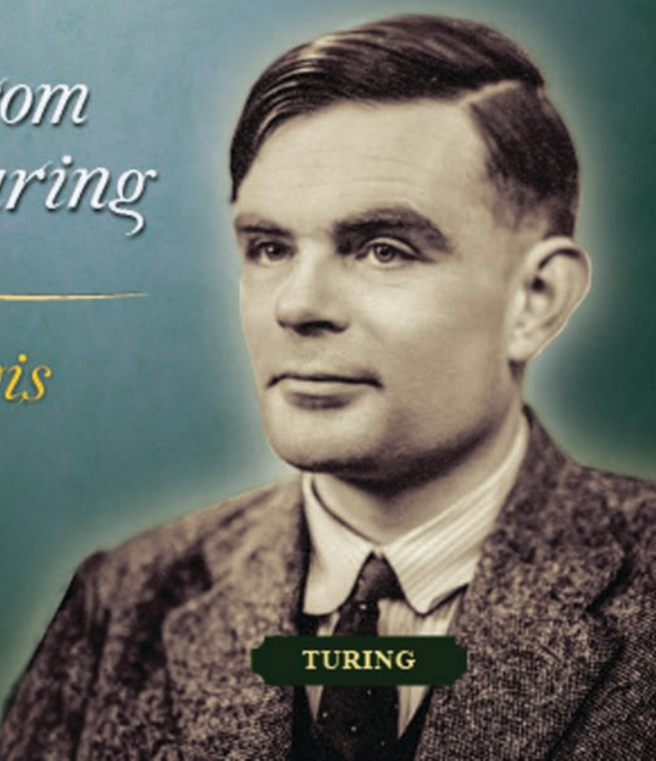
the Universal Computer

Third Edition

The Road from Leibniz to Turing

Martin Davis



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To Virginia, my life's companion

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Preface to Third Edition

For this edition, I have had the opportunity to write about deep learning algorithms and the astonishing AlphaGo program that defeats human masters of the ancient game of Go. I tried to disentangle what is known about the relationship between Georg Cantor and Leopold Kronecker from the widespread myths about their relationship. In addition, I have detailed the drama of ideas in Princeton during the 1930s involving Alonzo Church, his student Stephen Kleene, and Kurt Gödel, before Alan Turing's arrival.

I am grateful to Susan Dickie, Harold Edwards, Dana Scott and François Treves who helped me in various ways. I'm particularly grateful to Thore Graepelof DeepMind for patiently explaining AlphaGo to me. Finally, I want to thank Sarfraz Khan, an editor who understood what I was trying to do and shared my enthusiasm for the project.

Martin Davis
Berkeley, 2017

Preface to Second Edition

Alan Turing was born on June 23, 1912. The year 2012, the hundredth anniversary of his birth, is providing the occasion for events and publications reflecting on and celebrating his achievements. I am delighted that this updated version of my book will be part of the excitement, and I am very grateful to Klaus Peters and Alice Peters for their help in making it happen. For this edition, I've tidied up some loose ends and brought a few things up to date, even commenting on IBM's achievement in fielding its Watson computer as a successful contestant on the popular television quiz program *Jeopardy*.

The "universality" of the computers with which we interact today is evident in the myriad unrelated tasks for which the computers that are on our desks or our laps are used, but also in the hidden computers that are embedded in many other devices. If our cameras are computers with lenses and our telephones are computers with microphones and earphones, it could almost be said that the hybrid automobile I drive is a computer with four wheels.

Today it is widely recognized that this universality is an application of a fundamental insight from an article by Turing published in a mathematical journal in 1936. When I began researching these matters in the 1980s, much controversy surrounded credit for the first "stored program" electronic computers, but Turing's name was never mentioned. The argument was whether the credit was due to the mathematician John von Neumann or to the engineers John Presper Eckert and John Mauchly. David Leavitt kindly suggested that I am responsible for the recognition of Turing's role. While an article I wrote may have had some influence, probably the publication of some of Turing's previously unavailable work from the 1940s together with the availability of information about his secret work towards decrypting enemy military communications during the Second World War was more important.[1]

This is a book of stories about seven remarkable people, their ideas and discoveries, and their fascinating lives. They investigated the why and how of logical reasoning. They forced the exhilaration and pitfalls of trying to come to grips with the infinite. Their heroic efforts to buttress the claims of rationality encountered unforeseen obstacles. Finally, Alan Turing's radically new understanding of the nature of algorithmic processes and its potential to make a single "all-purpose" machine that could be programmed to carry out almost any process was a by-product of this tumultuous development. I had great fun writing this book, and I hope that you will have fun reading it.

Berkeley, June 30, 2011

Preface

This book is about the underlying concepts on which our modern computers are based and about the people who developed these concepts. In the spring of 1951, shortly after completing my doctorate in mathematical logic at Princeton University where Alan Turing worked a decade earlier, I was teaching a course at the University of Illinois based on his ideas. A young mathematician who attended my lectures called my attention to a pair of machines being constructed across the street from my classroom that he insisted were embodiments of Turing's conception. It was not long before I found myself writing software for early computers. My professional career spanning half a century revolved around this relationship between the abstract logical concepts underlying modern computers and their physical realization.

As computers evolved from the room-filling behemoths of the 1950s to the small powerful machines of today performing a bewildering variety of tasks, their underlying logic has remained the same. These logical concepts developed from the work of a number of gifted thinkers over centuries. In this book I tell the stories of the lives of these people and explain some of their thoughts. The stories are fascinating. My hope is that readers will not only enjoy them, but will come away with a better sense of what goes on inside their computers and with enhanced respect for the value of abstract thought.

In developing this book I benefited from help of various kinds. The John Simon Guggenheim Memorial Foundation provided welcome financial support during the early stages of the studies that led to this book. Patricia Blanchette, Michael Friedman, Andrew Hodges, Lothar Kreiser, and Benson Mates generously shared their expert knowledge with me. Tony Sale kindly acted as my guide to Bletchley Park where Turing played an important part in the decoding of secret German military communications during World War II. Eloise Segal, who alas did not live to see the book completed, was a devoted reader who helped me avoid expository pitfalls. My wife, Virginia, stubbornly refused to let me be obscure. Sherman Stein read the manuscript carefully and suggested many improvements while saving me from a number of errors. I benefited from help with translations by Egon Börger, William Craig, Michael Richter, Alexis Manaster Ramer, Wilfried Sieg, and Francois Treves. Other readers who provided useful comments were Harold Davis, Nathan Davis, Jack Feldman, Meyer Garber, Dick and Peggy Kuhns, and Alberto Policriti. My editor, Ed Barber at W.W. Norton, generously shared his knowledge of English prose style and is responsible for many improvements. Harold Rabinowitz introduced me to my agent, Alex Hoyt, who has been unfailingly helpful. Of course this long list of names is meant only to express gratitude and not to absolve myself of responsibility for the book's shortcomings. I would be grateful for comments or corrections from readers sent to me at: davis@eipye.com.

Martin Davis
Berkeley, January 2, 2000

Introduction

If it should turn out that the basic logics of a machine designed for the numerical solution of differential equations coincide with the logics of a machine intended to make bills for a department store, I would regard this as the most amazing coincidence I have ever encountered.

—Howard Aiken 1956[1]

Let us now return to the analogy of the theoretical computing machines ... It can be shown that a single special machine of that type can be made to do the work of all. It could in fact be made to work as a model of any other machine. ***The special machine may be called the universal machine*** *...*

—Alan Turing 1947[2]

In the fall of 1945, as the ENIAC, a gigantic calculating engine containing thousands of vacuum tubes, neared completion at the Moore School of Electrical Engineering in Philadelphia, a group of experts met regularly to discuss the design of its proposed successor, the EDVAC. As the weeks went by, the meetings grew increasingly acrimonious, with the experts finding themselves divided into two groups they dubbed the "engineers" and the "logicians." John Presper Eckert, leader of the "engineers," was justly proud of his accomplishment with the ENIAC. It had been thought impossible for 15,000 hot vacuum tubes to work together long enough without any of them failing, for anything useful to be accomplished. Nevertheless, by using careful conservative design principles, Eckert had succeeded brilliantly in accomplishing this feat. Things came to a head when, much to Eckert's displeasure, the group's leading "logician," the eminent mathematician John von Neumann, circulated, under his own name, a draft report on the proposed EDVAC that, paying little attention to engineering details, set forth the fundamental *logical* computer design known to this day as the von Neumann architecture.

Although an engineering tour de force, the ENIAC was a logical mess. It was von Neumann's expertise as a logician and what he had learned from the English logician Alan Turing that enabled him to understand the fundamental fact that a computing machine is a logic machine. In its circuits are embodied the distilled insights of a remarkable collection of logicians, developed over centuries. Nowadays, when computer technology is advancing with such breathtaking rapidity, as we admire the truly remarkable accomplishments of the engineers, it is all too easy to overlook the logicians whose ideas made it all possible. This book tells their story.

CHAPTER 1

Leibniz's Dream

Situated southeast of the German city of Hanover, the ore-rich veins of the Harz mountain region had been mined since the middle of the tenth century. Because the deeper parts tended to fill with water, they could only be mined so long as pumps kept the water at bay. During the seventeenth century water wheels powered these pumps. Unfortunately, this meant that the lucrative mining operations had to shut down during the cold mountain winter season when the streams were frozen.

During the years 1680–1685, the Harz mountain mining managers were in frequent conflict with a most unlikely miner. G. W. Leibniz, then in his middle thirties, was there to introduce windmills as an additional energy source to enable all-season operation of the mines. At this point in his life, Leibniz had already accomplished a lot. Not only had he made major discoveries in mathematics, he had also acquired fame as a jurist, and had written extensively on philosophical and theological issues. He had even undertaken a diplomatic mission to the court of Louis XIV in an attempt to convince the French "Sun King" of the advantages of conducting a military campaign in Egypt (instead of against Holland and German territories).[1]

Some 70 years earlier, Cervantes had written of the misadventures of a melancholy Spaniard with windmills. Unlike Don Quixote, Leibniz was incurably optimistic. To those who reacted bitterly to the widespread misery in the world, Leibniz responded that God, from His omniscient view of all possible worlds, had unerringly created the best that could be constructed, that all the evil elements of our world were balanced by good in an optimal manner.[*]

But Leibniz's involvement with the Harz Mountain mining project ultimately proved to be a fiasco. In his optimism, he had not foreseen the natural hostility of the expert mining engineers towards a novice proposing to teach them their trade. Nor had he allowed for the inevitable break-in period a novel piece of machinery requires or for the unreliability of the winds. But his most incredible piece of optimism was with respect to what he had imagined he would be able to accomplish with the proceeds he had expected from the project.

[*] Voltaire's Dr. Pangloss in Voltaire's *Candide* was a sendup of this Leibnizian doctrine.

GOTTFRIED WILHELM LEIBNIZ

Leibniz had a vision of amazing scope and grandeur. The notation he had developed for the differential and integral calculus, the notation still used today, made it easy to do complicated calculations with little thought. It was as though the notation did the work.

In Leibniz's vision, something similar could be done for the whole scope of human knowledge. He dreamt of an encyclopedic compilation, of a universal artificial mathematical language in which each facet of knowledge could be expressed, of calculational rules which would reveal all the logical interrelationships among these propositions. Finally, he dreamed of machines capable of carrying out calculations, freeing the mind for creative thought. Even with his optimism, Leibniz knew that the task of transforming this dream to reality was not something he could accomplish alone. But he did believe that a small number of capable people working together in a scientific academy could accomplish much of the design in a few years. It was to fund such an academy that Leibniz embarked on his Harz Mountain project.

Leibniz's Wonderful Idea

Leibniz was born in Leipzig in 1646 into a Germany divided into something like 1,000 separate, semiautonomous political units, and devastated by almost three decades of war. The Thirty Years War, which didn't end until 1648, was fought mainly on German soil, although all of the major European powers had participated. Leibniz's father, a professor of philosophy at the University of Leipzig, died when the child was only six. Over the opposition of his teachers, Leibniz gained access to his father's library at the age of eight, and soon became a fluent reader of Latin.

Leibniz, destined to become one of the greatest mathematicians of all time, got his first introduction to mathematical ideas from teachers who had no inkling of the exciting work elsewhere in Europe that was revolutionizing mathematics. In the Germany of that day, even the elementary geometry of Euclid was an advanced subject, studied only at the university level. However, in his early teens, his school teachers did introduce Leibniz to the system of logic that Aristotle had developed two millennia earlier, and this was the subject that aroused his mathematical talent and passion.

Fascinated by the Aristotelian division of concepts into fixed "categories," Leibniz thought of what he came to call his "wonderful idea": he would seek a special "alphabet" whose elements represented not sounds, but concepts. A language based on such an alphabet should make it possible to determine by symbolic calculation which sentences written in the language were true and what logical relationships existed among them. Leibniz remained under Aristotle's spell and held fast to this vision for the rest of his life.

Indeed, for his bachelor's degree at Leipzig, Leibniz wrote a thesis on Aristotelian metaphysics. His master's thesis at the same university dealt with the relationship between philosophy and law. Evidently attracted to legal studies, Leibniz obtained a second bachelor's degree, this time in law, writing a thesis emphasizing the use of systematic logic in dealing with the law. Leibniz's first real contribution to mathematics developed out of his *Habilitationsschrift* (in Germany, a kind of second doctoral dissertation) in philosophy also at Leipzig: As a first step towards his "wonderful idea" of an alphabet of concepts, Leibniz foresaw the need to be able to count the various ways of combining such concepts. This led him to a systematic study of the problem of counting complex arrangements of basic elements, first in his *Habilitationsschrift* and then in his more extensive monograph *Dissertatio de Arte Combinatoria*.[2]

Continuing his legal studies, Leibniz presented a dissertation for a doctorate in law at the University of Leipzig. The subject, so typical for Leibniz, was the use of reason to resolve cases in law thought too difficult for resolution by the normal methods. For reasons that are not clear the Leipzig faculty refused to accept the dissertation, so Leibniz presented it instead at the University of Altdorf, near Nuremberg where it was received with acclaim. At the age of 21, his formal education completed, Leibniz faced the common problem of the newly graduated: how to develop a career.

Paris

Not being interested in a career as a university professor in Germany, Leibniz pursued his only real alternative: to find a wealthy noble patron. Baron Johann von Boineburg, nephew of the Elector of Mainz, was quite willing to play this role. In Mainz, Leibniz worked on a project to update the legal system based on Roman civil law, was appointed a judge at the High Court of Appeal, and tried his hand at diplomatic intrigue. This last included an abortive attempt to influence the selection of a new king for Poland and a mission to the court of Louis XIV.

The Thirty Years War had left France as the "superpower" on the European continent. Mainz, situated on the banks of the Rhine, had known military occupation during the war. So, the burghers of Mainz understood very well the importance of forestalling hostile military action, and therefore, of good relations with France. It was in this context that Boineburg and Leibniz concocted the scheme, already mentioned, to convince Louis XIV and his advisers of the great advantages of making Egypt the object of their military endeavors. The most important historical effect of this proposition—essentially the same proposition that led Napoleon to a military disaster over a century later—was that it brought Leibniz to Paris.

Leibniz arrived in Paris in 1672 to press the Egyptian scheme and to help untangle some of Boineburg's financial affairs. Before the end of the year disaster struck: the news came that Boineburg had died of a stroke. Although he continued to perform some services for the Boineburg family, Leibniz was left without a reliable source of income. Nevertheless he managed to remain in Paris for another four extremely productive years that included two brief visits to London.[3] On the first visit in 1673, he was unanimously elected to the Royal Society of London based on his model of a calculating machine capable of carrying out the four basic operations of arithmetic. Although Pascal had designed a machine that could add and subtract, Leibniz's was the first that could multiply and divide as well.[*] Leibniz's machine incorporated an ingenious gadget that became known as a "Leibniz wheel." Calculating machines continued to be built incorporating this device well into the twentieth century. About his machine, Leibniz wrote:

> And now that we may give final praise to the machine we may say that it will be desirable to all who are engaged in computations which, it is well known, are the managers of financial affairs, the administrators of others' estates, merchants, surveyors, geographers, navigators, astronomers ... But limiting ourselves to scientific uses, the old geometric and astronomic tables could be corrected and new ones constructed by the help of which we could measure all kinds of curves and figures ... it will pay to extend as far as possible the major Pythagorean tables; the table of squares, cubes, and other powers; and the tables of combinations, variations, and progressions of all kinds, ... Also the astronomers surely will not have to continue to exercise the patience which is required for computation. ... For it is unworthy of excellent men to lose hours like slaves in the labor of calculation which could safely be relegated to anyone else if the machine were used.[4]

The machine Leibniz was "praising" was limited to ordinary arithmetic. But Leibniz grasped the broader significance of mechanizing calculation. In 1674 he described a machine that could solve algebraic equations. A year later, he wrote comparing logical reasoning to a mechanism, thus pointing to the goal of reducing reasoning to a kind of calculation and of ultimately building a machine capable of carrying out such calculations.[5]

A crucial event for Leibniz, then 26, was meeting the great Dutch scientist Christiaan Huygens then living in Paris. The 43-year-old Huygens had already invented the pendulum clock and discovered the rings of Saturn.

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[*]Blaise Pascal, born on June 19, 1623, at Clermont-Ferrand, France, one of the founders of the mathematical theory of probability, was a prolific mathematician, physicist, and religious philosopher. His calculating machine, designed and built circa 1643, brought him considerable fame. He died in 1662.

What was perhaps to be his most important contribution, the wave theory of light, was still to come. His conception—that light was fundamentally to be viewed like the waves spreading across a pond when a pebble is tossed into it—directly contradicted the great Newton's account of light as consisting of a stream of discrete bullet-like particles.* Huygens gave Leibniz a reading list enabling the younger man to quickly overcome his lack of knowledge of current mathematical research. Soon Leibniz was making fundamental contributions.

The explosion of mathematical research in the seventeenth century had been fueled by two crucial developments:

1. The technique of dealing with algebraic expressions (what is generally called "high-school algebra") had been systematized and became essentially the powerful technique we still use today.

2. Descartes and Fermat had shown how, by representing points by pairs of numbers, geometry could be reduced to algebra.

Various mathematicians were using this new power to solve problems that would not previously have been accessible. Much of this work involved what nowadays are called *limit processes.* Using limits means solving a problem by using approximations to the required answer that get systematically closer and closer to that answer. The idea was not to be satisfied with any particular approximation, but rather, by "going to the limit," to obtain an *exact* solution.

An example that may help to clarify this concept is one of Leibniz's own early results, one of which he was quite proud. This was the equation:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

On the left side of the equals sign is the familiar number π that occurs in the formulas for the circumference and the area of a circle.* On the right side is what is called an *infinite series*; the individual numbers alternately added and subtracted are called the *terms* of the series. The dots ... mean that it continues indefinitely. The full infinite pattern consists of fractions, with 1 as numerator and the successive odd numbers as denominators, being alternately added and subtracted, and is intended to be clear from the finite part shown: after subtracting $\frac{1}{11}$, add $\frac{1}{13}$, then subtract $\frac{1}{15}$, etc. But can one actually perform an infinite number of additions and subtractions? Not really. But, starting at the beginning and breaking off at any point, an

*Although Huygens's view came to be generally accepted, the coming of quantum physics in the twentieth century made it clear that both Newton and Huygens had been right; each grasped an essential characteristic of light.

*The number $\frac{\pi}{4}$ is in fact the area of a circle whose radius is $\frac{1}{2}$.

Number of terms	**Sum correct to eight decimal places**
10	0.76045990
100	0.78289823
1,000	0.78514816
10,000	0.78537316
100,000	0.78539566
1,000,000	0.78539792
10,000,000	0.78539816

Table of approximations to Leibniz's series

approximation to a "true" answer is obtained, and that approximation gets better and better as more terms are included. In fact, the approximation can be made as accurate as one wishes by including enough terms. In the table on page 7, it is shown how this works out for Leibniz's series. When including 10,000,000 terms, a value is obtained that agrees with the true value of $\frac{\pi}{4}$, namely 0.7853981634..., to eight places.[†]

Leibniz's series is so striking because it connects the number π, and therefore the area of a circle, with the succession of odd numbers in a particularly simple way. It is an example of one kind of problem that could be solved using limit processes, that of finding areas of figures with curved boundaries.

Another kind of problem susceptible to attack using limits was finding exact rates of change, such as the varying speed of a moving body. During the last months of 1675, towards the end of his stay in Paris, Leibniz made a number of conceptual and computational breakthroughs in the use of limit processes that, taken together, are called his "invention of the calculus":

1. Leibniz saw that the problems of finding areas and calculating rates of change were paradigmatic, in the sense that many different kinds of problems were reducible to one or the other of these two types.[*]

2. He also perceived that the mathematical operations required in calculating the solutions to problems of these two types were in fact *inverse* to each other in much the same sense that the operations of addition and subtraction (or multiplication and division) are inverse to one

[†]I used my PC to obtain the table of approximations to $\frac{\pi}{4}$ from Leibniz's series. A short Pascal program I wrote for the purpose runs for less than a second on a contemporary PC.

[*]Thus, finding volumes and centers of gravity are problems of the first kind, and computing accelerations and (in economic theory) marginal elasticity are problems of the second type.