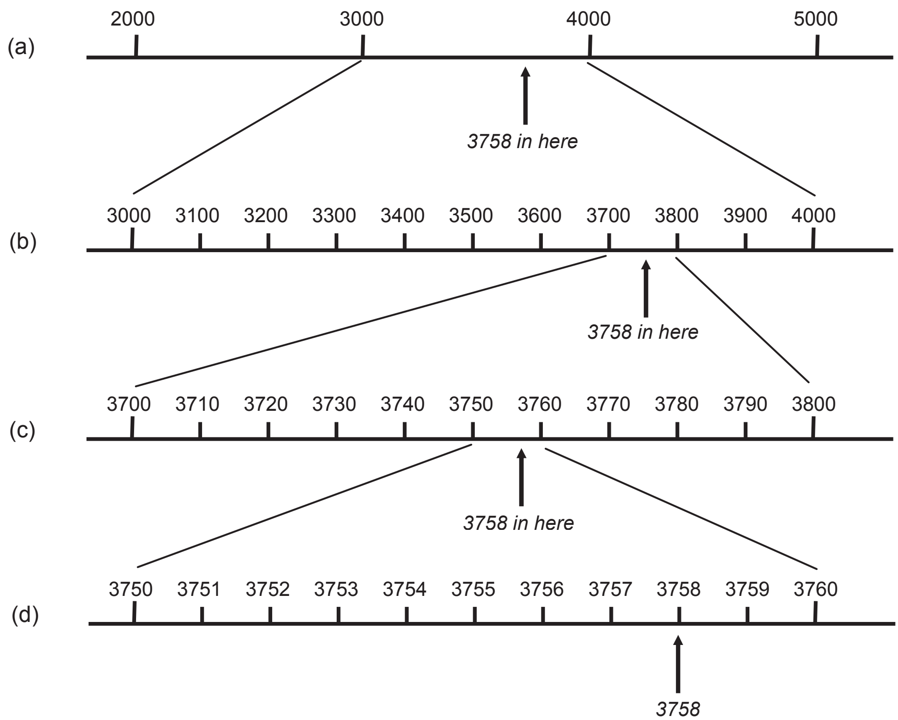

Figure 1.3 Locating 3758 on a number line

up to 4000. The number we are chasing lies between 3700 and 3800. So, in Figure 1.3(c) we have expanded this section of the line and again divided it up into ten equal sections. Each of these sections now represents a step of 10 along the line (because ten 10s equal a hundred). So the points marked on the line in (c) are 3700, 3710, 3720, 3730, and so on, up to 3800. Our number (3758) now lies between 3750 and 3760. So we repeat the process in 1.3(d), where the ten equal sections between 3750 and 3760 are now steps of one: 3750, 3751, 3752, and so on. Now we can see exactly where 3758 lies.

OK, so we have rather laboured the point here! But this way of seeing numbers is absolutely crucial in reading scales, as well as in having a sense of how numbers relate to each other in terms of order and relative position. And this process of repeatedly subdividing spaces between consecutive numbers on a scale into ten equal sections will be particularly important in your understanding of decimal numbers.

So, how does place value work with decimal numbers?

The building blocks of decimal numbers

A tenth = 0.1
A hundredth = 0.01
A thousandth = 0.001

Just as we can divide thousands into ten hundreds, hundreds into ten tens, and tens into ten ones, we can continue dividing sections of the number line repeatedly into ten smaller parts, generating tenths, hundredths, thousandths, and so on. A tenth is what you get if you divide a unit into ten equal parts. A hundredth is what you get if you divide a tenth into ten equal parts. A thousandth is what you get if you divide a hundredth into ten equal parts. And so on.

So this is how we are able to represent quantities less than 1, using decimal numbers. We put a 'decimal point' (a full stop) after the digit representing ones (units), and then continue with digits representing these smaller quantities. So, the 1 in the decimal number 0.1 means a tenth; in 0.01 it means a hundredth; in 0.001 it means a thousandth, and so on. These are the building blocks of decimal numbers.

Then, for example, in the decimal number 2.358, the 2 represents 2 ones (2 units); the 3 represents 3 tenths of a unit; the 5 represents 5 hundredths of a unit; and the 8 represents 8 thousandths of a unit.

Surprisingly, tenths, hundredths, thousandths, ten thousandths, and so on, can also be expressed as powers of ten. This is done by following the patterns in the table in Figure 1.4, continuing the powers of ten from zero into negative numbers: 0, −1, −2, −3, and so on. There's no need to be puzzled by these negative powers of ten: just accept the notation as a convenient (and actually rather clever) mathematical convention for representing the smaller and smaller subdivisions into ten equal parts.

Figure 1.4 is a table showing tenths, hundredths, and so on, how they are written as decimal numbers, and the corresponding negative powers of ten.

Negative powers of 10

An example:
$10^{-4} = 0.0001$
$= 1 \div 10 \div 10 \div 10 \div 10$

Again, there are a few things to note. First, just be aware of how this table is generated by continuing a pattern to produce smaller and smaller quantities, each a tenth of the preceding one, and how these can be expressed using the variety of language and notation. So, for example, 'a thousandth' is written as the decimal numeral 0.001; as a power

one	1	10^0	ten to the power 0	1
a tenth	0.1	10^{-1}	ten to the power –1	1 ÷ 10
a hundredth	0.01	10^{-2}	ten to the power –2	1 ÷ 10 ÷ 10
a thousandth	0.001	10^{-3}	ten to the power –3	1 ÷ 10 ÷ 10 ÷ 10
a ten thousandth	0.0001	10^{-4}	ten to the power –4	1 ÷ 10 ÷ 10 ÷ 10 ÷ 10
a hundred thousandth	0.000 01	10^{-5}	ten to the power –5	1 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10
a millionth	0.000 001	10^{-6}	ten to the power –6	1 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10

Figure 1.4 A table showing decreasing powers of 10 below zero

of 10 it is 10^{-3} (ten to the power negative 3); it is also what you get if you *divide* 1 by 10 three times (1 ÷ 10 ÷ 10 ÷ 10). To see this pattern in action, enter 1 onto a calculator then divide by 10 repeatedly. This helps us to interpret a negative power of 10, like 10^{-3}. It just means divide one by ten, 3 times!

Note also that the same convention applies for grouping digits after the decimal point in threes as applies for the digits in front of the decimal point.

Note further that we have written a zero in front of the decimal point in each of the numbers 0.1, 0.01, 0.001, and so on. This addition of a leading zero is not strictly necessary; it is there just to make sure we spot the decimal point! This is particularly important in healthcare practice, where a dose written, say, as .5 grams might be misread as 5 grams. The zeros after the decimal point in 0.01 and 0.001 are necessary place-holders, of course. (Grams and other units of weight are explained in Chapter 6.)

Then, of course, this table could continue for ever, producing smaller and smaller quantities, each one of them a tenth of the previous one. So, for example, one molecule of common salt (sodium chloride) would weigh about 10^{-22} grams. This means taking a gram and dividing it into 10 parts, dividing one of these into 10 parts, dividing one of these into 10 parts, and continuing to do this 22 times in total.

We now have all we need to represent numbers that have parts smaller than 1 – what are called decimal numbers. Consider, for example, the decimal numeral 3.758. This is read as 'three point seven five eight'. It stands for 3 ones (units), 7 tenths, 5 hundredths and 8 thousandths. This can also be written in expanded form as: 3 + 0.7 + 0.05 + 0.008.

Or, using powers of ten: $(3 \times 10^0) + (7 \times 10^{-1}) + (5 \times 10^{-2}) + (8 \times 10^{-3})$.

EXAMPLE 1.2

What are the values represented by the digits 3 and 2 in 5.3028? How do you read this number out loud? What is it expressed in powers of 10?

(a) The digit 3 represents 3 tenths, or 0.3, or 3×10^{-1}.

The digit 2 represents 2 thousandths, 0.002, or 2×10^{-3}.

(b) This number is read as: 'five point three zero two eight'.

(c) $5.3028 = (5 \times 10^{0}) + (3 \times 10^{-1}) + (0 \times 10^{-2}) + (2 \times 10^{-3}) + (8 \times 10^{-4})$.

Figure 1.5 shows how we can locate the number 3.758 on the number line. Notice that Figure 1.5 is identical to Figure 1.3 (apart from the labels). This shows that place value works in exactly the same way beyond the decimal point as it does before it.

So, in Figure 1.5(a) 3.758 lies somewhere between 3 and 4, because the first digit tells us that we have three units and the '.758' tells us we have some other bits of a unit to add to this. So, we follow the process we used in Figure 1.3: divide the section from 3 to 4 into ten equal steps, each of which is a tenth of a unit, giving us 3, 3.1, 3.2, 3.3, and so on. Our number (3.758) lies between 3.7 and 3.8, as shown in Figure 1.5(b). In (c) the section from 3.7 to 3.8 is divided into ten equal steps, this time using hundredths of a unit, giving us 3.7, 3.71, 3.72, 3.73, and so on. Our number (3.758) lies between 3.75 and 3.76. In (d) we divide this into ten equal steps (thousandths), and hence we locate 3.758. As we move from (a) to (d) in Figure 1.5, we can see that our number 3.758 is (a) 3 units + (b) 7 tenths + (c) 5 hundredths + (d) 8 thousandths.

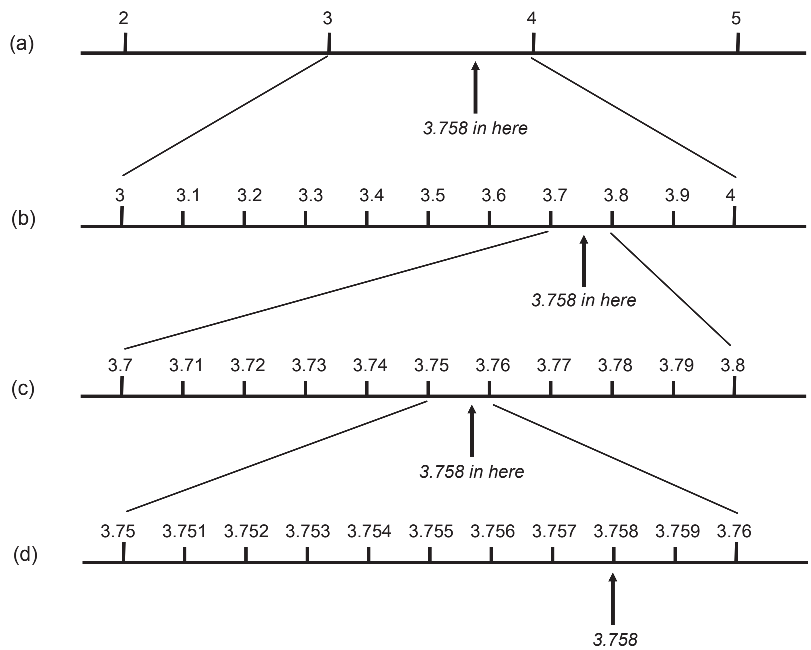

Figure 1.5 Locating 3.758 on a number line

EXAMPLE 1.3

On the section of a number line shown in Figure 1.6, (approximately) what are the numbers indicated by A, B and C?

The steps from 0.3 to 0.4, from 0.4 to 0.5, and so on, are steps of a tenth of a unit (0.1). These are then divided into 10 equal parts by the smaller gradations on the line, which therefore give steps of a hundredth (0.01). The points marked from 0.3 to 0.4, for example, are 0.3, 0.31, 0.32, 0.33, … 0.39, 0.4.

So, A is 0.32 and, similarly, B is 0.45. Note that B is the midpoint between 0.4 and 0.5.

C is about halfway between 0.56 and 0.57, so, imagining the gap between 0.56 and 0.57 divided into ten (even smaller) sections (thousandths), C would be at about 0.565. C represents 5 tenths, 6 hundredths and 5 thousandths.

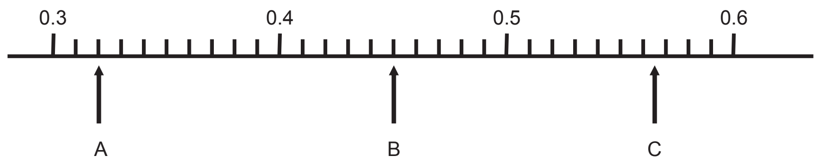

Figure 1.6 Number line section for Example 1.3

What else is there to know about zeros in decimal numbers?

In Figure 1.6, it is probably easier to identify the points A and B if we write the points on the scale as 0.30, 0.40, 0.50 and 0.60. Sticking a zero on the ends of these decimals doesn't change their values at all, because the zeros mean simply 'and no hundredths'. The points marked from 0.3 onwards (including 0.3) are then 0.30, 0.31, 0.32, 0.33, 0.34, 0.35, 0.36, 0.37, 0.38, 0.39, 0.40. Now this looks very similar to counting from 30 to 40! Notice also that it is much more obvious that B, being halfway between 0.40 and 0.50, must be 0.45. In the same way, in identifying C, if we think of 0.56 and 0.57 as 0.560 and 0.570, it is much easier to see that the point in the middle between them is 0.565. Have a look again at the correction to *Spot the errors* Statement 6 at the beginning of this chapter. The zeros we are exploiting here are called *trailing zeros*.

Zeros in numerals have some important roles to play! We have earlier commented on the role of zero in our number system as a place-holder, in numbers like 5007 (no hundreds, no tens) or 5.007 (no tenths, no hundredths). We have also noted the convention of placing an additional leading zero in front of a decimal point if there are no other digits there. Technically, we could place as many zeros as we wish at the front of a number – for example, 2.56 = 02.56 = 002.56 = 0002.56, and so on – because these zeros simply tell us that there are no tens no hundreds, no thousands, and so on – although it is not often that we would have a reason to do this. Occasionally it helps in doing some calculations, to ensure that we have all the ones, tens and hundreds, and so on, lined up correctly. Another area where we often use leading zeros is in recording time. At the time of writing this sentence it is 09:05 on 08.05.2012. Here we are using the convention of putting a leading zero in front of a single digit to give the hour, the number of minutes, the day and the month all with two digits – probably reflecting digital displays of dates and times.

But you will find that trailing zeros are quite often used in decimal numbers. A decimal number, such as 2.75, could also be written as 2.750, 2.7500, 2.75000, and so on, with as many of these trailing zeros as we wish. Trailing zeros like these are used particularly to ensure that that a group of numbers or measurements are all being given to the same level of accuracy. For example in a men's 100-metres race the times would be recorded in hundredths of a second, so they would appear with two digits after the decimal point. If one runner's time is 9.9 seconds, this would be given as 9.90, so that it can be more easily compared with the other times of 9.92, 9.89, 9.85 and 9.96 seconds. We have seen in Example 1.3 above how handy it is sometimes to stick one or two additional trailing zeros on decimal numbers.

Note that calculators do not show trailing zeros in the results of calculations. If you use a calculator to add £1.26 and £3.44, for example, the result displayed will be 4.7, which you then have to interpret as £4.70. (See *Spot the error* Statement 4 and its correction.)

EXAMPLE 1.4

(a) Put these numbers in order, from the smallest to the greatest: 0.9, 0.091, 0.91, 0.019, 0.19, 0.09.

Add trailing zeros so that all the numbers have three digits after the decimal points: 0.900, 0.091, 0.910, 0.019, 0.190, 0.090.

Now it is easier to put them in order: 0.019, 0.090, 0.091, 0.190, 0.900, 0.910.

Now dropping the trailing zeros, they are in order: 0.019, 0.09, 0.091, 0.19, 0.9, 0.91.

(b) What time comes halfway between 33.8 seconds and 33.9 seconds?

Add a trailing zero to each time so they become: 33.80 and 33.90 seconds. It is now clear that halfway between them is 33.85 seconds.

How do you read and interpret decimal numbers?

Although we might say ‘two pounds ninety’ when reading £2.90 – because we see the 90 as representing 90 pence – it is not correct to read the decimal number 2.90 as ‘two point ninety’. It should be read as ‘two point nine zero’. Similarly, 5.67 is not ‘five point sixty-seven’ is it ‘five point six seven’. The reason for this is that if you say ‘five point sixty-seven’ the response could be ‘sixty-seven what?’

The actual answer to that question is ‘sixty-seven hundredths’. When you see 5.67, the 6 means 6 tenths and the 7 means 7 hundredths. But, because a tenth is ten hundredths, the 6 tenths are equivalent to 60 hundredths. So, together with the 7 hundredths, the digits after the decimal point are equivalent to 67 hundredths in total. So, 5.67 (read as five point six seven) is equal to 5 units and 67 hundredths. Furthermore, since the 5 units are equivalent to 500 hundredths of a unit, the whole 5.67 is equal to 567 hundredths. That’s just like saying £5.67 is equal to 567 pence.

Here’s another example: 1.025 (one point zero two five). How can we interpret this? Well, as it stands it means 1 unit, 2 hundredths and 5 thousandths. But the 2 hundredths are equivalent to 20 thousandths, so 1.025 can be seen as 1 unit and 25 thousandths. Then, because the 1 unit is equivalent to 1000 thousandths, we could convert the entire number to 1025 thousandths. So, 1.025 = 1025 thousandths.

This way of looking at decimal numbers is very important when it comes to converting measurements between different units, as we shall see in Chapters 4 to 6. In Chapter 4, for example, we will see how a measurement of 1.75 metres is converted to 175 centimetres (hundredths of a metre). And, in Chapter 5, how 1.750 litres is converted to 1750 millilitres (thousandths of a litre).

EXAMPLE 1.5

(a) Write in tenths: 12.5

12.5 = 12 units and 5 tenths = 125 tenths

(b) Write in hundredths: 2.25; 12.33; 0.8

2.25 = 2 units and 25 hundredths = 225 hundredths

12.33 = 12 units and 33 hundredths = 1233 hundredths

0.8 = 0.80 = 80 hundredths.

(c) Write in thousandths: 0.175; 0.067; 5.67

0.175 = 0 units and 175 thousandths = 175 thousandths

0.067 = 0 units and 67 thousandths = 67 thousandths

5.67 = 5.670 = 5 units and 670 thousandths = 5670 thousandths.

1.1 Consider the numeral 72 405.

(a) In words, what does the 2 represent?
(b) Write your answer to part (a) using a power of 10.
(c) What does the zero represent?

1.2 In the decimal numeral 0.9208,

(a) in words, what does the 2 represent?
(b) Write your answer to part (a) using a power of 10;
(c) what does the zero after the 2 represent?

1.3 Which is the largest and which the smallest in this list of numbers:

5.601, 5.61, 5.061, 5.16, 5.016, 5.106?

1.4 Which number lies halfway between:

(a) 0.08 and 0.09?
(b) 0.1 and 0.11?

1.5 (a) Which is the larger: 1×10^7 or a million?
(b) Which is the smaller: 1×10^{-7} or a millionth?

1.6 In Figure 1.2 the main points on the scale are labelled 0.006, 0.007, 0.008, 0.009.

(a) How would the next main point in this sequence be marked?
(b) Roughly, where would the point labelled 100 be on this scale?

1.7 Identify (approximately) the numbers labelled A, B and C in Figure 1.7.

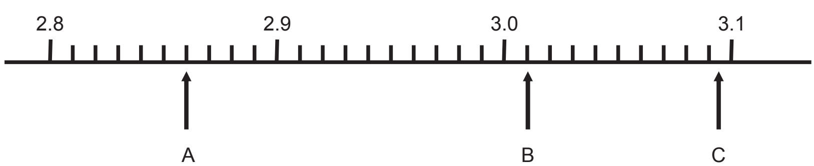

Figure 1.7 Number line section for check-up question 1.7

1.8 The labels on the section of the number line shown in Figure 1.8 have been removed. But the arrows indicate where on this line 0.301 and 0.309 would come. What are the labels missing from the four boxes?

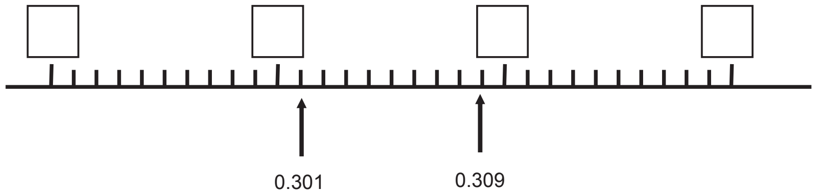

Figure 1.8 Number line section for check-up question 1.8

1.9 (See Example 1.5.) Write:
(a) 0.275 as thousandths;
(b) 25.6 as tenths;
(c) 12.5 as hundredths;
(d) 1.05 as thousandths.