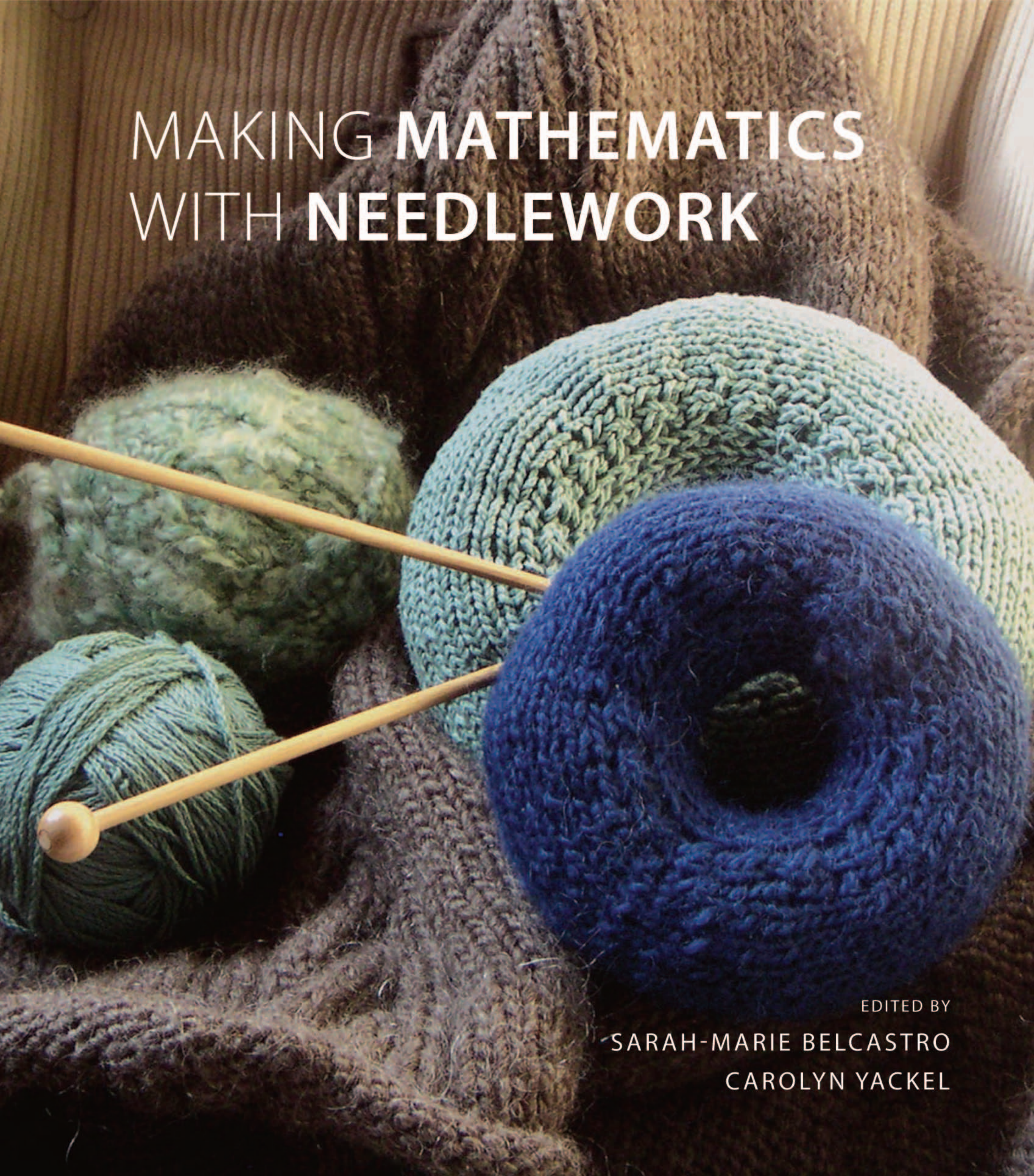
MAKING **MATHEMATICS** WITH **NEEDLEWORK**



EDITED BY

SARAH-MARIE BELCASTRO

CAROLYN YACKEL

Welcome to the first book to contain both mathematics papers and fiber arts project instructions.

Making Mathematics with Needlework will inspire mathematicians, mathematics educators, and crafters; every chapter has an overview as well as sections on mathematics and mathematics education and detailed instructions for completing the chapter's project. All readers will be able to understand the overview sections, as they include introductions to the various fiber arts as well as lay summaries of the mathematical content. While the mathematics sections are written for mathematicians, the authors have made a special effort to make their work accessible to lay readers by providing definitions of mathematical terms and many diagrams. The project sections are written for crafters, so that non-mathematician readers can have a tangible experience with mathematical concepts.

BELCASTRO
YACKEL

MAKING MATHEMATICS WITH NEEDLEWORK

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PETERS

making mathematics with needlework

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ten papers and ten projects

edited by

SARAH-MARIE BELCASTRO

CAROLYN YACKEL

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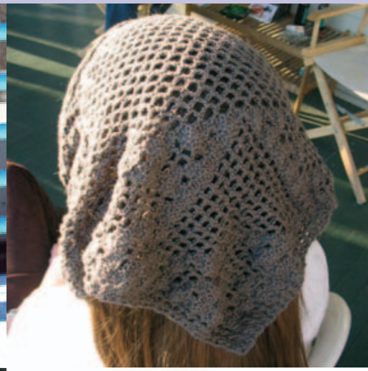
To my grandmothers, who were excellent needleworkers
but didn't know they were mathematicians.
—Carolyn Yackel

To my mother, who knits, and my father, who sews—both needleworkers
who ought to know by now that they're mathematicians.
—sarah-marie belcastro





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The community of fiber artists with a significant interest in mathematics and mathematicians who think about fiber arts would be poorer without people like

- Lara Neel of the *Math4Knitters* blog and podcast;
- Brenda Dayne, who wrote the "Geek Chic" article in *Interweave Knits* that seemed to take the world by storm (and who now does the popular podcast *Cast On*);
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- Lesley Perg of "thomasina's Guide to Geeky Knitting," who maintains a web index, broken out by discipline, of scientifically-related knitting;
- Ruth Stewart of the *Impulse of Delight* website who is a physician, a knitter, and a jeweler who knits and crochets with wire (and creates Möbius bands, trefoil knots, hyperbolic orchids...);
- Mark Shoulson, Claire Irving, and Nate Berglund, who knit and crochet a variety of embeddings of topological objects;
- Hinke Osinga, who created the crocheted Lorenz manifold;
- Miss Lime and Miss Violet (of *Lime & Violet* podcast fame) for liking algebra and spacial relations, respectively, and for saying loudly both that girls should not be discouraged from mathematics and that mathematics is necessary for knitting;

- the hundred or so members of our mathematical fiber arts email list, most of whom have attended Knitting Networks at national mathematics conferences. (There's not enough room to list all of your names here.)

We are grateful to all of these folks for contributing to the confluence of mathematics and fiber arts.

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INTRODUCTION

an overview of mathematics and fiber arts

SARAH-MARIE BELCASTRO
CAROLYN YACKEL

1 Welcome!

Welcome to the first book to contain both mathematics papers and fiber arts project instructions. This volume brings together eight mathematicians to present a wide variety of mathematics research and work in mathematics education related to the fiber arts. It is structured to be of interest to mathematicians, mathematics educators, and crafters; every chapter has an overview as well as sections on mathematics and mathematics education and the chapter's project. All readers will be able to understand the overview sections, as they include introductions to the various fiber arts as well as lay summaries of the mathematical content. While the mathematics sections are written for mathematicians, the authors have made a special effort to make their work accessible to lay readers by providing definitions of mathematical terms and many diagrams. Instructors at all levels—ranging from elementary school through graduate programs—will find ways to use fiber arts in their classrooms in the teaching ideas sections. Not every topic is suitable for every class, of course, but there really is something for everyone. Finally, the project sections are written for crafters, so that our non-mathematician readers can have a tangible experience with mathematical concepts.

At the same time, this is a very specialized volume. First, there are many fiber arts that are not represented (for example, tatting, felting/fulling, and weaving) but are associated with interesting mathematics. The selection of fiber arts represented here is a direct result of the book's origin—it grew out of the American Mathematical Society Special Session in Mathematics and Mathematics Education in Fiber Arts, held at the January 2005 Joint Mathematics Meetings in Atlanta, Georgia. The needlework discussed in the session (knitting, crochet, cross-stitch, embroidery, and sewing/quilting) determined what we could include in this book. Second, for each fiber art there is a wide range of mathematics and mathematics education to be discussed, but only a sampling is shown here. Again, this is somewhat an artifact of what work mathematicians chose to present in the Special Session. However, it is also true that there has been very little published work available in mathematics and fiber arts—aside from quilting and weaving, that is. On pages 7–10 we give bibliographies of all published material on mathematics or mathematics education and fiber arts known to the editors as this volume went to press.

The rest of this introduction contains a brief history of mathematical work on fiber arts, a discussion of fiber arts in mathematics education, and an overview of present and future mathematics research related to fiber arts.

2 A Short History

Elementary mathematics in the form of arithmetic and geometry has been used in fiber arts throughout human history. While that's not so interesting to professional mathematicians, in terms of mathematics education it's very interesting. For students who have prior experience with fiber arts, we can make explicit the mathematics they already use as a way to grow their experiences. Further, teaching various fiber arts to children can be a vehicle for introducing or reinforcing many mathematical concepts. Mary Harris created a sequence of mathematics teaching activities based on textile arts. Unfortunately, this work, called *Cabbage*, seems to be unavailable either in print or electronically. A couple of examples from the collection in her book *Common Threads* [14] highlight the significant mathematics present in simple practical issues in fiber arts.

That said, what sort of work *has* been done on mathematics and fiber arts? This miniature history is based only on published work—and only on those that we have been able to find—so it won't be surprising if readers know of sources we have missed. (Please contact us if you have additional information!)

There has been more published on the mathematics of weaving than on mathematics and any other fiber art—and apparently more than on mathematics and *all* other fiber arts. This began with three papers by E. Lucas published between 1867 and 1911 [45–47], followed by a paper of S. A. Shorter in 1911 [52] and a section of a more general work on pattern design by H. J. Woods in 1935 [54]. As far as we know, nothing more was done in the following 45 years, until Grünbaum and Shephard wrote in 1980 about the mathematics of weaving in satins and twills [30]. The approach is similar to their work on tilings and patterns. Their article spawned many other papers on the topic in the 1980s and early 1990s, including four by C. R. J. Clapham [26–29], two by Jean Pedersen [48, 49], eleven by Janet Hoskins [34–44] involving five varying collaborators, two by Richard Roth [50, 51], and three more by Grünbaum and Shephard themselves [31–33]. The *Mathematical Reviews* review (MR0674144) by G. Ewald of an early article by Hoskins [34] mentions that the mathematics of weaving had been of great interest for nearly a decade. Very recently, R. S. D. Thomas (a collaborator of Hoskins) released a preprint on isonemal prefabrics [53].

The history of mathematical publications on other fiber arts is much more recent. It begins in 1971 with a paper on knitting topological surfaces by Miles Reid [20], followed nearly fifteen years later by a quartet of papers in secondary mathematics education journals on crochet, quilting, sewing, and knitting [5, 7, 13, 21] and a blurb on random processes in knitting in a popular science magazine [12]. Another decade-plus passed before the current wave of publications began. In knitting, we have papers by Dan Isaksen and Al Petrofsky [17] and Claire Irving [16] on topological surfaces; in crochet, there are papers by David Henderson and Daina Taimina [15] and Hinke Osinga and Bernd Krauskopf [19] on geometric surfaces; in quilting, Gwen Fisher [8–10] and Mary Williams [23, 24] have written about depictions of mathematical concepts and objects; and Therese Biedl, John Horton, and Alejandro Lopez-Ortiz did work on thread optimization in cross-stitch [4].

A computer search of the literature yields basically nothing. MathSciNet (the comprehensive database of mathematics research papers) lists one article about knitting from 1999 [11], and it's really about notation for drawing knitted fabric. A few papers deal with the "knitting ansatz" of braid theory (for basic information on braid groups, see Chapter 8), which has nothing to do with fiber arts. MathSciNet also lists many promising references to crochet, except that "crochet" is French for "bracket" and so they're really all about Lie theory. (The two exceptions are [15] and [19].) Even quilting gets short shrift. There are two articles [6, 22] whose titles refer to quilts, but they are about dissections of rectangles into squares; neither appears to have been inspired by actual quilts. Conway and Hsu have named a group-theoretical concept a "quilt." And that's it. Completely. Nothing about tatting, or cross-stitch, or lace-making. So, mathematicians and fiber artists, let's get going! The mathematical world is ripe for our work. The next two sections provide some ideas for how to proceed.

3 Fiber Arts and Mathematics Education

Fiber arts problems and applications can be useful in a variety of classroom teaching styles. They can be used as applications by teachers who prefer to present theory first and give applications afterwards, as described below. They can also be used as motivation for development of mathematics, including development from a constructivist perspective. Indeed, through very careful design one can even create lessons that conform to Gravemeijer's notion of realistic mathematics education. (See [56, pp. 82–83].)

One common response of mathematics educators to students desiring to understand the utility of the mathematics being taught in the classroom is to assign applications problems. However, as fascinating as we may find the fact that it is possible to calculate the escape velocity from the moon, arguably a student may not find that applicable to his or her life. Is he or she really going to launch something at that velocity off of the face of the moon in order to send it into orbit? Likely not. How about the classic linear constraint of butter versus guns? This oversimplification of military spending is unsatisfactory to our students. The fact is that applications problems are often not grounded in the reality of students' lives. Further, they are usually so complicated that only a trivial version can be presented. With problems arising from fiber arts, often both difficulties are avoided. At least some portion of the audience can relate to the scenario, and with careful problem selection, the problems can be real, complete, and at the appropriate level.

Some instructors like to begin with a problem, real-world or otherwise, to motivate the development of a topic in mathematics. Fiber arts problems are wonderful venues for this, because they are real-world problems that real people actually want answered. Through successive passes at the problem, during each of which the scenario is formulated more and more mathematically, teachers model the process of mathematizing the situation, using Freudenthal's terminology [9, pp. 30–31]. (For a detailed explanation of the term, see [56].) At some point the problem is stated mathematically. Through this process the teacher has motivated the topic development. However, if the instructor wishes students to take the lead, careful planning of questions leading them along a learning trajectory [59] may help students to successfully transform the scenario into a relevant mathematical problem on their own.

At some stage, student autonomy in this regard should be a goal, because without the ability to reframe an everyday situation as mathematical, a person will never be mathematically fully functioning. That is, for math to be of maximal use to a person, that person must have a belief in the utility of viewing situations from a mathematical perspective and the skills and initiative to make successive translations of an everyday problem that is not presented in a mathematical format into the underlying essential question, which *is* mathematically stated. Of course, not all everyday scenarios can be developed into math problems, so the adroit user of mathematics must also have a sense of which questions are at heart mathematical and which are not.

This leads us to the question: Is it possible to teach another person to mathematize? This question is similar to, but not the same as, the old question of whether or not one can teach problem solving. Of course, teachers can present techniques and strategies. The most famous endeavor in this regard is Polya's *How to Solve It* [58]. Yet, creativity plays a large role, and often all we can do as instructors is to model that behavior ourselves. Indeed, the brilliance of the authors contributing to this book was to recognize which parts of mathematics applied to their arts and precisely how that mathematics should be applied. Throughout the book, the authors link the details of particular mathematics with a particular fiber art, focussing alternately on an aspect of the craft and an aspect of the mathematics, refining the mathematics to better describe the situation, and so forth. All of these activities and the development of general mathematical explanations and formulas are included in the term "mathematizing."

Nonetheless, the authors noticed the mathematics they did in their particular fiber arts because they had prior knowledge of that mathematics. (For more on *noticing*, see [57].) Therefore, it is through that knowledge that the fiber art became a mode of expression of the mathematics for that mathematician. Alternatively, a fiber artist who has no knowledge of the mathematics in question could make the same piece and admire the