

D. H. Stamatis

SIX SIGMA AND BEYOND

**Statistics
and
Probability**



St. Lucie Press

**SIX SIGMA
AND BEYOND**
**Statistics
and
Probability**

SIX SIGMA AND BEYOND

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D. H. Stamatis

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To Stephen

Preface

The long-range contribution of statistics depends not so much upon getting a lot of highly trained statisticians into industry, as it does in creating a statistically minded generation of physicists, chemists, engineers, and others who will in any way have a hand in developing and directing the production processes of tomorrow.

W.A. Shewhart and W.E. Deming

Much has been said about statistics and their use. Often, though, we statisticians overlook the discussion of the obvious as soon as we move away from the academic arena. We expect researchers and professionals in all walks of life to use the many tools offered by the statistical world, but we have failed to educate them appropriately both in concept and application. The focus of most statistics books seems to be formula utilization.

This volume will attempt to explain the tools of statistics and to provide guidance on how to use them appropriately and effectively. The structure of this work is going to follow (1) the conceptual domain of some useful statistical tools, (2) appropriate formulas for specific tools, and (3) the connection between statistics and probability.

This volume is not intended to be a textbook. It is intended to be a general manual for people who are interested in using statistical, probability, and reliability concepts to improve processes and profitability in their organizations.

The discussion begins with very elementary issues and progresses to some very advanced tools for decision-making. Specifically, the book begins by delineating the importance of collecting, analyzing, and interpreting data, from a practical perspective rather than an academic point of view. The assumption is that you (the reader) are about to begin a study of something, and you want to do it well. You want to design a good study, analyze the results properly, and prepare a cogent report that summarizes what you have found.

Because of these assumptions, this book does not dwell on formulas and significance tables or proofs for that matter. The assumption is that a statistical software package will be utilized, and that the reader will benefit more from learning to understand and interpret the results generated by that software than from memorizing formulas.

About the Author

D. H. Stamatis, Ph.D., ASQC-Fellow, CQE, CMfgE, is currently president of Contemporary Consultants, in Southgate, Michigan. He received his B.S. and B.A. degrees in marketing from Wayne State University, his Master's degree from Central Michigan University, and his Ph.D. degree in instructional technology and business/statistics from Wayne State University.

Dr. Stamatis is a certified quality engineer for the American Society of Quality Control, a certified manufacturing engineer for the Society of Manufacturing Engineers, and a graduate of BSI's ISO 9000 lead assessor training program.

He is a specialist in management consulting, organizational development, and quality science and has taught these subjects at Central Michigan University, the University of Michigan, and Florida Institute of Technology.

With more than 30 years of experience in management, quality training, and consulting, Dr. Stamatis has served and consulted for numerous industries in the private and public sectors. His consulting extends across the United States, Southeast Asia, Japan, China, India, and Europe. Dr. Stamatis has written more than 60 articles and presented many speeches at national and international conferences on quality. He is a contributing author in several books and the sole author of 12 books. In addition, he has performed more than 100 automotive-related audits and 25 pre-assessment ISO 9000 audits, and has helped several companies attain certification. He is an active member of the Detroit Engineering Society, the American Society for Training and Development, the American Marketing Association, and the American Research Association, and a fellow of the American Society for Quality Control.

Acknowledgments

In a typical book, the author begins by thanking several individuals who have helped to complete it. In this mammoth work, so many people have helped that I am concerned that I may forget someone.

The writing of a book is a collective undertaking by many people. To write a book that conveys hundreds of thoughts, principles, and ways of doing things is truly a Herculean task for one individual. Since I am definitely not a Hercules or a Superman, I have depended on many people over the years to guide me and help me formulate my thoughts and opinions about many things, including this work. To thank everyone by name who has contributed to this work would be impossible, although I am indebted to all of them for their contributions. However, some organizations and individuals do stand beyond the rest, and without them, this series would not be possible.

Special thanks go to Dr. A. Stuart for granting me permission to use and adopt much of the discussion on discrete random variables, continuous RVs, uniform and beta distributions, functions of random variables (tolerances), exponential distribution and reliability, and hypothesis testing and OC curves in Part II of this volume. The work was adapted from the notes of *Statistics and Probability for Engineers* used as training material at Ford Motor Company.

Special thanks also go to Duxbury Press for granting me permission to use the material on Holt's Model for trend and Winters' Model for seasonality and econometric models. The work is based on *Managerial Statistics* by S.C. Albright, W.L. Winston, C.J. Zappe and P. Kolesar, published in 2001.

In addition, special thanks go to Prentice Hall for granting me permission to use the material on the summary of differences between MANOVA and discriminant analysis, what is conjoint analysis, uses of conjoint analysis, what is canonical correlation, and what is cluster analysis. The work is based on *Multivariate Data Analysis*, 5th ed., by J.F. Hair, R.E. Anderson, R.L. Tathan, and W.C. Black, published in 1998.

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Part I

Essential Concepts of Statistics

Introduction

This introduction will discuss the basic concepts of all statistics. The intent of the introduction is to sensitize the reader to the importance of taking statistics into consideration in the design and planning of experiments. Unless the experimenter plans a study appropriately, accounts for certain issues that are inherent in any study, and understands what is needed for a successful experiment, all will be for naught.

WHAT ARE DATA?

Everything we do is based on data. So, the question quite often is: should the word be datum or data? Grammatically speaking, the singular word is datum and the plural is data. However, because generally speaking we have more than one, the convention is that we use data. In common usage, data are any materials that serve as a basis for drawing conclusions. (Notice that the word we use is “materials.” That is because materials may be quantifiable or numerical and measurable or on the other hand may be attribute or qualitative. In either case they can be used for drawing conclusions.) Drawing conclusions from data is an activity in which everyone engages — bankers, scholars, politicians, doctors, and corporate presidents. In theory, we base our foreign policy, methods of treating diseases, corporate marketing strategies, and process efficiency and quality on “data.”

Data come from many sources. We can conduct our own surveys or experiments, look at information from surveys other people have conducted, or examine data from all sorts of existing records — such as stock transactions, election tallies, or inspection records. But acquiring data is not enough. We must determine what conclusions are justified based on the data. That is known as “data analysis.” People and organizations deal with data in many different ways. Some people accumulate data but do not bother to evaluate it objectively. They think that they know the answers before they start. Others want to examine the data but do not know where to begin. Sometimes people carefully analyze data, but the data are inappropriate for the conclusions that they want to draw. Unless the data are correctly analyzed, the “conclusions” based on them may be in error. A superior treatment for a disease may be dismissed as ineffectual; you may purchase stocks that do not perform well and lose your life’s savings; you may target your marketing campaign to the wrong audience, costing your company millions of dollars; or you may adjust the wrong item in a process, and as a consequence, you may affect the response of the customer

in a very unexpected way. The consequences of bad data analysis can be severe and far-reaching. That is why you need to know how to analyze data well.

You can analyze data in many different ways. Sometimes all you need to do is describe the data. For example, how many people say they are going to buy a new product you are introducing? What proportion of them are men and what proportion are women? What is their average income? What product characteristic is the customer delighted with? In other situations, you want to draw more far-reaching conclusions based on the data you have at hand. You want to know whether your candidate stands a chance of winning an election, whether a new drug is better than the one usually used, or how to improve the design of a product so that the customer will be really excited about it. You do not have all of the information you would like to have. You have data from some people or samples, but you would like to draw conclusions about a much larger audience or population.

At this juncture your answer may be, “I do not have to worry about all this because the computer will do it for me.” That is not an absolute truth. Computers simplify many tasks, including data analysis. By using a computer to analyze your data, you greatly reduce both the possibility of error and the time required. Learning about computers and preparing data for analysis by computer do require time, but in the long run they substantially decrease the time and effort required. Using a computer also makes learning about data analysis much easier. You do not have to spend time learning formulas. The computer can do the calculating for you. Instead, your effort can go into the more interesting components of data analysis — generating ideas, choosing analyses, and interpreting their results.

Because calculations are the computer’s job, not yours, this volume does not emphasize formulas. It emphasizes understanding the concepts underlying data analysis. The computer can be used to calculate results. You need to learn how to interpret them.

DESCRIBING DATA

Once you have prepared a data file, you are ready to start analyzing the data. The first step in data analysis is describing the data. You look at the information you have gathered and summarize it in various ways. You count the number of people giving each of the possible responses. You describe the values by calculating averages and seeing how much the responses vary. You look at several characteristics together. How many men and how many women are satisfied with your new product? What are their average ages? You also identify values that appear to be unusual, such as ages in the one hundreds or incomes in the millions, and you check the original records to make sure that these values were picked up correctly. You do not want to waste time analyzing incorrect data.

TESTING HYPOTHESES

Sometimes you have information available for everyone or everything that you are interested in drawing conclusions about, and all you need to do is summarize your

data. But usually that is not the case. Instead, you usually want to draw conclusions about much larger groups of people or objects than those included in your study. You want to know what proportion of all purchasers of your product are satisfied with it, based on the opinions of the relatively small number of purchasers included in your survey. You want to know whether buyers of your product differ from nonbuyers. Are they younger, richer, better educated? You want to be able to draw conclusions about all buyers and nonbuyers based on the people you have included in your study.

To do this (and understand it), you have to learn something about statistical inference. Later chapters in this volume will show you how to test hypotheses and draw conclusions about populations based on samples. You will learn how to test whether you have sufficient evidence to believe that the differences or relationships you find in your sample are true for the whole population.

DESCRIBING RELATIONSHIPS

You often want to determine what the relationship is between two variables. For example, what is the relationship between dollars spent on advertising and sales? How can you predict how many additional sales to expect if you increase your advertising budget by 25%? What is the relationship between the dosage of a drug and the reduction in blood pressure? How can you predict the effect on blood pressure if you cut the dose in half? You can study and model the relationship between pairs of variables in many different ways. You can compute indexes that estimate the strength of the relationship. You can build a model that allows you to predict values of one variable based on the values of another. That is what the last part of the book is about.

You must state your ideas clearly if you plan to evaluate them. This advice applies to any kind of work but especially to research design and statistical analysis. Before you begin working on design and analysis, you need to have a clearly defined topic to investigate.

ASKING A QUESTION

You may have a general suspicion that smoking less makes people feel better. You may think that component A is better than component B. Or you may have an idea for a study method that will make people learn more. Before you begin a study about such intuitions, you should replace vague concepts such as “feeling better” or “smoking less” or “learning more” with definitions that describe measurements that you can make and compare. You might define “better” with a specific performance improvement or a reduction in failure. You might replace “feeling better” with an objective definition such as “the subject experiences no pain for a week.” Or you might record the actual dosage of medication required to control pain. If you are interested in smoking, you need a lot of information to describe it. What does each of the subjects smoke — a pipe, cigars, or cigarettes? How much tobacco do the subjects use in a day? How long have they been smoking? Has the number of cigarettes (or cigars or pipes) that they smoke changed?

On the other hand, you must balance your scientific curiosity with the practical problems of obtaining information. If you must rely on people's memory, you cannot ask questions like "What did you have for dinner ten years ago?" You must ask questions that people will be able to answer accurately. If you are trying to show a relationship between diet and disease, for example, you cannot rely on people's memory of what they ate at individual meals. Instead, you have to be satisfied with overall patterns that people can recall. Some information is simply not available to you, however much you would like to have it. It is better to recognize this fact before you begin a study than when you get your questionnaires back and find that people were not able to answer your favorite question. If you think about your topic in advance, you can substitute a better question — one that will give you information you can use, even if it is not the information you wish you could have.

WHAT INFORMATION DO YOU NEED?

A critical step in the design of any study is the decision about what information you are going to record. Of course, you cannot record every possible piece of information about your subjects and their environment. Therefore, you should think hard about what information you will try to get. If you accidentally forget to find out about an important characteristic of your subjects, you may be unable to make sense of the patterns you find in your data. When in doubt, it is usually better to record more information than less. It is easy to leave unnecessary variables out of your data analysis, but it is often difficult (and expensive) to go back and gather additional information. For example, if you are studying what types of people are likely to buy a high-priced new product, you may not be able to adequately compare buyers with nonbuyers if you forget to include information about income.

DEFINING A POPULATION

When you conduct a study, you want your conclusions to be far-reaching. If you are a psychology student, you may want your results to apply to all laboratory rats, not just the ones in your lab. Similarly, if you are doing a market research survey on whether people in Los Angeles would buy disposable umbrellas, you may want to draw conclusions about everybody in the city. If you are an engineer and you are involved in the development of a particular product, you want to know what kind of a base or population the product is for. The people or objects about whom you want to draw conclusions are called a *population*.

One of the early steps in any study is nailing down exactly what you want your population to be. The more definite you are in defining populations, the better your understanding of samples and the results of your study will be.

Defining a population may seem straightforward, but often it is not. Suppose that you are a company personnel manager, and you want to study why people miss work. You probably want to draw conclusions only about employees in your particular company. Your population is well defined. However, if you are a graduate student writing a dissertation about the same topic, you face a much more complicated

problem. Do you want to draw conclusions about professionals, laborers, or clerical staff? About men or women? Which part of the world is of interest — a city, a country, or the world as a whole? No doubt, you (and your advisor) would be delighted if you could come up with an explanation for absenteeism that would apply to all sorts of workers in all sorts of places. You are not likely to come up with that kind of explanation, though. Even if you do, you are not likely to come up with the evidence to support it.

All kinds of people miss work because they are sick, but unlike others, the president of Major Corporation probably does not need to stay home waiting for a phone to be installed. The afternoons he takes off to play golf with his buddies are probably not recorded by the personnel office as absenteeism, either. People miss work for lots of reasons, and the reasons are quite different for different kinds of employees. Be realistic and study only a part of the labor force. Absenteeism among laborers in auto factories in Detroit, for example, is a problem with a well-defined population about which you would have a fighting chance to draw some interesting conclusions.

DESIGNING A STUDY

Even when the population of interest seems to be well defined, you may not actually be able to study it. If you are evaluating a new method for weight loss, you would ideally like to draw conclusions about how well it works for all overweight people. You cannot really study all overweight people, though, or even a group that is typical of all overweight people. People who do not want to lose weight or who have been disheartened by past efforts to reduce may not agree to try yet another method. You will probably be able to try out your new method only on people who want to lose weight and who have not given up trying. These people, not all overweight people, form your population.

Remember that a population defined realistically in this way may be different from the ideal population. For example, the population in your weight loss study may be lighter, younger, or healthier than the ideal population of all overweight people. Therefore, your conclusions from studying people who want to lose weight do not necessarily apply to people who are not motivated. For example, the treatment may have some unpleasant consequences, such as making people want to chew on the nearest thing available, such as gum, a pencil, or the corner of a desk. People who really want to lose weight may be willing to put up with such minor inconveniences in order to reach their goal. People who do not care much about their weight probably will not be. Thus, the new treatment may work quite differently for those who are motivated versus those who are not.

SAMPLING

Although you may want to draw conclusions about all rats or all residents of Los Angeles, you certainly do not want to have to train all of the world's rats or personally visit every Los Angeles home. What you want to do is to study some rats or some

people, draw conclusions based on what you have observed for them, and have the conclusions apply to the population in which you are really interested.

The rats or people (or other creatures or objects) that you actually observe in your study are called the *sample*. You can select a sample from a particular population in countless ways. How you do it is very important because if you do not do it correctly, you will not be able to draw conclusions about your population. That is a pretty serious shortcoming. For the most part, interesting studies are those that allow you to draw conclusions about a much larger group of subjects than that actually included in the sample.

RANDOM SAMPLES

What is a good sample? A sample is supposed to let you draw conclusions about the population from which it is taken. Therefore, a good sample is one that is similar to the population you are studying. But you should not go out and just look for animals, vegetables, or minerals that you think are “typical” of your population. With that kind of a sample (a judgment sample), the reliability of the conclusions you draw depends on how good your judgment was in selecting the sample — and you cannot assess the selection scientifically. If you want to back up your research judgments with statistics (one of the reasons, I hope, why you are reading this book), you need a *random sample*. Statisticians have studied the behavior of random samples thoroughly. As you will learn in later chapters, the very fact that a sample is random means that you can determine what conclusions about the population you can reasonably draw from the sample.

So what is a random sample, if it is so important? It is a sample that gives every member of the population (animal, vegetable, mineral, or whatever) a fair chance of selection. Everyone or everything in the population has the same chance. No particular type of creature or thing is systematically *excluded* from the study, and no particular type is more likely than any other to be *included*. Also, each unit is selected independently: including one particular unit does not affect the chance of including another.

If you are interested in the opinions of all the adults in Los Angeles, do not rely on a door-to-door poll in mid-afternoon or ask questions of people as they leave church services on a rainy Sunday. Such samples exclude many of the types of people you want to draw conclusions about. People who have jobs are usually not home on weekday afternoons, so their opinions would not be included in your results. Similarly, people standing in the rain may express different opinions (especially about umbrellas, for example) than they would if they were warm and dry. Polling in the rain would lead you to a bad guess about the proportion of the city’s residents interested in your new product (disposable umbrellas). To make things worse, you cannot tell what the effects of excluding dry people will be. You cannot tell whether your observed results are biased one way or another, and you cannot tell by how much. You might even be on target, but you do not know that, either.

From any particular random sample, of course, the results are not exactly the same as the results you would get if you included the entire population. Later chapters will show you how statistical methods take into account the fact that different

samples lead to somewhat different results. You will then understand how much you can say about a population from the results you observe in a sample.

VOLUNTEERS

To make it easier to have people participate in your study, you may be tempted to rely on volunteers. But you should not rely on any special types of people, and volunteers are one of those special types. Many studies have shown that people who volunteer differ in important ways from those who do not.

By the same token, if you are interested in testing a particular product, you should not base your decisions only on bad samples just because they have failed. You do not know enough yet about the causes of the failure or the conditions under which it occurred. Conversely, you do not test only good samples because they have no failures. In both cases the results will be erroneous.

USING SURVEYS

Generally, there are two major categories of studies: (1) surveys and (2) experiments. Other categories of studies also exist, but these two are predominant. The two types of studies differ in important ways.

In a survey, one records information. You ask people questions and record their answers, or you take some kind of a measurement. The important thing is that the experimenter does not actually do anything to the subjects or objects of the study. In fact, the experimenter tries very hard not to exert any influence whatsoever.

To conduct a good survey, the experimenter must phrase the questions so they do not suggest “correct” answers. In the case of surveying products, the experimenter must be conscious of their location, category, and so on, so that a general profile may be reconstructed with the results obtained and not by limited selection or discrimination of the product.

The great advantage to conducting your own survey is that you can tailor it for your own research project. You can ask the questions you want to ask in the way you want to ask them. You can choose the exact population that you want to study and select just the kind of sample you need. You can control the training of interviewers, and you can deal with all of the problems that come up during the actual survey. In short, you can do everything possible to make sure the survey will help you answer your specific questions of interest.

Doing all of these things takes a great deal of time and often a great deal of money. If you are going to invest a lot of time and money in a study, you owe it to yourself to get expert advice. Show your plans to someone who has actually carried out similar surveys, and ask for advice — *before* you take any big steps such as printing the questionnaires. If in doubt, consult a statistician or a book on data analysis.

ANALYZING AN EXISTING SURVEY

Without a doubt, the best way to get survey data is to design and carry out a survey focused on precisely the research questions you want to study. Realistically, though,

you often have to settle for “re-using” a survey that somebody else has carried out. Using data from a survey that was not designed for your study is often called *secondary analysis* to distinguish it from the *primary analysis* that was the purpose of the original survey.

Secondary analysis lets you do research that you could not otherwise do all on your own. But you must keep in mind that the data were not collected specifically for your purposes. The survey questions may not have measured exactly what you wanted them to, but you are stuck with them nonetheless. Remember to interpret them as they were asked, not as you wish they had been asked.

When you plan to use existing data, you do not have to worry about the thousands of details that go into conducting a survey. Instead, you have to make sure that the survey was carried out properly in the first place. Was it conducted by a reputable organization? Were the questions well phrased? Was the sample well chosen? Were the forms carefully processed? Most important, have you formulated research questions that you can reasonably hope to answer with the existing data?

DESIGNING EXPERIMENTS

Unlike a survey, an experiment involves actually doing something to the subjects or objects rather than just soliciting answers to questions or making measurements. For example, instead of asking people whether they think that vitamin C is effective for preventing colds, you might give them vitamin C and observe how many colds they develop. Or you may want to try product A and product B and then compare the results to see which one is better. Sometimes you study the subjects before and after your experimental treatment. Sometimes, instead, you take several groups of subjects, do something different to each of the groups, and then compare the results.

Experimentation on people poses ethical questions that deserve careful thought. Many responsible institutions have committees that regulate experiments involving human subjects. If an experiment exposes a subject to risks, such as possible side effects from a new drug, you must certainly inform the subjects in advance. Usually you must have them sign forms to give their consent. Needless to say, that is not a concern when you test products — even though the test may be a destructive one.

In experiments as well as surveys, the subjects must come from the population that you are interested in. (As you have probably gathered by now, proper sampling is much easier with animals, processes, or products in a laboratory setting than with people in a survey or products in a real world application.) When you design an experiment, you need to fret about some other things as well. For example, to compare different treatments or techniques, you must make sure that the groups receiving them are as similar as possible. Again, randomness is the key. The best way to make groups similar is to assign subjects or objects to the groups randomly. This procedure does not guarantee that the groups will be exactly the same, but it does increase the likelihood.

RANDOM ASSIGNMENT

Random does not mean “any old way.” You cannot assign subjects or objects to groups according to whatever strikes your fancy or let others make the assignment

TABLE I.1
A Limited Table of Random Numbers

| | | | | | |
|------|------|------|------|------|----------|
| 8588 | 5171 | 0775 | 7818 | 8683 | 3168 ... |
| 7185 | 8645 | 1537 | 3754 | 0201 | 2450 ... |
| 1053 | 9728 | 3028 | 8725 | 4855 | 0218 ... |
| 7517 | 0826 | 7257 | 5527 | 2668 | 8157 ... |
| 3551 | 3316 | 3584 | 9439 | 0011 | 7365 ... |
| ⋮ | | | | | |
| 7405 | 7764 | 6131 | 6204 | 8835 | 0345 ... |

decisions for you. Randomness requires a very specific, systematic approach to minimize the chance of distortion of groups due to the inclusion of disproportionate numbers of particular types of individuals or products.

If you allowed teachers to select which of their students receive personal computers, for example, they might select well-behaved students to reward them for past efforts. These students may be more intelligent or more diligent than the students who do not get to use the special equipment. Any evaluation of the effect of personal computers would be tainted by the differences between the selected students and the population as a whole.

Or consider this example: An engineer is trying to study customers' perceptions of the effect of adjustable brakes in vehicles. The results would be very different if the sample was based only on individuals with a height of more than 5 feet 11 inches, rather than a random sample of drivers of different heights.

A good way to assign people, animals, or objects to groups is to use a table of random numbers. You cannot just make up a table of numbers that you think are random. You are likely to have certain number biases. Unlike experimenters, random number tables do not have birthdays, license plates, children, or any other reasons to prefer one number over another. In a properly constructed table of random numbers, every number from zero to nine has the same chance of appearing in any position in the table.

Table I.1 shows a small random number table. The table has the numbers grouped into fours, but the grouping is just for convenience. It has no other significance. To randomly assign subjects to groups, you start at an arbitrary place in the table and assign the digit at that place to the first subject or object. Each new subject gets a digit from successive places in the table. If you start at the fourth digit of the first vertical group in the fourth position in Table I.1, for example, and then proceed to the right, the first subject gets the number 7, the next subject the number 0, and the next subject the number 8. (These three digits have been printed in bold type.) Since everything is random, it really does not matter whether you read the table across or down. However, once you have selected a starting point, stay in sequence. Using the table in this systematic way prevents you from choosing "favorite" numbers as starting points or as the next numbers in the sequence. You can never be too careful when you are trying to be random.

You use the numbers you assigned to the subjects to assign them to experimental groups. For example, if you have two groups, you can assign subjects with even

numbers to one group and subjects with odd numbers to another group. This procedure should result in about the same number of subjects in the two groups. But if you want the groups to be exactly equal in size, you can assign two- or three-digit random numbers to each of the subjects. Then arrange the numbers in order, from smallest to largest. Subjects with numbers in the lower half go to one group, and subjects with numbers in the upper half go to the other. You can use all sorts of systems with a random number table to assign subjects to groups, even in very complicated experimental designs. It is customary nowadays to use a computer generator program to generate random numbers.

Does randomness really matter? Yes, it does. Unless you use a procedure that assigns your subjects randomly, the results of your study may be difficult or impossible to interpret. Many assignment schemes that appear random to the inexperienced investigator turn out to have hidden flaws. For example, on one occasion, researchers at a hospital compared two treatments for a particular disease. Patients who were admitted on even-numbered days received one treatment, and those admitted on odd-numbered days received the other. That assignment sounds random enough, but it failed. The number of patients admitted with the disease on even days gradually became larger than the number admitted on odd days. Why? What happened is that some of the physicians figured out the scheme and made it a point to admit their patients on days when the procedure they preferred was in use. A bias such as this makes it possible for the patients admitted on even and odd days to be quite different. You cannot rely on the results of a study that used nonrandom assignment.

“BLIND” EXPERIMENTS

In experiments, as in surveys, you must not bias your observations or treatments with your own opinions or preconceptions about which group or treatment should yield better results. Some events, of course, are not disputable, such as the fact that a rat has died. However, when making observations that are not as clear-cut, such as assessing the happiness of a person’s marriage, it is all too easy to let unreliable judgment creep in — even though you are trying to be objective and “scientific.”

Not only you as an experimenter but also your subjects (especially if they are humans) can influence the outcome of an experiment without even trying. An example of a biasing influence is the *placebo effect*, a well-known effect in medical research. A placebo (such as a brightly colored pill that has no real effect) and a pep talk from a sympathetic physician are enough to cure many ailments. In an experiment on alertness, for example, if students believe that the vitamin supplements they get with their math lessons are intended to make them less sleepy during class, they may actually feel more alert (or more drowsy if they have a bias against the experiment’s success). In an experiment on anxiety, if the patients believe that the pill they are getting contains a drug with a powerful relaxing effect, they will feel more tranquil than if they believe that they are just getting breath mints.

The placebo effect can occur in many kinds of experiments, not just in medical research. To avoid the effect, you should prevent subjects from knowing which experimental group they are in, and you should not tell them anything about the expected results. Keep them “blind” as much as possible. Ethical considerations