PSYCHOLOGY REVIVALS

The Origin of the Idea of Chance in Children

Jean Piaget and
Bärbel Inhelder

Psychology Press

Psychology Revivals

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Although originally published in France in 1951 this English translation was not published until 1975. The book supplements the authors' previous publications on the development of thought in the child and is the result of two preoccupations: how thought that is in the process of formation acts to assimilate those aspects of experience that cannot be assimilated deductively – for example, the randomly mixed; and the necessity of discovering how the mental processes work in the totality of spontaneous and experimental searchings that make up what is called the problem of 'induction'. Induction is a sifting of our experiences to determine what depends on regularity, what on law, and what on chance.

The authors examine the formation of the physical aspects of the notion of chance; they study groups of random subjects and of 'special' subjects; and they analyse the development of combining operations which contributes to determining the relationship between chance, probability, and the operating mechanisms of the mind.

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Jean Piaget and Bärbel Inhelder

Translated by Lowell Leake, Jr., Paul Burrell and Harold D. Fishbein

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Translators' Preface

Jean Piaget, born in 1896, is an intellectual giant in the field of developmental psychology. Although he earned his doctoral degree in the natural sciences (primarily biology) in 1918, his knowledge ranges in great depth through philosopy, religion, sociology, logic, mathematics, and, of course, psychology. Currently, the major emphases of his research and writings are on perception and memory experiments and on a complete analysis of the problems of genetic epistemology. During the past ten to fifteen years, the prolific contributions of Piaget and his collaborators, with Bärbel Inhelder being the most prominent and important of these, have attracted the rapt attention of psychologists and educators in the United States. His name is now well-known here, but the extent of his contributions is less well-known. John Flavell [1] in 1963 mentioned that more than twenty-five major books and over one hundred fifty articles exist for scholars to examine; few of the articles have been translated and not all of the books.

This work, *The Origin of the Idea of Chance in Children* fits in with earlier works concerning quantity, logic, number, time, movement and velocity, space, geometry, and adolescent reasoning. Some of these works are included in the Bibliography, but Flavell's book offers a comprehensive listing of works up to about 1962. Needless to say, many more publications have appeared since then.

The Origin of the Idea of Chance in Children was first

[1] John H. Flavell, *The Developmental Psychology of Jean Piaget* (Princeton, N. J.: Van Nostrand, 1963).

published in 1951 in Paris, France, but it has remained mysteriously untranslated until now. This twenty-three-year lag seems even more remarkable since the work is a major one by Piaget and Inhelder. Furthermore, translations of other early major works appeared as long ago as 1926 (*The Language and Thought of the Child*) and as recently as 1970 (*The Child's Conception of Movement and Speed*). And finally, this lack of a translation until now seems particularly strange since the work deals with chance and probability, mathematical concepts which have come to play a more and more important role in the mathematics curriculum of our schools, from kindergarten through graduate university education.

Hopefully this translation will provide interested psychologists and mathematics educators with a new catalyst for research in cognitive development. It should also prove useful to the mathematics teacher at the precollegiate level who has an interest in the theory of cognitive development and who teaches probability in the classroom. It is, in fact, one of the few works by Piaget that the beginner can read profitably, without needing too much help from the critics, because it gives a relatively clear example of the development of his techniques and thought. Flavell expressed the warning, "Furthermore, most of Piaget's writings are very difficult to read and understand, in French or in English. For one thing, there are many new and unfamiliar theoretical concepts, and they intertwine with one another in complicated ways to make the total theoretical structure. In addition, much of the theoretical content requires some sophistication in mathematics, logic, and epistemology." [1] Happily, Flavell softens this caveat somewhat by saying "Fortunately, *The Genesis of the Idea of Chance in Children* is a fairly easy book to read (as Piaget books go), and its concluding chapter offers an excellent summary of the principal findings and Piaget's interpretation of them." [2] Most readers of any of Piaget's works would do well to read Professor Flavell's book in conjunction with reading Piaget's own books; Piaget himself recommends Flavell's book in its Foreword, and it seems to be the single, most sophisticated analysis of Piaget's theories available in English.

[1] Ibid., p. 11.
[2] Ibid., p. 341.

Foreword

The present work is a supplement to our previous publications on the development of thought in the child and is the result of two preoccupations.

After having studied how logical, mathematical, and physical operations develop in the mind of the child and are adapted to that part of his experience which can be structured deductively, our next question was how thought which is in the process of formation acts to assimilate those aspects of experience which cannot be assimilated deductively—for example, the fortuitous or the randomly mixed.

Our second concern is that an analysis of intellectual operations and related phenomena in no way exhausts the genetic functions of the thought of the child (and still less that of the adolescent). What remains to be discovered, therefore, is how the mental processes work in the totality of spontaneous and experimental searchings which make up what is commonly called the problem of induction. Induction is first, and perhaps foremost, an effort at sifting our experiences to discover what depends on regularity, what on law, and what part remains outside these—i.e., what is simply chance. As an introduction to our investigations of intuition, we have recognized that a study of the origins of the idea of chance is unavoidable.

We are presenting here from this double perspective the results of some experiments on the psychology of the idea of chance. The reader will find in these pages material complementary to the operational analysis of the thought of the child; he will find, in addition, a sketch of further possible research into the formation of experimental induction.

Jean Piaget
Bärbel Inhelder

Introduction

The Intuition of Probabilities

A mathematician known for his work on probability theory suggested to us one day a study of the following problem: Could there be in a normal man an intuition of probability just as fundamental and just as frequently used as, say, the intuition of whole numbers? Almost every common action seems, in fact, to require the notion of chance as well as a sort of spontaneous estimate of the more or less probable character of feared or expected events. We know, for example, that there is a better chance of finding an object lost in a small space than in a large one. To avoid getting hit while crossing a street, we are making judgments every moment about the speed and position of the cars. After noticing a halo around the sun, if it rains the next day, we will say that it is not chance, while on the other hand rain on three Sundays in a row will not make us conclude any sort of natural law concerning rain on Sundays. We recognize, finally, that the inextricable mixture of facts and causal sequences forces us to take a probabilist attitude. It is only in theory, and at times in the laboratory, that phenomena are simplified to the point of allowing for the formation of a particular hypothesis. In our daily lives, all occurrences are complex: The fantastic path of a falling leaf is seen more often than is a straight line, and this is why we are reduced all our lives to guessing or to basing our expectancies on empirical frequencies and on contingencies.

But if we cannot deny that there is an intuition of probability in the normal civilized adult, and if we can correctly compare the role of this intuition to that of several practical operations such as number and space, there are nevertheless two questions which must be asked at the start: Is such an intuition in-born or does it develop later and, if so, how is it acquired?

Leaving aside both those psychopathological states in which the notion of chance is lost in a welter of obsessional or frenzied interpretations which load the most fortuitous events with subjective meanings and also those states of passion of a lover or gambler which are characterized by a like regression in favor of symbolism or a play of imaginary tokens, there are still two perfectly normal psychological states in which the understanding of chance and probability seem more or less absent: the primitive mind and the mind of a small child.

We are well aware that M. Levy-Bruhl considered the absence of the idea of chance an essential characteristic of the primitive mind. Since the primitive saw every event as the result of hidden as well as visible causes, and since he lacked the rational or experimental criteria to rule out even the strangest and most unforeseen connections, the prescientific mind could not have an intuition of probability as we have. The modern idea of chance is contrary to both types of causality, to determinism and to miracles. On the one hand, it differs from purely mechanical determinism, whose spatial and temporal links in the ideal state are reversible, because probability implies the intervention of irreversible phenomena. On the other hand, by interference within causal series, probability excludes categorically the concept of miracle. It assumes precisely that this mixture has its laws, while a miracle is the negation of such laws. In considering the primitive mind, a further question is, At what point does primitive thought perceive the possibility of mechanical causality? The question of miracle, however, cannot even be asked, since anything for the primitive can be a miracle. Here then are two reasons why the primitive remains innocent of the idea of chance, certainly to a greater degree than we do.

To use the primitive mind solely, however, in our analysis of the genesis of these notions leaves us uneasy. Levy-Bruhl's works have shown brilliantly the ideological and even mythic nature of primitive collective concepts. The whole technical side as well as the individual's daily differentiated use of these primitive concepts escape us still. In our examination of the idea of chance these aspects of their thought have an importance. We know quite well how the primitive attributes death, sickness, accident, and misfortune to the intervention of hidden powers and excludes the idea of chance. But we would certainly like more information on how the Arunta or the Bororo goes about finding a misplaced tool, or how he reacts when, taking aim, he is caught both by those kinematic laws which control the flight of his arrows and also by the fortuitous arrangement of things around his target.

It is at this point that our observations of the child are relevant. Certainly the mind of the child is always dependent on the surroundings, and each person, therefore, moves through an ensemble of collective representations in his development which the family and the school impose. But this circumstance is far from being a hindrance to our particular problem in psychogenetic studies. Suppose, for example, that in spite of the fact that intuitions of chance and probability are an integral part of our society's common sense, a small child, even one raised in an intellectual family, were resistant to such notions up to a certain age. This would prove first that mental operations are not entirely dependent on the collective milieu, and secondly, would permit us to analyze precisely how the idea of chance develops.

Not only do we find that things happen this way, but we find also that the formation of the rational processes in the mental evolution of the child throws light on the nature and conditions of the genesis of the ideas of chance and elementary probability.

It is quite natural that the child does not have at the very beginning an idea of chance, because he must first construct a system of consequences, such as position and displacement, before he would be able to grasp the possibility of the interference of causal series or of the mixture of moving objects. This development of the idea of causality and of order in general assumes an attitude exactly opposite to the attitude which can recognize the contingent and the fortuitous. Thus the idea of chance and the intuition of probability constitute almost without a doubt secondary and derived realities, dependent precisely on the search for order and its causes. As we have shown elsewhere, this is seen when we examine spontaneous questions of children, especially the famous Why questions which adults have so much trouble answering. The Why is asking the reason for things in cases where a reason exists, but also quite often in cases where it does not; that is, in cases where the phenomenon is fortuitous but where the child sees a hidden cause. Questions such as the following are pseudo questions: Why doesn't Lake Geneva go all the way to Berne? Why is there a Big and a Little Salève? Why isn't there a spring in our garden? Why is this stick taller than you? Why are you so tall and yet have small ears? And so on. All of these are pseudo questions for us because the facts to be explained are all due to chance interferences in biology and geology, to chance encounters, and the like, while the child who supposes that there are reasons for everything asks for the reasons in exactly those cases where they are least apparent. He has not yet understood that these are precisely the cases where there are none.

And there is more to it. If the study of the thought of the child brings us to recognize that the idea of chance and the intuition of probability are not innate, not primitive, then the psychogenesis of these notions during mental development throws a particular light on the formation of these ideas which have become fundamental in contemporary scientific thought. One particular aspect of the intellectual development of the child is to establish a progression between irreversible actions (both motor and perceptive) and rational operations, actions which become reversible and interrelated. The central problem which the evolution of intelligence in each individual permits us to resolve is the question of the genesis and nature of logical and mathematical operations in so far as these operations derive from experience and are structured by a reversible process. From the prelogic of the child, which is characterized by irreversibility, to primitive modes of thought, to the beginnings of logical-mathematical reasoning, one can follow step by step, through the course of a dozen years or so, the mechanics of this sort of development of the human mind. And the essential aspect of this development of the individual's thought processes depends on his gradual recognition of the reversibility of operations.

One can see immediately the interest that this situation holds for the ideas of chance and probability. From the physical point of view, chance is an essential characteristic of irreversible mix, while at the other extreme is mechanical causality, characterized by its intrinsic reversibility. Could it not be then that the discovery of the idea of chance, that is to say, the very understanding of irreversibility, had to come after the understanding of the reversible operations, since we mean by chance that part of the phenomena which is not reducible to reversible operations? And are these not the only operations which will allow us to grasp their opposite? This means then that thought which is still irreversible, that is, thought which moves in a single direction and which is completely dominated by the temporal course of events, this thought is not touched by the idea of chance precisely because it lacks the mental organization capable of distinguishing reversible mental operations from fortuitous events.

In a word then we must visualize chance as a domain complementary to the area in which logic works and, therefore, not to be understood until reversible operations are understood and then only by comparison with them. In such a case probability would be a counterpart of mental operations; that is, an assimilation of chance with the combining operations. Since we are unable quite simply to deduce each interference, it is the mixture taken as a whole which the mind assimilates. After this complex operation we have the reduction of the particular cases to all of their possible combinations.

These are the hypotheses our study has yielded on the development of the ideas of chance and probability at their most elementary stages—that is, in children from the age of four or five to eleven or twelve. The usefulness of this study lies not in any theoretical analysis of our data; rather, it has seemed to us necessary to make the facts themselves available in the order of their appearance. The facts will tell us then as we examine the correlative development of operations, of the idea of chance as their opposite, and of the probabilistic assimilation of chance with the combinatoric mechanisms.

In the first part of this work we will examine the formation of the physical aspects of the notion of chance: that is, the idea of an irreversible mixture, of diverse distributions (uniform or centered), which characterize fortuitous events, and the relationship between chance and induction.

The second part will examine one group of random subjects of various ages and another group of special subjects. At this point we will move from the interpretation of chance to an examination of the beginnings of the quantification of probabilities.

Finally, a third part will consist of the analysis of the development of combining operations—combinations, permutations, and arrangements. The juxtaposition of this development with the preceding facts will allow us to determine, as a conclusion, the relationship between chance, probability, and the operating mechanisms of the mind.

PART ONE

Chance in Physical Reality

I. Notions of Random Mixture and Irreversibility

It is quite probable that the concept of chance starts from the idea of an increasing and irreversible combination of phenomena. Cournot's famous interpretation conceived of physical chance as the interaction of independent causal series. But this complex notion, which needed an understanding of both interaction and independence, could hardly be grasped by the uninformed mind except in those cases where an intentionalist interpretation was eliminated because of the large number of elements involved. When a child is struck by a door which a gust of wind has closed, the child will find it difficult to believe that neither the wind nor the door had the intent of hurting him; he will certainly see the interaction of causes which brought him near the door, and also what caused the door to move, but he will not admit their independence. It is this fact which will not let him see the event as fortuitous. On the other hand, he does not recognize that chance characterizes daily happenings (social, meteorological, etc.) because he fails to notice the interactions of phenomena (e.g., the relationship between night frosts and the flowering of a fruit tree). In brief, the alternatives which have kept the child (as well as the primitive mind) from constructing the idea of chance are his recognition of either an interaction of causes with no recognition of their independence, or their independence without realizing their interaction. On the other hand, a combination of a sufficiently large number of elements seems to give a situation favorable to the intuition of causal series which both interact and are independent since the sequence of events is easily established with no need for imagining that there is anything intentional in the details.

The problem then is to determine if the child, in the presence of an obvious mixture of material objects, will perceive it as an increasing and irreversible mixture of the objects; or if, in spite of the obvious disorder, he will imagine the different objects as still being linked by invisible connections. In other words, is the intuition of the random mixture primary or does the concept of chance have a history? And if so, what is its history? This is what we mean to establish by experiment.

1. *Technique of the experiment and general results.*

The child is given a rectangular box which rests on a transversal pivot, allowing it to seesaw. In a state of rest, the box is inclined to one or the other of its shorter sides, and along this width are arranged eight red balls and eight white balls, each group separated

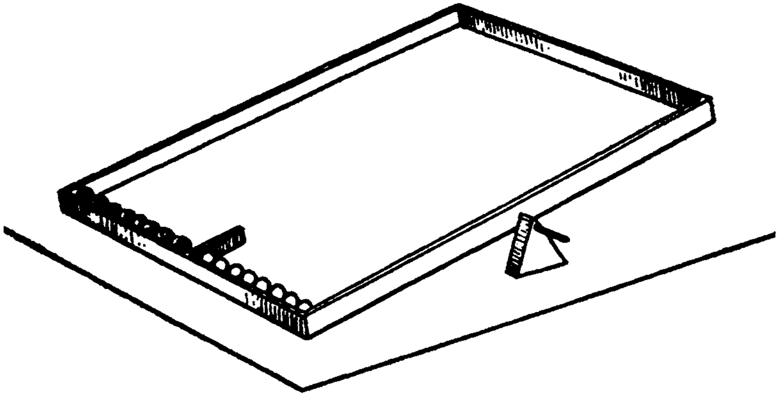

Figure 1.

by a divider (Figure 1). At each seesaw movement, the balls will roll to the opposite side and then will return to the original side when the box is tipped back, but in a series of possible permutations. The successive movements of the box ought not to be done too brusquely; in this way the mixture will proceed in gradual steps. For example, in the beginning, two or three of the red balls will be mixed with the white ones, and vice versa, and then the mixture will, little by little, be greater.

Before the box is first tipped (but telling the child or showing him how it moves while holding the balls in place), the child is asked a question: What will be the arrangement of the balls when they return to their starting places? Will the red ones stay on one side and the white ones on the other? Or will they get mixed up, and in approximately what proportion? We then proceed to tip the box and have the child note that two or three of the balls are in different positions. Then he is asked to predict the result of a second move which we then make, and so on. After some tries, he is asked to predict the result of a large number of moves of the box, and we note especially if he expects a progressive random mixture or a general crisscrossing of the red balls to the side of the white ones, and vice versa, and finally a return to their original order (that is, a final reordering).

Making a drawing of the arrangement of the balls can help in the questioning: He can draw his prediction of the first tipping of the box, and then after the first trial, make predictions by drawing the outcomes of successive trials. We will ask him in particular to draw an arrangement of the balls which he thinks is the best possible mixture. There is not always a connection between the drawing of the balls in their mixed positions and the trajectories of the balls which he is asked to draw. But an examination of this lack of agreement is quite useful for an interpretation of the thought process of the subject.

One notices that the different questions lead quite naturally to examining also the manner in which the subject conceives of the operation of permutations. We will return to this point in Chapter VIII, indicating there the technique to be used for the study of the development of these operations. We will note here only that it is a help to question the same children simultaneously by means of the tipping box of balls and also by means of the experiment with the permutations of counters (Chapter VIII). The correlation between the evolution of ideas concerning mixture and the development of the operations of permutation is an instructive factor in the interpretation of responses.

To remain for the moment with the mixture of the balls, the reactions observed at the time of the preceding questions permit us to distinguish three different stages. During the first stage (up to seven years), the mixture is conceived of as a total displacement of the elements, but without any intuition of a permutation of the individual positions nor any anticipation of an interaction in the trajectories. This total displacement certainly yields a state of disorder, but for the child it is not final and he often predicts that the balls will ultimately return to their original order. There is not, in the strictest sense, either a real mixture or chance. In the course of the second stage (from seven to eleven years as median), there is a progressive individualization of the positions, then of the trajectories, with the gradual construction of an intuitive scheme of permutations, but without a complete generalization. In the course of the third stage (over eleven to twelve years), the mixture is conceived of as a system of permutations due to the fortuitous collisions in the trajectories.

2. *The first stage (four to seven years): Failure to understand the random nature of the mixture.*

The reactions of the first stage are characterized by a very significant conflict between the facts noticed by the child which force him to see a progressive disorder and the interpretations which he searches for and which remain foreign to the idea of random mixture. In other words, the subject is obliged to accept the evidence of the changes of position due to the mixing, but he refuses to see in this a fortuitous mixing and decides to look for uniformities contrary to chance. Thus, in spite of the first permutations which the subject foresees, he predicts the return of the balls to their original places or perhaps a regular change of position, but not haphazard arrangements. For example, each ball is supposed to follow only a single trajectory—in particular, the red balls are to replace the white ones, and vice versa (Figure 2). A large number of movements of the box will not increase the mixture, and frequently the subject even expects a return of the balls to their original places. As for the drawing of the trajectories, rather than giving the child the idea of permutations, on the contrary, it leads him to the idea that the ball will come back to its starting place even when the fact of mixing has been previously established.

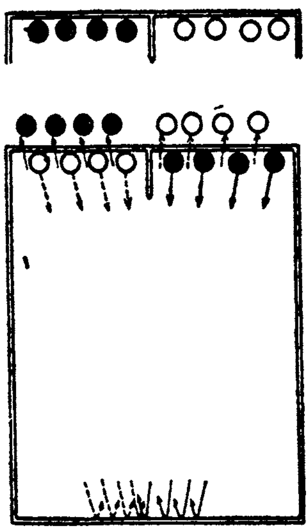

Figure 2.

Here are a few examples:

ELI (4; 4) begins by predicting that the balls will return to their places. *"They will go there* (a few centimeters from the point of rest)." And then: *'They will all come back in place* (perfectly arranged)." "Watch (the box is tipped and one white ball passes into the area of the red ones)." *"See, it's just as I said."* "Look again." *"One white ball went over there* (to the red side), *but all the red ones are there."* "And when I tip it again?" *"The white ones will be here and the red ones there* (complete crossing of all the balls)." "Take a look (another tipping and the mixture is greater)." *"Ah, it's all mixed now."* "And if I keep it up, will it be more or less mixed?" *"No. Don't do that* (he puts the balls back in place as if the mixture had bothered him)." "Now draw them for me as mixed up as they can be (he makes a drawing showing six white balls on one side, and on the other side three white ones together and three red ones)."

FER (5; 3): "What if I tip the box?" *"The balls will get all mixed up."* "How?" *"The white will go there and the red there* (crossing of reds to one side and white to the other)." (He makes a drawing of six reds on the right and six white ones on the left.) "Where will that ball go if I tip the box?" *"There* (return to the same place)." "You had said that the white balls would go there and the red ones here?" *"No, they will return to their places."* "Watch (the experiment)." *"They got mixed up. They crossed over and the red ones came over here and the white ones went there* (in fact, this only happened to two)." "And if we tip again?" *"They're going to get mixed up again. They'll go there and then there."* "Draw me a picture of how it will happen (he draws again six white balls on the left and six red ones on the right)." "Where will that white ball go?" (He draws a line which takes it to the other side on the right.) "And this red one?" *"It will go over and stop there* (inverse movement)." Fer continues the same drawing for the balls as follows: All the red ones move now to the left and all the white ones to the right making a complete crossover. At first there is a symmetry in the courses drawn, but after drawing some awkward lines, Fer puts the balls wherever he finds a free space which gives the appearance of there having been collisions in the trajectories, but the drawing is not done with the intention of showing any collisions.

VEI (5; 6): "If I tip the box, how will the balls then be lined up?" *"Just as they are now."* "Take a look (we tip the box: one red passes into the white section and one white into the red." *"They can't come back the same."* "And if we continue?" *"Then they will get even more mixed up."* "Why?" *"Because two of them will roll to the other side."* "And if I tip the box again?" *"Another one will move* (three red balls in with the whites and vice versa)." "And the next move?" *"Again one will move. They*