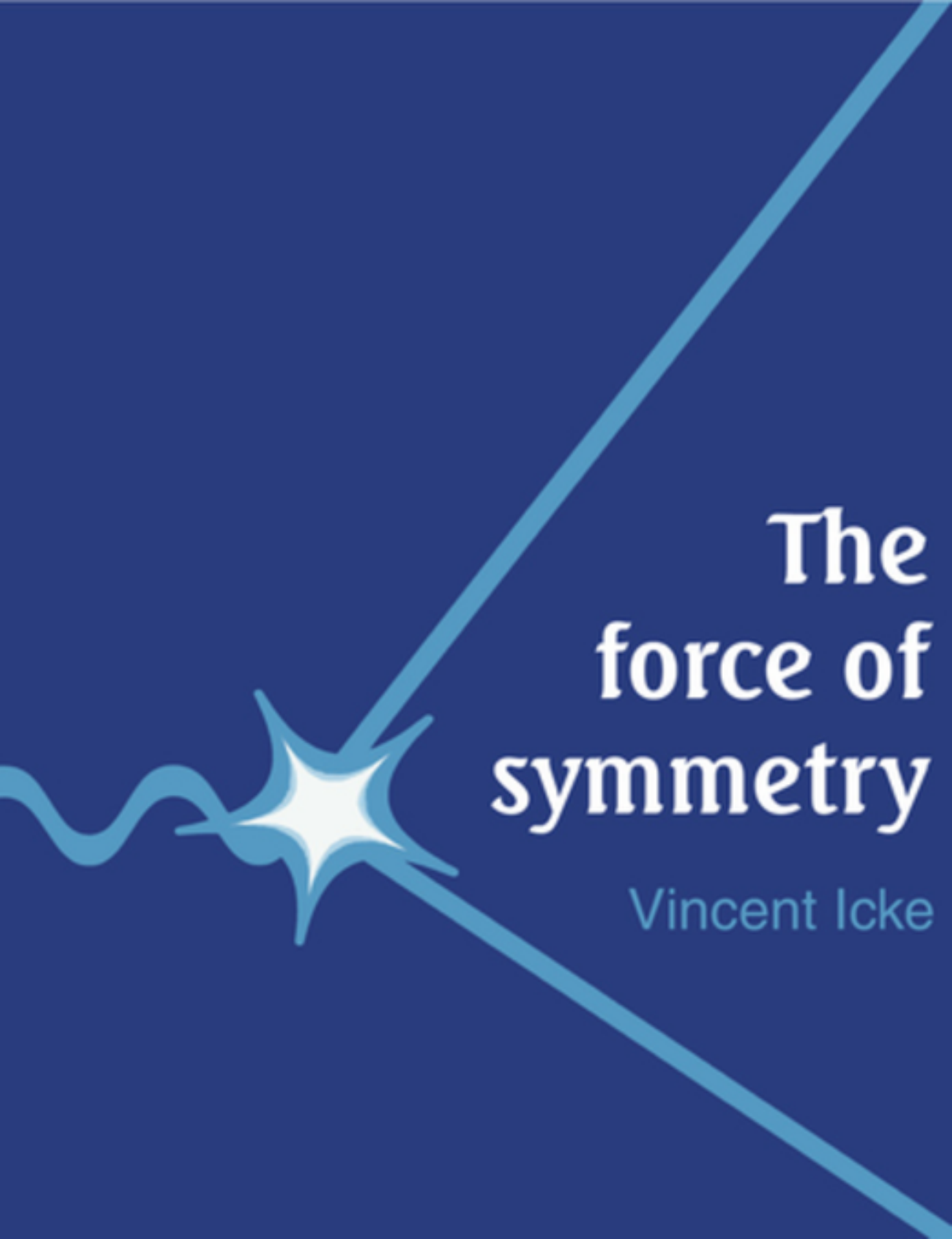
The force of symmetry
Vincent Icke

The force of symmetry gives an elementary introduction to the spectacular interplay between the three great themes of contemporary physics: quantum behaviour, relativity and symmetry. In clear, non-technical language, though without oversimplification, it explores many fascinating aspects of modern physics, discussing the nature and interaction of force and matter.

Through the examination of relevant physical effects, and analogies from daily experience, the book presents in some detail the workings and implications of special relativity, quantum mechanics and symmetries. In so doing, the importance of these fields and their influence on the everyday world is highlighted. Towards the end of the book, its major themes are drawn together to describe the most successful physics theory in history, the 'standard model' of subatomic particles. The strange, counter-intuitive world of the very fast and the very small provides an excellent illustration of many of the topics discussed in earlier chapters.

The lively and non-technical approach of this book will make it suitable for first-year undergraduates in the physical sciences and mathematics, or those just about to embark on such courses, and for anyone with a general interest in these topics. It will also be a valuable accompaniment to more advanced texts on quantum mechanics and particle physics.

Vincent Icke is professor of cosmology at the University of Amsterdam and at Leiden University in the Netherlands. He studied theoretical physics and astronomy at Utrecht, and in 1972 obtained a doctorate at Leiden. He has held postdoctoral positions at the University of Sussex, Cambridge University, and the California Institute of Technology. After serving as a junior faculty member at the University of Minnesota, he came to Leiden in 1983. At present, his main interests are cosmology, the formation of structure in the Universe, astrophysical applications of chaos theory, and high energy hydrodynamics. Besides his academic pursuits, he takes an active interest in the popularization of science and in the arts. He is free-lance employee of VPRO Broadcasting and has participated in many productions on radio and television. In 1994 he was admitted as a student to the Gerrit Rietveld Academy for the Arts in Amsterdam.

THE FORCE OF SYMMETRY

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VINCENT ICKE
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'But if anybody says he can think about quantum problems without getting giddy, that only shows that he has not understood the first thing about them.'

Niels Bohr

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for Ieske
Without you, Earth is
an insignificant speck
in the Universe

Contents

Preface

The wings of a butterfly shimmer spectacularly in the sun. But when you pulverize these wings – please use only animals that have died of natural causes – the resulting powder is greyish-brown, not vividly coloured. That's quantum mechanics in action: the colour is not due to pigment, but it is due to the fact that light is made of particles that behave like waves.

Plants do not grow in the dark, even if you keep them warm. That's quantum mechanics in action: heat radiation, even if there is lots of it, cannot trigger the proper chemical reactions in leaves, because an individual particle of that radiation doesn't carry enough energy.

When you look out into the street through a window of an evening, you will see yourself reflected in the window, and yet the passers-by can see you, too. That's quantum mechanics in action: the same causes, apparently, do not always have the same effect.

Clearly, you do not need an expensive laboratory or a huge accelerator to see quantum effects. A healthy dose of curiosity will do very well; I think that it will in fact do much better, at a fraction of the price and at a billion times the enjoyment.

Physics is not difficult; it's just weird. Physics, contrary to the opinion of many journalists and parents of scientists, is not particularly hard to explain or to learn. To learn how to use relativity, you do not have to be Einstein; nor do you have to be Heisenberg to do quantum mechanics.

Physics is weird because intuition is false. To understand what an electron's world is like, you've got to be an electron, or jolly nearly. Intuition is forged in the hellish fires of the everyday world, which makes it so eminently useful in our daily struggle for survival. For anything else, it is hopeless. Our intuitive fear of heights would be ridiculous for an albatross; our intuitive appreciation of the flight of a ball is silly if we want to trace a quark. Intuition gives us plausible nonsense like astrology, homeopathy, or

quantum-mechanics-turned-into-Zen. Intuition does not help us much in doing physics, be it quantum theory or classical mechanics (ever tried to understand the motions of a spinning top intuitively?)

Physics is powerful. That is a very good reason to write about it. Physics allows us to understand things: why the stars shine, why water is wet, why the sky is blue on Earth and orange on Mars. Physics teaches us to make things: from your contact lenses to the soles of your shoes, physics is at work. Physics is responsible for telephones, X-ray machines, gyroscopes, computers. Used wisely, like any power tool, treated with respect and understanding, physics benefits everyone.

Physics is beautiful. It makes me sad beyond words to know that so many people think of the physical sciences as barren, boring, bone-dry. Not so: when you lie outside in the grass on a clear dark night and look up at the stars, what you see is splendid. It is also physics. Understanding can lift you off the Earth, safer and faster and further than any rocket. The mind can travel among the stars, even enter them to see what causes those fires inside. To the beauty of seeing, we can add the beauty of understanding. And there is another level of beauty beyond that: the beauty of discovery, of creation, of *doing* physics. This beauty I love the most.

Physics is simplification, and so is explanation. Thus, insecure scientists need not fear that explaining physics to anyone is demeaning. The only thing that makes an explanation wrong (for experts or for novices alike) is if the reader has to unlearn something later. You will *not* have to unlearn anything when, having read this book, you go on to further study. This is the real stuff, even though there is a two-semester lecture series of heavy technical material behind every chapter. In fact, it is my intention that you could read this book right alongside a textbook on gauge field theory.

But no unlearning – just further reading. If I have to take anything back of what is in here, then either I blundered, or something really new has been discovered. Let us hope for the latter.

Physics may be easy to learn, but it is nearly impossible to do, to create. The weirdness of physics blocks the way. This prevents most of us from making new science, and it is what prevents most novices from understanding the old. Creating science is like creating poetry, of which Proust said: 'If it isn't easy, it is impossible'. To *do* physics, one has to be Bohr, or Curie, or Feynman. But people like me can write about it. And people like you can understand it (probably you're not like Bohr, but secretly I hope that you are).

But why this subject? I wanted to write about things that are well understood, by the standards of professional physics. That still leaves plenty

of choice. I might have written about gravity, black holes, and all that, and someday maybe I will. I might even fit the words 'Einstein' or 'dinosaur' into the title. But for now, I am writing about matter and forces as they occur in the world of the very small, because, in a sense, the interaction of particles is the most physical of physics subjects. Black holes may be rare in our everyday world, but colourful butterflies, green plants, and shining stars are not. These common things do what they do because of the way their particles interact. Matter and forces perform a ballet; this book is a brief account of that amazing dance.

In the ten years it took me to write this book I have accumulated an enormous debt of gratitude to quite a few people. First there is Martinus Veltman with his profound, fascinating, chaotic, hilarious and inspiring lectures on fundamental physics. Second is Fran Verter who induced me to stop talking (no small feat) and to start writing (even harder). Third is an anonymous referee who caught a large variety of inaccuracies in the first draft and who advised CUP to go ahead with the book none the less. Then there's Donald Knuth with his awesome TEX typesetting language, the designers of the Macintosh computer and the folks who wrote the *Canvas* drawing software for it: I am amazed and humbled by people who can express clarity and beauty in bits and bytes. Next, Sheila Shepherd and Philip Meyler helped me along with their editorial advice and their inexhaustible patience. Dap Hartmann went through the last-but-one draft with a fine-toothed comb. In the final year the support of Frans Icke and Erica Ott was crucial. I hope that all of them will find that this book is a suitable payment of my debts.

Second printing

Many readers asked for the inclusion of more material on the mathematical properties of symmetry groups, and for more details on fundamental symmetries such as chirality. I am afraid, however, that too many potential readers have already been put off by what little mathematics there is in this book. Moreover, my knowledge of this part of the subject is far too shallow.

Numerous people kindly reported errors big and small in the first printing. I am grateful to David Broadhurst, Tony Hey, Jan Hilgevoord, Ray Lahr, Johan Lugtenburg and D.H. Rouvray for their contributions. I am especially indebted to Dolf de Vries for a no-nonsense lecture on basic chemistry and to Howard Chang for his very detailed criticisms. I also thank the latter for quoting Confucius: *If you believe everything in a book uncritically, then it would have been very much better if the book had never been written. The*

author is a human being and is likely to make mistakes and utter complete nonsense.

Somewhat to my surprise, the most controversial statement in the book appears to be my contention *Physics is not difficult; it's just weird.* Maybe I should have omitted the qualification 'just', to emphasize my feeling than weirdness is a greater obstacle to understanding than anything else. But I stick to my guns. The fact that anything at all in the universe is comprehensible means either that we are very intelligent or that the basics of nature are very simple. Given that we are some sort of chimpanzee, carrying a mere kilogram of glop between our ears, I opt for the latter alternative.

Introduction

In the seventeenth century, great physicists and mathematicians like Descartes and Newton made the first real progress in understanding the interplay of matter and force, and in the description of the motions due to this. It must have been a marvellous epoch: for the first time, the workings of Nature on a grand range of scales were encompassed by theories that actually predicted things correctly. Since then, four forces have been discovered, and at present we are in the middle of another monumental advance: for the first time, there is a real prospect of a theory that explains *all* forces on the basis of one mechanism. We live in a marvellous epoch, and if you miss out on this revolution, you deprive yourself of a big piece of the action in the twentieth century. That is why I wrote this book: to instruct myself and to share with you the delight of this wonderfully intricate and powerful view of Nature.

This book is about experiences in the world that you and I live in, but the action takes place in an extremely special corner, which is not the one of our daily life. Thus, in reading what I have to say, your belief will be put to a severe test. What are you going to accept of all the extraordinary explanations that lie here before you? As stated in the splendid book by Abell and Singer,

> 'We accept that the light from stars in the night sky is thousands or even millions of years old, even though that concept contradicts our intuitions; we believe those same stars are unimaginably huge balls of glowing gases whose light is maintained by nuclear fusion [. . .] even though they look like tiny, pristine jewels. We believe that this book we are holding is solid, even though at the atomic level we know that it is almost 100 percent empty space'.

And then I come along, telling you that, on the subatomic level, this book *is* empty space, in a sense, and that everything we see can be reduced to the behaviour of a kind of stuff called 'vacuum'! All these things may seem

bizarre, but to the best of today's knowledge they are true. You need not take my word (or anyone else's) for this; one of the fun things about physical facts is that you can verify them yourself if you want to spend the time and effort. This book is mostly about facts (and conjectures whenever we are dealing with frontline stuff); some will seem even more bizarre than those just quoted and are more difficult to verify.

The emphasis of this book is on the theory of matter and force; only incidentally will I mention specific observations, such as the mass or the decay time of a certain particle. I like the theoretical side of physics best, and theory has got rather short shrift in popular writing. By the way, it would be wonderful if someone were to write a parallel book that is *really* about accelerators, detectors, computers and all that. It ought to contain not just a few incidental pictures from bubble- and spark-chambers, larded with thumbnail sketches of personalities attending them, but should be packed from cover to cover with the all-stops-out high technology that is the glory of today's experimental science.

There are quite a few books that place more emphasis than I do on the experimental findings in particle physics. A reference list is given at the back of this volume. These books also contain more history than is presented here.

Because this book is about theory, it might be unsettling in several ways. First, the emphasis on theory will take us into somewhat uncertain territory, because this is a field in which there is an intense activity at present. Thus, you may encounter statements like 'This was thought to be absolutely impossible, but then someone realized that ....' Such is progress in theoretical physics. Second, you will probably find many things here that are radically new to you. I have tried to soften the impact of this, by introducing concepts first in the form of a sketch, for example in the discussion of the differences and similarities of 'matter' and 'force'. This sketch is filled in and detailed later, when more of the ingredients of the overall picture are available. Thus, you will sometimes want more information than is given a few chapters hence; if you find that you haven't got it by the end of the book, you're ready for reading the more technical work specified in the reading list. Third, unless there's a major bug in the typesetting software, this book does not contain the same stuff twice. Moreover, it follows a line of development and argument. So you will have to read it much more carefully than a coffee-table book. Maybe (horrors!) you'll even have to reread it.

I have tried to relate the current state of affairs in the theory of forces, but I did not write a book to convince you of anything: I have no philosophy to sell and I refuse to get bogged down in sterile speculation on what 'reality'

means, and that sort of thing. For me, reality is what we have in common – you can do experiments and calculations just as well as anyone and possibly better. Thus, if you don't believe the physics, I demand that you study the original publications I used – there is a reference list – or, better yet, go and study physics yourself, and contribute to its advance.

Today's understanding of forces rests on three pillars: relativity, quantization and symmetry. These may seem formidable, but they aren't so bad; your worst obstacle is yourself, or rather the prejudices you acquired by being a certain size. On human scales of length, mass and speed, most effects that dominate the lives of subatomic particles are imperceptible, even though we'll see later that the very solidity of matter depends entirely on a quantum property. Our intuitions, and hence our philosophies, have a devastating bias against anything that lies outside the minuscule compass of gross everyday physics. Conquer the chauvinism due to slowness and bigness and you will be free to enter the spectacular world of the very fast and the very small, where relativity and quantization reign. Your reward will be a picture of forces that is, perhaps surprisingly, not too difficult to accept.

Our everyday experience of the world may delude us into thinking that 'classical' physics is obvious. But, in fact, almost nothing is self-evident. When we take a few steps down the street, we occupy an entirely different location in space and yet we do not change. When we turn a corner, we are not suddenly transformed into a giant penguin. This is so commonplace that it has ceased to amaze us, if indeed we ever stopped to notice. And yet we know of no compelling reason why these things should be so. In other words, we don't know why they are necessary in order to obtain a properly working Universe. Indeed, if you were to construct your own universe, you might well be able to arrange things differently; quite possibly, there are excellent reasons why space and time behave the way they do, but we haven't discovered them yet.

As the story unfolds, we will see that the most elementary question about the working of the Universe – namely *where is what when* – is answered in different ways, which reveal more and more about the way in which the Universe is put together. First, the three ingredients of the question ('where', 'when' and 'what') are considered in the form of classical mechanics, the theory of everyday life with which you are most familiar. In that case, each of these three items must be introduced separately: `space` for 'where', `time` for 'when' and `particle` for 'what'. In this picture, there is no connection or kinship whatever between these ingredients.

Second, the introduction of relativity forges a bond between 'where' and 'when'; thus, we learn to work with one ingredient, `space-time`, which

takes over the role of what formerly was the duo 'where' and 'when'. Third, quantization allows us to give meaning to 'small' and 'large', which is necessary if we want to explain large and complex systems in terms of smaller and simpler ones. But we are compelled thereby to replace particles with `quanta` as entities of 'what'. Also, we discover that there is a basic uncertainty in the behaviour of a quantum, which brings us dangerously close to allowing chaos to reign the world. Fourth, the introduction of `symmetry` imposes order on the quantum chaos and enables us to merge all three ingredients into one: we find that quanta and space-time obey rules that are imposed by symmetries. Thus, the 'where', 'when' and 'what' are folded together into one, the `vacuum` (perhaps, in the most modern theories, this is a ten-dimensional kind of stuff). In this amazing picture, the properties of the vacuum encompass all the workings of the Universe.

I have suppressed almost all mathematics in the presentation, in part to accommodate my readers, in part to accommodate myself: although some facets of the relevant mathematics (especially those on symmetry) have great beauty, the nitty-gritty of relativistic quantum theory is often singularly repulsive (the cognoscenti who have seen, for example, the full expression of the electroweak Lagrangian, with its horrid tangle of typefaces and indices, may know what I mean). Even so, the stuff presented in this book is quite deep, but we mustn't confuse deep with difficult. Working out the quantitative details of the processes described here requires an enormous amount of difficult calculation, but the problem is often just technical. The depth of our subject lies not there, but in the insights of the physics. The basics of these I have tried to present; the obstacle to our understanding is not the technical complexity, but our hidebound intuition. What gets in our way is that the concepts are unusual, not that they are exceedingly difficult.

Even so, there are a few places in the text where I have used mathematical symbols. Most of the time, this is a matter of economy. It is very cumbersome to write 'Planck's constant' all the time when simply jotting down $\hbar$ will do. And I'm sure that you cannot be bothered to read that sort of verbose text. In some other places I have written a few elementary equations of the type $E = mc^2$. If such things *really* bother you, just skip them, and you will hardly be worse off. Or, preferably, just follow the text and re-read the occasional paragraph or two, just like all of us physicists do.

The price to be paid for leaving out the mathematics is that many things are said in the form of plausibility arguments or metaphors, and some things remain unsaid altogether: in those cases, precision is lost. This is sadly inevitable when using ordinary language. It is known that the great Niels Bohr struggled mightily with the problem of conveying physics in words,

and for lesser mortals this is certainly impossible. To me, narrating physics is like casting shadows of a very complex object: one never gets the whole shape, but illuminating it from many different angles helps.

Plausibility arguments are all one can give at *any* level in physics, because the laws cannot be found by deduction. But there is always the danger of carrying a metaphor too far; in the end, we cannot escape the fact that to us, who are so very much heavier and bigger than particles, the world of the very small will forever be an alien place. I can well imagine that metaphoric writing annoys some readers, especially those with knowledge in the physical sciences. Yet I make no apologies: I'll be happy if this book induces them to acquire the necessary skills in maths and to pass on to the real stuff in the source texts. In fact, that would be a gratifying function of my song of praise for these wonders of Nature: just as a museum visit should induce you to practise art, I hope that my book will encourage you to study physics.

1

A matter of force

1.1 The law of inertia

The way the world works is mostly the way things move: *Where is what when*? is just about the most basic question one can ask about the Universe. Everyday experience gives us a rough-and-ready answer: the motion of matter is governed by forces. A puck may lie still on the ice until it is struck with a stick, after which it glides straight along until it hits something else. Without being struck, bumped, caught, or otherwise interfered with, it will follow its own path.

This description is horribly vague. On the ice, the puck moves with very nearly constant speed in a straight line. But the same object, struck in the same way, moves very differently on the pavement: almost as soon as the blow that sets it in motion is over, the puck lies still again. At the very least, then, it is unclear what an object's true path is: the smooth gliding along the ice, or the state of rest on the pavement, or what?

We cannot specify what we mean by 'force' until we have specified what ideal state of motion that force is supposed to perturb. Some four centuries ago, it was generally assumed that motion with constant speed along a circle is the ideal motion that can maintain itself indefinitely without external influence. This idea (although we now know it to be wrong) is not in itself absurd: we cannot deduce from first principles whether or not uniform circular motion is ideal in the above sense. Indeed, if we were to build our own universe, maybe we could arrange it that way.

But we *observe* that in the actual Universe a circular motion cannot maintain itself indefinitely, as is evident in the operation of a slingshot (Fig. 1.1). In another universe, constructed along the lines of the classical philosophers, maybe David's stone would have continued to buzz around in a circle, rather than fly off on a tangent to strike Goliath down.

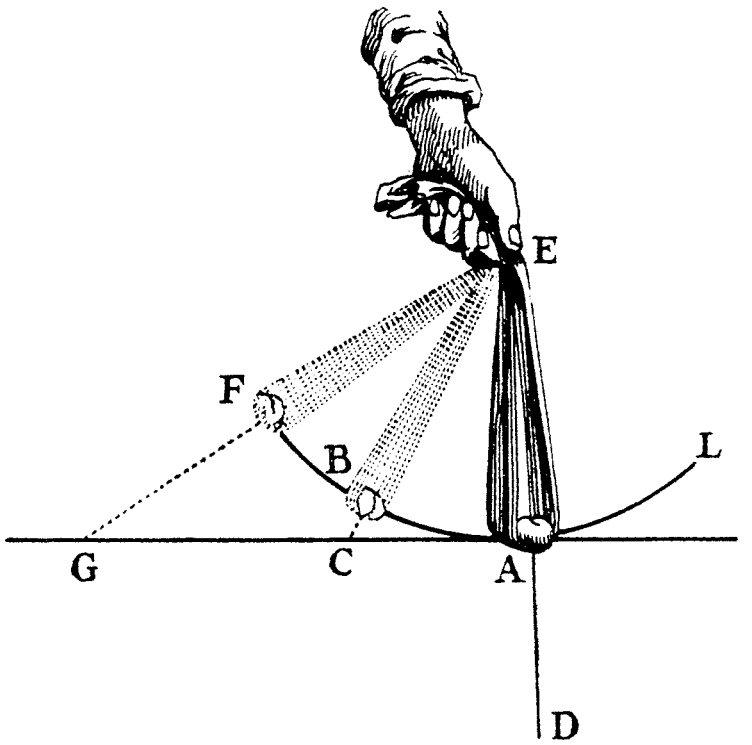

Fig. 1.1 Engraving illustrating the action of a slingshot, showing the law of inertia at work: the stone, when released from the constraining force of the sling, moves away in a straight line. (From Descartes's *Principes de la Philosophie*, part two, Art.39.)

A major advance towards today's theories of Nature was made by Descartes, who formulated what we now know as the Law of Inertia: *a piece of matter moves in a straight line with constant speed, unless a force acts on it.* This introduces force as something that causes the state of motion of an object to become different from the ideal constant-velocity motion. The pivotal question is then: what is that something?

Descartes himself thought that a force comes about through the immediate physical contact between objects. To anyone who limits the observation of Nature's workings to an occasional billiards game, this idea is self-evident. Descartes knew that the motions of the planets must be governed by some sort of force, even though they do not seem to be in contact with anything (pinned to crystalline spheres, or whatever). Planetary orbits are *curved*, so the law of inertia implies that there is a force which acts on them constantly.

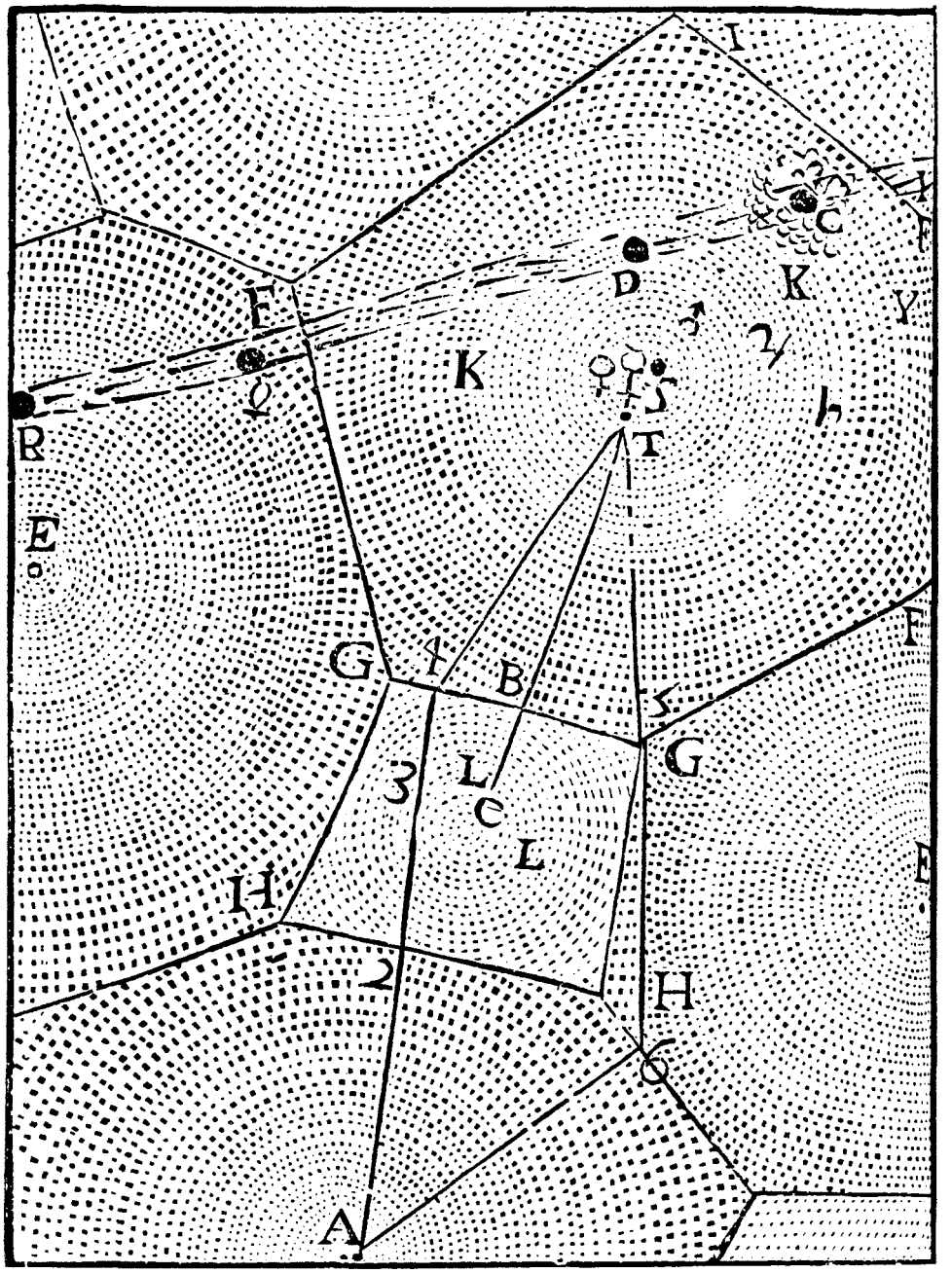

Fig. 1.2 Vortices in the 'subtle matter' or aether which, according to Descartes, was responsible for transmitting the forces that keep the universe together. (From *Le Monde, ou Traité de la Lumière.*)

Accordingly, Descartes assumed that all of space was filled, to the complete exclusion of emptiness, with a novel kind of substance, a subtle matter that transmitted the forces in the Solar System through direct contact. This matter was supposed carry the celestial bodies around in its swirling eddies (Fig. 1.2). In the second part of his *Principes de la Philosophie*, Article 16, Descartes wrote:

> ... from the sole fact that an object is extended in length, height and depth, we have reason to conclude that it is a substance, [and so] we must conclude the same about space which is supposedly empty: namely that, because it possesses [spatial] extent, it also has substance.

In other words: space has physical attributes, namely its three dimensions, so it must be regarded as real stuff. This powerful notion lay hidden for three hundred years, until it was rediscovered independently by Einstein.

One could make some objections of principle, for example that the constituents of Descartes's subtle matter must not have any internal structure, and hence must be infinitely small, wherefore – by Descartes's own definition – they have no spatial extent and consequently do not exist. Indeed, this problem had been spotted in antiquity, and gave rise to the hypothesis that the world is made of atoms: small particles that have no inner structure and therefore cannot be further divided, but that do have a finite extent. Of course, the atoms we now know do not fit that description at all, even though they have the same name. Later I will return in detail to the consideration of smallness.

Unfortunately for its inventor, the Cartesian hypothesis about the direct-contact origin of forces did not lead to calculable results. Descartes proposed that the planets are kept in their orbits by swirling, vortical motions in the subtle matter between them; but this assumption did not give a quantitative prescription for the behaviour of the force. Thus, nothing could be calculated; for example, Kepler's laws (which were known to describe planetary motion very well) could not be explained in terms of the motion of the subtle matter.

It was Newton who first realized that, for the description of planetary motions, it is not necessary to know what a force 'is', as long as one can give a precise description of what it does, i.e. formulate how it depends algebraically on physical quantities. In fact, it had already been pointed out by Hooke that Kepler's Third Law implies that the force between the Sun and a planet acts along the line connecting them and decreases in inverse proportion to the square of the distance between them. Newton extended this with the prescription that the force be proportional to the product of

the masses of the attracting bodies. This was an important advance, because it established a symmetry between the objects involved. It isn't as if one object, for example the Sun, is the boss that does all the attracting; in the Newtonian description of a force, both objects attract *each other*, so that we can truly speak of an interaction. Moreover, in keeping with Descartes's hint that there is only one force in Nature, Newton presumed that the dominant force in the Solar System (to be known as gravitation) is universal and acts between all objects.

The success of this approach is well known, and it has been praised beyond measure. And yet the Newtonian idea of force had some uncomfortable features. It was conceived to be an instantaneous interaction: a mutual working, at exactly the same moment, between two spatially separated objects. The instantaneous nature of the action was not, at first, recognized as a problem; but the objections to the 'action at a distance', across supposedly empty space, were loud from the beginning. Still, it worked. Planetary motions could be calculated; Kepler's laws were explained in terms of the force of gravity. The Cartesian hypothesis of direct contact was completely eclipsed by the Newtonian action at a distance.

It was clear even in Newton's days that there must be more forces in Nature than gravity alone. Whereas Descartes's forces could appear in many guises – attractive or repulsive – depending on the detailed workings of the subtle matter, the gravitational force is always attractive, and hence cannot make stable objects: all things always fall down, so to speak. Thus, all many-particle systems in our Universe, when acted upon by gravity alone, must inevitably collapse, even though this might require a very long time. The apparent solidity and stability of matter is proof of the existence of other forces, so there was scope for an extension of the Newtonian system by finding those forces. Some were found, such as the magnetic and electrostatic forces; the experiments of Cavendish and Coulomb even showed that the electrostatic force can be described by exactly the same mathematical form as the force of gravity. But nobody questioned the underlying concept of instantaneous action at a distance any more.

1.2 The speed of light

A dramatic and fundamental step forward was made by Maxwell. This advance was wholly within the Newtonian world view but, interestingly, was also one of the first nails in the coffin of that view. Maxwell showed that the forces of electricity and magnetism are not two totally different ones, but instead are two aspects of one force (albeit a more complex one), the

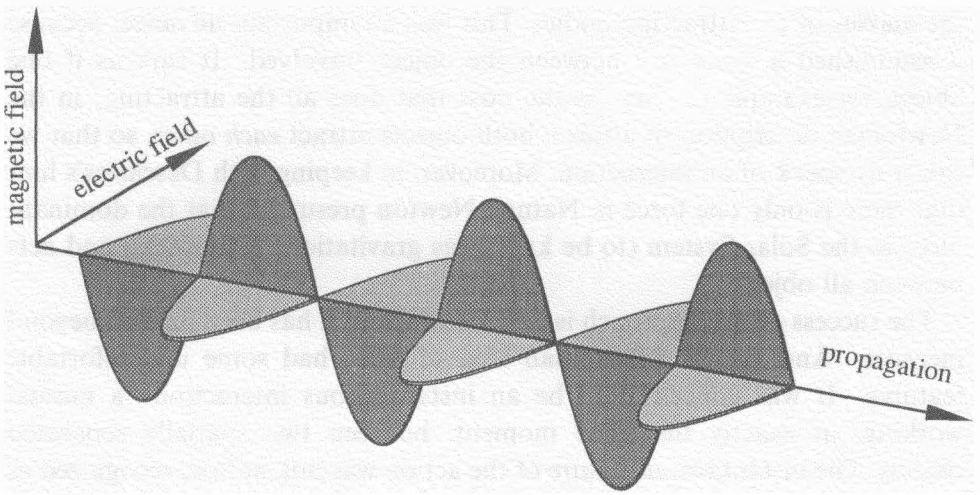

Fig. 1.3 Electromagnetic wave. The electric field (light grey), the magnetic field (dark grey) and the direction of propagation are all mutually perpendicular.

electromagnetic force. From the experiments of Ampère, Faraday and others it was known that the motion of an electric charge generates a magnetic force, and that the motion of a magnet induces an electric current.

Maxwell showed in his theory that this leads to the unification of the two phenomena, and predicted that therefore these two can continue to jump over each other (Fig. 1.3): a changing electric force generates a magnetic one, which in turn induces an electric force, and so on. The resulting electromagnetic waves are now familiar: for example light, radio waves and X-rays. In vacuum, all move with the same speed: the speed of light.

Once electromagnetic waves were recognized as an important constituent of our Universe, a thorough study of their behaviour was undertaken. In particular, people wondered how such waves could propagate: what was it that did the waving, so to speak? Some suggested that space was filled with an invisible kind of matter, a luminiferous aether, thereby bringing back Descartes's subtle matter in a different guise and for a different purpose. Here the problems associated with the Newtonian notion of action at a distance returned with a vengeance; the presence of an aether – even though its constituent particles, whatever they might be, could not be observed – has measurable consequences for the propagation of light. If the waves are carried by any subtle matter whatever, then their observed propagation must depend on the motion of their source (or the observer) through that aether.

Experiments to detect the effects of such an aether drift were undertaken. Soon it appeared, from the experiments of Michelson and Morley in 1887, that there is no apparent motion at all with respect to an aether; they found that the speed of light does not depend on the state of motion of emitter or observer, but that it is the same for everyone, regardless of their behaviour.

This is often expressed by saying that *the speed c of light is invariant*. The invariance of c is a certified observational fact, and it is surely very odd; how odd, can be glimpsed as follows. Light behaves, at least in part, like a wave. Now from daily experience we know that the speed of a wave depends on how we move. When a wave rolls towards the shore, you see the crests move at a certain speed; but a seagull can adjust its airspeed in such a way that the wave seems to be standing still below it. Now I can ask: do light waves behave like that?

Michelson and Morley took a light beam, split it in two with a half-silvered mirror, and sent the two half-beams along on different tracks, perpendicular to each other. One path was placed along the direction of Earth's orbital motion, and the other at right-angles to it. After the light rays had traversed their paths, they were brought together again, and their travel times were compared by means of an optical effect called *interference* (we will see much more of that later).

The results showed conclusively that, whatever you try, the travel time of light in vacuum along equal distances is always exactly the same. In other words, *the speed of light does not depend on the motion of the emitter or the receiver* of the light.

In daily life you won't notice this bizarre fact or its bizarre consequences. The reason that you do not observe anything particularly odd when you are cycling along is that you move so very, very slowly. No insult intended, but as far as light is concerned (which travels at 300 million metres per second) you are almost standing still. It is this, and nothing else, that makes the consequences of the invariance of the speed of light so contrary to intuition.

Notice, by the way, that the fact that c is invariant provides a standard of speed for the entire Universe! If the difference in speed between two objects were only a relative thing, it would make no sense to distinguish between fast and slow. But, contrary to popular opinion, it is false to say that 'everything is relative'. Because of the invariance of the speed of light, it is more nearly correct to say that everything is absolute!

This observed invariance is a most curious fact; not so much because we must abandon the idea of an aether, but rather because it leads to an amazing conclusion about motion in general. The remarkable consequences of the constancy of the speed of light can be qualitatively understood by

asking: 'If c is really invariant, then how does one light ray see another one move?' Well, if there is no conceivable superseagull that can fly in such a way that a light wave appears to be standing still around it, the answer must be: 'Any light ray sees any other one move with the speed c.'

If a light ray, travelling at speed c, encounters another one head-on, also travelling at c, then the above can be written symbolically (not algebraically!) as "$c+c=c$." Similarly, if two light rays that travel in the same direction see each other move with the speed c, we must, in some sense, likewise require that $c-c=c$. If it is really true that c is invariant, then we cannot escape the conclusion that $c+c=c-c=c$! *Most* remarkable: one would have expected $2c$, or 0, or something in between, depending on circumstances. But the Michelson–Morley experiment has shown that the speed c does *not* depend on circumstances.

Einstein, who first asked the question about the relative motion of light rays, had the courage and the insight not to reject $c+c=c$ out of hand as absurd; all it means is that the invariance of the speed of light compels us to accept that speed is a more complicated beast than we suspected. However, the implications are staggering: we can, in fact, add speeds in such a seemingly contradictory way, but only at the expense of a drastic revision of the classical concepts of space and time.

Because it is required that $c+c=c-c=c$, speed cannot be a simple algebraic number for which the normal rules of addition and subtraction hold, as in the case of money or apples in the market. We must reconsider what exactly is meant by the addition of speeds. Note that this is no cause for dismay; even in everyday experience, we deal with quantities for which two plus two does not always equal four. Your position on Earth is a case in point: if you walk two kilometres, then another two, and then two more, you are not necessarily six kilometres from your point of departure. In fact, if you have walked along an equilateral triangle, you are back where you started. If distances added like money, you could never mail a letter or walk the dog: you would never get back home.

1.3 Relativity and fields

Einstein showed how we ought to define addition in such a way that the addition of any two velocities leaves the speed of light invariant. In order to be able to do this, he had to abandon the idea that time and space can be measured independently: if speeds add in a curious way, then this is due to the underlying behaviour of the ingredients of speed, namely space and time (remember, speed = distance/time!) If space and time cannot

be measured independently of one another, then there is no such thing as universal time.

Subsequently, Einstein found a number of other results that are, to our intuition conditioned by always moving much slower than light about as bizarre as $c + c = c$. One of these results is that c is an absolute *maximum* speed: nothing can travel faster than light. This conclusion can be glimpsed from the above. If the addition of speeds is defined such that $c+c=c$, then we must also have $c+c+c=c$, and so forth: we can never exceed the speed of light. Notice, by the way, that I have hereby shown that c must be an upper limit because of the observed fact that the speed of light is always the same; I do *not* say that c is the maximum speed because I have tried hard to exceed it and have failed! The c limit is an inescapable consequence of the experimental fact of c invariance, and so we can prove that any attempt to exceed the speed of light must fail. Because c is finite and maximal, and because it is the same no matter what you do, it serves as a universal standard of speed. There is an *absolute* meaning to the expressions fast and slow: motion with a speed that is much smaller than the speed of light is slow, any other motion is fast.

Another result is that mass and energy are essentially the same. This, too, can be appreciated on the basis of the above. I will show this by means of a `space-time diagram`, which is a graphical summary of 'where is what when' in a particular case (Fig. 1.4).

Suppose that we have a particle that has no internal structure, and that is acted upon by one force only. We know that the more energy is transferred by the force to the particle, the faster it travels. But because c is the maximum speed, I can transfer all the energy I want, I will never exceed the speed of light. Then where did all that energy go? Recall from everyday mechanics that the transfer of a given amount of energy gives a large boost to the speed of a low-mass object, and a small speed increment to one with a high mass. Let us look at the behaviour of a football and a railway engine (Fig. 1.5) when each is given a standard kick, for example one delivered by the goalkeeper of the Dutch national team.

The ball is propelled to a high speed, which in a space-time diagram means that it lies on a line that is inclined strongly with respect to the time axis. But the same kick, delivered to a railway engine, has almost no effect; even if all eleven team members were to hurl themselves at the machine, its speed would barely differ from zero, and its space-time path would be practically parallel to the time axis. We summarize these experimental facts by saying that a football has a small mass and a railway engine has a large mass.

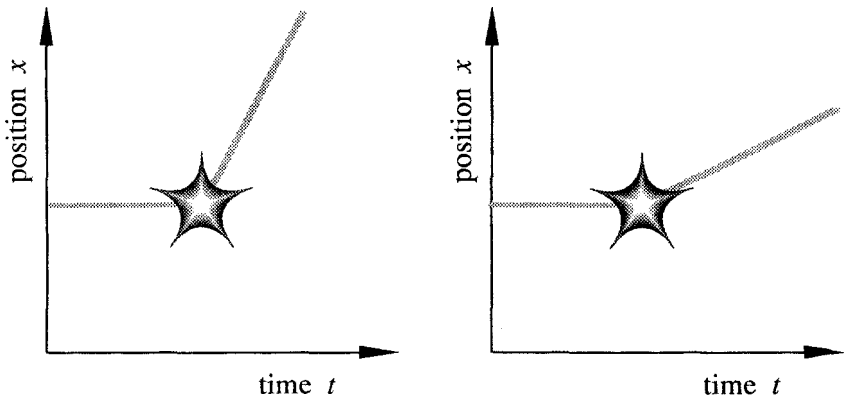

Fig. 1.4 Sequence of events of two different objects during a kick, as shown by their tracks in space-time. Such space-time diagrams occur frequently in this book, but the axes will usually be omitted. The convention I will use is that time runs to the right and spatial distance increases upwards on the page. This is different from what you usually find in the physics literature: there, time runs upwards on the page. That convention is typographically inept and I will almost never use it in this book.

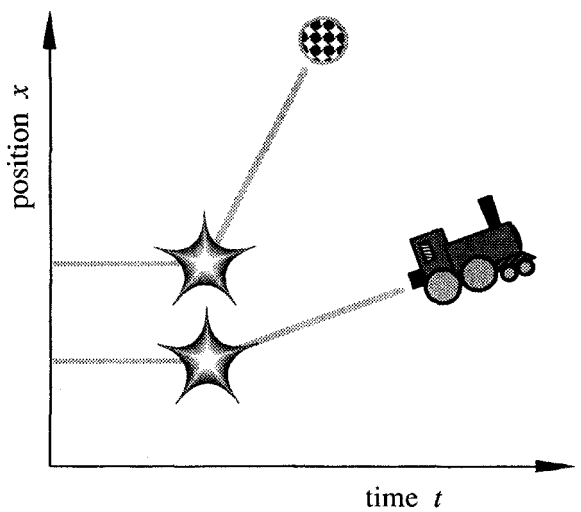

Fig. 1.5 Space-time diagrams of two different objects during a kick. The top track shows a football being kicked; its low mass shows up in a large change in velocity (i.e. a large acceleration, producing a sharp kink in the space-time track). The other track is not as sharply deflected, showing what happens when the same kick is delivered to a railway engine.

Now if c is to be a strict upper limit, the space-time paths of objects cannot be arbitrarily steep. If the football has a small speed, the kick delivered by our goalie makes a noticeable bend in its space-time path. But if the ball moves close to c, the same kick must make a much smaller bend: because c is a strict upper limit, the closer we get to the speed of light, the smaller the increase of the velocity becomes for a given addition of energy. Therefore, the more energy a football has, the more it resembles a railway engine: its energy acts as if it were mass! Because the c limit forces us to accept that energy is essentially the same as mass, we must also accept that even the mass m of an object at rest is equivalent to a certain quantity of energy E. A precise calculation of this equivalence leads to the famous $E = mc^2$.

These consequences of the experimental fact that the speed of light is the same for all observers are described precisely in the theory of `relativity`. In our discussion of forces, the facts of relativity have a very profound influence, mostly because of the findings that: first, the speed of light cannot be exceeded; second, mass is equivalent to energy; third, because a universal time does not exist, the order in time of events can be different for different observers; fourth, the speed of light can be kept constant only if we treat space and time on an equal footing. These points will be discussed in more detail later.

The facts and conclusions of relativity slash all support from under the Newtonian concept of instantaneous action at a distance. Because the speed of light is a maximum, there is no such thing as an instantaneous connection between spatially separated points. Accordingly, if there is to be any influence across a spatial distance, we must accept that that influence is underway for a while. Relativity prevents a force from acting instantaneously, so that there must be something that transmits it, some sort of messenger substance that carries the information about the action of the force from one point to another. This something is called a `field`, and because of the c limit we are compelled to accept the field as a physically real object, not merely as an aid to calculation. It is beginning to look after all as if we need some sort of direct contact to transmit a force (at this point, Descartes smiles).

Fields can have different forms, from very simple to very complex. We may imagine a field as follows: at every instant in time, each point in space is provided with a little label, on which we can read the strength of the force. When the field is simple, the label contains just one number ('scalar' field; Fig. 1.6). In a more complex case, the tag contains more numbers.

On this two-dimensional sheet of paper, I can represent a scalar field by a greyscale: the darker the picture, the stronger the field, and a single number (the percentage of paper covered by ink) describes the field at each

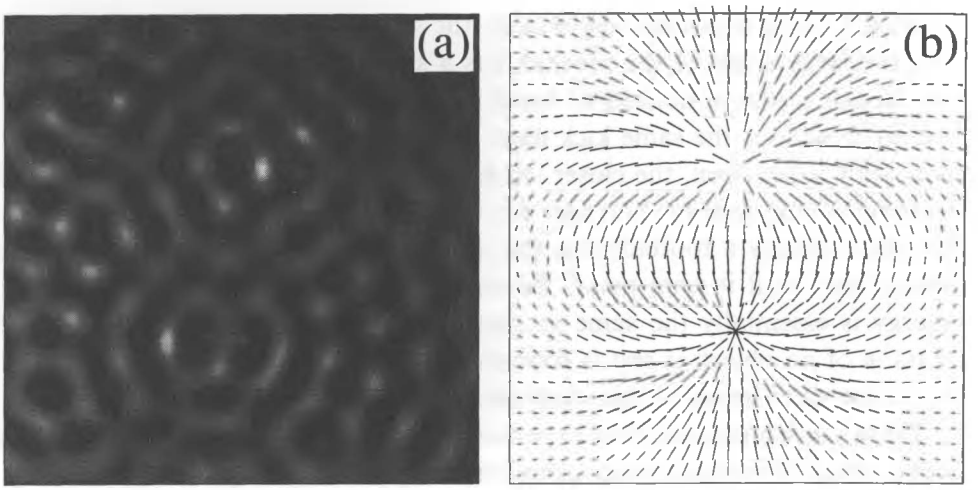

Fig. 1.6 (a) A scalar field. The field amplitude is indicated by a single quantity which, in this diagram, is represented by the intensity of the grey shading, as on a black-and-white photograph: the blacker the print, the stronger the field. (b) A vector field. Here, the field amplitude has two components: a strength and a two-dimensional direction. The strength is indicated by the length of the line segments, the direction by their orientation.

point. A somewhat more complicated field requires more numbers (called components) to describe it at each point. In two dimensions, I could represent a two-component field by a colour scale, where each component is shown as the percentage of paper covered by a different ink (say, blue for one component and yellow for the other – this would allow us to speak truly of 'green fields'). In practice, it is much clearer to represent the field at a given point by means of an arrow, which is why this type is called a *vector field*. The base of the arrow is the point where the field is measured; the directional angle is the first field component, and the arrow's length represents the second component. Thus, we obtain a field of arrows.

Complicated fields are more difficult to represent graphically, so that we usually prefer to work with an array of numbers that indicate the values of the various field components. Notice that we have a certain amount of leeway as to how we choose the components; for example, instead of taking the direction and the length of an arrow as the field components, we could have taken the lengths of the projections of the arrow in two different directions.

Fields are like weather maps. The temperature at ground level is a scalar field: single numbers all over the map suffice to specify it. The wind velocities

form a vector field: notice the little arrows on weather maps. The wind shear is a still more complicated field, and cannot be represented easily on a map. I'm sure that airplane pilots wish it could!

If we require the fields to behave properly according to the laws of relativity, it turns out that we are obliged to arrange their components in powers of four: a single number (four to the power zero, `scalar field`), a row of 4 numbers (4^1, `vector field`), a square array of 4×4 numbers (4^2, `tensor field`), a cubic array of $4 \times 4 \times 4$ numbers, and so forth. The magical number 4 comes in because of the combined $3 + 1 = 4$ dimensions of space and time.

1.4 Feynman diagrams

Our everyday experiences, limited as they are to speeds that are very much smaller than c, have shaped our thinking in ways that make relativistic phenomena very counter-intuitive; and now we encounter yet another oddity. Not only does relativity compel us to accept force fields as physical entities, but it turns out that the action of a force at a given space-time point cannot be varied continuously. Therefore, the above description of a field as an infinite collection of tags with numbers on them is found to be incorrect (Fig. 1.7).

Instead of having, at each point in space-time, a number that indicates the *strength* of the force, we find written on each label the *probability* that the recipient of the force gets a certain standard push (say, the kick of the goalkeeper of the Quark Quantum Football Club). The field is `quantized` and the entity that delivers the push is a `quantum`.

A quantized field is very different from a continuous one. Suppose one were to interpret the force exerted by all railway trains as a continuous field. Then we could stand between the rails anywhere in the country and be at ease: the average force is so small that one's equilibrium can be maintained by just leaning forward a little. But in reality, the force is not continuous but quantized: there is either no train at all, or a very heavy one, so that – depending on the quirks of fate – you are either completely safe or completely shattered.

So it is with fields: they are quantized, that is to say, they are built up from quanta that are exchanged between the participants in the force. The particle that emits the field quanta thus resembles a mad gunner, spraying a rain of bullets in arbitrary directions. We saw that a force is symmetric between its source and its recipient: it is a mutual affair, so that, properly speaking, we