

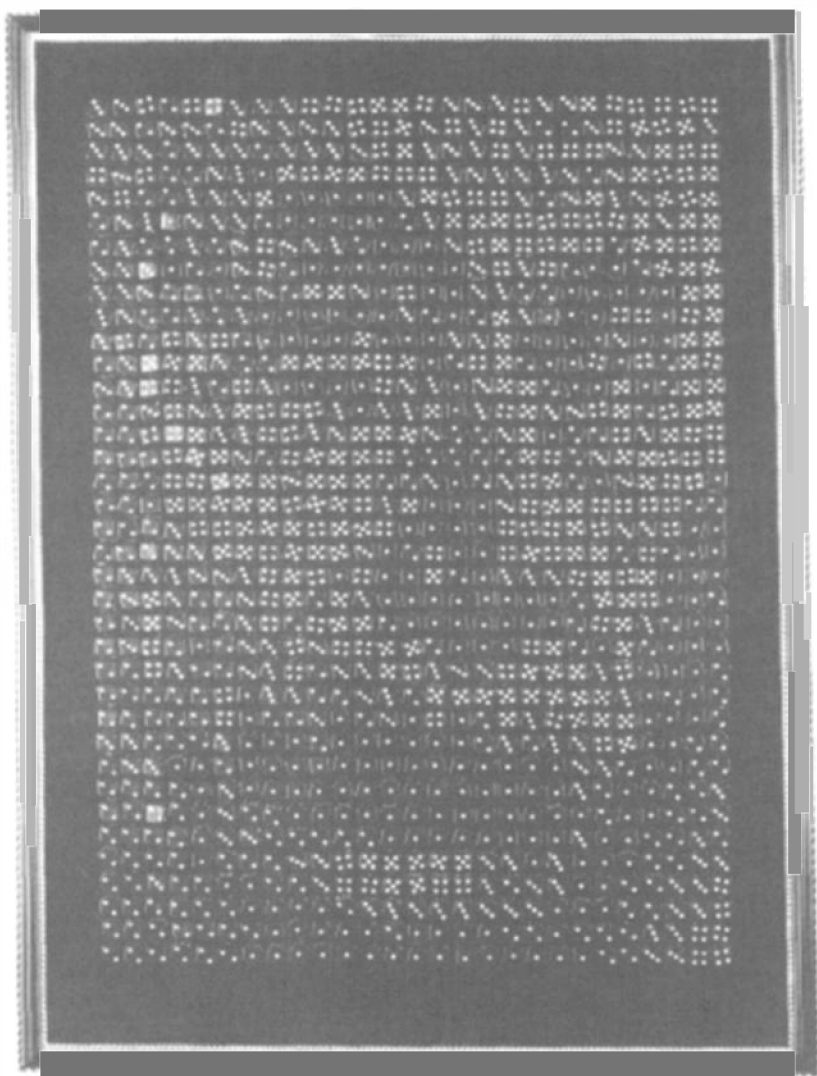
PUZZIERS' TRIBUTE

A FEAST *for the* MIND

DAVID WOLFE, TOM RODGERS, EDS.



Puzzlers' Tribute



“I shall never believe that God plays dice with the world.” — Albert Einstein

Ken Knowlton is widely known for his development of computer graphics languages and techniques, and for his computer-assisted artwork. All rights reserved. Used with permission of the artist.

Puzzlers' Tribute

A Feast for the Mind

Edited by
David Wolfe and Tom Rodgers



CRC Press

Taylor & Francis Group

Boca Raton London New York

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Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

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Preface

Mathematicians, magicians, and puzzlists are masters of the unsolvable, the unbelievable and the undoable. Their currency is paradox. They reach enlightenment through bewilderment.

Members of these three communities meet every two or three years to honor the man at the forefront and nexus of all three, Martin Gardner. At this remarkable Gathering for Gardner (the four so far are dubbed G4G1 through G4G4), participants share talks and performances; problems and puzzles; knowledge and ideas. We invite you to read this compendium of contributions so that you too can be bewildered and enlightened.

The Mathematician and Pied Puzzler was published in 1999 as a tribute to Martin Gardner based on contributions to G4G1. Many of the Gatherings' participants and other fans of Martin Gardner expressed the wish for a second volume. We were glad to have another opportunity to share some of group's favorite paradoxes, problems and puzzles. The articles enthusiastically proffered by G4G participants honoring Gardner have led to the creation of a website and the start of a third volume. Emily DeWitt Rodgers set up and maintains the dedicated website, <http://www.g4g4.com>, which includes the full text of the first tribute book and a maze of puzzles, illusions, and problems. Many contributions and materials are placed there with the permission of the authors along with links to homepages of G4G participants.

All of the things that we said in the introduction to *The Mathematician and Pied Puzzler* remain true and relevant for this book so we have included words from its preface here:

Martin Gardner has had no formal education in mathematics, but he has had an enormous influence on the subject. His writings exhibit

an extraordinary ability to convey the essence of many mathematically sophisticated topics to a very wide audience. In the words first uttered by the mathematician John Conway, Gardner has brought “more mathematics, to more millions, than anyone else.” It is a moving testimony that many professional mathematicians feel that Martin Gardner sparked and guided their early interest in mathematics.

In January 1957, Martin Gardner began writing a monthly column called “Mathematical Games” in *Scientific American*. He soon became the influential center of a large network of research mathematicians with whom he corresponded frequently. On browsing through Gardner’s old columns, one is struck by the large number of now-prominent names that appear therein. Some of these people wrote Gardner to suggest topics for future articles; others wrote to suggest novel twists on his previous articles. Gardner personally answered all of their correspondence.

Gardner’s interests extend well beyond the traditional realm of mathematics. His writings have featured mechanical puzzles as well as mathematical ones, Lewis Carroll, and Sherlock Holmes. He has had a life-long interest in magic, including tricks based on mathematics, on sleight of hand, and on ingenious props. He has played an important role in exposing charlatans who have tried to use their skills not for entertainment but to assert supernatural claims. Although he nominally retired as a regular columnist at *Scientific American* in 1982, Gardner’s prolific output has continued.

Martin Gardner’s influence has been so broad that a large percentage of his fans had only infrequent contacts with each other, until Tom Rodgers conceived of the idea of hosting a weekend gathering in honor of Gardner to bring some of these people together. The first “Gathering for Gardner” (G4G1) was held in January 1993. Elwyn Berlekamp helped publicize the idea to mathematicians. Mark Setteducati took the lead in reaching the magicians. Tom Rodgers contacted the puzzle community. Out of this first gathering grew a series of events; a second gathering, G4G2, was held in January 1994, G4G3 in January 1998, and G4G4 in February 2000.

The success of these gatherings has depended on the generous donations of time and talents of many people. The organizers, Elwyn Berlekamp, Tom Rodgers, Mark Setteducati would like to acknowledge the work of many people who have helped make the Gatherings for Gardner successful, including Scott Kim, Jeremiah Farrell, Karen Farrell, Emily DeWitt Rodgers, David Singmaster, and many others.

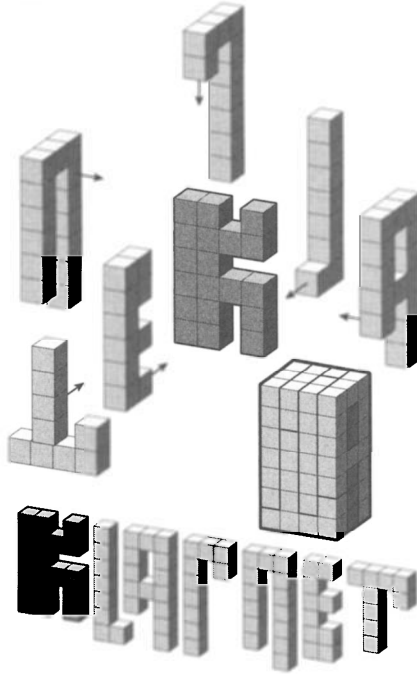
Of course, this book could not exist without the efforts of a large group of contributors. Scott Kim conceived of and assembled the first

tribute volume, and created the graphically spectacular cover in this second volume; Scott's generous contribution will remain engraved in our memories. Emily DeWitt Rodgers has done an excellent job of designing and maintaining the g4g4.com website. In addition, we owe thanks as well to a number of anonymous reviewers for reviewing articles outside the realms of expertise of the editors. David Wolfe is indebted to his wife, Susan Hirshberg, for her incredible support and writing expertise, even while he neglected wedding and honeymoon plans to work on this book. As with any book, only the competence and professionalism of our publisher, A K Peters, Ltd., has allowed this project to overcome the difficult transition from an idea spoken of over wine to a real book which is now in your hands.

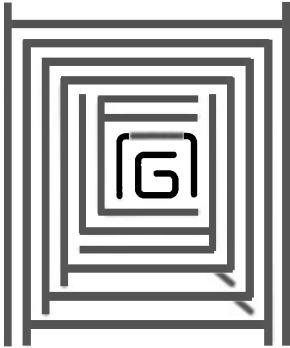
All of us feel honored by this opportunity to join together in tribute to the man in whose name we gathered, Martin Gardner.

David Wolfe
St. Peter, Minnesota

Tom Rodgers
Atlanta, Georgia



David Klarner



Harry Eng



Mel Stover

A Playful Tribute—Scott Kim

In Memoriam

Even mathemagicians are mortal. This book is dedicated to the memory of three participants in The Gathering for Gardner, Mel Stover, Harry Eng, and David Klarner. Each was an admirer of Gardner and has gathered into the folds of another dimension since last we met.

Mel Stover, a fifty-plus-year friend and correspondent of Martin Gardner, lived magic and created his own deliciously pernicious magic-puzzles. Both Martin Gardner and Max Maven wrote articles not only honoring Mel, but also remembering many of Mel's illusions.

"The Impossible Just Takes Longer" was oft quoted by Harry Eng, whose magic tricks, impossible objects, and memory feats pushed the limits of man's capabilities, as described in Mark Setteducati's article. The solution of Harry Eng's Impossible Bottle with inserted coins larger than the bottle mouth is explained in Gary Foshee's article.

David Klarner, who is remembered in articles by Solomon Golomb, C. J. Bouwkamp, and David Singmaster, helped develop the mathematics of box-packing problems and created masterful box-packaging puzzles.

We will miss them.

The Participants
G4G1, G4G2, G4G3, G4G4



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A Greeting to Martin Gardner

Sir Arthur Clarke CBE

I am very happy to send my greetings to Martin on the occasion of the 'gathering' in his honour.

In particular, I notice that the theme of the event is the "Fourth Dimension"—something that has always fascinated me. I had almost forgotten that it was featured in one of my earliest (1946!) stories *Technical Error* (now in *Reach for Tomorrow*). And my very first television programme (BBC TV, 4 May 1950) was a thirty minutes talk on the Fourth Dimension—live of course, because there was no video tape in those days! After that ordeal, no camera has ever had any fears for me.

I am very grateful to Martin for creatively disrupting my life on at least two occasions, thanks to his columns in *Scientific American*. Thirty years ago he turned me on to Pentominos, with results you'll see in *Imperial Earth*. However, even more important, he opened my eyes to the infinite universe of the Mandelbrot Set, which I cleverly managed to combine with SS Titanic, in *The Ghost from the Grand Banks*. (Isn't the connection obvious?)

Finally, it was Martin's *The Night is Large* that inspired me to put together my own collection of non-fiction, *Greetings, Carbon-Based Biped!*

I would be hard put to think of anyone else to whom I owe so great an intellectual debt, and I wish him many more years of happy puzzling!

and C Clarke

6 December 1999
Colombo, Sri Lanka

Sir Arthur Clarke



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Part I

The Toast: Tributes



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Harry Eng: A Tribute

Mark Setteducati

Harry Eng was a teacher, inventor, minister, artist, magician, and musician whose life was about thinking and inspiring others to think. He was born in California, but the details are vague because he was adopted. For more than 30 years he lived in La Mesa with his wife Betty, raising two children, Greg and Diana.

Harry's house is filled with his creations and described by all who visit as the most unique home they've been in. An extraordinarily innovative school teacher in the San Diego area, in recent years he acted as a consultant to schools giving special lectures to gifted students on creativity and thinking. Harry created many original teaching techniques and devices such as using play money in class printed with his students' pictures on it, and a dummy named Ziggy that had clear plastic lungs attached to a vacuum cleaner. Harry would light up a cigarette for Ziggy and turn on the vacuum to teach kids not to smoke. He used magic and more recently his impossible bottles to inspire students to think.

Harry had a lifelong interest in magic. During the late 1970s he was actively involved in the San Diego magic scene, including being president of a local IBM ring. He never read books on magic and everything Harry performed was original. He invented the "PK Factor," a magnetic principle that was marketed by magic dealers. His hands were chubby and twisted, yet out of these would come incredible feats of magic, inventions, and impossible bottles. Magicians would show him a trick and Harry would often have an insight on a better way to do it—always in a humble way. Harry never made anybody feel he was better or smarter.

Mark Setteducati is a magician and inventor of magic, games and puzzles. He created Milton Bradley's *Magic Works*® and is co-author of *The Magic Show*. This article originally appeared in *Genii* in October, 1996.

One of Harry's passions was his feats of memory which included books he made that contained 10,000 numbers or thousands of words that he had memorized. He would have you turn to any page and he would recite the contents without looking. He created a cardboard name computer that has over 1000 names programmed into a few cards. With a few questions he could guess anybody's name. He created his own stacked deck and was constantly working on new and more impossible card effects with it. He was a master at origami and would fold an ordinary paper bag into a hat, a pair of shoes or a wallet. My favorite routine that Harry performed was with a single piece of heavy duty rope, sometimes with his trademark button knot tied at the bottom. He would pass the rope behind his back while talking about a courtroom. The first time a knot would appear in the center of the rope. Harry would say, "Knot Guilty." The second time Harry would amazingly be able to snap the rope behind his back so it created a noose as he would exclaim, "Guilty!... Hang 'em!" He would then go on to thread a needle, shoot the rope as a bow and arrow, penetrate the rope through a spectator's thumbs, and arrange the rope between his fingers to create a frog and then a dog. Each trick would be accompanied by silly patter and clever puns, all executed with precision skill and a sense of humor that exemplified Harry's personality.

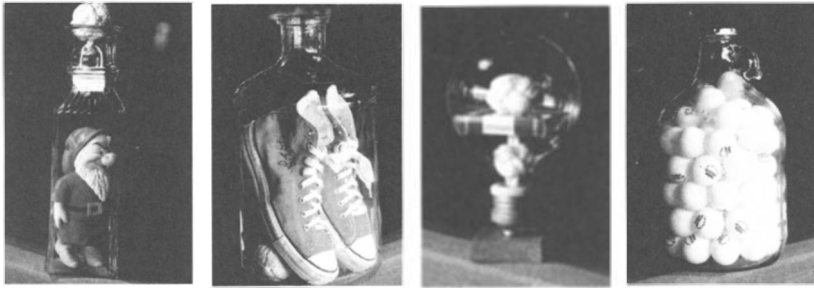


Figure 1. Harry Eng's bottles. Harry explained that cutting the bottle is the hard way to fit the items in for it's nearly impossible to disguise the cut when resealing. (See Color Plate II.)

And then there are the bottles. If Harry never made a single bottle he would still be one of the most remarkable men I have known—but it's the bottles that certify him as a true genius. Impossible bottles date back more than a hundred years, and Hoffman describes an arrow through a bottle, as well as a bottle containing a dowel with a screw

through it. And of course there is the classic ship in a bottle. But about ten years ago Harry Eng put a deck of cards in a bottle and it went on from there: ping pong balls, tennis balls, coins, sneakers, padlocks, baseballs, light bulbs, scissors.... You name it and Harry has probably put it in a bottle. He brought to his bottles a level of originality and diversity that astonished and delighted top puzzle experts in the world. The beauty of the way he would tie knots inside the bottles using the same rope he would perform his tricks with and the clever puns he would incorporate in many of the bottles (a deck of cards with a bullet through it is titled a “loaded deck”) make them more than just puzzles—they are works of art. When people look at the bottles Harry would say they became Indians because they always say “How.” After giving one of his standard humorous answers such as “trained cockroaches” he would tell you his real secret which is: He “thinks his way into the bottle.”

Most people assume the bottles were cut, which, according to Harry, would be the hard way to do it—there would be burn marks or evidence on the bottles. Then, after a little thought, people would figure he put the cards in one at a time or he took the lock apart and assembled it inside the bottle and would be satisfied with their answer. But it’s exactly at this point, if you keep thinking about it, the more impossible they become. How did he get the steel nut through the deck—there’s no clearance to screw the bolt on. How did he lock the padlock through the wooden plug capping the bottle—it doesn’t move. Harry put things in bottles to challenge himself and to make people think.

After suffering a major heart attack more than 15 years ago, the doctors gave Harry about a year to live. Thankfully he didn’t listen to them then—but he did suffer health problems that made him face his mortality every day, never letting it get in his way of living and enjoying life to the fullest. Ironically it was in the middle of performing his rope routine for a group of friends in Northern California on the afternoon of July 29, 1996 at age 64, Harry felt a little faint, sat down and passed away.

Harry truly loved people. He traveled extensively and had friends all over the world—there isn’t a person who knew him who hasn’t been influenced by his inspirational mind and personality. He was one of those rare people who I never saw get angry and who would always look at the positive side of things. Since his passing I think of him every day and the image that keeps coming to me is not of his incredible mind or creations, but of a happy man with a great sense of humor that was reflected in everything he did—always telling silly jokes and laughing. Harry loved to laugh.



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The Eng Coin Vise

Gary Foshee

Of all the objects Harry Eng has placed inside of a bottle, a solid metal coin is one of the most amazing. The effect is one of total impossibility. I pondered over this for some time until Harry let me in on his secret: he bent the coin, placed it in the bottle, then used a specially constructed metal vise *inside* the bottle to flatten the coin. To accomplish this, some of the vise pieces are fed into the bottle one at a time, and then reassembled while inside the bottle. The remaining pieces of the vise are assembled outside of the bottle and joined to the pieces inside. The bent coin is now maneuvered into the vise inside the bottle, the vise closed, and the coin pressed flat. The vise is now disassembled and removed from the bottle. The real beauty of the vise is that the force applied to flatten the coin is generated *outside* of the bottle.

The basic principles of Harry's vise are presented here. There is sufficient detail to build a vise from this description, but it will prove quite difficult. Proper high-strength steels must be used for certain components to withstand the considerable force required to flatten the coin. Consulting someone skilled at metalworking is highly recommended.

The vise consists of an inside piece, an outside piece, and a connecting rod joining the inside and outside pieces. The inside piece is shown unassembled in Figure 1 and assembled in Figure 2. The central hole in the upper bar of Figure 1 is threaded to accept the connecting rod. The two lower bars of Figure 2 slide freely on the two bolts.

The parts of the outside piece are shown in Figure 4. The block has a vertical hole through the center. The hole is threaded at the top to accept a bolt, and threaded at the bottom to accept the connecting rod. A solid push rod fits inside of the connecting rod. The part to the right

Gary Foshee is a mechanical puzzle collector and designer living in Seattle, Washington.

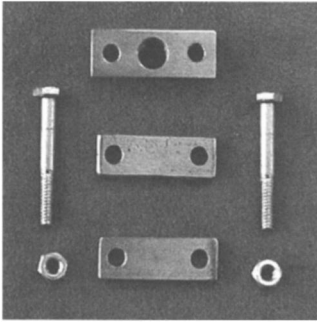


Figure 1.

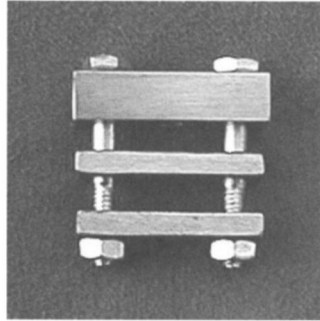


Figure 2.

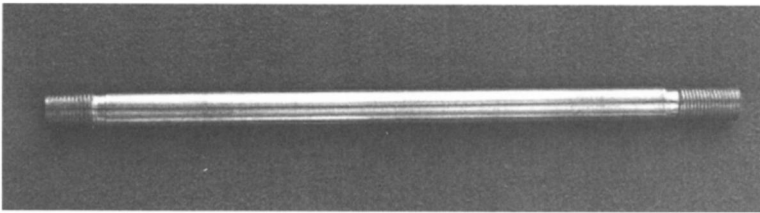


Figure 3.

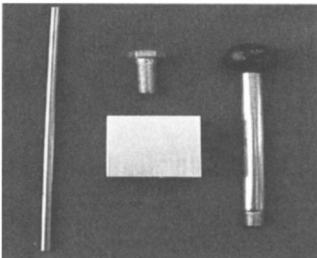


Figure 4.

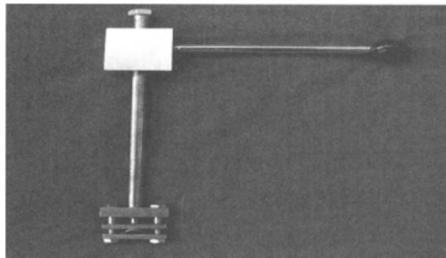


Figure 5.

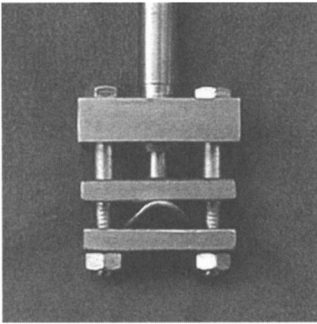


Figure 6.

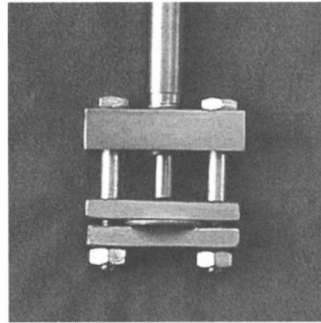


Figure 7.

of the block is a handle that threads into a hole in one side of the block. The connecting rod is shown in Figure 3.

To assemble the vise, feed the parts of the inside piece into the bottle, and join them as shown in Figure 2. This will require considerable dexterity, patience, and many hours of practice. Join the inside and outside pieces with the connecting rod. Next, slip the push rod in through the top of the block and push it all the way down into the inside piece. Thread the bolt into the upper part of the block. Attach the handle. The vise should now appear as in Figure 5.

Place the bent coin in the bottle and maneuver it into the vise as shown in Figure 6. This is very difficult because the connecting rod keeps getting in the way. Place a wrench on the bolt and tighten, gripping the handle with your other hand. This force will be transmitted via the push rod to the coin, and flatten it as shown in Figure 7. Several pressings are needed to completely flatten the coin. When the pressing is complete, disassemble and remove the vise, and your coin bottle is ready to amaze all that view it.



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David A. Klarner—A Memorial Tribute

Solomon W. Golomb

I may have been the Prophet of Polyominoes, but David Klarner was their most faithful Apostle.

It was Christmastime in 1959 (or thereabouts) when I received a large oblong rectangular wooden box in the mail. The return address was one D. A. Klarner from the far north of California. The box had a sliding lid, which I opened carefully, and dumped the wooden contents out on a tabletop. These turned out to be the 29 “pentacubes,” and it took several hours to get them back in the box as neatly as they had arrived. David’s enclosed letter revealed that he was a student at Humboldt State College who had learned about polyominoes from Martin Gardner’s columns in *Scientific American*.

Several years later, when I had gone from JPL to USC, I was invited by Professor Leo Moser of the University of Alberta, in Edmonton, Canada, to be the “outside reader” of the Ph.D. dissertation of one of his students. The student was David Klarner, and the doctoral thesis contained a proof that the number $P(n)$ of n -ominoes lies between a^n and b^n , where $a = 2$ and $b = 8$ was an acceptable choice. It was a day in mid-March in 1964 (or thereabouts) when I flew from Los Angeles around noon (80°F), and with several intermediate stops arrived in Edmonton late at night (−40°F) and met David for the first time. It had been a severe winter, and he had been ill with pneumonia much of the time. The next day was his successful thesis defense.

David had an important article, “Packing a Rectangle with Congruent N -ominoes,” published in Volume 7 of the *Journal of Combinatorial Theory*, in 1969, where he introduced the concept of the **order** of a

Solomon W. Golomb is USC’s University Professor, renowned for his work in shift register sequences, radar, number theory, fluency in numerous languages, and many other things.

polyomino, defined as the minimum number of congruent copies which can be assembled to form a rectangle. (If the given polyomino does not tile any rectangle, its order is undefined.) In the same article, he defined the **odd-order** of a polyomino to be the smallest odd number (if any) of congruent copies which can be assembled to form a rectangle. He had several beautiful illustrative examples, and the subject has inspired important research ever since.

When David was on the faculty at Stanford, he invited me to give a seminar talk (on polyominoes, of course). The date of my talk should be easy to establish, because the news item of the day was the death of former president Lyndon B. Johnson.

After Stanford, David was at SUNY-Binghamton, and also spent more than one sabbatical year visiting the Technical University of Eindhoven, in the Netherlands, where he interacted with N. G. de Bruijn and L. E. J. Bouwkamp, among others. From Binghamton, his next academic position was at the University of Nebraska, in Lincoln.

David had suffered since childhood from “type 1” diabetes (formerly called “juvenile onset” diabetes), and had a lifelong battle with the many complications of this ailment. Driving from New York to Nebraska, around the time he arrived in Lincoln he had a near-fatal heart attack. He was also the recipient of a kidney transplant.

It was in 1993 (or thereabouts) that I was invited (no doubt at David’s suggestion) to be a member of a team of visiting experts to evaluate the progress of the University of Nebraska in the several areas of science and technology that the State Legislature had identified for special funding. I spent most of the free time I had during that period visiting with David Klarner—as it turned out, for the last time.

Ever since the first edition of *Polyominoes* appeared in 1965 (published by Charles Scribner and Sons), I was compiling material for a second edition, which finally appeared in 1994 (with Princeton University Press). I had considerable help from David in the preparation of the final manuscript for the new edition.

David had been the editor of the *Mathematical Gardner*, to which I contributed a chapter; and he was asked to edit the proceedings of the first “Gathering for Gardner”, but by that time his health problems seriously interfered.

In mid-March of 1999, I was visiting the University of Waterloo, in Ontario, Canada, for several days. Douglas Stinson had been at the University of Nebraska when I had visited there, but was now at Waterloo. I asked him what he knew of David’s whereabouts. He told me that David had retired, for health reasons, and had relocated, with

his wife, back to Humboldt, California. I resolved to contact David when I returned to Los Angeles, but I never got the chance. Shortly after coming home, I learned the sad news of his demise.

David Klarner made an important and very distinctive contribution to the literature of combinatorial mathematics in general, and to polyominoes in particular. He will be long remembered by many mathematicians who never actually met him for the quality of his work, and he will be sorely missed by those of us who knew him.



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David Klarner's Pentacube Towers

C. J. Bouwkamp

Dedicated to the memory of David A. Klarner

David, the Inventor: He writes, in a letter to Solomon W. Golomb dated November 27, 1959, "The box which this letter is attached to contains the mathematical blocks that I invented and named pentacubes. They are so named because they are all of the combinations of five cubes joined face to face." The total number of distinct pentacubes is known to be 29. The 12 planar pentacubes are also called solid pentominoes.

David, the Constructor: One of his many constructions with solid pentominoes is, what I call, the pentacube tower. To my knowledge, David never published or mentioned it to his friends before 1968. Among his many drawings, I was fortunate enough to find two different towers which are copied in Figure 1 (Type I). The reader, if in possession of a set of pentacubes, will certainly feel rewarded to successfully construct either tower.¹

At the end of 1968, David came to Eindhoven University of Technology for the first time; his third and last visit to Eindhoven University was in 1991. He found colleagues and friends not only in Mathematics but also in the Building and Architecture Departments, and lectured about combinatorial problems.

C. J. Bouwkamp, from the Netherlands, is a long time reviewer for *Mathematical Reviews*. He has had lectureships at institutions throughout the U.S. and Europe.

¹Pentomino aficionados assign each block a letter name. David used the letters C and S instead of Golomb's U and Z, probably because U might be taken for V, and similarly quarter-turned Z for N.

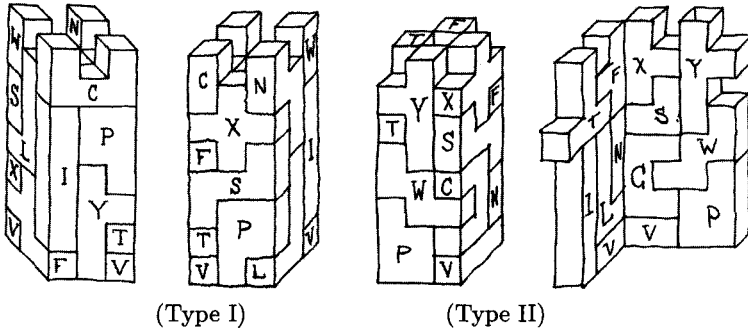


Figure 1. On the left are two distinct pentacube towers of Type I due to Klarner. On the right, Klarner's first known pentacube tower of Type II along with the same tower shown unfolded.

In September 1968, I became aware of the existence of yet another pentacube tower, as depicted in Figure 1 (Type II). Type I and Type II differ in the positions of the turrets, and both have an empty column at the inside. The reconstruction of the Type II tower is not that easy, so David added the fold-up diagram.

Once David was able to find one or two solutions to his own challenges, his interest quickly turned to other problems; finding all solutions was left to the computer. Type I has 10 solutions and Type II has 27, modulo reflection, as obtained by me on a "big" computer in October 1968. They are presented in plane diagrams in Figures 2 and 3, in terms of pentominoes.

The $\square+$ indicates a \square turned on its side with the vertical stem pointing into the page. Similarly $\square\cdot$ is a \square turned on its side. How to convert the diagrams to towers is a puzzle in itself! The method is shown in Figure 4.

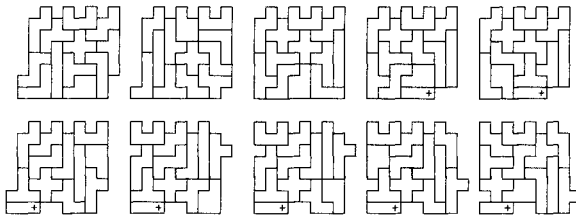


Figure 2. The 10 pentacube towers of Type I.

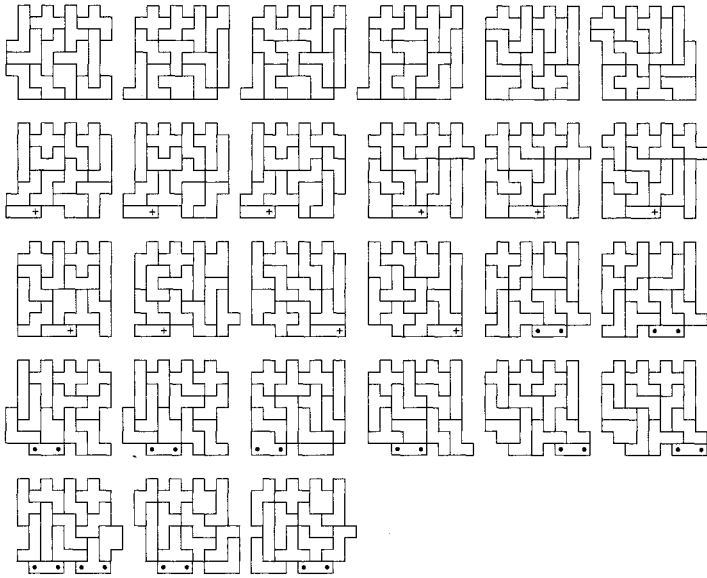


Figure 3. The complete set of 27 pentacube towers of Type II.

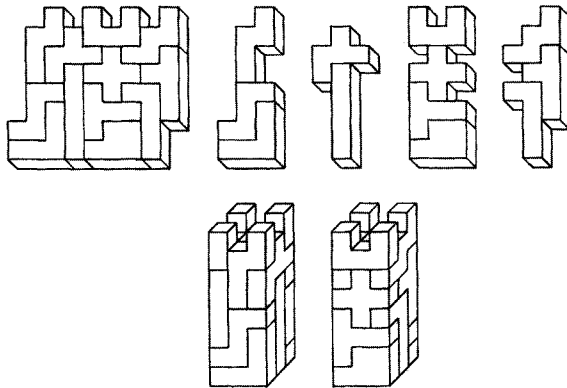
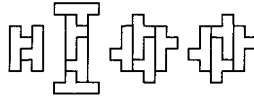


Figure 4. The upper left is the first diagram of Type I from Figure 2. The diagram is split into four parts, folded, and reassembled to form the pentacube tower. Two views of the same tower are shown here.

Y2K Tribute to Martin Gardner

I'd like to close with a creation of my own.

Figure 5 shows a tiling of a square with 2000 congruent Y-pentominoes. The “Y” of Y2K refers both to the year and to the Y-pentomino. The tiling is simple (not compound), in that no subset of Y-pentominoes tiles a smaller rectangle. Moreover, it is rotationally symmetric, and does not contain any of the patterns shown below, patterns which are commonly used to aid in tilings:



Rotations and reflections of these patterns are also forbidden.

The reader is invited to find the Y-pentomino patterns hidden in the tiling!

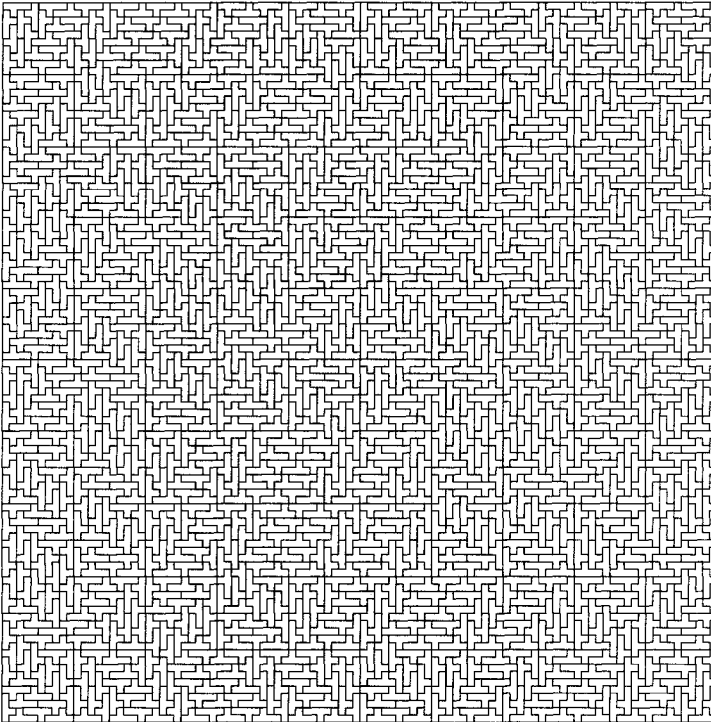


Figure 5. Y2K tiling.

Some Reminiscences of David Klarner

David Singmaster

I first met David in 1970 or 1971, when he was visiting Reading University. We discussed box-packing and I found I had solved a problem that he'd been considering. In two dimensions, if a brick packs a box (larger than itself), then one can divide the box into two smaller boxes such that each smaller box can be packed (indeed with its bricks all in the same direction). The simplest illustration is filling a 6×5 box with 3×2 bricks, where the only packings exhibit this divisibility property.

David had wondered if this still held in three dimensions and I had found that 25 $1 \times 3 \times 4$ bricks can pack into a $5 \times 5 \times 12$ box but could not pack $5 \times 5 \times c$, for $c = 1, \dots, 11$, nor $1 \times 5 \times 12$ nor $2 \times 5 \times 12$ in any way. This is the smallest example of this behavior. David later mentioned this in his classic "Brick-packing puzzles" [Kla73], but he cited an different example: $2 \times 3 \times 7$ in $8 \times 11 \times 21$, apparently having forgotten the numbers in my example.

In 1978, Dean Hoffman proposed the following. Can one fit 27 bricks, all $a \times b \times c$, into a cube of side $a + b + c$? The planar version is to use 4 bricks of size $a \times b$ to fit into a square of side $a + b$. This is easy to do and is a way of showing the arithmetic-geometric mean inequality

$$\sqrt{ab} \leq (a + b)/2$$

in the form

$$4ab \leq (a + b)^2.$$

The corresponding inequality for three variables gives us $27abc \leq (a + b + c)^3$ so that a solution of Hoffman's problem gives a geometric proof of the arithmetic-geometric mean inequality for three variables.

David Singmaster was the leading expositor of the mathematics of the Rubik's Cube and is presently the principal historian of recreational mathematics.

Hoffman tried to do this using pencil and paper and found it too hard to do, so he rang up David and asked if he could do it. David had inherited a fine table saw from his father and used it to make up a set of 27 blocks. As he made each one, he stacked it in the corner and found a solution as he went.

In the early 1980s, I visited David at Binghamton. Dean Hoffman was present and David made me a set of 27 blocks from a lovely redwood. He also made a three-cornered frame to hold them. This set is one of the treasures of my collection.

It was on this visit that I saw the bedspread made by Kara Lynn Klarner showing two orthogonal Latin squares of order 10. This is basically a 10×10 array of squares, using ten colors such that each color occurs once in each row and column. Then each square has a circle on it, using the same ten colors so that each color occurs once in each row and column, and further, so that each pair of colors occurs just once as a square-circle pair. They said the hardest part of making the spread was finding ten sufficiently contrasting colors.

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Just for the Mel of It

Max Maven

Let's get this part out of the way: In late March, 1999, he became the late Mel Stover. I'm not happy about that, and I'd guess that Mel wasn't particularly thrilled about it either.

When Houdini was hospitalized, he held out past the doctors' predictions and managed to die on Halloween. If Mel had held out just a few days longer, he'd have died on April Fool's Day. Then again, when Houdini was Mel's age he'd been dead for 34 years.

My introduction to Mel Stover was via Martin Gardner's seminal *Mathematics Magic and Mystery*, which I encountered a few years after its 1956 publication. There were several exceedingly clever contributions from Mel, including some of his groundbreaking work on geometric vanishes. I was quite taken with his trick entitled "Gargantua's Ten-Pile Problem," which required a deck of 10 billion cards. Mel's suggestion as to how such a pack could be most easily assembled: "Buy 200 million decks of 52 cards each, then discard two cards from each deck."

Clearly, this was no ordinary inventor.

Over the ensuing years, I came across Mel Stover's name in the pages of such learned journals as *Scientific American*, *The New Phoenix*, and *Ibidem*. And in the mid-1970s, I became friends with the Winnipeg wag himself.

As indicated above, Mel was a prolific contributor to a range of periodicals in the fields of puzzles, gaming, and magic. Most recently, during the last three years of the Larsen era, readers of this magazine were treated to his monthly "Braintwisters" column. In books, not

Orson Welles wrote that **Max Maven** has "the most original mind in magic." Fortunately, Mr. Welles died before he could revise his opinion. This article first appeared in *Genii* in September, 1999. The text is copyright by Max Maven, and used with permission.

infrequently, his contributions were simply appropriated. That was all the more unfortunate, given that Mel was a particularly generous fellow who, in most cases, surely would have granted permission had it been sought.

Despite this general largesse, there were certain creations Mel held back from releasing to a wider audience, preferring to retain the option of using them to bedevil friends and acquaintances.

Mel was especially fond of devious variants on old puzzles. The scenario would usually run something like this:

MEL: “Say, y’ever seen this one?”

VICTIM: “Um, yeah, I used to know that, years ago. Let me see, I think I sort of vaguely remember...”

And that was it. You were screwed. Because whatever you vaguely remembered, what Mel had given you was different. Oh, it looked the same—at least, you thought it did—but it wasn’t.

Of these mischievous pranks, Mel’s proudest accomplishment was clearly his take on the classic horse-and-rider puzzle. The origin is a type of ambiguous novelty picture that dates back at least as far as the 17th century; examples have been found from Persia, China, and Japan. The idea was transformed into a puzzle by the American inventor Sam Loyd in 1858, when he was just 17 years old. Millions were distributed by P. T. Barnum, and versions have shown up in countless books and puzzle kits. It’s one of those things that most of us have encountered at some point or other, but not recently—which was perfect grist for the Mel.

Exactly when Mel’s insidious variant was developed is not known. At various times he had at least three versions printed up. One rendering, believed to be his most recent, is shown in Figure 1.¹

Okay, so take a look at the layout. Seem familiar? Sure; you kind of remember this, don’t you? The challenge is to cut along the dotted lines, and rearrange the pieces so that each clown is riding a zebra. Right. It’s, uh, hey—guess it’s not as easy as it appeared.

Indeed, for the simple reason that it can’t be done.

Figure 2 shows the Loyd puzzle.² Do you see what’s different about this compared to the Stover layout?

¹Copyright and marketing rights for these reproductions are retained by the Stover estate.

²This particular artwork was used as an advertising premium in the late 1800s; it is reproduced here courtesy of the Slocum collection.

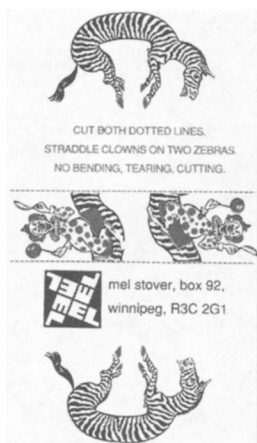


Figure 1.



Figure 2.

Yes, of course, donkeys and jockeys are different from zebras and clowns, but in this context that's merely a cosmetic distinction. The important difference is in the orientation of the quadrupeds. In the Loyd composition, the donkey pieces have identical profiles; that, in turn, enables the solution, as depicted in Figure 3.³ The card is cut into three pieces. The animals are positioned back-to-back, and then the rider strip is placed crosswise on top to produce the solution.

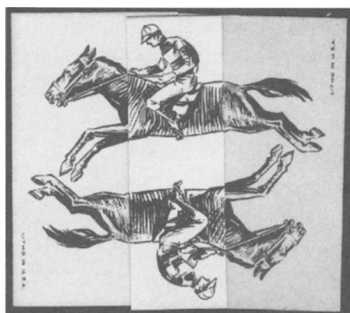


Figure 3.

³Another graphic from the Slocum collection, this one used for restaurant promotions circa 1890.

Let's give the Stover layout another perusal. See it now? If you cut the card into three pieces, the zebras will be headed in opposite directions, and Loyd's crosswise solution won't work.

But Mel was, in truth, even sneakier than his victims imagined, for this was a *double* whammy, hiding in plain sight.

The crafty Canadian would always magnanimously hand you *two* of the cards, with the seemingly offhand comment "No need to cut the second one." And you, chomping promptly, interpreted that to mean that the kindly old duffer was providing the two cards for separate purposes: One to slice up and reorganize, the other to keep as an unmarred souvenir.

And that's where he nailed you the second time, because when you reached the teeth-gnashing realization that the pieces of this modified layout couldn't be rearranged successfully, you assumed the sting was over. But in fact, Mel's puzzle *could* be solved, if you used *all* of the materials he'd given you.

Go back and read the text on the zebra card. It's deliberately terse; there's no determinate article in the challenge to "Straddle clowns on two zebras." It's never specified *which* two zebras. So the real solution—the existence of which Mel didn't always bother to mention—is to cut one of the cards along its dotted lines, place one of those zebras back-to-back with its identical counterpart on the whole card, then set the clown strip crosswise over those two zebras to create the desired outcome.

Yet another example of Stoverian stealth was his take on an item of more recent vintage, the pyramid puzzle, which seems to go back less than a century. Despite its relative youth, it has become accepted as one of those venerable conundra that virtually everyone almost remembers. The challenge involves a set of identical pieces that must be assembled to form an equilateral tetrahedron (i.e., a three-dimensional pyramid with four matching triangular sides).

In its best known format, there are two five-sided pieces, as shown in Figure 4. Despite the simplicity of the apparatus, the solution is trickier than one would expect. I won't spoil it by explaining it here; if you want to make a set of pieces with which to experiment, you can use the template in Figure 5 to fold your own.

The arrangement is not easy to remember, as it is rather counterintuitive. Further impeding the task of cerebral retention is the fact that, somewhere along the line, someone came up with the idea of bisecting each piece, thus producing a version wherein *four* identical pieces have to be tetrahedonally assembled.

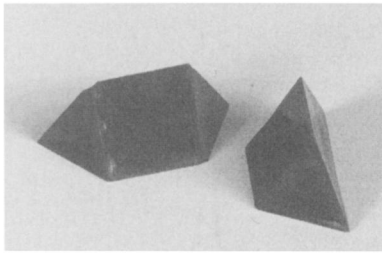


Figure 4.

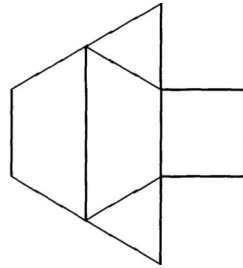


Figure 5.

This is where Stover stepped in. He made use of the two-piece version, in a common small plastic edition just under an inch and a half wide. That's conveniently small enough to make it an easy endeavor to fingerpalm a third piece. (Perhaps you see where this is going; Mel's victims certainly didn't.)

He'd begin by bringing out the three pieces, using two to form a completed pyramid. As the pieces were so small, his hands hid the details of what he was actually doing. He'd move his hands away, revealing the assembled pyramid while keeping the extra piece concealed in his fingers (Figures 6 and 7).

With his trademark tone of lethargic enthusiasm, he'd remark, "Say, y'veer seen this one?" And, as you were acknowledging that you had, while beginning to ponder the location of that long unvisited mental cranny where the proper configuration was stored, Mel would casually brush his hand against the pyramid, knocking it apart—and, in so doing, he'd add the extra piece, as shown in Figure 8.

You can't get there from here. Welcome to Melville.

These intellectual Chinese Finger Traps had an almost mystical allure; Mel's spells were irresistible. A further enticement was the distant, ethereal possibility that one might just be able to beat him at his

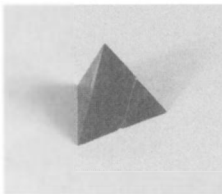


Figure 6.

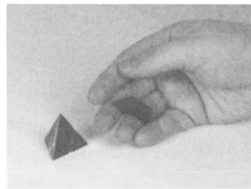


Figure 7.

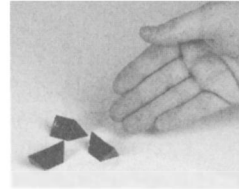


Figure 8.

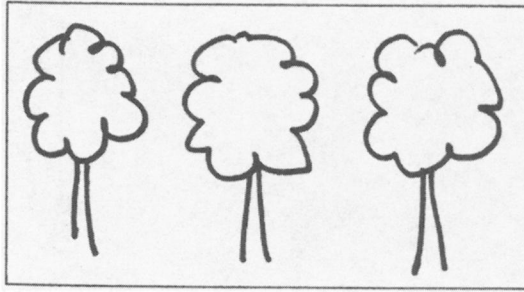


Figure 9.

own game, by discovering a linguistic loophole or conceptual corner that might afford an alternative solution.

It is with great fondness that I recall the first of the very few times I managed to trump Mel. This momentous event took place in a coffee shop, around 1977. Mel turned over his paper placemat, took out a ball-point pen, and produced a scrawl much like the one shown in Figure 9. This, he contended, was a row of oak trees. The riddle: What number did this picture represent?

Warily, I decided to take the least embarrassing route, and plead ignorance. This, of course, elicited a gleeful cackle from the Manitoba mortifier, who revealed that the answer was nine. “See?” he exclaimed, stabbing his finger onto each oak. “*Tree, tree and tree—that makes nine.*”

Without pause, he drew a little smudge at the base of each tree as he continued, “A dog comes along, and urinates on each tree. Now what number does it represent?”

Inescapably, I knew that I was going to feel excruciating chagrin at missing what would surely turn out to be a conspicuously obvious answer, but opted to proclaim my continued ignorance. This provoked another self-satisfied snicker as he declared that it was, quite plainly, ninety-nine. He gestured again toward the crude illustration and elucidated: “*Dirty tree, dirty tree plus dirty tree.*”

He furthered the story, enlarging the smudge at the base of each oak while describing the return of the dog, who this time took the additional effort to defecate on the base of each tree. “Okay,” he taunted, “what number is represented now?”

By this time, my dander had been roused from its normally benign and prone posture, and I insisted that Mel wait at least a few moments before charging ahead to announce the solution.

Time stood still, and then, epiphany. “Aha.” I crowed. “I’ve got it! It represents one hundred and eighty-nine.”

Irked, my now scowling tormentor said, “No. The answer is one hundred: *Dirty tree and a turd, dirty tree and a turd* plus *dirty tree and a turd*. Now, how the hell did you get a hundred and eighty-nine out of this?”

I leaned back against the curved plastic seat—if memory serves, it was a light orange color not found in nature—and produced my own vainglorious smile, as I explained: “*Shits de tree, shits de tree and shits de tree.*”

Life was good.

Mel’s last few weeks were spent at the Cedars-Sinai Hospital in Los Angeles. During one of our final meetings, as he lay festooned with tubes and wires, he suddenly brightened, and began the actions of transferring a small object back and forth from hand to hand. Then he stopped, and extended both palm-up fists in front of him.

“Okay,” he said, “which hand has the Viagra pill?”

I paused to consider. Slowly, the middle finger of his left hand rose to vertical position.

We all smiled.

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