Third Edition

Quantum Non-Locality & Relativity

Tim Maudlin

WILEY-BLACKWELL

QUANTUM NON-LOCALITY AND RELATIVITY

For Vishnya, who always believed in it

Quantum Non-Locality and Relativity

Metaphysical Intimations of Modern Physics

Third Edition

Tim Maudlin

WILEY-BLACKWELL
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Preface to First Edition

> *I state my case, even though I know it is only part of the truth, and I would state it just the same if I knew it was false, because certain errors are stations on the road to the truth. I am doing all that is possible on a definite job at hand.*
>
> Robert Musil

If the introductory chapter of a book is the overture to the ensuing score, a brief, undeveloped melange of themes and leitmotifs destined to appear again and again, the preface serves as program notes. Here may one find some small account of the events which propelled the project; some acknowledgment of the many friends who encouraged and nourished it; and some explanation of idiosyncratic elements which arise from the author's own peculiarities. And as an intriguing introduction may encourage the reader to warm to the subject, so the successful preface may inspire some sympathy and understanding in the reader for the author's plight, for the many compromises, lapses, and errors that attend the writing of a book. So how did this book come about?

In October 1990, John Stewart Bell succumbed quite suddenly and unexpectedly to a hemorrhage of the brain. Anyone who had studied Bell's works mourned the passing of an incisive intellect; those who had had the pleasure of discussing the knotty problems of quantum theory with him felt even more sharply the loss of a figure of inspiring integrity, clarity, and humor. Here at Rutgers, Renée Weber suggested that we honor Dr Bell's memory with a symposium on his work. David Mermin treated us to a non-technical exposition of Bell's Theorem, Shelly Goldstein spoke of the relationship between Bell's work and that of David Bohm, and Professor Weber recounted some parts of her recent interview with Bell. My part was to be a short discussion of the compatibility between Relativity theory and the violation of Bell's inequality.

When I originally agreed to the assignment, I thought that I knew just what I was going to say: Relativity has been interpreted in two quite different ways, as forbidding superluminal effects and as demanding Lorentz invariance, and one must sort out how to construe Relativity before one can address the question of compatibility with quantum theory. But after a few days I realized that another construal of Relativity was available (no superluminal signals), then another (no superluminal energy transmission), then yet another (no superluminal information transmission). Since all of these interpretations of Relativity were provably non-equivalent, this situation posed a straightforward analytical task: how do the various interpretations relate to one another and how does each fare if Bell's inequality is violated? This manuscript is my attempt to work through that analytical problem.

In writing the book I have been constantly surprised by the variety and beauty of the interconnections between these various questions. But I have been even more impressed by Bell's deep and steady understanding of the problematic. Over and over I found some terse passage in Bell's work to contain exactly what needed to be said on a subject, the decisive pronouncement. I have often felt that whatever is of value in this book could be found in Bell's "The Theory of Local Beables" (1987, ch. 7), and have consoled myself that this book will have served a great purpose if it does no more than encourage people to read Bell with the care and attention he deserves.

My foremost goal in composing the book has been to make it comprehensible to the non-specialist. The sparks which fly when quantum theory collides with Relativity ignite conceptual brushfires of particular interest to philosophers, problems about causation, time, and holism, among others. Unfortunately, much of the work done by philosophers presupposes a considerable amount of familiarity with the physics. This is particularly sad since the physics is not, in most cases, very complicated. I fear that many readers may be frightened off from the topic by unnecessary formalization, so I have tried to keep the mathematical complexity of the discussion to a minimum. But on the other hand, I have not wished to drop to the level of vague metaphor which sometime infects popularizations. Every compromise between rigor and simplicity is a bargain with the devil, and I have struck mine as follows. The presentation of Bell's inequality needs no more than some algebra, and is quite rigorous. Understanding Relativity also requires no more than algebraic manipulation, but enough that a purely mathematical account would tax the patience of the average reader. So I have tried to present Relativity pictorially, so far as possible. The figures in the book present the concepts of Relativity accurately, but demand of the reader some skill in interpretation. Pictures of space-time look misleadingly like pictures of space, and the novice must unlearn some of the conventions of normal pictorial representation to avoid being misled. Newcomers should therefore take great care with the pictures in chapter 2: if those are properly understood, the sequel will be easy.

Quantum theory itself has been another matter. Most of the content before chapter 7 can be understood without much discussion of quantum formalism. That formalism itself also uses no more than linear algebra and vector spaces. Interested neophytes can find enough technical detail in any standard introductory text. A particularly nice and accessible presentation of the requisite mathematics is provided in David Albert's *Quantum Mechanics and Experience* (1992, ch. 2).

Just as professional physics scares off the uninitiated, so does professional philosophy. Philosophers have developed many languages of technical analysis which permit concise communication among the cognoscenti but which make amateurs feel like unwelcome guests. But most clear philosophical ideas can be presented intuitively, shorn of the manifold qualifications, appendices, and terminological innovations that grow like weeds in academic soil. I have been very selective in my discussions of the philosophical corpus, usually focusing on a single proposal which illuminates a region of logical space. I do not pretend to comprehensiveness in my review of the philosophical literature, and can only plead for understanding that my decisions reflect a desire for a short, provocative text.

Finally, I feel I should explain the "metaphysical intimations" of my subtitle. Metaphysics has acquired rather a bad reputation in this century, following the insistence of Kant that all metaphysical speculations must be pursued *a priori*. It was not always so. The fount of metaphysics as a philosophical pursuit is the treatise on First Philosophy by Aristotle which has come down to us as the *Metaphysics*. Aristotle was concerned with analysis of what there is into its most generic categories: substance, quality, quantity, etc. I see no reason to believe that Aristotle thought such an examination could not be informed by experience. At its most fundamental level, physics tells us about what there is, about the categories of being. And modern physics tells us that what there is ain't nothing like what we thought there is.

I have used "intimations" rather than "implications" because we still do not know how this story ends. Quantum theory and Relativity have not yet been reconciled, and so we can now at best only guess what picture of the world will prevail. But we do know enough to make some guesses.

This book would not have come to be without help of all sorts. David Albert, Nick Huggett, Martin Jones, Bert Sweet, Paul Teller, and Robert Weingard all devoted their own time and insight reviewing the manuscript and generously shared their views with me. Abner Shimony pushed me to clarify the models in chapter 6, and thereby saved me from repeating some errors in print. Steve Stich expended considerable effort finding the manuscript a home, and always had a word of encouragement. The National Endowment for the Humanities graciously provided financial support in the form of Summer Stipend FT-36726-92 (money = time). And the atmosphere in which the book was completed was lightened by Clio Maudlin, who also improvised some emendations with her feet.

Preface to Second Edition

Publication of the second edition of this tome affords the opportunity, beside typographical corrections, for two more substantial changes. The first is a new derivation of the Relativistic mass increase formula, to be found in chapter 3. The new derivation is somewhat simpler than that in the first edition, and has the advantage of allowing the exact formula to be obtained by means of a few lines of algebra. There are many methods for deriving the formula, but to my knowledge this one is novel. The second is the addition of an Overview of Quantum Mechanics. The overview contains just the bare mathematical bones of the theory, but that is enough to explain how violations of Bell's inequality are implied by the theory. It is hoped that the overview, while not a complete account of quantum theory, helps make this study more self-sufficient.

Beyond providing the chance for small improvements, the issuing of the second edition invites reflection, at some years' remove, on the plan of the original. Perhaps the most vexing question confronting any study of Bell's inequality is how the role of quantum theory ought to be treated. On the one hand, there is little doubt that Bell's inequality, and the experimental observation of violations of that inequality, would never have been discovered if not for the existence of the quantum formalism. On the other hand, the inequality itself is derived without any mention of quantum theory and the violations are matters of plain experimental fact. So the explication and analysis of the importance of Bell's work can in principle proceed without mentioning quantum mechanics at all. Should an account of Bell's inequality emphasize its historical roots in the great mysteries of quantum mechanics or rather sever those ties in the interest of logical clarity?

In composing this book, I chose the second option, playing down the role of quantum theory in favor of pure experimental results. In retrospect, I stand by that decision: the interpretation of quantum theory is troublesome enough in its own right to overshadow and confuse the relatively straightforward proof of non-locality. But once the main points have been made, the connections between non-locality and the interpretive problems of quantum theory are both intriguing and instructive. In particular, non-locality appears at exactly the point where the "measurement problem" which infects standard quantum theory is resolved. If one resolves the measurement problem by allowing a real physical process of wave collapse, it is the collapse dynamics which manifests the non-locality, and which resists a fully Relativistic formulation. If one resolves the measurement problem by postulating additional variables beside the wave function, it is the dynamics of these variables which manifests the non-locality and which resists a fully Relativistic formulation. The regrettably widespread opinion that there is no real non-locality inherent in the quantum theory is therefore deeply intertwined with the regrettably widespread opinion that the measurement problem can painlessly be solved without postulating either additional variables or any real collapse process.

Having thrown some rocks at the hornet's nest of the interpretation of quantum theory in this preface, I am obliged now to do more than turn heel and walk away. Although this book is not the place to thrash out those issues, I have thrashed them from time to time in other venues. Some discussions may be found in Maudlin (1995), (1996), (1997), and (1998).

Finally, I must note that although there has been some discussion of Bell's theorem and non-locality in the eight years between the two editions of this book, there had been, to my knowledge, no fundamental change in the basic logic of the situation, and no real progress in reconciling quantum theory and Relativity.

Preface to Third Edition

The impetus for a third edition of *Quantum Non-Locality and Relativity* arises from several different circumstances. The most important is the development of a fully relativistic, precise physical theory that can produce violations of Bell's inequality for experiments at space-like separation. This theory, called flashy Relativistic GRW, was discovered by Roderich Tumulka, and is described in the new chapter 10. Tumulka's theory settles the logical question about the possibility of fully reconciling quantum theory and Relativity. But as the reader will see, the theory also suggests that the consummation was perhaps not so devoutly to be wished. That is, there is a price of plausibility to be paid by this theory, a price so high that it is unlikely to gain converts. One nice thing about flashy GRW is that it forces us to confront deep issues about what exactly makes for a plausible physical theory.

Interest in Bell's inequality has been on the rise in recent years for other unrelated reasons. John Conway and Simon Kochen produced the so-called "Free Will Theorem," whose title alone is enough to raise eyebrows. The theorem utilizes a situation in which there is a violation of Bell's inequality, and Conway and Kochen have made some rather astonishing claims about its significance. This theorem is also discussed in the new chapter.

More generally, interest in Bell's theorem has been reinvigorated by the rise of quantum information theory and the experimental verification of such quantum effects as the teleportation of quantum states. The information-theoretic analysis of quantum theory has had the entirely salubrious effect of focusing attention on the key aspect of quantum theory called *entanglement*. The entanglement of distant systems is what produces violations of Bell's inequality in quantum theory, and physicists have come to routinely accept this entanglement as a quantifiable, exploitable physical resource. There is little dispute any more that the entanglement of distant systems is somehow physically real.

The reason that this development is so cheering is that it deflects attention from other less important aspects of quantum theory. For example, it has been repeated *ad nauseam* that Einstein's main objection to quantum theory was its lack of determinism: Einstein could not abide a God who plays dice. But what annoyed Einstein was not lack of determinism, it was the apparent failure of *locality* in the theory on account of entanglement. Einstein recognized that, given the predictions of quantum theory, only a deterministic theory could eliminate this non-locality, and so he realized that a local theory must be deterministic. But it was the locality that mattered to him, not the determinism. We now understand, due to the work of Bell, that Einstein's quest for a local theory was bound to fail.

Schrödinger, in his famous "cat" paper, remarked on the "*entanglement* of our knowledge of [...] two bodies" (1935, p. 161) found in the quantum-mechanical formalism, but denied, as Einstein did, that this could reflect any real physical connection between separated systems: "Measurements on separated systems cannot directly influence each other – that would be magic" (ibid., p. 164). Bell's work has shown that the magic is real, and physicists who study entanglement have accepted the non-locality that Einstein and Schrödinger could not abide. I have briefly adverted to recent work on the information-theoretic implications of quantum theory in chapter 6 of this new edition.

When I first wrote *Quantum Non-Locality and Relativity*, I tried to keep discussion of the foundations of quantum theory to a minimum. All that is relevant to Bell's theorem are the predictions of quantum theory, not how the theory itself is understood. Separating these issues was especially important at the time because discussions of quantum theory *per se* contained a wealth of distractions and confusion. Perhaps the time is at last ripe to open up the dialog again, and to recover an understanding of Einstein's and Schrödinger's real concerns through the lens of Bell's theorem. I hope that chapter 10 provides a small step in the direction of a clearer understanding of what a comprehensible presentation of any physical theory (and hence a comprehensible presentation of quantum theory) demands.

Introduction

In the 1930s, Otto Neurath was one among many philosophers engaged in the project of purifying scientific language of its ambiguities, its vagueness, and its “metaphysical” contents. One might hope to accomplish this task by an act of radical innovation, building anew from elements of perfect clarity and precision. Neurath realized that such hopes are unattainable, that at best we can only successively improve the language we have, always retaining some of its deficiencies. He illustrated our situation with a resonant image:

> No *tabula rasa* exists. We are like sailors who must rebuild their ship on the open sea, never able to dismantle it in dry-dock and reconstruct it there out of the best materials. (Neurath 1959, p. 201)

The physical sciences themselves suffer the same fate. Fundamental conceptual changes occur, but they are always modifications of a previously existing structure. The entire edifice is not reconstituted anew; rather, tactical adjustments are made in order to render the whole consistent. The *ad hoc* nature of this procedure may leave us with lingering doubts as to whether the whole really is consistent.

During the past century our physical picture of the world has undergone two revolutionary modifications. The Theory of Relativity has overthrown classical presumptions about the structure of space and time. The quantum theory has provided us with intimations of a new conception of physical reality. Classical notions of causality, of actuality, and of the role of the observer in the universe have all come under attack. The ultimate outcome of the revolutions is now but dimly seen, at best. The final reconciliation of

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quantum theory and Relativity is a theoretical problem of the first magnitude. No quantum version of General Relativity exists, and the prospects for one are murky. But even apart from that hurdle, problems about the consistency of our two fundamental physical theories may appear.

The problem that will concern us here is easily stated. It arises from the remarkable results derived by John Stewart Bell in 1964 concerning the behavior of certain pairs of particles that are governed by quantum laws. Bell showed that observable correlations between the particles could not be accounted for by any theory which attributes only locally defined physical states to them. The particles appear to remain "connected" or "in communication" no matter how distantly separated they may become. The outcome of experiments performed on one member of the pair appears to depend not just on that member's own intrinsic physical state but also on the result of experiments carried out on its twin.

Many features of this quantum connection are puzzling. It is, for example, entirely undiminished by distance. This distinguishes it from any connection mediated by a classical force, such as gravity or electromagnetism. But even more amazingly, the connection exists even when the observations carried out occupy positions in space and time which cannot be connected by light rays. The particles communicate faster than light.

It is this last feature which raises questions about the consistency of our fundamental theories. Relativity is commonly taken to prohibit anything from traveling faster than light. But if nothing can go faster than light, how can the particles continue to display the requisite correlations even when greatly separated? The two pillars of modern physics seem to contradict one another.

The predicted correlations have been experimentally confirmed. Indeed, they have been seen even in conditions where the communication between the particles would require superluminal velocities. So we are presented with the problem of determining whether Relativity has been violated, and, if so, whether our present account of space-time structure must be modified or abandoned.

The question of whether the quantum correlations are consistent with Relativity seems precise enough to admit a decisive answer, but on closer examination this appearance of clarity dissolves. Exactly what sort of constraints Relativity imposes on physical processes is a matter of much dispute. Many physicists and philosophers would agree that Relativity prohibits *something* from going faster than light but disagree over just what that something is. Among the candidates we may distinguish:

> Matter or energy cannot be transported faster than light.
> Signals cannot be sent faster than light.
> Causal processes cannot propagate faster than light.
> Information cannot be transmitted faster than light.

Most of these prohibitions are easily seen to be non-equivalent. For example, signals could in principle be sent without any accompanying transmission of matter or energy. Or again, superluminal causal processes could exist which, due to their uncontrollability, could not be used to send signals.

Yet another interpretation holds that Relativity requires only that

Theories must be Lorentz invariant.

This requirement is compatible with the violation of every one of the prohibitions listed above.

Not surprisingly, the various prohibitions are justified by different considerations. In one case it is claimed that a violation of the prohibition would require an infinite amount of energy, in another than it would engender paradox, in yet another that some relativity principle would be abrogated. We are therefore left with a rather tangled thicket of problems. We must consider each of the proposed prohibitions and ask whether it is violated by the quantum connection. We must ask how each prohibition is justified and how it connects with the formalism of the Theory of Relativity. We would also like to see how the prohibitions relate to one another. Until this work is done we cannot begin to evaluate the implications of the quantum correlations for our picture of the world.

This problematic directly dictates the structure of our inquiry. Chapter 1 presents Bell's results with a minimum of technical machinery. Chapter 2 is a short intuitive account of Special Relativity. The following four chapters examine the four prohibitions listed above, tracing their connection with Special Relativity on the one hand and their compatibility with quantum non-locality on the other. Chapter 7 delves into the technical requirement of Lorentz invariance and its implications. Chapter 8 touches on the difficulties involved in passing from the space-time of Special Relativity to that of General Relativity.

Any book which attempts to deal with quantum theory, Special Relativity and General Relativity courts various forms of disaster. Technical and mathematical detail can easily push the discussion beyond the ready grasp of the general reader, and the philosophical interpretation of the mathematical formulae can be even more daunting. In this last respect an asymmetry regarding our two fundamental theories should be noted. Relativity is quite well understood. Although it employs ideas that depart radically from those of classical physics, the concepts are themselves unproblematic and become quite transparent with use. Quantum theory, in contrast, still presents deep and basic interpretational problems, the discussion of which could fill several volumes. Fortunately, our concerns will not draw us much into these controversies. Bell's theorem can be proven without so much as a mention of quantum theory, and although one uses quantum theory to predict the violation of Bell's inequalities, the violation itself is confirmed by straightforward laboratory technique. The observed facts, not merely some interpretation of the theory, stand against locality, so the thorny problems surrounding the interpretation of quantum formalism can be almost entirely avoided.[1] For aficionados, more detailed remarks concerning the interpretation of quantum theory will be provided in appendices or in end-of-chapter notes.

Technical details of physics are not the only casualties of our approach. The philosophical literature on this subject is large and growing, and we will be forced to pass over much of it with little examination. I hope that the philosophical views discussed will be accepted by my colleagues as simplifications rather than caricatures.

For those interested in the fundamental structure of the physical world, the experimental verification of violations of Bell's inequality constitutes the most significant event of the past half-century. In some way our basic picture of space, time, and physical reality must change. These results, and the mysteries they engender, should be the common property of all who contemplate with wonder the universe we inhabit. So in telling this tale I have tried to leave behind the arcane technicalia of the academy. In doing so, I have sacrificed no small degree of precision, and perhaps also some important subtleties. But I hope at least to have provided a framework sturdy enough and correct enough to serve both professional and amateur naval architects who propose to redesign the craft which carries us on our journey.

Note

1 To be precise, the only assumption we will be making is that when one does, for example, a polarization experiment and gets some result (photon passed or absorbed), there is, after the experiment is finished, something in the physical state of the universe which picks out that result over the other possible results. Our assumption is held in common by all wave-collapse theories, whether collapse is caused by interaction with macroscopic devices, by conscious experience, or by random "hits" as in the theory of Ghirardi, Rimini, and Weber (1986). It is also held by no-collapse theories such as Bohm's which use additional variables to describe the world. Indeed, I know of only two interpretations which deny the assumption: the many-worlds interpretation of Everett and Wheeler (De Witt and Graham 1973) and the many-minds interpretation of David Albert and Barry Loewer (1988, 1989; Albert 1992). The many-worlds theory is incoherent for reasons which have been often pointed out: since there are no frequencies in the theory there is nothing for the numerical predictions of quantum theory to mean. This fact is often disguised by the choice of fortuitous examples. A typical Schrödinger-cat apparatus is designed to yield a 50 percent probability for each of two results, so the "splitting" of the universe in two seems to correspond to

the probabilities. But the device could equally be designed to yield a 99 percent probability of one result and 1 percent probability of the other. Again the world "splits" in two; wherein lies the difference between this case and the last?

Defenders of the theory sometimes try to alleviate this difficulty by demonstrating that in the long run (in the limit as one repeats experiments an infinite number of times) the quantum probability assigned to branches in which the observed frequencies match the quantum predictions approaches unity. But this is a manifest *petitio principii*. If the connection between frequency and quantum "probability" has not already been made, the fact that the assigned "probability" approaches unity cannot be interpreted as approach to certainty of an outcome. All of the branches in which the observed frequency diverges from the quantum predictions still exist, indeed they are certain to exist. It is not highly likely that I will experience one of the frequencies rather than another, it is rather certain that for each possible frequency some descendants of me (descendants through world-splitting) will see it. And in no sense will "more" of my descendants see the right frequency rather than the wrong one: just the opposite is true. So approach of some number to unity cannot help unless the number already has the right interpretation. It is also hard to see how such limiting cases help us: we never get to one since we always live in the short run. If the short-run case can be solved, the theorems about limits are unnecessary; if they can't be then the theorems are irrelevant.

The many-minds theory does not have this problem, and may be the only existing interpretation of quantum theory which requires no non-local effects. We will discuss the many-minds theory in chapter 7.

1

Bell's Theorem: The Price of Locality

According to our naive, everyday conception, and even according to most of our refined theories, the physical world is composed of separate individually existing objects. The book on my desk sits apart from the glass, each constituted separate from the other and with its own intrinsic properties. The book has its mass, shape, number of pages, the marks of its history engraved on it. It is made up of atoms, each with its own physical constitution, tied together by chemical bonds. The glass similarly exists on its own, constructed from a separate complement of particles. There are, of course, relations between the book and the glass. The book is heavier and occupies more volume; there is a certain definite distance between them. Spatial separation plays a unique role: as an external relation it is not determined by any facts about the book and the glass taken individually. But once we have taken into account their intrinsic properties and their situation in space we appear to have exhausted the facts about the pair. All other facts about them are determined by these.

Each of the pair may influence the other. The glass, full of steaming tea, raises the temperature of the book which is in its proximity. But this interaction is mediated by other localized bits of matter. Air molecules around the glass are made more energetic through interactions with the tea, some wander off and communicate their energy with the book, heating it. The book exerts a slight gravitational pull on the glass and vice versa. This is a subtle matter, but we come to think of this too as a mediated interaction, an effect of a gravitational field.

The fields of classical physics are not so familiar as books or atoms but they too are local entities. Although an electric field may spread out and

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permeate the universe, the state of the field is determined entirely by its value at each point of space. Disturbances propagate through the field, but they do so by local interactions: changes in the field quantities induce other changes nearby and so ripple off to infinity. Like the transmission of heat, this process takes time as the vibrations of the field are passed along.

Einstein set great store by the idea that the physical state of the universe is determined by a set of locally defined physical magnitudes so that the state of any localized entity exists independently of all spatially separated systems. As he expressed it in a letter to Max Born:

> If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas of physics, one is first of all struck by the following: the concepts of physics relate to a real outside world, that is, ideas are established relating to things such as bodies, fields, etc., which claim "real existence" that is independent of the perceiving subject – ideas which, on the other hand, have been brought into as secure a relationship as possible with the sense-data. It is further characteristic of these physical objects that they are thought of as arranged in a space-time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects "are situated in different parts of space". Unless one makes this kind of assumption about the independence of the existence (the "being-thus") of objects which are far apart from one another in space – which stems in the first place from everyday thinking – physical thinking in the familiar sense would not be possible. It is also hard to see any way of formulating and testing the laws of physics unless one makes a clear distinction of this kind. This principle has been carried to extremes in the field theory by localizing the elementary objects on which it is based and which exist independently of each other, as well as the elementary laws which have been postulated for it, in the infinitely small (four-dimensional) elements of space.
>
> The following idea characterizes the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B; this is known as the "principle of contiguity," which is used consistently in the field theory. If this axiom were to be completely abolished, the idea of the existence of (quasi-) enclosed systems, and thereby the postulation of laws which can be checked empirically in the accepted sense, would become impossible. (Born 1971, pp. 170–1)

Bell's theorem addresses the implications, and ultimately the tenability, of this picture.

Given the extreme generality of the local conception of reality it is hard to imagine that it could, by itself, have any testable empirical consequences. No constraints have been put on the nature or complexity of the locally defined quantities. The locality condition allows, for example, that every particle in the universe could retain traces of every interaction it has ever undergone. It allows a system to be governed by laws which are deterministic or are probabilistic, placing no limit on the subtlety or sophistication of the laws. Nonetheless, Bell was able to show that some behavior of separated pairs of systems cannot be explained by *any* local physical theory if the systems do not interact. Although Bell's results can be derived in different ways and with great generality, we will begin by focusing on a singular fact about light.

Polarization

When one passes a beam of sunlight through a polarized filter, such as the material used in Polaroid sunglasses, two things happen. First, about half of the light is absorbed and half transmitted, as is immediately evident. Second, the light which is transmitted displays an entirely new and surprising characteristic: it shows a particular directionality. This directionality can be most easily observed if one passes the new beam through a second polarized filter. The effect of the second filter depends critically on its orientation with respect to the first. In one orientation the second polarizer will have no effect at all, allowing the entire beam to pass. But as it is rotated, the second filter allows less and less of the light through. By the time it has been turned 90°, it absorbs the beam entirely; as it is rotated further it permits ever more light to pass until, at 180°, the whole beam passes again.

The directionality that the sunlight acquires depends on the orientation of the first polarizer. When the first filter is rotated, the characteristic orientation at which the transmitted beam passes the second filter rotates with it. So light which has passed through a Polaroid filter acquires a new property, a polarization, which is associated with some direction perpendicular to its line of motion.

All that really concerns us is the behavior recounted above; the explanation of the phenomena will ultimately be irrelevant to our concerns. But to help fix our ideas it may help to recall the classical theory of polarization. The classical theory provides us with a simple picture of polarization which should, however, be taken *cum grano sails*, for it cannot be straightforwardly extended when quantum phenomena are taken into consideration.

According to classical physics, light is an electromagnetic wave, a propagating disturbance of the electric and magnetic fields. The fields that vary always point perpendicular to the direction of motion of the light. At any given moment the electric and magnetic fields are also perpendicular to each other, but as time goes on their direction and magnitude may change in any number of ways. For example, if we look at a ray of light head on as it comes toward us, the electric field may rotate in a circle, either clockwise or counterclockwise (circularly polarized light); or it may trace out an ellipse,

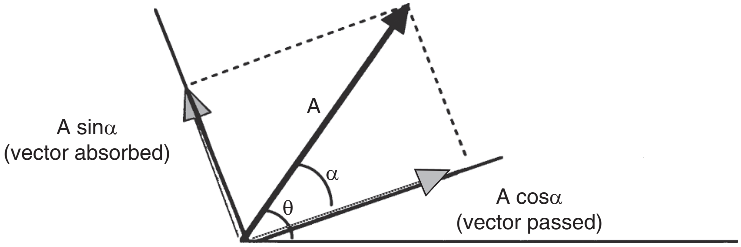

Figure 1.1 Resolving a Vector

rotating and varying in length; or it may simply oscillate back and forth without rotating, always remaining in a plane that points in a given direction. This last possibility, plane polarized light, is the case of interest to us. Plane polarized light has a characteristic direction, the direction of the plane in which the electric field vector always lies. Furthermore, for any direction θ we choose, light of any sort can be analyzed into a component plane polarized in that direction and a component polarized in the perpendicular (that is, θ + 90°) direction. Even circularly polarized light can be constructed from two such elements, if they are added together in the right way, with the right phase relations.

The phenomena recounted above are now easily explained. A Polaroid filter in effect analyzes all incoming light waves into two parts: one plane polarized in the direction of the filter's polarization, the other perpendicular to that direction. It then absorbs the perpendicular component, allowing only the plane polarized remainder to pass through. If the incoming light is unpolarized, this means that on average half of it will pass through and half be absorbed. The effect of the second filter then depends crucially on its orientation relative to the first. If they are perfectly aligned, the light which passes the first is already polarized in the direction of the second and so all gets through. If the second is misaligned by 90°, then exactly the component which passes the first will be absorbed by the second, and none will get through.

What if the two filters are misaligned by some angle α between 0° and 90°? We can represent the light coming through the first filter by a vector pointing in the θ direction whose length A represents the maximum amplitude of the electric field. The second filter resolves this vector into two components, one parallel to θ – α, the other perpendicular (see figure 1.1). The perpendicular component is absorbed by the filter, so the amplitude of the transmitted light is A cos α.

We now must appeal to a seemingly minor but highly significant fact. The energy of plane polarized light is proportional to the *square* of the amplitude of its electric field vector. So if we measure the amount of light

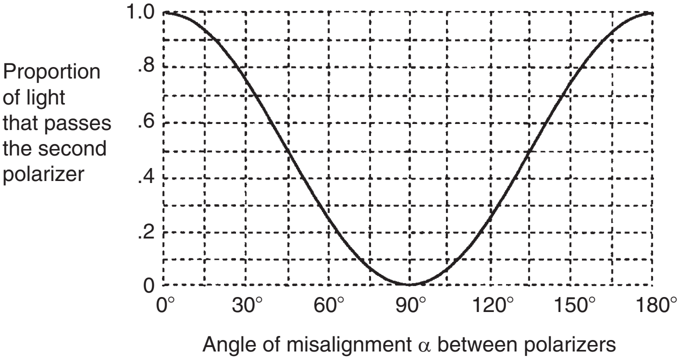

Figure 1.2 Proportion of Light Passing Second Polarizer

which passes the second polarizer by the energy of the beam, the proportion of the beam that gets through is $A^2 \cos^2 \alpha / A^2 = \cos^2\alpha$. Figure 1.2 shows the proportion of the beam which passes the second filter as a function of the angle of misalignment α.

As expected, when $\alpha = 0°$ and the filters are aligned, all the beam is transmitted. When the filters are misaligned by 90° none of the light gets through. But the most significant behavior is found between these extremes. For the moment we need only note that when $\alpha = 30°$, $\cos^2 30° = \left(\sqrt{3}/2\right)^2 = \frac{3}{4}$ of the beam gets through, while when $\alpha = 60°$, $\cos^2 60° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ of the light is transmitted.

Light Quanta

According to the classical conception light is a wave, spread out in space. Whenever a plane polarized beam impinges on a filter oriented at, say, 30° off of the polarization plane of the incoming beam the same thing happens: $\frac{3}{4}$ of the beam passes, and what gets through is polarized in the direction of the filter. A beam always comes out with its amplitude and energy reduced by a fixed proportion.

But as Einstein observed in 1905, light does not always behave like a wave. For example, when light falls on certain metals it can knock out electrons causing a current to flow, the so-called photoelectric effect. When one measures the energy of the electrons so liberated one finds that the energy of the incident light is not delivered uniformly over the surface of the metal as one would expect. The energy rather comes in small but discrete packets.

At fine levels of analysis light behaves as if it is made up of particles. These light quanta, or photons, can be individually registered and counted by photomultiplier tubes.

The exact nature of the wave/particle duality of light need not detain us. We need only note two experimentally verifiable facts. First, light from certain sources has the effect of causing discrete, countable events in certain detection equipment. Second, if this light is passed through a polarizer, the resulting beam also behaves as if made up of photons and *the photons are each of exactly the same energy as those in the incoming beam.*

Nothing we have said so far could have prepared us for this new piece of information. It would have been plausible to guess instead that the photons coming through the polarizer would all have had their energy reduced by the same proportion as the energy of the beam as a whole. But in fact the transmitted photons are as energetic as the incoming ones, only the orientation of their polarization is changed by passage.

If the photons which survive the second polarizer each have the same energy as the incoming light quanta, how is the overall energy of the beam reduced? The only possibility is that the light coming out of the polarizer contains *fewer* photons than the light going in. Photons appear to be either transmitted complete through the filter or else swallowed whole.

It is worthwhile to note that all of this talk about light quanta need not be made precise. We could instead refer only to the observable behavior of pieces of laboratory equipment. When light from certain sources is directed at photomultiplier tubes discrete and countable events occur. When the light is passed through a filter fewer such events occur. A second filter again reduces the number, and the proportion of the reduction is the square of the cosine of the angle between the filters. These are the sorts of facts that we will be concerned to explain. The photon picture provides a convenient model of the underlying process, but the correctness of that model need not concern us. If the reader is puzzled by the particulate nature of light it may help to note that experiments similar to the ones we will describe can also be carried out on protons and electrons, archetypical particles. In those cases one measures the so-called "spin" of the particles by passing them through an inhomogeneous magnetic field.

If light behaves as if made up of quanta and if each such quantum which survives a filter has the same energy as it had coming in, then figure 1.2 takes on a new significance. The quantity $\cos^2 \alpha$, which previously measured the proportion of the beam that passes the filter, now represents *the probability for each photon to pass.* If the energy of the beam is to be reduced to one quarter of its previous value when passed through a polarizer oriented at 60° it must be that only one quarter as many photons will compose the passed beam. And if we can create individual photons, it must be that their individual probability for surviving the polarizer is one out of four. As we turn the second polarizer from perfect alignment to perfect misalignment, the likelihood of each photon to get through the second polarizer drops in accord with the graph in figure 1.2.

The Entangled State

So far nothing much mysterious has happened. Polarization phenomena are not particularly strange, and the quantization of light, if unexpected, seems perfectly comprehensible. But one final observation, also apparently rather pedestrian, turns out to be enough to destroy our accustomed picture of physical reality.

When calcium vapor is exposed to lasers tuned to a certain frequency it fluoresces. As excited electrons in the atoms cascade down to their ground state they give off light. In particular, each atom emits a pair of photons which travel off in opposite directions. The polarization of the photons individually shows no preferred direction: for any randomly chosen direction θ the photons will pass a polarizer oriented in that direction half the time. But although the photons individually show no particular polarization, the pairs exhibit some striking correlations. Roughly, each member of a pair always acts as if it has the same polarization as its partner.

More precisely, the following can be observed.[1] Suppose that one photon, R, goes off to the right while its partner, L, goes off to the left. R and L will each eventually impinge on a filter which sits before a photomultiplier tube. If the two filters are set in the same direction then either both photons will pass the filter or both will be absorbed. When the filters are aligned, in whatever direction, the photons are *perfectly correlated*: each does what the other does. If the filters are misaligned, then the photons still behave as if they have the same polarization. That is, suppose R passes through its polarizer, which is oriented in direction θ. Then L will act as if it is polarized in direction θ. If the left polarizer is also oriented in direction θ then L will pass, as we have seen. If the left polarizer is oriented at $\theta + 90°$ then L will be absorbed. And if the angle of misalignment θ is between 0° and 90° then L will pass the filter a proportion $\cos^2 \alpha$ of the time. Similarly, if R is absorbed by its polarizer, L will act as if it is polarized in the $\theta + 90°$ direction. It will always pass a polarizer oriented at $\theta + 90°$, always be absorbed by one at θ, and generally if the filter is oriented at $\theta + \alpha$ the photon will pass $\sin^2 \alpha$ of the time.

Let us say that a pair of photons *agree* if they are either both passed or both absorbed by their respective filters and *disagree* if one is transmitted while the other is not. Then if the two filters are aligned in the same direction the photons will always agree, half of the pairs being jointly passed, the other

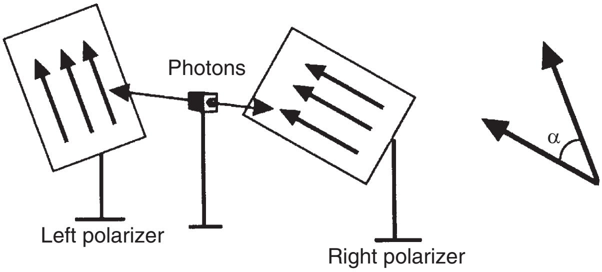

Figure 1.3 Experimental Set-up

half jointly absorbed. If the polarizers are misaligned by 90° the photons always disagree, one being absorbed, the other not. And for any other angle of misalignment α the percentage of pairs which agree (in the long term) is $\cos^2 \alpha$, as shown in figure 1.2.

Note that when we set up a particular experiment we have two choices to make. First we must choose the angle θ of the right-hand polarizer. Then we choose the degree of misalignment α of the left-hand polarizer. If we decide to examine a case of perfect alignment ($\alpha = 0°$) we are still at liberty to set the pair of filters in any direction θ we choose. The fact that we have two free variables, θ and α, is just a reflection of the fact that we have two decisions to make: the angle of the right polarizer and the angle of the left. But no matter how we set the two, the only relevant parameter for calculating the probability of agreement is α, the degree of misalignment (see figure 1.3). If $\alpha = 30°$ the photons will agree $\frac{3}{4}$ of the time; if $\alpha = 60°$ they will agree one time out of four. These simple facts about pairs of photons emitted by calcium vapor are enough to destroy any theory according to which physical reality is local.

How Do They Do It?

Suppose that you and a friend are set the task of reproducing the behavior of the photons: one of you will play photon L, the other photon R. These are the rules of the game: you and your friend start out together in a room (the "calcium atom"). You know that each of you will leave the room by a different door, and after some period of time you will each be asked a question. The question will consist of a number between 0 and 180 written on a piece of paper. Your answer must be either the word "passed" or "absorbed." Before you leave the room, you have no idea which question either of you will be asked. However, while in the room you and your friend are permitted to devise any strategy you please in order to coordinate your answers. Your aim is to ensure that after many repetitions of the game (you are permitted to adopt an entirely new strategy each time) your answers display exactly the same sorts of correlations as the photons show. That is, your strategies must ensure that, in the long run, when the question asked you differs from that asked your friend by an amount α, your answers agree $\cos^2 \alpha$ of the time.

For the moment, we will simplify your task even further. Unlike the photons, which have no information at all about which question will be $\cos^2 \alpha$ asked, you and your friend can know that only one of three possible questions, "0?", "30?" or "60?", will be asked. (We will eventually simplify the task even more, but it is easiest to begin here.) Of course, while you are in the room you still have no idea which of the three questions either of you will be asked. And once you leave the room, we suppose *you have no way of knowing what question has been asked (or will be asked) of your partner*. Your behavior may be determined by your agreed upon strategy and by the question you are asked, but not by the question which your friend happens to be asked. Once again, you must each respond either "Passed" or "Absorbed" when a question is asked.

Over a long run of this game you are aiming to reproduce the behavior of the photons in similar circumstances. That is, after a long series of plays you want to ensure that

Fact 1: When you and your friend happen to be asked the same question you always give the same answer.

Fact 2: When your questions differ by 30, that is, when one is asked "0?" and the other "30?" or one is asked "30?" and the other "60?", you and your friend agree $\frac{3}{4}$ of the time.

Fact 3: When your questions differ by 60, that is, when one of you is asked "0?" and the other "60?", your answers agree $\frac{1}{4}$ of the time.

After all, this is what the photons manage to do.

You and your friend are free to agree on any strategy you like, and you are free to vary your strategy from experiment to experiment. We may suppose that the questions to be asked are chosen at random, so that the pair of the questions "0?" to R and "30?" to L, for example, occurs $\frac{1}{9}$ of the time. It is not, however, important that the questions be asked equal amounts of the time, only that the choices be made at random, so that you can have no idea what is to come. How might you go about settling on a strategy?

The first obvious point is that there is no advantage, and much disadvantage, to using any sort of random element after you have left the room. For suppose your strategy demands that if asked the question "0?" you will decide your answer by a flip of the coin. Since you are unable to communicate with your partner there would be no way for your friend to know how you have answered the question, and so no way to be sure of matching your answer if asked the same question. In general, there is no possible way of satisfying Fact 1 above without deciding in the room *how each of you will answer each question if asked*. For without the knowledge of how your partner would answer a question you cannot act so as to ensure that your answers will match if you happened to be asked the same question.

Besides, no possible advantage can be gained by the introduction of random elements. If one of you may have to flip a coin when asked a question, why not flip it beforehand in the room and share the result with your partner? Or flip it three times, one for each possible contingency. Your partner would then have more information than would be available if you only appeal to the random element when actually asked the question. That excess information cannot possibly *degrade* your performance since, in the worst case, the information can just be ignored. Thus we have the simple result that *any strategy which involves local stochastic elements can do no better than a corresponding strategy where the random choices are made at the source*. A "local stochastic element" is a random process which takes place outside the room and whose outcome cannot be communicated to one's partner. In your case, "at the source" means "in the room"; for the photons it means "in the calcium atom." So in the first place, strategies utilizing local stochastic elements cannot ensure the perfect correlation when identical questions are asked, and in the second place, for every strategy using such elements an equally effective strategy which eschews them exists. If you have a penchant for flipping coins, you may as well flip them in the room. Given these two facts we may now narrow our search to strategies which involve no local stochastic elements. This means that when you leave the room each of you knows exactly what the other will do in each possible situation.

Furthermore, not any such deterministic strategy will do. Since you always run the risk of being asked identical questions, you and your friend must resolve to give the same answer as each other to each question. Only in this way can you assure that when answering identical questions your answers will tally.

So our situation has been greatly simplified. Only eight possible strategies are available, corresponding to the possible ways of answering the three questions. You might, for example, decide to answer "passed" no matter which of the three questions is asked. We will represent that strategy as

⟨P, P, P⟩, where the first slot represents the answer to "0?", the second to "30?" and the third to "60?". The eight possible strategies are then:

(1)	⟨P, P, P⟩	(2)	⟨A, A, A⟩	(A)
(3)	⟨A, P, P⟩	(4)	⟨P, A, A⟩	(B)
(5)	⟨P, A, P⟩	(6)	⟨A, P, A⟩	(C)
(7)	⟨P, P, A⟩	(8)	⟨A, A, P⟩	(D)

Since we are only interested in whether the answers given by you and your friend agree or differ, we can regard each of the corresponding mirror-image strategies above as equivalent. That is, if you choose either strategy (1) or strategy (2) you will agree no matter what pair of questions is asked, if you choose (3) or (4) you will disagree if exactly one person is asked "0?" and agree otherwise, and so on. (Of course there are *other* facts, such as that in the long run approximately half the photons pass and half are absorbed, that would demand a judicious choice between the strategies in the right column and those in the left, but those facts have been omitted from our list.) So we may lump together strategies (1) and (2) calling each "strategy (A)," either (3) or (4) will be "strategy (B)," (5) or (6) "strategy (C)," (7) or (8) "strategy (D)." In order to ensure the strict correlations of Fact 1, you and your friend must choose among strategies (A), (B), (C), and (D) every time a new experiment is run. The only real option that is left open to you, then, is what proportion of the time each strategy will be chosen.

Let us suppose that your decisions over the long run result in choosing strategy (A) a proportion α of the time, strategy (B) β of the time, strategy (C) γ of the time, and strategy (D) δ of the time. α, β, γ, and δ must all be positive numbers (or zero), and of course $\alpha + \beta + \gamma + \delta$ must equal unity.

You and your friend must make your choice of which strategy to adopt in complete ignorance of what questions you are to be asked. Further, we may assume that the choice of questions is determined by a process which is random with respect to your choice of strategy. The experimenters, however they decide which questions to ask, do not do so by predicating their choice on your predetermined strategy. In these circumstances, the long-run results of many repetitions of these experiments will depend solely on the values of α, β, γ, and δ. For example, suppose we wish to know how often the pair of questions "0?", "60?" will receive answers which disagree. They will do so exactly when you have chosen strategy (B) or strategy (D), as can be verified by inspection. In the long run, you choose those strategies $\beta + \delta$ proportion of the time. And since the selection of experiments in which that pair of questions is asked constitutes a random selection from the sequence of strategies you choose, in the long run that pair of questions will receive disagreeing answers $\beta + \delta$ of the time.