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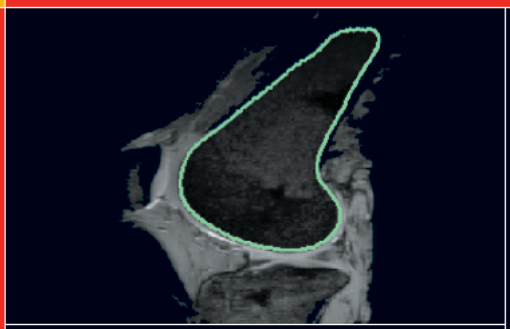

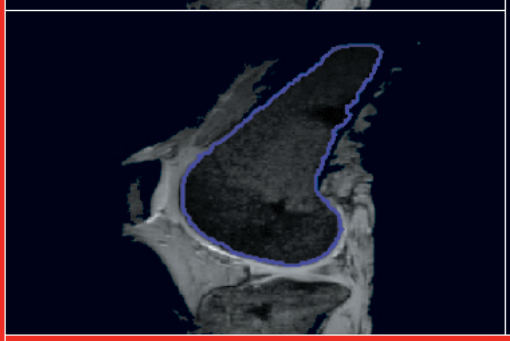

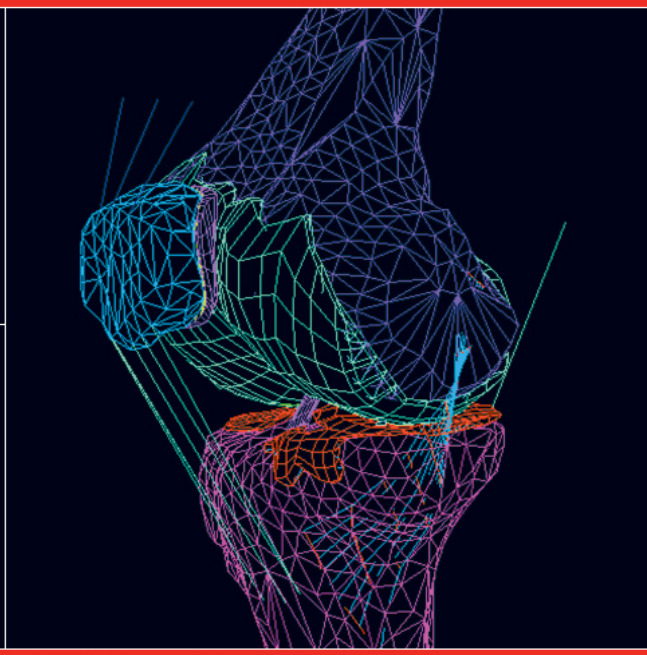

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The *Medical Science Series* is the official book series of the International Federation for Medical and Biological Engineering (IFMBE) and the International Organization for Medical Physics (IOMP).

IFMBE

The IFMBE was established in 1959 to provide medical and biological engineering with an international presence. The Federation has a long history of encouraging and promoting international cooperation and collaboration in the use of technology for improving the health and life quality of man.

The IFMBE is an organization that is mostly an affiliation of national societies. Transnational organizations can also obtain membership. At present there are 42 national members, and one transnational member with a total membership in excess of 15 000. An observer category is provided to give personal status to groups or organizations considering formal affiliation.

Objectives

- To reflect the interests and initiatives of the affiliated organizations.
- To generate and disseminate information of interest to the medical and biological engineering community and international organizations.
- To provide an international forum for the exchange of ideas and concepts.
- To encourage and foster research and application of medical and biological engineering knowledge and techniques in support of life quality and cost-effective health care.
- To stimulate international cooperation and collaboration on medical and biological engineering matters.
- To encourage educational programmes which develop scientific and technical expertise in medical and biological engineering.

Activities

The IFMBE has published the journal *Medical and Biological Engineering and Computing* for over 34 years. A new journal *Cellular Engineering* was established in 1996 in order to stimulate this emerging field in biomedical engineering. In *IFMBE News* members are kept informed of the developments in the Federation. *Clinical Engineering Update* is a publication of our division of Clinical Engineering. The Federation also has a division for Technology Assessment in Health Care.

Every three years, the IFMBE holds a World Congress on Medical Physics and Biomedical Engineering, organized in cooperation with the IOMP and the IUPESM. In addition, annual, milestone, regional conferences are organized in different regions of the world, such as the Asia Pacific, Baltic, Mediterranean, African and South American regions.

The administrative council of the IFMBE meets once or twice a year and is the steering body for the IFMBE. The council is subject to the rulings of the General Assembly which meets every three years.

For further information on the activities of the IFMBE, please contact Jos A E Spaan, Professor of Medical Physics, Academic Medical Centre, University of Amsterdam, PO Box 22660, Meibergdreef 9, 1105 AZ, Amsterdam, The Netherlands. Tel: 31 (0) 20 566 5200. Fax: 31 (0) 20 691 7233. E-mail: IFMBE@amc.uva.nl. WWW: http://vub.vub.ac.be/~ifmbe.

IOMP

The IOMP was founded in 1963. The membership includes 64 national societies, two international organizations and 12 000 individuals. Membership of IOMP consists of individual members of the Adhering National Organizations. Two other forms of membership are available, namely Affiliated Regional Organization and Corporate Members. The IOMP is administered by a Council, which consists of delegates from each of the Adhering National Organization; regular meetings of Council are held every three years at the International

Conference on Medical Physics (ICMP). The Officers of the Council are the President, the Vice-President and the Secretary-General. IOMP committees include: developing countries, education and training; nominating; and publications.

Objectives

- To organize international cooperation in medical physics in all its aspects, especially in developing countries.
- To encourage and advise on the formation of national organizations of medical physics in those countries which lack such organizations.

Activities

Official publications of the IOMP are *Physiological Measurement*, *Physics in Medicine and Biology* and the *Medical Science Series*, all published by Institute of Physics Publishing. The IOMP publishes a bulletin *Medical Physics World* twice a year.

Two Council meetings and one General Assembly are held every three years at the ICMP. The most recent ICMPs were held in Kyoto, Japan (1991), Rio de Janeiro, Brazil (1994) and Nice, France (1997). The next conference is scheduled for Chicago, USA (2000). These conferences are normally held in collaboration with the IFMBE to form the World Congress on Medical Physics and Biomedical Engineering. The IOMP also sponsors occasional international conferences, workshops and courses.

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PREFACE

This book is based upon *Medical Physics and Physiological Measurement* which we wrote in 1981. That book had grown in turn out of a booklet which had been used in the Sheffield Department of Medical Physics and Clinical Engineering for the training of our technical staff. The intention behind our writing had been to give practical information which would enable the reader to carry out a very wide range of physiological measurement and treatment techniques which are often grouped under the umbrella titles of medical physics, clinical engineering and physiological measurement. However, it was more fulfilling to treat a subject in a little depth rather than at a purely practical level so we included much of the background physics, electronics, anatomy and physiology relevant to the various procedures. Our hope was that the book would serve as an introductory text to graduates in physics and engineering as well as serving the needs of our technical staff.

Whilst this new book is based upon the earlier text, it has a much wider intended readership. We have still included much of the practical information for technical staff but, in addition, a considerably greater depth of material is included for graduate students of both medical physics and biomedical engineering. At Sheffield we offer this material in both physics and engineering courses at Bachelor's and Master's degree levels. At the postgraduate level the target reader is a new graduate in physics or engineering who is starting postgraduate studies in the application of these disciplines to healthcare. The book is intended as a broad introductory text that will place the uses of physics and engineering in their medical, social and historical context. Much of the text is descriptive, so that these parts should be accessible to medical students with an interest in the technological aspects of medicine. The applications of physics and engineering in medicine have continued to expand both in number and complexity since 1981 and we have tried to increase our coverage accordingly.

The expansion in intended readership and subject coverage gave us a problem in terms of the size of the book. As a result we decided to omit some of the introductory material from the earlier book. We no longer include the basic electronics, and some of the anatomy and physiology, as well as the basic statistics, have been removed. It seemed to us that there are now many other texts available to students in these areas, so we have simply included the relevant references.

The range of topics we cover is very wide and we could not hope to write with authority on all of them. We have picked brains as required, but we have also expanded the number of authors to five. Rod and I very much thank Rod Hose, Pat Lawford and David Barber who have joined us as co-authors of the new book.

We have received help from many people, many of whom were acknowledged in the preface to the original book (see page xxiii). Now added to that list are John Conway, Lisa Williams, Adrian Wilson, Christine Segasby, John Fenner and Tony Trowbridge. Tony died in 1997, but he was a source of inspiration and we have used some of his lecture material in Chapter 13. However, we start with a recognition of the encouragement given by Professor Martin Black. Our thanks must also go to all our colleagues who tolerated our hours given to the book but lost to them. Sheffield has for many years enjoyed joint University and Hospital activities in medical physics and biomedical engineering. The result of this is a large group of professionals with a collective knowledge of the subject that is probably unique. We could not have written this book in a narrow environment.

We record our thanks to Kathryn Cantley at Institute of Physics Publishing for her long-term persistence and enthusiasm. We must also thank our respective wives and husband for the endless hours lost to them. As before, we place the initial blame at the feet of Professor Harold Miller who, during his years as Professor of Medical Physics at Sheffield and in his retirement until his death in 1996, encouraged an enthusiasm for the subject without which this book would never have been written.

Brian Brown and Rod Smallwood

Sheffield, 1998

PREFACE TO '*MEDICAL PHYSICS AND PHYSIOLOGICAL MEASUREMENT*'

This book grew from a booklet which is used in the Sheffield Department of Medical Physics and Clinical Engineering for the training of our technical staff. The intention behind our writing has been to give practical information which will enable the reader to carry out the very wide range of physiological measurement and treatment techniques which are often grouped under the umbrella title of medical physics and physiological measurement. However, it is more fulfilling to treat a subject in depth rather than at a purely practical level and we have therefore included much of the background physics, electronics, anatomy and physiology which is necessary for the student who wishes to know why a particular procedure is carried out. The book which has resulted is large but we hope it will be useful to graduates in physics or engineering (as well as technicians) who wish to be introduced to the application of their science to medicine. It may also be interesting to many medical graduates.

There are very few hospitals or academic departments which cover all the subjects about which we have written. In the United Kingdom, the Zuckermann Report of 1967 envisaged large departments of 'physical sciences applied to medicine'. However, largely because of the intractable personnel problems involved in bringing together many established departments, this report has not been widely adopted, but many people have accepted the arguments which advocate closer collaboration in scientific and training matters between departments such as Medical Physics, Nuclear Medicine, Clinical Engineering, Audiology, ECG, Respiratory Function and Neurophysiology. We are convinced that these topics have much in common and can benefit from close association. This is one of the reasons for our enthusiasm to write this book. However, the coverage is very wide so that a person with several years' experience in one of the topics should not expect to learn very much about their own topic in our book—hopefully, they should find the other topics interesting.

Much of the background introductory material is covered in the first seven chapters. The remaining chapters cover the greater part of the sections to be found in most larger departments of Medical Physics and Clinical Engineering and in associated hospital departments of Physiological Measurement. Practical experiments are given at the end of most of the chapters to help both individual students and their supervisors. It is our intention that a reader should follow the book in sequence, even if they omit some sections, but we accept the reality that readers will take chapters in isolation and we have therefore made extensive cross-references to associated material.

The range of topics is so wide that we could not hope to write with authority on all of them. We considered using several authors but eventually decided to capitalize on our good fortune and utilize the wide experience available to us in the Sheffield University and Area Health Authority (Teaching) Department of Medical Physics and Clinical Engineering. We are both very much in debt to our colleagues, who have supplied us with information and made helpful comments on our many drafts. Writing this book has been enjoyable to both of us and we have learnt much whilst researching the chapters outside our personal competence. Having said that, we nonetheless accept responsibility for the errors which must certainly still exist and we would encourage our readers to let us know of any they find.

Our acknowledgments must start with Professor M M Black who encouraged us to put pen to paper and Miss Cecile Clarke, who has spent too many hours typing diligently and with good humour whilst looking after a busy office. The following list is not comprehensive but contains those to whom we owe particular debts: Harry Wood, David Barber, Susan Sherriff, Carl Morgan, Ian Blair, Vincent Sellars, Islwyn Pryce, John Stevens, Walt O'Dowd, Neil Kenyon, Graham Harston, Keith Bomford, Alan Robinson, Trevor Jenkins, Chris Franks, Jacques Hermans and Wendy Makin of our department, and also Dr John Jarratt of the Department of Neurology and Miss Judith Connell of the Department of Communication. A list of the books which we have used and from which we have profited greatly is given in the Bibliography. We also thank the Royal Hallamshire Hospital and Northern General Hospital Departments of Medical Illustration for some of the diagrams.

Finishing our acknowledgments is as easy as beginning them. We must thank our respective wives for the endless hours lost to them whilst we wrote, but the initial blame we lay at the feet of Professor Harold Miller who, during his years as Professor of Medical Physics in Sheffield until his retirement in 1975, and indeed since that time, gave both of us the enthusiasm for our subject without which our lives would be much less interesting.

Brian Brown and Rod Smallwood

Sheffield, 1981

NOTES TO READERS

Medical physics and biomedical engineering covers a very wide range of subjects, not all of which are included in this book. However, we have attempted to cover the main subject areas such that the material is suitable for physical science and engineering students at both graduate and postgraduate levels who have an interest in following a career either in healthcare or in related research.

Our intention has been to present both the scientific basis and the practical application of each subject area. For example, Chapter 3 covers the physics of hearing and Chapter 15 covers the practical application of this in audiology. The book thus falls broadly into two parts with the break following Chapter 14. Our intention has been that the material should be followed in the order of the chapters as this gives a broad view of the subject. In many cases one chapter builds upon techniques that have been introduced in earlier chapters. However, we appreciate that students may wish to study selected subjects and in this case will just read the chapters covering the introductory science and then the application of specific subjects. Cross-referencing has been used to show where earlier material may be needed to understand a particular section.

The previous book was intended mainly for technical staff and as a broad introductory text for graduates. However, we have now added material at a higher level, appropriate for postgraduates and for those entering a research programme in medical physics and biomedical engineering. Some sections of the book do assume a degree level background in the mathematics needed in physics and engineering. The introduction to each chapter describes the level of material to be presented and readers should use this in deciding which sections are appropriate to their own background.

As the book has been used as part of Sheffield University courses in medical physics and biomedical engineering, we have included problems at the end of each chapter. The intention of the short questions is that readers can test their understanding of the main principles of each chapter. Longer questions are also given, but answers are only given to about half of them. Both the short and longer questions should be useful to students as a means of testing their reading and to teachers involved in setting examinations.

The text is now aimed at providing the material for taught courses. Nonetheless we hope we have not lost sight of our intention simply to describe a fascinating subject area to the reader.

ACKNOWLEDGMENTS

We would like to thank the following for the use of their material in this book: the authors of all figures not originated by ourselves, Butterworth–Heinemann Publishers, Chemical Rubber Company Press, Churchill Livingstone, Cochlear Ltd, John Wiley & Sons, Inc., Macmillian Press, Marcel Dekker, Inc., Springer-Verlag GmbH & Co. KG, The MIT Press.

CHAPTER 1

BIOMECHANICS

1.1. INTRODUCTION AND OBJECTIVES

In this chapter we will investigate some of the biomechanical systems in the human body. We shall see how even relatively simple mechanical models can be used to develop an insight into the performance of the system. Some of the questions that we shall address are listed below.

- What sorts of loads are supported by the human body?
- How strong are our bones?
- What are the engineering characteristics of our tissues?
- How efficient is the design of the skeleton, and what are the limits of the loads that we can apply to it?
- What models can we use to describe the process of locomotion? What can we do with these models?
- What are the limits on the performance of the body?
- Why can a frog jump so high?

The material in this chapter is suitable for undergraduates, graduates and the more general reader.

1.2. PROPERTIES OF MATERIALS

1.2.1. Stress/strain relationships: the constitutive equation

If we take a rod of some material and subject it to a load along its axis we expect that it will change in length. We might draw a load/displacement curve based on experimental data, as shown in figure 1.1.

We could construct a curve like this for any rod, but it is obvious that its shape depends on the geometry of the rod as much as on any properties of the material from which it is made. We could, however, chop the rod up into smaller elements and, apart from difficulties close to the ends, we might reasonably assume that each element of the same dimensions carries the same amount of load and extends by the same amount. We might then describe the displacement in terms of extension per unit length, which we will call *strain* (ε), and the load in terms of load per unit area, which we will call *stress* (σ). We can then redraw the load/displacement curve as a stress/strain curve, and this should be independent of the dimensions of the bar. In practice we might have to take some care in the design of a test specimen in order to eliminate end effects.

The shape of the stress/strain curve illustrated in figure 1.2 is typical of many engineering materials, and particularly of metals and alloys. In the context of biomechanics it is also characteristic of bone, which is studied in more detail in section 1.2.2. There is a linear portion between the origin O and the point Y. In this

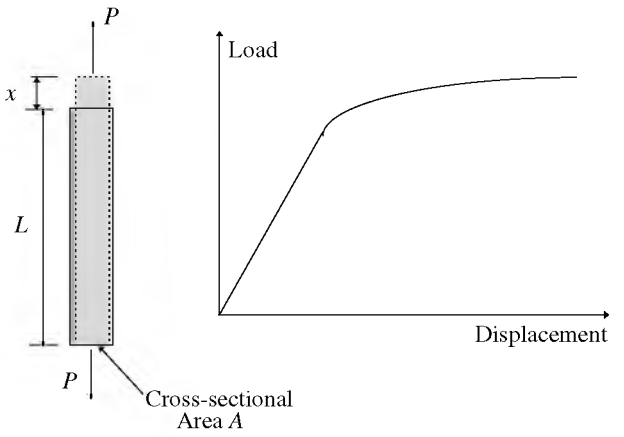


Figure 1.1. *Load/displacement curve: uniaxial tension.*

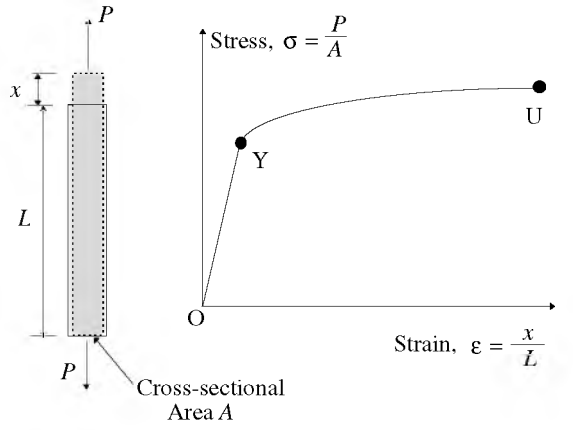


Figure 1.2. *Stress/strain curve: uniaxial tension.*

region the stress is proportional to the strain. The constant of proportionality, E, is called *Young's modulus*,

$$\sigma = E\varepsilon.$$

The linearity of the equivalent portion of the load/displacement curve is known as Hooke's law.

For many materials a bar loaded to any point on the portion OY of the stress/strain curve and then unloaded will return to its original unstressed length. It will follow the same line during unloading as it did during loading. This property of the material is known as *elasticity*. In this context it is not necessary for the curve to be linear: the important characteristic is the similarity of the loading and unloading processes. A material that exhibits this property and has a straight portion OY is referred to as *linear elastic* in this region. All other combinations of linear/nonlinear and elastic/inelastic are possible.

The linear relationship between stress and strain holds only up to the point Y. After this point the relationship is nonlinear, and often the slope of the curve drops off very quickly after this point. This means

that the material starts to feel 'soft', and extends a great deal for little extra load. Typically the point Y represents a critical stress in the material. After this point the unloading curve will no longer be the same as the loading curve, and upon unloading from a point beyond Y the material will be seen to exhibit a permanent distortion. For this reason Y is often referred to as the *yield point* (and the stress there as the yield stress), although in principle there is no fundamental reason why the limit of proportionality should coincide with the limit of elasticity. The portion of the curve beyond the yield point is referred to as the *plastic region*.

The bar finally fractures at the point U. The stress there is referred to as the (uniaxial) *ultimate tensile stress* (UTS). Often the strain at the point U is very much greater than that at Y, whereas the ultimate tensile stress is only a little greater (perhaps by up to 50%) than the yield stress. Although the material does not actually fail at the yield stress, the bar has suffered a permanent strain and might be regarded as being damaged. Very few engineering structures are designed to operate normally above the yield stress, although they might well be designed to move into this region under extraordinary conditions. A good example of post-yield design is the 'crumple zone' of an automobile, designed to absorb the energy of a crash. The area under the load/displacement curve, or the volume integral of the area under the stress/strain curve, is a measure of the energy required to achieve a particular deformation. On inspection of the shape of the curve it is obvious that a great deal of energy can be absorbed in the plastic region.

Materials like rubber, when stretched to high strains, tend to follow very different loading and unloading curves. A typical example of a uniaxial test of a rubber specimen is illustrated in figure 1.3. This phenomenon is known as *hysteresis*, and the area between the loading and unloading curves is a measure of the energy lost during the process. Over a period of time the rubber tends to creep back to its original length, but the capacity of the system as a shock absorber is apparent.

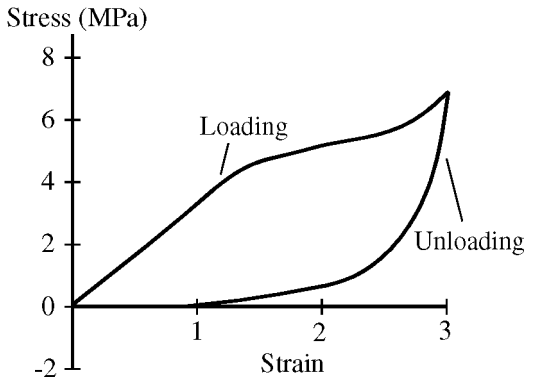


Figure 1.3. *Typical experimental uniaxial stress/strain curve for rubber.*

We might consider that the uniaxial stress/strain curve describes the behaviour of our material quite adequately. In fact there are many questions that remain unanswered by a test of this type. These fall primarily into three categories: one associated with the nature and orientation of loads; one associated with time; and one associated with our definitions of stress and strain. Some of the questions that we should ask and need to answer, particularly in the context of biomechanics, are summarized below: key words that are associated with the questions are listed in italics. We shall visit many of these topics as we discuss the properties of bone and tissue and explore some of the models used to describe them. For further information the reader is referred to the works listed in the bibliography.

- Our curve represents the response to tensile loads. Is there any difference under compressive loads? Are there any other types of load?

Compression, Bending, Shear, Torsion.

- The material is loaded along one particular axis. What happens if we load it along a different axis? What happens if we load it along two or three axes simultaneously?
  Homogeneity, Isotropy, Constitutive equations.
- We observe that most materials under tensile load contract in the transverse directions, implying that the cross-sectional area reduces. Can we use measures of this contraction to learn more about the material?
  Poisson's ratio, Constitutive equations.
- What happens if the rod is loaded more quickly or more slowly? Does the shape of the stress/strain curve change substantially?
  Rate dependence, Viscoelasticity.
- What happens if a load is maintained at a constant value for a long period of time? Does the rod continue to stretch? Conversely, what happens if a constant extension is maintained? Does the load diminish or does it hold constant?
  Creep, Relaxation, Viscoelasticity.
- What happens if a load is applied and removed repeatedly? Does the shape of the stress/strain curve change?
  Cyclic loads, Fatigue, Endurance, Conditioning.
- When calculating increments of strain from increments of displacement should we always divide by the original length of the bar, or should we recognize that it has already stretched and divide by its extended length? Similarly, should we divide the load by the original area of the bar or by its deformed area prior to application of the current increment of load?
  Logarithmic strain, True stress, Hyperelasticity.

The concepts of homogeneity and isotropy are of particular importance to us when we begin a study of biological materials. A *homogeneous* material is one that is the same at all points in space. Most biological materials are made up of several different materials, and if we look at them under a microscope we can see that they are not the same at all points. For example, if we look at one point in a piece of tissue we might find collagen, elastin or cellular material; the material is inhomogeneous. Nevertheless, we might find some uniformity in the behaviour of a piece of the material of a length scale of a few orders of magnitude greater than the scale of the local inhomogeneity. In this sense we might be able to construct characteristic curves for a 'composite material' of the individual components in the appropriate proportions. Composite materials can take on desirable properties of each of their constituents, or can use some of the constituents to mitigate undesirable properties of others. The most common example is the use of stiff and/or strong fibres in a softer matrix. The fibres can have enormous strength or stiffness, but tend to be brittle and easily damaged. Cracks propagate very quickly in such materials. When they are embedded in an elastic matrix, the resulting composite does not have quite the strength and stiffness of the individual fibres, but it is much less susceptible to damage. Glass, aramid, carbon fibres and epoxy matrices are widely used in the aerospace industries to produce stiff, strong and light structures. The body uses similar principles in the construction of bone and tissue.

An *isotropic* material is one that exhibits the same properties in all directions at a given point in space. Many composite materials are deliberately designed to be anisotropic. A composite consisting of glass fibres aligned in one direction in an epoxy matrix will be stiff and strong in the direction of the fibres, but its properties in the transverse direction will be governed almost entirely by those of the matrix material. For such a material the strength and stiffness obviously depend on the orientation of the applied loads relative to the orientation of the fibres. The same is true of bone and of tissue. In principle, the body will tend to orientate

its fibres so that they coincide with the load paths within the structures. For example, a long bone will have fibres orientated along the axis and a pressurized tube will have fibres running around the circumference. There is even a remodelling process in living bone in which fibres can realign when load paths change.

Despite the problems outlined above, simple uniaxial stress/strain tests do provide a sound basis for comparison of mechanical properties of materials. Typical stress/strain curves can be constructed to describe the mechanical performance of many biomaterials. In this chapter we shall consider in more detail two very different components of the human body: bones and soft tissue. Uniaxial tests on bone exhibit a linear load/displacement relationship described by Hooke's law. The load/displacement relationship for soft tissues is usually nonlinear, and in fact the gradient of the stress/strain curve is sometimes represented as a linear function of the stress.

1.2.2. Bone

Bone is a composite material, containing both organic and inorganic components. The organic components, about one-third of the bone mass, include the cells, osteoblasts, osteocytes and osteoid. The inorganic components are hydroxyapatites (mineral salts), primarily calcium phosphates.

- The osteoid contains collagen, a fibrous protein found in all connective tissues. It is a low elastic modulus material ($E \approx 1.2$ GPa) that serves as a matrix and carrier for the harder and stiffer mineral material. The collagen provides much of the tensile strength (but not stiffness) of the bone. Deproteinized bone is hard, brittle and weak in tension, like a piece of chalk.
- The mineral salts give the bone its hardness and its compressive stiffness and strength. The stiffness of the salt crystals is about 165 GPa, approaching that of steel. Demineralized bone is soft, rubbery and ductile.

The skeleton is composed of *cortical* (*compact*) and *cancellous* (*spongy*) bone, the distinction being made based on the porosity or density of the bone material. The division is arbitrary, but is often taken to be around 30% porosity (see figure 1.4).

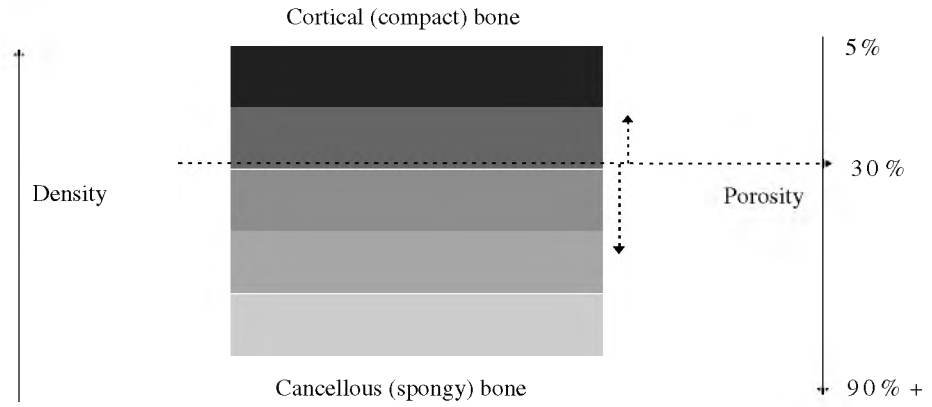


Figure 1.4. *Density and porosity of bone.*

Cortical bone is found where the stresses are high and cancellous bone where the stresses are lower (because the loads are more distributed), but high distributed stiffness is required. The aircraft designer uses honeycomb cores in situations that are similar to those where cancellous bone is found.

Cortical bone is hard and has a stress/strain relationship similar to many engineering materials that are in common use. It is anisotropic, and the properties that are measured for a bone specimen depend on the orientation of the load relative to the orientation of the collagen fibres. Furthermore, partly because of its composite structure, its properties in tension, in compression and shear are rather different. In principle, bone is strongest in compression, weaker in tension and weakest in shear. The strength and stiffness of bone also vary with the age and sex of the subject, the strain rate and whether it is wet or dry. Dry bone is typically slightly stiffer (higher Young's modulus) but more brittle (lower strain to failure) than wet bone. A typical uniaxial tensile test result for a wet human femur is illustrated in figure 1.5. Some of the mechanical properties of the femur are summarized in table 1.1, based primarily on a similar table in Fung (1993).

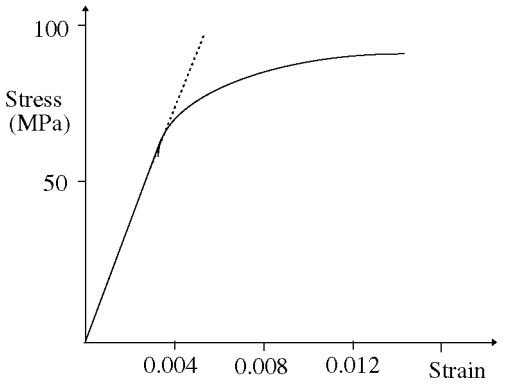


Figure 1.5. *Uniaxial stress/strain curve for cortical bone.*

Table 1.1. *Mechanical properties of bone (values quoted by Fung (1993)).*

	Tension			Compression			Shear		Poisson's ratio
Bone	σ (MPa)	ε (%)	E (GPa)	σ (MPa)	ε (%)	E (GPa)	σ (MPa)	ε (%)	ν
Femur	124	1.41	17.6	170	1.85		54	3.2	0.4

For comparison, a typical structural steel has a strength of perhaps 700 MPa and a stiffness of 200 GPa. There is more variation in the strength of steel than in its stiffness. *Cortical bone is approximately one-tenth as stiff and one-fifth as strong as steel.* Other properties, tabulated by Cochran (1982), include the yield strength (80 MPa, 0.2% strain) and the fatigue strength (30 MPa at 10^8 cycles).

Living bone has a unique feature that distinguishes it from any other engineering material. It remodels itself in response to the stresses acting upon it. The re-modelling process includes both a change in the volume of the bone and an orientating of the fibres to an optimal direction to resist the stresses imposed. This observation was first made by Julius Wolff in the late 19th Century, and is accordingly called Wolff's law. Although many other workers in the field have confirmed this observation, the mechanisms by which it occurs are not yet fully understood.

Experiments have shown the effects of screws and screw holes on the energy-storing capacity of rabbit bones. A screw inserted in the femur causes an immediate 70% decrease in its load capacity. This is

consistent with the stress concentration factor of three associated with a hole in a plate. After eight weeks the stress-raising effects have disappeared completely due to local remodelling of the bone. Similar re-modelling processes occur in humans when plates are screwed to the bones of broken limbs.

1.2.3. Tissue

Tissue is the fabric of the human body. There are four basic types of tissue, and each has many subtypes and variations. The four types are:

- epithelial (*covering*) tissue;
- connective (*support*) tissue;
- muscle (*movement*) tissue;
- nervous (*control*) tissue.

In this chapter we will be concerned primarily with connective tissues such as tendons and ligaments. Tendons are usually arranged as ropes or sheets of dense connective tissue, and serve to connect muscles to bones or to other muscles. Ligaments serve a similar purpose, but attach bone to bone at joints. In the context of this chapter we are using the term tissue to describe soft tissue in particular. In a wider sense bones themselves can be considered as a form of connective tissue, and cartilage can be considered as an intermediate stage with properties somewhere between those of soft tissue and bone.

Like bone, soft tissue is a composite material with many individual components. It is made up of cells intimately mixed with intracellular materials. The intracellular material consists of fibres of collagen, elastin, reticulin and a gel material called ground substance. The proportions of the materials depend on the type of tissue. Dense connective tissues generally contain relatively little of the ground substance and loose connective tissues contain rather more. The most important component of soft tissue with respect to the mechanical properties is usually the collagen fibre. The properties of the tissue are governed not only by the amount of collagen fibre in it, but also by the orientation of the fibres. In some tissues, particularly those that transmit a uniaxial tension, the fibres are parallel to each other and to the applied load. Tendons and ligaments are often arranged in this way, although the fibres might appear irregular and wavy in the relaxed condition. In other tissues the collagen fibres are curved, and often spiral, giving rise to complex material behaviour.

The behaviour of tissues under load is very complex, and there is still no satisfactory first-principles explanation of the experimental data. Nevertheless, the properties can be measured and constitutive equations can be developed that fit experimental observation. The stress/strain curves of many collagenous tissues, including tendon, skin, resting skeletal muscle and the scleral wall of the globe of the eye, exhibit a stress/strain curve in which the gradient of the curve is a linear function of the applied stress (figure 1.6).

1.2.4. Viscoelasticity

The tissue model considered in the previous section is based on the assumption that the stress/strain curve is independent of the rate of loading. Although this is true over a wide range of loading for some tissue types, including the skeletal muscles of the heart, it is not true for others. When the stresses and strains are dependent upon time, and upon rate of loading, the material is described as *viscoelastic*. Some of the models that have been proposed to describe viscoelastic behaviour are discussed and analysed by Fung (1993). There follows a brief review of the basic building blocks of these viscoelastic models. The nomenclature adopted is that of Fung. The models that we shall consider are all based on the assumption that a rod of viscoelastic material behaves as a set of linear springs and viscous dampers in some combination.

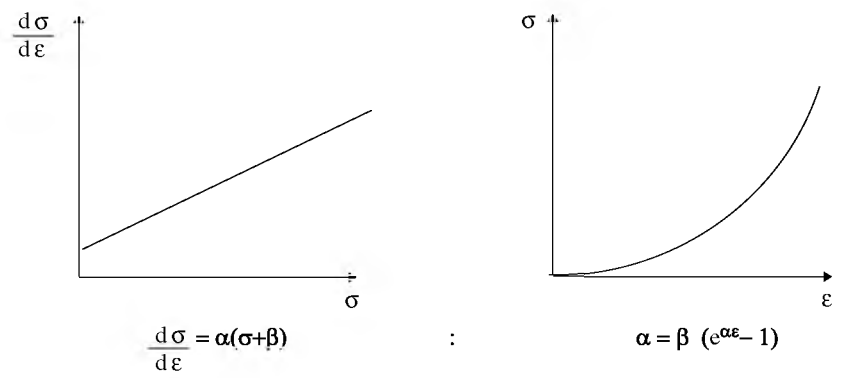


Figure 1.6. *Typical stress/strain curves for some tissues.*

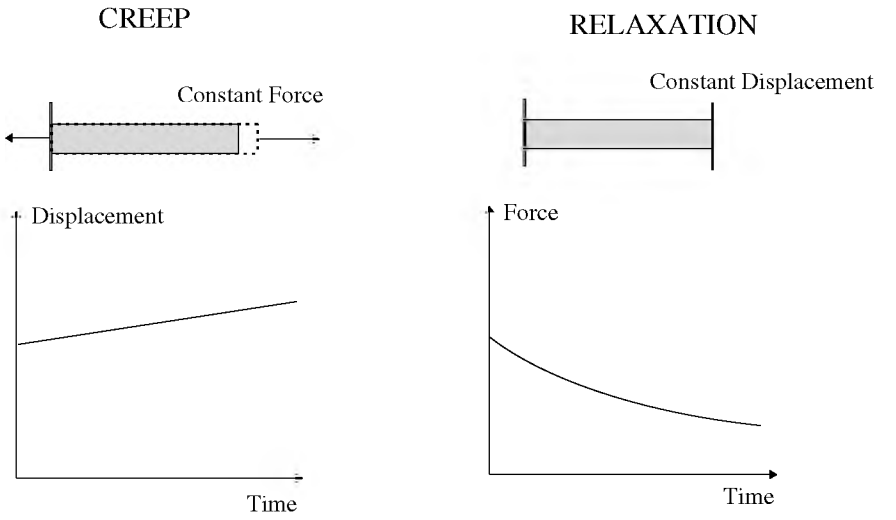


Figure 1.7. *Typical creep and relaxation curves.*

Creep and relaxation

Viscoelastic materials are characterized by their capacity to *creep under constant loads and to relax under constant displacements* (figure 1.7).

Springs and dashpots

A linear spring responds instantaneously to an applied load, producing a displacement proportional to the load (figure 1.8).

The displacement of the spring is determined by the applied load. If the load is a function of time, $F = F(t)$, then the displacement is proportional to the load and the rate of change of displacement is

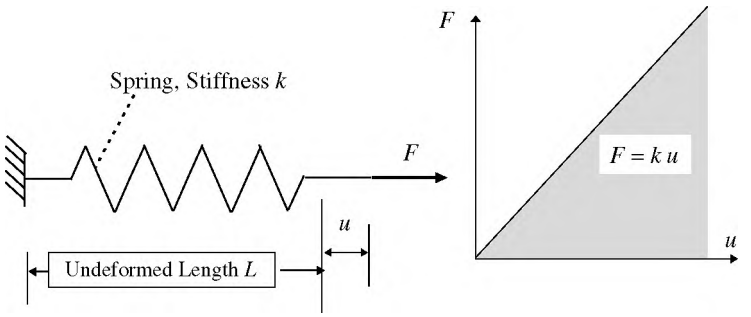


Figure 1.8. *Load/displacement characteristics of a spring.*

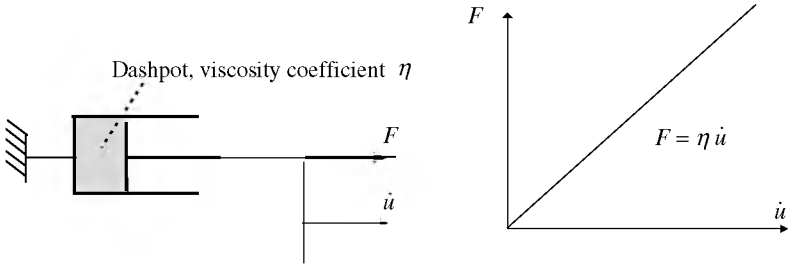


Figure 1.9. *Load/velocity characteristics of a dashpot.*

proportional to the rate of change of load,

$$u_{\text{spring}} = \frac{F}{k} \qquad \dot{u}_{\text{spring}} = \frac{\dot{F}}{k}.$$

A dashpot produces a velocity that is proportional to the load applied to it at any instant (figure 1.9).

For the dashpot the velocity is proportional to the applied load and the displacement is found by integration,

$$\dot{u}_{\text{dashpot}} = \frac{F}{\eta} \qquad u_{\text{dashpot}} = \int \frac{F}{\eta}\,\mathrm{d}t.$$

Note that the displacement of the dashpot will increase forever under a constant load.

Models of viscoelasticity

Three models that have been used to represent the behaviour of viscoelastic materials are illustrated in figure 1.10.

The *Maxwell model* consists of a spring and dashpot in series. When a force is applied the velocity is given by

$$\dot{u} = \dot{u}_{\text{spring}} + \dot{u}_{\text{dashpot}}$$

$$\dot{u} = \frac{\dot{F}}{k} + \frac{F}{\eta}.$$

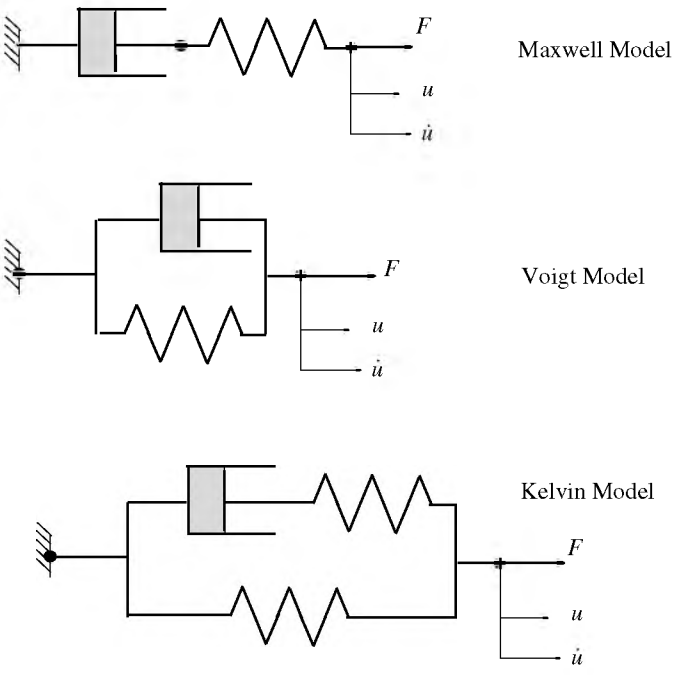


Figure 1.10. *Three 'building-block' models of viscoelasticity.*

The displacement at any point in time will be calculated by integration of this differential equation.

The *Voigt model* consists of a spring and dashpot in parallel. When a force is applied, the displacement of the spring and dashpot is the same. The total force must be that applied, and so the governing equation is

$$F_{\text{dashpot}} + F_{\text{spring}} = F$$
$$\eta\dot{u} + ku = F.$$

The *Kelvin* model consists of a Maxwell element in parallel with a spring. The displacement of the Maxwell element and that of the spring must be the same, and the total force applied to the Maxwell element and the spring is known. It can be shown that the governing equation for the Kelvin model is

$$E_{\text{R}}\left(\tau_\sigma \dot{u} + u\right) = \tau_\varepsilon \dot{F} + F$$

where

$$E_{\text{R}} = k_2 \qquad \tau_\sigma = \frac{\eta_1}{k_2}\left(1 + \frac{k_2}{k_1}\right) \qquad \tau_\varepsilon = \frac{\eta_1}{k_1}.$$

In this equation the subscript 1 applies to the spring and dashpot of the Maxwell element and the subscript 2 applies to the parallel spring. τ_ε is referred to as the relaxation time for constant strain and τ_σ is referred to as the relaxation time for constant stress.

These equations are quite general, and might be solved for any applied loading defined as a function of time. It is instructive to follow Fung in the investigation of the response of a system represented by each of these models to a unit load applied suddenly at time $t = 0$, and then held constant. The unit step function $\mathbf{1}(t)$ is defined as illustrated in figure 1.11.

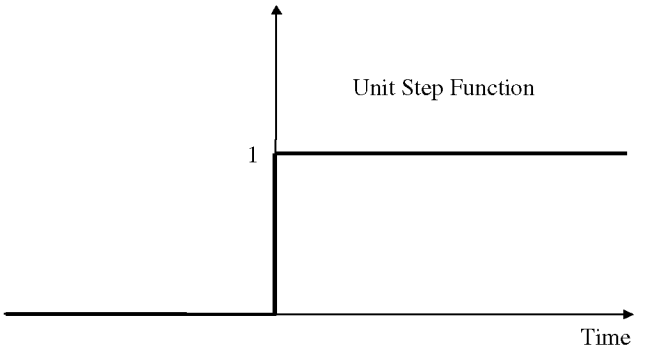


Figure 1.11. *Unit step function.*

For the Maxwell solid the solution is

$$u = \left(\frac{1}{k} + \frac{1}{\eta}t\right)\mathbf{1}(t).$$

Note that this equation satisfies the initial condition that the displacement is $1/k$ as soon as the load is applied.

For the Voigt solid the solution is

$$u = \frac{1}{k}\left(1 - \mathrm{e}^{-(k/\eta)t}\right)\mathbf{1}(t).$$

In this case the initial condition is that the displacement is zero at time zero, because the spring cannot respond to the load without applying a velocity to the dashpot. Once again the solution is chosen to satisfy the initial conditions.

For the Kelvin solid the solution is

$$u = \frac{1}{E_{\mathrm{R}}}\left(1 - \left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right)\mathrm{e}^{-t/\tau_\sigma}\right)\mathbf{1}(t).$$

It is left for the reader to think about the initial conditions that are appropriate for the Kelvin model and to demonstrate that the above solution satisfies them.

The solution for a load held constant for a period of time and then removed can be found simply by adding a negative and phase-shifted solution to that shown above. The response curves for each of the models are shown in figure 1.12. These represent the behaviour of the models under constant load. They are sometimes called creep functions. Similar curves showing force against time, sometimes called relaxation functions, can be constructed to represent their behaviour under constant displacement. For the Maxwell model the force relaxes exponentially and is asymptotic to zero. For the Voigt model a force of infinite magnitude but infinitesimal duration (an impulse) is required to obtain the displacement, and thereafter the force is constant. For the Kelvin model an initial force is required to displace the spring elements by the required amount, and the force subsequently relaxes as the Maxwell element relaxes. In this case the force is asymptotic to that generated in the parallel spring.

The value of these models is in trying to understand the observed performance of viscoelastic materials. Most soft biological tissues exhibit viscoelastic properties. The forms of creep and relaxation curves for the materials can give a strong indication as to which model is most appropriate, or of how to build a composite model from these basic building blocks. Kelvin showed the inadequacy of the simpler models in accounting

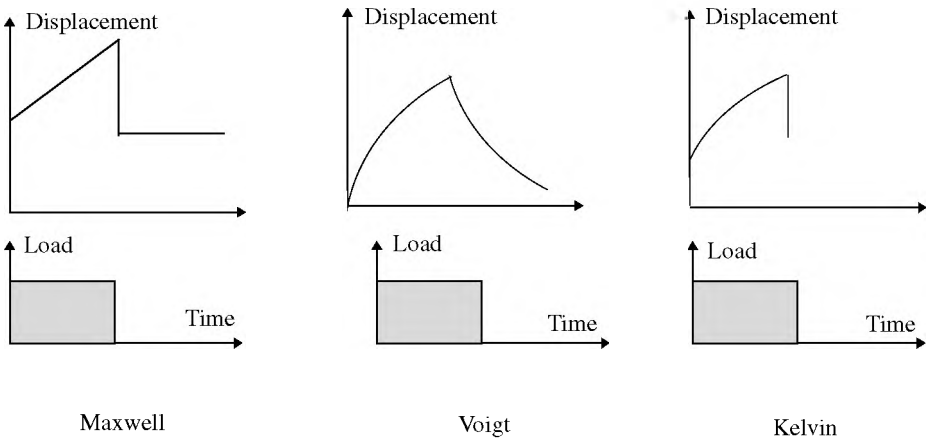


Figure 1.12. *Creep functions for Maxwell, Kelvin and Voigt models of viscoelasticity.*

for the rate of dissipation of energy in some materials under cyclic loading. The Kelvin model is sometimes called the standard linear model because it is the simplest model that contains force and displacement and their first derivatives.

More general models can be developed using different combinations of these simpler models. Each of these system models is *passive* in that it responds to an externally applied force. Further *active* (load-generating) elements are introduced to represent the behaviour of muscles. An investigation of the characteristics of muscles is beyond the scope of this chapter.

1.3. THE PRINCIPLES OF EQUILIBRIUM

1.3.1. Forces, moments and couples

Before we begin the discussion of the principles of equilibrium it is important that we have a clear grasp of the notions of force and moment (figure 1.13).

A *force* is defined by its *magnitude, position* and *direction*. The SI unit of force is the newton, defined as the force required to accelerate a body of mass 1 kg through 1 m s^{-2}, and clearly this is a measure of the magnitude. In two dimensions any force can be resolved into components along two mutually perpendicular axes.

The *moment* of a force about a point describes the tendency of the force to turn the body about that point. Just like a force, a moment has position and direction, and can be represented as a vector (in fact the moment can be written as the cross-product of a position vector with a force vector). The magnitude of the moment is the force times the perpendicular distance to the force,

$$M = |F|d.$$

The SI unit for a moment is the newton metre (N m). A force of a given magnitude has a larger moment when it is further away from a point—hence the principle of levers. If we stand at some point on an object and a force is applied somewhere else on the object, then in general we will feel both a force and a moment. To put it another way, any force applied through a point can be interpreted at any other point as a force plus a moment applied there.

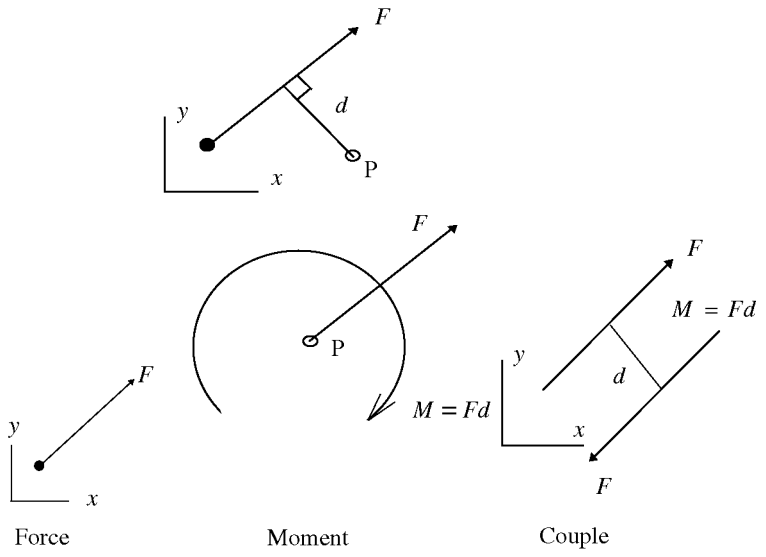


Figure 1.13. *Force, moment and couple.*

A *couple* is a special type of moment, created by two forces of equal magnitude acting in opposite directions, but separated by a distance. The magnitude of the couple is independent of the position of the point about which moments are taken, and no net force acts in any direction. (Check this by taking moments about different points and resolving along two axes.) Sometimes a couple is described as a pure bending moment.

1.3.2. Equations of static equilibrium

When a set of forces is applied to any structure, two processes occur:

- the body deforms, generating a system of internal stresses that distribute the loads throughout it, and
- the body moves.

If the forces are maintained at a constant level, and assuming that the material is not viscoelastic and does not creep, then the body will achieve a deformed configuration in which it is in a state of static equilibrium.

By definition: *A body is in static equilibrium when it is at rest relative to a given frame of reference.*

When the applied forces change only slowly with time the accelerations are often neglected, and the equations of static equilibrium are used for the analysis of the system. In practice, many structural analyses in biomechanics are performed based on the assumption of static equilibrium.

Consider a two-dimensional body of arbitrary shape subjected to a series of forces as illustrated in figure 1.14.

The body has three potential rigid-body movements:

- it can translate along the x-axis;
- it can translate along the y-axis;
- it can rotate about an axis normal to the plane, passing through the frame of reference (the z-axis).

Any other motion of the body can be resolved into some combination of these three components. By definition, however, if the body is in static equilibrium then it is at rest relative to its frame of reference. Thus the resultant

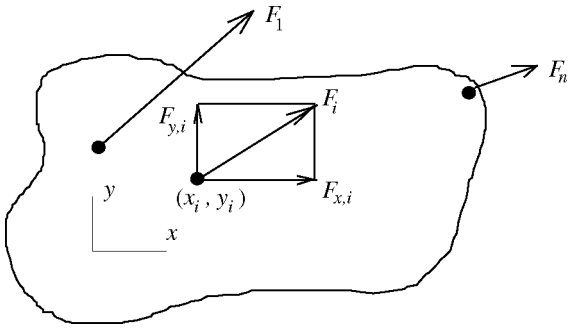


Figure 1.14. *Two-dimensional body of arbitrary shape subjected to an arbitrary combination of forces.*

load acting on the body and tending to cause each of the motions described must be zero. There are therefore three equations of static equilibrium for the two-dimensional body.

Resolving along the x-axis:

$$\sum_{i=1}^{n} F_{x,i} = 0.$$

Resolving along the y-axis:

$$\sum_{i=1}^{n} F_{y,i} = 0.$$

Note that these two equations are concerned only with the magnitude and direction of the force, and its position on the body is not taken into account.

Taking moments about the origin of the frame of reference:

$$\sum_{i=1}^{n} \left(F_{y,i}x_i - F_{x,i}y_i\right) = 0.$$

The fact that we have three equations in two dimensions is important when we come to idealize physical systems. We can only accommodate three unknowns. For example, when we analyse the biomechanics of the elbow and the forearm (figure 1.15), we have the biceps force and the magnitude and direction of the elbow reaction. We cannot include another muscle because we would need additional equations to solve the system.

The equations of static equilibrium of a three-dimensional system are readily derived using the same procedure. In this case there is one additional rigid-body translation, along the z-axis, and two additional rigid-body rotations, about the x- and y-axes, respectively. There are therefore six equations of static equilibrium in three dimensions.

By definition: *A system is statically determinate if the distribution of load throughout it can be determined by the equations of static equilibrium alone.*

Note that the position and orientation of the frame of reference are arbitrary. Generally the analyst will choose any convenient reference frame that helps to simplify the resulting equations.

1.3.3. Structural idealizations

All real structures including those making up the human body are three-dimensional. The analysis of many structures can be simplified greatly by taking idealizations in which the three-dimensional geometry is

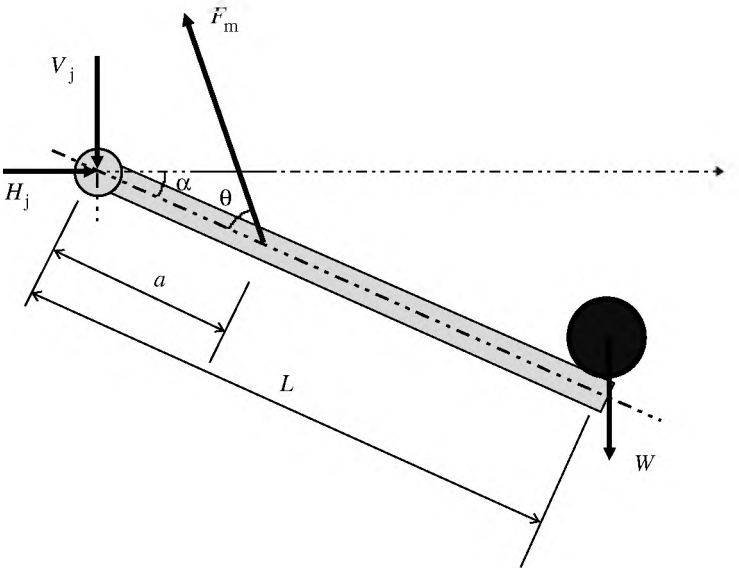


Figure 1.15. *A model of the elbow and forearm. The weight W is supported by the force of the muscle* F_m *and results in forces on the elbow joint* V_j *and* H_j.

Table 1.2. *One-dimensional structural idealizations.*

Element type	Loads	Example
Beams	Tension Compression Bending Torsion	Bones
Bars or rods	Tension Compression	
Wires or cables	Tension	Muscles, ligaments, tendons

represented by lumped properties in one or two dimensions. One-dimensional line elements are used commonly in biomechanics to represent structures such as bones and muscles. The following labels are commonly applied to these elements, depending on the loads that they carry (table 1.2).

1.3.4. Applications in biomechanics

Biomechanics of the elbow and forearm

A simple model of the elbow and forearm (figure 1.15) can be used to gain an insight into the magnitudes of the forces in this system.

Taking moments about the joint:

$$F_m a \sin\theta = WL\cos\alpha$$

$$F_m = W\frac{L}{a}\frac{\cos\alpha}{\sin\theta}.$$

Resolving vertically:

$$V_j = W - F_m \sin(\alpha+\theta) = W\left(1 - \frac{L}{a}\frac{\cos\alpha}{\sin\theta}\sin(\alpha+\theta)\right).$$

Resolving horizontally:

$$H_j = F_m \cos(\alpha+\theta) = W\frac{L}{a}\frac{\cos\alpha}{\sin\theta}\cos(\alpha+\theta).$$

The resultant force on the joint is

$$R = \sqrt{H_j^2 + V_j^2}.$$

For the particular case in which the muscle lies in a vertical plane, the angle $\theta = \pi/2 - \alpha$ and

$$F_m = W\frac{L}{a}.$$

For a typical person the ratio L/a might be approximately eight, and the force in the muscle is therefore eight times the weight that is lifted. The design of the forearm appears to be rather inefficient with respect to the process of lifting. Certainly the force on the muscle could be greatly reduced if the point of attachment were moved further away from the joint. However, there are considerable benefits in terms of the possible range of movement and the speed of hand movement in having an 'inboard' attachment point.

We made a number of assumptions in order to make the above calculations. It is worth listing these as they may be unreasonable assumptions in some circumstances.

- We only considered one muscle group and one beam.
- We assumed a simple geometry with a point attachment of the muscle to the bone at a known angle. In reality of course the point of muscle attachment is distributed.
- We assumed the joint to be frictionless.
- We assumed that the muscle only applies a force along its axis.
- We assumed that the weight of the forearm is negligible. This is not actually a reasonable assumption. Estimate the weight of the forearm for yourself.
- We assumed that the system is static and that dynamic forces can be ignored. Obviously this would be an unreasonable assumption if the movements were rapid.

1.4. STRESS ANALYSIS

The loads in the members of statically determinate structures can be calculated using the methods described in section 1.3. The next step in the analysis is to decide whether the structure can sustain the applied loads.

1.4.1. Tension and compression

When the member is subjected to a simple uniaxial tension or compression, the stress is just the load divided by the cross-sectional area of the member at the point of interest. Whether a tensile stress is sustainable can often be deduced directly from the stress/strain curve. A typical stress/strain curve for cortical bone was presented in figure 1.5.

Compressive stresses are a little more difficult because there is the prospect of a *structural instability*. This problem is considered in more detail in section 1.5. In principle, long slender members are likely to be subject to structural instabilities when loaded in compression and short compact members are not. Both types of member are represented in the human skeleton. Provided the member is stable the compressive stress/strain curve will indicate whether the stress is sustainable.

1.4.2. Bending

Many structures must sustain bending moments as well as purely tensile and compressive loads. We shall see that a bending moment causes both tension and compression, distributed across a section. A typical example is the femur, in which the offset of the load applied at the hip relative to the line of the bone creates a bending moment as illustrated in figure 1.16. W is the weight of the body. One-third of the weight is in the legs themselves, and each femur head therefore transmits one-third of the body weight.

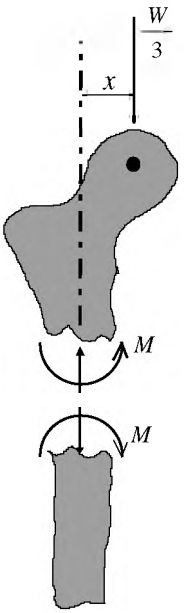


Figure 1.16. *Moment on a femur: two-leg stance.*

This figure illustrates an important technique in the analysis of structures. The equations of static equilibrium apply not only to a structure as a whole, but also to any part of it. We can take an arbitrary cut and apply the necessary forces there to maintain the equilibrium of the resulting two portions of the structure. The forces at the cut must actually be applied by an internal system of stresses within the body. For equilibrium of the head of the femur, the internal bending moment at the cut illustrated in figure 1.16 must be equal to $Wx/3$, and the internal vertical force must be $W/3$.

Engineer's theory of bending

The most common method of analysis of beams subjected to bending moments is the *engineer's theory of bending*. When a beam is subjected to a uniform bending moment it deflects until the internal system of

stresses is in equilibrium with the externally applied moment. Two fundamental assumptions are made in the development of the theory.

- It is assumed that every planar cross-section that is initially normal to the axis of the beam remains so as the beam deflects. This assumption is often described as 'plane sections remain plane'.
- It is assumed that the stress at each point in the cross-section is proportional to the strain there, and that the constant of proportionality is Young's modulus, E. The material is assumed to be linearly elastic, homogeneous and isotropic.

The engineer's theory of bending appeals to the principle of equilibrium, supplemented by the assumption that plane sections remain plane, to derive expressions for the curvature of the beam and for the distribution of stress at all points on a cross-section. The theory is developed below for a beam of rectangular cross-section, but it is readily generalized to cater for an arbitrary cross-section.

Assume that a beam of rectangular cross-section is subjected to a uniform bending moment (figure 1.17).

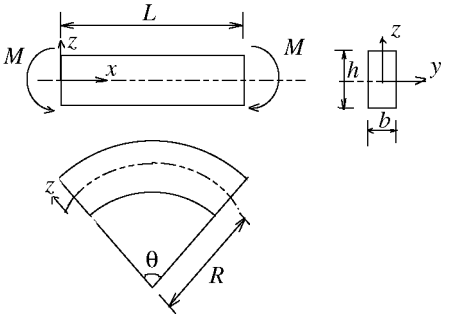


Figure 1.17. *Beam of rectangular cross-section subjected to a uniform bending moment.*

A reference axis is arbitrarily defined at some point in the cross section, and the radius of curvature of this axis after bending is R. Measuring the distance z from this axis as illustrated in figure 1.17, the length of the fibre of the beam at z from the reference axis is

$$l = (R + z)\theta.$$

If the reference axis is chosen as one that is unchanged in length under the action of the bending moment, referred to as the neutral axis, then

$$l_{z=0} = R\theta = L \qquad \theta = \frac{L}{R}.$$

The strain in the x direction in the fibre at a distance z from the neutral axis is

$$\varepsilon_x = \frac{(l_z - L)}{L} = \frac{(R + z)L/R - L}{L} = \frac{z}{R}.$$

If the only loads on the member act parallel to the x-axis, then the stress at a distance z from the neutral axis is

$$\sigma_x = E\varepsilon = E\frac{z}{R}.$$

Hence the stress in a fibre of a beam under a pure bending moment is proportional to the distance of the fibre from the neutral axis, and this is a fundamental expression of the engineer's theory of bending.

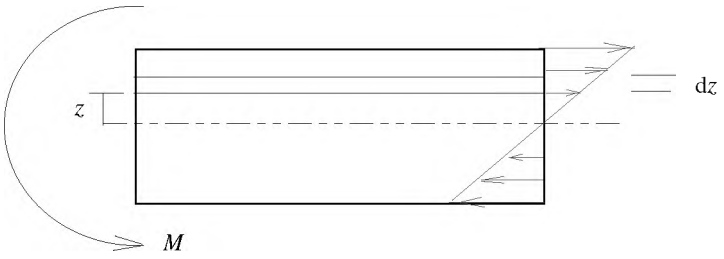


Figure 1.18. *Internal forces on beam.*

Consider now the forces that are acting on the beam (figure 1.18). The internal force F_x acting on an element of thickness $\mathrm{d}z$ at a distance z from the neutral axis is given by the stress at that point times the area over which the stress acts,

$$F_x = \sigma_x b\,\mathrm{d}z = E\frac{z}{R}b\,\mathrm{d}z.$$

The moment about the neutral axis of the force on this elemental strip is simply the force in the strip multiplied by the distance of the strip from the neutral axis, or $M_y = F_x z$. Integrating over the depth of the beam,

$$M_y = \int_{-h/2}^{h/2} E\frac{z}{R}bz\,\mathrm{d}z = \frac{E}{R}\int_{-h/2}^{h/2} z^2 b\,\mathrm{d}z = \frac{E}{R}\left[\frac{bz^3}{3}\right]_{-h/2}^{h/2}$$
$$= \frac{E}{R}\frac{bh^3}{12} = \frac{E}{R}I.$$

The term

$$\int_{-h/2}^{h/2} z^2 b\,\mathrm{d}z\left[=\int_{\text{area}} z^2\,\mathrm{d}A\right] = \frac{bh^3}{12} = I$$

is the *second moment of area* of the rectangular cross-section.

The equations for stress and moment both feature the term in E/R and the relationships are commonly written as

$$\frac{\sigma_x}{z} = \frac{E}{R} = \frac{M_y}{I}.$$

Although these equations have been developed specifically for a beam of rectangular cross-section, in fact they hold for any beam having a plane of symmetry parallel to either of the y- or z-axes.

Second moments of area

The application of the engineer's theory of bending to an arbitrary cross-section (see figure 1.19) requires calculation of the second moments of area.

By definition:

$$I_{yy} = \int_{\text{area}} z^2\,\mathrm{d}A \qquad I_{zz} = \int_{\text{area}} y^2\,\mathrm{d}A \qquad I_{yz} = \int_{\text{area}} yz\,\mathrm{d}A.$$

↑ product moment of area

The cross-sections of many practical structures exhibit at least one plane of geometrical symmetry, and for such sections it is readily shown that $I_{yz} = 0$.

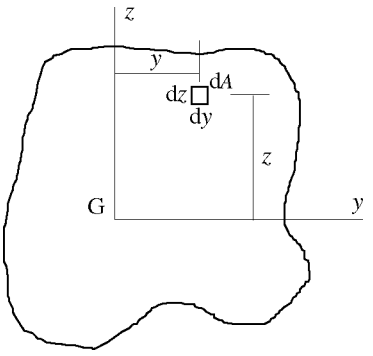


Figure 1.19. *Arbitrary cross-section.*

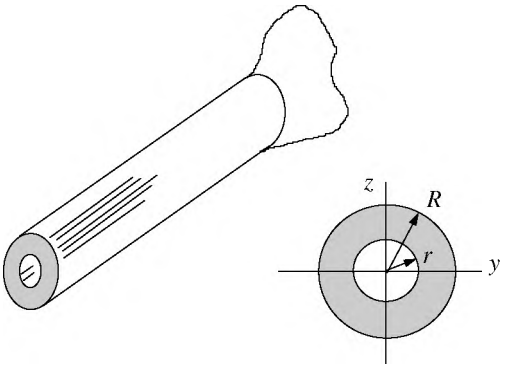


Figure 1.20. *Hollow circular cross-section of a long bone.*

The second moment of area of a thick-walled circular cylinder

Many of the bones in the body, including the femur, can be idealized as thick-walled cylinders as illustrated in figure 1.20.

The student is invited to calculate the second moment of area of this cross-section. It is important, in terms of ease of calculation, to choose an element of area in a cylindrical coordinate system ($dA = r\,d\theta\,dr$). The solution is

$$I_{\text{long bone}} = \frac{\pi(R^4 - r^4)}{4}.$$

Bending stresses in beams

The engineer's theory of bending gives the following expression for the bending stress at any point in a beam cross-section:

$$\sigma = \frac{Mz}{I}.$$

The second moment of area of an element is proportional to z^2, and so it might be anticipated that the optimal design of the cross-section to support a bending moment is one in which the elemental areas are separated as

far as possible. Hence for a given cross-sectional area (and therefore a given weight of beam) the stress in the beam is inversely proportional to the separation of the areas that make up the beam. Hollow cylindrical bones are hence a good design.

This suggests that the efficiency of the bone in sustaining a bending moment increases as the radius increases, and the thickness is decreased in proportion. The question must arise as to whether there is any limit on the radius to thickness ratio. In practice the primary danger of a high radius to thickness ratio is the possibility of a local structural instability (section 1.5). If you stand on a soft-drinks can, the walls buckle into a characteristic diamond pattern and the load that can be supported is substantially less than that implied by the stress/strain curve alone. The critical load is determined by the bending stiffness of the walls of the can.

Deflections of beams in bending

The deflections of a beam under a pure bending moment can be calculated by a manipulation of the results from the engineer's theory of bending. It was shown that the radius of curvature of the neutral axis of the beam is related to the bending moment as follows:

$$\frac{E}{R} = \frac{M}{I}.$$

It can be shown that the curvature at any point on a line in the (x, z) coordinate system is given by

$$-\frac{1}{R} = \frac{\mathrm{d}^2w/\mathrm{d}x^2}{(1 + (\mathrm{d}w/\mathrm{d}x)^2)^{3/2}} \approx \frac{\mathrm{d}^2w}{\mathrm{d}x^2}.$$

The negative sign arises from the definition of a positive radius of curvature. The above approximation is valid for small displacements.

Then

$$\frac{\mathrm{d}^2w}{\mathrm{d}x^2} = -\frac{M}{EI}.$$

This equation is strictly valid only for the case of a beam subjected to a uniform bending moment (i.e. one that does not vary along its length). However, in practice the variation from this solution caused by the development of shear strains due to a non-uniform bending moment is usually small, and for compact cross-sections the equation is used in unmodified form to treat all cases of bending of beams. The calculation of a displacement from this equation requires three steps. Firstly, it is necessary to calculate the bending moment as a function of x. This can be achieved by taking cuts normal to the axis of the beam at each position and applying the equations of static equilibrium. The second step is to integrate the equation twice, once to give the slope and once more for the displacement. This introduces two constants of integration; the final step is to evaluate these constants from the boundary conditions of the beam.

1.4.3. Shear stresses and torsion

Shear stresses can arise when tractile forces are applied to the edges of a sheet of material as illustrated in figure 1.21.

Shear stresses represent a form of biaxial loading on a two-dimensional structure. It can be shown that for any combination of loads there is always an orientation in which the shear is zero and the direct stresses (tension and/or compression) reach maximum and minimum values. Conversely any combination of tensile and compressive (other than hydrostatic) loads always produces a shear stress at another orientation. In the uniaxial test specimen the maximum shear occurs along lines orientated at 45° to the axis of the specimen.

Pure two-dimensional shear stresses are in fact most easily produced by loading a thin-walled cylindrical beam in torsion, as shown in figure 1.22.

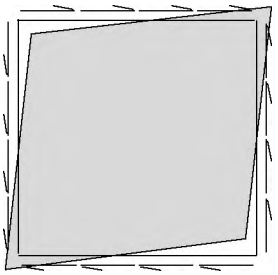


Figure 1.21. *Shear stresses on an elemental area.*

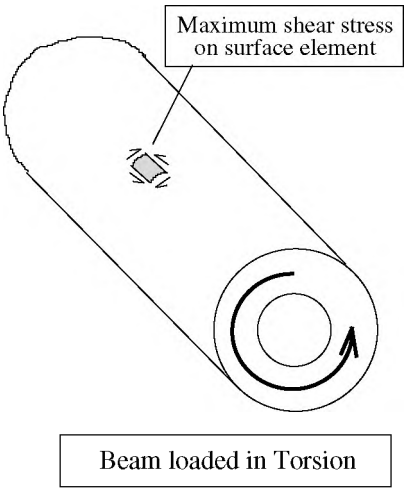


Figure 1.22. *External load conditions causing shear stresses.*

The most common torsional failure in biomechanics is a fracture of the tibia, often caused by a sudden arrest of rotational motion of the body when a foot is planted down. This injury occurs when playing games like soccer or squash, when the participant tries to change direction quickly. The fracture occurs due to a combination of compressive, bending and torsional loads. The torsional fracture is characterized by spiral cracks around the axis of the bone.

Combined stresses and theories of failure

We have looked at a number of loading mechanisms that give rise to stresses in the skeleton. In practice the loads are usually applied in combination, and the total stress system might be a combination of direct and shear stresses. Can we predict what stresses can cause failure? There are many theories of failure and these can be applied to well defined structures. In practice in biomechanics it is rare that the loads or the geometry are known to any great precision, and subtleties in the application of failure criteria are rarely required: they are replaced by large margins of safety.

1.5. STRUCTURAL INSTABILITY

1.5.1. Definition of structural instability

The elastic stress analysis of structures that has been discussed so far is based on the assumption that the applied forces produce small deflections that do not change the original geometry sufficiently for effects associated with the change of geometry to be important. In many practical structures these assumptions are valid only up to a critical value of the external loading. Consider the case of a circular rod that is laterally constrained at both ends, and subjected to a compressive loading (figure 1.23).

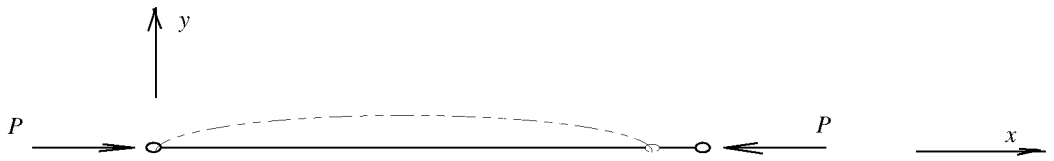


Figure 1.23. *Buckling of a thin rod.*

If the rod is displaced from the perfectly straight line by even a very small amount (have you ever seen a perfectly straight bone?) then the compressive force P produces a moment at all except the end points of the column. As the compressive force is increased, so the lateral displacement of the rod will increase. Using a theory based on small displacements it can be shown that there is a critical value of the force beyond which the rod can no longer be held in equilibrium under the compressive force and induced moments. When the load reaches this value, the theory shows that the lateral displacement increases without bound and the rod collapses. This phenomenon is known as buckling. It is associated with the *stiffness* rather than the strength of the rod, and can occur at stress levels which are far less than the yield point of the material. Because the load carrying capacity of the column is dictated by the lateral displacement, and this is caused by bending, it is the bending stiffness of the column that will be important in the determination of the critical load.

There are two fundamental approaches to the calculation of buckling loads for a rod.

- The *Euler theory* is based on the engineer's theory of bending and seeks an exact solution of a differential equation describing the lateral displacement of a rod. The main limitation of this approach is that the differential equation is likely to be intractable for all but the simplest of structures.
- An alternative, and more generally applicable, approach is to estimate a buckled shape for a structural component and then to apply the *principle of stationary potential energy* to find the magnitude of the applied loads that will keep it in equilibrium in this geometric configuration. This is a very powerful technique, and can be used to provide an *approximation* of the critical loads for many types of structure. Unfortunately the critical load will always be overestimated using this approach, but it can be shown that the calculated critical loads can be remarkably accurate even when only gross approximations of the buckled shape are made.

1.5.2. Where instability occurs

Buckling is always associated with a compressive stress in a structural member, and whenever a light or thin component is subjected to a compressive load, the possibility of buckling should be considered. It should be noted that a *pure shear* load on a plate or shell can be resolved into a tensile and a *compressive component*, and so the structure might buckle under this load condition.

In the context of biomechanics, buckling is most likely to occur:

- in long, slender columns (such as the long bones);
- in thin shells (such as the orbital floor).

1.5.3. Buckling of columns: Euler theory

Much of the early work on the buckling of columns was developed by Euler in the mid-18th Century. Euler methods are attractive for the solution of simple columns with simple restraint conditions because the solutions are closed-form and accurate. When the geometry of the columns or the nature of the restraints are more complex, then alternative approximate methods might be easier to use, and yield a solution of sufficient accuracy. This section presents analysis of columns using traditional Euler methods. Throughout this section the lateral deflection of a column will be denoted by the variable y, because this is used most commonly in textbooks.

Long bone

Consider the thin rod illustrated in figure 1.23, with a lateral deflection indicated by the broken line. If the column is in equilibrium, the bending moment, M, at a distance x along the beam is

$$M = Py.$$

By the engineer's theory of bending:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{-M}{EI} = \frac{-Py}{EI}.$$

Defining a variable $\lambda^2 = (P/EI)$ and re-arranging:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \lambda^2 y = 0.$$

This linear, homogeneous second-order differential equation has the standard solution:

$$y = C\sin\lambda x + D\cos\lambda x.$$

The constants C and D can be found from the boundary conditions at the ends of the column. Substituting the boundary condition $y = 0$ at $x = 0$ gives $D = 0$. The second boundary condition, $y = 0$ at $x = L$, gives

$$C\sin\lambda L = 0.$$

The first and obvious solution to this equation is $C = 0$. This means that the lateral displacement is zero at all points along the column, which is therefore perfectly straight. The second solution is that $\sin\lambda L = 0$, or

$$\lambda L = n\pi \qquad n = 1, 2, \ldots, \infty.$$

For this solution the constant C is indeterminate, and this means that the magnitude of the lateral displacement of the column is indeterminate. At the value of load that corresponds to this solution, the column is just held in equilibrium with an arbitrarily large (or small) displacement. The value of this critical load can be calculated by substituting for λ,

$$\sqrt{\frac{P_{\mathrm{cr}}}{EI}}L = n\pi$$

from which

$$P_{cr} = \frac{n^2\pi^2 EI}{L^2}.$$

Although the magnitude of the displacement is unknown, the shape is known and is determined by the number of half sine-waves over the length of the beam. The lowest value of the critical load is that when $n = 1$,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

and this is a well-known formula for the critical load of a column.

It is interesting to illustrate the results of this analysis graphically (see figure 1.24).

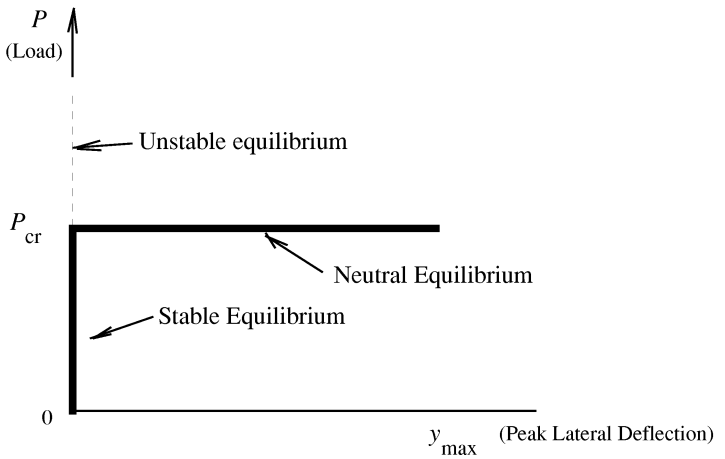


Figure 1.24. *Euler load/displacement curve for a slender column.*

If the load is less than the critical load, then the only solution to the differential equation is $C = 0$, and the column is stable. At the critical load, two solutions become possible. Either the column remains straight ($C = 0$) and the load can continue to increase, or the lateral displacement become indeterminate and no more load can be sustained. The first solution corresponds to a situation of unstable equilibrium and any perturbation (even one caused by the initial imperfection of the column) will cause collapse. Although this curve can be modified using more sophisticated theories based on finite displacements, and it can be demonstrated that loads higher than the critical load can be sustained, the associated lateral displacements are high and design in this region is normally impractical. The Euler instability load represents a realistic upper bound to the load that the column can carry in compression. The graph can be re-drawn with the axial deflection on the x-axis, and then it is apparent that there is a sudden change in axial stiffness of the column at the critical load.

1.5.4. Compressive failure of the long bones

It has been demonstrated that the stability of a uniform column is proportional to its flexural stiffness (EI) and inversely proportional to the square of its length. It is likely, therefore, that the most vulnerable bones in the skeleton will be the long bones in the leg. Since the governing property of the cross-section is the second moment of area, as for bending stiffness, it is apparent that a hollow cylindrical cross-section might be most efficient in resisting buckling. A compressive load applied to the head of the femur is illustrated in figure 1.25.

In a two-leg stance approximately one-third of the body weight is applied to the head of each femur. The remaining weight is distributed in the legs, and this makes the analysis more difficult. For simplicity (and conservatively) we might assume that one-half of the body weight is applied to the top of the femur. We have the usual problem in biomechanics in deciding which parts of the structure to include in our model. We might calculate separately the buckling loads of the femur and of the tibia, but this assumes that each provides a lateral support to the other at the knee. It might be more appropriate to take the femur/tibia structure as a continuous member, assuming that there is continuity of bending moment across the knee. Taking the skeleton of the leg as a whole, the length is approximately 1 m.

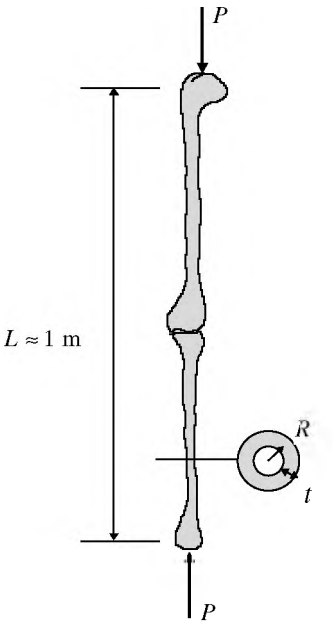


Figure 1.25. *Compressive buckling of the skeleton of the leg.*

The buckling strength of a non-uniform column will always be higher than that of a uniform column of its minimum dimensions. The buckling stress is simply the buckling load divided by the cross-sectional area.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \sigma_{cr} = \frac{\pi^2 EI}{AL^2}.$$

The buckling stress can be calculated and compared with the ultimate compressive stress to determine whether failure will be by buckling or by simple overloading. In either case the result will be the same—a broken leg—but the appearance of the fracture might be different, and the design steps required to prevent a recurrence will certainly be so.

1.6. MECHANICAL WORK AND ENERGY

1.6.1. Work, potential energy, kinetic energy and strain energy

We have seen that we can gain an insight into the forces and stresses involved in many biomechanical systems by using the equations of static equilibrium alone. However, if we are to study problems that involve dynamic

actions such as running and jumping, we shall need to develop additional techniques. One of the most powerful tools available to us is the principle of conservation of energy.

Energy is a scalar quantity with SI units of joules (J). Joules are derived units equivalent to newton metres (N m). We should not confuse energy with moment or torque, which are vector quantities whose magnitudes also happen to be measured in newton metres. The concept of work is similar to that of energy. We usually associate energy with a body and work with a force. In this chapter we shall concern ourselves only with mechanical energy, and not with other forms such as heat energy.

The *work done* by a constant force acting on a body is equal to the force multiplied by the distance moved in the direction of the force. If the force and the distance moved are taken to be vectors, then the work done is their dot product. By definition, therefore, a force that produces no displacement does no work.

In a *conservative* system all of the work that is done on the body is recoverable from it. In some sense it is 'stored' within the body as a form of energy.

Potential energy is associated with the potential of a body to do work under the action of a force (see figure 1.26). In order for a body to have potential energy it must be subjected to a force (or a field). Potential energy can perhaps best be understood by considering the work done by the force acting on the body. The work done by the force is equal to the potential energy lost by the body.

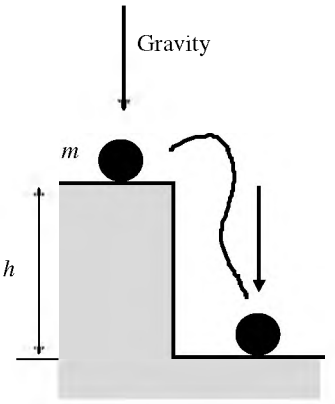


Figure 1.26. *Potential energy is a function of position and applied force.*

The weight of a mass in gravity is mg. In order to lift the mass (slowly) from one stair to another we would have to do work on the body. This work could, however, be recovered if we allowed the body to fall back to the lower step. The work that would be done by the gravitational force during the process of falling would be the force times the distance moved, or $work = mgh$. We might therefore consider that the body has more potential to do work, or potential energy, when it is on the top step relative to when it is on the bottom step. Obviously we can choose any position as the reference position, and potential energy can only be a relative quantity.

Kinetic energy is associated with the motion of a body. Newton's second law tells us that a body subjected to a constant force will accelerate at a constant rate. The work done by the force on the body is converted into kinetic energy of the body. The kinetic energy K of a body of mass m travelling at velocity v is

$$K = \tfrac{1}{2}mv^2.$$

Strain energy is associated with the deformation of a body. If the deformation is elastic then all of the strain energy is recoverable if the load is removed. The strain energy in a bar can be defined in terms of the area under the load/displacement curve (see figure 1.27).

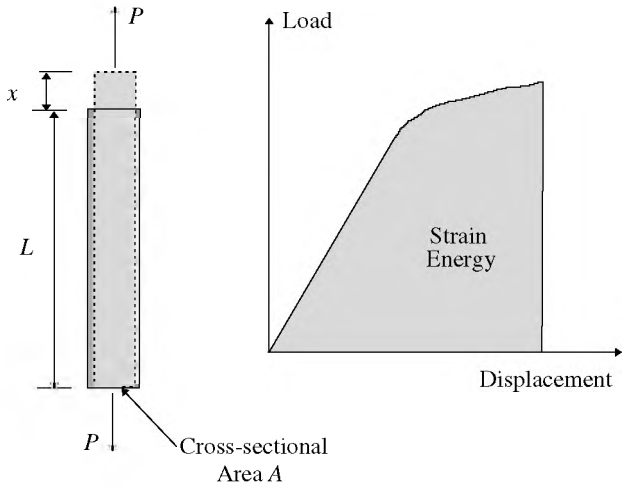


Figure 1.27. *Strain energy in a uniaxial test.*

Once again the energy of the body can be calculated from the work done by the force acting on it. In this case the force is not constant but is applied gradually, so that a particular value of the force is associated only with an increment of displacement.

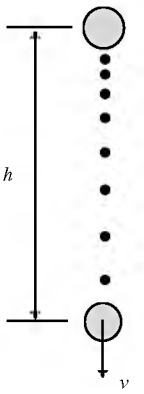
Using the above definitions, potential energy and kinetic energy are terms that we might use in studying the dynamics of rigid particles. Deformable bodies have strain energy as well as potential and kinetic energy. When we apply the principles of conservation of energy we tacitly assume that all of the work done to a body is converted into a form of energy that can be recovered. In some situations this is not true, and the energy is converted into a form that we cannot recover. The most common examples include plastic deformations and frictional forces. When a bar is bent beyond the elastic limit it suffers permanent deformation, and we cannot recover the energy associated with the plastic deformation: indeed, if we wished to restore the bar to its original condition we would have to do more work on it. Similarly when a body is pulled along a rough surface, or through a fluid, we must do work to overcome the frictional forces which we can never recover. Systems in which the energy is not recoverable are called *non-conservative*. Sometimes, particularly in assessment of problems in fluid dynamics, we might modify our equations to cater for an estimated irrecoverable energy loss.

We should not always regard non-conservative systems in a negative light. If, for example, we need to absorb the energy of an impact, it might be most appropriate to use a dashpot system that dissipates the energy using the viscous characteristics of the fluid. Cars are designed with crush zones that absorb energy in gross plastic deformation—unfortunately they can only be used once!

1.6.2. Applications of the principle of conservation of energy

Gravitational forces on the body: falling and jumping

The reader will be familiar with the calculation of the velocities of rigid bodies falling under the action of gravity (see figure 1.28). The principle of conservation of energy dictates that the sum of the potential and kinetic energy is constant. In this case the strain energy is negligible.



Consider a body of mass m dropped from a height h under the action of gravity.
Change in potential energy, $\mathrm{d}U = -mgh$.
Change in kinetic energy, $\mathrm{d}K = \frac{1}{2}mv^2$.
For a conservative system, $\mathrm{d}U + \mathrm{d}K = 0$.

$$v = \sqrt{2gh}$$

Figure 1.28. *Body falling under gravity.*

We can extend the principle to estimate the forces applied to the body during falling and jumping actions. Consider, for example, the case of a person dropping from a wall onto level ground. There is a three-stage process.

- There is a period of free-fall during which potential energy of the body is converted into kinetic energy.
- The feet hit the ground and the reaction force from the ground does work to arrest the fall.
- The energy stored as elastic strain energy in the bones and muscles during the arresting process will be released causing a recoil effect.

The first two stages of the process are indicated in figure 1.29. Two possibilities are illustrated: in the first the knees are locked and the body is arrested rather abruptly, and in the second the knees are flexed to give a gentler landing.

We can apply the principle of conservation of energy from the time at which the fall commences to that at which it is arrested and the body is again stationary. At both of these times the kinetic energy is zero. The work done on the body by the reaction to the ground must therefore be equal to the potential energy that the body has lost in the falling process. We have an immediate problem in that we do not know how the reaction force changes with time during the arresting process (F_1 and F_2 are functions of time).

The second problem, landing on a bended knee, is addressed in section 1.9. The method adopted could produce unreliable results for the first problem because of the difficulty in estimating the dynamic compression of the skeleton. An alternative approach would be to take the legs as a spring, with the compressive force being proportional to the displacement, and to equate the strain energy in the legs at the end of the arresting process with the potential energy lost by the body.

1.7. KINEMATICS AND KINETICS

The degrees of freedom that govern the motion of a system can often be determined from the geometry alone. The branch of mechanics associated with the study of such systems is called kinematics. Kinematics plays an important part in our understanding of the function of the human skeleton. The study of the motions and forces generated is called kinetics.

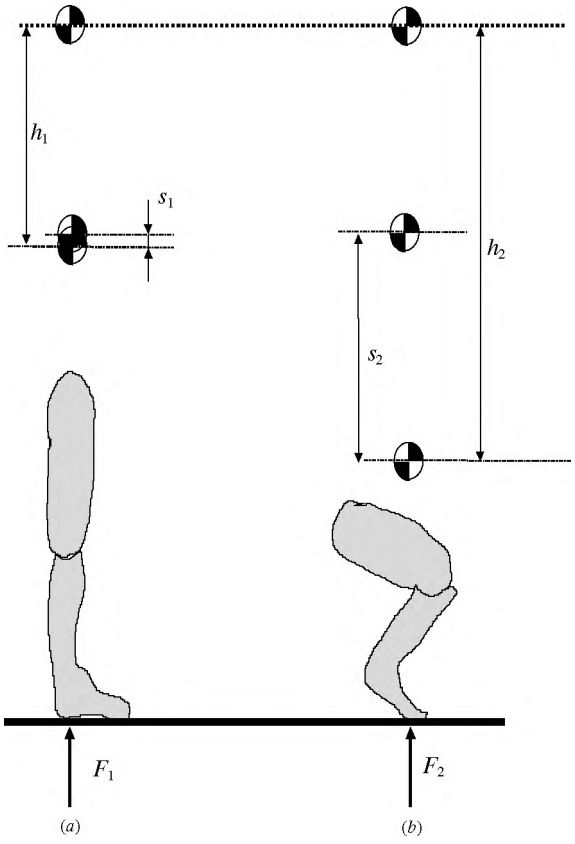


Figure 1.29. *Arrest of a fall: (a) knees locked; (b) knees bent.*

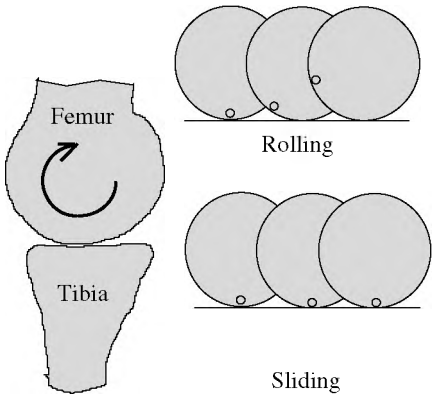


Figure 1.30. *Rolling and sliding action of the knee.*

1.7.1. Kinematics of the knee

The knee is a very complex joint that features many degrees of freedom in three dimensions (see figure 1.30). Its primary purpose, however, is to allow a flexion in the sagittal plane, and it is most instructive to develop a simple two-dimensional kinematic model of the knee in this plane. The femoral condyle rotates relative to the tibial plateau to give the leg the freedom to bend at the knee. We can quickly show, however, that the degree of rotation (approximately 150°) cannot be achieved by a rolling action alone: the femur would simply roll right off the tibia. Similarly, if the action were sliding alone the geometry of the bones would limit the degree of flexure to approximately 130°. In practice the femoral condyle must roll and slide on the tibial plateau.

It is illuminating to consider how one might go about the task of designing a system to achieve the required two-dimensional motion of the knee. One possibility is to make an engaging mechanism, like a standard door hinge, in which effectively one structure is embedded inside the other. On inspection of the anatomy of the femur and the tibia it is apparent that this is not the type of mechanism that is featured in the knee. The two bones do not overlap to any substantial degree, and certainly neither 'fits inside' the other. Clearly then the kinematics of the knee must involve straps (or ligaments) that attach the two bodies together. Assuming that the point of attachment of the straps on the bone cannot move once made, and that the straps cannot stretch, one strap (or two attached to the same point on one of the members) could be used only to provide a centre for the pure rolling action. We have seen that this cannot describe the motion of the knee. The simplest arrangement of rigid links that can provide the degree of rotation required is the four-bar link. When we study the anatomy of the knee, we see that there is indeed a four-bar link mechanism, formed by the tibial plateau, the femoral condyle and the two cruciate ligaments (see figure 1.31).

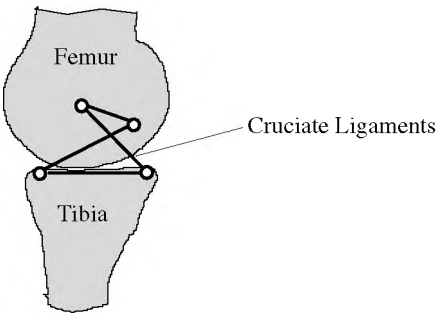


Figure 1.31. *The knee as a four-bar linkage.*

An understanding of the mechanics of this system can be gained by constructing a simple mechanical model using taut strings to represent the ligaments and a flat bar to represent the face of the tibia. As the tibia is rotated a line can be drawn illustrating its current position, and it is found that the resulting set of lines trace out the shape of the head of the femur. If the lengths of the strings or the mounting points are changed slightly the effect on the shape of the head of the femur can be seen. Consider the implications for the orthopaedic surgeon who is changing the geometry of a knee joint.

1.7.2. Walking and running

The study of the forces on the skeleton during the processes of walking and running is called gait analysis. The analysis of gait provides useful information that can assist in clinical diagnosis of skeletal and muscular abnormalities, and in the assessment of treatment. It has even been used to help to design a running track

on which faster speeds can be attained. The following discussion is based on Chapter 8, 'Mechanics of Locomotion', of McMahon (1984), and this source should be studied for more comprehensive information.

For the purposes of definition, walking constitutes an action in which there is always at least one foot in contact with the ground. In the process of running there is an interval of time in which no part of the body is in contact with the ground. It is interesting to investigate the changes of the kinetic energy and the potential energy of the body during the processes of running and walking. It is conventional in the analysis of gait to separate the kinetic energy into a vertical and a horizontal component. In the simplest possible model we might consider only the motion of the centre of gravity of the body, as illustrated in figure 1.32.

- The kinetic energy associated with the forward motion of the centre of mass is $K_x = \frac{1}{2}mv_x^2$.
- The kinetic energy associated with the vertical motion of the centre of mass is $K_y = \frac{1}{2}mv_y^2$.
- The potential energy associated with the vertical position of the centre of mass is $U_y = mgh$.

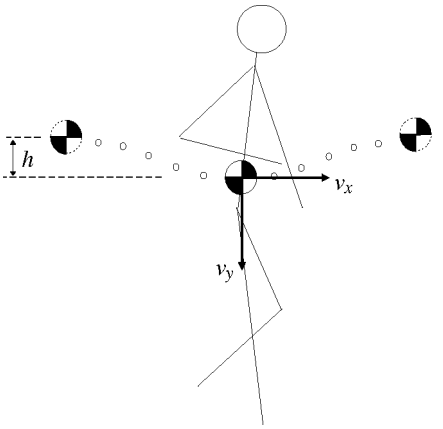


Figure 1.32. *Motion of the centre of gravity in running or walking.*

A record of the external reactions on the feet from the ground can be obtained from force-plate experiments. From these records the three components of energy can be derived, and each varies as a sine wave or as a trapezoidal wave. It turns out that:

- In the walking process the changes in the forward kinetic energy are of the same amplitude as and 180° out of phase with those in the vertical potential energy, and the vertical kinetic energy is small. Hence the total of the three energies is approximately constant.
- In the running process the vertical components of the energies move into phase with the forward kinetic energy. The total energy has the form of a trapezoidal wave, and varies substantially over the gait cycle.

The difference in the energy profiles during running and walking suggests that different models might be appropriate. For walking, the system appears simply to cycle between states of high potential energy and high kinetic energy, with relatively little loss in the process (in fact at a normal walking speed of 5 km hr^{-1} about 65% of the vertical potential energy is recovered in forward kinetic energy). In running there is effectively no recovery of energy in this manner, and alternative mechanisms such as muscle action must be responsible for the energy cycle. This gives us a strong clue as to how we might model the two systems: an appropriate walking model might be based on the conservation of energy, whereas a running model might feature viscoelastic and dynamically active muscle.

Ballistic model of walking

We have seen that there is an efficient mechanism of conversion of potential to kinetic energy in the walking process. This suggests that the action of walking might be described in terms of some sort of compound pendulum. One attempt to describe the process based on this conception is the ballistic model of walking, illustrated in figure 1.33.

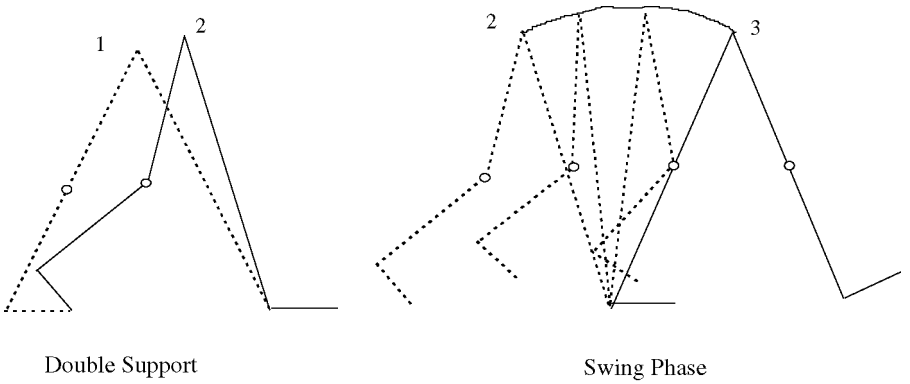


Figure 1.33. *Ballistic model of walking (based on McMahon (1984)).*

In this model it is assumed that the support leg remains rigid during the double support and swing phase periods, with the required rotation occurring at the ankle to keep the foot flat on the ground. The swing leg hinges at the knee. The swing phase commences when the toe leaves the ground and ends when the heel strikes the ground as illustrated above. During this period the foot remains at right angles to the swing leg. There follows another period of double support (not shown). It is assumed that all muscular action occurs during the double-support period and that thereafter the legs swing under the action of gravity alone (hence the term ballistic model). The mass of the body might be assumed to act at the hip joint, whilst the mass in the legs might be distributed in a realistic way to give appropriate centroids and mass moments of inertia.

The model is a three-degrees-of-freedom system, comprising the angle of the stiff leg and the two angles of the swing leg. All other parameters are determined by the geometry of the system. The equations of motion for the system can be written down. The initial conditions at the start of the swing phase are determined by the muscular action and are thus unknown. However, choosing an arbitrary set of initial conditions, the system can be traced (probably numerically) until the point at which the heel strikes, signalling the end of the swing phase. The initial conditions can be adjusted until the swing leg comes to full extension at the same moment at which the heel strikes the ground. Other constraints are that the toe must not stub the ground during the swing phase and that the velocity must not get so high that the stiff leg leaves the ground due to the centrifugal force of the body moving in an arch. The model can be used to find a correspondence between step length and time taken for the swing. Although the solution of the equations for any particular set of initial conditions is determinate, there is more than one set of initial conditions that corresponds to each step length, and therefore there are a range of times that the step can take. Typical results from the model, showing the range of swing time against subject height for a step length equal to the length of the leg, are presented by McMahon. Experimental data for a number of subjects are also shown, and there is a good correspondence with the results of the theoretical model.

A model of running

The components of energy that we considered for the walking model are demonstrably not conserved when we run. In fact, their sum has a minimum in the middle of the support phase and then reaches a maximum as we take off and fly through the air. This suggests that perhaps there is some form of spring mechanism becoming active when we land. This spring absorbs strain energy as our downwards progress is arrested, and then recoils propelling us back into the air for the next stride. McMahon analyses the model illustrated in figure 1.34. This idealization is strictly valid only during the period in which the foot is in contact with the track. We can analyse this system using standard methods for a two-degrees-of-freedom system. To gain a preliminary insight into the performance of the system we might first of all investigate its damped resonant frequencies, assuming that the foot remains permanently in contact with the track. The period during which the foot actually remains in contact with the track would be expected to be about one-half of the period of the resonant cycle. We can plot foot contact time against track stiffness. The surprising result of this exercise is that there is a range of track stiffnesses for which the foot contact time is less than it would be for a hard track. There is experimental evidence that running speed is inversely proportional to track contact time, and so the above analysis suggests that a track could be built that would improve times for running events. A further benefit of such a track would be that its compliance would reduce the peak forces on the foot relative to those associated with a hard surface, thus leading to a reduction in injuries. The first track constructed to test this hypothesis, a 220 yard indoor track built at Harvard University in 1977, was a real success. Training injuries were reduced to less than one-half of those previously sustained on a cinder track, and times were reduced by about 2%.

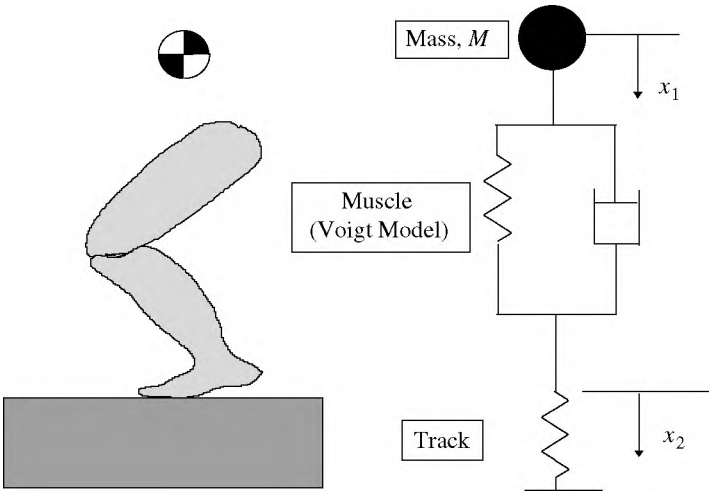


Figure 1.34. *Model of running.*

Experimental gait analysis

Experimental techniques used to measure gait include:

- visual systems, perhaps including a video camera;
- position, strain and force measurement systems;
- energy measurement systems.

Some of the techniques for defining and measuring normal gait are described in section 20.3.2. All of the techniques are of interest to the physicist working in medical applications of physics and engineering.

1.8. DIMENSIONAL ANALYSIS: THE SCALING PROCESS IN BIOMECHANICS

When we look around at the performance of other creatures we are often surprised at the feats that they can achieve. A flea can jump many times its own height, whilst we can only manage about our own (unless we employ an external device such as a pole). An ant can carry enormous weights relative to its own body weight and yet we cannot. We can often explain these phenomena using simple arguments based on our understanding of biomechanics.

1.8.1. Geometric similarity and animal performance

The simplest scaling process that we can imagine is that based on *geometric similarity*. If objects scale according to this principle then the only difference between them is their size. All lengths are scaled by the same amount, and we should not be able to tell the difference between them geometrically. If a characteristic length in the object is L, then the volume of the object is proportional to L^3.

Using the principle of geometric similarity together with Newton's second law, and assuming that the stress that can be sustained by a muscle is independent of its size, Hill reasoned in 1949 (see McMahon) that the absolute running and jumping performance of geometrically similar animals should be independent of their size.

There are three fundamental parameters in structural mechanics:

M	L	T
mass	length	time

All other parameters such as velocity, acceleration, force, stress, etc can be derived from them.

The *stress* developed in muscle is constant, independent of muscle size,

$$\sigma \propto L^0.$$

The *force* developed by the muscle is the stress multiplied by the cross-sectional area, and so is proportional to the square of the length,

$$F = \sigma A \propto L^2.$$

Newton's second law tells us that force is mass multiplied by acceleration. Assuming that the density of the tissue is constant for all animals, the force can be written as

$$F = ma \propto L^3 L T^{-2} \equiv L^2 \left(L T^{-1}\right)^2.$$

Comparing these two expressions for the force, it is apparent that the group (LT^{-1}) must be independent of the body size. The velocity of the body has these dimensions, and so must be independent of the body size. Hence the maximum running speed of geometrically similar animals is independent of their size! Hill supported his argument by the observation that the maximum running speeds of whippets, greyhounds and horses are similar despite the wide variation in size. In fact, the maximum running speeds of animals as diverse as the rat, the elephant, man and the antelope is constant to within a factor of about three, despite a weight ratio of up to 10 000.

The argument can readily be extended to assess the maximum height to which an animal might be able to jump. The principle of conservation of energy tells us that the maximum height will be proportional to the square of the take-off velocity. Since the velocity is independent of size, so will be the maximum jumping height. This is true to within a factor of about two for a wide range of animals.

1.8.2. Elastic similarity

When we apply the principle of geometric similarity we assume that the body has a single characteristic length, L, and that all dimensions scale with L. We can conceive alternative rules featuring more than one characteristic dimension. For example, we might assume that a long bone is characterized by its length and by its diameter, but that the proportion of length to diameter is not constant in a scaling process. A more general scaling process could be based on the assumption that the length and diameter scales are based on the relationship

$$L \propto d^q, \quad \text{where } q \text{ is an arbitrary constant.}$$

One rule that we might construct is based on the assumption that the structure will scale so that the stability of the column is independent of the animal size. In section 1.4.2 we developed a method of calculating the deflection of a beam. The deflection at the tip of a cantilever beam under self-weight can be shown to be proportional to $AL^4 = I$, where L is the length of the beam, A its cross-sectional area and I its second moment of area. For a beam of circular cross section the area is proportional to d^2 and the second moment of area is proportional to d^4,

$$\frac{\delta_{\text{tip}}}{L} \propto \frac{1}{L}\frac{L^4 d^2}{d^4} = \frac{L^3}{d^2}.$$

Hence if the deflection of the tip is to be proportional to the length of the beam, the relationship between d and L must be $d \propto L^{3/2}$.

This scaling rule applies to other elastic systems. The critical length of column with respect to buckling under self-weight is one example. This process of scaling is called elastic similarity.

The weight, W, of the cylindrical body is proportional to the square of the diameter multiplied by the length,

$$W \propto d^2 L = d^{q+2} = L^{(q+2)/q} \quad \text{or} \quad L \propto W^{q/(q+2)} \qquad d \propto W^{1/(q+2)}.$$

For *geometric similarity*:

$$L \propto W^{1/3} \qquad d \propto W^{1/3}.$$

For *elastic similarity*:

$$L \propto W^{1/4} \qquad d \propto W^{3/8}.$$

Evidence from the animal kingdom suggests that *elastic similarity is the governing process of scaling in nature.*

1.9. PROBLEMS

1.9.1. Short questions

a Is the average pressure under the foot when standing higher or lower than normal systolic arterial pressure?
b Is bone weakest when subjected to tension, compression or shear stress?
c What is the 'yield point' of a material?
d Is most tissue homogeneous or inhomogeneous in terms of its mechanical properties?
e Categorize bone and soft tissue as typically having linear or nonlinear mechanical properties.
f What is a viscoelastic material?
g If a spring is considered as being equivalent to an electrical resistance where *force* $\equiv$ *voltage* and *displacement* $\equiv$ *current*, then what electrical component is equivalent to a dashpot?

h What three basic models can be used to describe 'creep'?
i Approximately what fraction of body weight is applied to the head of each femur when standing?
j Is energy conserved during running?
k Why can a frog jump so high?
l What is the characteristic property of an elastic material?
m What are the approximate magnitudes of the ultimate tensile stress and of the ultimate tensile strain of cortical bone?
n What process in living bone is described by Wolff's law?
o What is the difference between creep and relaxation?
p Sketch the arrangement of springs and dampers for the Kelvin model of viscoelasticity.
q What is the equation describing the relationship between stress and applied moment, commonly referred to as the engineer's theory of bending?
r State one of the fundamental assumptions of the engineer's theory of bending.
s If a thin longitudinal slit is cut into a hollow tube of circular cross-section the cross-sectional area will be essentially unchanged. Is there any significant change in the torsional stiffness?
t What are the relationships between body weight, W, and (a) the length, (b) the diameter of a cylindrical body if the scaling process is governed by the law of elastic similarity?

1.9.2. Longer questions (answers are given to some of the questions)

Question 1.9.2.1

Write a brief description of each of the following quantities:

(a) potential energy;
(b) kinetic energy;
(c) strain energy.

Discuss the process of falling to the ground, indicating which of the above energies the body has at different stages in the fall. Draw a sketch illustrating a model that might be used to analyse the process when landing and allowing the knees to flex to cushion the fall. State clearly the assumptions inherent in the model, and discuss whether the assumptions are reasonable.

Derive an expression for the force generated at the feet during the arrest of the fall.

Assuming that the geometrical arrangement of the skeleton and musculature is such that the bones experience a load eight times higher than the load at the foot, and that the fracture load of the tibia is 36 kN, calculate the maximum height from which a 70 kg body might be able to fall before a fracture is likely.

Answer

- *Potential energy* is the potential of a body to do work under the action of a force. It is associated with the position of the body and the force acting on it. It is a relative quantity, because some arbitrary position will be taken as the reference point where the potential energy is zero. Gravitational p.e. $= mgh$.
- *Kinetic energy* is associated with the motion of a body. k.e. $= \frac{1}{2}mv^2$.
- *Strain energy* is associated with the deformation of a body. It is the area under a load/deflection curve, or the integral over volume of the area under a stress/strain curve.

Consider the case of a person dropping from a wall onto level ground. There is a three-stage process.

- There is a period of free-fall during which the potential energy of the body is converted into kinetic energy.

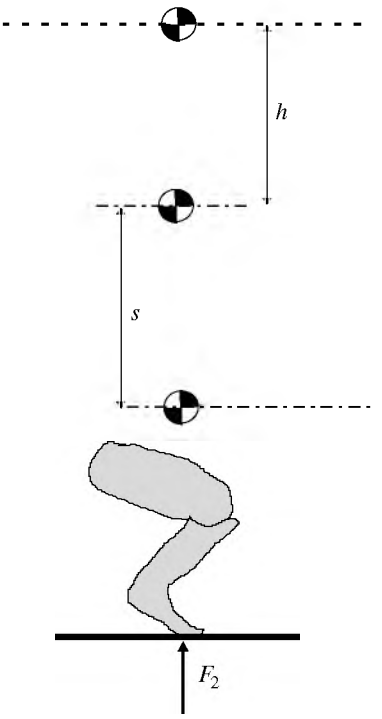


Figure 1.35. *Model of falling, bending knees to arrest.*

- The feet hit the ground and the reaction force from the ground does work to arrest the fall.
- The energy stored as elastic strain energy in the bones and muscles during the arresting process will be released, causing a recoil effect.

The first two stages of the process are indicated in figure 1.35.

We can apply the principle of conservation of energy from the time at which the fall commences to that at which it is arrested and the body is again stationary. At both of these times the kinetic energy is zero. The work done on the body by the reaction to the ground must therefore be equal to the potential energy that the body has lost in the falling process. We have an immediate problem in that we do not know how the reaction force changes with time during the arresting process (F_1 and F_2 are functions of time).

Assume that:

- The body is just a mass concentrated at one point in space, and that the surroundings contain some mechanism for absorbing its energy.
- The instant that the body makes contact with the ground a constant reaction force is applied by the ground, and that this force remains constant until motion is arrested (figure 1.36).

The work done on the body by the reaction force is

$$\text{work} = Fs.$$

The change in potential energy of the body is

$$\Delta U = -mg(h + s).$$

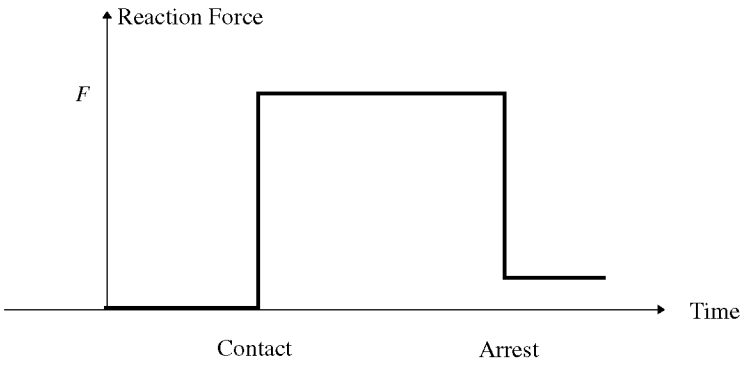


Figure 1.36. *Force versus time on a body during arrest of falling.*

The change in kinetic energy of the body is zero.

For conservation of energy:

$$Fs - mg(h + s) = 0$$
$$F = mg\left(\frac{h}{s} + 1\right)$$

but how big is s?

If we bend our knees, then we know that we can arrest the fall over a distance of perhaps 0.5 m. In this condition we are going to use our muscles, ligaments and tendons to cushion the fall. We are told that the geometrical arrangement of the skeleton and muscles is such that bone forces are eight times higher than external loads, and that the maximum force on the bone is 36 kN. This implies that the maximum force that can be exerted at one foot is 4.5 kN, or 9 kN on the body assuming that force is distributed evenly between the two feet.

Using the assumptions of our current model, this implies that the maximum height from which we can fall and land on bended knees is

$$9000 = 70 \times 10 \times \left(\frac{h}{0.500} + 1\right)$$
$$h \approx 5.9 \text{ m}.$$

Question 1.9.2.2

Determine expressions for the forces in the erector spinae and on the fifth lumbar vertebra in the system illustrated in figure 1.37. Produce a graph of spine elevation against forces for the case $W = 20$ kg and $W_0 = 75$ kg.

Question 1.9.2.3

You are preparing to dive into a pool (see figure 1.38). The mechanical system at the instant of maximum deflection of the diving board is illustrated in the figure. If you are launched 1 m into the air above the static equilibrium position of the board calculate the strain energy that is recovered from the board to achieve this height. Calculate the maximum velocity during the launch assuming that this occurs when the board is level. Calculate the maximum force that is applied to the foot of the diver, assuming that the force is proportional

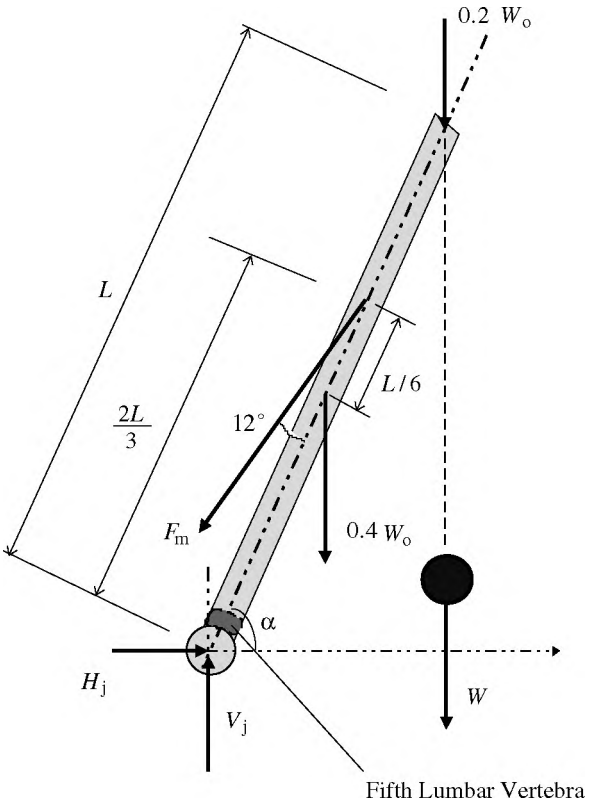


Figure 1.37. *Loads on the spine when lifting.*

Figure 1.38. *Diver on a board.*

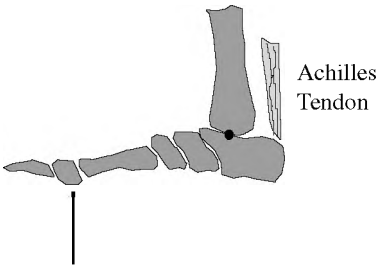


Figure 1.39. *Diagrammatic illustration of the geometry of the foot and the force on the foot applied by the diving board.*

to the downward deflection of the board and that the maximum deflection of the board is 0.25 m. In any of these calculations you may assume that the mass of the diving board is negligible.

At the instant of maximum load the attitude of the foot is that illustrated in figure 1.39. Estimate the dimensions of your own foot. Calculate in this system the force in your Achilles tendon and the force in your tibia at this instant. Calculate the stress in the Achilles tendon.

Question 1.9.2.4

In a simple model of pole-vaulting we might assume that the kinetic energy of the vaulter that is generated during the run-up is translated into potential energy at the height of the jump. Derive an expression relating the peak velocity in the run-up to the height achieved in the vault. Calculate the energy involved for a person with a mass of 70 kg.

The mechanism for the transfer of energy is assumed to be the pole, and we might assume that at some point in the process the whole of the energy of the vault is contained within the pole as strain energy. Assuming that a 5 m pole (of uniform cross-section) buckles elastically into a shape represented by one half of a sine wave, and that the peak transverse deflection is 1 m, deduce the flexural stiffness (EI) that is required to absorb the energy. Write down an expression for the peak stress in the pole at maximum flexion, and derive an expression for the upper limit on the radius of the pole assuming that it is of circular cross-section.

If the pole is made of a carbon fibre material with a Young's modulus of 100 GPa and a failure stress of 1500 MPa, design a cross-section for the pole to sustain the loads imposed. Calculate the weight of the pole that you have designed if the density is 1600 kg m^{-3}.

Comment on the assumptions, and discuss whether you believe this to be a realistic model. (The strain energy in a beam under flexion can be written as $\mathrm{SE} = \frac{1}{2}\int_0^L EI\chi^2\,\mathrm{d}x$).

Answer

Equating potential and kinetic energies,

$$mgh = \tfrac{1}{2}mv^2$$

$$h = \frac{v^2}{2g}.$$

For a height of 6 m, the peak running velocity would have to be 10.8 m s^{-1}; about the peak velocity achievable by man. The energy would be 4.12 kJ.

Assume

$$y = Y \sin \frac{\pi x}{L}.$$

Then

$$\chi \approx \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{\pi^2 Y}{L^2} \sin \frac{\pi x}{L}.$$

Strain energy

$$\mathrm{SE} = \tfrac{1}{2} \int_0^L EI\chi^2 \,\mathrm{d}x = \frac{\pi^4 Y^2 EI}{2L^4} \int_0^L \sin^2 \frac{\pi x}{L} \mathrm{d}x = \frac{\pi^4 Y^2 EI}{4L^3}$$

$$EI = \frac{4L^3(\mathrm{SE})}{\pi^4 Y^2}.$$

Assuming that $Y = 1.0$ m,

$$EI = \frac{4 \times 5^3 \times 4120}{\pi^4 1.0^2} = 21\,148 \text{ N m}^2.$$

The bending moment is $EI\chi$,

$$M_{\max} = EI\chi_{\max} = \frac{\pi^2 YEI}{L^2}.$$

The bending stress in the tube is Mr/I,

$$\sigma_{\max} = \frac{M_{\max} r}{I} = \frac{\pi^2 YEr}{L^2} = 0.395Er.$$

If the stress is to be less than two-thirds of its ultimate value for safety, then

$$r < \frac{0.67\sigma_{\mathrm{ult}}}{0.273Er} = \frac{1000 \times 10^6}{0.395 \times 100 \times 10^9} = 0.253 \text{ m}.$$

If we set the radius to 25 mm the stress is 988 MPa.

Returning to the stiffness requirement:

$$EI = E\pi r^3 t = 21\,148 \quad \rightarrow \quad t = \frac{21\,148}{100 \times 10^9 \times \pi \times 0.025^3} = 0.0043 \text{ m}.$$

The suggested design is a tube of radius 25 mm and wall thickness 4.3 mm.

The weight of this pole would be $2 \times \pi \times 0.025 \times 0.0043 \times 5 \times 1600 = 5.4$ kg.

Question 1.9.2.5

The head of the femur is offset from the axis of the bone, producing a bending moment in the bone when standing in a relaxed position. Calculate the magnitude of this moment for the system shown in figure 1.40 assuming that the offset x is 50 mm and that the body weight is 70 kg.

Write down the general expression for the second moment of area of a cross-section as illustrated in figure 1.41, and show from first principles that the second moment of area of a thin-walled hollow cylinder of radius r and thickness t is $\pi r^3 t$.

For the applied moment calculated, and using the thin-walled approximation, calculate the maximum bending stress in the femur assuming a mean radius of 20 mm and a wall thickness of 12 mm.

Compare the calculated bending stress with the compressive stress that arises due to end load in the femur. Is the bending stress significant?

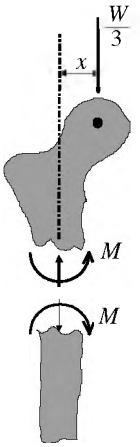


Figure 1.40. *Moment on femur. Two-leg stance.*

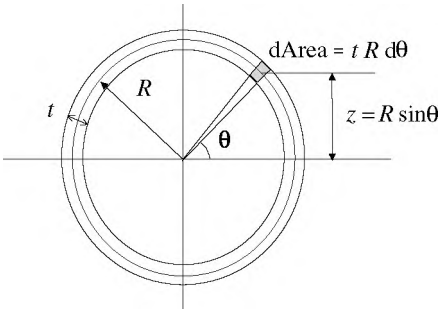


Figure 1.41. *Thin-walled cylindrical section.*

Answer

Force on one hip, two-leg stance $= \dfrac{70 \times 10}{3} = 233$ N: Bending moment on femur $= 233 \times 50 = 11\,667$ N mm.
By definition:

$$I = \int_{\text{area}} z^2 \, \mathrm{d}A$$

$$I = \int_{\text{area}} (R \sin\theta)^2 Rt \, \mathrm{d}\theta$$

$$I = \int_{\text{area}} R^3 t \sin^2\theta \, \mathrm{d}\theta \qquad I = R^3 t \int_0^{2\pi} \sin^2\theta \, \mathrm{d}\theta$$

$$I = R^3 t \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2}\right) \mathrm{d}\theta \qquad I = R^3 t \left[\frac{\theta}{2} - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$I = \pi R^3 t.$$

For the geometry specified,

$$I = \pi R^3 t = \pi \times 20^3 \times 12 = 302\,000 \text{ mm}^4.$$

The bending stress is calculated from the engineer's theory of bending:

$$\sigma = \frac{Mz}{I}.$$

At the extreme fibre, $z = R + t/2 = 20 + 12/2 = 26$ mm,

$$\sigma = \frac{Mz}{I} = \frac{11\,667 \times 26}{302\,000} = 1.0 \text{ N mm}^{-2}.$$

The failure stress is about 120 N mm^{-2}, and so there is, as expected, a high margin of safety in this condition.

For comparison, the compressive stress in the bone is simply the end load divided by the area:

$$\sigma_{\text{comp}} = \frac{P}{A} \approx \frac{P}{2\pi Rt} = \frac{233}{2 \times \pi \times 20 \times 12} = 0.15 \text{ N mm}^{-2}.$$

Hence the bending stress is some six or seven times higher than the simple compressive stress, and so bending is important in the analysis of the femur.

Answers to short questions

a The average pressure under the foot is higher than normal arterial systolic pressure.
b Bone is weakest when subject to shear stress.
c The point on a stress/strain curve beyond which permanent strain occurs is the yield point.
d Tissue is inhomogeneous in terms of mechanical properties.
e Bone, linear; soft tissue, nonlinear.
f One where the stress/strain relationship depends upon time.
g An inductance is equivalent to a dashpot.
h Maxwell, Voigt and Kelvin models.
i Approximately one-third of body weight is applied to the head of each femur.
j No, energy is not conserved during running.
k Because it can be shown dimensionally that for geometrically similar animals jumping performance should be independent of size.
l Loading and unloading follow the same path.
m 80 MPa and 1%; see figure 1.5.
n Stress remodelling is described by Wolff's law.
o Creep is the process whereby displacement will change with time under a constant stress. Relaxation is the process whereby the stress will fall with time under constant strain; see figure 1.7.
p See figure 1.10 for the Kelvin model.
q Stress = applied moment/second moment of area
r 'Plane sections remain plane' and 'stress proportional to strain' are assumed in the engineer's theory of bending.
s Yes, there will be a large change in the torsional stiffness.
t (a) Weight proportional to the fourth power of length. (b) Weight proportional to diameter raised to the power 2.67.

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CHAPTER 2

BIOFLUID MECHANICS

2.1. INTRODUCTION AND OBJECTIVES

In Chapter 1 we looked at biomechanical systems, focusing in particular on the fundamental mechanical properties of bone and tissue and on how loads are supported by the skeleton and by the muscles. In this chapter we will investigate some basic fluid-mechanical systems in the human body. Once again the emphasis will be on the use of relatively simple models to describe gross performance characteristics of the systems. The primary difference between a solid and a fluid is that a stationary fluid cannot sustain shear stresses.

There are three scalar parameters that describe the conditions in a small volume of a fluid at rest. Moving fluids have one additional parameter, the velocity vector, which is conveniently expressed in terms of its components in three mutually perpendicular directions.

Table 2.1. *Fluid parameters.*

Variable	Symbol	SI unit
Density	ρ	kg m^{-3}
Temperature	T	K
Velocity	u, v, w	m s^{-1}
Pressure	p	Pa

In our analyses of human fluid-dynamic systems we shall normally consider that densities and temperatures are constant, and so we shall be concerned only with pressure and velocity distributions.

Most of the examples of fluid dynamics discussed in this chapter are drawn from the cardiovascular system. This is simply a reflection of the areas of interest and expertise of the authors, but the basic methodologies described are equally applicable to other areas of biofluid mechanics. Some of the questions that we shall address are listed below.

- What levels of fluid pressure are present in the human body?
- What parameters govern the flow of blood in the arteries and veins?
- Is blood thicker than water?
- How can the fluid-dynamic performance of the heart be measured?
- What are the fluid-dynamic effects of disease processes such as the narrowing of the arteries?

As in Chapter 1 there is material here that should be of interest to all readers.

2.2. PRESSURES IN THE BODY

We can often learn a great deal about the condition and performance of the body by studying the distribution of pressures in and around it. We all know many people who have problems with high blood pressure, and will be aware that blood pressure is taken routinely in the course of a general medical examination. There are many other pressure measurements made by physicians—and sometimes by medical physicists. These include respiratory pressures, bladder pressure, foot pressure, ocular pressure and middle-ear pressure.

We will recognize that pressure has the same units as stress, and indeed that a pressure is just a constant normal stress applied around all surfaces bounding a body. As physicists we will naturally use SI units of N m^{-2}, or Pascals. Our clinical colleagues, however, will demand pressures in millimetres of mercury or sometimes in millimetres (or centimetres) of water, and we need to be familiar with the following conversion factors.

Table 2.2. *Pressure conversion factors.*

	Atmosphere	N m^{-2} (Pa)	mmHg	mmH_2O
1 Atmosphere	1	1.01×10^5	760	10 300
1 N m^{-2} (Pa)	9.87×10^{-6}	1	0.0075	0.102
1 mmHg	0.001 32	133	1	13.6
1 mmH_2O	9.68×10^{-5}	9.81	0.0735	1

When we measure pressures using devices such as manometers we, in fact, obtain relative pressures. It is customary to quote pressures relative to atmospheric pressure, and we then refer to them as gauge pressures. Hence a pressure that is lower than atmospheric pressure is usually quoted as a negative pressure. Typical levels of pressure in and around the body are presented in table 2.3. All measures are quoted in millimetres of mercury for ease of comparison, but other units are standard for many of the systems.

2.2.1. Pressure in the cardiovascular system

There are essentially two circuits in the cardiovascular system. The systemic circuit carries oxygen-rich blood to the major organs, tissue, etc and returns the oxygen-depleted blood to the heart. The pulmonary circuit carries oxygen-depleted blood to the lungs to be recharged and returns oxygen-rich blood to the heart. The pressure in each of the two circuits is not constant. It varies with time and it varies with position—*temporal* and *spatial* variation. The pressure is generated by the muscular action of the heart, and is used to overcome viscous and other resistive forces as it drives the blood around the body. During *systole* the muscle of the heart contracts and blood is ejected into the aorta and pulmonary trunk. During *diastole* the muscle relaxes and the heart fills.

The pressure distribution in the cardiovascular system is governed by the equations of fluid dynamics, and later in this chapter we shall be looking at this variation in more detail. For now a qualitative description of the pressure variations will suffice. The spatial variation of pressure is a decrease along the arteries and veins as blood flows away from the heart. The pressure is used to overcome the viscosity of the blood. All of the branches and orifices in the circulatory system cause further decreases in pressure. We will soon learn how to quantify these losses. At positions close to the heart there is a substantial temporal variation of pressure as the heart beats. The temporal variation of pressure decreases at increasing distance from the heart, so that by the time the blood reaches the smaller vessels there is very little pulsatility. The damping of the pulsatility is attributable to the elasticity of the walls of the vessels in the cardiovascular system.

Table 2.3. *Pressures in the body (some of the values are taken from Cameron and Skofronik (1978)).*

Site		Pressure (mm Hg)
Arterial blood pressure:	systole	100–140
	diastole	60–90
Capillary blood pressure:	arterial end	~30
	venous end	~10
Venous blood pressure:	smaller veins	3–7
	great veins	<1
Cerebrospinal pressure in brain (lying down)		5–12
Gastrointestinal pressure		10–20
Bladder pressure		5–30
Lungs:	during inspiration	minus 2–3
	during expiration	2–3
Intrathoracic cavity (between lung and chest wall)		minus 10
Joints in skeleton		up to 10 000
Foot pressure:	static	up to 1200
	dynamic	up to 7500
Eye		12–23
Middle ear		<1

The blood pressure of a patient is used as a routine clinical measure of health. This pressure is often measured using a device called a sphygmomanometer. A standard procedure for use of this device is outlined below.

- Place cuff around upper arm.
- Rest arm on flat surface just below heart level.
- Place stethoscope on hollow of elbow over brachial artery.
- Inflate cuff to a pressure at which the blood does not flow, usually about 180 mmHg is sufficient.
- Release pressure slowly until sounds of blood squirting through restricted artery are heard. These sounds are called Korotkoff or 'K' sounds. *This is systolic pressure.*
- Continue to release pressure until no further sound is heard. *This is diastolic pressure.*

We will return to the subject of blood pressure and in particular how to measure it, in Chapter 18.

2.2.2. *Hydrostatic pressure*

The pressure in the liquid in a tank varies with the height. At the bottom of the tank the pressure is higher than it is at the top simply because of the weight of fluid supported on the lower layers. The pressure distribution that arises in a stationary fluid due to the action of gravity is called the hydrostatic pressure distribution. The hydrostatic pressure at all points within a stationary continuous fluid varies only with the vertical distance from the surface of the fluid,

$$\frac{\mathrm{d}p}{\mathrm{d}z} = \rho g \qquad \text{hydrostatic pressure distribution}$$

The rate of change of pressure due to the weight of gases is small because the density is small, and we would certainly not be concerned, for example, with the variation of atmospheric pressure outside the body over the height of a person. The hydrostatic pressure distribution is important clinically only with respect to liquids, and to blood in particular. The density of blood is close to that of water, about 1000 kg m^{-3}.

The pressure in a vessel the shape and size of the human body, if filled with blood, would be expected to vary by more than 100 mm of mercury because of the hydrostatic pressure effect when a person is standing. We would expect the pressure in the blood vessels in the skull to be some 35 mmHg lower than that at the heart and that in the blood vessels in the feet perhaps 100 mmHg higher (see figure 2.1). When a physician takes our blood pressure he places the cuff around the upper arm at about the level of the heart. Since typical blood pressure measurements are of the order of 100 mmHg, the hydrostatic effect is clearly an important one. It is not surprising that lying down is beneficial when a person feels faint due to a reduced blood supply to the brain. The pumping pressure of the heart is not excessive compared to the height through which blood must be lifted, even without accounting for any dynamic losses, and we operate on a relatively low factor of safety.

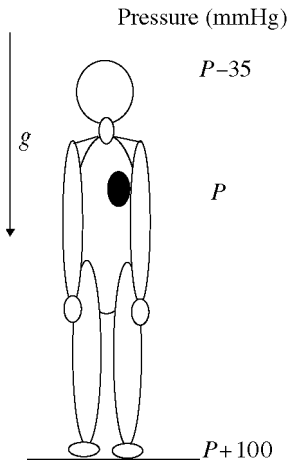

Figure 2.1. *Hydrostatic pressure when standing.*

2.2.3. *Bladder pressure*

The measurement of pressure in the bladder is a useful diagnostic tool for a range of urological problems. The pressure within the bladder has a passive component due to the viscoelastic nature of the bladder wall, and an active component due to the contraction of the muscular wall (the detrusor muscle). The normal rate of filling of the bladder is about 1 ml s^{-1} from each kidney. With this slow rate of filling, the detrusor muscle stays relaxed and the pressure rises very little, to about 10 cmH_2O. The sensation of fullness is triggered by stretch receptors in the wall of the bladder. The emptying of the bladder involves an active contraction of the detrusor muscle which, in a normal subject, will result in a pressure rise of about 25 cmH_2O (figure 2.2). A higher pressure in a male subject may be caused by enlargement of the prostate gland, which will increase the resistance to flow through the urethra.

Clinically we might measure these pressures using a catheter inserted through the urethra or a needle through the wall of the bladder (*cystometry*). As physicists we might devise very simple techniques based on

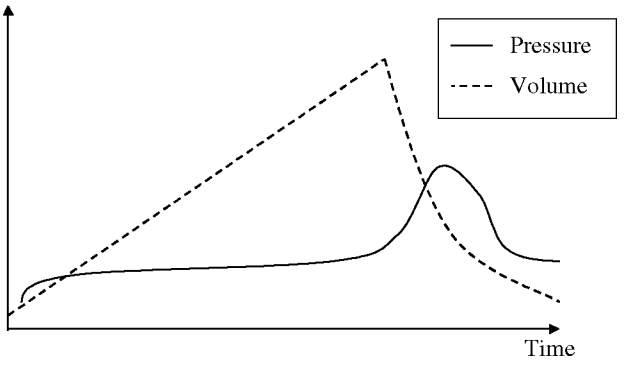

Figure 2.2. *Bladder pressure and volume during filling and voiding.*

principles such as energy conservation. The dynamics of urination present some very interesting problems in the analysis of collapsible elastic tubes, extending into the range of sonic or critically choked flows.

2.2.4. Respiratory pressures

The pressures in the lungs are required only to move air. Typical respiration rates at rest are about 12 breaths per minute, and typical inspiration volume is about 300 ml (see Chapter 17). The mass of the air to be moved is relatively small, and the viscosity of air is small, and so only small pressures are required. Typically the pressure in the lung is less than 5 mmHg below atmospheric for inspiration, and 5 mmHg above for expiration. The variation in pressure is achieved in principle by movement of the diaphragm, but to assist the process the pressure in the intrathoracic cavity, between the wall of the chest and the lung, is sub-atmospheric. This helps to keep the lungs inflated at all times during the breathing cycle.

2.2.5. Foot pressures

At the Royal Hallamshire Hospital in Sheffield one of the services offered by the Department of Medical Physics and Clinical Engineering is a dynamic foot pressure monitoring system. This can be of great benefit in diagnosis and treatment of problems with gait, as well as in the assessment of pathological conditions associated with diseases such as diabetes. In a normal adult the maximum pressure under the sole of the foot is about 100 kPa under static conditions, and perhaps some five times higher under dynamic conditions (see Chapter 20, section 20.2.2, for more detailed information). Higher pressures can indicate a tendency to ulceration, and can be indicative of neuropathological conditions.

2.2.6. Eye and ear pressures

The eyeball contains aqueous humour, a fluid comprised mostly of water. This fluid is produced continuously, and should drain to maintain normal pressure. A blockage in the drain causes an increase in pressure (*glaucoma*) and can lead to a restricted blood supply to the retina. In severe cases permanent damage might ensue. Sticking a finger in your eye to monitor pressure was one of the most common diagnostic techniques used by the local doctor up to the turn of the century. Eye pressure was regarded as a general indicator of physical well-being. Quite sophisticated methods for measuring intraocular pressure have now been developed (see Chapter 18, section 18.2.4).