

DUANE T. MCRUER  
DUNSTAN GRAHAM  
IRVING ASHKENAS

# Aircraft Dynamics and Automatic Control



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*AIRCRAFT  
DYNAMICS AND  
AUTOMATIC  
CONTROL*

DUANE McRUER  
IRVING ASHKENAS  
DUNSTAN GRAHAM

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## *PREFACE*

Flight control is a systems discipline that brings together the component dynamic characteristics of aircraft and flight controllers to form the system dynamic characteristics of the vehicle under the action of feedback control. Unfortunately, it has seemed to us that, by and large, the texts, monographs, and courses of instruction that treat these topics have tended to emphasize their disparities. There is certainly no lack of books on aircraft stability and control nor on feedback control systems. Our conviction, however, is that there is a field that comprises both of these subjects and that an understanding of either one can help to illuminate the other.

The purpose of this book is to present an integrated, analytical treatment of the dynamics of the vehicle (the controlled element) and its flight control systems. The book has been written by and for engineers concerned with the analysis of aircraft dynamics and the synthesis of aircraft flight controls. Such studies are at least as old as powered flight itself and they seem likely to remain pertinent as long as there are new and more advanced aeronautical vehicles.

Not long ago, the intellectual mathematical equipment of skilled stability and flight control analysts generally exceeded their physical ability to perform all the design and tradeoff calculations that might be needed or desired. Nowadays quite the opposite situation exists because advances in both analog and digital computation allow the consideration of problems that at one time would have been rejected as being too time-consuming. As a consequence, the analyst's physical means now often exceed his mental grasp; what he can compute may, possibly, far exceed his understanding or appreciation. This can lead to an excessively empirical approach to design which is similar to the one used by "practical" designers 30 or more years ago. Then airplane stability and control properties were evaluated only in flight test, and flight control equipment was also "designed" with the aid of extensive full scale testing. A difference, of course, lies in the abstractions involved, for, regardless of the detail and complexity of our mathematical models, they remain just that; whereas the physical equipment and the aircraft are the objects of our abstractions. Viewed in these terms, too great a reliance on a numerical-empirical approach to design is no better and may even be worse than

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the physical empiricism of earlier days. Inundated by computer printouts and strip chart recordings, the analyst is confronted with a crucial problem—what is the essence, what does it all mean?

For this reason we have strongly emphasized an analytical approach to flight control system design and have summarized an eclectic collection of efficient, neatly interconnected techniques that *inherently* and readily display the essential aspect of complex system problems. When skillfully applied, either with pencil and paper or using computer aids, these techniques enable one to attain a high level of insight and physical understanding with a minimum of effort. They are suitable for the establishment of nominal system designs, for the forecast of off-nominal problems, and for the diagnosis of the root causes of the problems that almost inevitably occur in the course of the design process.

While we have tried to be as definitive as possible on the subject of aircraft and flight control system dynamics and the procedures that are employed to accomplish automatic flight control system designs, the scope of our work has had to be limited to keep within the confines of one volume. The necessary limitation has been accomplished primarily by considering the aircraft only as a rigid body, and by almost exclusively emphasizing the theory of linear constant coefficient systems. The decisions on both these limitations were made somewhat reluctantly, since the flexible airframe and nonlinear features of flight control are always fascinating academically and are often important practically. We should hasten to remark, however, that regardless of the number of modes or nonlinearity of a problem, linearized solutions to comparatively low order problems almost always give reasonable approximate answers. They provide, as it were, a most useful species of limiting case solution, and limiting cases are, in general, the basis for much of our physical understanding of complex phenomena. With a solid grounding in linear theory, the extension of the results to nonlinear problems, especially of stability, is ordinarily rewarding and effective. Thus linear theory is, very generally, a theory of a first approximation which has the great virtue that it can be conceptually assimilated in its entirety. Further, as a practical matter, it is our observation that the great majority of the physical problems of aircraft flight control which are susceptible to mathematical treatment are, in fact, handled to a very good first approximation by linear treatments.

This book has a genealogy. Its immediate predecessors are the series of Bureau of Aeronautics volumes prepared in the early 1950s at Northrop Aircraft. The considerable success and the reputation of these volumes in industrial design departments, government laboratories, and in engineering schools prompted the original intent of the Naval Air Systems Command in sponsoring a large portion of the present work (Contract NOw 62-0781-c), so as to provide revisions and an updating for two of those volumes:

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“Dynamics of the Airframe,” Bureau of Aeronautics Report AE-61-4II, September 1952; “Automatic Flight Control Systems for Piloted Aircraft,” Bureau of Aeronautics Report AE-61-4VI, April 1956. Although some of the numerical data and examples from these earlier volumes have been used here, we present an essentially new effort rather than a revision. Furthermore, in order to provide an integrated treatment, we have included material that partially revises the first of the Bureau of Aeronautics volumes: “Methods of Analysis and Synthesis of Piloted Aircraft Flight Control Systems,” BuAer Report AE-61-4I, March 1952. Consequently, this book will, in the main, supersede the above three volumes of the Bureau of Aeronautics series.

We are indebted to many people and organizations for their assistance in the preparation of this book. First and foremost is the Naval Air Systems Command (NASC), which sponsored the preparation of much of the manuscript. The NASC project monitor, Jack Crowder, was an ideal supporter, continually interested and anxious to get the job done, yet patient and understanding, in spite of the inevitable delays that projects of this sort seem to incur. We also owe major debts to our colleagues, at Systems Technology and elsewhere, who have critically reviewed various versions and portions of the manuscript and have offered constructive criticisms and suggestions for its improvement. First in this group is Robert L. Stapleford of Systems Technology, who has gone through the book several times, exercising his penchant for clarity and his keen eye for error. Robert J. Woodcock of the Air Force Flight Dynamics Laboratory, who thoroughly reviewed several chapters, was also a great help. H. R. Hopkins of the Royal Aircraft Establishment, Farnborough, reviewed Chapter 1, making many helpful suggestions and very graciously offered us the use of his own extensive material on the history of flight control. Dr. Malcolm J. Abzug of Thompson-Ramo-Wooldridge also made a number of correcting and clarifying remarks related to the history presented in Chapter 1, for which we are very grateful; Ronald O. Anderson of the Air Force Flight Dynamics Laboratory made available to us his bibliography on the history of feedback controls. Gary Teper of Systems Technology was responsible for the collection and presentation of the data contained in Appendix A. Particular acknowledgment is further due to the publication staff of Systems Technology, who labored long and hard to prepare the manuscript for publication, and especially to their Publications Manager, Junichi Taira, whose meticulous attention to every detail is revealed on each page of the book.

Besides those who helped directly, there are others in the background. Most important, of course, are our many colleagues in the flight control and automatic control community whose original work is reflected here. We have tried to acknowledge them throughout the book with pertinent

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references to the published literature. As is evident from these footnotes, a great deal of the work summarized here was originally accomplished for the Control Criteria Branch of the Air Force Flight Dynamics Laboratory. In fact, some of the material appearing here for the first time is based on unpublished notes prepared in the course of work sponsored by the United States Air Force. We must also acknowledge our former colleagues at Northrop Aircraft, Warren Koerner and Robert E. Trudel, who were among the authors of the old BuAer "Dynamics of the Airframe" volume on which parts of Chapters 4, 5, and 6 are based. Finally, we wish to acknowledge our present or past Systems Technology co-workers, J. J. Best, T. S. Durand, D. E. Johnston, H. R. Jex, W. A. Johnson, L. G. Hofmann, J. D. McDonnell, R. A. Peters, R. J. Wasicko, D. H. Weir, and J. Wolkovitch, for their several original contributions to the material presented in the following pages.

The merits that this book may possess can, in large part, be attributed to all of these people. Any faults are the responsibility of the authors. We hope that this work will prove both instructive and useful to others who, like ourselves, wish to help solve the flight control system design problems of future generations of aircraft.

Duane McRuer  
*Hawthorne, California*

Irving Ashkenas  
*Hawthorne, California*

Dunstan Graham  
*Princeton, New Jersey*

February 1971

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*AIRCRAFT DYNAMICS AND  
AUTOMATIC CONTROL*



## CHAPTER 1

# INTRODUCTION AND ANTECEDENTS

We now know a method of mounting into the air, and, I think, are not likely to know more. The vehicles can serve no use till we can guide them; and they can gratify no curiosity till we mount with them to greater heights than we can reach without; till we rise above the tops of the highest mountains.<sup>1</sup>

The economic or military value of any vehicle depends fundamentally on its ability to traverse a controllable path between its point of departure and its destination or "target." Abstractly, the vehicle is a velocity vector in space. It has a direction in which it is going and a speed with which it is going there. The time integral of the velocity vector is the path. Each type of vehicle, however, is made to move and carry in a certain medium and its motions may be subject to constraints. Means for the control of the path vary widely and depend on the constraints. Thus a train, for example, is constrained to move along a track; the control that is provided is merely a speed control. The train is not steered. An automobile or a ship, on the other hand, while constrained to move on the surface of the land or the sea, must be steered as well. Aircraft share with submarines and torpedos an unusual freedom from constraints, and the problems of the control of aircraft are of unusual complexity. We do indeed "know a method of mounting into the air," but the solution of the problems of control still requires both sensibility and diligence.

An aeronautical vehicle or weapon system contains spatial sensors, and guidance and control devices (possibly all subsumed in the human pilot) whose purpose it is to develop three-dimensional flight path commands appropriate to steering so as to reach a destination or target, and then to execute those commands by maintaining or modifying the forces on the vehicle so as to maintain or modify the velocity vector. This allows an intended purpose or "mission" to be accomplished.

Qualities of an aircraft that tend to make it resist changes in the direction or magnitude of its velocity vector are referred to as *stability*, while the ease and expedition with which the vector may be altered are referred

<sup>1</sup> Samuel Johnson, "A Dissertation on the Art of Flying," Chapter 6 in *History of Rasselas*, originally published in 1759, republished by Clarendon Press, Oxford, 1931.

to as the qualities of *control*. Stability makes a steady unaccelerated flight path possible; maneuvers are made with control. The path of an aircraft, however, is never stable of itself; whether through the intervention of the human pilot or by means of automatic control, stability is actually secured with the mechanism of *feedback*, a principle by which cause and effect systems are modified to secure certain desirable properties. Information about the effect (or output) is fed back (or returned) to the input and is used to modify the cause. Typical of feedback control is its speed of response and its accuracy in following commands and in suppressing the effects of disturbances. Also typical, however, is its tendency to "hunt" or oscillate. The particular advantages of feedback are enhanced by high gain, but this is inimical to dynamic stability, and high gain also increases the susceptibility of the system to spurious signals or "noise." Therefore, a designer intending to exploit the potential advantages of feedback is compelled to strike a fine balance between the desirable properties that might be secured and the pressing danger of disastrous performance.

The earliest aeronautical experimenters had hoped to achieve "inherent" stability (i.e., without feedback), and while many, such as Cayley, Penaud, Lilienthal, Chanute, and Langley, pursued this goal and discovered how to set the incidence of the tailplane so as to achieve longitudinal stability with respect to the relative wind, and to use wing dihedral so as to achieve "lateral stability," it gradually became clear that configurations with a large amount of such inherent stability were particularly, and distressingly, susceptible to being upset by gusts.

Speaking before the Western Society of Engineers in 1901, Wilbur Wright said: "Men already know how to construct wings or aeroplanes, which when driven through the air at sufficient speed, will not only sustain the weight of the wings themselves, but also that of the engine, and of the engineer as well. Men also know how to build engines and screws of sufficient lightness and power to drive these planes at sustaining speed . . . . Inability to balance and steer still confronts students of the flying problem . . . . When this one feature has been worked out, the age of flying machines will have arrived, for all other difficulties are of minor importance."<sup>2</sup>

While this statement was somewhat optimistic with respect to the state of knowledge concerning airfoils and propellers, as the Wright brothers themselves soon discovered, it was correct in its essentials, and there is no doubt at all that suitable stability and control characteristics were the very last features of the first successful airplane to be developed. It is now generally agreed that the principal contribution of Wilbur and

<sup>2</sup> M. W. McFarland, ed., *The Papers of Wilbur and Orville Wright*, Vol. 1, McGraw-Hill, New York, 1953, pp. 99-100.

## INTRODUCTION AND ANTECEDENTS

Orville Wright was their recognition that the frustrating search for inherent stability might well be abandoned if only the operator were provided with sufficiently powerful controls with which to balance and steer, i.e., that the human pilot, operating on feedback signals, could use the controls to stabilize a neutrally stable or an inherently *unstable* aircraft.<sup>3</sup> Of course, the Wright brothers did not use this language, and indeed the recognition of the essential character of the airplane as an element in a feedback control loop came comparatively recently.

While the first *automatic* feedback control system for an airplane actually antedated the first successful flight by more than a decade, and the demonstration of completely automatic control of an airplane in full flight took place more than 50 years ago—in 1914, the means employed to secure satisfactory flying qualities of the aircraft themselves and to develop artificial stabilizers and automatic pilots were, at first, largely empirical arts. They seem to have made progress with a minimum amount of mathematics until after the end of the 1939–1945 war.

The modern view of the dynamics of aircraft and their control systems, in terms of the stability and response of the entire closed-loop (feedback) system, can be traced from its sources by way of three separate branches of technical knowledge, their confluence, and the recent advance and augmentation of the subject (see Fig. 1-1). During roughly the first 50 years of aviation's history, the study of the dynamics of aircraft and their control systems was of negligible interest to designers, who learned to get by with rules of thumb for proportioning the stabilizing and control surfaces and to develop automatic feedback controls by cut-and-try methods. This was in spite of the fact that a mathematical theory for the stability of the unattended motion and of the aircraft's response to control was developed at an early date. On the other hand, design trends since World War II, which have greatly extended the flight envelope of fixed-wing airplanes and introduced new types of vehicles such as helicopters, VTOL airplanes, ground effect machines, hydrofoil boats, winged missiles, and space launchers, have so enormously multiplied the number and type of problems that the techniques formerly employed in practice would have been totally inadequate. Very fortunately, wartime pressures produced two developments that fundamentally altered techniques for the design of automatic flight control systems. The first of these was the theory of servomechanisms; the second was the electronic computer. Analysis and simulation are today the twin pillars on which the entablature of aircraft flight control system design stands.

There has been an explosive growth in the practice of "experimenting" with mathematical models. It has been urged by both the expanding complexity of the problems and the increasing availability of appropriate

<sup>3</sup> C. S. Draper, "Flight Control," *J. Roy. Aeron. Soc.*, 59, 451–477 (July 1955).

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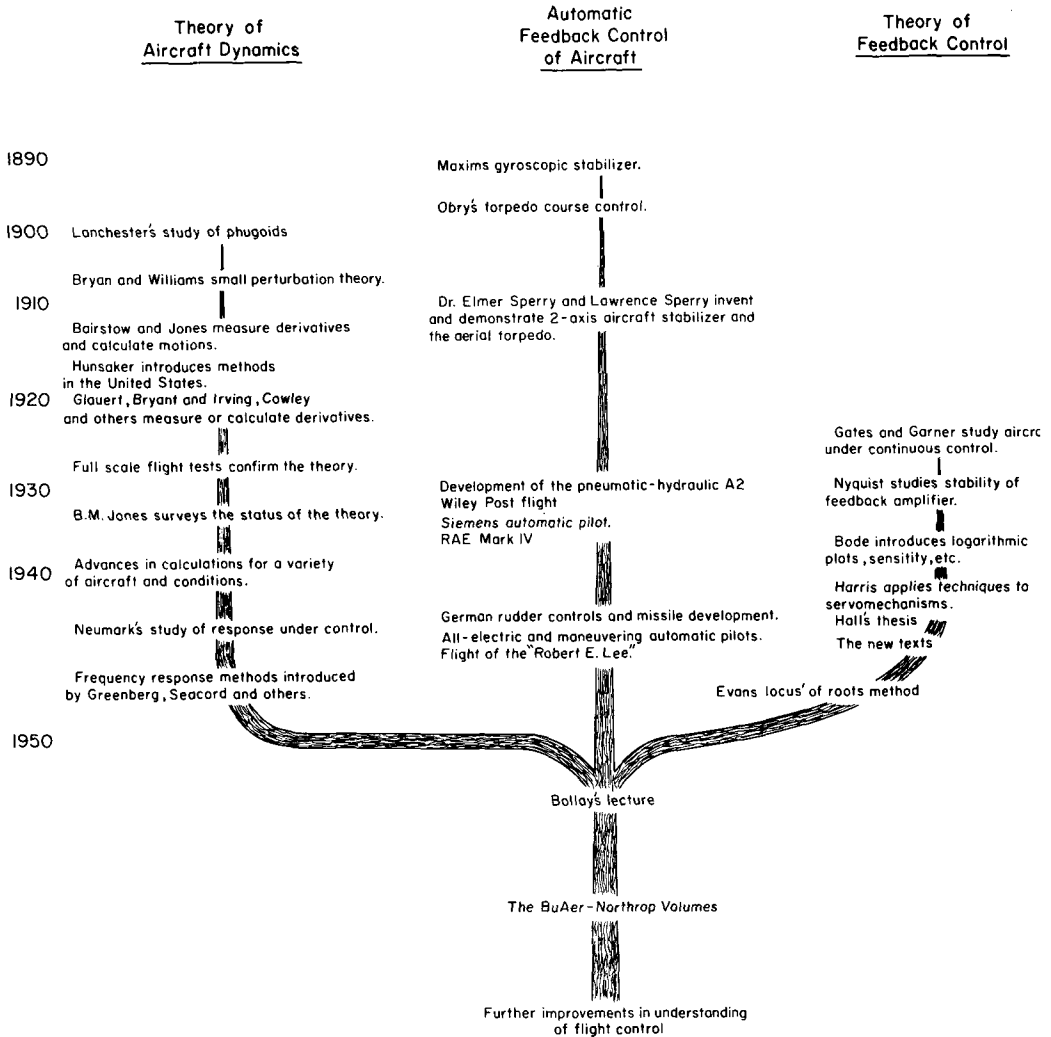


Fig. 1-1. Confluence and augmentation of the theory and practice of automatic feedback control of aircraft.

methods and techniques. Further, the mathematical theory has served for the classification, interpretation, and extrapolation of the growing number of results of physical experiments.

It is to the development, exposition, and demonstration of methods of analysis and synthesis for aircraft automatic flight control systems that this monograph is addressed. It is not a text on design but is rather a guide to the consideration of the effects of vehicle and equipment features on the dynamic performance of the system. Where possible, the emphasis in treating the elements of the system is on the largest entities. Thus attention is directed to the response of the airplane to elevator motion, rather than to the change in airflow over the tail, and to the input/output characteristics of a rate gyro, rather than to detailed consideration of the torques acting on the gimbal. The vehicles considered are the ones that are heavier than the fluid in which they operate but which are acted on by significant fluid dynamical forces. This class includes at least the following types of vehicles:

Airplanes  
Helicopters  
Vertical takeoff and landing aircraft  
Ground effect machines  
Hydrofoil boats

Control, as somewhat distinct from guidance, is taken to be the subject of interest. For this reason it will ordinarily be possible to consider the motions in moving coordinate systems fixed in the vehicle and to avoid the coordinate axis transformations required to obtain the vehicle motion in, for example, a coordinate system fixed in the earth. When the origin of the moving coordinate system is in an "equilibrium" state of motion along a nominal trajectory, the equations of motion of the vehicle can be linearized for small perturbations and the linearized equations will have constant coefficients. Then it is possible to use the convenient *transfer function* models for the dynamics of the vehicle, and all the analytical techniques for the study of linear feedback systems can be brought to bear on the problem.

Although there are a number of modern treatments of the stability and control of aircraft,<sup>4</sup> all of which emphasize the same approach to the

<sup>4</sup> Among the more recent texts are C. D. Perkins and R. E. Hage, *Airplane Performance, Stability, and Control*, Wiley, New York, 1949; W. J. Duncan, *Control and Stability of Aircraft*, Cambridge University Press, London and New York, 1952; B. Etkin, *Dynamics of Flight*, Wiley, New York, 1959; W. R. Kolk, *Modern Flight Dynamics*, Prentice-Hall, New York, 1961; A. W. Babister, *Aircraft Stability and Control*, Pergamon Press, New York, 1961; R. L. Halfman, *Dynamics*, Vol. 1, *Particles, Rigid Bodies, and Systems*, Addison-Wesley, Reading, Mass., 1962; E. Seckel, *Stability and Control of Airplanes and Helicopters*, Academic Press, New York, 1964.

linearized dynamics that is to be adopted here, and there is also a very wide selection of both introductory texts and more advanced treatises on automatic feedback control,<sup>5</sup> there has been a conspicuous lack of any significant treatment of these subjects in concert and therefore no proper introduction to the area between these fields. It is a fact that the methods of servomechanism analysis can be used as a powerful tool in the study of aircraft dynamics, and, additionally, that the characteristics of aircraft and their control systems provide a series of both subtle and complex problems that are likely to carry the student of feedback systems beyond what he may have learned in connection with the customary examples of remote position control, speed regulation, process control, and instrumentation. The discussion that follows will serve to bridge a gap between existing technical disciplines and to make more readily available some of the results contained in a scattered engineering report literature which is now familiar only to a small group of specialists.

The authors have adopted an eclectic view, taking from several fields what best appeals and suits but attempting, at the same time, to provide a unified treatment. Where a completely unified view is not feasible, the dominant theme is stated and the minor theme is contraposed.

It is the conviction of the authors that only the most thorough understanding of the dynamics of each element is a suitable basis for system synthesis. While digital and analog computers are now generally available to produce "solutions," even a sheaf of solutions may not clearly show the designer how to obtain the most satisfactory behavior and to avoid unpleasant surprises when the machinery is built. It is for this reason that the mathematical analysis of aircraft feedback control systems is emphasized throughout the treatment here. Of course, simulation and flight testing are valuable tools in the development of aircraft control systems, but, to an extent, a good theory is a summary of, and substitute for, experience, and the understanding which is conferred by analysis is a short-cut to the best results. It may seem, however, that a linearized theory is unrealistic because practical aircraft feedback control systems inevitably include nonlinear elements. The results that are achieved justify its use. Restrictions that are implicit in the use of linear theory are

<sup>5</sup> See, e.g., H. M. James, N. B. Nichols, and R. S. Phillips, *Theory of Servomechanisms*, McGraw-Hill, New York, 1947; H. S. Tsien, *Engineering Cybernetics*, McGraw-Hill, New York, 1954; J. G. Truxal, *Automatic Feedback Control System Synthesis*, McGraw-Hill, New York, 1955; H. Chestnut and R. W. Mayer, *Servomechanisms and Regulating System Design*, 2nd edn., Vol. 1, McGraw-Hill, New York, 1959; J. J. D'Azzo and C. H. Houpis, *Feedback Control System Analysis and Synthesis*, McGraw-Hill, New York, 1960; R. N. Clark, *Introduction to Automatic Control Systems*, Wiley, New York, 1962; E. C. Barbe, *Linear Control Systems*, International Textbook, Scranton, Pa., 1963; I. M. Horowitz, *Synthesis of Feedback Systems*, Academic Press, New York, 1963; C. J. Savant, Jr., *Control System Design*, McGraw-Hill, New York, 1964.

nowhere nearly as severe as might be imagined. In part, this is because linear approximations often have a substantial validity; in part it is so because feedback, in itself, tends to "linearize" the system.

Finally, it may or may not be true, as George Santayana said, that "those who cannot remember the past are condemned to repeat it," but there is enough truth there so that the history of the present subject can be studied with considerable profit. It is evident upon knowledgeable consideration that some costly mistakes might have been avoided with a better appreciation of the difficulties that confronted previous investigators of the problems of flight control.

### 1-1. Outline of the Volume: A Guide for the Reader

The subject of the feedback control of flight has a considerable scope and variety, and there is no canonical approach to its understanding. Its students will typically have acquired a considerable knowledge of the theory of linear feedback systems, and of the dynamic stability of aircraft and their response to control, as substantially independent subjects. The background of the typical reader will probably include some knowledge of operational or Laplace transform techniques for the solution of ordinary linear differential equations with constant coefficients, conventional servo analysis techniques such as the root locus and frequency response methods, response calculations with either deterministic or random inputs, and the describing function method for the treatment of common control system nonlinearities. While many of these matters are reviewed here before they are applied, the pace is brisk and the treatment is not intended as an introduction to the elements of the theory. The reader is further presumed to have some acquaintance with the dynamics of rigid bodies, although it is not, strictly speaking, necessary to have studied the dynamics of aircraft. Again, the latter subject is treated here *ab initio* but with a purpose not shared with the conventional texts cited in note 4.

Figure 1-2 is a graphical representation of the outline for this volume.

The book begins, in this first chapter, with a definition of control appropriate to aeronautical vehicles and a distinction between control and *guidance*. This is followed by a brief summary of the advantages of feedback for control and an introduction to some of the earliest examples of feedback control. Historical sketches of the development of aircraft dynamic stability and control, practical automatic flight control systems, and feedback system analysis complete the introduction.

Chapter 2 comprises a review of those aspects of applied mathematics pertinent to the construction and use of linear mathematical models of aircraft and their control systems. The Laplace transform method and

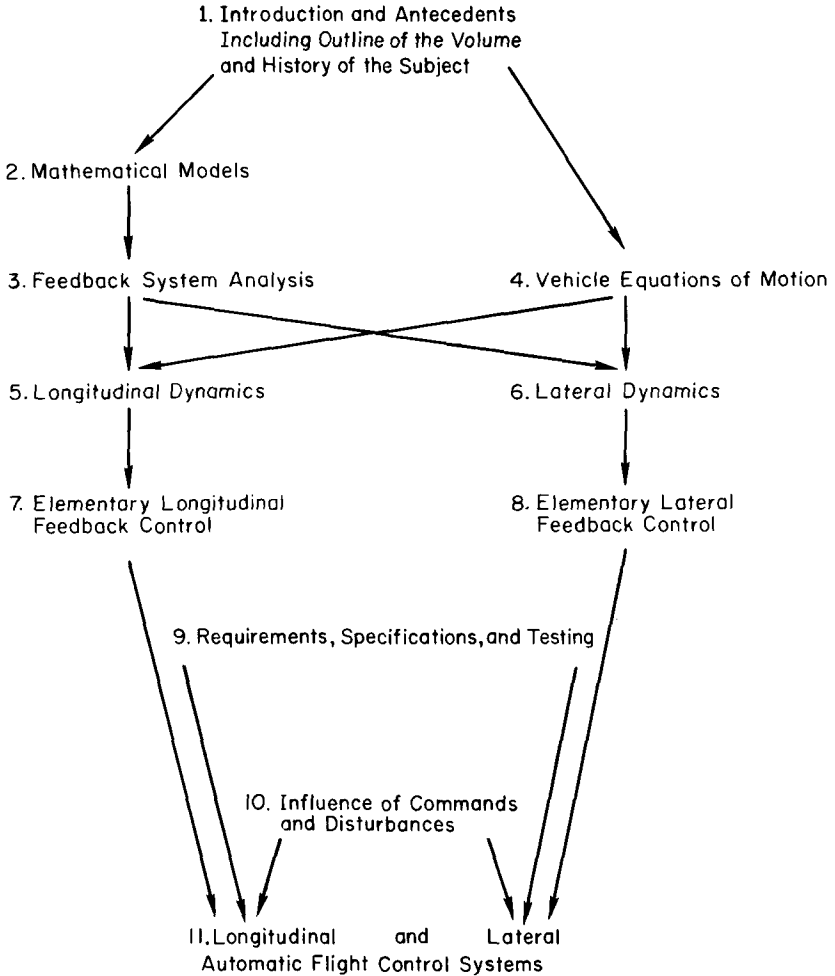


Fig. 1-2. Graphical outline of the volume showing interrelationship between topics.

the transfer function model, which play such a prominent part later, are discussed in detail, and considerable emphasis is placed on graphical representations and graphical constructions. While the typical reader is assumed already to have a considerable familiarity with this material so that he should be able to move ahead rapidly, he is likely to find that certain matters such as time vectors and the steady-state response to

polynomial inputs are treated here in a unique way that provides a background for subsequent developments.

The material of Chapter 3 is a condensed account of the particular topics in feedback system analysis on which the remainder of the monograph strongly depends. Here the reader will find not only a review of the root locus method and the conventional open-loop/closed-loop logarithmic frequency response methods but also their presentation as elements of a unified servanalysis method that is a complete generalization of the semigraphical analytical techniques. The reader will also find here an exposition of multiloop analysis procedures particularly appropriate to the study of vehicular control systems and, finally, a discussion of sensitivity, including the connection between gain sensitivity and the modal response coefficients (time vectors or eigenvectors) of the system response. This chapter is one of the most unusual features of the volume because many of the techniques, and especially their highly organized connections, are not explained in the conventional textbooks on linear feedback system analysis.

The main issue is discussed in Chapter 4, in which the equations of motion of aeronautical vehicles are developed from first principles. It is shown there that these equations can be linearized about a nominal flight path and that, when this flight path lies in the plane of symmetry of the unperturbed vehicle, the equations can normally be separated into two independent sets, the longitudinal equations and the lateral equations. This simplification is the basis for the division of the greater part of the balance of the discussion.

Still with the intention of studying the aircraft under active control, the longitudinal dynamics of the aircraft-alone are explored in Chapter 5. The transfer functions for the aircraft's response to control are evolved from the equations of motion, and *approximate factors* for the numerators and denominators are presented in terms of coefficients in the equations. Although approximate factors for parts of the characteristic functions (denominators) of airplanes have been known for about 40 years, it was only a few years ago that a similar understanding of the numerators was developed, and a similar approach to VTOL aircraft has only been successful even more recently. Here again, the presentation in Chapter 5 departs considerably from the conventional practice because little or no attention is paid to transfer function factors in the existing texts, and the developments summarized there represent a part of the novel approach of this volume.

Chapter 6 does for the lateral motions what Chapter 5 does for the longitudinal motions. The treatment is exactly parallel, although the results are different because of the distinction between the typical motions in the several degrees of freedom.

In Chapters 7 and 8 the discussion finally turns to the feedback control of airplanes and helicopters. The stability and response of vehicles under continuously active control are considered with the assumption of ideal proportional control, i.e., no account is yet taken of the practical imperfections such as lags, which inevitably are associated with real sensors, amplifiers, and actuators. The possible ideal feedback systems for control of the longitudinal motions are canvassed in Chapter 7, while a similar presentation on ideal feedback systems for control of the lateral motions is made in Chapter 8.

Chapter 9 is on the subject of general requirements, specifications, and testing. These subjects are presented in the context of a design process outline. Emphasis is placed on the sources of operational requirements and the logical evolution of the requirements from these origins. The requirements that derive from a consideration of flight control systems as feedback devices are also treated at length.

In Chapter 10 the effects of inputs and disturbances are treated as a performance consideration. Up to that point, the inputs to the system are considered to be relatively simple test signals such as an impulse or a sine wave. Now the influence on design of considering the structure of the inputs and disturbances is introduced for the first time. Actually, the inputs and disturbances are approximated by either deterministic signals more complicated than the ones previously considered, or, where their nature demands it, in probabilistic terms. The first probability density function and the second probability distribution function are reviewed; their use in system performance calculations is explained in some detail for the cases in which the signals have a Gaussian distribution and are stationary. In that case convenient calculations of the performance of linear systems can be carried out in the frequency domain by making use of the power and cross-spectral density functions. The "transient analog" and adjoint technique, which underlie the computer approach to more complex problems, are also introduced.

Finally, much of the material of all the previous chapters is used in discussions of longitudinal and lateral automatic flight control systems in Chapter 11. The influence of requirements and of imperfections in the components is particularly pointed out. Multiloop flight control systems of several types are treated as illustrative examples.

At the end of the book are two appendices and a bibliography. The bibliography supplements this book by providing references to those aspects of aircraft dynamics and automatic control that are not extensively treated here. It covers vehicle flexibility and other higher order dynamic effects, components, and descriptions of flight control systems. The first appendix presents tabulations of dynamic characteristics for some representative aircraft, and the second serves as a brief introduction to probability theory.

## 1-2. A Definition of Flight Control

It is not surprising that, when considered in detail, the abstract or physical attributes of an aeronautical vehicle or weapon system and its elements are so interrelated as almost to preclude discussion of any one aspect of the system without simultaneously treating most of the others. Still, it ultimately becomes necessary to stake out definite domains that can be treated more or less individually. This can be accomplished with some generality if other factors and entities in the system can be considered either precursory or *by definition* separated from the subject of special attention.

As a first step in separating the automatic flight control area from other aspects of the overall aeronautical vehicle or weapon system, it is necessary to distinguish control from guidance. Unfortunately, the boundary between these two areas is seldom inherently sharp because of basic functional, operational, and equipment interactions that they may share. As a practical matter, however, the following definitions can ordinarily be used:

- Guidance: The action of determining the course and speed, relative to some reference system, to be followed by a vehicle.
- Control: The development and application to a vehicle of appropriate forces and moments that
1. Establish some equilibrium state of vehicle motion (*operating point control*).
  2. Restore a disturbed vehicle to its equilibrium (operating point) state and/or regulate, within desired limits, its departure from operating point conditions (*stabilization*).

To apply these definitions to a specific example, consider the air to surface missile system shown in Fig. 1-3. In this figure the blocks inscribed with capital letters in square brackets are not simple transfer functions relating outputs to inputs, but instead are matrix operations. It is readily apparent that the complete system, when viewed in the large, is complicated and analytically intractable. However, two major types of loops are seen to be present: one a series of inner loops involving the feedback of airframe motion quantities; the other an outer loop containing the kinematic transformations required to generate the relative orientation between target and vehicle, and closed through a geometry sensor and computer that generates flight path commands. By use of the definitions given above, it is now possible to separate the guidance and the control areas, at least in terms of the matrix operators shown in the block diagram.

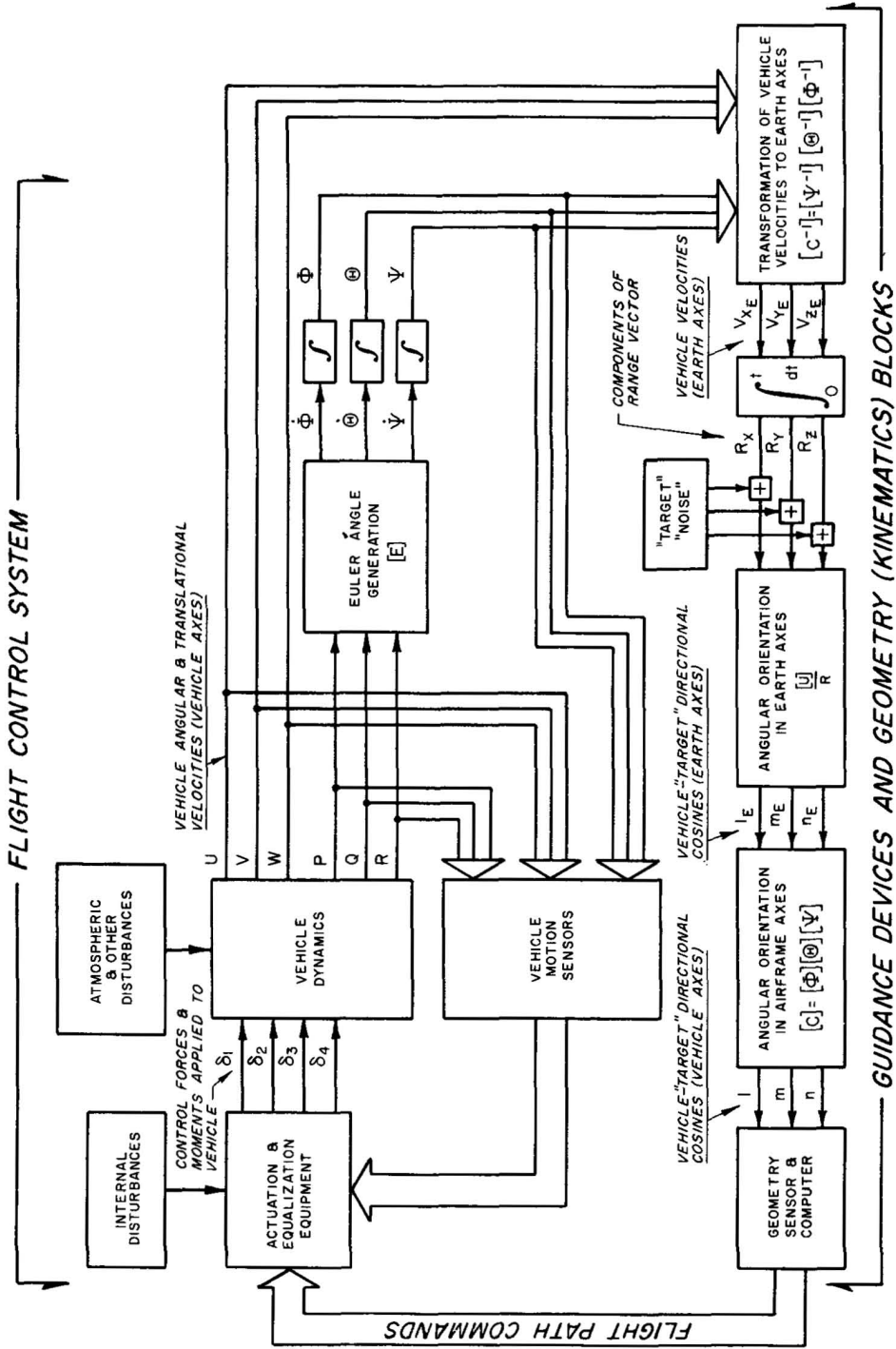


Fig. 1-3. Air to surface missile system block diagram.

Note, parenthetically, that an abstract or functional picture, rather than one drawn in terms of physical equipment, is preferred at this stage. If, e.g., in Fig. 1-3, the Euler angles,  $\Phi$ ,  $\Theta$ , and  $\Psi$ , used as measures of vehicle motion, were obtained from a stable platform, this equipment would have to be considered a part of the flight control system; yet to many people the very words "stable platform" imply an item of guidance equipment.

On a physical basis Fig. 1-3 makes apparent an important distinction between the two types of loops. The flight control loop is concerned only with vehicle motion quantities measured in the aircraft (although two reference axis systems are necessary), while the guidance loop involves axis system transformations that put the vehicle and target on comparable terms. For many systems this distinction is quite helpful in separating guidance from control. There is little doubt that the control of aircraft attitude angles is one of the functions of flight control, while the control of the *path* is, strictly speaking, a guidance function. Later it will become clear, however, that there are pseudo path variables such as pressure altitude and heading which are measured in the aircraft, and whose control, therefore, is logically considered to be a part of the domain of flight control. Further, it is often possible to formulate guidance problems such as terrain avoidance and approach to a runway on a localizer beam without involving more than linear approximations to the kinematic transformations in the guidance loop; then, with a single notable exception, guidance problems can be considered as minor extensions to the problem of flight control.

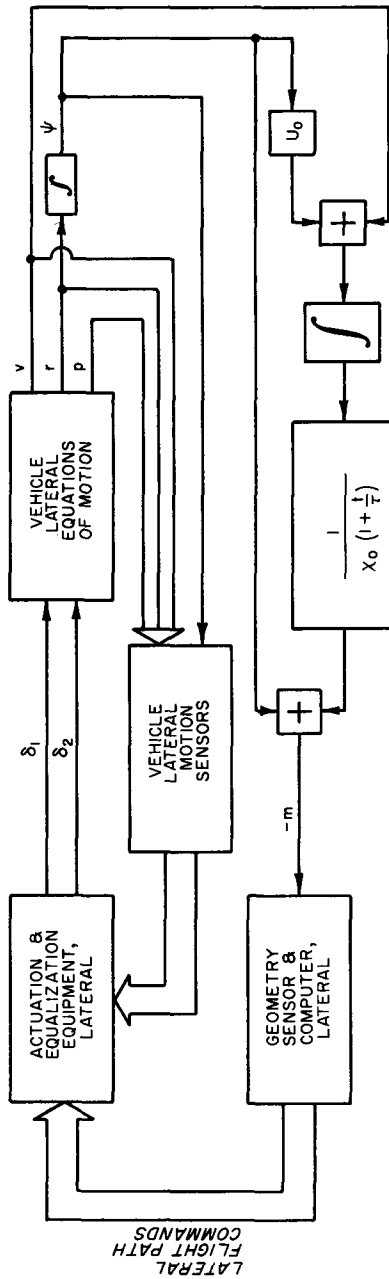
The exception is in those cases in which there are important dynamic interactions between the control and guidance loops. The complex diagram of Fig. 1-3 can be simplified by specifying the general type of guidance to be used and defining ideal steady-state "trajectories." The desired steady-state conditions can then be used as operating points, and all of the equations indicated by the block diagram of Fig. 1-3 can be *linearized* about these operating points. A simplified block diagram, emphasizing the system dynamics in a form suitable for dynamic analysis, can finally be drawn. Figure 1-4 shows linearized block diagrams (derived from Fig. 1-3) that relate perturbed quantities when the vehicle is on a straight line collision course with the target and is operating about straight and level flight condition.<sup>6</sup> Figure 1-5(a) results when the longitudinal control system block diagram is redrawn so as to use flight path angle,  $\gamma$ , instead of pitching velocity,  $q$ , and plunging velocity,  $w$ , as the motion

<sup>6</sup> While the implied assumption is surely a tremendously simplifying one, aeronautical vehicles do, in fact, spend most of their time in the air in straight and level flight, and the control system must be made to work for that flight condition first. The choice of operating point, however, is illustrative and is not necessary to the argument.

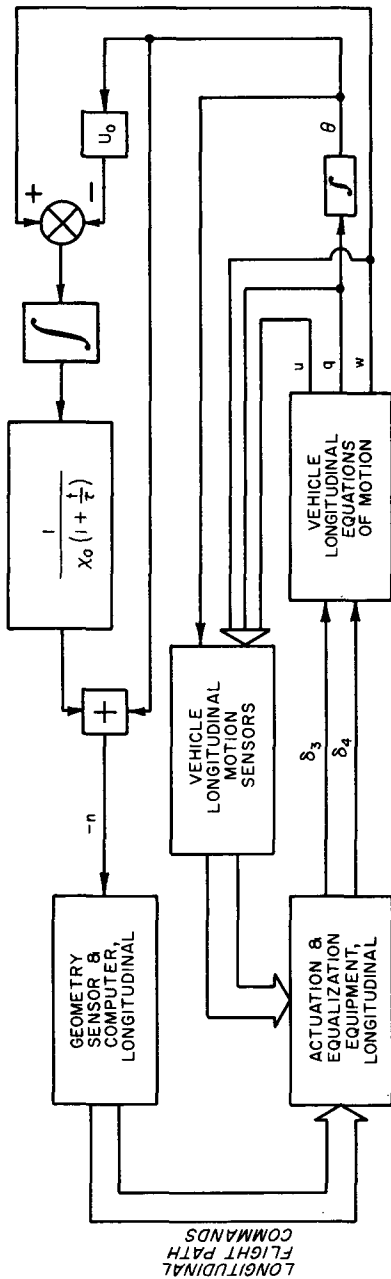
$$P_0 = Q_0 = R_0 = V_0 = W_0 = \dot{\Psi}_0 = R_{\dot{\Psi}_0} = \dot{\Phi}_0 = 0$$

NO LATERAL - LONGITUDINAL CONTROL CROSSFEEDS

### LATERAL FLIGHT CONTROL SYSTEM



### GUIDANCE DEVICES & GEOMETRY BLOCKS



### LONGITUDINAL FLIGHT CONTROL SYSTEMS

Fig. 1-4. Air to surface missile system linearized block diagram.

variables. Here the geometry relationships are shown in a single block, while the flight control system portion of the diagram is separated into functional divisions. Figure 1-5(b) goes one step further and shows a single closed-loop flight control system block with the geometry block broken into two parallel channels. Both diagrams in Fig. 1-5 assume unity dynamics for the geometry sensor and the computer.

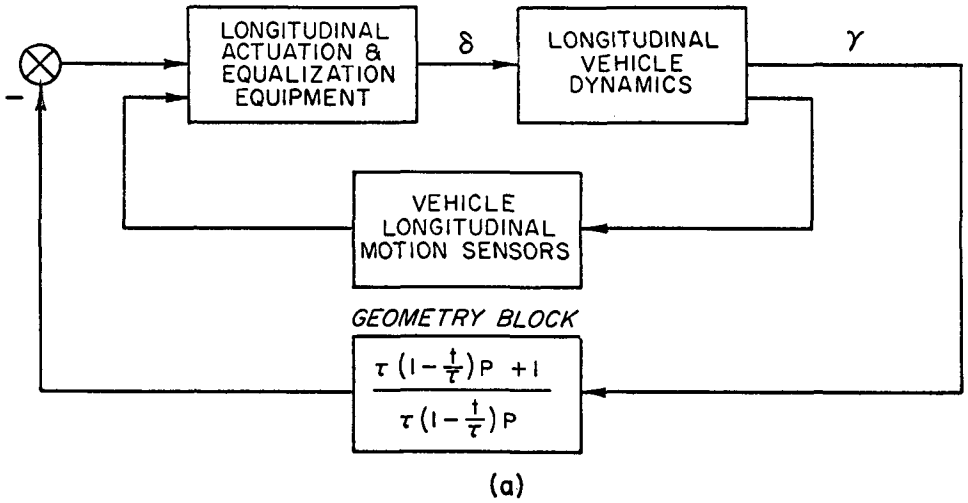
Figure 1-5 emphasizes the fact that the geometry block contains a time-varying parameter  $(1 - t/\tau)$ , where the time variable,  $t$ , appears explicitly. The magnitude of the parameter defines the relative degree of dynamic interaction between the flight control and the guidance. When the ratio time:time-to-go,  $t/\tau$ , is very small, the sole dynamic effect of the guidance elements is to add a unity feedback path to the closed-loop flight control system. In most cases this effect, while certainly worthy of consideration, does not complicate the problem. *It can easily be taken into account as just another loop in the flight control system.* On the other hand, as  $t/\tau$  approaches unity, the geometry block gain approaches infinity and the dynamic interactions between guidance and flight control may become the most prominent feature of system performance.

If the considerations exemplified in the discussions above are generalized for a variety of guidance system types, it is found that guidance and control can interact in only three ways. These are illustrated in Fig. 1-6 as connections between the two parts of a guidance and control dichotomy. The interaction with *operating point control*, as illustrated in the example above, is the starting point in the development of diagrams, such as the ones shown, which emphasize the dynamics. Guidance system effects on *limiting* involve the characteristics of particular physical mechanizations and, in any event, relate only to conditions outside the realm of linear theory. Finally, *dynamic interaction* between guidance and control occurs only in homing guidance, and even then the interaction is slight until the target is close. On these bases it should be clear that linear constant approximations to flight control and guidance systems can be treated completely separately, as far as their dynamics are concerned, for all guidance modes except final "homing" maneuvers. Therefore, if a particular operating point—straight and level flight—is assumed and a possible interest in homing maneuvers is neglected, the subject of control can be separated from guidance and the synthesis of automatic flight control systems can be studied in its own right.

### 1-3. Why Feedback?

The flight control systems in Figs. 1-3 to 1-5 are, quite apparently, shown as *feedback* systems, in which a portion of each output is fed back

FLIGHT CONTROL SYSTEM



Note:  $P$  is the operator,  $\frac{d}{dt}$

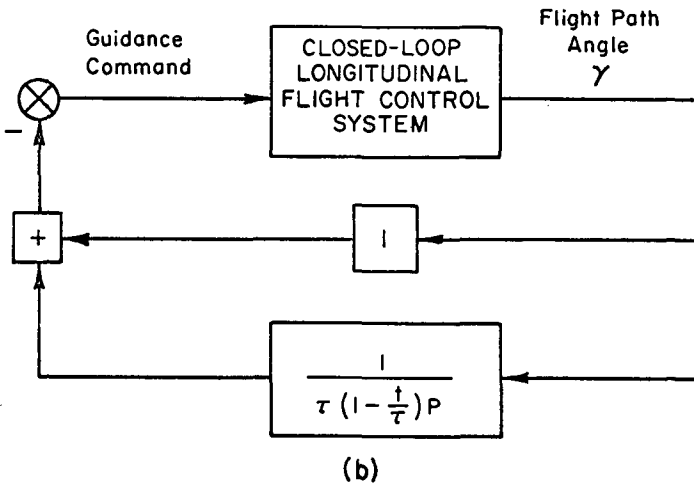


Fig. 1-5. Simplified longitudinal collision course guidance and control system.

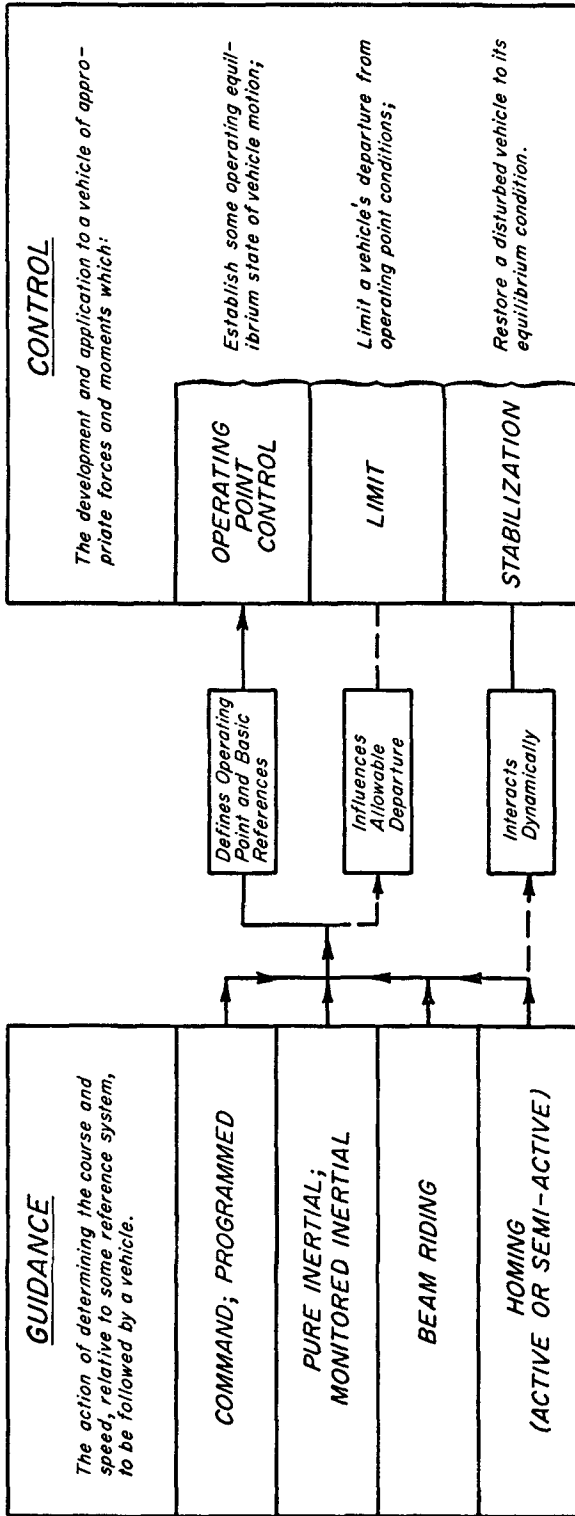


Fig. 1-6. Guidance/control dichotomy.

so as to modify the input. Effective flight control systems invariably are feedback systems, for a number of important reasons.

Even if invention had ultimately produced a satisfactory, inherently stable aircraft, the disadvantage of open-loop, i.e., nonfeedback, control would probably still preclude its use for the control of flight. Open-loop controls are programmed and calibrated. Their proper operation depends on the computation of an appropriate program and on maintaining the calibration of the controlled element or object of control. In flight control applications the appropriate program is often of considerable complexity, and a most notable feature of the aircraft's response to control is that it changes markedly with speed, altitude, and loading.

The advantages of feedback control are:

1. The provision of stability.
2. The adjustment of dynamic response, including
  - a. Reduction of lags.
  - b. Provision of desired or specified command/response relationships, especially as regards the improvement of linearity and the reduction of the effect of vehicle cross-coupling forces.
3. The suppression of unwanted inputs and disturbances.
4. The suppression of the effects of variations and uncertainties in the characteristics of the controlled element.

Feedback can make an unstable system stable. (It can, of course, also make a stable system unstable, a subject to which we shall have frequent occasion to return later.) Unfortunately, aircraft are never stable by themselves. At the very best they are neutrally stable in heading and altitude; continuous corrections must be made in order to fly a straight and level course. Otherwise, and this is especially true of modern configurations, a disturbance may start an aperiodically divergent motion such as the one pilots sometimes call the "graveyard spiral," or a similar disturbance may initiate weakly damped or perhaps divergent oscillations in the rolling, yawing, and pitching degrees of freedom. It is for the repair of any such deficiencies in stability that the classes of automatic flight control systems known as stability augmenters and automatic pilots are principally useful.

Feedback can improve the speed of response and may be used so as to enforce some desired correspondence between the input and output of the system. The series of figures that have been presented have served to emphasize the fact that one of the purposes of the flight control system is to follow the commands generated by the guidance system. Rapid and accurate response to commands, so that the commanded flight path is matched by the actual flight path, for example, is made possible or is largely enhanced by the feedback of aircraft motion quantities. These

feedbacks have been illustrated and are defined in the figures to comprise the flight control system.

It is not only the case that the (feedback) flight control system will improve the speed of response and accuracy in following commands but it will also tend to suppress the effect of disturbances such as the atmospheric (gust) disturbances illustrated in Fig. 1-3, as well as the effects of changes in the characteristics of the vehicle's response to control. These are not the least reasons for employing feedback. The aircraft must typically fly in atmospheric turbulence which tends to upset it and to alter its flight path, and the response to control may very well be substantially changed by the consumption of fuel, the release of stores, and changes in the flight speed or altitude.

Some of the earliest inanimate feedback controls, aside from water level controls which were known in antiquity, were speed regulators for prime movers.<sup>7</sup> These were primarily designed to *regulate* against changes in speed because of disturbances such as changes in the load, or changes in the response of the machine to control such as a change in speed at the same throttle setting because of an increase in steam pressure. The early governors secured some of the very practical advantages of feedback but they also tended to display the largest disadvantage: a tendency to hunt or oscillate. The phenomenon of hunting of engine governors motivated a number of authors to study the stability of feedback control systems and to lay the foundations of a mathematical theory of the subject. Among the earliest of these investigators was the physicist J. C. Maxwell,<sup>8</sup> who in his own paper on the subject conceded his inability to discover the criteria for the stability of higher order systems. Later, he was one of the examiners who set the subject "The Criterion of Dynamical Stability" for the Adams Prize Essay Contest in 1877. The prize was won by E. J. Routh,<sup>9</sup> who considered not only governors but the stability of general motion of rigid bodies. His studies in that field became the basis for the investigation of the dynamic stability of aircraft and for many years provided the principal tool for the study of feedback control systems.

#### 1-4. Early History of the Subject of Aircraft Dynamics

F. W. Lanchester was the first to investigate analytically the dynamic stability of aircraft. Before the turn of the century he experimented with glider models and studied the properties of the solutions to a simplified

<sup>7</sup> James Watt is commonly credited with the invention of the flyball governor, about 1784, but it seems likely that these were in use on windmills before his time. (See A. Wolf, *A History of Science, Technology, and Philosophy in the XVIIIth Century*, Macmillan, New York, 1939.)

<sup>8</sup> J. C. Maxwell, "On Governors," *Proc. Roy. Soc. series A. No. 100, 270-283, 1868.*

<sup>9</sup> E. J. Routh, *Stability of a Given State of Motion*, Macmillan, London, 1877.

set of equations for motion in the plane of symmetry.<sup>10</sup> He called the resulting flight paths "phugoids," a name that persists to this day.

In the year of the first powered flight, 1903, Bryan and Williams, using more conventional mathematical methods, introduced the linearized equations of motion that have been the foundation of studies of dynamic stability and response to control ever since.<sup>11</sup> Later, the theory of both the longitudinal and lateral motions was presented by Bryan.<sup>12</sup> The six Euler equations for the general motion of a rigid body were considered for "small" departures from steady straight flight of an airplane with a plane of symmetry. Under these assumptions the equations were shown to be separable into two groups of three each. One group related the motion variables in the plane of symmetry, while the other group related the motion variables out of the plane of symmetry. Neither group contained any variable occurring in the other, so that they could be treated entirely separately. The separate groups of equations were called the "symmetric" or *longitudinal* and the "asymmetric" or *lateral* equations. A further consequence of the assumption of small perturbations was that the air forces on the airplane could be shown to depend on certain *constants* or "stability derivatives," as they were called; Bryan suggested that these might be determined experimentally.

As early as 1912, Bairstow and Melvill Jones, at the National Physical Laboratory in Great Britain, had taken up Bryan's suggestion and had developed some of the wind tunnel techniques for measuring the stability derivatives of models. They reported the results of their initial effort the following year, showing how features of the motion could be recognized in the mathematical solutions for the free response of a hypothetical airplane for which they had measured or calculated all the derivatives.<sup>13</sup> In both language and notation this report is thoroughly modern; it might be used as a text on the dynamic stability of airplanes today.

The theory and the experimental practice were subsequently extended by the original investigators and others. Bairstow considered the stability of more complicated motions such as circling flight and treated the motion of dirigible airships. He provided a comprehensive account of the subject in 1920.<sup>14</sup> Hunsaker, who had visited the National Physical Laboratory

<sup>10</sup> F. W. Lanchester, *Aerodromics*, Archibald Constable, London, 1908. See also B. Melvill Jones, "Dynamics of the Aeroplane," in W. F. Durand, ed., *Aerodynamic Theory*, Vol. 5, Durand Reprinting Committee, Pasadena, Calif., 1943; republished (Vols. 5 and 6 bound in one volume) by Dover, New York, 1963, pp. 2-3, 169.

<sup>11</sup> G. H. Bryan and W. E. Williams, "The Longitudinal Stability of Aerial Gliders," *Proc. Roy. Soc. Series A*, 73, No. 489, 100-116 (1904).

<sup>12</sup> G. H. Bryan, *Stability in Aviation*, Macmillan, London, 1911.

<sup>13</sup> L. Bairstow, B. Melvill Jones, and B. A. Thompson, "Investigation into the Stability of an Airplane," ARC R & M 77, 1913.

<sup>14</sup> L. Bairstow, *Applied Aerodynamics*, Longmans Green, London, 1st edn., 1920; 2nd edn., 1939.

## INTRODUCTION AND ANTECEDENTS

in 1914, introduced Bairstow's wind tunnel techniques and the method of Bryan and Bairstow for the calculation of dynamic stability in the United States. He collaborated on the first report of the United States National Advisory Committee for Aeronautics which was concerned with the response of aircraft to gusts.<sup>15</sup> Glauert calculated the stability derivatives of a running propeller and the motions of an airplane with the elevator free.<sup>16</sup> The model measurement<sup>17</sup> or calculation<sup>18</sup> of particular stability derivatives continued to attract attention, and a considerable effort was made to measure derivatives, free motions, and the response to controls<sup>19</sup> in full scale flight tests. The references given in the footnotes are only typical, not comprehensive. More details are given in the historical sketch appended to the 1947 paper by Milliken<sup>20</sup>; anecdotal accounts of the work of the British pioneers are presented in several contributions to the "Centenary Journal" of the Royal Aeronautical Society,<sup>21</sup> particularly the ones by A. V. Stevens, Sir Harry Garner, J. L. Mayler, and R. W. McKinnon Wood.

By 1935, when the survey by B. Melvill Jones appeared in Durand's *Aerodynamic Theory*, the classical approach of Bryan and Bairstow was well established but was very little used. Results of the full scale experiments had led to the conviction that the theory of infinitesimal motions was practical for the prediction of the stability of motion, the time history of the motion following a disturbance, and the response to the application of control. The effect of variations in the configuration of a typical airplane had been traced via their influence on the derivatives to the result in terms of stability of motion. Furthermore, these results were

<sup>15</sup> J. C. Hunsaker, "Experimental Analysis of Inherent Longitudinal Stability for a Typical Biplane," NACA TR 1, Pt. I, 1915. See also "Dynamic Stability of Aeroplanes," *Smithsonian Misc. Collection*, 1916.

<sup>16</sup> H. Glauert, "The Stability Derivatives of an Aircsrew," ARC R & M 642, Oct. 1919; see also "The Longitudinal Stability of an Airplane," ARC R & M 638, 1919.

<sup>17</sup> L. W. Bryant and H. B. Irving, "Apparatus for the Measurement of  $M_q$  on a Complete Model Airplane," ARC R & M 616, 1919.

<sup>18</sup> W. L. Cowley, "The Effect of the Lag of Downwash on the Longitudinal Stability of an Aeroplane," ARC R & M 718, Feb. 1918.

<sup>19</sup> H. Glauert, "Analysis of Phugoids Obtained by a Recording Airspeed Indicator," ARC R & M 576, Jan. 1919; E. P. Warner and F. H. Norton, "Preliminary Report on Free Flight Tests," NACA TR 70, 1919; F. H. Norton, "Practical Stability and Controllability of Airplanes," NACA TR 120, 1921; see also NACA TR 112, TR 167, and TR 170; M. A. Garner and S. B. Gates, "The Full-Scale Determination of the Lateral Resistance Derivatives of a Bristol Fighter," ARC R & M 987, Aug. 1925; see also ARC R & M 1068 and 1070; H. A. Soule, and J. B. Wheatley, "A Comparison Between the Theoretical and Measured Longitudinal Stability Characteristics of an Airplane," NACA TR 442, 1933.

<sup>20</sup> W. F. Milliken, Jr., "Progress in Dynamic Stability and Control Research," *J. Aeron. Sci.* 14, No. 9, 493-519 (Sept. 1947).

<sup>21</sup> "Centenary Journal, Royal Aeronautical Society 1866-1966," *J. Roy. Aeron. Sci.* 70, No. 661, pp. 71-78, 82-84, 89-90, 95-97 (Jan. 1966).

appreciated not only in terms of the solutions to specific numerical examples but more generally, at least in part, as approximate solutions given in terms of the dominant literal stability derivatives.

Melvill Jones himself, speaking of complete solutions to the equations of motion and of approximate solutions to the stability quartic equations, evaluated the state of affairs in the following words:

In spite . . . of the completeness of the experimental and theoretical structure . . . it is undoubtedly true that, at the time of writing, calculations of this kind are very little used by any but a few research workers. It is in fact rare for anyone actually engaged upon the design and construction of aeroplanes to make direct use of [such] computations . . . , or even to be familiar with the methods by which they are made . . . . In my own opinion it is the difficulty of computation . . . which has prevented designers of aeroplanes from making use of the methods.

Though the process . . . will, if worked correctly, give the final answer required, it is so involved that it is not easy to trace the connection between the final answer and the separate characteristics of the airplane which are represented by the various derivatives included in the equation of motion.

With regard to the response to specific disturbances no convenient means of tracing this connection has yet been devised; but when . . . the form of the solution of the quartic for  $\lambda$  [i.e., the frequencies and damping factors of the free modes] in normal flight is all that is required, the omission of certain terms, which are then relatively unimportant, allows such drastic simplifications to be made that the relation between cause and effect can be displayed with comparative ease.<sup>22</sup>

The situation was hardly altered during the next ten years. In spite of the introduction of the method of operators,<sup>23</sup> which *did* reduce the labor of computation, and in spite of earnest efforts to make the techniques as simple and general as possible by introducing a nondimensional

<sup>22</sup> B. Melvill Jones, "Dynamics of the Aeroplane," in W. F. Durand, ed., *Aerodynamic Theory*, Vol. 5, Durand Reprinting Committee, Pasadena, Calif., 1943, republished (Vols. V and VI bound in one volume) by Dover Publications, New York, 1963, pp. 2-3, 169.

<sup>23</sup> L. W. Bryant and D. H. Williams, "The Application of the Method of Operators to the Calculation of the Disturbed Motion of an Airplane," ARC R & M 1346, July 1930; R. T. Jones, "A Simplified Application of the Method of Operators to the Calculation of the Disturbed Motion of an Airplane," NACA TR-560, 1936; see also "Calculation of the Motion of an Airplane under the Influence of Irregular Disturbances," *J. Aeron. Sci.*, 3, No. 12, 419-425 (Oct. 1936); A. Klemin and B. F. Ruffner, "Operator Solutions in Airplane Dynamics," *J. Aeron. Sci.* 3, No. 7, 252-255 (May 1936).

## INTRODUCTION AND ANTECEDENTS

notation,<sup>24</sup> and by summarizing information on the stability factors in convenient charts,<sup>25</sup> and, further, in spite of hortatory expositions of the theory,<sup>26</sup> designers of airplanes continued to disdain dynamic stability analysis.

Nevertheless, research continued at a pace that was accelerated by the advent of the war, and some improvements were made in the understanding of, e.g., the importance of wing/fuselage interference and power effects on the stability derivatives, the effect of closely balanced free controls on the motion, the response to particular motions of particular controls such as spoilers, and the influence of changes in design on the character of the motions.

The point of view then current, however, did not permit one (with very rare and soon forgotten exceptions) to consider the response of the airplane under the continuous action of the controls, i.e., as a feedback system. The controls were, almost invariably, considered as:

1. Fixed, as in the earliest studies.
2. Free, i.e., restrained only by aerodynamic hinge moments (or later by friction as well).<sup>27</sup>
3. Programmed, i.e., moved as a simple function of time, such as a step or ramp function or a smooth pulse.<sup>28</sup>

It may have been not only the fact that the calculations were laborious but also that the assumptions of the analysis appeared unrealistic, which discouraged their use in design. The stability of unattended motion with the controls fixed or free and the response to programmed control motions were and are, indeed, of some interest in connection with the dynamics of an airplane, but it is evident that the human or an automatic pilot flies by operating the controls more or less continuously. The airplane plainly is an element in a system that includes a human pilot or an automatic

<sup>24</sup> H. Glauert, "A Non-Dimensional Form of the Stability Equations of an Aeroplane," ARC R & M 1093, 1927.

<sup>25</sup> S. B. Gates, "A Survey of Longitudinal Stability below the Stall, with an Abstract for Designers' Use," ARC R & M 1118, July 1927; C. H. Zimmerman, "An Analysis of Longitudinal Stability in Power-Off Flight with Charts for Use in Design," NACA TR-521, 1935; also "An Analysis of Lateral Stability in Power-Off Flight with Charts for Use in Design," NACA TR 589, 1937.

<sup>26</sup> O. C. Koppen, "Happier Landings," *Aviation*, Sept. 1934; "Control Sensitivity," *Aviation*, Oct. 1935; "Smart Airplanes for Dumb Pilots," paper presented to the SAE, Detroit, Mich., Jan. 1936; "Airplane Stability and Control from the Designer's Point of View," *J. Aeron. Sci.*, 7, No. 4, 135-140 (Feb. 1940).

<sup>27</sup> H. Glauert, "The Longitudinal Stability of an Aeroplane," ARC R & M 638, 1919; R. T. Jones and D. Cohen, "Analysis of the Stability of an Airplane with Free Controls," NACA TR 709, 1940.

<sup>28</sup> R. T. Jones, "A Simplified Application of the Method of Operators to the Calculation of the Disturbed Motion of an Airplane," NACA TR 560, 1936; K. Mitchell, "Lateral Response Theory," RAE Rept. Aero. 1952, Mar. 1944.

pilot. This view did not come to be generally accepted until after the war, and the understanding of a convenient means of tracing the connection between the response to specific disturbances, such as the operation of the controls, and the characteristics of the airplane that are represented by the various derivatives was of an even later date.

### 1-5. Early History of Automatic Flight Control

The development of automatic flight, like the development of airplanes themselves, proceeded for a long time with the benefit of very little theoretical knowledge.

Sir Hiram Maxim (1840–1916) was a prodigious inventor. In 1891, when he turned his attention to the design and construction of a heavier than air flying machine, he proposed to secure its longitudinal stability by means of a servo drive and automatic feedback. The devices that are described in Maxim's book and are illustrated there by a photograph of the installation in an airplane are surprisingly modern in concept and execution.<sup>29</sup>

A steam-driven, pendulous, vertical gyroscope was made to operate a valve that ported steam to a servo cylinder.<sup>30</sup> The motion of the piston drove the elevators, and the feedback link from the piston repositioned the gyro-operated valve body so as to close the valve. In principle, this "gyroscopic apparatus for automatically steering [the] machine in a vertical direction" is indistinguishable from the elevator control portion of automatic pilots of much more recent date. Easily recognizable are the elements of any automatic flight control system: the sensor (gyroscope), the amplifier (valve), and the control surface positioning servo. Unluckily, tests of the flying machine for which the gyroscopic control was designed ended in disaster when the aircraft lifted off the tracks designed to restrain it, turned over, and was destroyed. Maxim, who felt that his special contribution was to be the development of lifting surfaces and power plants, thought that the point about lift and power had been proved, and the inventor turned his energies in other directions.

Still in the nineteenth century, however, successful gyroscopic feedback control of the "flight" path was demonstrated by Ludwig Obry, an Austrian, who in 1894 introduced a course-keeping gyro as an improvement for the naval torpedo invented by Robert Whitehead in 1866. (Whitehead's torpedo had an automatic depth control.) A little later the

<sup>29</sup> H. S. Maxim, *Artificial and Natural Flight*, Whittaker, London, 1908, pp. 92–94.

<sup>30</sup> The principles of steam and hydraulic servomotors had already been known for some time. See A. B. Brown, British Patent 2253, 1871, and J. Farcot, *Le Servo Moteur ou Moteur Asservi. Gouvernails à Vapeur Farcot, Description Théorique et Pratique*, J. Baudry, Paris, 1873. Among early applications to vehicle control were steering engines for steamships.

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principle of gyroscopic stabilization of ships was introduced, and, although depending on a completely different principle, was the model for the next attempt at gyroscopic control of an aircraft. In 1909–1910 Dr. Elmer Sperry attempted to make a gyroscopic “stabilizer” for an airplane. This was a rigidly mounted engine-driven wheel with its spin axis vertical. It would have opposed rolling motions with a pitching torque, and vice versa. It was apparently never brought to a test because of the lack of success of the airplane in which it was installed but it served as the inspiration for further trials.<sup>31</sup>

Between 1910 and 1912 Dr. Sperry and his son Lawrence developed and installed, in an airplane belonging to Glenn H. Curtiss, an all-electric, two-axis automatic pilot. Roller contacts on a gyro platform, measuring the bank and pitch angles, actuated solenoid clutches which connected the ailerons and elevator to a propeller-driven “air turbine,” and motion of the surfaces repositioned the contactor segments<sup>32</sup> (e.g., see Fig. 1-7). The machine was announced to the public in October 1912. In 1914 the aircraft and its automatic pilot were entered in a safety contest sponsored by the Aero Club of France. Lawrence Sperry made a dramatic demonstration of automatic flight as he flew at low altitude along the Seine in the vicinity of Paris, standing upright in the cockpit of the Curtiss flying boat,

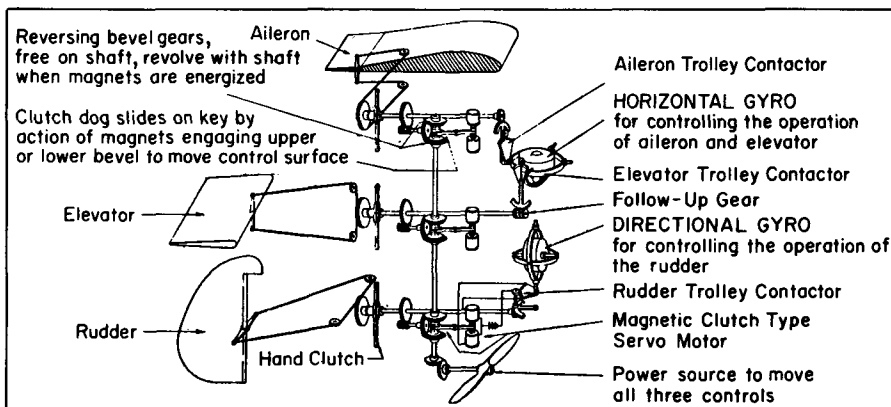


Fig. 1-7. Diagrammatic plan of Sperry automatic pilot.

<sup>31</sup> C. S. Draper, “Flight Control,” *J. Roy. Aeron. Soc.*, 59, 451–477 (July 1955).

<sup>32</sup> One observer vividly recalls the loud groaning noise that this type of automatic pilot made. Presumably, the noise came from the grinding of the toothed clutch faces. See the discussion by Dr. A. L. Rawlings in the article by F. W. Meredith and P. A. Cooke, “Aeroplane Stability and Automatic Control,” in *J. Roy. Aeron. Soc.*, 61, No. 318, 415–436 (June 1937).

AIRCRAFT DYNAMICS AND AUTOMATIC CONTROL

Table 1-1. Selected early inventions in the feedback control of aircraft (adapted from F. Haus, "Automatic Stabilization," NACA TM 802, Aug. 1936).

Feedback variable	Control	Inventor and date	Actuating means
Speed, $U$	Elevator deflection, $\delta_e$	Budig 1912 Etévé 1914	Mechanical connection to sensor
Incidence, $\alpha$	Elevator deflection, $\delta_e$	Etévé 1910	Mechanical connection to sensor
Inclination, $\theta$	Elevator deflection, $\delta_e$	Regnard 1910	Electric type of servo.
		Sperry 1912	Air-turbine-driven clutch servo.
		RAE 1927	Pneumatic servo
Angular velocity, $\dot{\theta}$	Elevator deflection, $\delta_e$	Lucas 1929 Girardville 1910	Mechanical connection to sensor
Direction of apparent gravity, $g \sin \theta + dU/dt$	Elevator deflection, $\delta_e$	Moreau 1912	Electric-motor-driven clutch servo
Speed, $U$ , and incidence, $\alpha$	Elevator deflection, $\delta_e$	Etévé 1914	Mechanical connection to sensor
Speed, $U$ , and direction of apparent gravity, $g \sin \theta + dU/dt$	Elevator deflection, $\delta_e$	Doutre 1911	Pneumatic servo
Speed, $U$ , and magnitude of apparent gravity, $a_z$	Elevator deflection, $\delta_e$	Doutre 1913	Pneumatic servo
Speed, $U$ , and inclination, $\theta$	Elevator deflection, $\delta_e$	Marmonier 1909	Unknown type of servo
Speed, $U$ , and angular velocity, $\dot{\theta}$	Elevator deflection, $\delta_e$	Boykow 1928	Hydraulic servo
Sideslip, $\beta$	Aileron deflection, $\delta_a$	Constantin 1920	Mechanical connection to sensor
Bank angle, $\varphi$	Aileron deflection, $\delta_a$	Sperry 1912	Air-turbine-driven clutch servo

INTRODUCTION AND ANTECEDENTS

Table 1-1 (Continued).

Feedback variable	Control	Inventor and date	Actuating means
Heading, $\psi$	Rudder deflection, $\delta_r$	RAE 1927	Pneumatic servo
Yawing velocity, $r$ , and side acceleration, $a_y$	Rudder deflection, $\delta_r$	Mazade and Aveline 1922	Pneumatic servos
Side acceleration, $a_y$ , and yawing velocity, $r$	Aileron deflection, $\delta_a$		
Bank angle, $\phi$	Aileron deflection, $\delta_a$	Sperry 1932	Air-turbine-driven clutch servos
Heading, $\psi$	Rudder deflection, $\delta_r$		

holding his hands over his head, while his mechanic walked out along the wing. A photograph of this unusual event has been reproduced by Bollay and by Richardson, among others.<sup>33</sup> A similar demonstration was planned for early the next year in New York, but there the aircraft was overturned and extensively damaged by wind before the demonstration of its performance could be satisfactorily completed.<sup>34</sup>

This first automatic pilot was intended as an aircraft stabilizer. In other words, it was intended to supply stability, as we now say, "artificially," to aircraft that were often deficient in this regard. Other inventors were pursuing the same goals by the same and other means. The feedback of angle of attack and angle of sideslip, speed, longitudinal, side, and normal acceleration, lift, and body axis rates, as well as attitude angles, were all tried singly and sometimes in combination. Clarke<sup>35</sup> in an early paper described some of his own experiments in Great Britain, while Haus<sup>36</sup> has sketched some of the history of early developments on the continent of Europe (see Table 1-1). However, none of these original inventors was successful enough for his device to pass immediately into common use.

The design of aircraft made giant strides during the 1914-1918 war and it was found that sufficient stability for the human pilot's use could be supplied by suitable choice of the size and shape of the aerodynamic surfaces. Actually, many aircraft were still unstable but not dangerously so, and, with reference to the ground, the human pilot performed the stabilizing and control functions of the (feedback) control and guidance

<sup>33</sup> W. Bollay, "Aerodynamic Stability and Automatic Control," *J. Aeron. Sci.*, 18, No. 9, 569-624 (Sept. 1951); K. I. T. Richardson, *The Gyroscope Applied*, Philosophical Library, New York, 1954.

<sup>34</sup> "The Sperry Gyroscopic Stabilizer," *Flight*, 7, No. 5, 74-76 (Jan. 29, 1915).

<sup>35</sup> T. W. K. Clarke, "Auto-mechanical Stability," *Aeron. J.*, 101-115 (Apr. 1912).

<sup>36</sup> F. Haus, "Automatic Stability of Airplanes," NACA TM 695, Dec. 1932. *Automatic Stabilization*, NACA TM 802, Aug. 1936; 815, Dec. 1936.

systems. Neither artificial stabilizers nor automatic pilots were found to be particularly useful on the manned warplanes. They, in effect, disappeared from view. Under the cover of military secrecy, however, the development of the automatic pilot was continued for possible application to pilotless aircraft; in 1917-1918 Lawrence Sperry completed the construction and test of an aerial torpedo for the United States Navy.<sup>37</sup> In an advanced version the aerial torpedo was even remotely controlled by radio. The success of the project, however, came too late for the use of a "flying bomb" in World War I. Following the war, and turning to more prosaic applications, the Sperry Gyroscope Company had, by 1932, developed an automatic pilot for possible commercial transport use<sup>38</sup> (see Fig. 1-7). Except for the gyroscopic references that comprised the then new directional and vertical gyroscopes and the fact that it provided for control about all three aircraft axes, this automatic pilot retained many of the features of the original one of 1910-1915. Better results, however, were just around the corner.

In 1933 the prototype of the A2 automatic pilot was under construction. It featured panel-mounted gyroscopes with pneumatic pickoffs and three-axis control with proportional hydraulic servos. When Wiley Post, visiting the Sperry factory, saw it, he insisted that it be installed in his Lockheed Vega 5-C.<sup>39</sup> During the period July 15-22, 1933 Post, flying alone, set a round-the-world record of 7 days, 18 hours. The performance and reliability of the automatic pilot, which allowed the human pilot to perform the navigator's function and even to nap in flight, played a considerable role in this feat. The prototype automatic pilot used by Wiley Post, together with his airplane, the "Winnie Mae," are in the National Aeronautical Collection of the Smithsonian Institute in Washington.

The A2 automatic pilot came into widespread use among the airlines during the 1930s. This was due partly to its demonstrated reliability, partly because the panel-mounted instruments then coming into extensive use for routine operations under low visibility conditions supplied its gyroscopic references (an obvious economy in cost and weight), and partly because airplanes had achieved a range performance that made pilot relief attractive. The A2 was first introduced to airline service on the Boeing Model 247 in 1934. Its defects, if any, were its virtues; it flew straight and level. It was not designed for maneuvering.

In effect this automatic pilot provided for control surface deflections

<sup>37</sup> P. R. Bassett, "Instruments and Control of Flight," *Aeron. Eng. Rev.*, 12, No. 12, 118-123, 133 (Dec. 1953).

<sup>38</sup> E. A. Sperry, Jr., "Description of the Sperry Automatic Pilot," *Aviation Eng.*, 6, No. 1, 16-18 (Jan. 1932); see also E. S. Ferry, *Applied Gyrodynamics*, Wiley, New York, 1932, pp. 123-125.

<sup>39</sup> P. R. Bassett, "Instruments and Control of Flight," *Aeron. Eng. Rev.*, 12, No. 12, 118-123, 133 (Dec. 1953).

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in three axes that were proportional to the departures from the reference attitude. A description of its operation is given by Richardson,<sup>40</sup> among others. It was as if the surfaces were "geared" to the instrument (see Fig. 1-8). This concept of gearing was frequently employed in some of the early mathematical studies of automatic control of aircraft but it lacks the generality offered by the concept of feedback.

During roughly the same period of time (1922-1937), somewhat similar automatic pilot developments were underway in Great Britain, although the aim there was the satisfaction of military requirements and the work was, at first, carried out in secrecy.<sup>41</sup> Interestingly, in both the earlier (Mark I) and later (Mark VII and Mark VIII) versions, the British pursued the design of two-axis controls with great diligence. The Mark I used a single free gyroscope measuring heading and pitch to control the rudder and elevators by means of low pressure pneumatic servos. An account of the results of the early efforts was presented by the pioneers Meredith and Cook of the Royal Aircraft Establishment (RAE) in England in 1937.<sup>42</sup> This account describes the use of the automatic pilot in aerial map-making and suggests its superiority over the Sperry three-axis design (Fig. 1-9) for applications requiring maneuvers. It also shows the considerable acquaintance of the authors with the theory, methods, and conclusions of their colleagues at the Royal Aircraft Establishment who had been engaged in the study of the dynamic stability of airplanes. The action of the automatic pilot was clearly explained in those terms. In fact the methods of dynamic stability analysis had been applied in the design of the RAE flight control equipment from 1924 on, but very few results were ever published in the open literature.

Following in the footsteps of Bairstow<sup>43</sup> and Glauert<sup>44</sup> and using a method developed by S. B. Gates,<sup>45</sup> who employed it in the obscure RAE

<sup>40</sup> K. I. T. Richardson, *The Gyroscope Applied*; see also P. R. Bassett, "Development and Principles of the Gyropilot," *Instruments*, 9, No. 9, 251-254 (Sept. 1936); *The Sperry Aircraft Gyropilot*, Sperry Gyroscope Co. Publication 15-665, July 1940.

<sup>41</sup> A comprehensive bibliography of British and foreign work on automatic flight control, both theoretical and experimental, from 1903-1957 has been prepared by the RAE. See R. C. Wright, A. T. E. Bray, and H. R. Hopkin, "List of Published and Unpublished References on the Remote and Automatic Control of Aircraft and Missiles, Pitotless Target Aircraft, Autopilots and Gyroscopic Flight Instruments, Inertial Guidance and Automatic Landing of Aircraft," RAE Library Bibliography No. 224, Ministry of Aviation, Sept. 1960.

<sup>42</sup> F. W. Meredith and P. A. Cooke, "Aeroplane Stability and the Automatic Pilot," *J. Roy. Aeron. Soc.*, 61, No. 318, 415-436 (June 1937).

<sup>43</sup> L. Bairstow, *Applied Aerodynamics*, 1st edn., Longmans Green, London, 1920.

<sup>44</sup> H. Glauert, "Summary of the Present State of Knowledge with Regard to Stability and Control of Aeroplanes, ARC R & M 710, Dec. 1920.

<sup>45</sup> S. B. Gates, "Notes on the Aerodynamics of Automatic Directional Control," RAE Rept. No. BA 487, Feb. 19, 1924; "Notes on the Aerodynamics of an Altitude Elevator Control," RAE No. BA 494, Mar. 19, 1924. (The latter report discusses the instability of altitude control with elevator at speeds below the speed for minimum power required.)

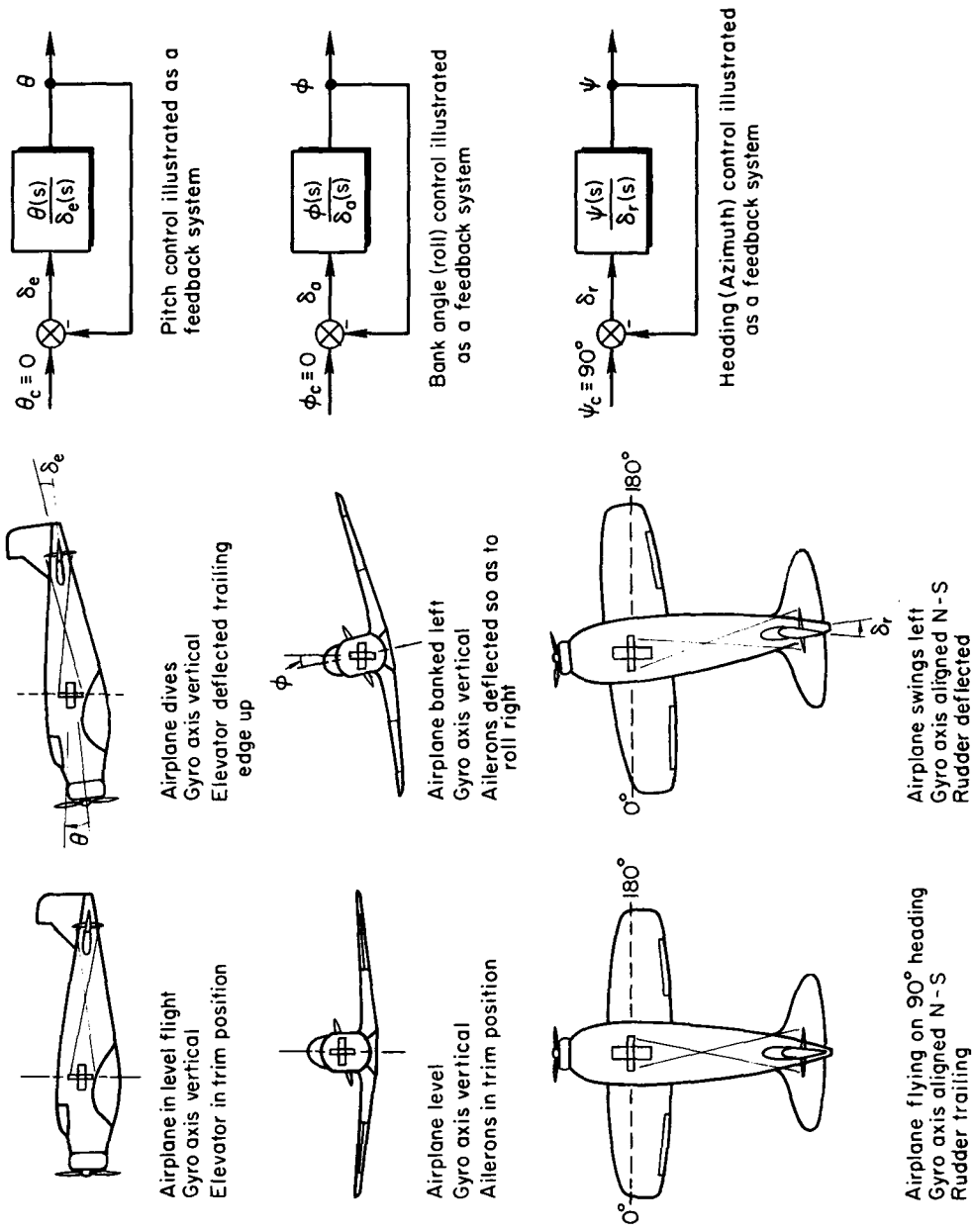


Fig. 1-8. Simple three-axis attitude control illustrating the concept of gearing.

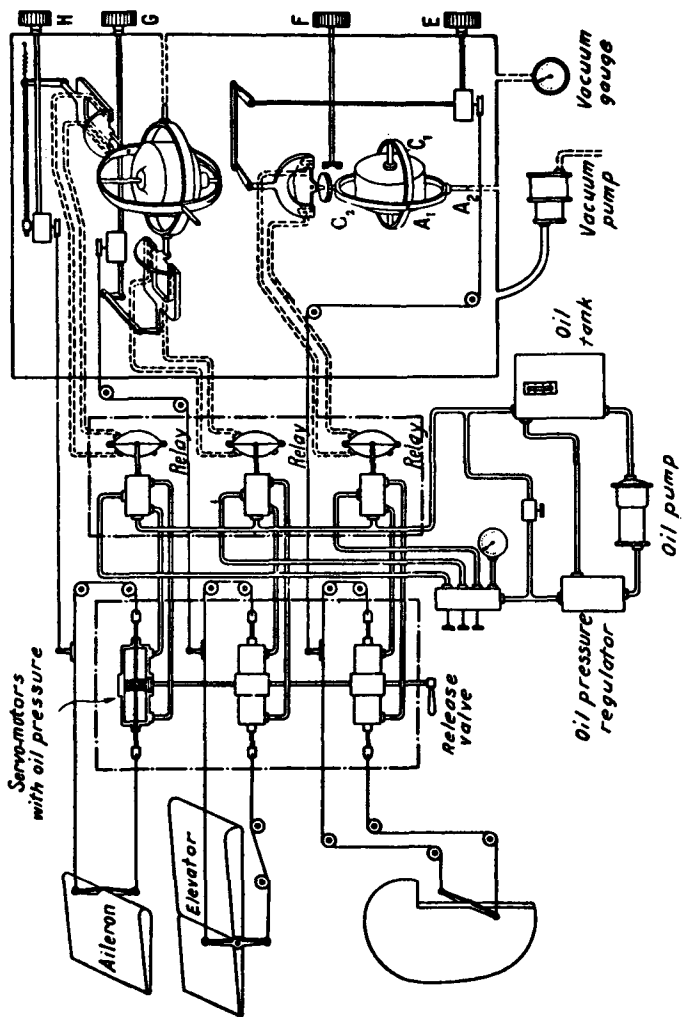


Fig. 1-9. Assembly sketch of Sperry stabilizer.

Reports BA 487 and 494, Garner,<sup>46</sup> in 1926, made an analysis of the lateral/directional motion of an airplane under the influence of feedback control. Gates assumed that the controls were moved according to certain laws, e.g., in proportion to certain output variables and their derivatives. He also stressed that good stability was not enough, since it was essential also to consider the amplitudes of the several modes of motion. With similar assumptions, Garner calculated the stability of the undisturbed motion and the transient motion, following an initial disturbance, under the influence of the feedback control system. It was specifically pointed out that the movements of the controls might be regarded as made either by the (human) pilot or by some mechanical means. Garner then further had the wit and vision to provide in the theoretical treatment for "lag" in the application of controls, and was able to point to a qualitative correspondence between his analytical results and flight tests of an RAE (automatic) rudder control that had appreciable reaction lag. Shortly after Garner's report, a further contribution by Cowley<sup>47</sup> appeared which proposed more elaborate methods of taking into account the time lag in the application of control. Both a pure time delay and a second-order lag were successfully treated.

It now seems surprising that these papers are not given more prominence in accounts of the development of the theory of automatic control systems. Perhaps they were simply too far ahead of their time. Perhaps, on the other hand, it was only in Great Britain, where automatic flight control system development was the responsibility of a government research establishment, that it was thought to be desirable to make response calculations in connection with the design of practical systems. In spite of an apparently adequate theory, however, stability difficulties attended the early flight trials of the RAE Mark IV automatic pilot about 1934.<sup>48</sup> A solution to the problem was apparently not found by analysis or simulation. The problem disappeared when the autopilots were installed in the larger aircraft for which they were intended, no doubt because of the larger inertia and slower response of the multi-engined bombers.

Comprehensive details of subsequent British automatic pilot development (1937-1947), as well as comments on American and German efforts, are set forth in the report by Hopkin and Dunn.<sup>49</sup> Included there is the

<sup>46</sup> H. M. Garner, "Lateral Stability with Special Reference to Controlled Motion," ARC R & M 1077, Oct. 1926.

<sup>47</sup> W. L. Cowley, "On the Stability of Controlled Motion," ARC R & M 1235, Dec. 1928.

<sup>48</sup> H. R. Hopkin and R. W. Dunn, *Theory and Development of Automatic Pilots, 1937-1947*, RAE Rept. IAP 1459, Monograph 2.5.03, Aug. 1947.

<sup>49</sup> Hopkin and Dunn, however, omit any mention of the Pollock/Brown all-hydraulic automatic pilot. This again was a two-axis unit, driving the elevators and rudder. It is described in the article "A New Automatic Pilot," *Flight*, 27, No. 1360, Mar. 14, 1935.

story of the uniquely conceived Mark VII autopilot. In this device the elevator was moved in response to airspeed error and error rate, while the ailerons were actuated by a combination of yaw and roll signals detected by a free gyroscope. The rudder was left free. Calculations showed that the stability properties of this arrangement should have been satisfactory, as indeed they were. Unfortunately, although the performance in average weather was good, in very rough air and in some aircraft at low speed, the elevator, responding to detected changes in the airspeed and airspeed rate, caused violent changes in pitch attitude. These were large enough in some cases so that the acceleration on the fuel system caused the engines to stop momentarily. Later, the matter was investigated theoretically by Neumark<sup>50</sup> and by Sudworth and Hopkin.<sup>51</sup> They were quite able to identify the source of the difficulty and to show that the calculation of damping factors alone was not sufficient to insure satisfactory performance. At the Royal Aircraft Establishment similar calculations were made both by hand and with a mechanical differential analyzer. Very few of the results, however, were published. The understanding of response to *specific* disturbances that Gates had stressed, and for which B. Melvill Jones had called, was thus only modestly enhanced.

Interestingly, by 1935 the German firm of Siemens had developed an elevator control that successfully used an airspeed reference and a rudder control with one of the earliest magnetic compass tie-ins.<sup>52</sup> Hydraulic positioning servos were employed to move the surfaces, but the key feature in both cases was the use of a rate gyro feedback. The rudder "course control" was an independent unit. In Germany it was argued that only the largest and heaviest airplanes would require a complete automatic pilot but that practically all small and medium machines could make good use of a course control.<sup>53</sup>

The firms of Siemens and Askania both developed three-axis automatic pilots that included an independent course control,<sup>54</sup> but during the war they concentrated on the production of the single-axis units. Eventually, more than 80 percent of the aircraft in the German Air Force were equipped with similar automatic stabilizers. A schematic diagram of the

<sup>50</sup> S. Neumark, "The Disturbed Longitudinal Motion of an Uncontrolled Aeroplane and of an Aeroplane with Automatic Control," ARC R & M 2078, Jan. 1943.

<sup>51</sup> J. Sudworth and H. R. Hopkin, "Influence of Automatic Pilots in Stabilization and Dynamic Stability in Pitch," RAE Tech. Note Instn. 775, July 1943.

<sup>52</sup> F. Haus, "Automatic Stabilization," NACA TM 802, Aug. 1936. See also "Siemens Autopilot," *Flight*, 27, No. 1359, 41-42 (Jan. 10, 1935).

<sup>53</sup> G. Klein, "Bedeutung automatischer Flugzeugsteuerungen für den Flugzeugbau," *Jahrbuch 1938 der Deutscher Luftfahrtforschung, Ergänzungsband*, R. Oldenbourg, Munich and Berlin, 1938, pp. 237-242.

<sup>54</sup> E. Fischel, "Vefahren und Bauglieder automatischer Flugzeugsteuerungen," *Jahrbuch 1938 der Deutscher Luftfahrtforschung, Ergänzungsband*, R. Oldenbourg, Munich and Berlin, 1938, pp. 231-236.

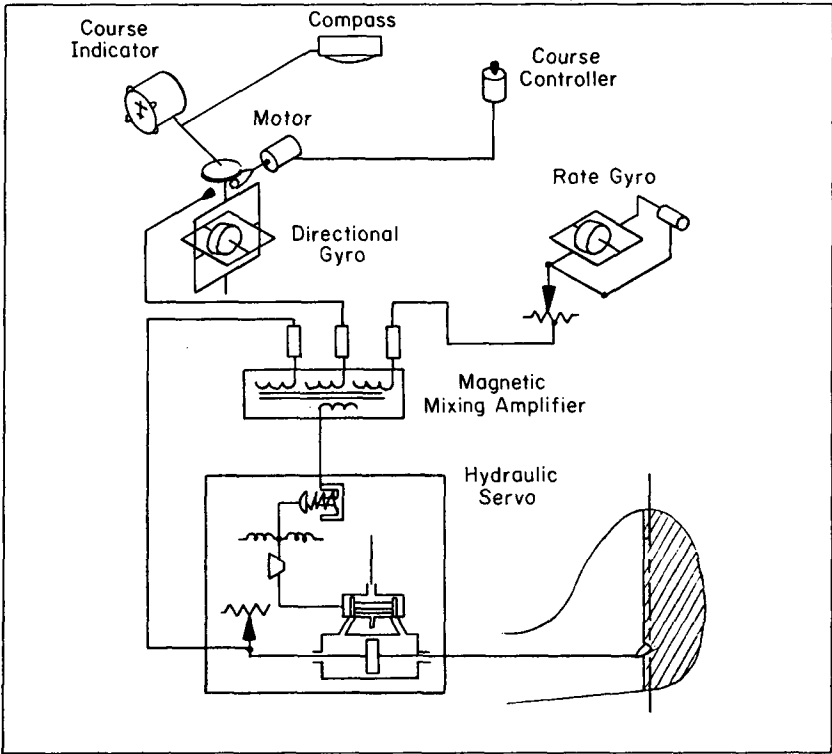


Fig. 1-10. Course control K-12.

Siemens K-12 unit is shown in Fig. 1-10. It was recognized at an early date that such a control could be used to supplement any deficiency in the aerodynamic damping about the yaw axis,<sup>55</sup> and that the course-holding feature, providing the airplane with a heading sense, would permit unattended operation, to a degree, so that even a pilot inexperienced in instrument flying could conduct operations in instrument weather in comparative safety. It may be noted that the methods of mathematical analysis that were introduced by Oppelt<sup>56</sup> for the study of automatic course-holding were rather sophisticated for their time. While he used much simplified linear equations to represent the aircraft in the 1937

<sup>55</sup> K. Wilde, "Über neuere Arbeiten auf dem Gebiet der automatischer Steuerungen," *Jahrbuch 1938 der Deutscher Luftfahrtforschung, Ergänzungsband*, R. Oldenbourg, Munich and Berlin, 1938, pp. 243-247.

<sup>56</sup> W. Oppelt, "Die Flugzeugkursteuerung im Geradeausflug," *Jahrbuch 1937 der Deutscher Luftfahrtforschung*, R. Oldenbourg, Munich and Berlin, 1937, pp. 3-22 to 3-34; also "Comparison of Automatic Control Systems," NACA TM 966, Feb. 1941.

paper, he also used phasor diagrams and approximate describing functions for friction and hysteresis to explore the deleterious effects on the action of the automatic control of these and other imperfections in the system such as quantized signals, and he pointed to the use of a rudimentary simulator as a means for exploring the effect of nonideal equipment characteristics in practice.

Later, during the war, the Germans introduced the rate-rate principle in the Siemens K-23 and Askania PKS-12 fighter course controls, and also in the experimental Patin three-axis automatic pilot. (Here the surfaces were made to move at a rate proportional to the rate of body axis rotation measured by a rate gyro. Damping was insured by electrical differentiation of the signal, in the case of the Siemens and Askania units, and by special design of the gyros to pick up a component of angular acceleration in the case of the Patin design. No feedback units measured the output of the servos.) All three of these control systems were all-electric.

It can be appreciated, even from this abbreviated account, that airplane automatic pilot development proceeded quite independently in Germany from its course in the United States and in Great Britain. Some idea of how it might have evolved can be obtained from Dudenhausen's<sup>57</sup> description of a three-axis, rate-rate automatic flight control system actually built in 1955 but based to a large extent on developments carried out just before the final collapse of Hitler's armies. (It is further amusing to note in the same issue of *Luftfahrttechnik* in which the Dudenhausen article appears that a speaker from an American company, tracing the history of automatic pilot development, says that course control was easily added after the more difficult problem of stabilizing the aircraft in pitch and roll was accomplished. His German translator felt constrained to correct him. From the German point of view the course control came first.<sup>58</sup>)

The Germans were also, of course, very active in the development of pilotless aircraft and missiles.<sup>59</sup> The V-1 flying bomb had a conventional two-axis (elevator and rudder) automatic pilot with altitude and compass tie-ins (see Fig. 1-11). An air mileage counter determined when the final dive should begin. In spite of its, in many ways, very advanced technology the V-2 (A-4) simply used two free gyros: the master control

<sup>57</sup> H. J. Dudenhausen, "Dreiaachsen-Flugregelung für Hochleistungsflugzeuge mit Integrations-Wendekreiseln als Haupttrichtgeber," *Luftfahrttechnik*, 4, No. 3, 49-58 (Mar. 18, 1958).

<sup>58</sup> J. F. Wiren, "Geschichtliches zur Entwicklung der Flugregelungen," *Luftfahrttechnik*, 4, No. 3, 46-68 (Mar. 18, 1958), and notes by the translator, Dpl. Ing. Manteufel.

<sup>59</sup> T. H. Benecke and A. W. Quick, eds., *History of German Guided Missiles*, Verlag E. Appelhans, Brunswick, Germany, 1959; J. N. Thiry, "Control Projects in the German Army, Air Force, and Navy," unpublished translation of a German report with the same title written in Aug. 1944, Sept. 10, 1958.

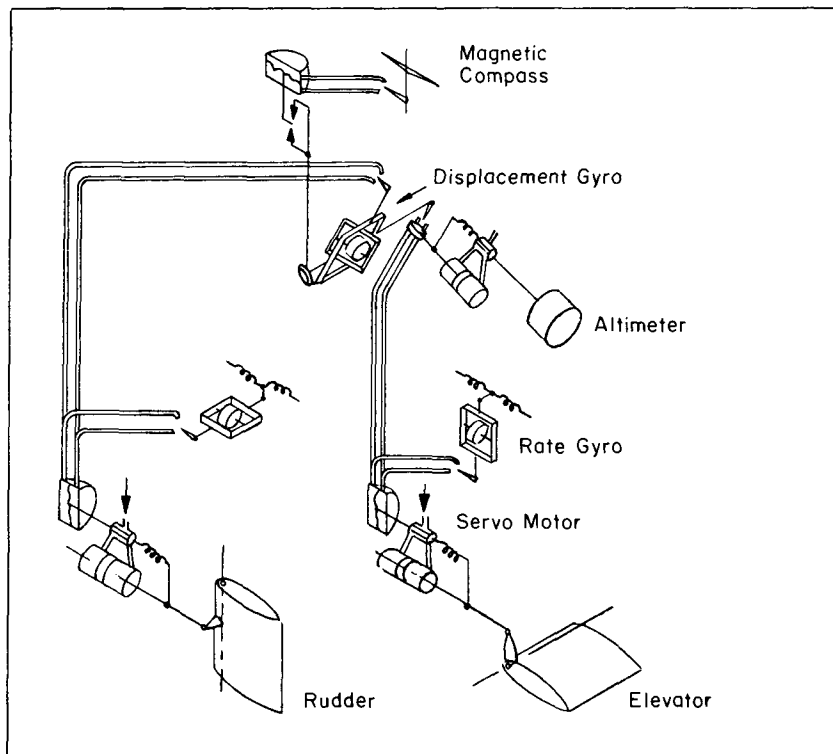


Fig. 1-11. The V-1.

gyro to control yaw and roll, and the “verticant” to control pitch by means of hydraulic servo-driven vanes in the exhaust blast of the rocket engine (see Fig. 1-12). A pitch maneuver was preprogrammed and thrust was cut off by an integrating accelerometer. Provision was made for monitoring and correcting the course with a radio beam. The gyro and vane arrangement was somewhat similar to the one evolved some time earlier (1932) by the pioneer R. H. Goddard for the control of his rockets.<sup>60</sup>

Beginning in 1941 there was a considerable amount of activity in the United States aimed at the development of electric automatic pilots ultimately capable of accepting maneuvering commands, either from the human pilot or from some other source of guidance information such as a bombsight. The first of these “all-electric” automatic pilots was the

<sup>60</sup> M. Lehman, *This High Man: The Life of Robert H. Goddard*, Farrar Straus, New York, 1963, pp. 202 ff.

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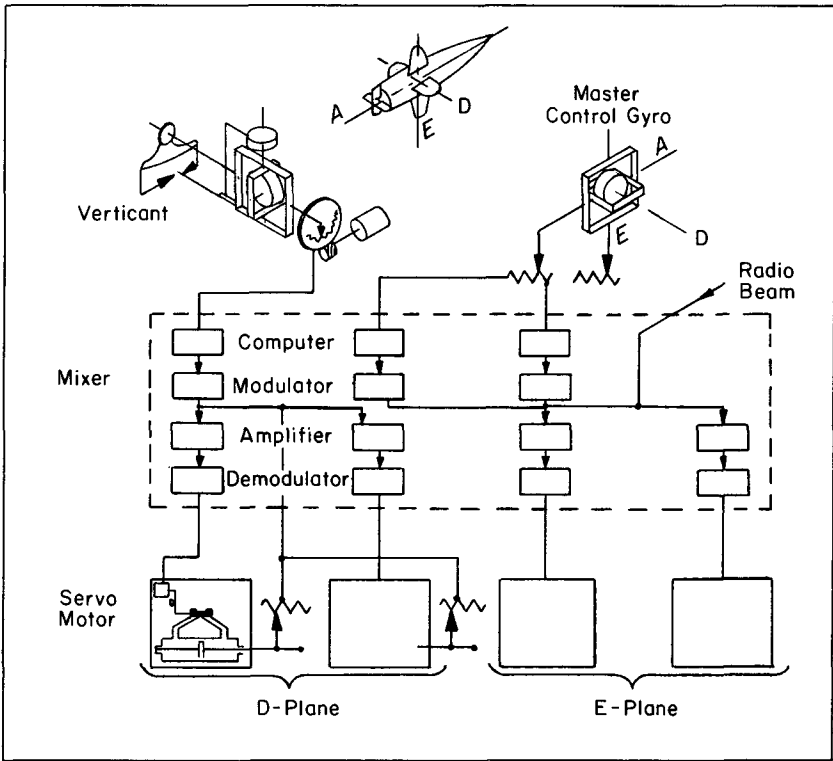


Fig. 1-12. The V-2.

C-1, built by the Minneapolis-Honeywell Regulator Company.<sup>61</sup> It was installed in all the American four-engined bombers such as the B-17, the B-24, and the B-29. In the C-1 deviations from the reference attitude in pitch, roll, and yaw were measured with vertical and directional gyroscopes, as in the Sperry A2 design, but the amplified signals were applied to electrical positioning servomotors driving the elevators, ailerons, and rudders. The automatic pilot unit was specifically designed to subject the aircraft to commands originating in the bombardier's operation of the bombsight, but a single-knob turn control, and later a "formation stick," was also provided for the pilot. Erection cutout in turns was one of the novel features of this automatic pilot. The electrically driven gyroscopes were not intended to be used as flight instruments. Later, in the C-1A

<sup>61</sup> W. H. Gille and R. J. Kutzler, "Application of Electronics to Aircraft Flight Control," *Trans. AIEE*, 63, 849-853 (Nov. 1944); see also W. H. Gille and H. T. Sparrow, "Electronic Autopilot Circuits," *Electronics*, 17, 110-117 (Oct. 1944).

(1945), a number of improvements were introduced, including the addition of a yaw rate gyro signal to the rudder.

This unit was closely followed in time by the General Electric design,<sup>62</sup> which was similar to the Sperry pneumatic/hydraulic A2 unit in its functions, and which differed from the C-1 in its design mainly in that the electrically driven gyroscopes were also the panel-mounted flight instruments, and in that electrohydraulic positioning servomotors provided the final stage of power amplification. At first the controls were merely trim knobs, one for each axis, but later a single-knob turn control was introduced.<sup>63</sup> This automatic pilot was notable for its light weight (74.5 lb), so that it was suitable for installation in fighters and light attack aircraft.

The Sperry Gyroscope Company also developed an electric automatic pilot during the war, the A5. It had a number of novel features such as altitude control, automatic elevator trim by means of an additional trim tab servo, and, notably, equalizing circuits that provided for phase advance of the servo actuating signals. The servos were electrohydraulic units with *force* feedback. This was thought to be a desirable feature in that, since control surface effectiveness and aerodynamic hinge moment vary in much the same way with speed and altitude, the closed-loop response with force feedback should tend to be invariant with flight condition.<sup>64</sup> Early flight tests in a Fairchild 22 were encouraging. Unfortunately, as it turned out in practice, the deleterious effect of control cable friction made it extremely difficult to secure satisfactory operation in the large aircraft,<sup>65</sup> such as the B-24E "Liberator," for which this automatic pilot was intended.

The Eclipse-Pioneer Division of the Bendix Corporation began work on the "flux-gate" compass in 1939 and, about 1943, introduced automatic heading control from the flux-gate compass in the all-electric P-1 (A-10) autopilot. This equipment obviated the very tiresome necessity of frequently resetting the directional gyro by reference to the magnetic compass. The P-1 also featured a yaw rate gyro signal fed to the rudder, automatic synchronization so that it could be engaged in any attitude, and a computed up-elevator signal in turns.

Somewhat later, considerable effort was expended in designing automatic pilots specifically for fighter aircraft; Lear introduced the F-5

<sup>62</sup> C. M. Young, E. E. Lynch, and E. R. Boynton, "Electrical Control in Automatic Pilots," *Trans. AIEE*, 63, 939-943 (Nov. 1944).

<sup>63</sup> H. R. Hopkin and R. W. Dunn, "Theory and Development of Automatic Pilots, 1937-1947," RAE Rept. IAP 1459, Monograph 2.5.03, Aug. 1947.

<sup>64</sup> P. Halpert and O. E. Esval, "Electric Automatic Pilots for Aircraft," *Trans. AIEE*, 63, 861-866 (Nov. 1944).

<sup>65</sup> B. Levine, Discussion of paper by P. Halpert and O. E. Esval, *Trans. AIEE*, 63, 1501 (Nov. 1944).

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automatic pilot about 1950.<sup>66</sup> In the F-5 the problem of supplying power amplification for the control surface positioning servos was solved with magnetic powder clutches. The use of these units eliminated much relatively heavy equipment such as electronic or rotating power amplifiers and provided for high performance servomechanisms in a compact and lightweight package. The gyroscopic references, however, were the conventional vertical and directional gyroscopes, and the maneuvers that could be made under automatic control were limited by the phenomenon of "gimbal lock." This problem was attacked by Westinghouse<sup>67</sup> and the Instrumentation Laboratory at the Massachusetts Institute of Technology (MIT),<sup>68</sup> both of whom constructed laboratory models of fighter airplane all-attitude sensors, using different kinds of single degree of freedom gyroscopes.

Improvements were also made in providing for guidance tie-ins. Immediately after the war the Sperry Gyroscope Company brought out the A-12 automatic pilot,<sup>69</sup> and Bendix introduced the PB-10.<sup>70</sup> Both of these were equipped with approach couplers, and the Bendix system had automatic throttle controls for the control of airspeed on the approach to the runway. In England the Smith's firm brought out the all-electric, rate-rate SEP-2 automatic pilot and approach coupler.<sup>71</sup>

All the elements of a modern automatic pilot were now at hand, and in 1947 the United States Air Force All-Weather Flying Division's C-54 "Robert E. Lee," equipped with a Sperry automatic pilot and approach coupler and Bendix throttle controls, made a dramatic demonstration of completely automatic flight. Taking off from Stephenville, Newfoundland, on the evening of September 21, it flew through the night across the Atlantic and landed the next day at Brize-Norton in England. From the time the brakes were released for the takeoff roll until the landing roll was complete, no human hand touched the controls. The selection of course, radio station, speed, flap setting, landing gear position, and the final application of wheel brakes were all accomplished automatically from a

<sup>66</sup> J. Harper, "George Turns Tiger," *Flying Safety*, Jan. 1956.

<sup>67</sup> C. R. Hanna, K. A. Oplinger, and G. R. Douglas, "Automatic Flight Control System Using Rate Gyros for Unlimited Maneuvering," *Electrical Engineering*, 73, No. 5, 443-448 (May 1954).

<sup>68</sup> H. P. Whitaker, J. A. Gautraud, and S. A. Wingate, "Flight Test Evaluation of the MIT Automatic Control System for Aircraft," MIT Instrumentation Lab. Rept. R-55, 1953.

<sup>69</sup> P. Halpert, *The A-12 Gyropilot*, paper presented to the SAE, New York, (Apr. 13-15, 1948).

<sup>70</sup> J. C. Owen, "Automatic Pilots," *Electrical Engineering*, 67, No. 6, 551-561 (June 1948); P. A. Noxon, "Flight Path Control," *Aeron. Eng. Rev.*, 7, No. 8, 36-45 (Aug. 1948).

<sup>71</sup> F. W. Meredith, "The Modern Autopilot," *J. Roy. Aeron. Soc.*, 409-428 (May 1949); see also W. H. Coulthard, *Aircraft Instrument Design*, Pitman, New York, 1952.

program stored on punched cards. The complete automation of aircraft flight seemed to be at hand. Anast described the performance and the prospects early in 1948.<sup>72</sup>

While the development of automatic pilots up to 1950 had, in general, been responsive to the needs of potential users, and while such features as single-knob turn control, erection cutout, automatic trim, altitude control, synchronizers, rate gyro feedbacks, compass tie-in, and approach coupling served useful purposes in some applications, in many cases they were introduced only as the technology became readily available and certainly not because they were *required* for any particular airplane. The automatic pilot was almost universally regarded as a useful but hardly an essential item of equipment; the day had not yet arrived when an automatic pilot would be designed for a specific airplane by taking into account, right from the beginning, particular and peculiar features of the mission of the airplane and of the design of its control system.

Perhaps because of the obvious necessity for a special design of the flight control system in a missile and the successes achieved with that approach, and certainly, in part, because the postwar generation of jet airplanes almost invariably needed some form of stability augmentation whose particular nature was often dictated by the unique configuration or mission of the vehicle, the most common design practice later came to be based on a careful enumeration of *requirements* to be met and *functions* to be performed by each system. This alteration in the methods of design was only one of several that, together, radically changed the nature of work in automatic flight control.

## 1-6. The Joining of Control Technology and Dynamic Analysis

While the mathematical tools for performing analyses of automatic flight control systems for aircraft had existed in at least a rudimentary form before the war of 1939–1945, there has been occasion to remark that they did not seem to be much used. The work of Gates, Garner, and Cowley<sup>73</sup> seems to have been nearly forgotten. Oppelt,<sup>74</sup> even though translated into English, does not seem to have attracted much attention, while Minorsky's paper<sup>75</sup> on the steering of ships was fairly widely known but did not seem to inspire other workers to follow similar lines. There had

<sup>72</sup> J. L. Anast, "Automatic Aircraft Control," *Aeron. Eng. Rev.*, 7, No. 7, 20–24 (July 1948).

<sup>73</sup> See notes 42–47.

<sup>74</sup> W. Oppelt, "Comparison of Automatic Control Systems," NACA TM 966 Feb. 1941.

<sup>75</sup> N. Minorsky, "Directional Stability of Automatically Steered Bodies," *J. Amer. Soc. of Naval Eng.*, 34, 280–309 (1922).

appeared, here and there, papers or monographs on the theory of servomechanisms,<sup>76</sup> the regulation of prime movers,<sup>77</sup> process control,<sup>78</sup> the dynamics of instruments,<sup>79</sup> and the Cauchy/Heaviside operational calculus applied to the dynamic response of aircraft,<sup>80</sup> but apparently the need to study the feedback control of aircraft for practical reasons was not yet felt.

The subject of the theory of automatic control of aircraft did receive some attention in the universities, technical institutes, and research laboratories, however. Longitudinal stability of an airplane under the action of a control system of the Sperry A2 or A3 automatic pilot type was investigated by Klemin, Pepper, and Wittner<sup>81</sup> at New York University, and H. K. Weiss<sup>82</sup> at Massachusetts Institute of Technology performed a comprehensive study of the stability of an automatically controlled airplane, including both the free longitudinal and lateral motions and the response to gusts, as his thesis research for the Master's degree. There was also a very original contribution from Imlay,<sup>83</sup> who explored the problem of selecting "optimum" gearings; but all this barely

<sup>76</sup> H. L. Hazen, "Theory of Servomechanisms," *J. Franklin Inst.*, 218, No. 3, 279-331 (Sept. 1934); see also "Design and Test of a High Performance Servomechanism," *J. Franklin Inst.*, 218, No. 5, 543-580 (Nov. 1934).

<sup>77</sup> A. Stodola, *Steam and Gas Turbines*, Vol. 1, translated from German 6th edn. by C. L. Loewenstein, McGraw-Hill, New York, 1927; W. Trinks, *Governors and the Governing of Prime Movers*, Van Nostrand, New York, 1919; H. K. Weiss, "Constant Speed Control Theory," *J. Aeron. Sci.*, 6, No. 4, 147-152 (Feb. 1939).

<sup>78</sup> G. Wünsch, *Regler für Druck und Menge*, R. Oldenbourg, Munich, 1930; A. Ivanoff, "Theoretical Foundations of the Automatic Regulation of Temperature," *J. Inst. of Fuel* (London), 7, 117-130 (Feb. 1934); S. D. Mitereff, "Principles Underlying the Rational Solution of Automatic Control Problems," *Trans. ASME*, 57, No. 4, 159-163 (May 1935); E. S. Smith, Jr., "Automatic Regulators, Their Theory and Application," *Trans. ASME*, 9, No. 4, 159-163 (May 1936); see also *Automatic Control Engineering*, McGraw-Hill, New York, 1944; A. Callender, D. R. Hartree, and A. Porter, "Time Lag in a Control System," *Phil. Trans. Roy. Soc. A235*, 415-444 (1936); C. E. Mason and G. A. Philbrick, "Automatic Control in the Presence of Process Lags," *Trans. ASME*, 62, 295-308 (1940).

<sup>79</sup> C. S. Draper and G. V. Schliestett, "General Principles of Instrument Analysis," *Instruments*, 12, No. 5, 137-142, (May 1939); C. S. Draper and G. P. Bentley, "Design Factors Controlling the Dynamic Performance of Instruments," *Trans. ASME*, 62, No. 5, 421-432 (July 1940).

<sup>80</sup> L. W. Bryant and D. H. Williams, "The Application of the Method of Operators to the Calculation of the Disturbed Motion of an Airplane," ARC R & M 1346, July 1930. R. T. Jones, "A Simplified Application of the Method of Operators to the Calculation of the Disturbed Motion of an Airplane," NACA TR 560, 1936; see also "Calculation of the Motion of an Airplane Under the Influence of Irregular Disturbances," *J. Aeron. Sci.*, 3, No. 12, 419-425 (Oct. 1936). A. Klemin and B. F. Ruffner, "Operator Solutions in Airplane Dynamics," *J. Aeron. Sci.*, 3, No. 7, 252-255 (May 1936).

<sup>81</sup> A. Klemin, P. A. Pepper, and H. A. Wittner, "Longitudinal Stability in Relation to the Use of an Automatic Pilot," NACA TN 666, Sept. 1938.

<sup>82</sup> H. K. Weiss, "Theory of Automatic Control of Airplanes," NACA TN 700, Apr. 1939.

<sup>83</sup> F. H. Imlay, "A Theoretical Study of Lateral Stability with an Automatic Pilot," NACA TR 693, 1940.

carried the matter further than the state in which it had been left by the British authors nearly 15 years before. The difficulty, as Weiss pointed out, lay in the necessity for factoring characteristic functions of the fifth, sixth, and seventh degree. The same problem, of course, plagued students of other feedback control devices, and a considerable effort was made to find convenient methods for accomplishing the tedious algebra.<sup>84</sup>

In 1944, discussing the subject of automatic flight and airplane stability (which he treated separately), Zand<sup>85</sup> still found the situation very unsatisfactory. He wrote,

a thorough knowledge of the stability of the airplane is a prerequisite toward the successful solution of the problem of automatic flight. We have sketched the difficult path which led airplane designers toward the understanding of fundamentals of dynamic stability. An equally thorny road full of obstacles has been conquered by the instrument engineer who succeeded against such odds as space and weight limitations, lack of power, etc. . . . In many instances the airplane engineering field and the instrument engineering fraternity have worked independently on the problem which concerns both branches directly. Pooling the knowledge of dynamic stability with the knowledge of instrument design for the general betterment of aeronautics is essential . . . Experience has shown that if the matching is performed theoretically first the number of experimental flying hours will be reduced to a minimum and the results obtained superior.

Indeed, the theoretical matching of the two subjects was shortly to become not merely a desideratum but an absolute necessity.

The war had seen the advent, on both sides, of the turbojet engine, so that suddenly the limits of the flight envelope were enormously extended in both speed and altitude, with concomitant configuration changes involving increased wing loadings, mass distributions concentrated in long thin fuselages, and the aerodynamic benefits of short span, swept wings. These changes led to a marked defect in the damping of the longitudinal short period and dutch roll oscillations in the airplanes of the immediate postwar period. However, it was not only the "classical" modes that were deficient in stability, previously unknown coupled modes made their

<sup>84</sup> Y. J. Liu, "Servomechanisms, Charts for Finding Their Stability and for Finding the Roots of Their Third and Fourth Degree Characteristic Equations," Dept. of Electrical Eng., Massachusetts Institute of Technology, Oct. 1941; Shi-Nge Lin, "Method of Successive Approximations of Evaluating the Real and Complex Roots of Cubic and Higher Order Equations," *J. Math. and Phys.*, 20, 231-242 (Aug. 1941); A. Porter and C. Mack, "New Methods for the Numerical Solution of Algebraic Equations," *Phil. Mag.* 40, No. 304, 578-585 (May 1949); H. R. Hopkin, "Routine Computing Methods for Stability and Response Investigations on Linear Systems," ARC R & M 2392, Aug. 1946.

<sup>85</sup> S. J. Zand, "Automatic Flight and Airplane Stability," *Aviation*, 43, No. 6, 140-141, 290-296 (June 1944).

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appearance. Among these were fuel slosh<sup>86</sup> and the rolling instability.<sup>87</sup> Furthermore, thinner wings and finer fuselages combined with advances in materials and manufacturing processes made for increased structural flexibility. Power-boosted controls<sup>88</sup> had also come into use to handle the large hinge moments of the control surfaces. Early hydraulic power units had stability difficulties of their own,<sup>89</sup> and the inevitable lag was inimical to the stability of automatic flight control systems. All these trends were bad news for the automatic flight control system designer, who now desperately wanted analytical help.

The joining of control technology and vehicle dynamic analysis which would, no doubt, have come about in any event, was forced by the marked deficiencies in stability of the new jet aircraft and also by the advent of the guided missile, in which it was obviously essential to match the dynamics of the airframe and the control system from the first flight on.<sup>90</sup> One of the first results was the "stability augments," a feedback control designed to modify the inherent aerodynamic stability of the airframe, augmenting one or more of the stability derivatives by imposing forces or moments through actuation of the controls in response to motion variables. Thus, in short order, the following devices were invented or reinvented:

- Yaw damper
- Sideslip stability augments
- Pitch damper
- Roll damper
- Transonic trim shifter
- Autothrottle

<sup>86</sup> A. Schy, "A Theoretical Analysis of the Effects of Fuel Motion on Airplane Dynamics," NACA Rept. 1080, 1952; H. Luskin and E. Lapin, "An Analytical Approach to the Fuel Sloshing and Buffeting Problems of Aircraft," *J. Aero. Sci.*, 19, No. 4, 217-228 (Apr. 1952).

<sup>87</sup> F. D. Graham and R. C. Uddenberg, "The Dynamic Stability and Control Problem of a Pivoted-Wing Supersonic Pilotless Aircraft," Boeing Airplane Co., Document D-8810, Feb. 1948; W. H. Phillips, "Effect of Steady Rolling on Longitudinal and Directional Stability," NACA TN 1627, June 1948.

<sup>88</sup> D. J. Lyons, "Present Thoughts on the Use of Powered Flying Controls in Aircraft," *J. Roy. Aeron. Soc.*, 53, 253-277 (Mar. 1949); T. A. Feeney, "Powered Control System Design Practice at Northrop Aircraft," Proc. Bureau of Aeronautics Symposium on Analysis and Design of Power Boosted and Power Operated Surface Control Systems, Oct. 6-7, 1949.

<sup>89</sup> D. T. McRuer, "An Analysis of Northrop Aircraft Powered Flight Controls," Proc. Bureau of Aeronautics Symposium on Analysis and Design of Power Boosted and Power Operated Surface Control Systems, Oct. 6-7, 1949.

<sup>90</sup> While it seems surprising that the developers of the V-2 were able to make do with stability diagrams and a rudimentary simulator, they did not entirely neglect analysis in the design of the control and guidance equipment. The frequency response method and Nyquist stability criterion were known but were not "popular." See O. Müller, "The Control System of the V-2," in T. Benecke and A. W. Quick, eds., *History of German Guided Missile Development*, Verlag E. Appelhans, Brunswick, Germany, 1959.

Since both the problem and the solution were inevitably connected with a particular aircraft and its control system, the old methods of designing general purpose equipment were totally inadequate. Extensive analysis and simulation for each application to a piloted aircraft or missile were found to be required.

In the interim, wartime pressures for very high performance servo-mechanisms and regulators for such uses as servo-controlled aircraft wing spar milling machines,<sup>91</sup> process controls in the manufacture of fissionable material,<sup>92</sup> and particularly for fire control and navigation computers,<sup>93</sup> as well as antenna drives for tracking radars,<sup>94</sup> had led to the widespread adoption of analytical design techniques originally developed for long distance telephone amplifiers.<sup>95</sup> While James, Nichols, and Phillips credit John F. Taplin, at the Massachusetts Institute of Technology, with frequency response technique work for servomechanisms as early as 1937, substantially nothing concerning the matter appeared in public print until after the war was over. It seems clear that the same or very similar ideas were shared by widely separated investigators. At both the Bell Telephone Laboratories and the Massachusetts Institute of Technology, classified memoranda were prepared on the eve of the United States involvement.<sup>96</sup> As far as the present authors know, these historic documents have still not been released. Later, at the Massachusetts Institute of Technology, Hall<sup>97</sup> prepared a dissertation that was initially classified but was released in 1947. The effort to keep all this information classified did not prevent the duplicate development and use of the ideas

<sup>91</sup> *Electronics*, Oct. 1944.

<sup>92</sup> H. Smyth, *Atomic Energy for Military Purposes*, Sections 7 and 27 and Appendix 4, Princeton University Press, Princeton, N.J., 1945.

<sup>93</sup> I. A. Greenwood, Jr., J. V. Holdam, Jr., and D. MacRae, Jr., *Electronic Instruments*, McGraw-Hill, New York, 1948.

<sup>94</sup> H. M. James, N. B. Nichols, and R. S. Phillips, *Theory of Servomechanisms*, McGraw-Hill, New York, 1947.

<sup>95</sup> H. Nyquist, "Regeneration Theory," *Bell Systems Tech. J.*, 11, No. 1, 126-147 (Jan. 1932); see also "The Regeneration Theory," *Trans. ASME*, 76, No. 8, 1151 (Nov. 1954); H. S. Black, "Stabilized Feedback Amplifiers," *Bell System Tech. J.*, 13, No. 1, 1-18; (Jan. 1934); see also United States Patent 2,102,671, Dec. 1937; E. Peterson, J. G. Kreer, and L. A. Ware, "Regeneration Theory and Experiment," *Bell System Tech. J.*, 13, No. 10, 680-700 (Oct. 1934); H. W. Bode, *Amplifiers*, United States Patent 2,123,17, July 12, 1938; see also "Relations Between Attenuation and Phase in Feedback Amplifier Design," *Bell System Tech. J.*, 19, No. 3, 421-454 (July 1940); *Network Analysis and Feedback Amplifier Design*, Van Nostrand, New York, 1945; "Feedback—The History of an Idea," in *Active Networks and Feedback Systems*, Polytechnic Press, Brooklyn, New York, 1961.

<sup>96</sup> D. C. Bomberger and B. T. Weber, "Stabilization of Servomechanisms," Bell Telephone Laboratories Restricted Publication M.M.-41-110152, Dec. 10, 1941; H. Harris, Jr., "The Analysis and Design of Servomechanisms," OSRD, NDRC (Section D-2), Rept. 454, Jan. 1942. (Brown and Campbell give the date of this report as Dec. 1941, but other authorities agree on the one given here.)

<sup>97</sup> A. C. Hall, *The Analysis and Synthesis of Linear Servomechanisms*, Technology Press, Cambridge, Mass., 1943.

on several sides. Almost simultaneously with Hall's dissertation, Profos<sup>98</sup> contributed his own at Zurich; a little later the book that we know in translation as *The Dynamics of Automatic Control*<sup>99</sup> was published in Munich. The system was not even airtight on our own side. The earliest published work in English that refers to the frequency response method in control seems to have been the 1944 paper by Prinz.<sup>100</sup>

Immediately upon the cessation of hostilities there appeared a rash of papers by the original contributors, and others.<sup>101</sup> Almost simultaneously with the papers a growing number of books began to appear, many of which are almost as valuable today as when they were first published.<sup>102</sup> Typically, these books not only expounded the new theory of the frequency response of automatic control systems but further connected it to the performance in the time domain via the correspondence between the transfer function and the transient response as revealed by the Laplace transform method. Operational techniques were not new but their rigorous and respectable foundation in the Laplace transformation was, at that time, a comparatively recent development.<sup>103</sup>

<sup>98</sup> P. Profos, *Die Behandlungen von Regel Problemen vermittels des Frequenzganges des Regelkreises*, Ph.D. dissertation, A. G. Gebr. Leeman, Zurich, 1943; also "A New Method of Regulating System Design," *Sulzer Tech. Rev.*, No. 2 (1945).

<sup>99</sup> R. C. Oldenbourg and H. Sartorius, *Dynamik Selbsttätiger Regelung*, R. Oldenbourg, Munich, 1944; see also *The Dynamics of Automatic Control*, ASME, New York, 1948.

<sup>100</sup> D. G. Prinz, "Contributions to the Theory of Automatic Controllers and Followers," *J. Sci. Instrum.* 21, No. 4, 53-64 (Apr. 1944).

<sup>101</sup> D. P. Campbell, "Theory of Automatic Control Systems," *Industrial Aviation*, 62-64, 94, 95 (Sept. 1945); E. B. Ferrell, "The Servo Problem as a Transmission Problem," *Proc. IRE*, 33, No. 11, 763-767 (Nov. 1945); G. S. Brown and A. C. Hall, "Dynamic Behavior and Design of Servomechanisms," *Trans. ASME*, 68, 503-524 (1946); A. C. Hall, "Application of Circuit Theory to Design of Servomechanisms," *J. Franklin Inst.*, 242, No. 4, 279-307 (Oct. 1946); see also "Early History of the Frequency Response Field," *Trans. ASME*, 76, No. 8, 1153-1154 (Nov. 1954); H. Harris, Jr., "Frequency Response of Automatic Control Systems," *Trans. AIEE*, 65, 539-545 (1946); R. E. Graham, "Linear Servo Theory," *Bell System Tech. J.*, 25, No. 4, 616-651 (Oct. 1946); H. T. Marcy, "Parallel Circuits in Servomechanisms," *Trans. AIEE*, 65, 521-529 (1946).

<sup>102</sup> L. A. McColl, *Fundamental Theory of Servomechanisms*, Van Nostrand, New York, 1945; H. Lauer, R. Lesnick, and L. E. Matson, *Servomechanism Fundamentals*, McGraw-Hill, New York, 1947; H. M. James, N. B. Nichols, and R. S. Phillips, *Theory of Servomechanisms*, McGraw-Hill, New York, 1947; W. R. Ahrendt and J. F. Taplin, *Automatic Regulation*, Vol. I, Ahrendt and Taplin, Washington, D.C., 1947; I. A. Greenwood, Jr., J. V. Holdam, Jr., and D. MacRae, Jr., *Electronic Instruments*, McGraw-Hill, New York, 1948; G. S. Brown and D. P. Campbell, *Principles of Servomechanisms*, Wiley, New York, 1948.

<sup>103</sup> G. Doetsch, *Theorie und Anwendung der Laplace Transformation*, Springer-Verlag, Berlin, 1937; N. W. MacLachlan, *Complex Variable and Operational Calculus*, Cambridge University Press, London, 1939; H. S. Carslaw and J. C. Jaeger, *Operational Methods in Applied Mathematics*, Clarendon Press, Oxford, 1941; M. F. Gardner and J. L. Barnes, *Transients in Linear Systems*, Vol. I, Wiley, New York, 1942; R. V. Churchill, *Modern Operational Mathematics in Engineering*, McGraw-Hill, New York, 1944.

There also appeared, immediately after the war, accounts of the improved mechanical analog computer developed at the Massachusetts Institute of Technology<sup>104</sup> and of the digital scientific calculators developed at Harvard University,<sup>105</sup> at the Bell Telephone Laboratories,<sup>106</sup> and at the University of Pennsylvania.<sup>107</sup> These machines had originally been employed primarily to compute ballistic tables but their potentialities for the solution of other problems, including the design of feedback control systems for aircraft, was quite plain. Furthermore, requirements for fire control computers had led to the development of a variety of new or improved components. Among these was the d-c or operational amplifier. In a prophetic paper published in 1947, Ragazzini, Randall, and Russell<sup>108</sup> pointed out that these might be used in a general-purpose machine for solving differential equations, an electronic analog computer. A number of firms almost immediately developed such machines for sale or for their own use, and by 1950 they were fairly common. Several universities also developed their own machines; among the first was the University of Michigan. In an early report on the feasibility of electronic analog computation,<sup>109</sup> autopilot control of the longitudinal motion of an airplane was given as one of the illustrative examples.

Knowledge of the development of the new methods of analysis and of the newly available computers spread very rapidly, and one could almost say that a new branch of the engineering profession came suddenly into being: men were proud to call themselves feedback systems engineers, or "systems engineers" for short, some of them became aircraft control system engineers. Application of the frequency response (or transfer function) technique to the design of aircraft and their control systems

<sup>104</sup> V. Bush and S. H. Caldwell, "A New Type of Differential Analyzer," *J. Franklin Inst.*, 240, No. 4, 255-326 (Oct. 1945).

<sup>105</sup> H. H. Aiken and G. M. Hopper, "The Automatic Sequence Controlled Calculator—I," *Elec. Eng.*, 65, No. 8-9, 384-391 (Aug.-Sept. 1946); "The Automatic Sequence Controlled Calculator—II," *Elec. Eng.*, 65, No. 10, 449-454 (Oct. 1946); "The Automatic Sequence Controlled Calculator—III," *Elec. Eng.*, 65, No. 11, 522-528 (Nov. 1946).

<sup>106</sup> F. L. Alt, "A Bell Telephone Laboratories Computing Machine—I," *Math. Tables and Other Aids to Computation*, 3, No. 21, 1-13 (Jan. 1948); "A Bell Telephone Laboratories Computing Machine—II," *Math. Tables and Other Aids to Computation*, 3, No. 22, 69-84 (Apr. 1948).

<sup>107</sup> H. H. Goldstine and A. Goldstine, "The Electronic Numerical Integrator and Computer (ENIAC)," *Math. Tables and Other Aids to Computation*, 2, No. 15, 97-110 (July 1946).

<sup>108</sup> J. R. Ragazzini, R. H. Randall, and F. A. Russell, "Analysis of Problems in Dynamics by Electronic Circuits," *Proc. IRE*, 35, No. 5, 442-452 (May 1947).

<sup>109</sup> D. W. Hagelbarger, C. E. Howe, and R. M. Howe, "Investigation of the Utility of an Electronic Analog Computer in Engineering Problems," External Memo. 28, Eng. Res. Inst., University of Michigan, Apr. 1, 1949.

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was pointed out early,<sup>110</sup> and it quickly became a part of the design process for actual aircraft (or missile) control systems.<sup>111</sup>

Further improvements and extensions of the analytical techniques were also discovered. By no means the least of these, Evans' locus of roots method<sup>112</sup> was inspired by consideration of the problems of aircraft and missile flight control. In the classic Fourteenth Wright Brothers Lecture for the year 1950, Bollay summarized the then existing state of the art and pointed to the use of the Laplace transformation, frequency response techniques, the Nyquist stability criterion, the root locus method, analog computers, and other tools of the systems engineer in the design departments of the major aircraft manufacturers.<sup>113</sup>

At Northrop Aircraft, based in part on the experiences gained with the power controls and stability augmentation system developments needed for tailless and other advanced designs,<sup>114</sup> and in a comprehensive study of the F-5 automatic pilot for the F-89A aircraft,<sup>115</sup> an attempt was made to summarize the most useful aspects of the new knowledge of aircraft control system engineering in a series of seven volumes, prepared for the U.S. Navy Bureau of Aeronautics.<sup>116</sup> These volumes began to appear in

<sup>110</sup> W. F. Milliken, Jr., "Progress in Dynamic Stability and Control Research," *J. Aeron. Sci.*, 14, No. 9, 493-519 (Sept. 1947); J. B. Rea, *Automatic Tracking Control of Aircraft*, Sc.D thesis, Massachusetts Institute of Technology, 1947; H. Greenberg, "Frequency-Response Method for Determination of Dynamic Stability Characteristics of Airplanes with Automatic Controls," NACA TN 1229, Mar. 1947; R. C. Seamans, Jr., B. G. Bromberg, and L. E. Payne, "Application of the Performance Operator to Aircraft Automatic Control," *J. Aeron. Sci.*, 15, No. 9, 535-555 (Sept. 1948); J. R. Moore, "Application of Servo Systems to Aircraft," *Aeron. Eng. Rev.*, 8, No. 1, 32-43, 71 (Jan. 1949); C. L. Seacord, "Application of Frequency Response Analysis to Aircraft Autopilot Stability," *J. Aeron. Sci.*, 17, No. 8, 481-498 (Aug. 1950).

<sup>111</sup> P. A. Noxon, "Flight Path Control," *Aeron. Eng. Rev.*, 17, No. 8, 38-45 (Aug. 1948); R. J. White, "Investigation of Lateral Dynamic Stability in the XB-47, Airplane," *J. Aeron. Sci.*, 17, No. 3, 133-148 (Mar. 1950).

<sup>112</sup> W. R. Evans, *Servo Analysis by Locus of Roots Method*, North American Aviation Rept. AL-787, Nov. 1, 1948; "Graphical Analysis of Control Systems," *Trans. AIEE*, 67, 547-551 (1948); "Control System Synthesis by the Root Locus Method," *Trans. AIEE*, 69, 66-69 (1950); "The Use of Zeros and Poles for Frequency Response or Transient Response," *Trans. ASME*, 76, No. 8, 1335-1342 (Nov. 1954); *Control System Dynamics*, McGraw-Hill, New York, 1954.

<sup>113</sup> W. Bollay, "Aerodynamic Stability and Automatic Control," *J. Aeron. Sci.* 18, No. 9, 569-624 (Sept. 1951).

<sup>114</sup> D. T. McRuer, "An Electronic Tail for the Flying Wing," *Flight Lines*, (Northrop Aircraft, Inc) Nov. 1950.

<sup>115</sup> "Analysis Final Report: Analysis of Type F-5 Automatic Pilot Applied to the Type F-89 Aircraft and Control System," Northrop Aircraft, Servomechanisms and Dynamics Section, Rept. SMD-3, Sept. 13, 1950.

<sup>116</sup> "Fundamentals of Design of Pilot Aircraft Flight Control Systems," Bureau of Aeronautics Rept. AE-61-4; Vol. 1, "Methods of Analysis and Synthesis of Piloted Aircraft Flight Control Systems," Mar. 1952; Vol. 2, "Dynamics of the Airframe," Sept. 1952; Vol. 3, "The Human Pilot," Aug. 1954; Vol. 4, "The Hydraulic System," Mar. 1953; Vol. 5, "The Artificial Feel System," May 1953; Vol. 6, "Automatic Flight Control Systems for Piloted Aircraft," Apr. 1956; Vol. 7, "Methods of Design and Evaluation of Interceptor Fire Control Systems," Oct. 1959.

March 1952. The genealogy of the present work can be traced directly to several of the "Northrop Volumes." In fact, it began, about 10 years after the initial summary, as an effort to revise and update Volumes 2 and 6 and to provide in one volume a comprehensive account of the theory and application of analytical techniques in the design of automatic flight control systems.

## CHAPTER 2

# *MATHEMATICAL MODELS OF LINEAR SYSTEM ELEMENTS*

### **2-1. Introduction**

A major task in systems analysis is the estimation of system response to commands or disturbances. The most concrete way to determine behavior is to test the actual system. This direct experimental approach is precluded in the early phases of design, when the "system" may be but one of a number of competing possibilities, or when the physical system may be unavailable. Fortunately, many of the potential results of actual physical measurements can be foreseen by performing experiments utilizing various models of the system.

As the underlying basis for system models consider the block diagram representation of Fig. 2-1. The input, stimulus, command, disturbance, or forcing function elicits an output or response from the system. The system might be one of a very large number of elements including a human being, an airplane, or a society, and the words appropriate to the several portions of the diagram of Fig. 2-1 are quite different in these different contexts. Nevertheless, it is assumed here that we are dealing with cause and effect elements, perhaps combined into larger systems; and that an input and output of a particular element can always be identified. It is further assumed that the relationship between the input and output can be represented by one or more ordinary differential equations. These equations become the mathematical attorneys for the physical elements or systems, and it is through them as intermediaries that the transactions in which we are to engage will ordinarily be conducted.

Many physical elements and systems are practically linear and time-invariant; that is, they can be described adequately over a limited range by linear differential equations with constant coefficients. Examples of linearization abound in the literature,<sup>1</sup> and Chapter 4 presents an example of the assumptions and techniques that are employed, in this case, to

<sup>1</sup> D. Graham and D. McRuer, *Analysis of Nonlinear Control Systems*, Wiley, New York, 1961, pp. 9-12, 445-454; R. W. Jones, "Stability Criteria for Certain Non-linear Systems," in A. Tustin, ed., *Automatic and Manual Control*, Butterworths London, 1952, pp. 319-324.

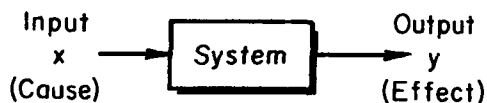


Fig. 2-1. A pattern.

linearize the equations of motion of an aircraft. It can be demonstrated that feedback control itself has the property of linearizing the performance of the systems or elements to which it is applied,<sup>2</sup> so that while all systems and elements are in fact nonlinear, the assumptions of our analyses are often not so restrictive as they may at first appear.

Proceeding on the assumption, for the moment, that we need only be concerned with systems that are linear with constant coefficients, or may legitimately be linearized, we shall introduce in this chapter the powerful and convenient concepts of the Laplace transformation. The rudiments of response calculations are first reviewed, including approximate calculations and modal response ratios. Following this is a discussion of the system descriptors: the weighting function or impulse response and its transform, and the transfer function. Graphical representations for both of these functions are emphasized: time vectors for the weighting function; pole-zero plots,  $j\omega$ -Bode diagrams, and  $\sigma$ -Bode diagrams for the transfer function.

## 2-2. Laplace Transformation

The system of Fig. 2-1 is, according to our assumptions, described by the equation

$$\left( \frac{d^{m+n}}{dt^{m+n}} + b_1 \frac{d^{m+n-1}}{dt^{m+n-1}} + \cdots + b_{m+n-1} \frac{d}{dt} + b_{m+n} \right) y(t) = \kappa \left( \frac{d^n}{dt^n} + a_1 \frac{d^{n-1}}{dt^{n-1}} + \cdots + a_{n-1} \frac{d}{dt} + a_n \right) x(t) \quad (2-1)$$

For any physical system,  $m \geq 1$ . We are interested in discovering certain aspects of the performance of the system such as the stability, accuracy, and speed of response of the output for certain inputs. The *analysis problem* is defined as follows: Given the input and the differential equation, find the output. If the analysis problem were solved for all the inputs to which the system might be subjected and these solutions were tabulated

<sup>2</sup> J. C. West, *Analytical Techniques for Non-linear Control Systems*, English Universities Press, London, 1960, pp. 16-23.

as input-response pairs, the analyst would have a complete description of the performance of the system. Luckily, this turns out to be unnecessary for linear systems.

In modern engineering analysis an equation such as Eq. 2-1 is most often solved, if it has to be, by a digital or an analog computer. On the other hand, a great deal of information concerning the nature of the solutions for a variety of inputs can be found without solving the equation itself. The techniques that are commonly employed, however, are intimately related to the method of solving Eq. 2-1 by means of the Laplace transformation.

The Laplace transformation of a function of time is defined as

$$\mathcal{L}[f(t)] = F(s) = \lim_{\substack{T_2 \rightarrow \infty \\ T_1 \rightarrow 0}} \int_{T_1}^{T_2} f(t)e^{-st} dt \quad (2-2)$$

where  $s$  is the complex variable  $s = \sigma + j\omega$ . Application of the definition allows the construction of tables of the transforms of operations such as differentiation and integration, and tables of the transforms of functions such as an impulse or a sine wave. The inverse transformation [i.e.,  $f(t)$ , given  $F(s)$ ] is usually carried out by finding the appropriate pair in a table so arranged that  $f(t)$  can be associated with a particular  $F(s)$ .

The most interesting and useful properties of the Laplace transformation are summarized (without proof) in Table 2-1.<sup>3</sup>

Common transform pairs for the time functions that occur in the analysis and testing of feedback control systems are catalogued in the short illustrated table of transform pairs presented in Table 2-2.<sup>4</sup> In many cases two alternative forms are given for the transform. In some problems one form is more suitable than the other, so they are used interchangeably as convenience may dictate. To emphasize the physical interpretations of the

<sup>3</sup> For more details on the properties of the transformation and its uses, see: M. F. Gardner and J. L. Barnes, *Transients in Linear Systems*, Wiley, New York, 1942; J. A. Aseltine, *Transform Method in Linear System Analysis*, McGraw-Hill, New York, 1958; R. V. Churchill, *Operational Mathematics*, 2nd edn., McGraw-Hill; New York, 1958; W. Kaplan, *Operational Methods for Linear Systems*, Addison-Wesley, Reading, Mass., 1962; G. Doetsch, *Guide to the Applications of Laplace Transforms*, Van Nostrand, London, 1961.

<sup>4</sup> A table of transform pairs of particular value for vehicular control problems is S. Neumark's *Operational Formulae for Response Calculations*, ARC Tech. Rept. R & M 3075, Her Majesty's Stationery Office, London, 1958. This report comprehensively treats operational fractions of the first, second, third, and fourth order and has additional tables on the reduction of every fraction of fifth or sixth order to a combination of fractions of lower order. Other useful tables of transform pairs are the references on the Laplace transformation and its uses cited previously; see also F. E. Nixon, *Principles of Automatic Controls*, Prentice-Hall, New York, 1953 (the same table has also been published separately as *Handbook of Laplace Transformations*, Prentice-Hall, New York, 1960); A. Erdélyi, F. Oberhettinger, and F. G. Tricomi, *Tables of Integral Transforms*, Vol. 1, McGraw-Hill, New York, 1954.

Table 2-1. Fundamental properties of the Laplace transformation.

Item	$F(s)$	$f(t)$	Special restrictions and remarks
Laplace transformation	$F(s)$	$f(t)$	$t$ is a real variable; $f(t)$ is known and single-valued almost everywhere for $0 \leq t$  $\lim_{\substack{t_2 \rightarrow \infty \\ t_1 \rightarrow 0}} \int_{t_1}^{t_2}  f(t)  e^{-\sigma t} dt < \infty$ for some real number $\sigma$ .
Differentiation	$sF(s) - f(0+)$  $s^2 F(s) - sf(0+) - \frac{df}{dt}(0+)$  $s^n F(s) - \sum_{k=1}^n s^{n-k} \frac{d^{k-1}f}{dt^{k-1}}(0+)$	$\frac{df(t)}{dt}$  $\frac{d^2 f(t)}{dt^2}$  $\frac{d^3 f(t)}{dt^3}$	$f(t)$ and $df(t)/dt$ are Laplace transformable;  $f(t)$ , $df(t)/dt$ and $d^2 f(t)/dt^2$ are Laplace transformable;  $f(t)$ , $df(t)/dt$ , $\dots$ , $d^n f(t)/dt^n$ are Laplace transformable.
Integration	$\frac{F(s)}{s} + \left[ \frac{\int f(t) dt}{s} \right]_{t=0+}$  $\frac{F(s)}{s^k} + \sum_{n=1}^k \frac{f^{-n}(0+)}{s^{k-n+1}}$	$\int f(t) dt$  $\iint \dots \int f(t) (dt)^k$	$f(t)$ is Laplace transformable;  $f(t)$ is Laplace transformable.
Linearity	$aF(s)$  $F_1(s) + F_2(s)$	$af(t)$  $f_1(t) + f_2(t)$	$a$ is a constant or a variable that is independent of $t$ and $s$ ; $f(t)$ is Laplace transformable;  $f_1(t)$ and $f_2(t)$ are Laplace transformable.

Scale change	$aF(as)$	$f\left(\frac{t}{a}\right)$	$f(t)$ is Laplace transformable; $a$ is positive and is a constant or variable independent of $t$ or $s$ .
Convolution or complex multiplication	$F_1(s)F_2(s)$	$\int_0^t f_1(t-\tau)f_2(\tau) d\tau$ or $f_1(t) * f_2(t)$	$f_1(t)$ and $f_2(t)$ are Laplace transformable.
Real multiplication	$\frac{1}{2\pi j} \int_{C_2-j\infty}^{C_2+j\infty} F_1(s-\lambda)F_2(\lambda) d\lambda$ or $F_1(s) * F_2(s)$ $\sum_{i=1}^n \left[ \frac{N_1(s_i)}{dD_1(s)} \right]_{s=s_i} F_2(s-s_i)$	$f_1(t)f_2(t)$	$f_1(t)$ and $f_2(t)$ are Laplace transformable. $\sigma_{a1} + \sigma_{a2} < \sigma$ $\sigma_{a1} < C_2 < \sigma - \sigma_{a2}$ In the special case where $F_1(s) = N_1(s)/D_1(s)$ , a rational fraction having no higher order and $k$ first-order poles, $s_1, s_2, \dots, s_k$
Real translation	$e^{-\tau s}F(s)$ $e^{\tau s}F(s)$	$f(t-\tau)$ $f(t+\tau)$	$f(t)$ is Laplace transformable; $\tau$ is nonnegative real number; and $f(t-\tau) = 0; \quad 0 < t < \tau$ $f(t+\tau) = 0; \quad -\tau < t < 0$
Complex translation	$F\left(s + \frac{1}{T}\right)$ $F\left(s - \frac{1}{T}\right)$ $F(Ts - b)$	$e^{-t/T}f(t)$ $e^{t/T}f(t)$ $\left(\frac{1}{T}\right)e^{bt/T} f\left(\frac{t}{T}\right)$	$f(t)$ is Laplace transformable; $T$ has a nonnegative real part.

Table 2.1 (Continued)

Item	$F(s)$	$f(t)$	Special restrictions and remarks
Final value	$\lim_{s \rightarrow 0} sF(s)$	$\lim_{t \rightarrow \infty} f(t)$	$f(t)$ and $df(t)/dt$ are Laplace transformable; $sF(s)$ is analytic in the right half-plane and on the axis of imaginaries.
Initial value	$\lim_{s \rightarrow \infty} sF(s)$	$\lim_{t \rightarrow 0} f(t)$	$f(t)$ and $df(t)/dt$ are Laplace transformable. $\lim_{s \rightarrow \infty} sF(s)$ exists.
Complex differentiation	$-\frac{d}{ds} F(s)$ $(-1)^n \frac{d^n F(s)}{ds^n}$	$tf(t)$ $t^n f(t)$	$f(t)$ is Laplace transformable.
Complex integration	$\int_s^\infty F(s) ds$	$\left(\frac{1}{t}\right) f(t)$	$F(t)$ and $f(t)/t$ are Laplace transformable $\int_s^\infty F(s) ds$ exists.
Periodic functions	$\int_0^T \frac{e^{-st} f(t) dt}{1 - e^{-Ts}}$	$f(t)$	$f(t)$ is Laplace transformable. $f(t) = f(t + T)$
Antiperiodic functions	$\int_0^T \frac{e^{-st} f(t) dt}{1 + e^{-Ts}} = F(s)$ $\frac{F(s)}{1 - e^{-Ts}}$ $F(s) \coth\left(\frac{Ts}{2}\right)$	$f(t)$ Half-wave rectification of $f(t)$ . Full-wave rectification of $f(t)$ .	$f(t)$ is Laplace transformable. $f(t) = -f(t + T)$