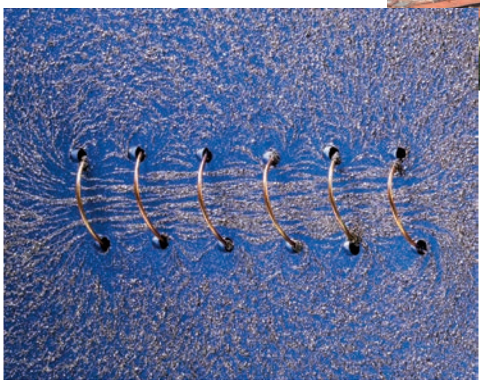
GLOBAL EDITION

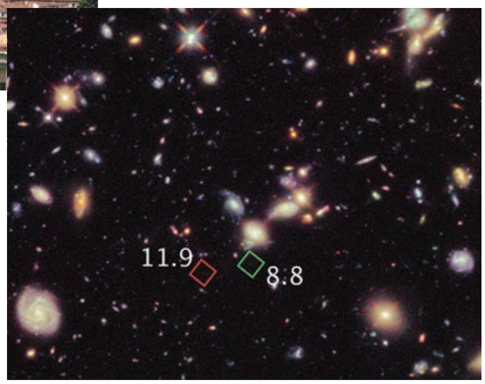
5th EDITION

PHYSICS

for SCIENTISTS and ENGINEERS

with Modern Physics





DOUGLAS
GIANCOLI

Fundamental Constants

Quantity	Symbol	Approximate Value	Current Best Value[†]
Speed of light in vacuum	c	3.00×10^{8} m/s	2.99792458×10^{8} m/s
Gravitational constant	G	6.67×10^{-11} N·m²/kg² → $6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$	$6.67430(15) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
Avogadro's number	N_A	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$	$6.02214076 \times 10^{23}\ \mathrm{mol^{-1}}$
Gas constant	R	$8.314\ \mathrm{J/mol \cdot K} = 1.99\ \mathrm{cal/mol \cdot K}$ $= 0.0821\ \mathrm{L \cdot atm/mol \cdot K}$	$8.314462618\ \mathrm{J/mol \cdot K}$
Boltzmann's constant	k	$1.38 \times 10^{-23}\ \mathrm{J/K}$	$1.380649 \times 10^{-23}\ \mathrm{J/K}$
Charge on electron	e	$1.60 \times 10^{-19}\ \mathrm{C}$	$1.602176634 \times 10^{-19}\ \mathrm{C}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$	$5.670374419 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$
Permittivity of free space	ϵ_0	$8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$	$8.8541878128(13) \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$
Permeability of free space	μ_0	$1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}$	$1.25663706212(19) \times 10^{-6}\ \mathrm{T \cdot m/A}$
Planck's constant	h	$6.63 \times 10^{-34}\ \mathrm{J \cdot s}$	$6.62607015 \times 10^{-34}\ \mathrm{J \cdot s}$
Electron rest mass	m_e	$9.11 \times 10^{-31}\ \mathrm{kg} = 0.000549\ \mathrm{u}$ $= 0.511\ \mathrm{MeV}/c^2$	$9.1093837015(28) \times 10^{-31}\ \mathrm{kg}$ $= 5.48579909065(16) \times 10^{-4}\ \mathrm{u}$
Proton rest mass	m_p	$1.6726 \times 10^{-27}\ \mathrm{kg} = 1.00728\ \mathrm{u}$ $= 938.27\ \mathrm{MeV}/c^2$	$1.67262192369(51) \times 10^{-27}\ \mathrm{kg}$ $= 1.007276466621(53)\ \mathrm{u}$
Neutron rest mass	m_n	$1.6749 \times 10^{-27}\ \mathrm{kg} = 1.008665\ \mathrm{u}$ $= 939.57\ \mathrm{MeV}/c^2$	$1.67492749804(95) \times 10^{-27}\ \mathrm{kg}$ $= 1.00866491595(49)\ \mathrm{u}$
Atomic mass unit (1 u)		$1.6605 \times 10^{-27}\ \mathrm{kg} = 931.49\ \mathrm{MeV}/c^2$	$1.66053906660(50) \times 10^{-27}\ \mathrm{kg}$ $= 931.49410242(28)\ \mathrm{MeV}/c^2$

[†]Numbers in parentheses indicate one-standard-deviation experimental uncertainties in final digits (2019, new SI). Values without parentheses are exact (i.e., defined quantities).

Other Useful Data

Joule equivalent (1 cal)	4.186 J
Absolute zero (0 K)	−273.15°C
Acceleration due to gravity at Earth's surface (avg.)	$9.80\ \mathrm{m/s^2}$ $(= g)$
Speed of sound in air (20°C)	343 m/s
Density of air (dry)	$1.29\ \mathrm{kg/m^3}$
Earth: Mass	5.98×10^{24} kg
Radius (mean)	6.38×10^{3} km
Moon: Mass	7.35×10^{22} kg
Radius (mean)	1.74×10^{3} km
Sun: Mass	1.99×10^{30} kg
Radius (mean)	6.96×10^{5} km
Earth–Sun distance (mean)	149.60×10^{6} km
Earth–Moon distance (mean)	384×10^{3} km

The Greek Alphabet

Alpha	Α	α	Nu	Ν	ν
Beta	Β	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	Ο	o
Delta	Δ	δ	Pi	Π	π
Epsilon	Ε	ϵ, ε	Rho	Ρ	ρ
Zeta	Ζ	ζ	Sigma	Σ	σ
Eta	Η	η	Tau	Τ	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	Ι	ι	Phi	Φ	ϕ, φ
Kappa	Κ	κ	Chi	Χ	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	Μ	μ	Omega	Ω	ω

Values of Some Numbers

$\pi = 3.1415927$	$\sqrt{2} = 1.4142136$	$\ln 2 = 0.6931472$	$\log_{10} e = 0.4342945$
$e = 2.7182818$	$\sqrt{3} = 1.7320508$	$\ln 10 = 2.3025851$	$1\ \mathrm{rad} = 57.2957795°$

Mathematical Signs and Symbols

$\propto$	is proportional to	$\leq$	is less than or equal to
$=$	is equal to	$\geq$	is greater than or equal to
$\approx$	is approximately equal to	Σ	sum of
$\neq$	is not equal to	$\bar{x}$	average value of x
$>$	is greater than	Δx	change in x
$\gg$	is much greater than	$\Delta x \rightarrow 0$	Δx approaches zero
$<$	is less than	$n!$	$n(n-1)(n-2)\ldots(1)$
$\ll$	is much less than		

Properties of Water

Density (4°C)	$1.000 \times 10^{3}\ \mathrm{kg/m^3}$
Heat of fusion (0°C)	334 kJ/kg (79.8 kcal/kg)
Heat of vaporization (100°C)	2260 kJ/kg (539.9 kcal/kg)
Specific heat (15°C)	$4186\ \mathrm{J/kg \cdot C°}$ $(1.00\ \mathrm{kcal/kg \cdot C°})$
Index of refraction	1.33

Unit Conversions (Equivalents)

Time

1 day = 8.640×10^4 s
1 year = 365.242 days = 3.156×10^7 s

Length

1 in. = 2.54 cm (defined)
1 cm = 0.3937 in.
1 ft = 30.48 cm
1 m = 39.37 in. = 3.281 ft
1 mi = 5280 ft = 1.609 km
1 km = 0.6214 mi
1 nautical mile = 1.151 mi = 6076 ft = 1.852 km
1 fermi = 1 femtometer (fm) = 10^{-15} m
1 angstrom (Å) = 10^{-10} m = 0.1 nm
1 light-year (ly) = 9.461×10^{15} m
1 parsec = 3.26 ly = 3.09×10^{16} m

Volume

1 liter (L) = 1000 mL = 1000 cm^3 = $1.0 \times 10^{-3}\ \text{m}^3$ = 1.057 qt (U.S.) = 61.02 in.^3
1 gal (U.S.) = 4 qt (U.S.) = 231 in.^3 = 3.785 L = 0.8327 gal (British)
1 quart (U.S.) = 2 pints (U.S.) = 946 mL
1 pint (British) = 1.20 pints (U.S.) = 568 mL
1 m^3 = 35.31 ft^3

Speed

1 mi/h = 1.4667 ft/s = 1.6093 km/h = 0.4470 m/s
1 km/h = 0.2778 m/s = 0.6214 mi/h
1 ft/s = 0.3048 m/s = 0.6818 mi/h = 1.0973 km/h
1 m/s = 3.281 ft/s = 3.600 km/h = 2.237 mi/h
1 knot = 1 nautical mile/h = 1.151 mi/h = 1.852 km/h = 0.5144 m/s

Angle

1 radian (rad) = 57.30° = 57°18′
1° = 0.01745 rad
1 rev/min (rpm) = 0.1047 rad/s

Mass

1 atomic mass unit (u) = 1.6605×10^{-27} kg
1 kg = 0.06852 slug
1 ton (metric) = 1000 kg
[1 kg has a weight of 2.20 lb where $g = 9.80\ \text{m/s}^2$.]

Force

1 lb = 4.44822 N
1 N = 10^5 dyne = 0.2248 lb
1 ton (U.S.) = 2000 lbs

Energy and Work

1 J = 10^7 ergs = 0.7376 ft · lb
1 ft · lb = 1.356 J = 1.29×10^{-3} Btu = 3.24×10^{-4} kcal
1 kcal = 4.19×10^3 J = 3.97 Btu
1 eV = 1.6022×10^{-19} J
1 kWh = 3.600×10^6 J = 860 kcal
1 Btu = 1.056×10^3 J

Power

1 W = 1 J/s = 0.7376 ft · lb/s = 3.41 Btu/h
1 hp = 550 ft · lb/s = 746 W

Pressure

1 atm = 1.01325 bar = $1.01325 \times 10^5\ \text{N/m}^2$ = 14.7 lb/in.^2 = 760 torr
1 lb/in.^2 = $6.895 \times 10^3\ \text{N/m}^2$
1 Pa = 1 N/m^2 = $1.450 \times 10^{-4}\ \text{lb/in.}^2$

Some SI Units in Terms of Base Units

Quantity	Unit	Abbreviation	In Terms of Base Units†
Force	newton	N	$\text{kg} \cdot \text{m/s}^2$
Energy and work	joule	J	$\text{kg} \cdot \text{m}^2/\text{s}^2$
Power	watt	W	$\text{kg} \cdot \text{m}^2/\text{s}^3$
Pressure	pascal	Pa	$\text{kg}/(\text{m} \cdot \text{s}^2)$
Frequency	hertz	Hz	s^{-1}
Electric charge	coulomb	C	$\text{A} \cdot \text{s}$
Electric potential	volt	V	$\text{kg} \cdot \text{m}^2/(\text{A} \cdot \text{s}^3)$
Electric resistance	ohm	Ω	$\text{kg} \cdot \text{m}^2/(\text{A}^2 \cdot \text{s}^3)$
Capacitance	farad	F	$\text{A}^2 \cdot \text{s}^4/(\text{kg} \cdot \text{m}^2)$
Magnetic field	tesla	T	$\text{kg}/(\text{A} \cdot \text{s}^2)$
Magnetic flux	weber	Wb	$\text{kg} \cdot \text{m}^2/(\text{A} \cdot \text{s}^2)$
Inductance	henry	H	$\text{kg} \cdot \text{m}^2/(\text{A}^2 \cdot \text{s}^2)$

† kg = kilogram (mass), m = meter (length), s = second (time), A = ampere (electric current).

Metric (SI) Multipliers

Prefix	Abbreviation	Value
quetta	Q	10^{30}
ronna	R	10^{27}
yotta	Y	10^{24}
zeta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deka	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}
ronto	r	10^{-27}
quecto	q	10^{-30}

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DOUGLAS GIANCOLI

Pearson

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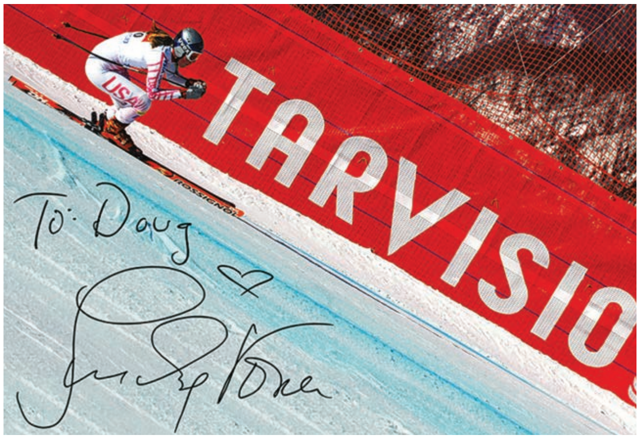
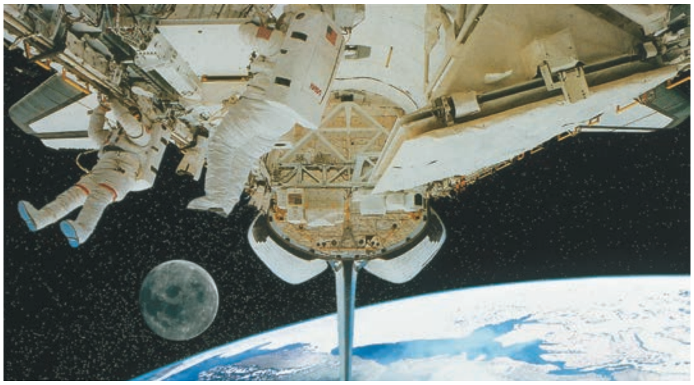
Contents


Estimate how much this lake weighs.





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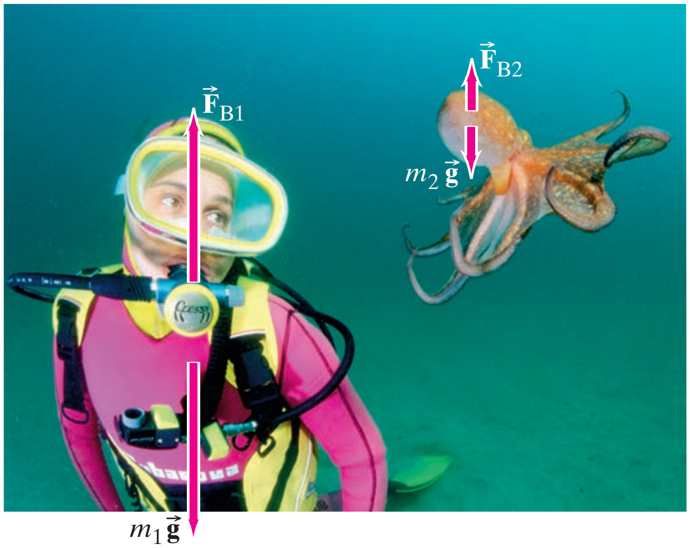

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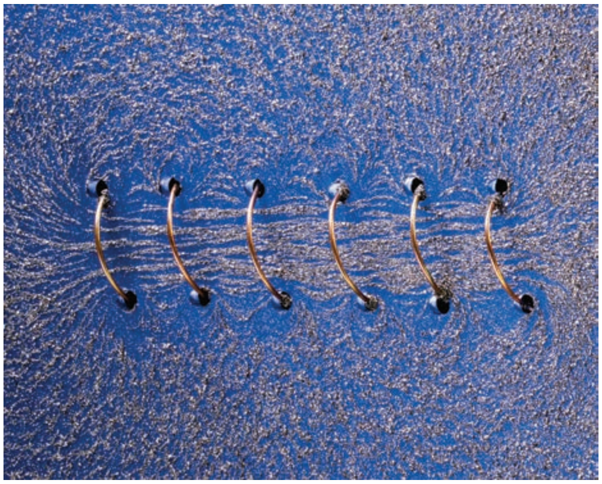

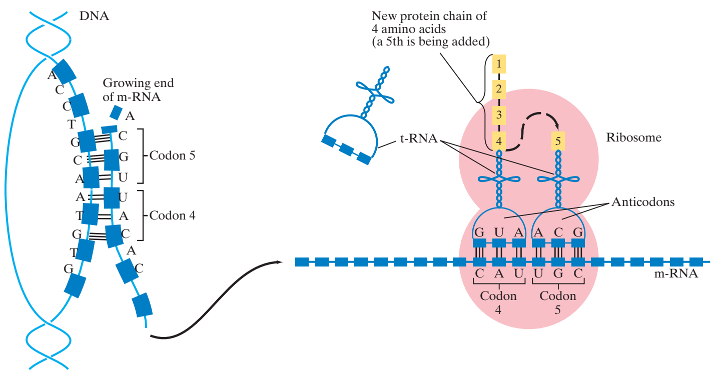

DNA
New protein chain of 4 amino acids (a 5th is being added)
Growing end of m-RNA
Codon 5
Codon 4
t-RNA
Ribosome
Anticodons
m-RNA
Codon 4
Codon 5

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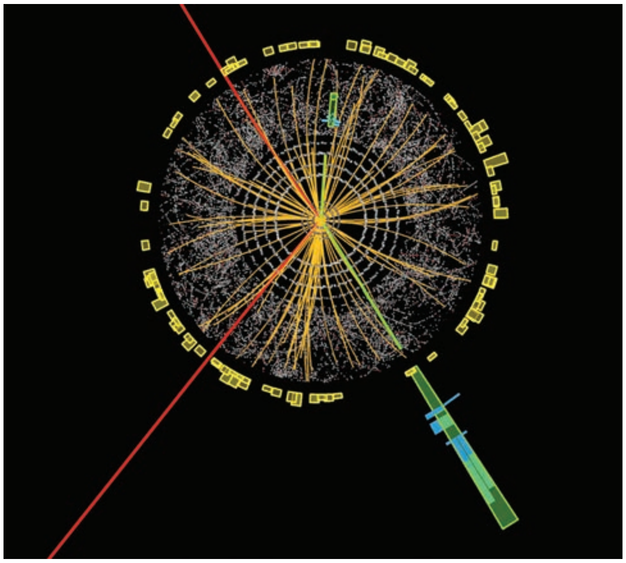

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Appendices

Applications (Selected) to Medicine and Biology and to Engineering, Environment, Everyday Life, Etc.

(Entries with a star * include material new to this edition)

Preface

New Stuff!

1. **MisConceptual Questions**, 10 or 15 at the end of each Chapter. The multiple-choice answers include common misconceptions as well as correct responses. Pedagogically, asking students to think, to consider the options, is more effective than just telling them what is valid and what is wrong. (These are in addition to the one at the start of each Chapter).
2. **Digital** is all around us. Yet that word is not always used carefully. In this new edition we have 20 new pages describing the basics from the ground up. **Binary** numbers, *bits* and *bytes*, are introduced in Chapter 23 along with analog-to-digital conversion (ADC), and vice versa, including *digital audio* and how video screens work. Also information **compression**, *sampling rate*, *bit depth*, *pixel addressing*, *digital transmission* and, in later chapters, information **storage** (RAM, DRAM, flash), *digital cameras* and their *sensors* (CCD, CMOS).
3. **Gravitational Assist** (Slingshot) to accelerate spacecraft (Chapter 8).
4. **Magnetic field** of a **single moving charge**, rarely treated (and if it is, maybe not well), and it shows the need for relativity theory.
5. Seeing **yourself** in a **magnifying mirror** (concave), angular magnification and blurriness with a paradox. Also **convex** (rearview) **mirrors** (Chapter 32).
6. Pedagogical clarification on defining **potential energy**, and energy itself (Chapter 8), and on hundreds of other topics.
7. The **Moon** rises an hour later each day (Chapter 6), its *phases*, *periods*, and diagram.
8. Efficiency of **lightbulbs** (Chapter 34).
9. **Idealization** vs. reality emphasized—such as PV diagrams (Chapter 19) as an idealized approximation.
10. Many new Problems (~ 500) plus new Questions as well as the 500 or so MisConceptual Questions (point 1 above).
11. Many new worked-out Examples.
12. More **math** steps included in derivations and Examples.
13. New **phrases** to remind students of our objective in the middle of a long discussion or derivation (it is so easy to lose track).
14. **State** of a system and *state variables* clarified (Chapter 17).
15. Contemporary physics: Gravitational waves, LIGO and Virgo, Higgs, WIMPS, OLEDS and other semiconductor physics, nuclear fusion updates, neutrino-less double beta decay.
16. New SI units (Chapters 1, 21, Tables).
17. *Boiling* temperature of water vs. *elevation* (Chapter 18).
18. Modern physics in earlier classical Chapters (sometimes in Problems): Light-years, observable universe (Chapter 1); optical tweezers (Chapter 4); uranium enrichment (Chapter 5); black holes and curved space, white dwarfs (Chapter 6); crystal structure (Chapter 7); Yukawa potential, Lennard-Jones potential (Chapter 8); neutrons, nuclear reactors, moderator, nuclear collisions, radioactive decay, neutron star collapse (Chapter 9); galaxy redshift (Chapter 16); gas diffusion of uranium (Chapter 18); quarks (Chapter 21); liquid-drop model of nucleus, Geiger counter, Van de Graaff (Chapter 23); transistors (Chapters 23, 29); isotopes, cyclotron (Chapter 27); MOSFET (Chapter 29); semiconductor (camera sensor), photon (Chapter 33); line spectra, X-ray crystallography (Chapter 35).
19. Second law of thermodynamics and heat energy reorganized (Chapter 20).
20. **Symmetry** emphasized throughout.
21. *Uranium enrichment*, % needed in reactors, bombs (Chapters 5, 42).
22. Mass excess, mass defect (Chapter 41).
23. The *mole*, more careful definition (Chapter 17).
24. Liquid-gas ambiguity above critical temperature (Chapter 18).
25. Measurement affects quantity measured, new emphasis.

26. New **clarifications** and **reminders** in longer discussions and derivations. Because students can lose track of what our aim is, it is mentioned again part way through (often replacing "it" or "this").

27. More New **Applications**:

- Ocean Tides (Chapter 6)
- Anticyclonic weather (Chapter 11)
- Jump starting a car safely (Chapter 26)
- Lightbulb efficiency (Chapter 34)
- Specialty microscopes and contrast (Chapter 35)
- Forces on Muscles and Joints (Chapter 12)
- Doppler ultrasound imaging (Chapter 16)
- Lake level change when rock thrown from boat (Chapter 13)
- Skier speed on snow vs. flying through the air (Chapter 5)
- Inductive charging (Chapter 29)
- Human body internal heat transfer is convection (blood) (Chapter 19)
- Blood pressure measurement (Chapter 13)
- Sports (lots)
- Voltage divider (Chapter 26, Problems)
- Flat screen TV (Chapters 23, 34, 40)
- Carbon footprint and climate (Chapter 20)
- Electrocardiogram (Chapter 23)
- Wireless from the Moon unimaginable (Chapter 31)
- Why snorkels are short (Chapter 17 Problem)
- Electric cars (Chapter 25)
- Digital (Chapters 23, 29, 33, 40) includes (in addition to details in point 2 above) quantization error, digital error correction, noise, bit error rate, digital TV data stream, refresh rate, active matrix, thin film transistors, digital memory, bit-line, reading and writing of memory cells (MOSFET), floating gate, volatile and nonvolatile memory, Bayer, JPEG, ASCII code, and more.
- Importance of Latin, in footnote on page 1021. Other references on pages 84, 354, 698, 805, 950, 997, 1069.

Seeing the World through Eyes that Know Physics

I was motivated to write a textbook for 2 reasons. First, non-physics students in engineering, pre-med, biology, and architecture sometimes asked me "why do I have to take physics." They were right a new textbook was needed that showed how physics is the basis for so much of their fields and in everyday life. More importantly, I saw that physics textbooks were in the style of an instruction manual. Even as a freshman in college I saw that physics books were not telling the truth about how physics is actually practiced. In this book I start each topic by appealing to the student's intuition, which is how physics developed and is actually practiced. Instead of beginning formally and dogmatically, I begin each topic with everyday observations and experiences the students can relate to: start with specifics, the real world, and then go to the great generalizations and more formal aspects of the physics, showing why we believe what we believe.

Much effort has gone into approaches for the practical techniques of solving problems: worked-out Examples, Problem Solving sections, and Problem Solving Strategies.

Chapter 1 is *not* a throwaway. It is fundamental to physics to realize that every measurement has an *uncertainty*, and how significant figures are used. Being able to make rapid *estimates* is a powerful tool useful for every student, and used throughout the book starting in Chapter 1 (*you* can estimate the Earth's radius!).

Mathematics can be an obstacle to students. I have aimed at including all steps in a derivation. Important mathematical tools, such as addition of vectors and vector product, are incorporated in the text where first needed, so they come with a context rather than in a forbidding introductory Chapter. Appendices contain a basic math review, derivatives and integrals, plus some more advanced topics including numerical integration, gravitational field of spherical mass distribution, Maxwell's equations in differential form, and a Table of selected nuclear isotopes (carefully updated, as are the Periodic Table and the Fundamental Constants found inside the back and front covers).

Versions of this Book

Complete version: 44 Chapters including 9 Chapters of modern physics.

Volume 1: Chapters 1–20 on mechanics, including fluids, oscillations, waves, plus heat and thermodynamics.

Volume 2: Chapters 21–35 on electricity and magnetism, plus light and optics.

Volume 3: Chapters 36–44 on modern physics: relativity, quantum theory, atomic physics, condensed matter, nuclear physics, elementary particles, cosmology and astrophysics.

Some instructors may find this book contains more material than can be covered in their courses. The text offers great flexibility. Sections marked with a star * may be considered optional. These contain slightly more advanced physics material, or material not usually covered in typical courses, or interesting applications; they contain no material needed in later Chapters (except perhaps in later optional Sections). For a brief course, all optional material could be dropped as well as significant parts of Chapters 13, 16, 26, 30, and 35, and selected parts of Chapters 9, 12, 19, 20, and 33. Topics not covered in class can be a valuable resource for outside study by students. Indeed, this text can serve as a useful reference for years because of its wide range of coverage.

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He has taught a wide range of undergraduate courses, traditional as well as innovative ones, and works to improve his textbooks meticulously, seeking ways to provide a better understanding of physics for students.

Doug loves the outdoors, especially climbing peaks. He says climbing peaks is like learning physics: it takes effort and the rewards are great.

Advice for Students

HOW TO STUDY

1. Read the Chapter. Learn new vocabulary and notation. Respond to questions and exercises as they occur. Follow carefully the steps of worked-out Examples and derivations. Avoid time looking at a screen. Paper is better than pixels when it comes to learning and thinking.
2. Attend all class meetings. Listen. Take notes. Ask questions (everyone wants to, but maybe you will have the courage). You will get more out of class if you read the Chapter first.
3. Read the Chapter again, paying attention to details. Follow derivations and worked-out Examples. Absorb their logic. Answer Exercises and as many of the end-of-Chapter Questions as you can, and all MisConceptual Questions.
4. Solve at least 10 to 20 end-of-Chapter Problems, especially those assigned. In doing Problems you may find out what you learned and what you didn't. Discuss them with other students. Problem solving is one of the great learning tools. Don' t just look for a formula—it might be the wrong one.

NOTES ON THE FORMAT AND PROBLEM SOLVING

1. Sections marked with a star (*) may be considered optional or advanced. They can be omitted without interrupting the main flow of topics. No later material depends on them except possibly later starred Sections. They may be fun to read, though.
2. The customary **conventions** are used: symbols for quantities (such as m for mass) are italicized, whereas units (such as m for meter) are not italicized. Symbols for vectors are shown in boldface with a small arrow above: $\vec{\mathbf{F}}$.
3. Few equations are valid in all situations. Where practical, the **range of validity** of important equations are stated in square brackets next to the equation. The equations that represent the great laws of physics are displayed with a tan background, as are a few other indispensable equations.
4. At the end of each Chapter is a set of **Questions** you should try to answer. Attempt all the multiple-choice **MisConceptual Questions**, which are intendend to get common misconceptions "out on the table" by including them as responses (temptations) along with correct answers. Most important are **Problems** which are ranked as Level I, II, or III, according to estimated difficulty. Level I Problems are easiest, Level II are standard Problems, and Level III are "challenge problems." These ranked Problems are arranged by Section, but Problems for a given Section may depend on earlier material too. There follows a group of **General Problems**, not arranged by Section or ranked. Problems that relate to optional Sections are starred (*). Answers to odd-numbered Problems are given at the end of the book.
5. Being able to solve **Problems** is a crucial part of learning physics, and provides a powerful means for understanding the concepts and principles. This book contains many aids to problem solving: (a) worked-out **Examples**, including an Approach and a Solution, which should be studied as an integral part of the text; (b) some of the worked-out Examples are **Estimation Examples**, which show how rough or approximate results can be obtained even if the given data are sparse (see Section 1–6); (c) **Problem Solving Strategies** placed throughout the text to suggest a step-by-step approach to problem solving for a particular topic—but the basics remain the same; most of these "Strategies" are followed by an Example that is solved by explicitly following the suggested steps; (d) special problem-solving Sections; (e) "Problem Solving" marginal notes which refer to hints within the text for solving Problems; (f) **Exercises** within the text that you should work out immediately, and then check your response against the answer given at the bottom of the last page of that Chapter; (g) the Problems themselves at the end of each Chapter.
6. **Conceptual Examples** pose a question which hopefully starts you to think about a response. Give yourself a little time to come up with your own response before reading the Response given.
7. Math review, plus additional topics, are found in **Appendices**. **Useful data**, **conversion factors**, and math **formulas** are found inside the front and back covers.

USE OF COLOR

Vectors

- A general vector
  - resultant vector (sum) is slightly thicker
  - components of any vector are dashed
- Displacement ($\vec{\mathbf{D}}, \vec{\mathbf{r}}$)
- Velocity ($\vec{\mathbf{v}}$)
- Acceleration ($\vec{\mathbf{a}}$)
- Force ($\vec{\mathbf{F}}$)
  - Force on second object
  - or third object in same figure
- Momentum ($\vec{\mathbf{p}}$ or $m\vec{\mathbf{v}}$)
- Angular momentum ($\vec{\mathbf{L}}$)
- Angular velocity ($\vec{\boldsymbol{\omega}}$)
- Torque ($\vec{\boldsymbol{\tau}}$)
- Electric field ($\vec{\mathbf{E}}$)
- Magnetic field ($\vec{\mathbf{B}}$)

Electricity and magnetism

- Electric field lines
- Equipotential lines
- Magnetic field lines
- Electric charge (+) — + or +
- Electric charge (–) — – or –

Electric circuit symbols

- Wire, with switch S — S
- Resistor
- Capacitor
- Inductor
- Battery
- Ground

Optics

- Light rays
- Object
- Real image (dashed)
- Virtual image (dashed and paler)

Other

- Energy level (atom, etc.)
- Measurement lines — 1.0 m
- Path of a moving object
- Direction of motion or current

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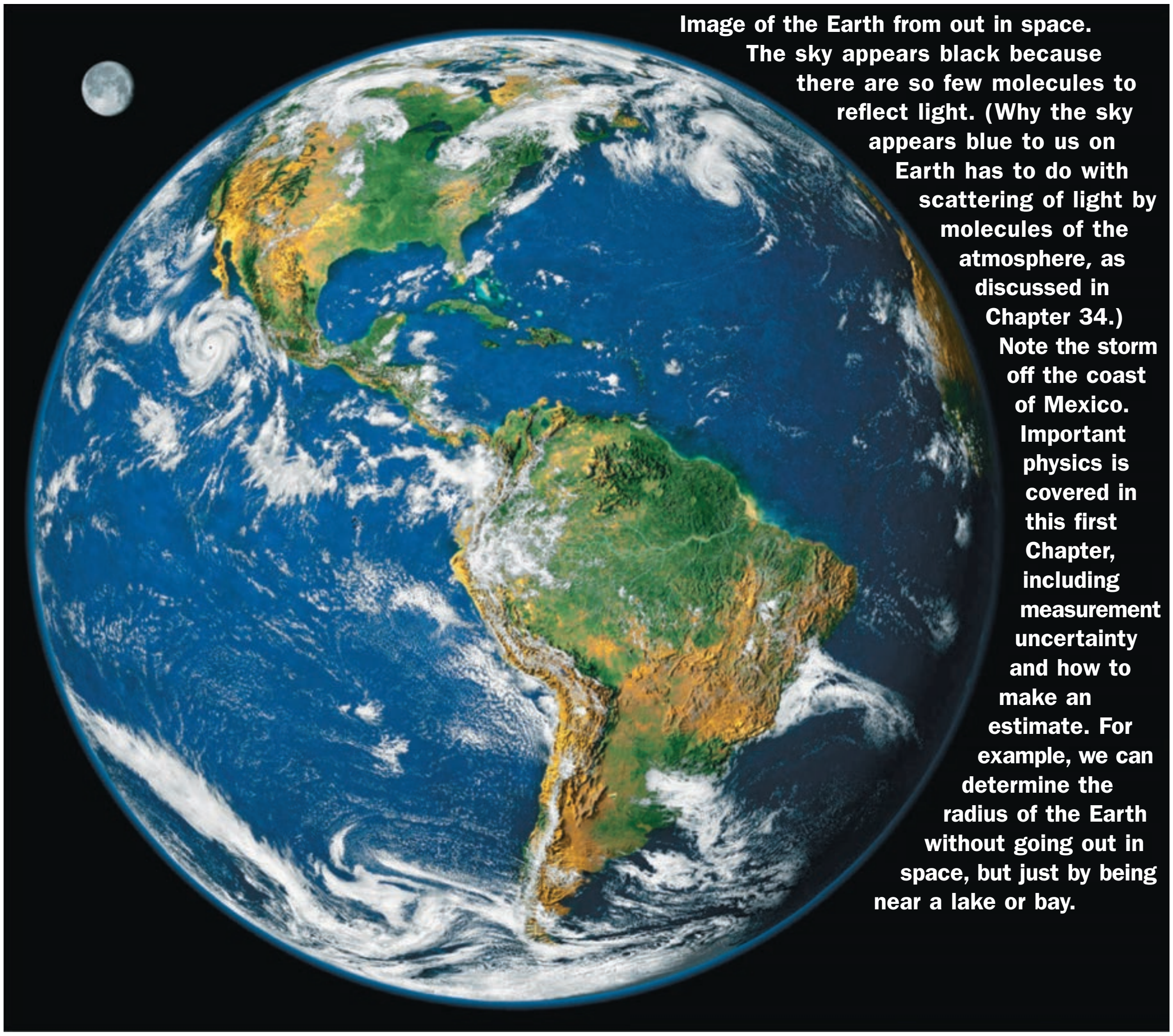


Image of the Earth from out in space. The sky appears black because there are so few molecules to reflect light. (Why the sky appears blue to us on Earth has to do with scattering of light by molecules of the atmosphere, as discussed in Chapter 34.) Note the storm off the coast of Mexico. Important physics is covered in this first Chapter, including measurement uncertainty and how to make an estimate. For example, we can determine the radius of the Earth without going out in space, but just by being near a lake or bay.

CHAPTER 1

Introduction, Measurement, Estimating

CHAPTER-OPENING QUESTIONS—Guess now!

1. How many cm^3 are in $1.0\ m^3$?

(a) 10. **(b)** 100. **(c)** 1000. **(d)** 10,000. **(e)** 100,000. **(f)** 1,000,000.

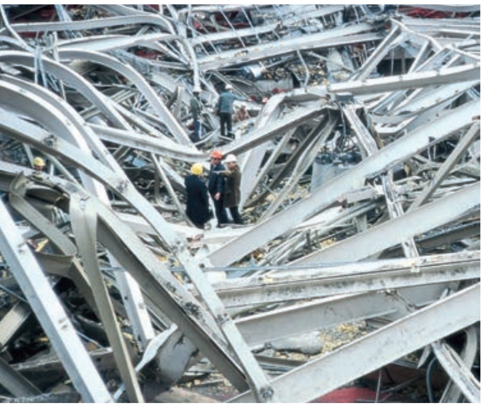
2. Suppose you wanted to actually measure the radius of the Earth, at least roughly, rather than taking other people's word for what it is. Which response below describes the best approach?

(a) Use an extremely long measuring tape.
(b) It is only possible by flying high enough to see the actual curvature of the Earth.
(c) Use a standard measuring tape, a stepladder, and a large smooth lake.
(d) Use a laser and a mirror on the Moon or on a satellite.
(e) Give up; it is impossible using ordinary means.

[*We start each Chapter with a Question—sometimes two. Try to answer right away. Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table. If they are misconceptions, we expect them to be cleared up as you read the Chapter. You will get another chance at the Question later in the Chapter when the appropriate material has been covered. These Chapter-Opening Questions will also help you see the power and usefulness of physics.*]

CONTENTS

(a)



(b)

FIGURE 1–1 (a) This bridge over the River Tiber in Rome was built 2000 years ago and still stands. (b) The Hartford Civic Center collapsed in 1978, just two years after it was built.

CAUTION

Science is not static. It changes and develops

Physics is the most basic of the sciences. It deals with the behavior and structure of matter. The field of physics is usually divided into *classical physics* which includes motion, fluids, heat, sound, light, electricity and magnetism; and *modern physics* which includes the topics of relativity, atomic structure, condensed matter, nuclear physics, elementary particles, and cosmology and astrophysics. We will cover all these topics in this book, beginning with motion (or mechanics, as it is often called) and ending with the most recent results in our study of the cosmos.

An understanding of physics is wonderfully useful for anyone making a career in science or technology. Engineers, for example, must know how to calculate the forces within a structure to design it so that it remains standing (Fig. 1–1a). Indeed, in Chapter 12 we will see a worked-out Example of how a simple physics calculation—or even intuition based on understanding the physics of forces—would have saved hundreds of lives (Fig. 1–1b). We will see many examples in this book of how physics is useful in many fields, and in everyday life.

1–1 How Science Works

There is a real physical world out there. We could just walk through it, not thinking much about it. Or, we can instead examine it carefully. That is what scientists do. The aim of science is the search for order in our observations of the physical world so as to provide a deeper picture or description of this world around us. Sometimes we just want to understand how things work.

Some people seem to think that science is a mechanical process of collecting facts and devising theories. But it is not so simple. Science is a creative activity, and in many ways resembles other creative activities of the human mind.

One important aspect of science is **observation** of events (which great writers and artists also do), and includes the design and carrying out of experiments. But observation and experiment require imagination, because scientists can never include everything in a description of what they observe. In other words, scientists must make judgments about what is relevant in their observations and experiments.

Consider, for example, how two great minds, Aristotle (384–322 B.C.) and Galileo (1564–1642), interpreted motion along a horizontal surface. Aristotle noted that objects given an initial push along the ground (or on a level tabletop) always slow down and stop. Consequently, Aristotle argued, the natural state of an object is to be at rest. Galileo, in his reexamination of horizontal motion in the 1600s, had the idea that friction is a kind of force like a push or a pull; and he imagined that if friction could be eliminated, an object given an initial push along a horizontal surface would continue to move indefinitely without stopping. He concluded that for an object to be in motion was *just as natural* as for it to be at rest. By inventing a new approach, Galileo founded our modern view of motion (Chapters 2, 3, and 4), and he did so with a leap of the imagination. Galileo made this leap conceptually, without actually eliminating friction.

Observation, with careful experimentation and measurement, is one side of the scientific process. The other side is the invention or creation of **theories** to explain and order the observations. Theories are never derived directly from observations. Observations may help inspire a theory, and theories are accepted or rejected based on the results of observation and experiment.

Theories are inspirations that come from the minds of humans. For example, the idea that matter is made up of atoms (the atomic theory) was not arrived at by direct observation of atoms. Rather, the idea sprang from creative minds. The theory of relativity, the electromagnetic theory of light, and Newton's law of universal gravitation were likewise the result of human imagination.

The great theories of science may be compared, as creative achievements, with great works of art or literature. But how does science differ from these other creative activities? One important difference is that science requires **testing** of its ideas or theories to see if their predictions are borne out by experiment.

But theories are not "proved" by testing. First of all, no measuring instrument is perfect, so exact confirmation is not possible. Furthermore, it is not possible to test a theory in every single possible circumstance. Hence a theory cannot be absolutely verified.

Indeed, the history of science tells us that long-held theories can often be replaced by new ones.

1–2 Models, Theories, and Laws

When scientists are trying to understand a particular aspect of the physical world, they often make use of a **model**. A model, in the scientist's sense, is a kind of analogy or mental image of the phenomena in terms of something we are familiar with. One example is the wave model of light. We cannot see waves of light as we can water waves. But it is valuable to think of light as made up of waves because experiments indicate that light behaves in many respects as water waves do.

The purpose of a model is to give us an approximate mental or visual picture—something to hold on to—when we cannot see what actually is happening in the real world. Models often give us a deeper understanding: the analogy to a known system (for instance, water waves in the above example) can suggest new experiments to perform and can provide ideas about what other related phenomena might occur.

You may wonder what the difference is between a theory and a model. Usually a model is relatively simple and provides a structural similarity to the phenomena being studied. A **theory** is broader, more detailed, and can give quantitatively testable predictions, often with great precision.

It is important not to confuse a model or a theory with the real world and the phenomena themselves. Theories are descriptions of the physical world, and they are made up by us. Theories are *invented*—usually by very smart people.

CAUTION

Theories and laws are NOT discovered. They are invented

Scientists give the title **law** to certain concise but general statements about how nature behaves (that energy is conserved, for example). Sometimes the statement takes the form of a relationship or equation between quantities (such as Newton's second law, $F = ma$).

To be called a law, a statement must be found experimentally valid over a wide range of observed phenomena. For less general statements, the term **principle** is often used (such as Archimedes' principle). We use "theory" to describe a more general picture of a large group of phenomena.

Scientific laws are different from political laws, which are *prescriptive*: they tell us how we ought to behave. Scientific laws are *descriptive*: they do not say how nature *should* behave, but rather are meant to describe how nature *does* behave. As with theories, laws cannot be tested in the infinite variety of cases possible. So we cannot be sure that any law is absolutely true. We use the term "law" when its validity has been tested over a wide range of situations, and when any limitations and the range of validity are clearly understood.

Scientists normally do their research as if the accepted laws and theories were true. But they are obliged to keep an open mind in case new information should alter the validity of any given law or theory. In other words, laws of physics, or the "laws of nature", represent our descriptions of reality and are not inalterable facts that last forever. Laws are not lying there in nature, waiting to be discovered. We humans, the brightest humans, invent the laws using observations and intuition as a basis. And we hope our laws provide a good description of nature, and at a minimum give us a reliable approximation of how nature really behaves.

1–3 Measurement and Uncertainty; Significant Figures

In the quest to understand the world around us, scientists seek to find relationships among physical quantities that can be measured.

Uncertainty

Reliable measurements are an important part of physics. But no measurement is absolutely precise. There is an uncertainty associated with every measurement. Among the most important sources of uncertainty, other than blunders, are the limited accuracy of every measuring instrument and the inability to read

FIGURE 1–2 Measuring the width of a board with a centimeter ruler. The uncertainty is about ± 1 mm.

an instrument (such as a ruler) beyond some fraction of the smallest division shown. For example, if you were to use a centimeter ruler to measure the width of a board (Fig. 1–2), the result could be claimed to be precise to about 0.1 cm (1 mm), the smallest division on the ruler, although half of this value might be a valid claim as well. The reason is that it is difficult for the observer to estimate (or *interpolate*) between the smallest divisions. Furthermore, the ruler itself may not have been manufactured to an accuracy very much better than this.

When giving the result of a measurement, it is important to state the **estimated uncertainty** in the measurement. For example, the width of a board might be written as 8.8 ± 0.1 cm. The ± 0.1 cm ("plus or minus 0.1 cm") represents the estimated uncertainty in the measurement, so that the actual width most likely lies between 8.7 and 8.9 cm. The **percent uncertainty** is the ratio of the uncertainty to the measured value, multiplied by 100. For example, if the measurement is 8.8 and the uncertainty about 0.1 cm, the percent uncertainty is

$$\frac{0.1}{8.8} \times 100\% \approx 1\%,$$

where $\approx$ means "is approximately equal to."

Often the uncertainty in a measured value is not specified explicitly. In such cases, scientists follow a general rule that

uncertainty in a numerical value is assumed to be *one or a few units* in the last digit specified.

For example, if a length is given as 5.6 cm, the uncertainty is assumed to be about 0.1 cm or 0.2 cm, or possibly 0.3 cm. It is important in this case that you do not write 5.60 cm, for this implies an uncertainty on the order of 0.01 or 0.02 cm; it assumes that the length is probably between about 5.58 cm and 5.62 cm, when actually you believe it is between about 5.4 and 5.8 cm.

Significant Figures

The number of reliably known digits in a number is called the number of **significant figures**. Thus there are four significant figures in the number 23.21 cm and two in the number 0.062 cm (the zeros in the latter are merely place holders that show where the decimal point goes). The number of significant figures may not always be clear. Take, for example, the number 80. Are there one or two significant figures? We need words here: If we say it is *roughly* 80 km between two cities, there is only one significant figure (the 8) since the zero is merely a place holder. If there is no suggestion that the 80 is a rough approximation, then we can often assume (as we will in this book) that it has two significant figures: so it is 80 km within an accuracy of about 1 or 2 km. If it is precisely 80 km, to within ± 0.1 or ± 0.2 km, then we need to write 80.0 km (three significant figures).

When specifying numerical results, you should avoid the temptation to keep more digits in the final answer than is justified: see boldface statement above. For example, to calculate the area of a rectangle 11.3 cm by 6.8 cm, the result of multiplication would be 76.84 cm^2. But this answer can not be accurate to the implied 0.01 cm^2 uncertainty. Why? Because (using the outer limits of the assumed uncertainty for each measurement) the result could be between $11.2\text{ cm} \times 6.7\text{ cm} = 75.04\text{ cm}^2$ and $11.4\text{ cm} \times 6.9\text{ cm} = 78.66\text{ cm}^2$. At best, we can quote the answer as 77 cm^2, which implies an uncertainty of about 1 or 2 cm^2. The other two digits (in the number 76.84 cm^2) must be dropped (rounded off) because they are not significant. As a rough general **significant figures rule**,

PROBLEM SOLVING

Significant figures rule: Number of significant figures in final result should be same as the least significant input value

the final result of a multiplication or division should have no more digits than the numerical value with the fewest significant figures.

In our example, 6.8 cm has the least number of significant figures, namely two. Thus the result 76.84 cm^2 needs to be rounded off to 77 cm^2.

EXERCISE A The area of a rectangle 4.5 cm by 3.25 cm is correctly given by (*a*) 14.625 cm^2; (*b*) 14.63 cm^2; (*c*) 14.6 cm^2; (*d*) 15 cm^2.

When *adding* or *subtracting* numbers, the final result should contain no more decimal places than the number with the fewest decimal places. For example, the result of subtracting 0.57 from 3.6 is 3.0 (not 3.03). Similarly $36 + 8.2 = 44$, not 44.2.

Be careful not to confuse significant figures with the number of decimal places. Significant figures are related to the expected uncertainty in any measured quantity.

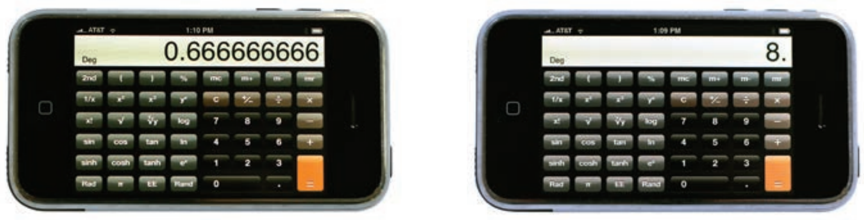
PROBLEM SOLVING
Significant figures when adding and subtracting

EXERCISE B For each of the following numbers, state the number of significant figures and the number of decimal places: (*a*) 1.23; (*b*) 0.123; (*c*) 0.0123.

Keep in mind when you use a calculator that all the digits it produces may not be significant. When you divide 2.0 by 3.0, the proper answer is 0.67, and not 0.666666666 as calculators give (Fig. 1–3a). Digits should not be quoted in a result unless they are truly significant figures. However, to obtain the most accurate result, you should normally *keep one or more extra significant figures throughout a calculation, and round off only in the final result*. (With a calculator, you can keep all its digits in intermediate results.) Calculators can also give too few significant figures. For example, when you multiply 2.5×3.2, a calculator may give the answer as simply 8. See Fig. 1–3b. But the answer is accurate to two significant figures, so the proper answer is 8.0.†

CAUTION
Calculators err with significant figures

PROBLEM SOLVING
Report only the proper number of significant figures in the final result. But keep extra digits during the calculation



(a) (b)

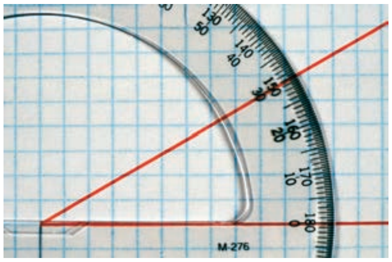
FIGURE 1–3 These two calculators show the wrong number of significant figures. In (a), 2.0 was divided by 3.0. The correct final result should be stated as 0.67. In (b), 2.5 was multiplied by 3.2. The correct result is 8.0.

CONCEPTUAL EXAMPLE 1–1 **Significant figures.** Using a protractor (Fig. 1–4), you measure an angle to be 30°. (*a*) How many significant figures should you quote in this measurement? (*b*) Use a calculator to find the cosine of the angle you measured.

RESPONSE (*a*) If you look at a protractor, you will see that the precision with which you can measure an angle is about one degree (certainly not 0.1°). So you can quote two significant figures, namely 30° (not 30.0°). (*b*) If you enter cos 30° in your calculator, you will get a number like 0.866025403. But the angle you entered is known only to two significant figures, so its cosine is correctly given by 0.87; you must round your answer to two significant figures.

NOTE Trigonometric functions, like cosine, are reviewed in Appendix A.

FIGURE 1–4 Example 1–1. A protractor used to measure an angle.



EXERCISE C Do 0.00324 and 0.00056 have the same number of significant figures?

Scientific Notation

We commonly write numbers in "powers of ten," or "scientific" notation—for instance 36,900 as 3.69×10^4, or 0.0021 as 2.1×10^{-3}. One advantage of scientific notation is that it allows the number of significant figures to be clearly expressed. For example, it is not clear whether 36,900 has three, four, or five significant figures. With powers of ten notation the ambiguity can be avoided: if the number is known to three significant figures, we write 3.69×10^4, but if it is known to four, we write 3.690×10^4.

†Be careful also about other digital read-outs. If a digital bathroom scale shows 85.6, do not assume the uncertainty is ±0.1 or ±0.2; the scale was likely manufactured with an accuracy of perhaps only 1% or so: that is, ±1 or ±2. For digital scientific instruments, also be careful: the instruction manual should state the accuracy.

EXERCISE D Write each of the following in scientific notation and state the number of significant figures for each: (*a*) 0.0258, (*b*) 42,300, (*c*) 344.50.

Percent Uncertainty versus Significant Figures

The significant figures rule is only approximate, and in some cases may underestimate the accuracy (or uncertainty) of the answer. Suppose for example we divide 97 by 92:

$$\frac{97}{92} = 1.05 \approx 1.1.$$

Both 97 and 92 have two significant figures, so the rule says to give the answer as 1.1. Yet the numbers 97 and 92 both imply an uncertainty of ± 1 if no other uncertainty is stated. Both 92 ± 1 and 97 ± 1 imply an uncertainty of about 1% $(1/92 \approx 0.01 = 1\%)$. But the final result to two significant figures is 1.1, with an implied uncertainty of ± 0.1, which is an uncertainty of $0.1/1.1 \approx 0.1 \approx 10\%$. In this case it is better to give the answer as 1.05 (which is three significant figures). Why? Because 1.05 implies an uncertainty of ± 0.01 which is $0.01/1.05 \approx 0.01 \approx 1\%$, just like the uncertainty in the original numbers 92 and 97.

SUGGESTION: Use the significant figures rule, but consider the % uncertainty too, and add an extra digit if it gives a more realistic estimate of uncertainty.

Approximations

Much of physics involves approximations, often because we do not have the means to solve a problem precisely. For example, we may choose to ignore air resistance or friction in doing a Problem even though they are present in the real world, and then our calculation is only an estimate or approximation. In doing Problems, we should be aware of what approximations we are making, and be aware that the precision of our answer may not be nearly as good as the number of significant figures given in the result.

Accuracy versus Precision

There is a technical difference between "precision" and "accuracy." **Precision** in a strict sense refers to the repeatability of the measurement using a given instrument. For example, if you measure the width of a board many times, getting results like 8.81 cm, 8.85 cm, 8.78 cm, 8.82 cm (interpolating between the 0.1 cm marks as best as possible each time), you could say the measurements give a *precision* a bit better than 0.1 cm. **Accuracy** refers to how close a measurement is to the true value. For example, if the ruler shown in Fig. 1–2 was manufactured with a 2% error, the accuracy of its measurement of the board's width (about 8.8 cm) would be about 2% of 8.8 cm or about ± 0.2 cm. Estimated uncertainty is meant to take both accuracy and precision into account.

1–4 Units, Standards, and the SI System

The measurement of any quantity is made relative to a particular standard or **unit**, and this unit must be specified along with the numerical value of the quantity. For example, we can measure length in British units such as inches, feet, or miles, or in the metric system in centimeters, meters, or kilometers. To specify that the length of a particular object is 18.6 is insufficient. The unit *must* be given, because 18.6 meters is very different from 18.6 inches or 18.6 millimeters.

For any unit we use, such as the meter for distance or the second for time, we need to define a **standard** which defines exactly how long one meter or one second is. It is important that standards be chosen that are readily reproducible so that anyone needing to make a very accurate measurement can refer to the standard in the laboratory and communicate results with other scientists.

Length

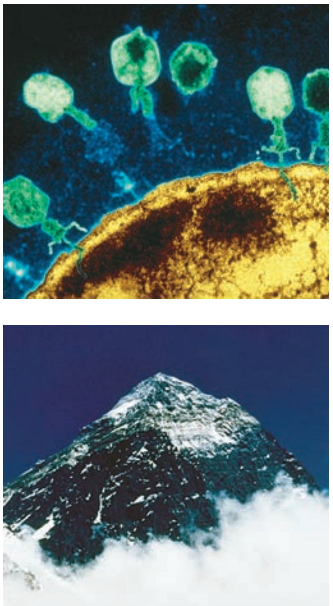
The first truly international standard was the **meter** (abbreviated m) established as the standard of **length** by the French Academy of Sciences in the 1790s. The standard meter was originally chosen to be one ten-millionth of the distance from the Earth's equator to either pole,† and a platinum rod to represent this length was made. (One meter is, very roughly, the distance from the tip of your nose to the tip of your finger, with arm and hand stretched out horizontally.) In 1889, the meter was defined more precisely as the distance between two finely engraved marks on a particular bar of platinum–iridium alloy. In 1960, to provide greater precision and reproducibility, the meter was redefined as 1,650,763.73 wavelengths of a particular orange light emitted by the gas krypton-86.

In 1983 the meter was again redefined, this time in terms of the speed of light (whose best measured value in terms of the older definition of the meter was 299,792,458 m/s, with an uncertainty of 1 m/s). The new definition reads: "The meter is the length of path traveled by light in vacuum during a time interval of 1/299,792,458 of a second." The new definition of the meter has the effect of giving the speed of light the exact value of 299,792,458 m/s. [The newer definitions provided greater precision than the 2 marks on the old platinum bar.]

New definition of the meter

British units of length (inch, foot, mile) are now defined in terms of the meter. The **inch** (in.) is defined as exactly 2.54 centimeters (cm; 1 cm = 0.01 m). One **foot** is exactly 12 in., and 1 **mile** is 5280 ft. Other conversion factors are given in the Table on the inside of the front cover of this book. Table 1–1 below presents some typical lengths, from very small to very large, rounded off to the nearest power of 10. (We call this rounded off value the **order of magnitude**.) See also Fig. 1–5. (Note that the abbreviation for inches (in.) is the only one with a period, to distinguish it from the word "in".) [The **nautical mile** = 6076 ft = 1852 km is used by ships on the open sea and was originally defined as 1/60 of a degree latitude on Earth's surface. A speed of 1 **knot** is 1 nautical mile per hour. Weather forecasts use it too.]

FIGURE 1–5 Some lengths: (a) viruses (about 10^{-7} m long) attacking a cell; (b) Mt. Everest's height is on the order of 10^4 m (8850 m, to be precise).



(a)

(b)

Time

The standard unit of **time** is the **second** (s). For many years, the second was defined as 1/86,400 of a mean solar day (24 h/day × 60 min/h × 60 s/min = 86,400 s/day). The standard second can be defined more precisely in terms of the frequency of radiation emitted by cesium atoms when they pass between two particular states. [Specifically, one second is the time required for 9,192,631,770 periods of this radiation. This number was chosen to keep "one second" the same as in the old definition.] There are, by definition, 60 s in one minute (min) and 60 minutes in one hour (h). Table 1–2 presents a range of time intervals, rounded off to the nearest power of 10.

†Modern measurements of the Earth's circumference reveal that the intended length is off by about one-fiftieth of 1%. Not bad!

TABLE 1–1 Some Typical Lengths or Distances (order of magnitude)

Length (or Distance)	Meters (approximate)
Neutron or proton (diameter)	10^{-15} m
Atom (diameter)	10^{-10} m
Virus [see Fig. 1–5a]	10^{-7} m
Sheet of paper (thickness)	10^{-4} m
Finger width	10^{-2} m
Football field length	10^{2} m
Height of Mt. Everest [see Fig. 1–5b]	10^{4} m
Earth diameter	10^{7} m
Earth to Sun	10^{11} m
Earth to nearest star	10^{16} m
Earth to nearest galaxy	10^{22} m
Earth to farthest galaxy visible	10^{26} m

TABLE 1–2 Some Typical Time Intervals (order of magnitude)

Time Interval	Seconds (approximate)
Lifetime of very unstable subatomic particle	10^{-23} s
Lifetime of radioactive elements	10^{-22} s to 10^{28} s
Lifetime of muon	10^{-6} s
Time between human heartbeats	10^{0} s (= 1 s)
One day	10^{5} s
One year	3×10^{7} s
Human life span	2×10^{9} s
Length of recorded history	10^{11} s
Humans on Earth	10^{14} s
Life on Earth	10^{17} s
Age of Universe	4×10^{17} s

TABLE 1–3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

Mass

The standard unit of **mass** is the **kilogram** (kg). The standard mass has been, since 1889, a particular platinum–iridium cylinder, kept at the International Bureau of Weights and Measures near Paris, France, whose mass is defined as exactly 1 kg. A range of masses is presented in Table 1–3. [For practical purposes, a 1 kg mass weighs about 2.2 pounds on Earth.]

1 metric **ton** is 1000 kg. In the British system of units, 1 ton is 2000 pounds.

When dealing with atoms and molecules, we usually use the **unified atomic mass unit** (u or amu). In terms of the kilogram,

$$1\ \text{u} = 1.6605 \times 10^{-27}\ \text{kg}.$$

(Precise values of this and other numbers are given inside the front cover.) The **density** of a uniform object is its mass divided by its volume, commonly expressed in kg/m^3.

Unit Prefixes

In the metric system, the larger and smaller units are defined in multiples of 10 from the standard unit, and this makes calculation particularly easy. Thus 1 kilometer (km) is 1000 m, 1 centimeter is $\frac{1}{100}$ m, 1 millimeter (mm) is $\frac{1}{1000}$ m or $\frac{1}{10}$ cm, and so on. The *prefixes* "centi-," "kilo-," and others are listed in Table 1–4 and can be applied not only to units of length but to units of volume, mass, or any other unit. For example, a centiliter (cL) is $\frac{1}{100}$ liter (L), and a kilogram (kg) is 1000 grams (g). An 8.2-megapixel camera has a detector with 8,200,000 pixels (individual "picture elements").

In common usage, $1\ \mu\text{m}\ (= 10^{-6}\ \text{m})$ is called 1 **micron**.

Systems of Units

PROBLEM SOLVING
Always use a consistent set of units

When dealing with the laws and equations of physics it is very important to use a consistent set of units. Several systems of units have been in use over the years. Today the most important is the **Système International** (French for International System), which is abbreviated SI. In SI units, the standard of length is the meter, the standard for time is the second, and the standard for mass is the kilogram. This system used to be called the MKS (meter-kilogram-second) system.

A second metric system is the **cgs system**, in which the centimeter, gram, and second are the standard units of length, mass, and time, as abbreviated in the title. The **British engineering system** (although more used in the U.S. than Britain) has as its standards the foot for length, the pound for force, and the second for time.

We use SI units almost exclusively in this book, although we often define the cgs and British units when a new quantity is introduced. In the SI, there have traditionally been seven *base* quantities, each defined in terms of a standard; seven is the smallest number of base quantities consistent with a full description of the physical world. See Table 1–5. All other quantities† can be defined in terms of seven base quantities; see the Table inside the front cover which lists many quantities and their units in terms of base units.

TABLE 1–4 Metric (SI) Prefixes

Prefix	Abbreviation	Value
quetta	Q	10^{30}
ronna	R	10^{27}
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}
ronto	r	10^{-27}
quecto	q	10^{-30}

† μ is the Greek letter "mu."

*A New SI

As always in science, new ideas and approaches can produce better precision and closer correspondence with the real world. Even for units and standards.

International organizations on units have proposed further changes that should make standards more readily available and reproducible. To cite one example, the standard kilogram (see above) has been found to have changed slightly in mass (contamination is one cause).

The new redefinition of SI standards follows the method already used for the meter as being related to the defined value of the speed of light, as we mentioned on page 29 under "Length." For example, the charge on the electron, *e*, instead of being a measured value, becomes *defined* as a certain value (its current value), and the unit of electric charge (the coulomb) follows from that. All units then become based on

†Some exceptions are for angle (radians—see Chapter 10), solid angle (steradian), and sound level (bel or decibel, Chapter 16).

*Some Sections of this book, such as this subsection, may be considered *optional* at the discretion of the instructor and they are marked with an asterisk (*). See the Preface for more details.

defined fundamental constants like e and the speed of light. Seven is still the number of basic standards. The new definitions maintain the values of the traditional definitions: the "new" meter is the same length as the "old" meter. The new definitions do not change our understanding of what length, time, or mass means.

For us, using this book, the difference between the new SI and the traditional SI is highly technical and does not affect the physics we study. We include the traditional SI because there is some good physics in explaining it. [The Table of Fundamental Constants inside the front cover would look slightly different using the new SI. The value of the charge e on the electron, for example, is *defined*, and so would have no uncertainty attached to it; instead, our Table inside the front cover includes the traditional SI measured uncertainty (updated) of $\pm 98 \times 10^{-29}$ C.]

TABLE 1–5 Traditional SI Base Quantities

Quantity	Unit	Unit Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

1–5 Converting Units

Any quantity we measure, such as a length, a speed, or an electric current, consists of a number *and* a unit. Often we are given a quantity in one set of units, but we want it expressed in another set of units. For example, suppose we measure that a shelf is 21.5 inches wide, and we want to express this in centimeters. We must use a **conversion factor**, which in this case is, *by definition*, exactly

$$1 \text{ in.} = 2.54 \text{ cm}$$

or, written another way,

$$1 = 2.54 \text{ cm/in.}$$

Since multiplying by the number one does not change anything, the width of our shelf, in cm, is

$$21.5 \text{ inches} = (21.5 \text{ in.}) \times \left(2.54 \frac{\text{cm}}{\text{in.}}\right) = 54.6 \text{ cm}.$$

Note how the units (inches in this case) cancelled out (thin red lines). A Table containing many unit conversions is found inside the front cover of this book. Let's consider some Examples.

FIGURE 1–6 The world's second highest peak, K2, whose summit is considered the most difficult of the "8000-ers." Example 1–2.

PHYSICS APPLIED
The world's tallest peaks

EXAMPLE 1–2 The 8000-m peaks. There are only 14 peaks whose summits are over 8000 m above sea level. They are the highest peaks in the world (Fig. 1–6 and Table 1–6) and are referred to as "eight-thousanders." What is the elevation, in feet, of an elevation of 8000 m?

APPROACH We need to convert meters to feet, and we can start with the conversion factor 1 in. = 2.54 cm, which is exact. That is, 1 in. = 2.5400 cm to any number of significant figures, because it is *defined* to be.

SOLUTION One foot is defined to be 12 in., so we can write

$$1 \text{ ft} = (12 \text{ in.})\left(2.54 \frac{\text{cm}}{\text{in.}}\right) = 30.48 \text{ cm} = 0.3048 \text{ m},$$

which is exact. Note how the units cancel (colored slashes). We can rewrite this equation to find the number of feet in 1 meter:

$$1 \text{ m} = \frac{1 \text{ ft}}{0.3048} = 3.28084 \text{ ft}.$$

(We could carry the result to 6 significant figures because 0.3048 is exact, $0.304800\cdots$.) We multiply this equation by 8000.0 (to have five significant figures):

$$8000.0 \text{ m} = (8000.0 \text{ m})\left(3.28084 \frac{\text{ft}}{\text{m}}\right) = 26{,}247 \text{ ft}.$$

An elevation of 8000 m is 26,247 ft above sea level.

NOTE We could have done the unit conversions all in one line:

$$8000.0 \text{ m} = (8000.0 \text{ m})\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)\left(\frac{1 \text{ in.}}{2.54 \text{ cm}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 26{,}247 \text{ ft}.$$

The key is to multiply conversion factors, each equal to one $(= 1.0000)$, and to make sure which units cancel.

TABLE 1–6 The 8000-m Peaks

Peak	Height (m)
Mt. Everest	8850
K2	8611
Kangchenjunga	8586
Lhotse	8516
Makalu	8462
Cho Oyu	8201
Dhaulagiri	8167
Manaslu	8156
Nanga Parbat	8125
Annapurna	8091
Gasherbrum I	8068
Broad Peak	8047
Gasherbrum II	8035
Shisha Pangma	8013

The first two equations in Example 1–2 on the previous page show how to change from feet to meters, or meters to feet. For practical purposes

$$1\text{ m} = 3.28\text{ ft} \approx 3.3\text{ ft}$$

which means that we can change any distance or height in meters to feet by multiplying by 3 and adding 10% (0.1). For example, a 3000-m-high peak in feet is $9000\text{ ft} + 900\text{ ft} \approx 10{,}000\text{ ft}$.

TABLE 1–6 The 8000-m Peaks

Peak	Height (m)
Mt. Everest	8850
K2	8611
Kangchenjunga	8586
Lhotse	8516
Makalu	8462
Cho Oyu	8201
Dhaulagiri	8167
Manaslu	8156
Nanga Parbat	8125
Annapurna	8091
Gasherbrum I	8068
Broad Peak	8047
Gasherbrum II	8035
Shisha Pangma	8013

EXERCISE E The names and elevations of the 14 eight-thousand-meter peaks in the world (see Example 1–2) are given in Table 1–6, repeated here. They are all in the Himalaya mountain range in India, Pakistan, Tibet, and China. Determine the elevation of the world's three highest peaks in feet.

EXAMPLE 1–3 Apartment area. You have seen a nice apartment whose floor area is 880 square feet (ft^2). What is its area in square meters?

APPROACH We use the same conversion factor, $1\text{ in.} = 2.54\text{ cm}$, but this time we have to use it twice.

SOLUTION Because $1\text{ in.} = 2.54\text{ cm} = 0.0254\text{ m}$, then

$$1\text{ ft}^2 = (12\text{ in.})^2(0.0254\text{ m/in.})^2 = 0.0929\text{ m}^2.$$

So

$$880\text{ ft}^2 = (880\text{ ft}^2)(0.0929\text{ m}^2/\text{ft}^2) \approx 82\text{ m}^2.$$

NOTE As a rule of thumb, an area given in ft^2 is roughly 10 times the number of square meters (more precisely, about 10.8×).

Rule of thumb: Floor area in ft^2 is about 10× area in m^2: $100\text{ m}^2 \approx 1000\text{ ft}^2$

EXERCISE F One **hectare** is defined as $1.000 \times 10^4\text{ m}^2$. There are 640 **acres** in a square mile. Both units are used for land area. (*a*) How many acres are in one hectare? (*b*) What would be an easy everyday rule-of-thumb conversion factor?

EXAMPLE 1–4 Speeds. Where the posted speed limit is 55 miles per hour (mi/h or mph), what is this speed (*a*) in meters per second (m/s) and (*b*) in kilometers per hour (km/h)?

APPROACH We again use the conversion factor $1\text{ in.} = 2.54\text{ cm}$, and we recall that there are 5280 feet in a mile and 12 inches in a foot; also, one hour contains $(60\text{ min/h}) \times (60\text{ s/min}) = 3600\text{ s/h}$.

SOLUTION (*a*) We can write 1 mile as

$$1\text{ mi} = (5280\text{ ft})\left(12\frac{\text{in.}}{\text{ft}}\right)\left(2.54\frac{\text{cm}}{\text{in.}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right) = 1609\text{ m}.$$

We also know that 1 hour contains 3600 s, so

$$55\frac{\text{mi}}{\text{h}} = \left(55\frac{\text{mi}}{\text{h}}\right)\left(1609\frac{\text{m}}{\text{mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 25\frac{\text{m}}{\text{s}},$$

where we rounded off to two significant figures.

(*b*) Now we use $1\text{ mi} = 1609\text{ m} = 1.609\text{ km}$; then

$$55\frac{\text{mi}}{\text{h}} = \left(55\frac{\text{mi}}{\text{h}}\right)\left(1.609\frac{\text{km}}{\text{mi}}\right) = 88\frac{\text{km}}{\text{h}}.$$

NOTE Each conversion factor is equal to one. You can look up most conversion factors in the Table inside the front cover.

PROBLEM SOLVING
Conversion factors = 1

EXERCISE G Return to the first Chapter-Opening Question, page 23, and answer it again now. Try to explain why you may have answered differently the first time.

PROBLEM SOLVING
Unit conversion is wrong if units do not cancel

When changing units, you can avoid making an error in the use of conversion factors by checking that units cancel out properly. For example, in our conversion of 1 mi to 1609 m in Example 1–4(*a*), if we had incorrectly used the factor $\left(\frac{100\text{ cm}}{1\text{ m}}\right)$ instead of $\left(\frac{1\text{ m}}{100\text{ cm}}\right)$, the centimeter units would not have cancelled out; we would not have ended up with meters.

1–6 Order of Magnitude: Rapid Estimating

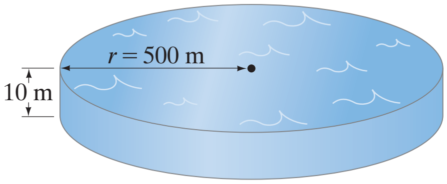
This is an exciting and powerful Section that will be useful throughout this book, and in real life. We will see how to make approximate calculations of quantities you may never have dreamed you could do.

Also, we are sometimes interested only in an approximate value for a quantity, maybe because an accurate calculation would take more time than it is worth or requires data that are not available. In other cases, we may want to make a rough estimate in order to check a calculation made on a calculator, to make sure that no blunders were made when the numbers were entered.

A rough estimate can be made by rounding off all numbers to one significant figure and its power of 10, and after the calculation is made, again keeping only one significant figure. Such an estimate is called an **order-of-magnitude estimate** and can be accurate within a factor of 10, and often better. In fact, the phrase "order of magnitude" is sometimes used to refer simply to the power of 10.

PROBLEM SOLVING
How to make a rough estimate

(a)



(b)

FIGURE 1–7 Example 1–5. (a) How much water is in this lake? (Photo is one of the Rae Lakes in the Sierra Nevada of California.) (b) Model of the lake as a cylinder. [We could go one step further and estimate the mass or weight of this lake. We will see later that water has a density of $1000\,\mathrm{kg/m^3}$, so this lake has a mass of about $(10^3\,\mathrm{kg/m^3})(10^7\,\mathrm{m^3}) \approx 10^{10}\,\mathrm{kg}$, which is about 10 billion kg or 10 million metric tons. (A metric ton is 1000 kg, about 2200 lb, slightly larger than a British ton, 2000 lb.)]

EXAMPLE 1–5 ESTIMATE **Volume of a lake.** Estimate how much water there is in a particular lake, Fig. 1–7a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

APPROACH No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate the volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder (Fig. 1–7b).

SOLUTION The volume V of a cylinder is the product of its height h times the area of its base: $V = h\pi r^2$, where r is the radius of the circular base.† The radius r is $\frac{1}{2}\,\mathrm{km} = 500\,\mathrm{m}$, so the volume is approximately

$$V = h\pi r^2 \approx (10\,\mathrm{m}) \times (3) \times (5 \times 10^2\,\mathrm{m})^2 \approx 8 \times 10^6\,\mathrm{m^3} \approx 10^7\,\mathrm{m^3},$$

where π was rounded off to 3. So the volume is on the order of $10^7\,\mathrm{m^3}$, ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate $(10^7\,\mathrm{m^3})$ is probably better to quote than the $8 \times 10^6\,\mathrm{m^3}$ figure.

NOTE To express our result in U.S. gallons, we see in the Table on the inside front cover that $1\text{ liter} = 10^{-3}\,\mathrm{m^3} \approx \frac{1}{4}\text{ gallon}$. Hence, the lake contains about $(8 \times 10^6\,\mathrm{m^3})(1\text{ gallon}/4 \times 10^{-3}\,\mathrm{m^3}) \approx 2 \times 10^9$ gallons of water.

PHYSICS APPLIED
Estimating the volume (or mass) of a lake; see also Fig. 1–7

†Formulas like this for volume, area, etc., are found inside the back cover of this book.

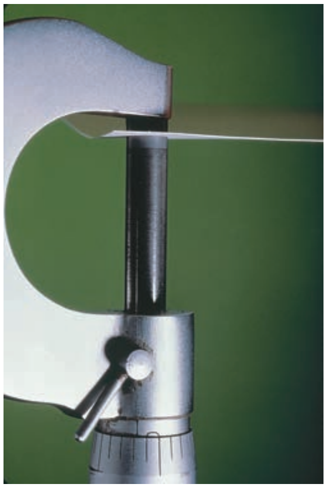


FIGURE 1–8 Example 1–6. Micrometer used for measuring small thicknesses.

EXAMPLE 1–6 ESTIMATE **Thickness of a sheet of paper.** Estimate the thickness of a page of this book.

APPROACH At first you might think that a special measuring device, a micrometer (Fig. 1–8), is needed to measure the thickness of one page since an ordinary ruler can not be read so finely. But we can use a trick or, to put it in physics terms, make use of a *symmetry*: we can make the reasonable assumption that all the pages of this book are equal in thickness.

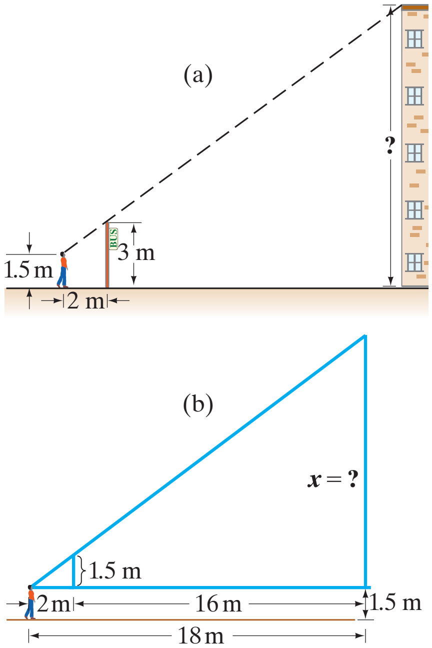
SOLUTION We can use a ruler to measure hundreds of pages at once. If you measure the thickness of the first 500 pages of this book (page 1 to page 500), you might get something like 1.5 cm. Note that 500 numbered pages, counted front and back, is 250 separate pieces of paper. So one sheet must have a thickness of about

$$\frac{1.5\,\text{cm}}{250\,\text{sheets}} \approx 6 \times 10^{-3}\,\text{cm} = 6 \times 10^{-2}\,\text{mm},$$

or less than a tenth of a millimeter (0.1 mm).

It cannot be emphasized enough how important it is to draw a diagram when solving a physics Problem, as the next Example shows.

FIGURE 1–9 Example 1–7. Diagrams are really useful!



EXAMPLE 1–7 ESTIMATE **Height by triangulation.** Estimate the height of the building shown in Fig. 1–9, by "triangulation," with the help of a bus-stop pole and a friend.

APPROACH By standing your friend next to the pole, you estimate the height of the pole to be 3 m. You next step away from the pole until the top of the pole is in line with the top of the building, Fig. 1–9a. You are 5 ft 6 in. tall, so your eyes are about 1.5 m above the ground. Your friend is taller, and when she stretches out her arms, one hand touches you and the other touches the pole, so you estimate that distance as 2 m (Fig. 1–9a). You then pace off the distance from the pole to the base of the building with big, 1-m-long steps, and you get a total of 16 steps or 16 m.

SOLUTION Now you draw, to scale, the diagram shown in Fig. 1–9b using these measurements. You can measure, right on the diagram, the last side of the triangle to be about $x \approx 13$ or 14 m. Alternatively, you can use similar triangles to obtain the height x:

$$\frac{1.5\,\text{m}}{2\,\text{m}} = \frac{x}{18\,\text{m}},$$

so

$$x \approx 13\tfrac{1}{2}\,\text{m}.$$

Finally you add in your eye height of 1.5 m above the ground to get your final result: the building is about 15 m tall.

EXAMPLE 1–8 ESTIMATE **Total number of heartbeats.** Estimate the total number of beats a typical human heart makes in a lifetime.

APPROACH A typical resting heart rate is 70 beats/min. But during exercise it can be a lot higher. A reasonable average might be 80 beats/min.

SOLUTION One year, in seconds, is $(24\,\text{h/d})(3600\,\text{s/h})(365\,\text{d}) \approx 3 \times 10^7\,\text{s}$. If an average person lives 70 years $= (70\,\text{yr})(3 \times 10^7\,\text{s/yr}) \approx 2 \times 10^9\,\text{s}$, then the total number of heartbeats would be about

$$\left(80\,\frac{\text{beats}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)(2 \times 10^9\,\text{s}) \approx 3 \times 10^9,$$

or 3 billion.

EXAMPLE 1–9 **ESTIMATE** **Estimating the radius of Earth.** Believe it or not, you can estimate the radius of the Earth without having to go into space (see the photograph on page 23). If you have ever been on the shore of a large lake, you may have noticed that you cannot see the beaches, piers, or rocks at water level across the lake on the opposite shore. The lake seems to bulge out between you and the opposite shore—a good clue that the Earth is round. Suppose you climb a stepladder and discover that when your eyes are 10 ft (3.0 m) above the water, you can just see the rocks at water level on the opposite shore. From a map, you estimate the distance to the opposite shore as $d \approx 6.1$ km. Use Fig. 1–10 with $h = 3.0$ m to estimate the radius R of the Earth.

APPROACH We use simple geometry, including the theorem of Pythagoras, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse of any right triangle, and a and b are the lengths of the other two sides.

SOLUTION For the right triangle of Fig. 1–10, the two sides are the radius of the Earth R and the distance $d = 6.1\text{ km} = 6100\text{ m}$. The hypotenuse is approximately the length $R + h$, where $h = 3.0$ m. By the Pythagorean theorem,

$$R^2 + d^2 \approx (R + h)^2$$
$$\approx R^2 + 2hR + h^2.$$

We solve algebraically for R, after cancelled R^2 on both sides:

$$R \approx \frac{d^2 - h^2}{2h} = \frac{(6100\text{ m})^2 - (3.0\text{ m})^2}{6.0\text{ m}}$$
$$= 6.2 \times 10^6\text{ m}$$
$$= 6200\text{ km}.$$

NOTE Precise measurements give 6380 km. But look at your achievement! With a few simple rough measurements and simple geometry, you made a good estimate of the Earth's radius. You did not need to go out in space, nor did you need a very long measuring tape.†

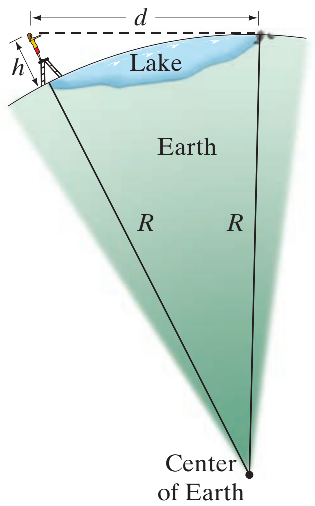
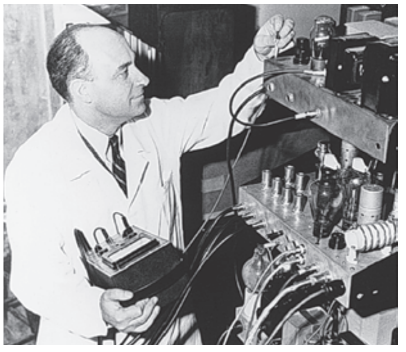


FIGURE 1–10 Example 1–9, but not to scale. You can just barely see rocks at water level on the opposite shore of a lake 6.1 km wide if you stand on a stepladder.

EXERCISE H Return to the second Chapter-Opening Question, page 23, and answer it again now. Try to explain why you may have answered differently the first time.

Another type of estimate, this one made famous by Enrico Fermi (1901–1954, Fig. 1–11), was to show his students how to estimate the number of piano tuners in a city, such as Chicago or San Francisco. To get a rough order-of-magnitude estimate of the number of piano tuners today in San Francisco, a city of about 800,000 inhabitants, we can proceed by estimating the number of functioning pianos, how often each piano is tuned, and how many pianos each tuner can tune. To estimate the number of pianos in San Francisco, we note that certainly not everyone has a piano. A guess of 1 family in 3 having a piano would correspond to 1 piano per 12 persons, assuming an average family of 4 persons.

FIGURE 1–11 Enrico Fermi. Fermi contributed significantly to both theoretical and experimental physics, a feat almost unique in modern times.



As an order of magnitude, let's say 1 piano per 10 people. This is certainly more reasonable than 1 per 100 people, or 1 per every person, so let's proceed with the estimate that 1 person in 10 has a piano, or about 80,000 pianos in San Francisco. Now a piano tuner needs an hour or two to tune a piano. So let's estimate that a tuner can tune 4 or 5 pianos a day. A piano ought to be tuned every 6 months or a year—let's say once each year. A piano tuner tuning 4 pianos a day, 5 days a week, 50 weeks a year can tune about 1000 pianos a year. So San Francisco, with its (very) roughly 80,000 pianos, needs about 80 piano tuners. This is, of course, only a rough estimate.‡ It tells us that there must be many more than 10 piano tuners, and surely not as many as 1000.

PROBLEM SOLVING
Estimating how many piano tuners there are in a city

†As a teenager I had a summer job washing dishes at a camp located 350 m above famous Lake Tahoe in California. Starting the drive down to Lake Tahoe, the beaches across the lake were visible. But approaching the level of Lake Tahoe, the beaches across the lake were no longer visible! I realized that Lake Tahoe was bulging up in the middle, blocking the view. ("The Earth is round.")

‡A search on the internet (done after this calculation) reveals over 50 listings. Each of these listings may employ more than one tuner, but on the other hand, each may also do repairs as well as tuning. In any case, our estimate is reasonable.

*1–7 Dimensions and Dimensional Analysis

When we speak of the **dimensions** of a quantity, we are referring to the type of base units that make it up. The dimensions of area, for example, are always length squared, abbreviated $[L^2]$ using square brackets; the units can be square meters, square feet, cm^2, and so on. Velocity, on the other hand, can be measured in units of km/h, m/s, or mi/h, but the dimensions are always a length $[L]$ divided by a time $[T]$: that is, $[L/T]$.

The formula for a quantity may be different in different cases, but the dimensions remain the same. For example, the area of a triangle of base b and height h is $A = \frac{1}{2}bh$, whereas the area of a circle of radius r is $A = \pi r^2$. The formulas are different in the two cases, but the dimensions of area are always $[L^2]$.

Dimensions can be used as a help in working out relationships, a procedure referred to as **dimensional analysis**. One useful technique is the use of dimensions to check if a relationship is *incorrect*. Note that we add or subtract quantities only if they have the same dimensions (we don't add centimeters and hours); and the quantities on each side of an equals sign must have the same dimensions. (In numerical calculations, the units must also be the same on both sides of an equation.)

For example, suppose you derived the equation $v = v_0 + \frac{1}{2}at^2$, where v is the velocity of an object after a time t, v_0 is the object's initial velocity, and the object undergoes an acceleration a. Let's do a dimensional check to see if this equation could be correct or is surely incorrect. Note that numerical factors, like the $\frac{1}{2}$ here, do not affect dimensional checks. We write a dimensional equation as follows, remembering that the dimensions of velocity are $[L/T]$ and (as we shall see in Chapter 2) the dimensions of acceleration are $[L/T^2]$:

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right][T^2]$$
$$\stackrel{?}{=} \left[\frac{L}{T}\right] + [L].$$

The dimensions are incorrect: on the right side, we have the sum of quantities whose dimensions are not the same. Thus we conclude that an error was made in the derivation of the original equation.

A dimensional check can only tell you when a relationship is wrong. It can not tell you if it is completely right. For example, a dimensionless numerical factor (such as $\frac{1}{2}$ or 2π) could be missing.

Dimensional analysis can also be used as a quick check on an equation you are not sure about. For example, consider a simple pendulum of length ℓ. Suppose that you can't remember whether the equation for the period T (the time to make one back-and-forth swing) is $T = 2\pi\sqrt{\ell/g}$ or $T = 2\pi\sqrt{g/\ell}$, where g is the acceleration due to gravity and, like all accelerations, has dimensions $[L/T^2]$. (Do not worry about these formulas—the correct one will be derived in Chapter 11; what we are concerned about here is a person's recalling whether it contains ℓ/g or g/ℓ.) A dimensional check shows that the former (ℓ/g) is correct:

$$[T] = \sqrt{\frac{[L]}{[L/T^2]}} = \sqrt{[T^2]} = [T],$$

whereas the latter (g/ℓ) is not:

$$[T] \neq \sqrt{\frac{[L/T^2]}{[L]}} = \sqrt{\frac{1}{[T^2]}} = \frac{1}{[T]}.$$

The constant 2π has no dimensions and so can't be checked using dimensions.

Further uses of dimensional analysis are found in Appendix D.

*Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor, and they are marked with an asterisk (*). See the Preface for more details.

EXAMPLE 1–10 **Planck length.** The smallest meaningful measure of length is called the "Planck length," and is defined in terms of three fundamental constants in nature: the speed of light $c = 3.00 \times 10^8\ \mathrm{m/s}$, the gravitational constant $G = 6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2}$, and Planck's constant $h = 6.63 \times 10^{-34}\ \mathrm{kg \cdot m^2/s}$. The Planck length λ_P (λ is the Greek letter "lambda") is given by the following combination of these three constants:

$$\lambda_P = \sqrt{\frac{Gh}{c^3}}.$$

Show that the dimensions of λ_P are length $[L]$, and find the order of magnitude of λ_P.

APPROACH We rewrite the above equation in terms of dimensions. The dimensions of c are $[L/T]$, of G are $[L^3/MT^2]$, and of h are $[ML^2/T]$.

SOLUTION The dimensions of λ_P are

$$\sqrt{\frac{[L^3/MT^2][ML^2/T]}{[L^3/T^3]}} = \sqrt{[L^2]} = [L]$$

which is a length. Good. The value of the Planck length is

$$\lambda_P = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2})(6.63 \times 10^{-34}\ \mathrm{kg \cdot m^2/s})}{(3.00 \times 10^8\ \mathrm{m/s})^3}} \approx 4 \times 10^{-35}\ \mathrm{m},$$

which is on the order of 10^{-34} or 10^{-35} m.

NOTE Some recent theories (Chapters 43 and 44) suggest that the smallest particles (quarks, leptons) have sizes on the order of the Planck length, 10^{-35} m. These theories also suggest that the "Big Bang," with which the Universe is believed to have begun, started from an initial size on the order of the Planck length.

Summary

[*The Summary that appears at the end of each Chapter in this book gives a brief overview of the main ideas of the Chapter. The Summary* cannot *serve to give an understanding of the material, which can be accomplished only by a detailed reading of the Chapter.*]

Physics, like other sciences, is a creative endeavor. It is not simply a collection of facts. Important **theories** are created with the idea of explaining **observations**. To be accepted, theories are **tested** by comparing their predictions with the results of actual experiments. Note that, in general, a theory cannot be "proved" in an absolute sense.

Scientists often devise models of physical phenomena. A **model** is a kind of picture or analogy that helps to describe the phenomena in terms of something we already know about. A **theory**, often developed from a model, is usually deeper and more complex than a simple model.

A scientific **law** is a concise statement, often expressed in the form of an equation, which quantitatively describes a wide range of phenomena.

Measurements play a crucial role in physics, but can never be perfectly precise. It is important to specify the **uncertainty** of a measurement either by stating it directly using the $\pm$ notation, and/or by keeping only the correct number of **significant figures**.

Physical quantities are always specified relative to a particular standard or **unit**, and the unit used should always be stated. The commonly accepted set of units today is the **Système International** (SI), in which the standard units of length, mass, and time are the **meter**, **kilogram**, and **second**.

When converting units, check all **conversion factors** for correct cancellation of units.

Making rough, **order-of-magnitude estimates** is a very useful technique in science as well as in everyday life.

[*The **dimensions** of a quantity refer to the combination of base quantities that comprise it. Velocity, for example, has dimensions of [length/time] or $[L/T]$. Working with only the dimensions of the various quantities in a given relationship—this technique is called **dimensional analysis**—makes it possible to check a relationship for correct form.]

Questions

1. What are the merits and drawbacks of using a person's foot as a standard? Consider both (*a*) a particular person's foot, and (*b*) any person's foot. Keep in mind that it is advantageous that fundamental standards be accessible (easy to compare to), invariable (do not change), indestructible, and reproducible.
2. What is wrong with this road sign:

   Memphis 7 mi (11.263 km)?
3. Why is it incorrect to think that the more digits you include in your answer, the more accurate it is?
4. For an answer to be complete, units need to be specified. Why?
5. You measure the radius of a wheel to be 4.16 cm. If you multiply by 2 to get the diameter, should you write the result as 8 cm or as 8.32 cm? Justify your answer.
6. Express the sine of 30.0° with the correct number of significant figures.
7. List assumptions useful to estimate the number of car mechanics in (*a*) San Francisco, (*b*) your hometown, and then make the estimates.

MisConceptual Questions

[List all answers that are valid.]

1. The laws of physics
   (*a*) are permanent and unalterable.
   (*b*) are part of nature and are waiting to be discovered.
   (*c*) can change, but only because of evidence that convinces the community of physicists.
   (*d*) apply to physics but not necessarily to chemistry or other fields.
   (*e*) were basically complete by 1900, and have undergone only minor revisions since.
   (*f*) are accepted by all major world countries, and cannot be changed without international treaties.

2. How should we write the result of the following calculation, being careful about significant figures?
$$(3.84\,\text{s})(37\,\text{m/s}) + (5.3\,\text{s})(14.1\,\text{m/s}) =$$
   (*a*) 200 m. (*b*) 210 m. (*c*) 216.81 m. (*d*) 217 m. (*e*) 220 m.

3. Four students use different instruments to measure the length of the same pen. Which measurement implies the greatest precision?
   (*a*) 160.0 mm. (*b*) 16.0 cm. (*c*) 0.160 m. (*d*) 0.00016 km. (*e*) Need more information.

4. The number 0.0078 has how many significant figures?
   (*a*) 1. (*b*) 2. (*c*) 3. (*d*) 4.

5. How many significant figures does $1.362 + 25.2$ have?
   (*a*) 2. (*b*) 3. (*c*) 4. (*d*) 5.

6. Accuracy represents
   (*a*) repeatability of a measurement, using a given instrument.
   (*b*) how close a measurement is to the true value.
   (*c*) an ideal number of measurements to make.
   (*d*) how poorly an instrument is operating.

7. Precision represents
   (*a*) repeatability of a measurement, using a given instrument.
   (*b*) how close a measurement is to the true value.
   (*c*) an ideal number of measurements to make.
   (*d*) how poorly an instrument is operating.

8. To convert from ft^2 to yd^2, you should
   (*a*) multiply by 3. (*b*) multiply by 1/3. (*c*) multiply by 9. (*d*) multiply by 1/9. (*e*) multiply by 6. (*f*) multiply by 1/6.

9. Which is *not* true about an order-of-magnitude estimation?
   (*a*) It gives you a rough idea of the answer.
   (*b*) It can be done by keeping only one significant figure.
   (*c*) It can be used to check if an exact calculation is reasonable.
   (*d*) It may require making some reasonable assumptions in order to calculate the answer.
   (*e*) It will always be accurate to at least two significant figures.

*10. $[L^2]$ represents the dimensions for which of the following?
   (*a*) cm^2. (*b*) square feet. (*c*) m^2. (*d*) All of the above.

Problems

[*The Problems at the end of each Chapter are ranked* I, II, *or* III *according to estimated difficulty, with* (I) *Problems being easiest. Level* III *are meant as challenges for the best students. The Problems are arranged by Section, meaning that the reader should have read up to and including that Section, but not only that Section—Problems often depend on earlier material. Next is a set of "General Problems" not arranged by Section and not ranked.*]

1–3 Measurement, Uncertainty, Significant Figures

(Note: *In Problems, assume a number like* 6.4 *is accurate to* ± 0.1; *and* 950 *is accurate to* 2 *significant figures* (± 10) *unless* 950 *is said to be "precisely" or "very nearly"* 950, *in which case assume* 950 ± 1.)

1. (I) How many significant figures do each of the following numbers have: (*a*) 777, (*b*) 81.60, (*c*) 7.03, (*d*) 0.03, (*e*) 0.0086, (*f*) 6465, and (*g*) 8700?

2. (I) Write the following numbers in powers of 10 notation: (*a*) 5.859, (*b*) 21.8, (*c*) 0.0068, (*d*) 328.65, (*e*) 0.219, (*f*) 444.

3. (I) Write out the following numbers in full with the correct number of zeros: (*a*) 8.69×10^5, (*b*) 9.1×10^3, (*c*) 2.5×10^{-1}, (*d*) 4.76×10^2, and (*e*) 3.62×10^{-5}.

4. (II) What is the percent uncertainty in the measurement 3.25 ± 0.35 m?

5. (II) Time intervals measured with a physical stopwatch typically have an uncertainty of about 0.2 s, due to human reaction time at the start and stop moments. What is the percent uncertainty of a hand-timed measurement of (*a*) 4.5 s, (*b*) 45 s, (*c*) 4.5 min?

6. (II) Add $(9.2 \times 10^3\,\text{s}) + (6.3 \times 10^4\,\text{s}) + (0.008 \times 10^6\,\text{s})$.

7. (II) Multiply 4.079×10^2 m by 0.057×10^{-1} m, taking into account significant figures.

8. (II) What, approximately, is the percent uncertainty for a measurement given as $1.27\,\text{m}^2$?

9. (II) For small angles θ, the numerical value of $\sin\theta$ is approximately the same as the numerical value of $\tan\theta$. Find the largest angle for which sine and tangent agree to within two significant figures.

10. (II) A report stated that "a survey of 215 students found that 37.2% had bought a sugar-rich soft drink the day before." (*a*) How many students bought a soft drink? (*b*) What is wrong with the original statement?

11. (II) A watch manufacturer claims that its watches gain or lose no more than 9 seconds in a year. How accurate are these watches, expressed as a percentage?

12. (III) What is the area, and its approximate uncertainty, of a circle of radius 5.1×10^4 cm?

13. (III) What, roughly, is the percent uncertainty in the volume of a spherical beach ball of radius $r = 0.64 \pm 0.04$ m?

1–4 and 1–5 Units, Standards, SI, Converting Units

14. (I) Write the following as full (decimal) numbers without prefixes on the units: (*a*) 286.6 mm, (*b*) 74 μV, (*c*) 430 mg, (*d*) 47.2 ps, (*e*) 22.5 nm, (*f*) 2.50 gigavolts.

15. (I) Express the following using the prefixes of Table 1–4: (*a*) 3×10^6 volts, (*b*) 2×10^{-6} meters, (*c*) 5×10^3 days, (*d*) 18×10^2 bucks, and (*e*) 9×10^{-7} seconds.

16. (I) Determine your own height in meters, and your mass in kg.

17. (II) To the correct number of significant figures, use the information inside the front cover of this book to determine the ratio of (*a*) the surface area of Earth compared to the surface area of the Moon, (*b*) the volume of Earth compared to the volume of the Moon.

18. (II) Would a driver traveling at 15 m/s in a 35 mi/h zone be exceeding the speed limit? Why or why not?

19. (II) The age of the universe is thought to be about 14 billion years. Assuming two significant figures, write this in powers of 10 in (*a*) years, (*b*) seconds.

20. (II) The Sun, on average, is 93 million miles from Earth. How many meters is this? Express (*a*) using powers of 10, and (*b*) using a metric prefix (km).

21. (II) Express the following sum with the correct number of significant figures: $1.90\,\text{m} + 142.5\,\text{cm} + 6.27 \times 10^5\,\mu\text{m}$.

22. (II) A typical atom has a diameter of about 1.0×10^{-10} m. (*a*) What is this in inches? (*b*) Approximately how many atoms are along a 1.0-cm line, assuming they just touch?

23. (II) Determine the conversion factor between (*a*) km/h and mi/h, (*b*) m/s and ft/s, and (*c*) km/h and m/s.

24. (II) What is the conversion factor between (*a*) ft^2 and yd^2, (*b*) m^2 and ft^2?

25. (II) A **light-year** is the distance light travels in one year $(\text{at speed} = 2.998 \times 10^8\,\text{m/s})$. (*a*) How many meters are there in 1.00 light-year? (*b*) An **astronomical unit** (AU) is the average distance from the Sun to Earth, 1.50×10^8 km. How many AU are there in 1.00 light-year?

26. (II) How much longer (percentage) is a one-mile race than a 1500-m race ("the metric mile")?

27. (II) How many wavelengths of orange krypton-86 light (Section 1–4) would fit into the thickness of one page of this book? See Example 1–6.

28. (II) Using the French Academy of Sciences' original definition of the meter, calculate Earth's circumference and radius in *those* meters. Give % error relative to today's accepted values (inside front cover).

29. (II) A passenger jet uses about 12 liters of fuel per km of flight. What is that value expressed as miles per gallon?

30. (II) American football uses a *field* that is 100.0 yd long, whereas a *soccer field* is 100.0 m long. Which field is longer, and by how much (give yards, meters, and percent)?

31. (II) (*a*) How many seconds are there in 1.00 year? (*b*) How many nanoseconds are there in 1.00 year? (*c*) How many years are there in 1.00 second?

32. (II) Use Table 1–3 to estimate the total number of protons or neutrons in (*a*) a bacterium, (*b*) a DNA molecule, (*c*) the human body, (*d*) our Galaxy.

33. (II) The diameter of the planet Mercury is 4879 km. (*a*) What is the surface area of Mercury? (*b*) How many times larger is the surface area of the Earth?

34. (III) A standard baseball has a circumference of approximately 23 cm. If a baseball had the same mass per unit volume (see Tables in Section 1–4) as a neutron or a proton, about what would its mass be?

1–6 Order-of-Magnitude Estimating

(Note: *Remember that for rough estimates, only round numbers are needed both as input to calculations and as final results.*)

35. (I) Estimate the order of magnitude (power of 10) of: (*a*) 3200, (*b*) 86.30×10^3, (*c*) 0.076, and (*d*) 15.0×10^8.

36. (II) Estimate how many books can be shelved in a college library with 6500 m^2 of floor space. Assume 8 shelves high, having books on both sides, with corridors 1.5 m wide. Assume books are about the size of this one, on average.

37. (II) Estimate how many hours it would take to run (at 10 km/h) across the U.S. from New York to California.

38. (II) Estimate the number of liters of water a human drinks in a lifetime.

39. (II) Estimate the number of *cells* in an adult human body, given that a typical cell has a diameter of about 10 μm, and the human body has a density of about 1000 kg/m^3.

40. (II) Estimate how long it would take one person to mow a football field using an ordinary home lawn mower (Fig. 1–12). (State your assumptions, such as the mower moves with a 1-km/h speed, and has a 0.5-m width.)

FIGURE 1–12 Problem 40.

41. (II) Estimate the number of gallons of gasoline consumed by the total of all automobile drivers in the U.S., per year.

42. (II) Estimate the number of dentists (*a*) in San Francisco and (*b*) in your town or city.

43. (II) Estimate how many kilograms of laundry soap are used in the U.S. in one year (and therefore pumped out of washing machines with the dirty water). Assume each load of laundry takes 0.1 kg of soap.

44. (II) How big is a *ton* (1000 kg)? That is, what is the volume of something that weighs a ton? To be specific, estimate the diameter of a 1-ton rock, but first make a wild guess: will it be 10 cm across, 1 m, or the size of a car? [*Hint*: Rock has mass per volume about 3 times that of water, which is 1 kg per liter $(10^3\,\text{cm}^3)$.]

45. (II) A hiking trail is 270 km long through varying terrain. A group of hikers cover the first 49 km in two and a half days. Estimate how much time they should allow for the rest of the trip.

46. (II) Estimate how many days it would take to walk around the circumference of the Earth, assuming 12 h walking per day at 4 km/h.

47. (II) Estimate the number of jelly beans in the jar of Fig. 1–13.

FIGURE 1–13 Problem 47. Estimate the number of jelly beans in the jar.

48. (II) Estimate the number of bus drivers (*a*) in Washington, D.C., and (*b*) in your town.

49. (III) You are in a hot air balloon, 300 m above the flat Texas plains. You look out toward the horizon. How far out can you see—that is, how far is your horizon? The Earth's radius is about 6400 km.

50. (III) I agree to hire you for 30 days. You can decide between two methods of payment: either (1) \$1000 a day, or (2) one penny on the first day, two pennies on the second day and continue to double your daily pay each day up through day 30. Use quick estimation to make your decision, and justify it.

51. (III) The rubber worn from tires mostly enters the atmosphere as *particulate pollution*. Estimate how much rubber (in kg) is put into the air in the United States every year. To get started, a good estimate for a tire tread's depth is 1 cm when new, and rubber has a mass of about 1200 kg per m^3 of volume.

52. (III) Many sailboats are docked at a marina 4.4 km away on the opposite side of a lake. You stare at one of the sailboats because, when you are lying flat at the water's edge, you can just see its deck but none of the side of the sailboat. You then go to that sailboat on the other side of the lake and measure that the deck is 1.5 m above the level of the water. Using Fig. 1–14, where $h = 1.5$ m, estimate the radius R of the Earth.

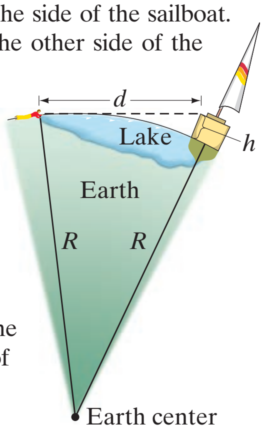


FIGURE 1–14 Problem 52. You see a sailboat across a lake (not to scale). R is the radius of the Earth. Because of the curvature of the Earth, the water "bulges out" between you and the boat.

53. (III) You are lying on a beach, your eyes 20 cm above the sand. Just as the Sun sets, fully disappearing over the horizon, you immediately jump up, your eyes now 150 cm above the sand, and you can again just see the top of the Sun. If you count the number of seconds $(= t)$ until the Sun fully disappears again, you can estimate the *Earth's radius*. But for this Problem, use the known radius of the Earth to calculate the time t.

*1–7 Dimensions

***54.** (I) What are the dimensions of density, which is mass per volume?

***55.** (II) The speed v of an object is given by the equation $v = At^3 - Bt$, where t refers to time. (*a*) What are the dimensions of A and B? (*b*) What are the SI units for the constants A and B?

***56.** (II) Three students derive the following equations in which x refers to distance traveled, v the speed, a the acceleration $(\mathrm{m/s^2})$, t the time, and the subscript zero $({}_0)$ means a quantity at time $t = 0$. Here are their equations: (*a*) $x = vt^2 + 2at$, (*b*) $x = v_0 t + \frac{1}{2}at^2$, and (*c*) $x = v_0 t + 2at^2$. Which of these could possibly be correct according to a dimensional check, and why?

***57.** (II) (*a*) Show that the following combination of the three fundamental constants of nature that we used in Example 1–10 (that is G, c, and h) forms a quantity with the dimensions of time:

$$t_{\mathrm{P}} = \sqrt{\frac{Gh}{c^5}}.$$

This quantity, t_{P}, is called the **Planck time** and is thought to be the earliest time, after the creation of the Universe, at which the currently known laws of physics can be applied. (*b*) Estimate the order of magnitude of t_{P} using values given inside the front cover (or Example 1–10).

General Problems

58. **Global positioning satellites (GPS)** can be used to determine your position with great accuracy. If one of the satellites is 20,000 km from you, and you want to know your position to ± 2 m, what percent uncertainty in the distance is required? How many significant figures are needed in the distance?

59. One mole of atoms consists of 6.02×10^{23} individual atoms. If a mole of atoms were spread uniformly over the Earth's surface, how many atoms would there be per square meter?

60. **Computer chips** (Fig. 1–15) can be etched on circular silicon wafers of thickness 0.300 mm that are sliced from a solid cylindrical silicon crystal of length 25 cm. If each wafer can hold 750 chips, what is the maximum number of chips that can be produced from one entire cylinder?

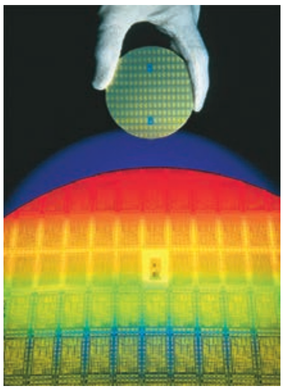


FIGURE 1–15 Problem 60. The wafer held by the hand is shown below, enlarged and illuminated by colored light. Visible are rows of integrated circuits (chips).

61. If you used only a keyboard to enter data, how many years would it take to fill up a *hard drive* in a computer that can store 1.0 terabytes $(1.0 \times 10^{12}$ bytes$)$ of data? Assume 40-hour work weeks, and that you can type 150 characters per minute, and that one byte is one keyboard character.

62. An average family of four uses roughly 1200 L (about 300 gallons) of water per day $(1\ \mathrm{L} = 1000\ \mathrm{cm^3})$. How much depth would a lake lose per year if it covered an area of 60 $\mathrm{km^2}$ with uniform depth and supplied a local town with a population of 40,000 people? Consider only population uses, and neglect evaporation, rain, creeks and rivers.

63. A certain compact disc (CD) contains 783.216 megabytes of digital information. Each byte consists of exactly 8 bits. When played, a CD player reads the CD's information at a constant rate of 1.4 megabits per second. How many minutes does it take the player to read the entire CD?

64. An *angstrom* (symbol Å) is a unit of length, defined as 10^{-10} m, which is on the order of the diameter of an atom. (*a*) How many nanometers are in 1.0 angstrom? (*b*) How many femtometers or fermis (the common unit of length in nuclear physics) are in 1.0 angstrom? (*c*) How many angstroms are in 1.0 m? (*d*) How many angstroms are in 1.0 light-year (see Problem 25)?

65. A typical adult human lung contains about 300 million tiny cavities called alveoli. Estimate the average diameter of a single alveolus.

66. Hold a pencil in front of your eye at a position where its blunt end just blocks out the Moon (Fig. 1–16). Make appropriate measurements to estimate the diameter of the Moon, given that the Earth–Moon distance is 3.8×10^5 km.

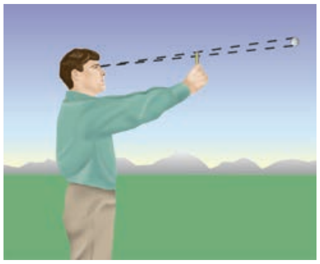


FIGURE 1–16 Problem 66. How big is the Moon?

67. A storm dumps 1.0 cm of rain on a city 5 km wide and 7 km long in a 2-h period. How many metric tons $(1\text{ metric ton} = 10^3\text{ kg})$ of water fell on the city? $(1\text{ cm}^3$ of water has a mass of $1\text{ g} = 10^{-3}\text{ kg.})$ How many gallons of water was this?

68. Greenland's ice sheet covers over $1.7 \times 10^6\text{ km}^2$ and is approximately 2.5 km thick. If it were to melt completely then by how much would you expect the ocean to rise? Assume $\frac{2}{3}$ of Earth's surface is ocean. See Tables inside front and back covers.

69. Noah's ark was ordered to be 300 cubits long, 50 cubits wide, and 30 cubits high. The cubit was a unit of measure equal to the length of a human forearm, elbow to the tip of the longest finger. Express the dimensions of Noah's ark in meters, and estimate its volume (m^3).

70. One liter (1000 cm^3) of oil is spilled onto a smooth lake. If the oil spreads out uniformly until it makes an oil slick just one molecule thick, with adjacent molecules just touching, estimate the diameter of the oil slick. Assume the oil molecules have a diameter of 2×10^{-10} m.

71. If you walked north along one of Earth's lines of longitude until you had changed latitude by 1 minute of arc (there are 60 minutes per degree), how far would you have walked (in miles)? This distance is a *nautical mile* (page 29).

72. Determine the percent uncertainty in θ, and in $\sin\theta$, when (*a*) $\theta = 15.0° \pm 0.5°$, (*b*) $\theta = 75.0° \pm 0.5°$.

73. Jim stands beside a wide river and wonders how wide it is. He spots a large rock on the bank directly across from him. He then walks upstream 85 strides and judges that the angle between him and the rock, which he can still see, is now at an angle of 30° downstream (Fig. 1–17). Jim measures his stride to be about 0.8 m long. Estimate the width of the river.

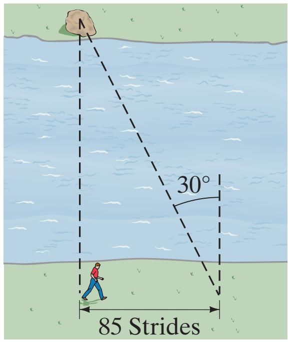


FIGURE 1–17 Problem 73.

74. Make a rough estimate of the volume of your body (in m^3).

75. Estimate the number of plumbers in San Francisco.

76. Estimate the ratio (order of magnitude) of the mass of a human to the mass of a DNA molecule. [*Hint*: Check the Tables in this Chapter.]

77. The following formula estimates an average person's lung capacity V $(\text{in liters, where } 1\text{ L} = 10^3\text{ cm}^3)$:

$$V = 4.1H - 0.018A - 2.7,$$

where H and A are the person's height (in meters) and age (in years), respectively. In this formula, what are the units of the numbers 4.1, 0.018, and 2.7?

78. The density of an object is defined as its mass divided by its volume. Suppose a rock's mass and volume are measured to be 6 g and 2.8325 cm^3. To the correct number of significant figures, determine the rock's density (mass/volume).

79. Recent findings in astrophysics suggest that the observable universe can be modeled as a sphere of radius $R = 13.7 \times 10^9$ light-years $= 13.0 \times 10^{25}$ m with an average total mass density of about $1 \times 10^{-26}\text{ kg/m}^3$. Only about 4% of total mass is due to "ordinary" matter (such as protons, neutrons, and electrons). Estimate how much ordinary matter (in kg) there is in the observable universe. (For the light-year, see Problem 25.)

ANSWERS TO EXERCISES

A: (*d*).

B: All three have three significant figures; the number of decimal places is (*a*) 2, (*b*) 3, (*c*) 4.

C: No: they have three and two, respectively.

D: (*a*) 2.58×10^{-2}, 3; (*b*) 4.23×10^4, 3 (probably); (*c*) 3.4450×10^2, 5.

E: Mt. Everest, 29,035 ft; K2, 28,251 ft; Kangchenjunga, 28,169 ft.

F: (*a*) 2.47 acres in 1 hectare; (*b*) $2\frac{1}{2}$ or even just 2 acres in 1 hectare.

G: (*f*) 1,000,000; that is, one million.

H: (*c*).

A space shuttle has released a parachute to reduce its speed quickly. The directions of the shuttle's velocity and acceleration are shown by the green ($\vec{\mathbf{v}}$) and gold ($\vec{\mathbf{a}}$) arrows.

Motion is described using the concepts of velocity and acceleration. In the case shown here, the velocity $\vec{\mathbf{v}}$ is to the right, in the direction of motion. The acceleration $\vec{\mathbf{a}}$ is in the opposite direction from the velocity $\vec{\mathbf{v}}$, which means the object is slowing down.

We examine in detail motion with constant acceleration, including the vertical motion of objects falling under gravity.

CHAPTER 2

Describing Motion: Kinematics in One Dimension

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also page 23 of Chapter 1 for more explanation.*]

Two small heavy balls have the same diameter but one weighs twice as much as the other. The balls are dropped from a second-story balcony at the exact same time. The time to reach the ground below will be:

(a) twice as long for the lighter ball as for the heavier one.
(b) longer for the lighter ball, but not twice as long.
(c) twice as long for the heavier ball as for the lighter one.
(d) longer for the heavier ball, but not twice as long.
(e) nearly the same for both balls.

The motion of objects—baseballs, automobiles, joggers, and even the Sun and Moon—is an obvious part of everyday life. It was not until the sixteenth and seventeenth centuries that our modern understanding of motion was established. Many individuals contributed to this understanding, particularly Galileo Galilei (1564–1642) and Isaac Newton (1642–1727).

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do. This Chapter and the next deal with kinematics.

For now we only discuss objects that move without rotating (Fig. 2–1a). Such motion is called **translational motion**. In this Chapter we will be concerned with describing an object that moves along a straight-line path, which is one-dimensional translational motion. In Chapter 3 we will describe translational motion in two (or three) dimensions along paths that are not straight. (Rotation, shown in Fig. 2–1b, is discussed in Chapters 10 and 11.)

We will often use the concept, or *model*, of an idealized **particle** which is considered to be a mathematical **point** with no spatial extent (no size). A point particle can undergo only translational motion. The particle model is useful in many real situations where we are interested only in translational motion and the object's size is not significant. For example, we might consider a billiard ball, or even a spacecraft traveling toward the Moon, as a particle for many purposes.

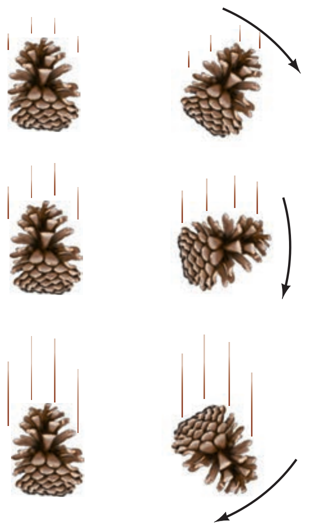
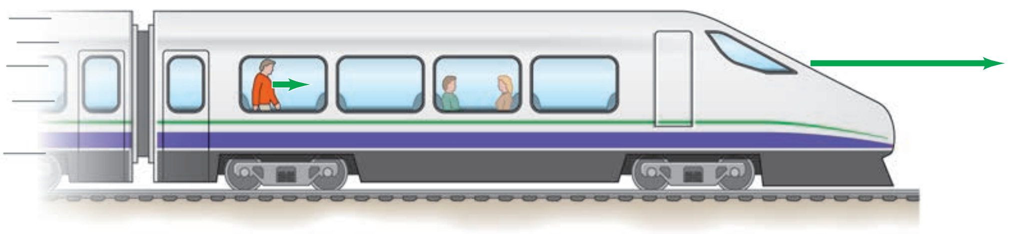


FIGURE 2–1 A falling pinecone undergoes (a) pure translation; (b) it is rotating as well as translating.

2–1 Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a **reference frame**, or **frame of reference**. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2–2). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of $80\text{ km/h} + 5\text{ km/h} = 85\text{ km/h}$. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.

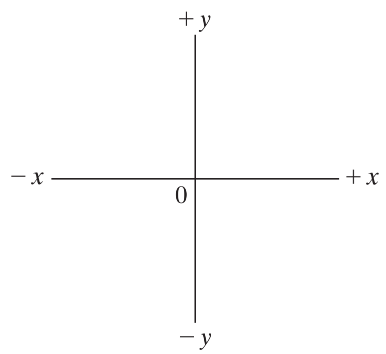
FIGURE 2–2 A person walks toward the front of a train at 5 km/h. The train is moving at 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.



When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 2–3, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. The x and y axes are always perpendicular to each other. The **origin** is where $x = 0$, $y = 0$. Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we almost always choose to be positive; objects at points to the left of 0 have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the xy plane can be specified by giving its x and y coordinates. In three dimensions, a z axis perpendicular to the x and y axes is added.

For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the **position** of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

FIGURE 2–3 Standard set of xy coordinate axes, sometimes called "rectangular coordinates." [Also called *Cartesian coordinates*, after René Descartes (1596–1650), who invented them.]



CAUTION
The displacement may not equal the total distance traveled

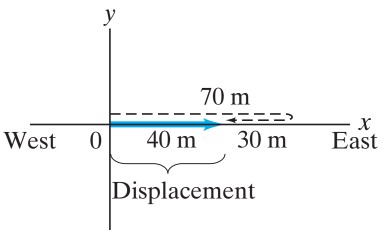


FIGURE 2–4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is 40 m to the east.

FIGURE 2–5 The arrow represents the displacement $x_2 - x_1$. Distances are in meters.

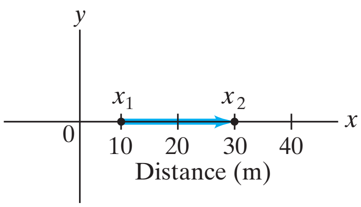
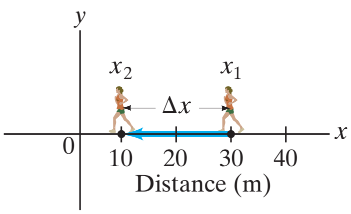


FIGURE 2–6 For the displacement $\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m}$, the displacement vector points left.



We need to make a distinction between the **distance** an object has traveled and its **displacement**, which is defined as the *change in position* of the object.

That is, *displacement is how far the object is from its starting point*. To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 2–4). The total distance traveled is 70 m + 30 m = 100 m, but the displacement is only 40 m since the person is now only 40 m from the starting point.

Displacement is a quantity that has both *magnitude* and *direction*. Such quantities are called **vectors**, and are represented by arrows in diagrams. For example, in Fig. 2–4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).

We will deal with vectors more fully in Chapter 3. For now, we deal only with motion in one dimension, along a line. In this case, vectors which point in one direction will be positive (usually to the right along the x axis). Vectors that point in the opposite direction will have a negative sign in front of their magnitude.

Consider the motion of an object over a particular time interval. Suppose that at some initial time, call it t_1, the object is on the x axis at the position x_1 in the coordinate system shown in Fig. 2–5. At some later time, t_2, suppose the object has moved to position x_2. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 2–5. It is convenient to write

$$\Delta x = x_2 - x_1,$$

where the symbol Δ (Greek letter delta) means "change in." Then Δx means "the change in x," or "change in position," which is in fact the displacement. The **change in** any quantity means the *final* value of that quantity, minus the *initial* value. Suppose $x_1 = 10.0\text{ m}$ and $x_2 = 30.0\text{ m}$, as in Fig. 2–5. Then

$$\Delta x = x_2 - x_1 = 30.0\text{ m} - 10.0\text{ m} = 20.0\text{ m},$$

so the displacement is 20.0 m in the positive direction, Fig. 2–5.

Now consider an object moving to the left as shown in Fig. 2–6. Here the object, a person, starts at $x_1 = 30.0\text{ m}$ and walks to the left to the point $x_2 = 10.0\text{ m}$. In this case her displacement is

$$\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m} = -20.0\text{ m},$$

and the blue arrow representing the vector displacement points to the left. For one-dimensional motion along the x axis, a vector pointing to the right is positive, whereas a vector pointing to the left has a negative sign.

EXERCISE A An ant starts at $x = 20\text{ cm}$ on a piece of graph paper and walks along the x axis to $x = -20\text{ cm}$. It then turns around and walks back to $x = -10\text{ cm}$. Determine (a) the ant's displacement and (b) the total distance traveled.

2–2 Average Velocity

An important aspect of the motion of a moving object is how *fast* it is moving—its speed or velocity.

The term "speed" refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the total distance traveled along its path divided by the time it takes to travel this distance*:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}. \qquad \textbf{(2–1)}$$

The terms "velocity" and "speed" are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and also the *direction* in which it is moving. Velocity is therefore a *vector*.

There is a second difference between speed and velocity: namely, the *average velocity* is defined in terms of *displacement*, rather than total distance traveled:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}.$$

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: recall the walk we described earlier, in Fig. 2–4, where a person walked 70 m east and then 30 m west. The total distance traveled was 70 m + 30 m = 100 m, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{time elapsed}} = \frac{100\text{ m}}{70\text{ s}} = 1.4\text{ m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{time elapsed}} = \frac{40\text{ m}}{70\text{ s}} = 0.57\text{ m/s}.$$

In everyday life, we are usually interested in average speed. If this second equation on average velocity seems strange, we will see its usefulness in the next Section.

CAUTION

Average speed is not necessarily equal to the magnitude of the average velocity

To discuss one-dimensional motion of an object in general, suppose that at some moment in time, call it t_1, the object is on the x axis at position x_1 in a coordinate system, and at some later time, t_2, suppose it is at position x_2. The **elapsed time** (= change in time) is $\Delta t = t_2 - t_1$. During this time interval the displacement of our object is $\Delta x = x_2 - x_1$. Then the **average velocity**, defined as *the displacement divided by the elapsed time*, can be written

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad \text{[average velocity]} \quad \textbf{(2–2)}$$

where v stands for velocity and the bar $(^{-})$ over the v is a standard symbol meaning "average."

It is always important to choose (and state) the *elapsed time*, or **time interval**, $t_2 - t_1$, the time that passes during our chosen period of observation.

CAUTION

Time interval = elapsed time

EXAMPLE 2–1 **Runner's average velocity.** The position of a runner is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from $x_1 = 50.0\text{ m}$ to $x_2 = 30.5\text{ m}$, as shown in Fig. 2–7. What is the runner's average velocity?

APPROACH We want to find the average velocity, which is the displacement divided by the elapsed time.

SOLUTION The displacement is

$$\begin{aligned}\Delta x &= x_2 - x_1 \\ &= 30.5\text{ m} - 50.0\text{ m} = -19.5\text{ m}.\end{aligned}$$

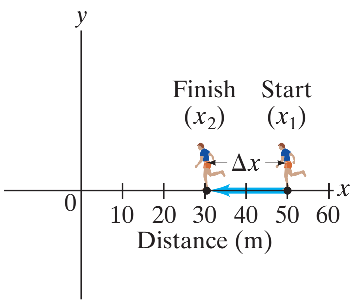
In this case the displacement is negative.

The elapsed time, or time interval, is given as $\Delta t = 3.00\text{ s}$. The average velocity (Eq. 2–2) is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\text{ m}}{3.00\text{ s}} = -6.50\text{ m/s}.$$

The displacement and average velocity are negative: that is, the runner is moving to the left along the x axis, as indicated by the arrow in Fig. 2–7. The runner's average velocity is 6.50 m/s to the left.

FIGURE 2–7 Example 2–1. A person runs from $x_1 = 50.0\text{ m}$ to $x_2 = 30.5\text{ m}$. The displacement is -19.5 m.



For one-dimensional motion in the usual case of the $+x$ axis to the right, if x_2 is less than x_1, then the object is moving to the left, and $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the average velocity, indicates the direction: the average velocity is positive for an object moving to the right along the x axis and negative when the object moves to the left. The direction of the average velocity is always the same as the direction of the displacement.

PROBLEM SOLVING

+ or − sign can signify the direction for linear motion

EXAMPLE 2–2 **Distance a cyclist travels.** How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

APPROACH We want to find the distance traveled, which in this case equals the displacement Δx, so we solve Eq. 2–2 for Δx.

SOLUTION In Eq. 2–2, $\bar{v} = \Delta x/\Delta t$, we multiply both sides by Δt and obtain

$$\Delta x = \bar{v}\,\Delta t = (18\,\text{km/h})(2.5\,\text{h}) = 45\,\text{km}.$$

EXAMPLE 2–3 **Car changes speed.** A car travels at a constant 50 km/h for 100 km. It then speeds up to 100 km/h and is driven another 100 km. What is the car's average speed for the 200-km trip?

APPROACH At 50 km/h, the car takes 2.0 h to travel 100 km. At 100 km/h, it takes only 1.0 h to travel 100 km. We use the definition of average velocity, Eq. 2–2.

SOLUTION Average velocity (Eq. 2–2) is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100\,\text{km} + 100\,\text{km}}{2.0\,\text{h} + 1.0\,\text{h}} = 67\,\text{km/h}.$$

NOTE Averaging the two speeds, $(50\,\text{km/h} + 100\,\text{km/h})/2 = 75\,\text{km/h}$, gives a wrong answer. Can you see why? You must use the definition of $\bar{v}$, Eq. 2–2.

2–3 Instantaneous Velocity

FIGURE 2–8 Car speedometer showing mi/h in white, and km/h in orange.

If you drive a car along a straight road for 150 km in 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To describe this situation we need the concept of *instantaneous velocity*, which is the velocity at any instant of time. (Its magnitude is the number, with units, indicated by a speedometer, Fig. 2–8.) More precisely, the **instantaneous velocity** at any moment is defined as *the average velocity over an infinitesimally short time interval*. That is, Eq. 2–2 is to be evaluated in the limit of Δt becoming extremely small, approaching zero. We can write the definition of instantaneous velocity, v, for one-dimensional motion as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \qquad \text{[instantaneous velocity]} \quad \textbf{(2–3)}$$

The notation $\lim_{\Delta t \to 0}$ means the ratio $\Delta x/\Delta t$ is to be evaluated in the limit of Δt approaching zero. But we do not simply set $\Delta t = 0$ in this definition, for then Δx would also be zero, and we would not be able to evaluate it. Rather, we consider the *ratio* $\Delta x/\Delta t$, as a whole. As we let Δt approach zero, Δx approaches zero as well. But the ratio $\Delta x/\Delta t$ approaches some definite value, which is the instantaneous velocity at a given instant.

In Eq. 2–3, the limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the **derivative** of x with respect to t:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \qquad \textbf{(2–4)}$$

This equation is the definition of instantaneous velocity for one-dimensional motion.

For instantaneous velocity we use the symbol v, whereas for average velocity we use $\bar{v}$, with a bar above. In the rest of this book, when we use the term "velocity" it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word "average."

Note that the *instantaneous speed* always equals the magnitude of the instantaneous velocity. Why? Because as the time interval becomes infinitesimally small ($\Delta t \to 0$), an object has no time to change speed or direction, and so the distance traveled and the magnitude of the displacement have to be the same.

If an object moves at a uniform (that is, constant) velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 2–9a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 2–9b. Also shown on the graph is the average velocity (dashed line), which is $\bar{v} = \Delta x/\Delta t = 15\,\text{km}/0.50\,\text{h} = 30\,\text{km/h}$.

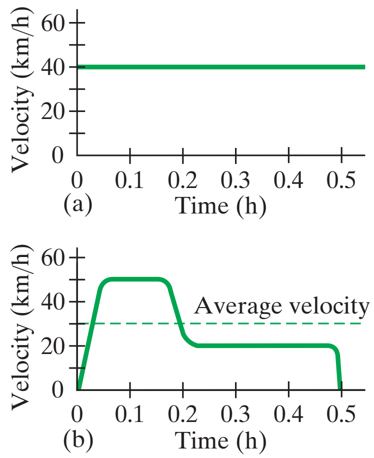
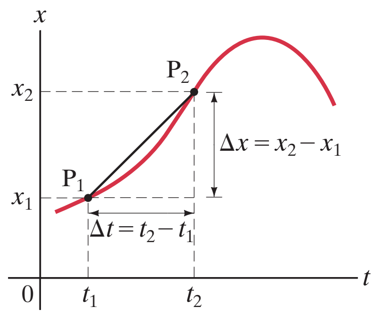


FIGURE 2–9 Velocity of a car as a function of time: (a) at constant velocity; (b) with velocity varying in time.

To better understand instantaneous velocity, let us consider a graph of the position versus time (x vs. t) of a particle moving along the x axis, as shown in Fig. 2–10. (Note that this is different from showing the "path" of a particle moving in two dimensions on an x vs. y plot.) The particle is at position x_1 at time t_1, and at position x_2 at time t_2. P_1 and P_2 represent these two points on the graph. A straight line drawn from point $P_1\left(x_1, t_1\right)$ to point $P_2\left(x_2, t_2\right)$ forms the hypotenuse of a right triangle whose sides are Δx and Δt. The ratio $\Delta x/\Delta t$ is the **slope** of the straight line P_1P_2. But $\Delta x/\Delta t$ is also the average velocity of the particle during the time interval $\Delta t = t_2 - t_1$. Therefore, we conclude that the average velocity of a particle during any time interval $\Delta t = t_2 - t_1$ is equal to the slope of the straight line (or *chord*) connecting the two points $\left(x_1, t_1\right)$ and $\left(x_2, t_2\right)$ on an x vs. t graph.

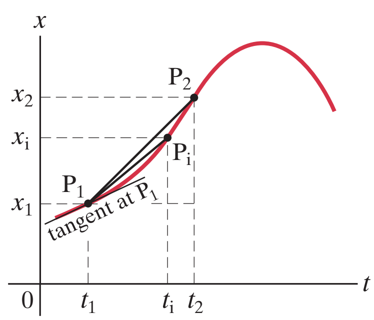
FIGURE 2–10 Graph of a particle's position x vs. time t. The slope of the straight line P_1P_2 represents the average velocity of the particle during the time interval $\Delta t = t_2 - t_1$.



Consider now a time t_i, intermediate between t_1 and t_2, at which time the particle is at x_i (Fig. 2–11). The slope of the straight line P_1P_i is less than the slope of P_1P_2 in this case. Thus the average velocity during the time interval $t_i - t_1$ is less than during the time interval $t_2 - t_1$.

Now let us imagine that we take the point P_i in Fig. 2–11 to be closer and closer to point P_1. That is, we let the interval $t_i - t_1$, which we now call Δt, become smaller and smaller. The slope of the line connecting the two points becomes closer and closer to the slope of a line tangent to the curve at point P_1. The average velocity (equal to the slope of the chord) thus approaches the slope of the tangent at point P_1. The definition of the instantaneous velocity (Eq. 2–3) is the limiting value of the average velocity as Δt approaches zero. Thus the *instantaneous velocity equals the slope of the tangent to the x vs. t curve* at that point (which we can simply call "the slope of the curve" at that point).

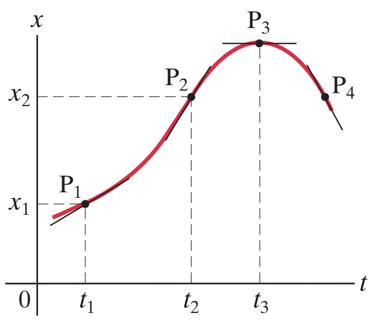
FIGURE 2–11 Same position vs. time curve as in Fig. 2–10, but including an intermediate time t_i. Note that the average velocity over the time interval $t_i - t_1$ (which is the slope of P_1P_i) is less than the average velocity over the time interval $t_2 - t_1$. The slope of the thin line tangent to the curve at point P_1 equals the instantaneous velocity at time t_1.



Because the velocity at any instant equals the slope of the tangent to the x vs. t graph at that instant, we can obtain the velocity at any instant from such a graph. For example, in Fig. 2–12 (which shows the same curve as in Figs. 2–10 and 2–11), the slope continually increases as our object moves from x_1 to x_2, so the velocity is increasing. For times after t_2, however, the slope begins to decrease and in fact reaches zero (so $v = 0$) where x has its maximum value, at point P_3 in Fig. 2–12. Beyond this point, the slope is negative, as for point P_4. The velocity is therefore negative, which makes sense since x is now decreasing—the particle is moving to the left on a standard xy plot, toward decreasing values of x.

If an object moves with constant velocity over a particular time interval, its instantaneous velocity is equal to its average velocity. The graph of x vs. t in this case will be a straight line whose slope equals the velocity. The curve of Fig. 2–10 has no straight sections, so there are no time intervals when the velocity is constant.

FIGURE 2–12 Same x vs. t curve as in Figs. 2–10 and 2–11, but here showing the slope at four different points: At P_3, the slope is zero, so $v = 0$. At P_4 the slope is negative, so $v < 0$.



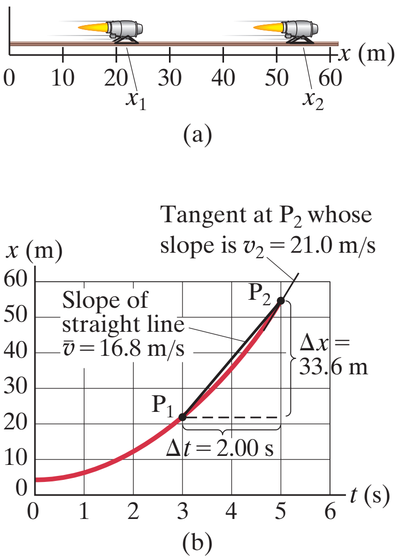
EXERCISE B What is your speed at the instant you turn around to move in the opposite direction? (*a*) Depends on how quickly you turn around; (*b*) always zero; (*c*) always negative; (*d*) none of the above.

The derivatives of various functions are studied in calculus courses, and you can find a review in this book in Appendix B. The derivatives of polynomial functions (which we use a lot) are:

$$\frac{d}{dt}\left(Ct^n\right) = nCt^{n-1} \quad \text{and} \quad \frac{dC}{dt} = 0,$$

where C is any constant.

FIGURE 2–13 Example 2–4. (a) Engine traveling on a straight track. (b) Graph of x vs. t: $x = At^2 + B$.



EXAMPLE 2–4 **Given x as a function of t.** A jet engine moves along an experimental track (which we call the x axis) as shown in Fig. 2–13a. We will treat the engine as if it were a particle. Its position as a function of time is given by the equation $x = At^2 + B$, where $A = 2.10\,\mathrm{m/s^2}$ and $B = 2.80\,\mathrm{m}$, and this equation is plotted in Fig. 2–13b. (*a*) Determine the displacement of the engine during the time interval from $t_1 = 3.00\,\mathrm{s}$ to $t_2 = 5.00\,\mathrm{s}$. (*b*) Determine the average velocity during this time interval. (*c*) Determine the magnitude of the instantaneous velocity at $t = 5.00\,\mathrm{s}$.

APPROACH (*a*) We substitute values for t_1 and t_2 in the given equation for x to obtain x_1 and x_2. (*b*) The average velocity can be found from Eq. 2–2. (*c*) To find the instantaneous velocity, we take the derivative of the given x equation with respect to t using the formulas given above.

SOLUTION (*a*) At $t_1 = 3.00\,\mathrm{s}$, the position (point P_1 in Fig. 2–13b) is

$$x_1 = At_1^2 + B = \left(2.10\,\mathrm{m/s^2}\right)(3.00\,\mathrm{s})^2 + 2.80\,\mathrm{m} = 21.7\,\mathrm{m}.$$

At $t_2 = 5.00\,\mathrm{s}$, the position (P_2 in Fig. 2–13b) is

$$x_2 = \left(2.10\,\mathrm{m/s^2}\right)(5.00\,\mathrm{s})^2 + 2.80\,\mathrm{m} = 55.3\,\mathrm{m}.$$

The displacement is thus

$$x_2 - x_1 = 55.3\,\mathrm{m} - 21.7\,\mathrm{m} = 33.6\,\mathrm{m}.$$

(*b*) The magnitude of the average velocity can then be calculated as

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{33.6\,\mathrm{m}}{2.00\,\mathrm{s}} = 16.8\,\mathrm{m/s}.$$

This equals the slope of the straight line joining points P_1 and P_2 shown in Fig. 2–13b.

(*c*) The instantaneous velocity at $t = t_2 = 5.00\,\mathrm{s}$ equals the slope of the tangent to the curve at point P_2 shown in Fig. 2–13b. We could measure this slope off the graph to obtain v_2. But we can calculate v more precisely for any time t, using the given formula

$$x = At^2 + B,$$

which is the engine's position x as a function of time t. We take the derivative of x with respect to time (see formulas at top of this page):

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At.$$

We are given $A = 2.10\,\mathrm{m/s^2}$, so for $t = t_2 = 5.00\,\mathrm{s}$,

$$v_2 = 2At = 2\left(2.10\,\mathrm{m/s^2}\right)(5.00\,\mathrm{s}) = 21.0\,\mathrm{m/s}.$$

2–4 Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how rapidly the velocity of an object is changing.

Average Acceleration

Average acceleration is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

In symbols, the average acceleration over a time interval $\Delta t = t_2 - t_1$ during which the velocity changes by $\Delta v = v_2 - v_1$ is defined as

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \qquad \textbf{(2–5)}$$

Because velocity is a vector, acceleration is a vector too. But for one-dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis.

EXAMPLE 2–5 **Average acceleration.** A car accelerates along a straight road from rest to 90 km/h in 5.0 s, Fig. 2–14. What is the magnitude of its average acceleration?

APPROACH Average acceleration is the change in velocity divided by the elapsed time, 5.0 s. The car starts from rest, so $v_1 = 0$. The final velocity is $v_2 = 90\,\text{km/h} = 90 \times 10^3\,\text{m}/3600\,\text{s} = 25\,\text{m/s}$.

SOLUTION From Eq. 2–5, the average acceleration is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{25\,\text{m/s} - 0\,\text{m/s}}{5.0\,\text{s}} = 5.0\,\frac{\text{m/s}}{\text{s}}.$$

This is read as "five meters per second per second" and means that, on average, the velocity changed by 5.0 m/s during each second. That is, assuming the acceleration was constant, during the first second the car's velocity increased from zero to 5.0 m/s. During the next second its velocity increased by another 5.0 m/s, reaching a velocity of 10.0 m/s at $t = 2.0$ s, and so on. See Fig. 2–14.

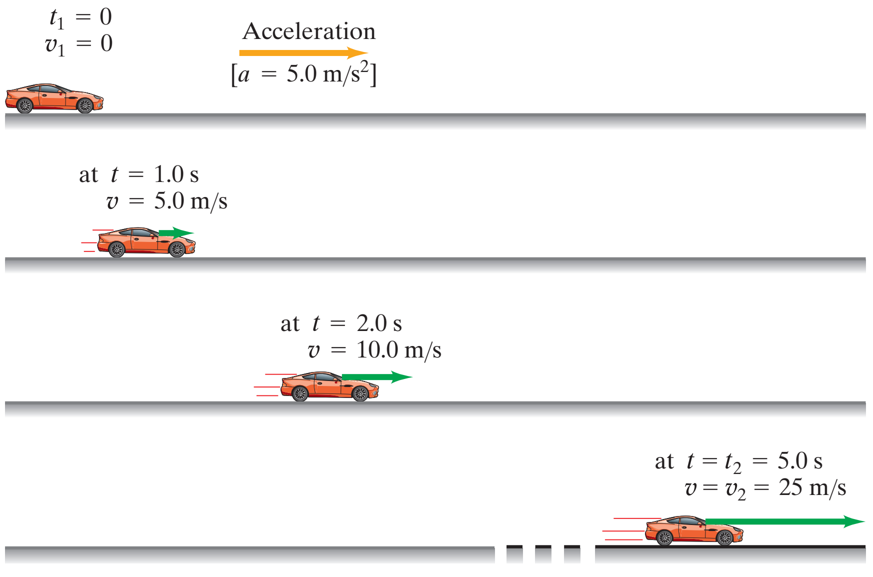


FIGURE 2–14 Example 2–5. The car is shown at the start with $v_1 = 0$ at $t_1 = 0$. The car is shown three more times, at $t = 1.0$ s, $t = 2.0$ s, and at the end of our time interval, $t_2 = 5.0$ s. The green arrows represent the velocity vectors, whose length represents the magnitude of the velocity at that moment and get longer with time. The acceleration vector is the orange arrow, whose magnitude is constant and is found to equal $5.0\,\text{m/s}^2$. Distances are not to scale.

We almost always write the units for acceleration as m/s² (meters per second squared) instead of m/s/s. This is possible because:

$$\frac{\mathrm{m/s}}{\mathrm{s}} = \frac{\mathrm{m}}{\mathrm{s \cdot s}} = \frac{\mathrm{m}}{\mathrm{s^2}}.$$

According to the calculation in Example 2–5, the velocity changed on average by 5.0 m/s during each second, for a total change of 25 m/s over the 5.0 s; the average acceleration was 5.0 m/s².

CAUTION
Distinguish velocity from acceleration

Note that *acceleration* tells us how quickly the *velocity* changes, whereas *velocity* tells us how quickly the *position* changes.

CAUTION
If v or a is zero, is the other zero too?

CONCEPTUAL EXAMPLE 2–6 **Velocity and acceleration.** (*a*) If the velocity of an object is zero, does it mean that the acceleration is zero? (*b*) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

RESPONSE A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (*a*) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but the acceleration is not zero since the velocity of the car changes. (How else could your car start forward if its velocity weren't changing—that is, accelerating?) (*b*) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero: $a = 0$, $v \neq 0$.

EXERCISE C A powerful car is advertised to go from zero to 60 mi/h in 5.4 s. What does this say about the car: (*a*) it is fast (high speed); or (*b*) it accelerates well?

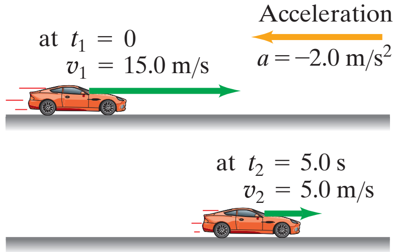


FIGURE 2–15 Example 2–7, showing the position of the car at times t_1 and t_2, as well as the car's velocity represented by the green arrows. We calculate that the acceleration vector (orange) points to the left as the car slows down while moving to the right.

EXAMPLE 2–7 **Car slowing down.** An automobile is moving to the right along a straight highway, which we choose to be the positive *x* axis (Fig. 2–15). Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is $v_1 = 15.0$ m/s, and it takes 5.0 s to slow down to $v_2 = 5.0$ m/s, what was the car's average acceleration?

APPROACH We put the given initial and final velocities, and the elapsed time, into Eq. 2–5 for $\bar{a}$.

SOLUTION In Eq. 2–5, we call the initial time $t_1 = 0$, and set $t_2 = 5.0$ s. (Note that our choice of $t_1 = 0$ doesn't affect the calculation of $\bar{a}$ because only $\Delta t = t_2 - t_1$ appears in Eq. 2–5.) Then

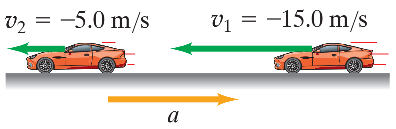
$$\bar{a} = \frac{5.0\,\mathrm{m/s} - 15.0\,\mathrm{m/s}}{5.0\,\mathrm{s}} = -2.0\,\mathrm{m/s^2}.$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left (in the negative *x* direction)—even though the velocity is always pointing to the right. We say that the acceleration is 2.0 m/s² to the left, and it is shown in Fig. 2–15 as an orange arrow.

FIGURE 2–16 The car of Example 2–7, now moving to the *left* and decelerating. The acceleration is

$$a = \frac{v_2 - v_1}{\Delta t}$$

$$a = \frac{(-5.0\,\mathrm{m/s}) - (-15.0\,\mathrm{m/s})}{5.0\,\mathrm{s}} = \frac{-5.0\,\mathrm{m/s} + 15.0\,\mathrm{m/s}}{5.0\,\mathrm{s}} = +2.0\,\mathrm{m/s^2}.$$



"Deceleration"

When an object is slowing down, we sometimes say it is **decelerating**. In physics, the concept of acceleration is all we need: it can be + or −. But if the word "deceleration" is used, be careful: deceleration does *not* mean that the acceleration is necessarily negative, as in Example 2–7. The velocity of an object moving to the right along the positive *x* axis is positive; if the object is slowing down (as in Fig. 2–15), the acceleration *is* negative. But the same car moving to the left (decreasing *x*), and slowing down, has positive acceleration that points to the right, as shown in Fig. 2–16. We have a deceleration whenever the magnitude of the velocity is decreasing; thus the *velocity and acceleration point in opposite directions* when there is deceleration.

EXERCISE D A car moves along the x axis. What is the sign of the car's acceleration if it is moving in the positive x direction with (a) increasing speed or (b) decreasing speed? What is the sign of the acceleration if the car moves in the negative x direction with (c) increasing speed or (d) decreasing speed?

Instantaneous Acceleration

The **instantaneous acceleration**, a, is defined as the *limiting value of the average acceleration as we let* Δt *approach zero*:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}. \qquad \textbf{(2–6)}$$

This limit, dv/dt, is the derivative of v with respect to t. We will use the term "acceleration" to refer to the instantaneous value. If we want to discuss the average acceleration, we will always include the word "average."

If we draw a graph of the velocity, v, vs. time, t, as shown in Fig. 2–17, then the *average* acceleration over a time interval $\Delta t = t_2 - t_1$ is represented by the slope of the straight line connecting the two points P_1 and P_2 in Fig. 2–17. [Compare this to the position vs. time graph of Fig. 2–10 for which the slope of the straight line represents the average velocity.] The *instantaneous* acceleration at any time, say t_1, is the slope of the tangent to the v vs. t curve at time t_1, which is also shown in Fig. 2–17. In Fig. 2–17, as we go from time t_1 to time t_2 the velocity continually increases, but the acceleration (the rate at which the velocity changes) is decreasing since the slope of the curve is decreasing.

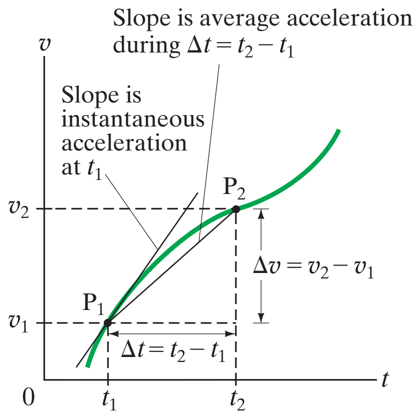


FIGURE 2–17 A graph of velocity v vs. time t. The average acceleration over a time interval $\Delta t = t_2 - t_1$ is the slope of the straight line P_1P_2: $\bar{a} = \Delta v/\Delta t$. The instantaneous acceleration at time t_1 is the slope of the v vs. t curve at that instant.

EXAMPLE 2–8 **Acceleration given $x(t)$.** A particle is moving in a straight line so that its position is given by the relation $x = (2.10\,\text{m/s}^2)t^2 + (2.80\,\text{m})$, as in Example 2–4. Calculate (a) its average acceleration during the time interval from $t_1 = 3.00\,\text{s}$ to $t_2 = 5.00\,\text{s}$, and (b) its instantaneous acceleration as a function of time.

APPROACH To determine acceleration, we first must find the velocity at t_1 and t_2 by differentiating x: $v = dx/dt$. Then we use Eq. 2–5 to find the average acceleration, and Eq. 2–6 to find the instantaneous acceleration.

SOLUTION (a) The velocity at any time t is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[(2.10\,\text{m/s}^2)t^2 + 2.80\,\text{m}\right] = (4.20\,\text{m/s}^2)t,$$

as we already saw in Example 2–4c. Therefore, at time $t_1 = 3.00\,\text{s}$, $v_1 = (4.20\,\text{m/s}^2)(3.00\,\text{s}) = 12.6\,\text{m/s}$ and at $t_2 = 5.00\,\text{s}$, $v_2 = 21.0\,\text{m/s}$. Therefore,

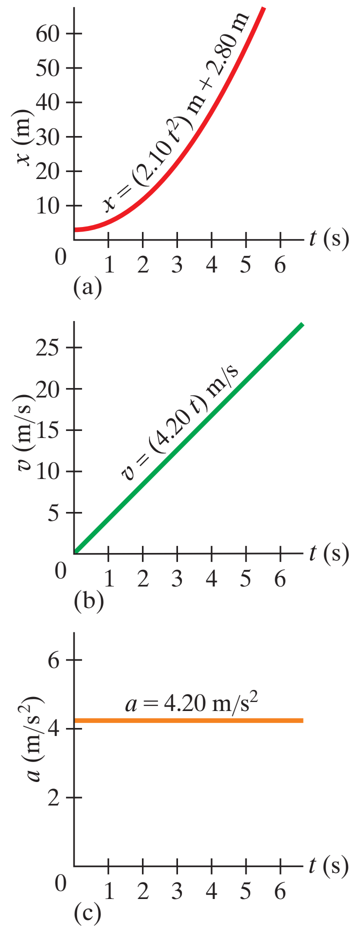
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{21.0\,\text{m/s} - 12.6\,\text{m/s}}{5.00\,\text{s} - 3.00\,\text{s}} = 4.20\,\text{m/s}^2.$$

(b) With $v = (4.20\,\text{m/s}^2)t$, the instantaneous acceleration at any time is

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[(4.20\,\text{m/s}^2)t\right] = 4.20\,\text{m/s}^2.$$

The acceleration in this case is constant; it does not depend on time. Figure 2–18 shows graphs of (a) x vs. t (the same as Fig. 2–13b), (b) v vs. t, which is linearly increasing as calculated above, and (c) a vs. t, which is a horizontal straight line because a = constant.

FIGURE 2–18 Example 2–8. Graphs of (a) x vs. t, (b) v vs. t, and (c) a vs. t for the motion $x = At^2 + B$. Note that v increases linearly with t and that the acceleration a is constant. Also, v is the slope of the x vs. t curve, whereas a is the slope of the v vs. t curve.



Like velocity, acceleration is a rate. The velocity of an object is the rate at which its displacement changes with time; its acceleration, on the other hand, is the rate at which its velocity changes with time. In a sense, acceleration is a "rate of a rate."

This can be expressed in equation form: since $a = dv/dt$ and $v = dx/dt$, then

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

Here d^2x/dt^2 is the *second derivative* of x with respect to time: we first take the derivative of x with respect to time (dx/dt), and then we again take the derivative with respect to time, $(d/dt)(dx/dt)$, to get the acceleration.

EXERCISE E The position of a particle is given by the following equation:

$$x = (2.00\,\text{m/s}^3)t^3 + (2.50\,\text{m/s})t.$$

What is the acceleration of the particle at $t = 2.00\,\text{s}$? Choose one: (*a*) $13.0\,\text{m/s}^2$; (*b*) $22.5\,\text{m/s}^2$; (*c*) $24.0\,\text{m/s}^2$; (*d*) $2.00\,\text{m/s}^2$; (*e*) $21.0\,\text{m/s}^2$.

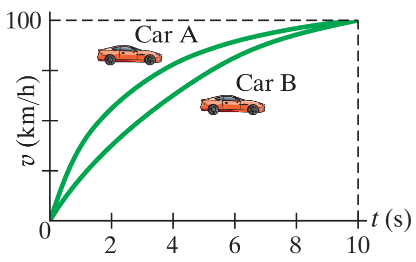


FIGURE 2–19 Example 2–9.

CONCEPTUAL EXAMPLE 2–9 **Analyzing with graphs.** Figure 2–19 shows the velocity as a function of time for two cars accelerating from 0 to 100 km/h in a time of 10.0 s. Compare for the two cars: (*a*) the average acceleration; (*b*) the instantaneous acceleration; and (*c*) the total distance traveled.

RESPONSE (*a*) Average acceleration is $\Delta v/\Delta t$. Both cars have the same Δv (100 km/h) and the same Δt (10.0 s), so the average acceleration is the same for both cars. (*b*) Instantaneous acceleration is the slope of the tangent to the v vs. t curve. For about the first 4 s, the top curve is steeper than the bottom curve, so car A has a greater instantaneous acceleration during this interval. The bottom curve is steeper during the last 6 s, so car B has the larger acceleration during this period. (*c*) Except at $t = 0$ and $t = 10.0\,\text{s}$, car A is always going faster than car B. Since it is going faster, it will go farther in the same time. Notice what marvelous information we can get from a graph.

2–5 Motion at Constant Acceleration

We now examine motion in a straight line when the magnitude of the acceleration is constant. In this case, the instantaneous and average acceleration are equal. We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate x, v, a, and t when a is constant, allowing us to determine any one of these variables if we know the others.

Notation in physics varies from book to book; and different instructors use different notation. We are now going to change our notation, to simplify it a bit for our discussion here of motion at **constant acceleration**. First we choose the initial time in any discussion to be zero, and we call it t_0. That is, $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at t_0.) We can then let $t_2 = t$ be the elapsed time. The initial position (x_1) and the initial velocity (v_1) of an object will now be represented by x_0 and v_0, since they represent x and v at $t = 0$. At time t the position and velocity will be called x and v (rather than x_2 and v_2). The average velocity during the time interval $t - t_0$ will be (Eq. 2–2)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose $t_0 = 0$. The acceleration, assumed constant in time, is $a = \Delta v/\Delta t$ (Eq. 2–5), so

$$a = \frac{v - v_0}{t}.$$

A common problem is to determine the velocity of an object after any elapsed time t, when we are given the object's constant acceleration. We can solve such problems by solving for v in the last equation: first we multiply both sides by t, which gives $at = v - v_0$, and then

$$v = v_0 + at. \qquad \text{[constant acceleration]} \quad \textbf{(2–7)}$$

If an object, such as a motorcycle, starts from rest $(v_0 = 0)$ and accelerates at $4.0\,\text{m/s}^2$, then after an elapsed time $t = 6.0\,\text{s}$ its velocity will be $v = 0 + at = (4.0\,\text{m/s}^2)(6.0\,\text{s}) = 24\,\text{m/s}$.

Next, let us see how to calculate the position x of an object after a time t when it undergoes constant acceleration. The definition of average velocity (Eq. 2–2) is $\bar{v} = (x - x_0)/t$, which we can rewrite by multiplying both sides by t:

$$x = x_0 + \bar{v}t. \qquad \textbf{(2–8)}$$

Because the velocity increases at a uniform rate, the average velocity, $\bar{v}$, will be midway between the initial and final velocities:

$$\bar{v} = \frac{v_0 + v}{2}. \qquad \text{[constant acceleration]} \quad \textbf{(2–9)}$$

CAUTION
Average velocity, but only if a = constant

(Careful: Equation 2–9 is not necessarily valid if the acceleration is not constant.) We combine the last two Equations with Eq. 2–7 and find, starting with Eq. 2–8,

$$\begin{aligned} x &= x_0 + \bar{v}t \\ &= x_0 + \left(\frac{v_0 + v}{2}\right)t \\ &= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t \end{aligned}$$

or

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2. \qquad \text{[constant acceleration]} \quad \textbf{(2–10)}$$

Equations 2–7, 2–9, and 2–10 are three of the four most useful equations for motion at constant acceleration. We now derive the fourth equation, which is useful in situations where the time t is not known. We substitute Eq. 2–9 into Eq. 2–8:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v + v_0}{2}\right)t.$$

Next we solve Eq. 2–7 for t, obtaining

$$t = \frac{v - v_0}{a},$$

and substituting this into the previous equation we have

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

We solve this for v^2 and obtain

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \text{[constant acceleration]} \quad \textbf{(2–11)}$$

which is the other useful equation we sought.

We now have four equations relating position, velocity, acceleration, and time, when the acceleration a is constant. We collect these *kinematic equations for constant acceleration* here in one place for further reference (the tan background is used to emphasize their importance):

$$v = v_0 + at \qquad [a = \text{constant}] \quad \textbf{(2–12a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \quad \textbf{(2–12b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \quad \textbf{(2–12c)}$$

$$\bar{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \quad \textbf{(2–12d)}$$

Kinematic equations for constant acceleration (we'll use them a lot)

These useful equations are not valid unless a is a constant. In many cases we can set $x_0 = 0$, and this simplifies the above equations a bit. Note that x represents position (not distance), and that $x - x_0$ is the displacement, whereas t is the elapsed time.

Equations 2–12 are useful also when a is approximately constant, in order to obtain reasonable estimates.

PHYSICS APPLIED
Airport design

EXAMPLE 2–10 **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at $2.00\ \text{m/s}^2$. (a) If the runway is 150 m long, can this airplane reach the required speed for takeoff? (b) If not, what minimum length must the runway have?

APPROACH Assuming the plane's acceleration is constant, we use the kinematic equations for constant acceleration. In (a), we want to find v, and we are given:

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\ \text{m}$	
$a = 2.00\ \text{m/s}^2$	

SOLUTION (a) Of the four kinematic equations on page 53, Eq. 2–12c will give us v when we know v_0, a, x, and x_0:

PROBLEM SOLVING
Equations 2–12 are valid only when the acceleration is constant, which we assume in this Example

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= 0 + 2(2.00\ \text{m/s}^2)(150\ \text{m}) = 600\ \text{m}^2/\text{s}^2 \\ v &= \sqrt{600\ \text{m}^2/\text{s}^2} = 24.5\ \text{m/s}. \end{aligned}$$

This runway length is *not* sufficient, because the minimum speed is not reached.
(b) Now we want to find the minimum runway length, $x - x_0$, for a plane to reach $v = 27.8\ \text{m/s}$, given $a = 2.00\ \text{m/s}^2$. We again use Eq. 2–12c, but rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\ \text{m/s})^2 - 0}{2(2.00\ \text{m/s}^2)} = 193\ \text{m}.$$

A 200-m runway is more appropriate for this plane.

NOTE We did this Example as if the plane were a particle, so we round off our answer to 200 m.

PHYSICS APPLIED
Car safety—air bags

EXAMPLE 2–11 **ESTIMATE** **Air bags.** Suppose you want to design an air bag system that can protect the driver at a speed of 100 km/h (60 mph) if the car hits a brick wall. Estimate how fast the air bag must inflate (Fig. 2–20) to effectively protect the driver. How does the use of a seat belt help the driver?

FIGURE 2–20 Example 2–11. An air bag deploying on impact.

APPROACH We assume the acceleration is roughly constant, so we can use Eqs. 2–12. Both Eqs. 2–12a and 2–12b contain t, our desired unknown. They both contain a, so we must first find a, which we can do using Eq. 2–12c if we know the distance x over which the car crumples. A rough estimate might be about 1 meter. We choose the time interval to start at the instant of impact with the car moving at $v_0 = 100\ \text{km/h}$, and to end when the car comes to rest ($v = 0$) after traveling 1 m.

SOLUTION We convert the given initial speed to SI units: $100\ \text{km/h} = 100 \times 10^3\ \text{m}/3600\ \text{s} = 28\ \text{m/s}$. We then find the acceleration from Eq. 2–12c:

$$a = -\frac{v_0^2}{2x} = -\frac{(28\ \text{m/s})^2}{2.0\ \text{m}} = -390\ \text{m/s}^2.$$

This enormous acceleration takes place in a time given by (Eq. 2–12a):

$$t = \frac{v - v_0}{a} = \frac{0 - 28\ \text{m/s}}{-390\ \text{m/s}^2} = 0.07\ \text{s}.$$

To be effective, the air bag would need to inflate faster than this.

What does the air bag do? It spreads the force over a large area of the chest (to avoid puncture of the chest by the steering wheel). The seat belt keeps the person in a stable position directly in front of the expanding air bag.

EXERCISE F A car starts from rest and accelerates at a constant $10\,\text{m/s}^2$ during a $\frac{1}{4}$-mile (402 m) race. How fast is the car going at the finish line? (*a*) 8040 m/s; (*b*) 90 m/s; (*c*) 81 m/s; (*d*) 804 m/s.

2–6 Solving Problems

Before doing more worked-out Examples, let us look at how to approach problem solving. First, it is important to note that physics is *not* a collection of equations to be memorized. Simply searching for an equation that might work can lead you to a wrong result and will not help you understand physics (Fig. 2–21). A better approach is to use the following (rough) procedure, which we present as a special "Problem Solving Strategy." (Other such Problem Solving Strategies will be found throughout the book.)

FIGURE 2–21 Read each Chapter of this book, study it by reading it again carefully, and work the Problems using your reasoning abilities.

PROBLEM SOLVING

1. Read and **reread** the whole problem carefully before trying to solve it.
2. Decide what **object** (or objects) you are going to study, and for what **time interval**. You can often choose the initial time to be $t = 0$.
3. **Draw** a **diagram** or picture of the situation, with coordinate axes wherever applicable. [You can place the origin of coordinates and the axes wherever you like to make your calculations easier. You also choose which direction is positive and which is negative. Usually we choose the x axis to the right as positive.]
4. Write down what quantities are "**known**" or "given," and then what you *want* to know ("unknowns"). Consider quantities both at the beginning and at the end of the chosen time interval. You may need to "translate" language into physical terms, such as "starts from rest" means $v_0 = 0$.
5. Think about which **principles of physics** apply in this problem. Use common sense and your own experiences. Then plan an approach.
6. Consider which **equations** (and/or definitions) relate the quantities involved. Before using them, be sure their **range of validity** includes your problem (for example, Eqs. 2–12 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, **solve** the equation algebraically for the unknown. Sometimes several sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.
7. Carry out the **calculation** if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answer(s) to the correct number of significant figures (Section 1–3).
8. Think carefully about the result you obtain: Is it **reasonable**? Does it make sense according to your own intuition and experience? A good check is to do a rough **estimate** using only powers of 10, as discussed in Section 1–6. Often it is preferable to do a rough estimate at the *start* of a numerical problem because it can help you focus your attention on finding a path toward a solution.
9. A very important aspect of doing problems is keeping track of **units**. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has been made. This can serve as a **check** on your solution (but it only tells you if you're wrong, not if you're right). Always use a consistent set of units.

PROBLEM SOLVING
"Starting from rest" means $v = 0$ at $t = 0$ [i.e., $v_0 = 0$]

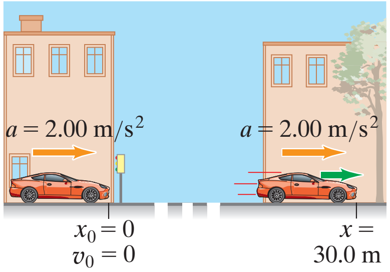


FIGURE 2–22 Example 2–12. (Not to scale.)

Known	Wanted
$x_0 = 0$	t
$x = 30.0\ \mathrm{m}$	
$a = 2.00\ \mathrm{m/s^2}$	
$v_0 = 0$	

EXAMPLE 2–12 **Acceleration of a car.** How long does it take a 4.0-m-long car to cross a 26.0-m-wide intersection after the light turns green, if the car accelerates from rest at a constant $2.00\ \mathrm{m/s^2}$? The car has to travel $26.0\ \mathrm{m} + 4.0\ \mathrm{m} = 30.0\ \mathrm{m}$ to clear the intersection.

APPROACH We follow the Problem Solving Strategy on the previous page.

SOLUTION

1. **Reread** the problem. Be sure you understand what it asks for (here, a time interval: "how long does it take").
2. The **object** under study is the car. We need to choose the **time interval** during which we look at the car's motion: we choose $t = 0$, the initial time, to be the moment the car starts to accelerate from rest $(v_0 = 0)$; the time t is the instant the car has traveled the full 30.0 m of the intersection.
3. **Draw a diagram**: the situation is shown in Fig. 2–22 where the car is shown moving along the positive x axis. We choose $x_0 = 0$ at the front bumper of the car before it starts to move.
4. The "**knowns**" and the "wanted" information are shown in the Table in the margin. Note that "starting from rest" means $v = 0$ at $t = 0$; that is, $v_0 = 0$. The wanted time t is how long it takes the car to travel 30.0 m.
5. The **physics**: the car, starting from rest $(\text{at } t_0 = 0)$, increases in speed as it covers more distance. The acceleration is constant, so we can use the kinematic equations, Eqs. 2–12.
6. **Equations**: we want to find the time, given the distance and acceleration; Eq. 2–12b $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$ is perfect since the only unknown quantity is t. Setting $v_0 = 0$ and $x_0 = 0$ in Eq. 2–12b, we have
$$x = \tfrac{1}{2}at^2.$$
We solve for t
$$t = \sqrt{\frac{2x}{a}}.$$
7. The **calculation**:
$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\ \mathrm{m})}{2.00\ \mathrm{m/s^2}}} = 5.48\ \mathrm{s}.$$
This is our answer. Note that the units come out correctly.
8. We can check the **reasonableness** of the answer by doing an alternate calculation: we use our result in step 7 and check if the distance traveled turns out to be 30.0 m. First we find the final velocity (Eq. 2–12a),
$$v = at = (2.00\ \mathrm{m/s^2})(5.48\ \mathrm{s}) = 10.96\ \mathrm{m/s},$$
and then find the distance traveled using the definition of average velocity (see Eq. 2–8).
$$x = x_0 + \bar{v}t = 0 + \tfrac{1}{2}(10.96\ \mathrm{m/s} + 0)(5.48\ \mathrm{s}) = 30.0\ \mathrm{m},$$
which checks with our given distance.
9. We checked the **units** in step 7, and they came out correctly (seconds).

PROBLEM SOLVING
Check your answer

NOTE In steps 6 and 7, when we took the square root, we should have written $t = \pm\sqrt{2x/a} = \pm 5.48\ \mathrm{s}$. Mathematically there are two solutions. But the second solution, $t = -5.48\ \mathrm{s}$, is a time *before* our chosen time interval and makes no sense physically. We say it is "unphysical" and ignore it.

PROBLEM SOLVING
"Unphysical" solutions

We explicitly followed the steps of the Problem Solving Strategy in Example 2–12. In upcoming Examples, we will use our usual "Approach" and "Solution" to avoid being wordy.

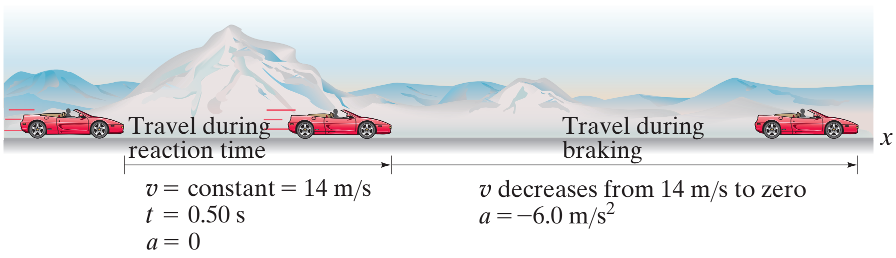


FIGURE 2–23 Example 2–13: stopping distance for a braking car.

EXAMPLE 2–13 **ESTIMATE** **Braking distances.** Estimate the minimum stopping distance for a car, which is important for traffic safety and traffic design. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time" during which the speed is constant, so $a = 0$. (2) The second time interval is the actual braking period when the vehicle slows down ($a \neq 0$) and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the deceleration of the car. For a dry road and good tires, good brakes can decelerate a car at a rate of about $5\,\mathrm{m/s^2}$ to $8\,\mathrm{m/s^2}$. Calculate the total stopping distance for an initial velocity of 50 km/h $(= 14\,\mathrm{m/s} \approx 31\,\mathrm{mi/h})$ and assume the acceleration of the car is $-6.0\,\mathrm{m/s^2}$ (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing). Reaction time for typical drivers varies from perhaps 0.3 s to about 1.0 s; take it to be 0.50 s.

PHYSICS APPLIED
Car stopping distances

APPROACH During the "reaction time," part (1), the car moves at constant speed of 14 m/s, so $a = 0$. Once the brakes are applied, part (2), the acceleration is $a = -6.0\,\mathrm{m/s^2}$ and is constant over this time interval. For both parts a is constant, so we can use Eqs. 2–12.

SOLUTION Part (1). We take $x_0 = 0$ for the first time interval, when the driver is reacting (0.50 s): the car travels at a constant speed of 14 m/s so $a = 0$. See Fig. 2–23 and the Table in the margin. To find x, the position of the car at $t = 0.50\,\mathrm{s}$ (when the brakes are first applied), we cannot use Eq. 2–12c because x is multiplied by a, which is zero. But Eq. 2–12b works:

$$x = v_0 t + 0 = (14\,\mathrm{m/s})(0.50\,\mathrm{s}) = 7.0\,\mathrm{m}.$$

Part 1: Reaction time

Known	Wanted
$t = 0.50\,\mathrm{s}$	x
$v_0 = 14\,\mathrm{m/s}$	
$v = 14\,\mathrm{m/s}$	
$a = 0$	
$x_0 = 0$	

Thus the car travels 7.0 m during the driver's reaction time, until the instant the brakes are applied. We will use this result as input to part (2).
Part (2). During the second time interval, the brakes are applied and the car is brought to rest. The initial position is $x_0 = 7.0\,\mathrm{m}$ (result of part (1)), and other variables are shown in the second Table in the margin. Equation 2–12a doesn't contain x; Eq. 2–12b contains x but also the unknown t. Equation 2–12c, $v^2 - v_0^2 = 2a(x - x_0)$, is what we want; after setting $x_0 = 7.0\,\mathrm{m}$, we solve for x, the final position of the car (where it stops):

Part 2: Braking

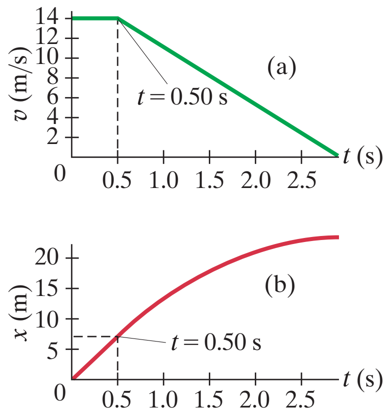
Known	Wanted
$x_0 = 7.0\,\mathrm{m}$	x
$v_0 = 14\,\mathrm{m/s}$	
$v = 0$	
$a = -6.0\,\mathrm{m/s^2}$	

$$\begin{aligned} x &= x_0 + \frac{v^2 - v_0^2}{2a} \\ &= 7.0\,\mathrm{m} + \frac{0 - (14\,\mathrm{m/s})^2}{2(-6.0\,\mathrm{m/s^2})} = 7.0\,\mathrm{m} + \frac{-196\,\mathrm{m^2/s^2}}{-12\,\mathrm{m/s^2}} \\ &= 7.0\,\mathrm{m} + 16\,\mathrm{m} = 23\,\mathrm{m}. \end{aligned}$$

The car traveled 7.0 m while the driver was reacting and another 16 m during the braking period before coming to a stop, for a total distance traveled of 23 m. Figure 2–24 shows graphs of (a) v vs. t (we were given that v is constant from $t = 0$ until $t = 0.50\,\mathrm{s}$, and after $t = 0.50\,\mathrm{s}$ it decreases linearly to zero), and (b) x vs. t.

NOTE From the equation above for x, we see that the stopping distance after the driver hit the brakes $(= x - x_0)$ increases with the *square* of the initial speed, not just linearly with speed. If you are traveling twice as fast, it takes *four* times the distance to stop (when braking at the same rate, a).

FIGURE 2–24 Example 2–13. Graphs of (*a*) v vs. t and (*b*) x vs. t.



EXAMPLE 2–14 ESTIMATE Two Moving Objects: Police and Speeder. A car speeding at 150 km/h (over 90 mph) passes a still police car which immediately takes off in hot pursuit. Using simple assumptions, such as that the speeder continues at constant speed, estimate how long it takes the police car to overtake the speeder. Then estimate the police car's speed at that moment and decide if the assumptions were reasonable.

PROBLEM SOLVING
Guess the acceleration

APPROACH When the police car takes off, it accelerates, and the simplest assumption is that its acceleration is constant. This may not be reasonable, but let's see what happens. We can estimate the acceleration if we have noticed automobile ads, which claim cars can accelerate from rest to 100 km/h in 5.0 s. So the average acceleration of the police car could be approximately

$$a_P = \frac{100\,\text{km/h}}{5.0\,\text{s}} = 20\,\frac{\text{km/h}}{\text{s}}\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 5.6\,\text{m/s}^2.$$

SOLUTION We need to set up the kinematic equations to determine the unknown quantities, and since there are two moving objects, we need two separate sets of equations. We denote the speeding car's position by x_S and the police car's position by x_P. Because we are interested in solving for the time when the two vehicles arrive at the same position on the road, we use Eq. 2–12b for each car ($x_0 = 0$ for both cars):

$$x_S = v_{0S}t + \tfrac{1}{2}a_S t^2 = (150\,\text{km/h})t + 0 = (42\,\text{m/s})t$$
$$x_P = v_{0P}t + \tfrac{1}{2}a_P t^2 = 0 + \tfrac{1}{2}(5.6\,\text{m/s}^2)t^2,$$

where we have set $v_{0P} = 0$ and $a_S = 0$ (speeder assumed to move at constant speed). We want the time when the cars meet, so we set $x_S = x_P$ and solve for t:

$$(42\,\text{m/s})t = (2.8\,\text{m/s}^2)t^2.$$

The solutions are

$$t = 0 \quad \text{and} \quad t = \frac{42\,\text{m/s}}{2.8\,\text{m/s}^2} = 15\,\text{s}.$$

The first solution corresponds to the instant the speeder passed the police car. The second solution tells us when the police car catches up to the speeder, 15 s later. This is our answer, but is it reasonable? The police car's speed at $t = 15$ s is

$$v_P = v_{0P} + a_P t = 0 + (5.6\,\text{m/s}^2)(15\,\text{s}) = 84\,\text{m/s}$$

or 300 km/h ($\approx$ 190 mi/h). Not reasonable, and highly dangerous.

CAUTION
Initial assumptions need to be checked out for reasonableness

NOTE More reasonable is to give up the assumption of constant acceleration. The police car surely cannot maintain constant acceleration at high speed. Also, the speeder, if a reasonable person, would slow down upon hearing the police siren. Figure 2–25 shows (a) x vs. t and (b) v vs. t graphs, based on the original assumption of a_P = constant, whereas (c) shows v vs. t for more reasonable assumptions.

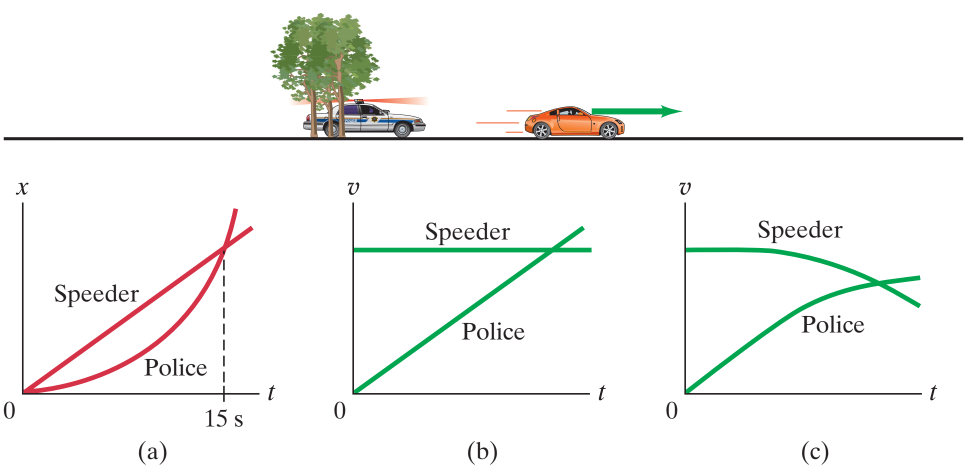


FIGURE 2–25 Example 2–14.

FIGURE 2–26 Painting of Galileo demonstrating to the Grand Duke of Tuscany his argument for the action of gravity being uniform acceleration. He used a wooden inclined plane (center) to slow down the action. A ball rolling down the plane still accelerates. Tiny bells placed at equal distances along the inclined plane would ring at shorter time intervals as the ball "fell," indicating that the speed was increasing.

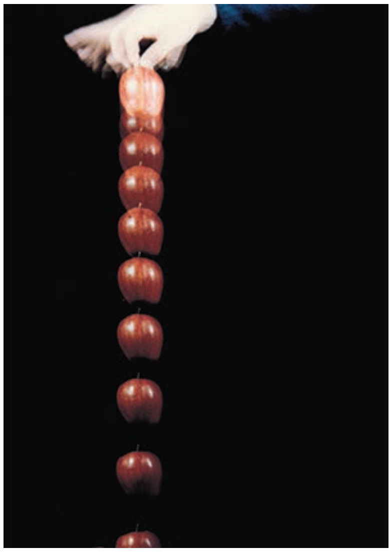


FIGURE 2–27 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

2–7 Freely Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed before the time of Galileo (Fig. 2–26), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is. Wrong. *The speed of a falling object is* not *proportional to its mass*.

Galileo made use of his new technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that *all objects would fall with the same constant acceleration in the absence of air or other resistance*. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 2–27); that is, $d \propto t^2$. We can see this from Eq. 2–12b for constant acceleration; but Galileo was the first to deduce this mathematical relation.

To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than if the same stone is dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

Galileo claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper flat and horizontal in one hand, and a heavier object like a baseball in the other, and release them at the same time as in Fig. 2–28a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad, you will find (see Fig. 2–28b) that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many ordinary circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 2–29). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the *content* of his science: astronomical discoveries, inertia, free fall; but also for his new methods of *doing* science: idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions.

FIGURE 2–28 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.

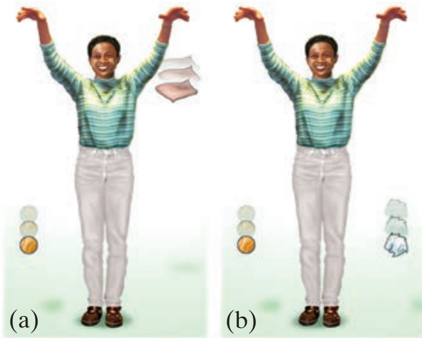
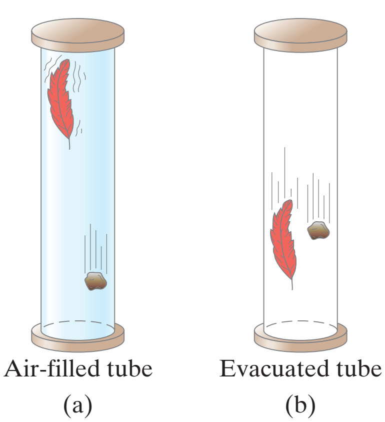


FIGURE 2–29 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.



Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

We call this acceleration the **acceleration due to gravity** at the surface of the Earth, and we give it the symbol g. Its magnitude is approximately

$$g = 9.80\,\text{m/s}^2. \qquad \begin{bmatrix}\text{acceleration due to gravity}\\ \text{at surface of Earth}\end{bmatrix}$$

In British units g is about $32\,\text{ft/s}^2$. Actually, g varies slightly according to latitude and elevation above sea level on the Earth's surface, but these variations are so small that we will ignore them for most purposes. (Acceleration of gravity out in space beyond the Earth's surface is treated in Chapter 6.) Air resistance acts to reduce the speed of a falling object, but this effect is often small, and we will neglect it for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.†

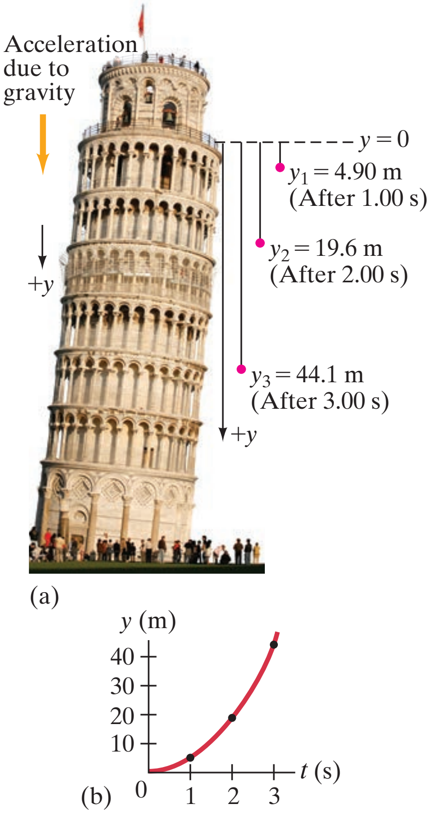
Acceleration due to gravity is a vector, as is any acceleration, and its direction is downward toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 2–12, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

PROBLEM SOLVING
You can choose y to be positive either up or down

EXERCISE G Return to the Chapter-Opening Question, page 42, and answer it again now, assuming minimal air resistance. Try to explain why you may have answered differently the first time.

FIGURE 2–30 Example 2–15. (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 2–27.) (b) Graph of y vs. t (y is + downward).



EXAMPLE 2–15 **Falling from a tower.** Suppose that a ball is dropped ($v_0 = 0$) from a tower. How far will it have fallen after a time $t_1 = 1.00\,\text{s}$, $t_2 = 2.00\,\text{s}$, and $t_3 = 3.00\,\text{s}$? Ignore air resistance.

APPROACH Let us take y as positive downward, so the acceleration is $a = g = +9.80\,\text{m/s}^2$. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position y of the ball after three different time intervals. Equation 2–12b, with x replaced by y, relates the given quantities (t, a, and v_0) to the unknown y.

SOLUTION We set $t = t_1 = 1.00\,\text{s}$ in Eq. 2–12b:

$$\begin{aligned} y_1 &= v_0 t_1 + \tfrac{1}{2}at_1^2 \\ &= 0 + \tfrac{1}{2}at_1^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 4.90\,\text{m}. \end{aligned}$$

The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\,\text{s}$. Similarly, after 2.00 s $(= t_2)$, the ball's position is

$$y_2 = \tfrac{1}{2}at_2^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 19.6\,\text{m}.$$

Finally, after 3.00 s $(= t_3)$, the ball's position is (see Fig. 2–30a)

$$y_3 = \tfrac{1}{2}at_3^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(3.00\,\text{s})^2 = 44.1\,\text{m}.$$

NOTE Whenever we say "dropped," it means $v_0 = 0$. Note also the graph of y vs. t (Fig. 2–30b): the curve is not straight but bends upward because y is proportional to t^2.

NOTE Because of air resistance, all of the distances in Fig. 2–30, y_1, y_2, y_3, would be smaller than shown (and as just calculated); but the difference will be small for a reasonably heavy (but small) object.

†The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity** due to air resistance. (Section 5–6 deals directly with air resistance.)

EXAMPLE 2–16 **Thrown down from a tower.** Suppose the ball in Example 2–15 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

APPROACH Again we use Eq. 2–12b, but now v_0 is not zero, it is $v_0 = 3.00\,\mathrm{m/s}$.

SOLUTION (*a*) At $t_1 = 1.00\,\mathrm{s}$, the position of the ball as given by Eq. 2–12b is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\mathrm{m/s})(1.00\,\mathrm{s}) + \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s})^2 = 7.90\,\mathrm{m}.$$

At $t_2 = 2.00\,\mathrm{s}$ (time interval $t = 0$ to $t = 2.00\,\mathrm{s}$), the position is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\mathrm{m/s})(2.00\,\mathrm{s}) + \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s})^2 = 25.6\,\mathrm{m}.$$

As expected, the ball falls farther each second than when it is dropped with $v_0 = 0$, Example 2–15.

(*b*) The velocity is obtained from Eq. 2–12a:

$$\begin{aligned} v &= v_0 + at \\ &= 3.00\,\mathrm{m/s} + (9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s}) = 12.8\,\mathrm{m/s} \quad [\text{at } t_1 = 1.00\,\mathrm{s}] \\ &= 3.00\,\mathrm{m/s} + (9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s}) = 22.6\,\mathrm{m/s}. \quad [\text{at } t_2 = 2.00\,\mathrm{s}] \end{aligned}$$

In Example 2–15, when the ball was dropped $(v_0 = 0)$, the first term (v_0) in these equations was zero, so

$$\begin{aligned} v &= 0 + at \\ &= (9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s}) = 9.80\,\mathrm{m/s} \quad [\text{at } t_1 = 1.00\,\mathrm{s}] \\ &= (9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s}) = 19.6\,\mathrm{m/s}. \quad [\text{at } t_2 = 2.00\,\mathrm{s}] \end{aligned}$$

NOTE For both Examples 2–15 and 2–16 the speed increases linearly in time by 9.80 m/s during each second. But the speed of the downwardly thrown ball at any instant is always 3.00 m/s (its initial speed) greater than that of a dropped ball.

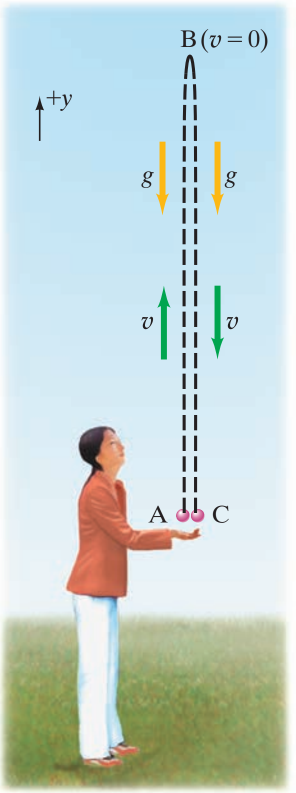


FIGURE 2–31 An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–17, 2–18, 2–19, 2–20, and 2–21.

EXAMPLE 2–17 **Ball thrown upward.** A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate how high it goes. Ignore air resistance.

APPROACH We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 2–31) at point A. Let us choose y to be positive in the upward direction and negative in the downward direction. (This is a different convention from that used in Examples 2–15 and 2–16, and so illustrates our options.) The acceleration due to gravity is downward and so will have a negative sign, $a = -g = -9.80\,\mathrm{m/s^2}$. As the ball rises, its speed decreases until it reaches the highest point (B in Fig. 2–31), where its speed is zero for an instant; then it descends, with increasing speed.

SOLUTION We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 2–31) we set $y_0 = 0$, $v_0 = 15.0\,\mathrm{m/s}$, and $a = -9.80\,\mathrm{m/s^2}$. At time t (maximum height), $v = 0$, $a = -9.80\,\mathrm{m/s^2}$, and we wish to find y. We use Eq. 2–12c, replacing x with y: $v^2 = v_0^2 + 2ay$. We solve this equation for y:

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 11.5\,\mathrm{m}.$$

The ball reaches a height of 11.5 m above the hand.

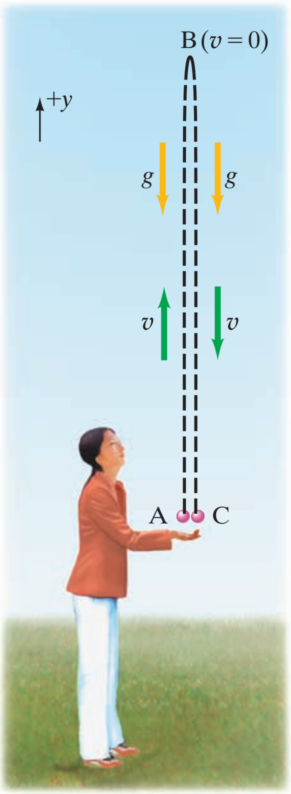


FIGURE 2–31 (Repeated.) An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–17, 2–18, 2–19, 2–20, and 2–21.

EXAMPLE 2–18 **Ball thrown upward, II.** In Example 2–17, Fig. 2–31 (shown here again), how long is the ball in the air before it comes back to the hand?

APPROACH We need to choose a time interval to calculate how long the ball is in the air before it returns to the hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 2–31) in one step and use Eq. 2–12b. We can do this because y is position (or displacement from the origin), and not the total distance traveled. Thus, at both points A and C we have $y = 0$.

SOLUTION We use Eq. 2–12b with $a = -9.80\,\text{m/s}^2$ and find

$$\begin{aligned} y &= y_0 + v_0 t + \tfrac{1}{2}at^2 \\ 0 &= 0 + (15.0\,\text{m/s})t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2. \end{aligned}$$

This equation can be factored (we factor out one t):

$$(15.0\,\text{m/s} - 4.90\,\text{m/s}^2\,t)\,t = 0.$$

There are two solutions:

$$t = 0 \quad \text{and} \quad t = \frac{15.0\,\text{m/s}}{4.90\,\text{m/s}^2} = 3.06\,\text{s}.$$

The first solution $(t = 0)$ corresponds to the initial point (A) in Fig. 2–31, when the ball was first thrown from $y_0 = 0$. The second solution, $t = 3.06\,\text{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

NOTE We have ignored air resistance in these last two Examples, which could be significant, so our result is only an approximation to a real, practical situation.

We did not consider the throwing action in these Examples. Why? Because during the throw, the thrower's hand is touching the ball and accelerating the ball at a rate unknown to us—the acceleration is *not* g. We consider only the time when the ball is in the air and the acceleration is equal to g.

CAUTION

Quadratic equations have two solutions. Sometimes only one corresponds to reality, sometimes both

Every quadratic equation (where the variable is squared) mathematically produces two solutions. In physics, sometimes only one solution corresponds to the real situation, as we saw in Example 2–12, in which case we ignore the "unphysical" solution. But here in Example 2–18, both solutions to our equation in t^2 are physically meaningful: $t = 0$ and $t = 3.06\,\text{s}$.

CONCEPTUAL EXAMPLE 2–19 **Two possible misconceptions.** Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 2–31).

CAUTION

(1) Velocity and acceleration are not always in the same direction; the acceleration (of gravity) always points down

(2) $a \neq 0$ even at the highest point of a trajectory

RESPONSE Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Fig. 2–31 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 2–31), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc points upward, then becomes zero for an instant (zero time) at the highest point, and then points downward. Gravity does not stop acting, so $a = -g = -9.80\,\text{m/s}^2$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration (= rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. Remember: the acceleration of gravity always points down toward the center of the Earth, even when the object is moving up.

EXAMPLE 2–20 **Ball thrown upward, III.** Let us consider again the ball thrown upward of Examples 2–17 and 2–18, and make more calculations. Calculate (*a*) how much time it takes for the ball to reach the maximum height (point B in Fig. 2–31), and (*b*) the velocity of the ball when it returns to the thrower's hand (point C).

APPROACH Again we assume the acceleration is constant, so we can use Eqs. 2–12. We have the maximum height of 11.5 m and initial speed of 15.0 m/s from Example 2–17. Again we take y as positive upward.

SOLUTION (*a*) We consider the time interval between the throw $(t_0 = 0,\ v_0 = 15.0\,\mathrm{m/s})$ and the top of the path $(y = +11.5\,\mathrm{m},\ v = 0)$, and we want to find t. The acceleration is constant at $a = -g = -9.80\,\mathrm{m/s^2}$. Both Eqs. 2–12a and 2–12b contain the time t with other quantities known. Let us use Eq. 2–12a with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, and $v = 0$:

$$v = v_0 + at;$$

setting $v = 0$ gives $0 = v_0 + at$, which we rearrange to solve for t: $at = -v_0$ or

$$t = -\frac{v_0}{a} = -\frac{15.0\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 1.53\,\mathrm{s}.$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in Example 2–18]. Thus it takes the *same time* to reach the maximum height as to fall back to the starting point. We might have guessed this from the *symmetry* of the motion. But be careful. When air resistance cannot be neglected, the symmetry is no longer perfect.

(*b*) To find the ball's velocity when it returns to the hand (point C), we consider the time interval from the throw $(t_0 = 0,\ v_0 = 15.0\,\mathrm{m/s})$ until the ball's return to the hand, which occurs at $t = 3.06\,\mathrm{s}$ (as calculated in Example 2–18). We use Eq. 2–12a again to find v when $t = 3.06\,\mathrm{s}$:

$$\begin{aligned} v &= v_0 + at \\ &= 15.0\,\mathrm{m/s} - \left(9.80\,\mathrm{m/s^2}\right)(3.06\,\mathrm{s}) = -15.0\,\mathrm{m/s}. \end{aligned}$$

NOTE The ball has the same speed (magnitude of velocity) when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). And, as we saw in part (*a*), the time is the same up as down. Thus the motion is *symmetrical* about the maximum height.

PROBLEM SOLVING
Acceleration in g's

The acceleration of objects such as rockets and fast airplanes is often given as a multiple of $g = 9.80\,\mathrm{m/s^2}$. For example, an airplane pulling out of a dive (see Fig. 2–32) and undergoing 3.00 g's would have an acceleration of $(3.00)\left(9.80\,\mathrm{m/s^2}\right) = 29.4\,\mathrm{m/s^2}$.

FIGURE 2–32 Several airplanes, in formation, are just coming out of a downward dive.

EXAMPLE 2–21 **Ball thrown upward, IV; the quadratic formula.** For the ball in Example 2–20, calculate at what time t the ball passes a point 8.00 m above the person's hand. (See Fig. 2–31, repeated below.)

APPROACH We choose the time interval from the throw $(t_0 = 0,\ v_0 = 15.0\ \text{m/s})$ until the time t (to be determined) when the ball is at position $y = 8.00\ \text{m}$, using Eq. 2–12b.

SOLUTION We want to find t, given $y = 8.00\ \text{m}$, $y_0 = 0$, $v_0 = 15.0\ \text{m/s}$, and $a = -9.80\ \text{m/s}^2$. We use Eq. 2–12b:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$8.00\ \text{m} = 0 + (15.0\ \text{m/s})t + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2.$$

To solve any quadratic equation of the form $at^2 + bt + c = 0$, where a, b, and c are constants (a is not acceleration here), we use the **quadratic formula**:

PROBLEM SOLVING
Quadratic formula is a very useful tool

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our y equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\ \text{m/s}^2)t^2 - (15.0\ \text{m/s})t + (8.00\ \text{m}) = 0.$$

So the coefficient a is $4.90\ \text{m/s}^2$, b is $-15.0\ \text{m/s}$, and c is 8.00 m. Putting these into the quadratic formula, we obtain

$$t = \frac{15.0\ \text{m/s} \pm \sqrt{(-15.0\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(8.00\ \text{m})}}{2(4.90\ \text{m/s}^2)},$$

which gives us $t = 0.69\ \text{s}$ and $t = 2.37\ \text{s}$. Are both solutions valid? Yes, because the ball passes $y = 8.00\ \text{m}$ when it goes up $(t = 0.69\ \text{s})$ and again when it comes down $(t = 2.37\ \text{s})$.

NOTE Figure 2–33 shows graphs of (a) y vs. t and (b) v vs. t for the ball thrown upward in Fig. 2–31, incorporating the results of Examples 2–17, 2–18, 2–20, and 2–21.

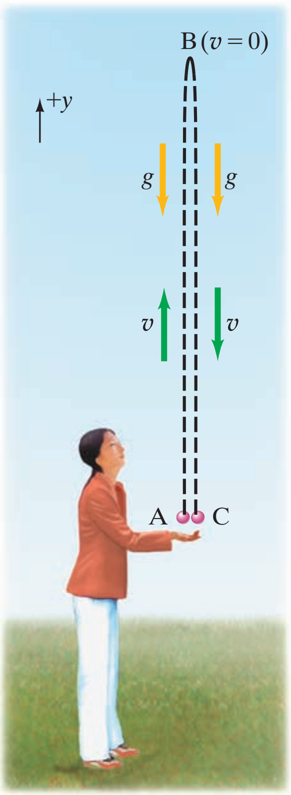
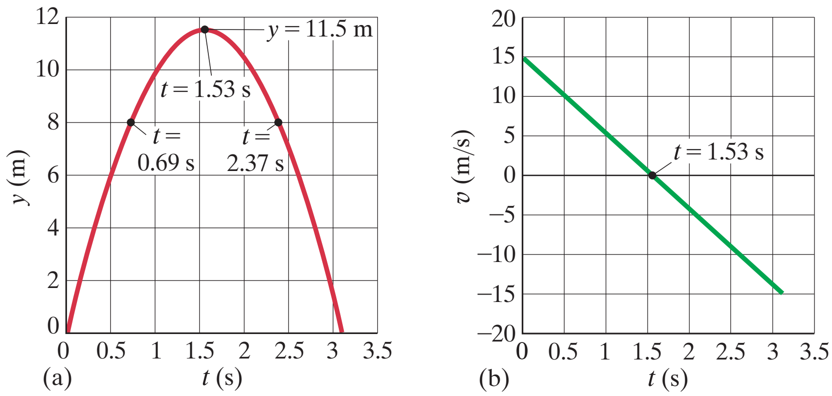


FIGURE 2–31 (Repeated.) An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–17, 2–18, 2–19, 2–20, and 2–21.

FIGURE 2–33 Graphs of (a) y vs. t, (b) v vs. t, for a ball thrown upward, Examples 2–17, 2–18, 2–20, and 2–21.



EXAMPLE 2–22 **Ball thrown upward at edge of cliff.** Suppose that the person of Examples 2–17, 2–18, 2–20, and 2–21 is standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below, as shown in Fig. 2–34. (a) How long does it take the ball to reach the base of the cliff? (b) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

APPROACH We again use Eq. 2–12b, but this time we set $y = -50.0\ \text{m}$, the bottom of the cliff, which is 50.0 m below the initial position $(y_0 = 0)$.

SOLUTION (*a*) We use Eq. 2–12b with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, $y_0 = 0$, and $y = -50.0\,\mathrm{m}$:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$-50.0\,\mathrm{m} = 0 + (15.0\,\mathrm{m/s})t - \tfrac{1}{2}(9.80\,\mathrm{m/s^2})t^2.$$

Rewriting in the standard form we have

$$(4.90\,\mathrm{m/s^2})t^2 - (15.0\,\mathrm{m/s})t - (50.0\,\mathrm{m}) = 0.$$

Using the quadratic formula, we find as solutions $t = 5.07\,\mathrm{s}$ and $t = -2.01\,\mathrm{s}$. The first solution, $t = 5.07\,\mathrm{s}$, is the answer we are seeking: the time it takes the ball to rise to its highest point and then fall to the base of the cliff. To rise and fall back to the top of the cliff took 3.06 s (Example 2–18); so it took an additional 2.01 s to fall to the base. But what is the meaning of the other solution, $t = -2.01\,\mathrm{s}$? This is a time before the throw, when our calculation begins, so it isn't relevant here. It is outside our chosen time interval, and so is an *unphysical* solution just as in Example 2–12.

(*b*) For the total distance traveled, Example 2–17 told us that the ball moves up 11.5 m, falls 11.5 m back down to the top of the cliff; it then falls down another 50.0 m to the base of the cliff, for a total distance traveled of $2\,(11.5\,\mathrm{m}) + 50.0\,\mathrm{m} = 73.0\,\mathrm{m}$. Note that the *displacement*, however, was −50.0 m. Figure 2–34b shows the y vs. t graph for this situation.

NOTE The solution $t = -2.01\,\mathrm{s}$ in part (*a*) could be meaningful in a different physical situation. Suppose that a person standing on top of a 50.0-m-high cliff sees a rock pass by her at $t = 0$ moving upward at 15.0 m/s; at what time did the rock leave the base of the cliff, and when did it arrive back at the base of the cliff? The equations will be precisely the same as for our original Example, and the answers $t = -2.01\,\mathrm{s}$ and $t = 5.07\,\mathrm{s}$ will be the correct answers. Note that we cannot put *all* the information for a problem into the mathematics, so we have to use common sense in interpreting results.

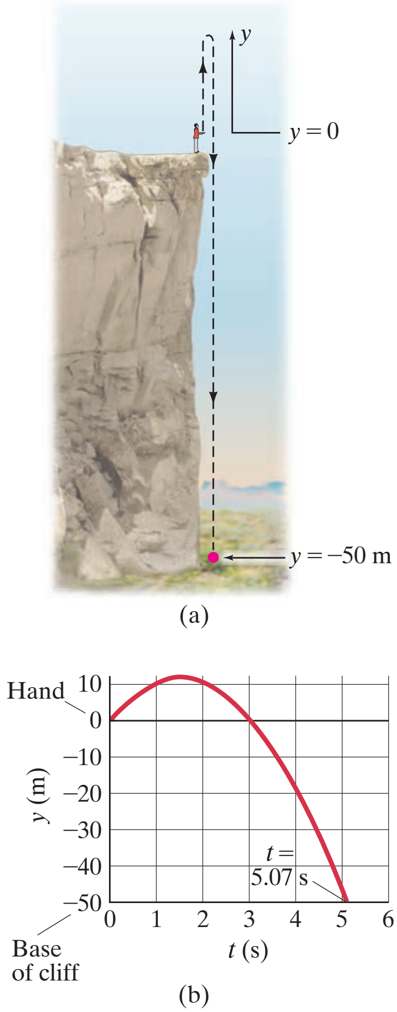


FIGURE 2–34 Example 2–22. (*a*) A person stands on the edge of a cliff. A ball is thrown upward, and then falls back down past the thrower to the base of the cliff, 50.0 m below. (*b*) The y vs. t graph.

EXERCISE H Two balls are thrown from a cliff. One is thrown directly up, the other directly down, each with the same initial speed, and both hit the ground below the cliff. Which ball hits the ground at the greater speed: (*a*) the ball thrown upward, (*b*) the ball thrown downward, or (*c*) both the same? Ignore air resistance.

*2–8 Variable Acceleration; Integral Calculus

In this brief optional Section we use integral calculus to derive the kinematic equations for constant acceleration, Eqs. 2–12a and b. We also show how calculus can be used when the acceleration is not constant. If you have not yet studied simple integration in your calculus course, you may want to postpone reading this Section until you have. We discuss integration in more detail in Section 7–3, where we begin to use it in the physics.

First we derive Eq. 2–12a, assuming as we did in Section 2–5 that an object has velocity v_0 at $t = 0$ and a constant acceleration a. We start with the definition of instantaneous acceleration, $a = dv/dt$, which we rewrite as

$$dv = a\,dt.$$

We take the definite integral of both sides of this equation, using the same notation we did in Section 2–5, from $v = v_0$ at $t_0 = 0$ to some velocity v at time t:

$$\int_{v=v_0}^{v} dv = \int_{t_0=0}^{t} a\,dt$$

which gives, since a = constant,

$$v - v_0 = at.$$

This is Eq. 2–12a, $v = v_0 + at$.

Next we derive Eq. 2–12b starting with the definition of instantaneous velocity, Eq. 2–4, $v = dx/dt$. We rewrite this as

$$dx = v\,dt$$

or

$$dx = (v_0 + at)\,dt$$

where we substituted in Eq. 2–12a.

Now we integrate from $x = x_0$ at $t_0 = 0$ to an arbitrary position x at time t:

$$\int_{x=x_0}^{x} dx = \int_{t_0=0}^{t} (v_0 + at)\,dt$$

$$x - x_0 = \int_{t_0=0}^{t} v_0\,dt + \int_{t_0=0}^{t} at\,dt$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

since v_0 and a are constants. This result is just Eq. 2–12b, $x = x_0 + v_0 t + \frac{1}{2}at^2$.

Finally let us use calculus to find velocity and displacement, given an acceleration that is not constant but varies in time.

EXAMPLE 2–23 **Integrating a time-varying acceleration.** An experimental vehicle starts from rest $(v_0 = 0)$ at $t = 0$ and accelerates for a few seconds at a rate given by $a = (7.00\,\text{m/s}^3)t$. What is (*a*) its velocity and (*b*) its displacement 2.00 s later?

APPROACH We cannot use Eqs. 2–12 because a is not constant. We integrate the acceleration $a = dv/dt$ over time to find v as a function of time; and then integrate $v = dx/dt$ to get the displacement.

SOLUTION From the definition of acceleration, $a = dv/dt$, we have

$$dv = a\,dt.$$

We take the integral of both sides from $v = 0$ at $t = 0$ to velocity v at an arbitrary time t:

$$\int_0^v dv = \int_0^t a\,dt$$

$$v = \int_0^t (7.00\,\text{m/s}^3)t\,dt$$

$$= (7.00\,\text{m/s}^3)\left(\frac{t^2}{2}\right)\Bigg|_0^t = (7.00\,\text{m/s}^3)\left(\frac{t^2}{2} - 0\right) = (3.50\,\text{m/s}^3)t^2.$$

At $t = 2.00\,\text{s}$, $v = (3.50\,\text{m/s}^3)(2.00\,\text{s})^2 = 14.0\,\text{m/s}$.

(*b*) To get the displacement, we assume $x_0 = 0$ and start with $v = dx/dt$ which we rewrite as $dx = v\,dt$. Then we integrate from $x = 0$ at $t = 0$ to position x at time t:

$$\int_0^x dx = \int_0^t v\,dt$$

$$x = \int_0^{2.00\,\text{s}} (3.50\,\text{m/s}^3)t^2\,dt = (3.50\,\text{m/s}^3)\,\frac{t^3}{3}\Bigg|_0^{2.00\,\text{s}} = 9.33\,\text{m}.$$

In sum, at $t = 2.00\,\text{s}$, $v = 14.0\,\text{m/s}$ and $x = 9.33\,\text{m}$.

Some problems in kinematics can be solved using Numerical Integration, which is discussed in Appendix C.

Summary

[*The Summary that appears at the end of each Chapter in this book gives a brief overview of the main ideas of the Chapter. The Summary* cannot *serve to give an understanding of the material, which can be accomplished only by a detailed reading of the Chapter.*]

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame**.

The **displacement** of an object is the change in position of the object.

Average speed is the distance traveled divided by the **elapsed time** or **time interval**, Δt, the time period over which we choose to make our observations. An object's **average velocity** over a particular time interval Δt is its displacement Δx during that time interval, divided by Δt:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \tag{2–2}$$

The **instantaneous velocity**, whose magnitude is the same as the *instantaneous speed*, is defined as the average velocity taken over an infinitesimally short time interval $(\Delta t \rightarrow 0)$:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}, \tag{2–4}$$

where dx/dt is the derivative of x with respect to t.

On a graph of position vs. time, the **slope** is equal to the instantaneous velocity.

Acceleration is the change of velocity per unit time. An object's **average acceleration** over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t}, \tag{2–5}$$

where Δv is the change of velocity during the time interval Δt.

Instantaneous acceleration is the average acceleration taken over an infinitesimally short time interval:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}. \tag{2–6}$$

On a graph of velocity vs. time, the slope is equal to the instantaneous acceleration.

If an object has position x_0 and velocity v_0 at time $t = 0$ and moves in a straight line with **constant acceleration**, the velocity v and position x at a later time t are related to the acceleration a, the initial position x_0, and the initial velocity v_0 by Eqs. 2–12:

$$\begin{aligned} v &= v_0 + at, \\ x &= x_0 + v_0 t + \tfrac{1}{2}at^2, \\ v^2 &= v_0^2 + 2a(x - x_0), \\ \bar{v} &= \frac{v + v_0}{2}. \end{aligned} \tag{2–12}$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward **acceleration due to gravity**, whose magnitude is $g = 9.80\ \mathrm{m/s^2}$ if air resistance can be ignored.

[*Integral calculus can be used to derive the kinematic Equations 2–12 and to solve Problems involving varying acceleration. Numerical integration is also a useful tool.]

Questions

1. Does a car speedometer measure speed, velocity, or both? Explain.
2. Can an object have a varying speed if its velocity is constant? Can it have varying velocity if its speed is constant? If yes, give examples in each case.
3. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant? Explain.
4. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain, using examples.
5. Compare the acceleration of a motorcycle that accelerates from 80 km/h to 90 km/h with the acceleration of a bicycle that accelerates from rest to 10 km/h in the same time.
6. Can an object have a northward velocity and a southward acceleration? Explain.
7. Can the velocity of an object be negative when its acceleration is positive? What about vice versa? If yes, give examples.
8. Give an example where both the velocity and acceleration are negative.
9. Two cars emerge side by side from a tunnel. Car A is traveling with a speed of 60 km/h and has an acceleration of 40 km/h/min. Car B has a speed of 40 km/h and has an acceleration of 60 km/h/min. Which car is passing the other as they come out of the tunnel? Explain your reasoning.
10. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.
11. A baseball player hits a ball straight up into the air. It leaves the bat with a speed of 120 km/h. Ignoring air resistance, how fast would the ball be traveling when the catcher catches it (at the same height it left the bat)? Explain.
12. As a freely falling object speeds up, what is happening to its acceleration—does it increase, decrease, or stay the same? (*a*) Ignore air resistance. (*b*) Consider air resistance.
13. You travel from point A to point B in a car moving at a constant speed of 70 km/h. Then you travel the same distance from point B to another point C, moving at a constant speed of 90 km/h. Is your average speed 80 km/h for the entire trip from A to C? Explain why or why not.
14. Can an object have zero velocity and nonzero acceleration at the same time? Give examples.
15. Can an object have zero acceleration and nonzero velocity at the same time? Give examples.
16. Which of these motions is *not* at constant acceleration: a rock falling from a cliff, an elevator moving from the second floor to the fifth floor making stops along the way, a dish resting on a table? Explain your answers.
17. Discuss two conditions given in Section 2–7 for being able to use a constant acceleration of magnitude $g = 9.8\ \mathrm{m/s^2}$. Give an example in which one of these conditions would not be met and would not even be a reasonable approximation of motion. [*Hint*: Carefully read Section 2–7, especially page 60.]

18. Describe in words the motion plotted in Fig. 2–35 in terms of velocity, acceleration, etc. [*Hint*: First try to duplicate the motion plotted by walking or moving your hand.]

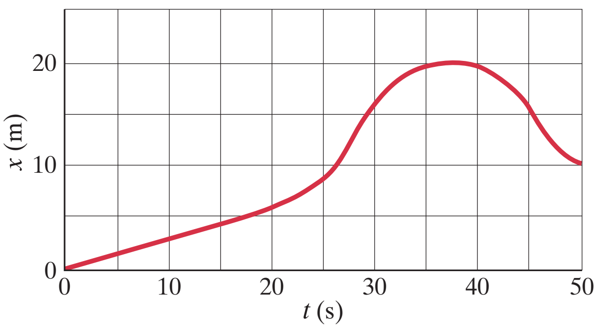


FIGURE 2–35 Question 18.

19. Describe in words the motion of the object graphed in Fig. 2–36.

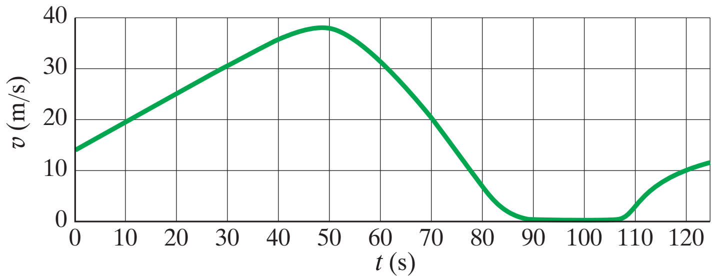


FIGURE 2–36 Question 19.

MisConceptual Questions

[List all answers that are valid, and ignore air resistance.]

1. In which of the following cases does a car have a negative velocity and a positive acceleration? A car that is traveling in the
(*a*) $-x$ direction at a constant 20 m/s.
(*b*) $-x$ direction increasing in speed.
(*c*) $+x$ direction increasing in speed.
(*d*) $-x$ direction decreasing in speed.
(*e*) $+x$ direction decreasing in speed.

2. At time $t = 0$ an object is traveling to the right along the $+x$ axis at a speed of 10.0 m/s with constant acceleration of $-2.0\ \text{m/s}^2$. Which statement is true?
(*a*) The object will slow down, eventually coming to a complete stop.
(*b*) The object cannot have a negative acceleration and be moving to the right.
(*c*) The object will continue to move to the right, slowing down but never coming to a complete stop.
(*d*) The object will slow down, momentarily stopping, then pick up speed moving to the left.

3. You drive 4 km at 30 km/h and then another 4 km at 50 km/h. What is your average speed for the whole 8-km trip?
(*a*) More than 40 km/h.
(*b*) Equal to 40 km/h.
(*c*) Less than 40 km/h.
(*d*) Not enough information.

4. Two cars start from rest and travel a distance *d* with constant acceleration. The acceleration of car B is four times that of car A. After each has traveled distance *d*,
(*a*) car B is moving 16 times as fast as car A.
(*b*) car B is moving 8 times as fast as car A.
(*c*) car B is moving 4 times as fast as car A.
(*d*) car B is moving 2 times as fast as car A.

5. A rock is thrown straight up rising to a maximum height before falling back down.
(*a*) The acceleration is constant for the entire trip.
(*b*) The velocity is constant for the entire trip.
(*c*) The magnitudes of both the velocity and acceleration decrease on the way up and increase on the way down.
(*d*) The acceleration is constant for the entire trip except at the top where it is 0.

6. A ball is dropped from the top of a tall building. At the same instant, a second ball is thrown upward from ground level. When the two balls pass one another, one on the way up, the other on the way down, compare the magnitudes of their acceleration:
(*a*) The acceleration of the dropped ball is greater.
(*b*) The acceleration of the ball thrown upward is greater.
(*c*) The acceleration of both balls is the same.
(*d*) The acceleration changes during the motion, so you cannot predict the exact value when the two balls pass each other.
(*e*) The accelerations are in opposite directions.

7. You drop a rock off a bridge. When the rock has fallen 4 m, you drop a second rock. As the two rocks continue to fall, what happens to their velocities?
(*a*) Both increase at the same rate.
(*b*) The velocity of the first rock increases faster than the velocity of the second.
(*c*) The velocity of the second rock increases faster than the velocity of the first.
(*d*) Both velocities stay constant.

8. Two objects are dropped from a bridge, an interval of 1.0 s apart. During the time that both objects continue to fall, their separation
(*a*) decreases at first, but then stays constant.
(*b*) increases at first, but then stays constant.
(*c*) decreases.
(*d*) stays constant.
(*e*) increases.

9. A ball is thrown downward at a speed of 20 m/s. Choosing the $+y$ axis pointing up and neglecting air resistance, which equation(s) would correctly describe the motions? The acceleration due to gravity is $g = 9.8\ \text{m/s}^2$ downward.
(*a*) $v = (20\ \text{m/s}) - gt$.
(*b*) $y = y_0 + (-20\ \text{m/s})t - (1/2)gt^2$.
(*c*) $v^2 = (20\ \text{m/s})^2 - 2g(y - y_0)$.
(*d*) $(20\ \text{m/s}) = (v + v_0)/2$.
(*e*) All of the above.

10. A car travels along the x axis with increasing speed. We are not sure if it is moving to the left or to the right. Which of the graphs in Fig. 2–37 could possibly represent the motion of the car?

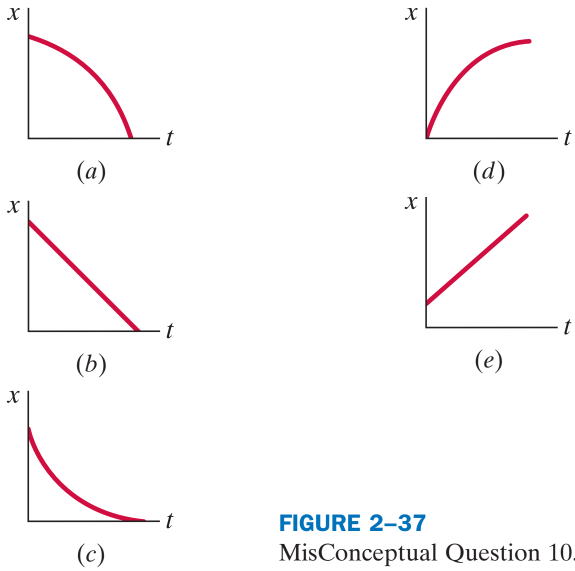


FIGURE 2–37 MisConceptual Question 10.

11. Two objects start at the same place at the same time and move along the same straight line (the x axis). Figure 2–38 shows the position x as a function of time t for each object. At point A, what must be true about the motion of the objects? (More than one statement may be correct.)
(a) Both have the same instantaneous speed.
(b) Both have the same instantaneous velocity.
(c) Both are at the same position.
(d) Both have traveled the same total distance.
(e) With respect to the start, both have the same average velocity.

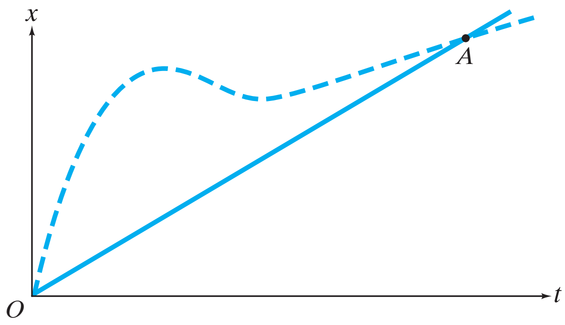


FIGURE 2–38 MisConceptual Question 11.

Problems

[*The Problems at the end of each Chapter are ranked* I, II, *or* III *according to estimated difficulty, with level* I *Problems being easiest. Level* III *are meant as challenges for the best students. The Problems are arranged by Section, meaning that the reader should have read up to and including that Section, but not only that Section—Problems often depend on earlier material. Next is a set of "General Problems" not arranged by Section and not ranked.*]

(Note: *In Problems, assume a number like* 6.4 *is accurate to* ± 0.1; *and* 950 *is* ± 10 *unless* 950 *is said to be "precisely" or "very nearly"* 950, *in which case assume* 950 ± 1. *See Section 1–3.*)

2–1 to 2–3 Speed and Velocity

1. (I) If you are driving 85 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?

2. (I) What must your car's average speed be in order to travel 235 km in 2.85 h?

3. (I) A particle at $t_1 = -2.0\,\text{s}$ is at $x_1 = 5.2\,\text{cm}$ and at $t_2 = 3.4\,\text{s}$ is at $x_2 = 8.5\,\text{cm}$. What is its average velocity over this time interval? Can you calculate its average speed from these data? Why or why not?

4. (II) According to a rule-of-thumb, each five seconds between a lightning flash and the following thunder gives the distance to the flash in miles. (a) Assuming that the light from the flash arrives in essentially no time at all, estimate the speed of sound in m/s from this rule. (b) What would be the rule for kilometers?

5. (II) You are driving home from school steadily at 95 km/h for 210 km. It then begins to rain and you slow to 65 km/h. You arrive home after driving 4.5 h. (a) How far is your hometown from school? (b) What was your average speed?

6. (II) A horse trots away from its trainer in a straight line, moving 38 m away in 7.4 s. It then turns abruptly and gallops halfway back in 1.8 s. Calculate (a) its average speed and (b) its average velocity for the entire trip, using "away from the trainer" as the positive direction.

7. (II) A person jogs eight complete laps around a 400-m track in a total time of 14.5 min. Calculate (a) the average speed and (b) the average velocity, in m/s.

8. (II) Every year the Earth travels about 10^9 km as it orbits the Sun. What is Earth's average speed in km/h?

9. (II) A car traveling 95 km/h is 310 m behind a truck traveling 75 km/h. How long will it take the car to reach the truck?

10. (II) Calculate (a) the average speed and (b) average velocity of a round trip: the outgoing 280 km is covered at 95 km/h, followed by a 1.0-h lunch break, and the return 280 km is covered at 55 km/h.

11. (II) Two locomotives approach each other on parallel tracks. Each has a speed of 155 km/h with respect to the ground. If they are initially 9.5 km apart, how long will it be before they reach each other? (See Fig. 2–39.)

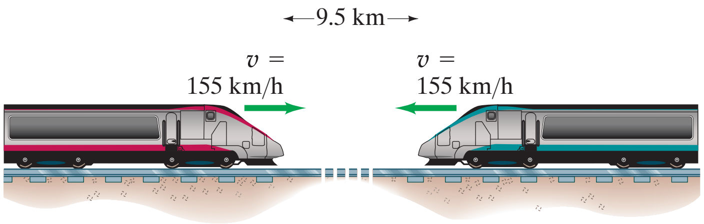


FIGURE 2–39 Problem 11.

12. (II) Digital bits on a 12.0-cm diameter audio CD are encoded along an outward spiraling path that starts at radius $R_1 = 2.5$ cm and finishes at radius $R_2 = 5.8$ cm. The distance between the centers of neighboring spiral-windings is $1.6\,\mu$m $(= 1.6 \times 10^{-6}\,\text{m})$. (*a*) Determine the total length of the spiraling path. [*Hint*: Imagine "unwinding" the spiral into a straight path of width $1.6\,\mu$m, and note that the original spiral and the straight path both occupy the same area.] (*b*) The CD player adjusts the rotation of the CD so that the player's readout laser reads along the spiral path at a constant rate of about 1.2 m/s. Estimate the maximum playing time of such a CD.

13. (II) The position of a small object is given by $x = 27 + 10t - 2t^3$, where t is in seconds and x in meters. (*a*) Plot x as a function of t from $t = 0$ to $t = 3.0$ s. (*b*) Find the average velocity of the object between 0 and 3.0 s. (*c*) At what time between 0 and 3.0 s is the instantaneous velocity zero?

14. (II) An airplane travels 1900 km at a speed of 720 km/h, and then encounters a tailwind that boosts its speed to 990 km/h for the next 2700 km. What was the total time for the trip? What was the average speed of the plane for this trip? [*Hint*: Does Eq. 2–12d apply?]

15. (II) Two ships need to arrive at a site in the middle of the ocean at the same time. They start out at the same time from positions equally distant from the arrival site. They travel at different velocities but both go in a straight line. The first ship travels at an average velocity of 20 km/h for the first 600 km, 40 km/h for the next 800 km, and 20 km/h for the final 600 km. The second ship can only sail at constant velocity. What is the magnitude of that velocity?

16. (II) The position of an object along a straight tunnel as a function of time is plotted in Fig. 2–40. What is its instantaneous velocity (*a*) at $t = 10.0$ s and (*b*) at $t = 30.0$ s? What is its average velocity (*c*) between $t = 0$ and $t = 5.0$ s, (*d*) between $t = 25.0$ s and $t = 30.0$ s, and (*e*) between $t = 40.0$ s and $t = 50.0$ s?

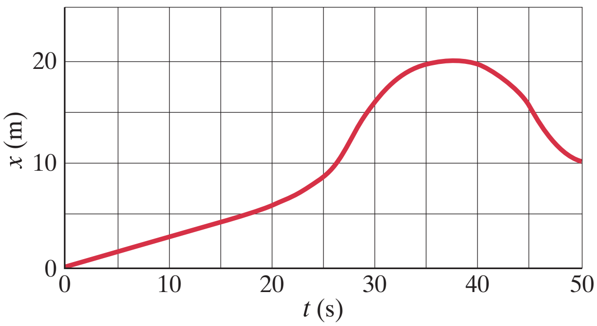


FIGURE 2–40 Problems 16, 17, and 18.

17. (II) In Fig. 2–40, (*a*) during what time intervals, if any, is the velocity constant? (*b*) At what time is the velocity greatest? (*c*) At what time, if any, is the velocity zero? (*d*) Does the object move in one direction or in both directions during the time shown?

18. (III) Sketch the v vs. t graph for the object whose displacement as a function of time is given by Fig. 2–40.

19. (III) A bowling ball traveling with constant speed hits the pins at the end of a bowling lane 16.5 m long. The bowler hears the sound of the ball hitting the pins 2.75 s after the ball is released from his hands. What is the speed of the ball, assuming the speed of sound is 340 m/s?

20. (III) An automobile traveling 95 km/h overtakes a 1.50-km-long train traveling in the same direction on a track parallel to the road. If the train's speed is 75 km/h, how long does it take the car to pass it, and how far will the car have traveled in this time? See Fig. 2–41. What are the results if the car and train are traveling in opposite directions?

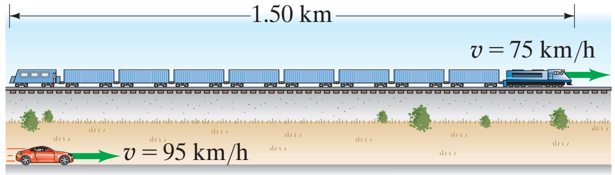


FIGURE 2–41 Problem 20.

2–4 Acceleration

21. (I) A sprinter accelerates from rest to 9.00 m/s in 1.48 s. What is her acceleration in (*a*) m/s^2; (*b*) km/h^2?

22. (I) A bicyclist in the Tour de France crests a mountain pass as he moves at 15 km/h. At the bottom, 4.0 km farther, his speed is 65 km/h. Estimate his average acceleration (in m/s^2) while riding down the mountain.

23. (II) A sports car moving at constant velocity travels 120 m in 5.0 s. If it then brakes and comes to a stop in 3.7 s, what is the magnitude of its acceleration (assumed constant) in m/s^2, and in g's $(g = 9.80\,\text{m/s}^2)$?

24. (II) At highway speeds, a particular automobile is capable of an acceleration of about $1.8\,\text{m/s}^2$. At this rate, how long does it take to accelerate from 65 km/h to 120 km/h?

25. (II) A car moving in a straight line starts at $x = 0$ when $t_0 = 0$. It passes the point $x = 25.0$ m with a speed of 11.0 m/s at $t = 3.00$ s. It passes the point $x = 385$ m with a speed of 45.0 m/s at $t = 20.0$ s. Find (*a*) the average velocity, and (*b*) the average acceleration, between $t = 3.00$ s and $t = 20.0$ s.

26. (II) A particle moves along the x axis. Its position as a function of time is given by $x = 4.8t + 7.3t^2$, where t is in seconds and x is in meters. What is the acceleration as a function of time?

27. (II) The position of an object is given by $x = At + Bt^2$, where x is in meters and t is in seconds. (*a*) What are the units of A and B? (*b*) What is the acceleration as a function of time? (*c*) What are the velocity and acceleration at $t = 6.0$ s? (*d*) What is the velocity as a function of time if $x = At + Bt^{-3}$?

28. (II) The position of a race car, which starts from rest at $t = 0$ and moves in a straight line, is given as a function of time in the following Table. Estimate (*a*) its velocity and (*b*) its acceleration as a function of time. Display each in a Table and on a graph.

t(s)	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50
x(m)	0	0.11	0.46	1.06	1.94	4.62	8.55	13.79

t(s)	3.00	3.50	4.00	4.50	5.00	5.50	6.00
x(m)	20.36	28.31	37.65	48.37	60.30	73.26	87.16

29. (II) A car traveling 25.0 m/s passes a second car which is at rest. When the cars are right next to each other, the first car slows down at a constant rate of $2.0\,\text{m/s}^2$ and the second car starts to accelerate at the same constant rate. When will the two cars be next to each other again?

30. (II) Figure 2–42 shows the velocity of a train as a function of time. (*a*) At what time was its velocity greatest? (*b*) During what periods, if any, was the velocity constant? (*c*) During what periods, if any, was the acceleration constant? (*d*) When was the magnitude of the acceleration greatest?

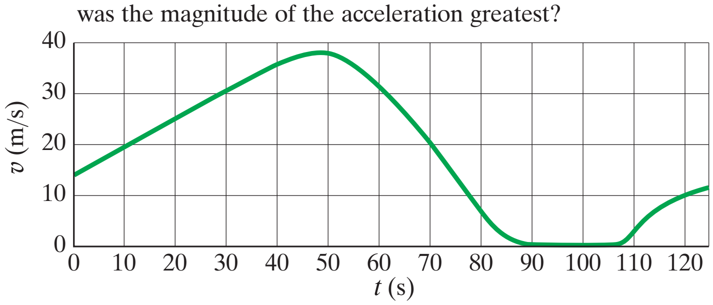


FIGURE 2–42 Problem 30.

31. (II) A sports car accelerates approximately as shown in the velocity–time graph of Fig. 2–43. (The short flat spots in the curve represent manual shifting of the gears.) Estimate the car's average acceleration in (*a*) second gear and (*b*) fourth gear.

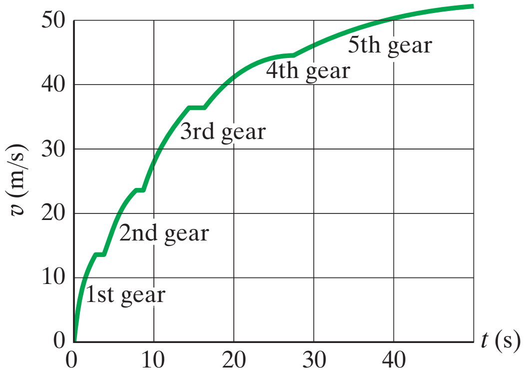


FIGURE 2–43 Problem 31. The velocity of a car as a function of time, starting from a dead stop. The flat spots in the curve represent gear shifts.

32. (III) A fugitive tries to hop on a freight train traveling at a constant speed of 5.0 m/s. Just as an empty box car passes him, the fugitive starts from rest and accelerates at $a = 1.4\,\text{m/s}^2$ to his maximum speed of 6.0 m/s, which he then maintains. (*a*) How long does it take him to catch up to the empty box car? (*b*) What is the distance traveled to reach the box car?

2–5 to 2–6 Motion at Constant Acceleration

33. (I) A car slows down from 26 m/s to rest in a distance of 88 m. What was its acceleration, assumed constant?

34. (I) A car slows down from 28 m/s to rest in 6.3 s. What was its (constant) acceleration?

35. (I) A car accelerates from 13 m/s to 22 m/s in 6.5 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

36. (II) A world-class sprinter can reach a top speed (of about 11.5 m/s) in the first 18.0 m of a race. What is the average acceleration of this sprinter and how long does it take her to reach that speed?

37. (II) A car slows down uniformly from a speed of 28.0 m/s to rest in 8.60 s. How far did it travel in that time?

38. (II) In coming to a stop, an old truck leaves skid marks 45 m long on the highway. Assuming a deceleration of $6.00\,\text{m/s}^2$, estimate the speed of the truck just before braking.

39. (II) A baseball pitcher throws a baseball with a speed of 43 m/s. Estimate the average acceleration of the ball during the throwing motion. In throwing the baseball, the pitcher accelerates it through a displacement of about 3.5 m, from behind the body to the point where it is released (Fig. 2–44).

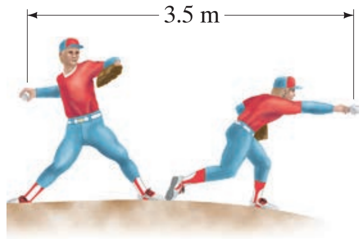


FIGURE 2–44 Problem 39.

40. (II) A car traveling at 95 km/h strikes a tree. The front end of the car compresses and the driver comes to rest after traveling 0.80 m. What was the magnitude of the average acceleration of the driver during the collision? Express the answer in terms of "*g*'s," where $1.00\,g = 9.80\,\text{m/s}^2$.

41. (II) A car traveling 85 km/h slows down at a constant $0.50\,\text{m/s}^2$ just by "letting up on the gas." Calculate (*a*) the distance the car coasts before it stops, (*b*) the time it takes to stop, and (*c*) the distance it travels during the first and fifth seconds.

42. (II) Determine the stopping distances for an automobile going a constant initial speed of 95 km/h in the $+x$ direction, and human reaction time of 0.40 s: (*a*) for an acceleration $a = -2.5\,\text{m/s}^2$; (*b*) for $a = -5.5\,\text{m/s}^2$.

43. (II) A driver is traveling 18.0 m/s when she sees a red light ahead. Her car is capable of decelerating at a rate of $3.65\,\text{m/s}^2$. If it takes her 0.380 s to get the brakes on and she is 24.0 m from the intersection when she sees the light, will she be able to stop in time? How far from the beginning of the intersection will she be, and in which direction?

44. (II) Show that $\bar{v} = (v + v_0)/2$ (see Eq. 2–12d) is not valid when the acceleration $a = A + Bt$, where A and B are non-zero constants.

45. (II) An 85-m-long train begins uniform acceleration from rest. The front of the train has a speed of 18 m/s when it passes a railway worker who is standing 180 m from where the front of the train started. What will be the speed of the last car as it passes the worker? (See Fig. 2–45.)

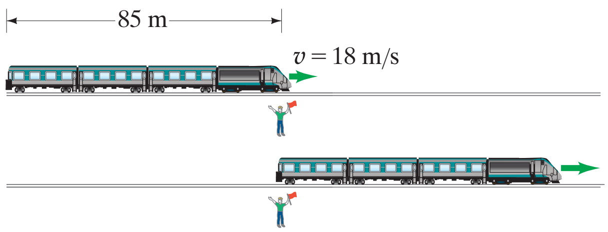


FIGURE 2–45 Problem 45.

46. (II) A space vehicle accelerates uniformly from 85 m/s at $t = 0$ to 162 m/s at $t = 10.0$ s. How far did it move between $t = 2.0$ s and $t = 7.0$ s?

47. (II) A runner hopes to complete the 10,000-m run in less than 30.0 min. After running at constant speed for exactly 27.0 min, there are still 1200 m to go. The runner must then accelerate at $0.20\,\text{m/s}^2$ for how many seconds in order to achieve the desired time?

48. (III) Mary and Sally are in a foot race (Fig. 2–46). When Mary is 22 m from the finish line, she has a speed of 4.0 m/s and is 5.0 m behind Sally, who has a speed of 5.0 m/s. Sally thinks she has an easy win and so, during the remaining portion of the race, slows down at a constant rate of $0.40\ \mathrm{m/s^2}$ to the finish line. What constant acceleration does Mary now need during the remaining portion of the race, if she wishes to cross the finish line side-by-side with Sally?

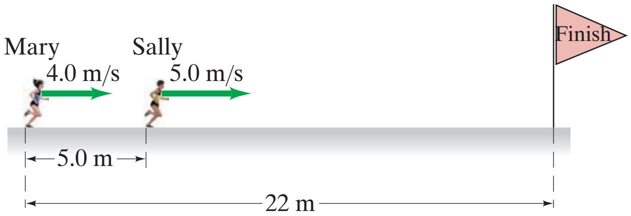


FIGURE 2–46 Problem 48.

49. (III) An unmarked police car traveling a constant 95 km/h is passed by a speeder traveling 135 km/h. Precisely 1.00 s after the speeder passes, the police officer steps on the accelerator; if the police car's acceleration is $2.60\ \mathrm{m/s^2}$, how much time passes before the police car overtakes the speeder (assumed moving at constant speed)?

50. (III) Assume in Problem 49 that the speeder's speed is not known. If the police car accelerates uniformly at $2.60\ \mathrm{m/s^2}$ and overtakes the speeder after accelerating for 7.00 s, what was the speeder's speed?

51. (III) A runner completes a 400-meter race in 55.0 s. The 55.0 seconds is made up of a 0.15 s reaction time from the starting sound until the runner starts moving followed by 30.0 m of constant acceleration and then 370 m at constant speed. What are the values of the acceleration and the constant speed?

2–7 Freely Falling Objects (neglect air resistance)

52. (I) A stone is dropped from the top of a cliff. It is seen to hit the ground below after 3.25 s. How high is the cliff?

53. (I) Estimate (*a*) how long it took King Kong to fall straight down from the top of the Empire State Building (380 m high), and (*b*) his velocity just before "landing."

54. (I) If a car rolls gently $(v_0 = 0)$ off a vertical cliff, how long does it take it to reach 55 km/h?

55. (II) A ball player catches a ball 2.6 s after throwing it vertically upward. With what speed did he throw it, and what height did it reach?

56. (II) A baseball is hit almost straight up into the air with a speed of 22 m/s. Estimate (*a*) how high it goes, and (*b*) how long it is in the air. (*c*) What factors make this an estimate?

57. (II) A kangaroo jumps straight up to a vertical height of 1.45 m. How long was it in the air before returning to Earth?

58. (II) The best *rebounders* in *basketball* have a vertical leap (that is, the vertical movement of a fixed point on their body) of about 120 cm. (*a*) What is their initial "launch" speed off the ground? (*b*) How long are they in the air?

59. (II) A stone is thrown vertically upward with a speed of 18.0 m/s. (*a*) How fast is it moving when it is at a height of 11.0 m? (*b*) How much time is required to reach this height? (*c*) Why are there two answers to (*b*)?

60. (II) For an object falling freely from rest, show that the distance traveled *during* each successive second increases in the ratio of successive odd integers (1, 3, 5, etc.). (This was first shown by Galileo.) See Figs. 2–27 and 2–30.

61. (II) If there were no air resistance, how long would it take a free-falling skydiver to fall from a plane at 3800 m to an altitude of 450 m, where she will open her parachute? What would her speed be at 450 m? (In reality, air resistance will restrict her speed to perhaps 150 km/h.)

62. (II) Pelicans tuck their wings and free-fall straight down when diving for fish. Suppose a pelican starts its dive from a height of 16.0 m and cannot change its path once committed. If it takes a fish 0.20 s to perform evasive action, at what minimum height must it spot the pelican to escape? Assume the fish is at the surface of the water.

63. (II) A stone is thrown vertically upward with a speed of 15.5 m/s from the edge of a cliff 75.0 m high (Fig. 2–47). (*a*) How much later does it reach the bottom of the cliff? (*b*) What is its speed just before hitting? (*c*) What total distance did it travel?

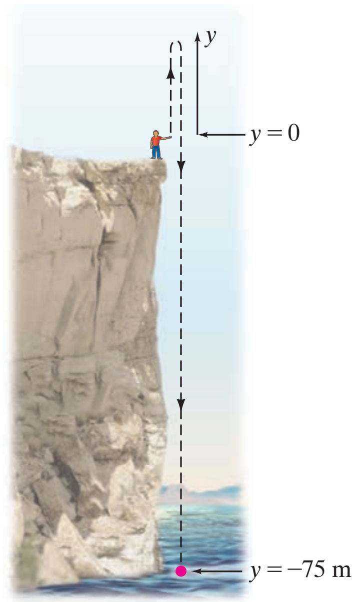


FIGURE 2–47 Problem 63.

64. (II) A rocket rises vertically, from rest, with an acceleration of $3.2\ \mathrm{m/s^2}$ until it runs out of fuel at an altitude of 725 m. After this point, its acceleration is that of gravity, downward. (*a*) What is the velocity of the rocket when it runs out of fuel? (*b*) How long does it take to reach this point? (*c*) What maximum altitude does the rocket reach? (*d*) How much time (total) does it take to reach maximum altitude? (*e*) With what velocity does it strike the Earth? (*f*) How long (total) is it in the air?

65. (II) Suppose you adjust your garden hose nozzle for a fast stream of water. You point the nozzle vertically upward at a height of 1.8 m above the ground (Fig. 2–48). When you quickly turn off the nozzle, you hear the water striking the ground next to you for another 2.5 s. What is the water speed as it leaves the nozzle?

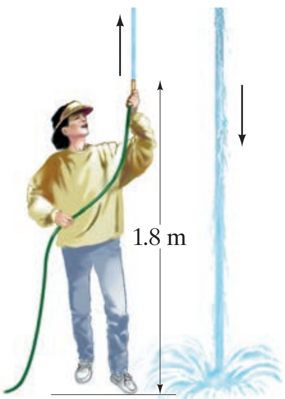


FIGURE 2–48 Problem 65.

66. (II) A helicopter is ascending vertically with a constant speed of 6.40 m/s. At a height of 105 m above the Earth, a package is dropped from the helicopter. How much time does it take for the package to reach the ground? [*Hint*: What is v_0 for the package?]

67. (II) Roger sees water balloons fall past his window. He notices that each balloon strikes the sidewalk 0.83 s after passing his window, 15 m above the sidewalk. (*a*) How fast are the balloons traveling when they pass Roger's window? (*b*) Assuming the balloons are being released from rest, from what height are they being released?

68. (II) A baseball is seen to pass upward by a window with a vertical speed of 13 m/s. If the ball was thrown by a person 18 m below on the street, (*a*) what was its initial speed, (*b*) what altitude does it reach, (*c*) when was it thrown, and (*d*) when does it reach the street again?

69. (III) A falling stone takes 0.28 s to travel past a window 2.2 m tall (Fig. 2–49). From what height above the top of the window did the stone fall?

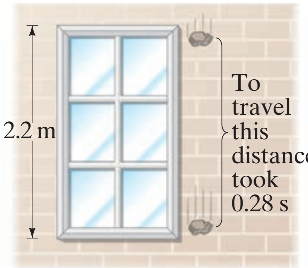


FIGURE 2–49 Problem 69.

70. (III) A toy rocket moving vertically upward passes by a 2.0-m-high window whose base is 8.0 m above the ground. The rocket takes 0.15 s to travel the 2.0 m height of the window. What was the launch speed of the rocket, and how high will it go? Assume the propellant is burned very quickly at blastoff.

71. (III) A ball is dropped from the top of a 55.0-m-high cliff. At the same time, a carefully aimed stone is thrown straight up from the bottom of the cliff with a speed of 24.0 m/s. The stone and ball collide part way up. How far above the base of the cliff does this happen?

*2–8 Variable Acceleration; Calculus

***72.** (II) Given $v(t) = 25 + 18t$, where v is in m/s and t is in s, use calculus to determine the total displacement from $t_1 = 1.3\,\text{s}$ to $t_2 = 3.6\,\text{s}$.

***73.** (III) The acceleration of a particle is given by $a = A\sqrt{t}$ where $A = 3.0\,\text{m/s}^{5/2}$. At $t = 0$, $v = 7.5\,\text{m/s}$ and $x = 0$. (*a*) What is the velocity as a function of time? (*b*) What is the displacement as a function of time? (*c*) What are the acceleration, velocity, and displacement at $t = 5.0\,\text{s}$?

***74.** (III) Air resistance acting on a falling body can be taken into account by the approximate relation for the acceleration:

$$a = \frac{dv}{dt} = g - kv,$$

where k is a constant. (*a*) Derive a formula for the velocity of the body as a function of time assuming it starts from rest ($v = 0$ at $t = 0$). [*Hint*: Change variables by setting $u = g - kv$.] (*b*) Determine an expression for the terminal velocity, which is the maximum value the velocity reaches.

General Problems

75. The acceleration due to gravity on the Moon is about one-sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

76. A person jumps out a fourth-story window 18.0 m above a firefighter's safety net. The survivor stretches the net 1.0 m before coming to rest, Fig. 2–50. (*a*) What was the average deceleration experienced by the survivor when she was slowed to rest by the net? (*b*) What would you do to make it "safer" (that is, to generate a smaller deceleration): would you stiffen or loosen the net? Explain.

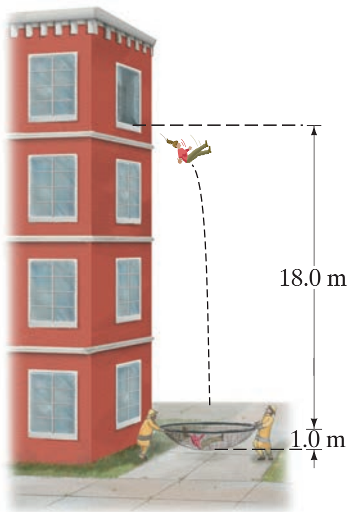


FIGURE 2–50 Problem 76.

77. A person who is properly restrained by an over-the-shoulder seat belt has a good chance of surviving a car collision if the deceleration does not exceed 30 "g's" $(1.00\,g = 9.80\,\text{m/s}^2)$. Assuming uniform deceleration at 30 g's, calculate the distance over which the front end of the car must be designed to collapse if a crash brings the car to rest from 95 km/h.

78. The position of a ball rolling in a straight line is given by $x = 2.0 - 3.6t + 1.7t^2$, where x is in meters and t in seconds. (*a*) What do the numbers 2.0, 3.6, and 1.7 refer to? (*b*) What are the units of each of these numbers? (*c*) Determine the position of the ball at $t = 1.0\,\text{s}$, 2.0 s, and 3.0 s. (*d*) What is the average velocity over the interval $t = 1.0\,\text{s}$ to $t = 3.0\,\text{s}$?

79. In a lecture demonstration, a 3.0-m-long vertical string with ten bolts tied to it at equal intervals is dropped from the ceiling of the lecture hall. The string falls on a tin plate, and the class hears the clink of each bolt as it hits the plate. (*a*) The sounds will not occur at equal time intervals. Why? (*b*) Will the time between clinks increase or decrease as the string falls? (*c*) How could the bolts be tied so that the clinks occur at equal intervals? (Assume the string is vertical with the bottom bolt touching the tin plate when the string is released.)

80. Two students are asked to find the height of a particular building using a barometer. Instead of using the barometer as an altitude-measuring device, they take it to the roof of the building and drop it off, timing its fall. One student reports a fall time of 2.0 s, and the other, 2.3 s. What % difference does the 0.3 s make for the estimates of the building's height?

81. Consider the street pattern shown in Fig. 2–51. Each intersection has a traffic signal, and the speed limit is 40 km/h. Suppose you are driving from the west at the speed limit. When you are 10.0 m from the first intersection, all the lights turn green. The lights are green for 13.0 s each. (*a*) Calculate the time needed to reach the third stoplight. Can you make it through all three lights without stopping? (*b*) Another car was stopped at the first light when all the lights turned green. It can accelerate at the rate of $2.00\,\mathrm{m/s^2}$ to the speed limit. Can the second car make it through all three lights without stopping? By how many seconds would it make it, or not make it?

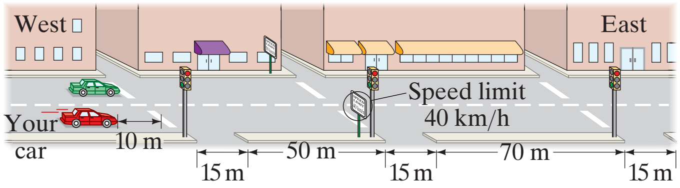


FIGURE 2–51 Problem 81.

82. Suppose a car manufacturer tested its cars for front-end collisions by hauling them up on a crane and dropping them from a certain height. (*a*) Show that the speed just before a car hits the ground, after falling from rest a vertical distance H, is given by $\sqrt{2gH}$. What height corresponds to a collision at (*b*) 35 km/h? (*c*) 95 km/h?

83. A stone is dropped from the roof of a high building. A second stone is dropped 1.50 s later. How far apart are the stones when the second one has reached a speed of 12.0 m/s?

84. A person jumps off a diving board 4.0 m above the water's surface into a deep pool. The person's downward motion stops 1.8 m below the surface of the water. Estimate the average deceleration of the person while under the water.

85. A police car at rest is passed by a speeder traveling at a constant 140 km/h. The police officer takes off in hot pursuit and catches up to the speeder in 850 m, maintaining a constant acceleration. (*a*) Qualitatively plot the position vs. time graph for both cars from the police car's start to the catch-up point. Calculate (*b*) how long it took the police officer to overtake the speeder, (*c*) the required police car acceleration, and (*d*) the speed of the police car at the overtaking point. (*e*) This last result is unrealistic—so which assumptions do we have to reconsider?

86. Agent Bond is standing on a bridge, 15 m above the road below, and his pursuers are getting too close for comfort. He spots a flatbed truck approaching at 25 m/s, which he measures by knowing that the telephone poles the truck is passing are 25 m apart in this region. The roof of the truck is 3.5 m above the road, and Bond quickly calculates how many poles away the truck should be when he drops down from the bridge onto the truck, making his getaway. How many poles is it?

87. Two children are playing on two trampolines. The first child bounces up one-and-a-half times higher than the second child. The initial speed upwards of the second child is 4.0 m/s. (*a*) Find the maximum height the second child reaches. (*b*) What is the initial speed of the first child? (*c*) How long was the first child in the air?

88. In putting, the force with which a golfer strikes a ball is planned so that the ball will stop within some small distance of the cup, say 1.0 m long or short, in case the putt is missed. Accomplishing this from an uphill lie (that is, putting the ball downhill, see Fig. 2–52) is more difficult than from a downhill lie. To see why, assume that on a particular "green" the ball decelerates constantly at $1.8\,\mathrm{m/s^2}$ going downhill, and constantly at $2.6\,\mathrm{m/s^2}$ going uphill. Suppose we have an uphill lie 7.0 m from the cup. Calculate the allowable range of initial velocities we may impart to the ball so that it stops in the range 1.0 m short to 1.0 m long of the cup. Do the same for a downhill lie 7.0 m from the cup. What in your results suggests that the downhill putt is more difficult?

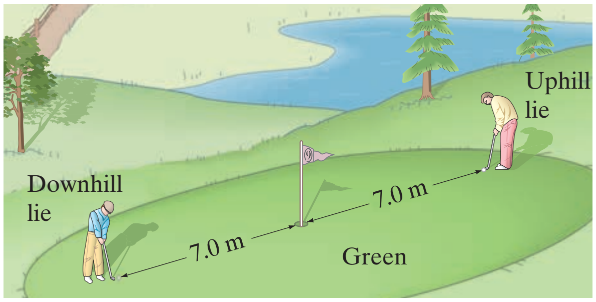


FIGURE 2–52 Problem 88.

89. A person driving her car at 35 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning to red, and she is 28 m away from the near side of the intersection (Fig. 2–53). Should she try to stop, or should she speed up to cross the intersection before the light turns red? The intersection is 15 m wide. Her car's maximum deceleration is $-5.8\,\mathrm{m/s^2}$, whereas it can accelerate from 45 km/h to 65 km/h in 6.0 s. Ignore the length of her car and her reaction time.

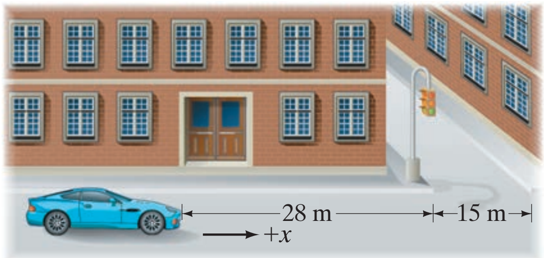


FIGURE 2–53 Problem 89.

90. A car is behind a truck going 18 m/s on the highway. The car's driver looks for an opportunity to pass, guessing that his car can accelerate at $0.60\,\mathrm{m/s^2}$ and that he has to cover the 20-m length of the truck, plus 10 m of extra space at the rear of the truck and 10 m more at the front of it. In the oncoming lane, he sees a car approaching, probably at the speed limit, 25 m/s (55 mph). He estimates that the car is about 500 m away. Should he attempt the pass? Give details.

91. A rock is dropped from a sea cliff and the sound of it striking the ocean is heard 4.1 s later. If the speed of sound is 340 m/s, how high is the cliff?

92. A conveyor belt is used to send burgers through a grilling machine. If the grilling machine is 1.2 m long and the burgers require 2.8 min to cook, how fast must the conveyor belt travel? If the burgers are spaced 25 cm apart, what is the rate of burger production (in burgers/min)?

93. A rock is thrown vertically upward with a speed of 15.0 m/s. Exactly 1.00 s later, a ball is thrown up vertically along the same path with a speed of 22.0 m/s. (*a*) At what time will they strike each other? (*b*) At what height will the collision occur? (*c*) Answer (*a*) and (*b*) assuming that the order is reversed: the ball is thrown 1.00 s before the rock.

94. Figure 2–54 is a position versus time graph for the motion of an object along the x axis. Consider the time interval from A to B. (*a*) Is the object moving in the positive or negative x direction? (*b*) Is the object speeding up or slowing down? (*c*) Is the acceleration of the object positive or negative? Now consider the time interval from D to E. (*d*) Is the object moving in the positive or negative x direction? (*e*) Is the object speeding up or slowing down? (*f*) Is the acceleration of the object positive or negative? (*g*) Finally, answer these same three questions for the time interval from C to D.

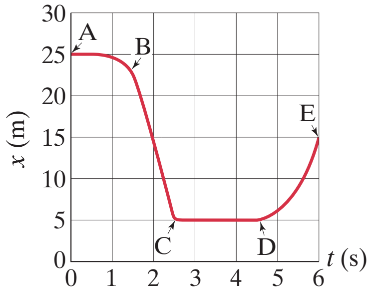


FIGURE 2–54 Problem 94.

95. In the design of a **rapid transit system**, it is necessary to balance the average speed of a train against the distance between station stops. The more stops there are, the slower the train's average speed. To get an idea of this problem, calculate the time it takes a train to make a 15.0-km trip in two situations: (*a*) the stations at which the trains must stop are 3.0 km apart (a total of 6 stations, including those at the ends); and (*b*) the stations are 5.0 km apart (4 stations total). Assume that at each station the train accelerates at a rate of 1.1 m/s^2 until it reaches 95 km/h, then stays at this speed until its brakes are applied for arrival at the next station, at which time it decelerates at -2.0 m/s^2. Assume it stops at each intermediate station for 22 s.

96. A race car driver must average 200.0 km/h over the course of a time trial lasting ten laps. If the first nine laps were done at an average speed of 196.0 km/h, what average speed must be maintained for the last lap?

97. A parachutist bails out of an airplane, and freely falls 75 m (ignore air friction). Then the parachute opens, and her acceleration is -1.5 m/s^2 (up). The parachutist reaches the ground with a speed of 1.5 m/s. (*a*) From how high did she bail out of the plane? (*b*) How much time did her fall take?

98. You stand at the top of a cliff while your friend stands on a beach below you. You drop a ball from rest and see that she catches it 1.4 s later. Your friend then throws the ball up to you, and it comes to rest just as it reaches your hand. What is the speed with which your friend threw the ball?

99. A robot used in a pharmacy picks up a medicine bottle at $t = 0$. It accelerates at 0.20 m/s^2 for 4.5 s, then travels without acceleration for 68 s and finally decelerates at -0.40 m/s^2 for 2.5 s to reach the counter where the pharmacist will take the medicine from the robot. From how far away did the robot fetch the medicine?

100. Bill can throw a ball vertically at a speed 1.5 times faster than Joe can. How many times higher will Bill's ball go than Joe's?

101. On an audio compact disc (CD), digital bits of information are encoded sequentially along a spiral path. Each bit occupies about 0.28 μm. A CD player's readout laser scans along the spiral's sequence of bits at a constant rate of about 1.2 m/s as the CD spins. (*a*) Determine the number N of digital bits that a CD player reads every second. (*b*) The audio information is sent to each of the two loudspeakers 44,100 times per second. Each of these samplings requires 16 bits, and so you might expect the required bit rate for a CD player to be

$$N_0 = 2\left(44{,}100\,\frac{\text{samplings}}{\text{s}}\right)\left(16\,\frac{\text{bits}}{\text{sampling}}\right) = 1.4\times10^6\,\frac{\text{bits}}{\text{s}},$$

where the 2 is for the 2 loudspeakers (the 2 stereo channels). Note that N_0 is less than the number N of bits actually read per second by a CD player. The excess number of bits $(= N - N_0)$ is needed for encoding and error correction. What percentage of the bits on a CD are dedicated to encoding and error correction?

102. Figure 2–55 shows the position vs. time graph for two bicycles, A and B. (*a*) Identify any instant at which the two bicycles have the same velocity. (*b*) Which bicycle has the larger acceleration? (*c*) At which instant(s) are the bicycles passing each other? Which bicycle is passing the other? (*d*) Which bicycle has the larger instantaneous velocity? (*e*) Which bicycle has the larger average velocity?

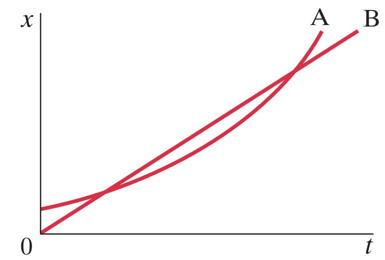


FIGURE 2–55 Problem 102.

103. You are traveling at a constant speed v_M, and there is a car in front of you traveling with a speed v_A. You realize that $v_M > v_A$, so you start slowing down with a constant acceleration a when the distance between you and the other car is x. What relationship between a and x determines whether or not you run into the car in front of you?

ANSWERS TO EXERCISES

A: (*a*) displacement $= -30$ cm; (*b*) total distance $= 50$ cm.
B: (*b*).
C: (*b*).
D: (*a*) +; (*b*) −; (*c*) −; (*d*) +.
E: (*c*).
F: (*b*).
G: (*e*).
H: (*c*).

A basketball flying through the air is an example of motion in two dimensions. In the absence of air resistance, the path would be a perfect parabola. The gold arrow represents the downward acceleration of gravity, $\vec{\mathbf{g}}$.

Galileo analyzed the motion of objects in two dimensions under the action of gravity near the Earth's surface (now called "projectile motion") by separating its horizontal and vertical components.

We will discuss how to manipulate vectors and how to add them. Besides analyzing projectile motion, we will also discuss unit vectors and vector kinematics, plus see how to work with relative velocity.

CHAPTER 3

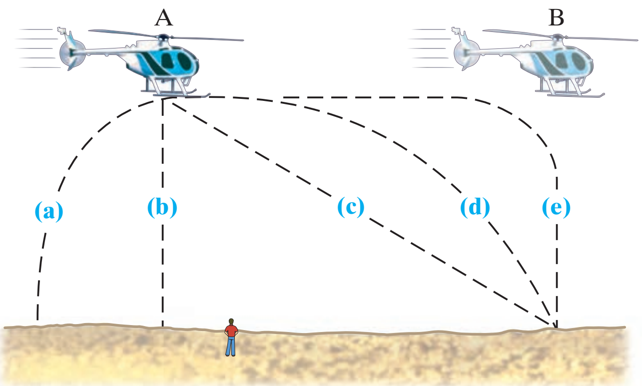
Kinematics in Two or Three Dimensions; Vectors

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also page 23 of Chapter 1 for more explanation.*]

A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies at constant speed in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting wind and air resistance) as seen by a person standing on the ground?



In Chapter 2 we dealt with motion along a straight line. We now consider the motion of objects that move in paths in two (or three) dimensions. In particular, we discuss an important type of motion known as *projectile motion*: objects projected outward near the Earth's surface, such as struck baseballs and golf balls, kicked footballs, and other projectiles. Before beginning our discussion of motion in two dimensions, we will need to discuss vectors, and how to add them.

3–1 Vectors and Scalars

We mentioned in Chapter 2 that the term *velocity* refers not only to how fast an object is moving but also to its direction. A quantity such as velocity, which has *direction* as well as *magnitude*, is a **vector** quantity. Other quantities that are also vectors are displacement, force, and momentum. However, many quantities have no direction associated with them, such as mass, time, and temperature. They are specified completely by a number and units. Such quantities are called **scalar** quantities.

Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, each vector is represented by an arrow. The arrow is always drawn so that it points in the direction of the vector quantity it represents. The *length* of the arrow is drawn proportional to the magnitude of the vector quantity. For example, in Fig. 3–1, green arrows have been drawn representing the velocity of a car at various places as it rounds a curve. The magnitude of the velocity at each point can be read off Fig. 3–1 by measuring the length of the corresponding arrow and using the scale shown ($1\text{ cm} = 90\text{ km/h}$).

When we write the symbol for a vector, we will always use boldface type, with a tiny arrow over the symbol. Thus for velocity we write $\vec{\mathbf{v}}$. If we are concerned only with the magnitude of the vector, we will write simply v, in italics, as we do for other symbols.

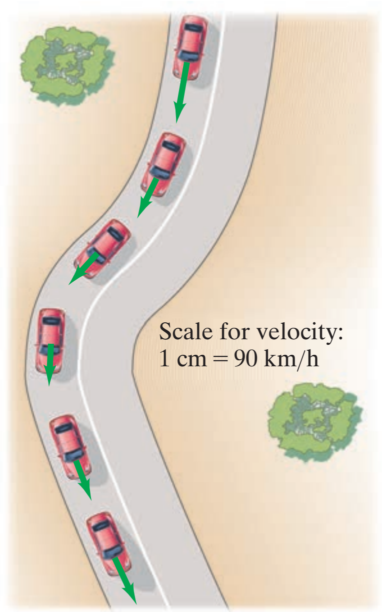


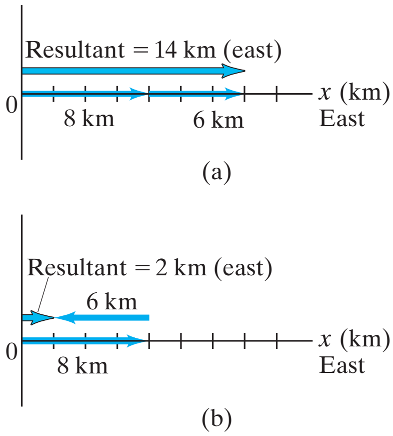
FIGURE 3–1 Car traveling on a road, slowing down to round the curve. The green arrows represent the velocity vector at each position.

3–2 Addition of Vectors—Graphical Methods

Because vectors are quantities that have direction as well as magnitude, they must be added in a special way. In this Chapter, we will deal mainly with displacement vectors, for which we now use the symbol $\vec{\mathbf{D}}$, and velocity vectors, $\vec{\mathbf{v}}$. But the results apply for acceleration and other vectors we will encounter later.

We use simple arithmetic for adding scalars. Simple arithmetic can also be used for adding vectors if they are in the same direction. For example, if a person walks 8 km east one day, and 6 km east the next day, the person will be $8\text{ km} + 6\text{ km} = 14\text{ km}$ east of the point of origin. We say that the *net* or *resultant* displacement is 14 km to the east (Fig. 3–2a). If, on the other hand, the person walks 8 km east on the first day, and 6 km west (in the reverse direction) on the second day, then the person will end up 2 km from the origin (Fig. 3–2b), so the resultant displacement is 2 km to the east. In this case, the resultant displacement is obtained by subtraction: $8\text{ km} - 6\text{ km} = 2\text{ km}$.

FIGURE 3–2 Combining vectors in one dimension.



But simple arithmetic cannot be used if the two vectors are not along the same line. For example, suppose a person walks 10.0 km east and then walks 5.0 km north. These displacements can be represented on a graph in which the positive y axis points north and the positive x axis points east, Fig. 3–3. On this graph, we draw an arrow, labeled $\vec{\mathbf{D}}_1$, to represent the 10.0-km displacement to the east. Then we draw a second arrow, $\vec{\mathbf{D}}_2$, to represent the 5.0-km displacement to the north. Both vectors are drawn to scale, as shown in Fig. 3–3.

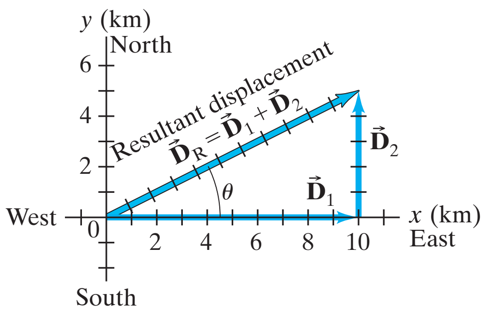


FIGURE 3–3 A person walks 10.0 km east and then 5.0 km north. These two displacements are represented by the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$, which are shown as arrows. Also shown is the resultant displacement vector, $\vec{\mathbf{D}}_R$, which is the vector sum of $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. Measurement on the graph with ruler and protractor shows that $\vec{\mathbf{D}}_R$ has a magnitude of 11.2 km and points at an angle $\theta = 27°$ north of east.

After taking this walk, the person is now 10.0 km east and 5.0 km north of the point of origin. The **resultant displacement** is represented by the arrow labeled $\vec{\mathbf{D}}_{\mathrm{R}}$ in Fig. 3–3. (The subscript R stands for resultant.) Using a ruler and a protractor, you can measure on this diagram (Fig. 3–3 on previous page) that the person is 11.2 km from the origin at an angle $\theta = 27°$ north of east. In other words, the resultant displacement vector has a magnitude of 11.2 km and makes an angle $\theta = 27°$ with the positive x axis. The magnitude (length) of $\vec{\mathbf{D}}_{\mathrm{R}}$ can also be obtained using the theorem of Pythagoras in this case, because D_1, D_2, and D_{R} form a right triangle with D_{R} as the hypotenuse. Thus

$$D_{\mathrm{R}} = \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0\,\mathrm{km})^2 + (5.0\,\mathrm{km})^2}$$
$$= \sqrt{125\,\mathrm{km}^2} = 11.2\,\mathrm{km}.$$

You can use the Pythagorean theorem only when the vectors are *perpendicular* to each other.

The resultant displacement vector, $\vec{\mathbf{D}}_{\mathrm{R}}$, is the sum of the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. That is,

$$\vec{\mathbf{D}}_{\mathrm{R}} = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2.$$

This is a *vector* equation. An important feature of adding two vectors that are not along the same line is that the magnitude of the resultant vector is not equal to the sum of the magnitudes of the two separate vectors, but is smaller than their sum. That is,

$$D_{\mathrm{R}} \leq (D_1 + D_2),$$

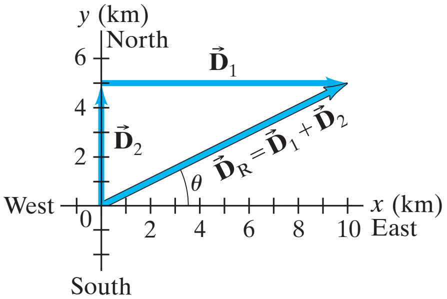
where the equals sign applies only if the two vectors point in the same direction. In our example (Fig. 3–3), $D_{\mathrm{R}} = 11.2\,\mathrm{km}$, whereas $D_1 + D_2$ equals 15 km, which is the total distance traveled. Note also that we cannot set $\vec{\mathbf{D}}_{\mathrm{R}}$ equal to 11.2 km, because we have a vector equation and 11.2 km is only a part of the resultant vector, its magnitude. We could write something like this, though: $\vec{\mathbf{D}}_{\mathrm{R}} = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 = (11.2\,\mathrm{km}, 27°\ \mathrm{N\ of\ E})$.

Figure 3–3 illustrates the general rules for graphically adding two vectors together, no matter what angles they make, to get their sum. The rules are as follows:

1. On a diagram, draw one of the vectors—call it $\vec{\mathbf{D}}_1$—to scale.
2. Next draw the second vector, $\vec{\mathbf{D}}_2$, to scale, placing its tail at the tip of the first vector and being sure its direction is correct.
3. The arrow drawn from the tail of the first vector to the tip of the second vector represents the *sum*, or **resultant**, of the two vectors.

The length of the resultant vector represents its magnitude. Note that vectors can be moved parallel to themselves on paper (maintaining the same length and angle) to accomplish these manipulations. The length of the resultant can be measured with a ruler and compared to the scale. Angles can be measured with a protractor. This method is known as the **tail-to-tip method of adding vectors**.

FIGURE 3–4 If the vectors are added in reverse order, the resultant is the same. (Compare to Fig. 3–3.)



The resultant is not affected by the order in which the vectors are added. For example, a displacement of 5.0 km north, to which is added a displacement of 10.0 km east, yields a resultant of 11.2 km and angle $\theta = 27°$ (see Fig. 3–4), the same as when they were added in reverse order (Fig. 3–3). Thus we can write, using $\vec{\mathbf{V}}$ to represent any type of vector,

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1, \qquad \text{[commutative property]} \quad \textbf{(3–1a)}$$

which is known as the *commutative* property of vector addition.

The tail-to-tip method of adding vectors can be extended to three or more vectors. The resultant is drawn from the tail of the first vector to the tip of the last

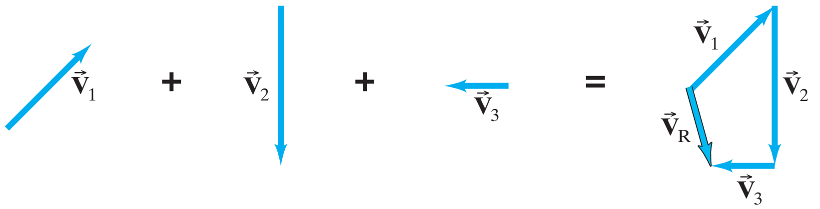


FIGURE 3–5 The resultant of three vectors: $\vec{V}_R = \vec{V}_1 + \vec{V}_2 + \vec{V}_3$.

one added. An example is shown in Fig. 3–5; the three vectors could represent displacements (northeast, south, west) or perhaps three forces. Check for yourself that you get the same resultant no matter in which order you add the three vectors; that is,

$$\left(\vec{V}_1 + \vec{V}_2\right) + \vec{V}_3 = \vec{V}_1 + \left(\vec{V}_2 + \vec{V}_3\right), \quad \text{[associative property]} \quad \textbf{(3–1b)}$$

which is known as the *associative* property of vector addition.

A second way to add two vectors is the **parallelogram method**. It is fully equivalent to the tail-to-tip method. In this method, the two vectors are drawn starting from a common origin, and a parallelogram is constructed using these two vectors as adjacent sides as shown in Fig. 3–6b. The resultant is the diagonal drawn from the common origin. In Fig. 3–6a, the tail-to-tip method is shown, and we can see that both methods yield the same result.

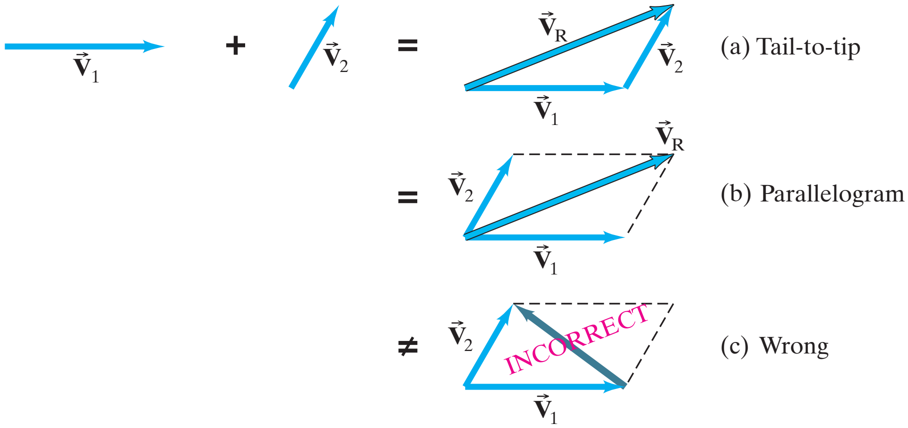


FIGURE 3–6 Vector addition by two different methods, (a) and (b). Part (c) is incorrect.

It is a common error to draw the sum vector as the diagonal running between the tips of the two vectors, as in Fig. 3–6c. *This is incorrect*: it does not represent the sum of the two vectors. (In fact, it represents their difference, $\vec{V}_2 - \vec{V}_1$, as we will see in the next Section.)

CAUTION

Be sure to use the correct diagonal on the parallelogram to get the resultant

CONCEPTUAL EXAMPLE 3–1 **Range of vector lengths.** Suppose two vectors each have length 3.0 units. What is the range of possible lengths for the vector representing the sum of the two?

RESPONSE The sum can take on any value from 6.0 $(= 3.0 + 3.0)$ where the vectors point in the same direction, to 0 $(= 3.0 - 3.0)$ when the vectors are antiparallel. Magnitudes between 0 and 6.0 occur when the two vectors are at an angle other than 0° and 180°.

EXERCISE A If the two vectors of Example 3–1 are perpendicular to each other, what is the resultant vector length?

3–3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

Given a vector $\vec{V}$, we define the *negative* of this vector $(-\vec{V})$ to be a vector with the same magnitude as $\vec{V}$ but opposite in direction, Fig. 3–7. Note, however, that no vector is ever negative in the sense of its magnitude: the magnitude of every vector is positive. Rather, a minus sign tells us about its direction.

FIGURE 3–7 The negative of a vector is a vector having the same length but opposite direction.

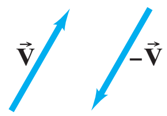
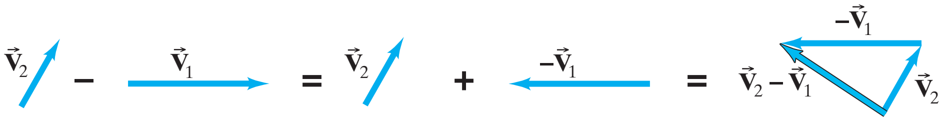


FIGURE 3–8 Subtracting two vectors: $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$.



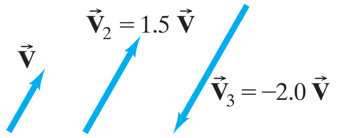
We can now define the subtraction of one vector from another: the difference between two vectors $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$ is defined as

$$\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = \vec{\mathbf{V}}_2 + \left(-\vec{\mathbf{V}}_1\right).$$

That is, the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus our rules for addition of vectors can be applied as shown in Fig. 3–8 using the tail-to-tip method.

A vector $\vec{\mathbf{V}}$ can be multiplied by a scalar c. We define their product so that $c\vec{\mathbf{V}}$ has the same direction as $\vec{\mathbf{V}}$ and has magnitude cV. That is, multiplication of a vector by a positive scalar c changes the magnitude of the vector by a factor c but doesn't alter the direction. If c is a negative scalar, the magnitude of the product $c\vec{\mathbf{V}}$ is still $|c|V$ (where $|c|$ means the magnitude of c), but the direction is precisely opposite to that of $\vec{\mathbf{V}}$. See Fig. 3–9.

FIGURE 3–9 Multiplying a vector $\vec{\mathbf{V}}$ by a scalar c gives a vector whose magnitude is c times greater and in the same direction as $\vec{\mathbf{V}}$ (or opposite direction if c is negative).



EXERCISE B What does the "incorrect" vector in Fig. 3–6c represent? (*a*) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$, (*b*) $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$, (*c*) something else (specify).

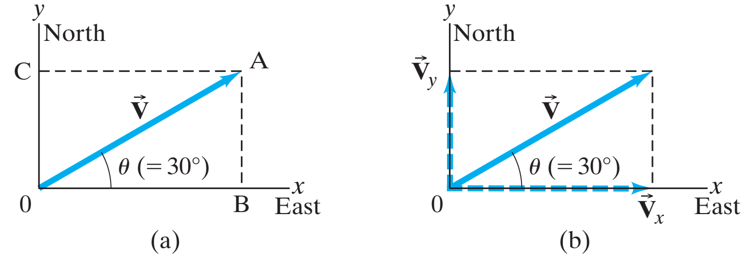
3–4 Adding Vectors by Components

Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors. But do not forget graphical methods—they are useful for visualizing, for checking your math, and thus for getting the correct result.

Consider first a vector $\vec{\mathbf{V}}$ that lies in a particular plane, and we have chosen an x and a y axis on this plane. Then $\vec{\mathbf{V}}$ can be expressed as the sum of two other vectors, called the **components** of the original vector, which are usually chosen to be along the x and y axes. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 3–10; the vector $\vec{\mathbf{V}}$ could be a displacement vector that points at an angle $\theta = 30°$ north of east, where we have chosen the positive x axis to be to the east and the positive y axis north. This vector $\vec{\mathbf{V}}$ is resolved into its x and y components by drawing dashed lines out from the tip (A) of the vector (lines AB and AC), making them perpendicular to the x and y axes. Then the lines 0B and 0C represent the x and y components of $\vec{\mathbf{V}}$, respectively, as shown in Fig. 3–10b. These *vector components* are written $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$. In this book we usually show vector components as arrows, like vectors, but dashed. The *scalar components*, V_x and V_y, are the magnitudes of the vector components, with units, accompanied by a positive or negative sign depending on whether they point along the positive or negative x or y axis. As can be seen in Fig. 3–10, $\vec{\mathbf{V}}_x + \vec{\mathbf{V}}_y = \vec{\mathbf{V}}$ by the parallelogram method of adding vectors.

Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are $\vec{\mathbf{V}}_x$, $\vec{\mathbf{V}}_y$, and $\vec{\mathbf{V}}_z$.

FIGURE 3–10 Resolving a vector $\vec{\mathbf{V}}$ into its components along a chosen set of x and y axes. The components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.



The use of trigonometric functions for finding the components of a vector is illustrated in Fig. 3–11, where a vector and its two components are thought of as making up a right triangle. (See also Appendix A for other details on trigonometric functions and identities.) We then see that the sine, cosine, and tangent are as given in Fig. 3–11, where θ is the angle $\vec{\mathbf{V}}$ makes with the $+x$ axis, measured positive counterclockwise. If we multiply the definition of $\sin\theta = V_y/V$ by V on both sides, we get

$$V_y = V\sin\theta. \tag{3–2a}$$

Similarly, from the definition of $\cos\theta$, we obtain

$$V_x = V\cos\theta. \tag{3–2b}$$

Keep in mind that θ is chosen (by convention) to be the angle that the vector makes with the positive x axis, measured positive counterclockwise.

The components of a given vector will be different for different choices of coordinate axes. It is therefore crucial to specify the choice of coordinate system when giving the components.

There are two ways to specify a vector in a given coordinate system:

1. We can give its components, V_x and V_y.
2. We can give its magnitude V and the angle θ it makes with the positive x axis.

We can shift from one description to the other using Eqs. 3–2, and, for the reverse, by using the theorem of Pythagoras† and the definition of tangent:

$$V = \sqrt{V_x^2 + V_y^2} \tag{3–3a}$$

$$\tan\theta = \frac{V_y}{V_x} \tag{3–3b}$$

as can be seen in Fig. 3–11.

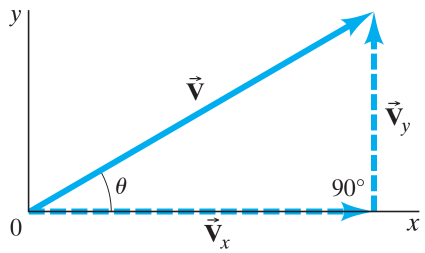
We can now discuss how to add vectors using components. The first step is to resolve each vector into its components, Eqs. 3–2. Next we can see, using Fig. 3–12, that the addition of any two vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ to give a resultant $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, implies that

$$\begin{aligned} V_{Rx} &= V_{1x} + V_{2x} \\ V_{Ry} &= V_{1y} + V_{2y}. \end{aligned} \tag{3–4}$$

That is, the sum of the x components equals the x component of the resultant vector, and the sum of the y components equals the y component of the resultant vector, as can be verified by a careful examination of Fig. 3–12. Note that we do *not* add x components to y components.

If the magnitude and direction of the resultant vector are desired, they can be obtained using Eqs. 3–3.

†In three dimensions, the theorem of Pythagoras becomes $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$, where V_z is the component along the third, or z, axis.



$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

FIGURE 3–11 Finding the components of a vector using trigonometric functions. The equations are valid only if θ is the angle $\vec{\mathbf{V}}$ makes with the positive x axis.

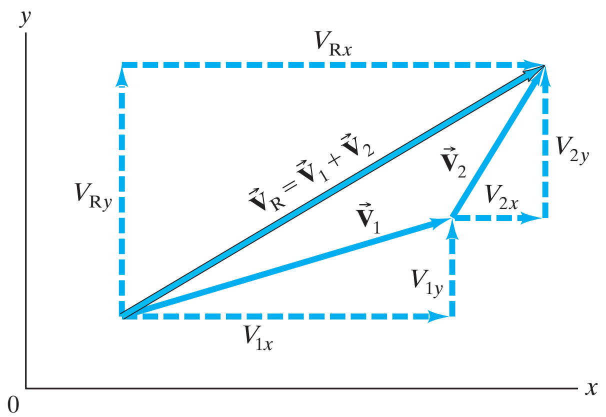


FIGURE 3–12 The components of $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ are
$V_{Rx} = V_{1x} + V_{2x}$
$V_{Ry} = V_{1y} + V_{2y}$.

The components of a given vector depend on the choice of coordinate axes. You can often reduce the work involved in adding vectors by a good choice of axes—for example, by choosing one of the axes to be in the same direction as one of the vectors. Then that vector will have only one nonzero component.

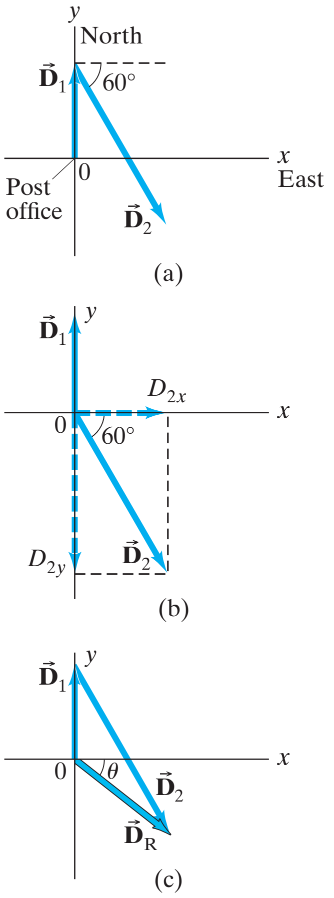


FIGURE 3–13 Example 3–2. (a) The two displacement vectors, $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. (b) $\vec{\mathbf{D}}_2$ is resolved into its components. (c) $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$ are added graphically to obtain the resultant $\vec{\mathbf{D}}_R$. The component method of adding the vectors is explained in the Example.

EXAMPLE 3–2 **Mail carrier's displacement.** A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km (Fig. 3–13a). What is her displacement from the post office?

APPROACH We choose the positive x axis to be east and the positive y axis to be north, since those are the compass directions used on most maps. The origin of the xy coordinate system is at the post office. We resolve each vector into its x and y components. We add the x components together, and then the y components together, giving us the x and y components of the resultant.

SOLUTION Resolve each displacement vector into its components, as shown in Fig. 3–13b. Since $\vec{\mathbf{D}}_1$ has magnitude 22.0 km and points north, it has only a y component:

$$D_{1x} = 0, \qquad D_{1y} = 22.0\text{ km}.$$

$\vec{\mathbf{D}}_2$ has both x and y components:

$$D_{2x} = +(47.0\text{ km})(\cos 60°) = +(47.0\text{ km})(0.500) = +23.5\text{ km}$$
$$D_{2y} = -(47.0\text{ km})(\sin 60°) = -(47.0\text{ km})(0.866) = -40.7\text{ km}.$$

Notice that D_{2y} is negative because this vector component points along the negative y axis. The resultant vector, $\vec{\mathbf{D}}_R$, has components:

$$D_{Rx} = D_{1x} + D_{2x} = 0\text{ km} + 23.5\text{ km} = +23.5\text{ km}$$
$$D_{Ry} = D_{1y} + D_{2y} = 22.0\text{ km} + (-40.7\text{ km}) = -18.7\text{ km}.$$

This specifies the resultant vector completely:

$$D_{Rx} = 23.5\text{ km}, \qquad D_{Ry} = -18.7\text{ km}.$$

We can also specify the resultant vector by giving its magnitude and angle using Eqs. 3–3:

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5\text{ km})^2 + (-18.7\text{ km})^2} = 30.0\text{ km}$$

$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-18.7\text{ km}}{23.5\text{ km}} = -0.796.$$

A calculator with a key labeled INV TAN, or ARC TAN, or TAN^{-1} gives $\theta = \tan^{-1}(-0.796) = -38.5°$. The negative sign means $\theta = 38.5°$ below the x axis, Fig. 3–13c. So, the resultant displacement is 30.0 km directed at 38.5° in a southeasterly direction.

NOTE Always be attentive about the quadrant in which the resultant vector lies. An electronic calculator does not fully give this information, but a good diagram does.

PROBLEM SOLVING
Identify the correct quadrant by drawing a careful diagram

As we saw in Example 3–2, any component that points along the negative x or y axis gets a minus sign. The signs of trigonometric functions depend on which "quadrant" the angle falls in: for example, the tangent is positive in the first and third quadrants (from 0° to 90°, and 180° to 270°), but negative in the second and fourth quadrants; see Appendix A–9, Fig. A–6. The best way to keep track of angles, and to check any vector result, is always to draw a vector diagram, like Fig. 3–13. A vector diagram gives you something tangible to look at when analyzing a problem, and provides a check on the results.

The following Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do to get you thinking and involved in the problem at hand.

PROBLEM SOLVING — Adding Vectors

Here is a brief summary of how to add two or more vectors using components:

1. **Draw a diagram**, adding the vectors graphically by either the parallelogram or tail-to-tip method.
2. **Choose** x and y **axes**. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors, which then will have only one component.)
3. **Resolve** each vector into its x and y **components**, showing each component along its appropriate (x or y) axis as a (dashed) arrow.
4. **Calculate each component** (when not given) using sines and cosines. If θ_1 is the angle that vector $\vec{\mathbf{V}}_1$ makes with the positive x axis, then:
$$V_{1x} = V_1 \cos\theta_1, \qquad V_{1y} = V_1 \sin\theta_1.$$
Pay careful attention to **signs**: any component that points along the negative x or y axis gets a minus sign.
5. **Add** the x **components** together to get the x component of the resultant. Similarly for y:
$$V_{Rx} = V_{1x} + V_{2x} + \text{any others}$$
$$V_{Ry} = V_{1y} + V_{2y} + \text{any others.}$$
This is the answer: the components of the resultant vector. Check signs to see if they fit the quadrant shown in your diagram (point 1 above).
6. If you want to know the **magnitude and direction** of the resultant vector, use Eqs. 3–3:
$$V_R = \sqrt{V_{Rx}^2 + V_{Ry}^2}, \qquad \tan\theta = \frac{V_{Ry}}{V_{Rx}}.$$
The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle θ.

EXAMPLE 3–3 **Three short trips.** An airplane trip involves three legs, with two stopovers, as shown in Fig. 3–14a. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?

APPROACH We follow the steps in the Problem Solving Strategy above.

SOLUTION

1. **Draw a diagram** such as Fig. 3–14a, where $\vec{\mathbf{D}}_1$, $\vec{\mathbf{D}}_2$, and $\vec{\mathbf{D}}_3$ represent the three legs of the trip, and $\vec{\mathbf{D}}_R$ is the plane's total displacement.
2. **Choose axes**: Axes are also shown in Fig. 3–14a: x is east, y north.
3. **Resolve components**: It is imperative to draw a good diagram. The components are drawn in Fig. 3–14b. Instead of drawing all the vectors starting from a common origin, as we did in Fig. 3–13b, here we draw them "tail-to-tip" style, which is just as valid and may make it easier to see.
4. **Calculate the components**:
$$\begin{aligned}
\vec{\mathbf{D}}_1: D_{1x} &= +D_1 \cos 0^\circ = D_1 = 620\text{ km} \\
D_{1y} &= +D_1 \sin 0^\circ = 0\text{ km} \\
\vec{\mathbf{D}}_2: D_{2x} &= +D_2 \cos 45^\circ = +(440\text{ km})(0.707) = +311\text{ km} \\
D_{2y} &= -D_2 \sin 45^\circ = -(440\text{ km})(0.707) = -311\text{ km} \\
\vec{\mathbf{D}}_3: D_{3x} &= -D_3 \cos 53^\circ = -(550\text{ km})(0.602) = -331\text{ km} \\
D_{3y} &= -D_3 \sin 53^\circ = -(550\text{ km})(0.799) = -439\text{ km.}
\end{aligned}$$
We have given a minus sign to each component that in Fig. 3–14b points in the $-x$ or $-y$ direction. The components are shown in the Table in the margin.
5. **Add the components**: We add the x components together, and we add the y components together to obtain the x and y components of the resultant:
$$D_{Rx} = D_{1x} + D_{2x} + D_{3x} = 620\text{ km} + 311\text{ km} - 331\text{ km} = 600\text{ km}$$
$$D_{Ry} = D_{1y} + D_{2y} + D_{3y} = 0\text{ km} - 311\text{ km} - 439\text{ km} = -750\text{ km.}$$
The x and y components of the resultant are 600 km and −750 km, and point respectively to the east and south. This is one way to give the answer.
6. **Magnitude and direction**: We can also give the answer as
$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(600)^2 + (-750)^2}\text{ km} = 960\text{ km}$$
$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-750\text{ km}}{600\text{ km}} = -1.25, \qquad \text{so } \theta = -51^\circ.$$
Thus, the total displacement has magnitude 960 km and points 51° below the x axis (south of east), as was shown in our original sketch, Fig. 3–14a.

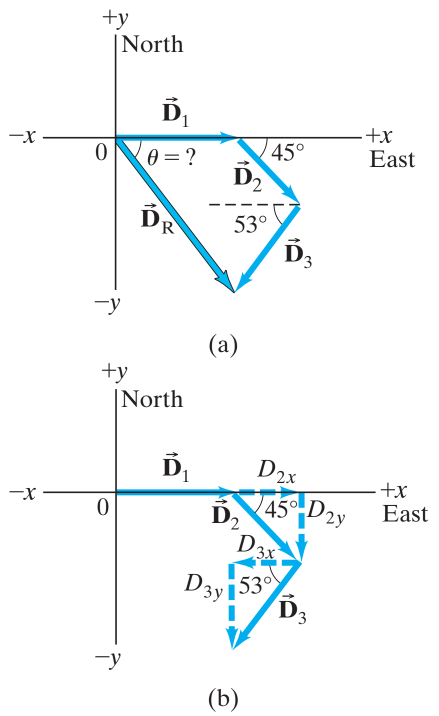


FIGURE 3–14 Example 3–3.

Vector	Components x (km)	y (km)
$\vec{\mathbf{D}}_1$	620	0
$\vec{\mathbf{D}}_2$	311	−311
$\vec{\mathbf{D}}_3$	−331	−439
$\vec{\mathbf{D}}_R$	600	−750

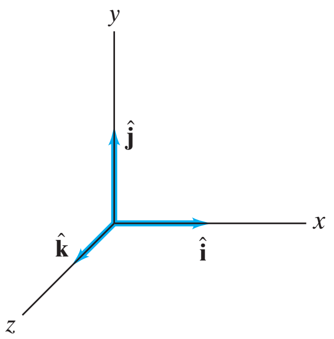


FIGURE 3–15 Unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ along the x, y, and z axes.

3–5 Unit Vectors

Vectors can be conveniently written in terms of *unit vectors*. A **unit vector** is defined to have a magnitude exactly equal to one (1). (The word "unit" comes from the Latin, unus, meaning "one".) It is useful to define unit vectors that point along coordinate axes, and in an x, y, z rectangular coordinate system these unit vectors are called $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$. They point, respectively, along the positive x, y, and z axes as shown in Fig. 3–15. Like other vectors, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ do not have to be placed at the origin, but can be placed elsewhere as long as the direction and unit length remain unchanged. It is common to write unit vectors with a "hat": $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ (and we will do so in this book) as a reminder that each has magnitude of exactly one unit.

Because of the definition of multiplication of a vector by a scalar (Section 3–3), the components of a vector $\vec{\mathbf{V}}$ can be written $\vec{\mathbf{V}}_x = V_x\hat{\mathbf{i}}$, $\vec{\mathbf{V}}_y = V_y\hat{\mathbf{j}}$, and $\vec{\mathbf{V}}_z = V_z\hat{\mathbf{k}}$. Hence any vector $\vec{\mathbf{V}}$ can be written in terms of its components as

$$\vec{\mathbf{V}} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}}. \qquad \textbf{(3–5)}$$

Unit vectors are helpful when adding vectors analytically by components. For example, Eq. 3–4 can be seen to be true by using unit vector notation for each vector (which we write for the two-dimensional case, with the extension to three dimensions being straightforward):

$$\begin{aligned}\vec{\mathbf{V}}_{\mathrm{R}} = (V_{\mathrm{R}x})\hat{\mathbf{i}} + (V_{\mathrm{R}y})\hat{\mathbf{j}} &= \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\\ &= (V_{1x}\hat{\mathbf{i}} + V_{1y}\hat{\mathbf{j}}) + (V_{2x}\hat{\mathbf{i}} + V_{2y}\hat{\mathbf{j}})\\ &= (V_{1x} + V_{2x})\hat{\mathbf{i}} + (V_{1y} + V_{2y})\hat{\mathbf{j}}.\end{aligned}$$

Comparing the first line to the third line, we get Eqs. 3–4.

EXAMPLE 3–4 **Using unit vectors.** Write the vectors of Example 3–2 in unit vector notation, and perform the addition.

APPROACH We use the components we found in Example 3–2,

$$D_{1x} = 0, \quad D_{1y} = 22.0\,\mathrm{km}, \quad \text{and} \quad D_{2x} = 23.5\,\mathrm{km}, \quad D_{2y} = -40.7\,\mathrm{km},$$

and we now write them in the form of Eq. 3–5.

SOLUTION We have

$$\begin{aligned}\vec{\mathbf{D}}_1 &= 0\hat{\mathbf{i}} + 22.0\,\mathrm{km}\,\hat{\mathbf{j}}\\ \vec{\mathbf{D}}_2 &= 23.5\,\mathrm{km}\,\hat{\mathbf{i}} - 40.7\,\mathrm{km}\,\hat{\mathbf{j}}.\end{aligned}$$

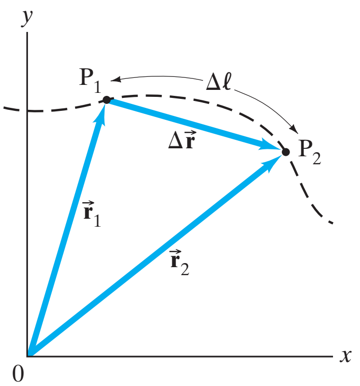
Then the resultant displacement is

$$\begin{aligned}\vec{\mathbf{D}}_{\mathrm{R}} = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 &= (0 + 23.5)\,\mathrm{km}\,\hat{\mathbf{i}} + (22.0 - 40.7)\,\mathrm{km}\,\hat{\mathbf{j}}\\ &= 23.5\,\mathrm{km}\,\hat{\mathbf{i}} - 18.7\,\mathrm{km}\,\hat{\mathbf{j}}.\end{aligned}$$

The components of the resultant displacement, $\vec{\mathbf{D}}_{\mathrm{R}}$, are $D_x = 23.5\,\mathrm{km}$ and $D_y = -18.7\,\mathrm{km}$. The magnitude of $\vec{\mathbf{D}}_{\mathrm{R}}$ is $D_{\mathrm{R}} = \sqrt{(23.5\,\mathrm{km})^2 + (18.7\,\mathrm{km})^2} = 30.0\,\mathrm{km}$, just as in Example 3–2.

3–6 Vector Kinematics

FIGURE 3–16 Path of a particle in the xy plane. At time t_1 the particle is at point P_1 given by the position vector $\vec{\mathbf{r}}_1$; at t_2 the particle is at point P_2 given by the position vector $\vec{\mathbf{r}}_2$. The displacement vector for the time interval $t_2 - t_1$ is $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$. The actual distance traveled along the path between P_1 and P_2 is $\Delta\ell$.



We can now extend our definitions of velocity and acceleration in a formal way to two- and three-dimensional motion. Suppose a particle follows a path in the xy plane as shown in Fig. 3–16. At time t_1, the particle is at point P_1, and at time t_2, it is at point P_2. The vector $\vec{\mathbf{r}}_1$ is the position vector of the particle at time t_1 (it represents the displacement of the particle from the origin of the coordinate system). And $\vec{\mathbf{r}}_2$ is the position vector at time t_2.

In one dimension, we defined displacement as the *change in position* of the particle. In the more general case of two or three dimensions, the **displacement vector** is defined as the vector representing change in position. We call it $\Delta\vec{\mathbf{r}}$,† where

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1.$$

This represents the displacement during the time interval $\Delta t = t_2 - t_1$.

†We used $\vec{\mathbf{D}}$ for the displacement vector earlier in the Chapter for illustrating vector addition. The new notation here, $\Delta\vec{\mathbf{r}}$, emphasizes that it is the difference between two position vectors.

In unit vector notation, we can write

$$\vec{\mathbf{r}}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}, \qquad \textbf{(3–6a)}$$

where x_1, y_1, and z_1 are the coordinates of point P_1. Similarly,

$$\vec{\mathbf{r}}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}.$$

Hence

$$\Delta\vec{\mathbf{r}} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}. \qquad \textbf{(3–6b)}$$

If the motion is along the x axis only, then $y_2 - y_1 = 0,\ z_2 - z_1 = 0$, and the magnitude of the displacement is $\Delta r = x_2 - x_1$, which is consistent with our earlier one-dimensional equation (Section 2–1). Even in one dimension, displacement is a vector, as are velocity and acceleration.

The **average velocity vector** over the time interval $\Delta t = t_2 - t_1$ is defined as

$$\text{average velocity} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}. \qquad \textbf{(3–7)}$$

Now let us consider shorter and shorter time intervals—that is, we let Δt approach zero so that the distance between points P_2 and P_1 also approaches zero, Fig. 3–17a. We define the **instantaneous velocity vector** as the limit of the average velocity as Δt approaches zero:

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}. \qquad \textbf{(3–8)}$$

The direction of $\vec{\mathbf{v}}$ at any moment is along the line tangent to the path at that moment (Fig. 3–17b).

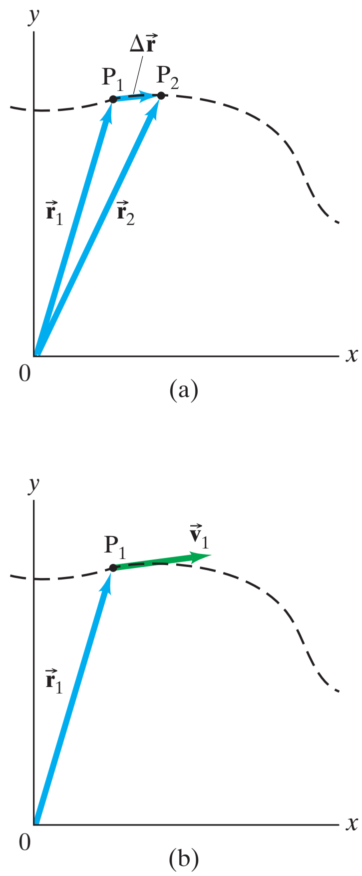


FIGURE 3–17 (a) As we take Δt and $\Delta\vec{\mathbf{r}}$ smaller and smaller [compare to Fig. 3–16] we see that the direction of $\Delta\vec{\mathbf{r}}$ and of the instantaneous velocity ($\Delta\vec{\mathbf{r}}/\Delta t$, where $\Delta t \to 0$) is (b) tangent to the curve at P_1.

Note that the magnitude of the average velocity in Fig. 3–16 is not equal to the average speed, which is the actual distance traveled along the path, $\Delta \ell$, divided by Δt. In some special cases, the average speed and average velocity are equal in magnitude (such as motion along a straight line in one direction), but in general they are not. However, in the limit $\Delta t \to 0$, Δr always approaches $\Delta \ell$, so the instantaneous speed *always* equals the magnitude of the instantaneous velocity at any time.

The instantaneous velocity (Eq. 3–8) is equal to the derivative of the position vector with respect to time. Equation 3–8 can be written in terms of components starting with Eq. 3–6a as:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}, \qquad \textbf{(3–9)}$$

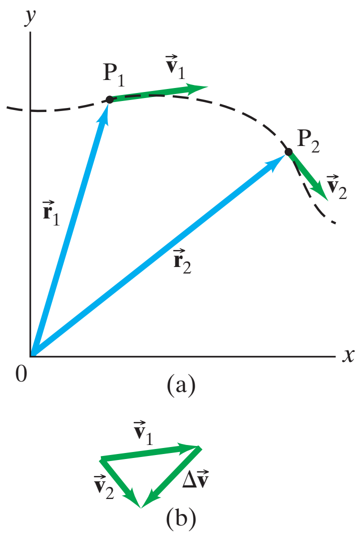
where $v_x = dx/dt,\ v_y = dy/dt,\ v_z = dz/dt$ are the x, y, and z components of the velocity. Note that $d\hat{\mathbf{i}}/dt = d\hat{\mathbf{j}}/dt = d\hat{\mathbf{k}}/dt = 0$ since these unit vectors are constant in both magnitude and direction.

Acceleration in two or three dimensions is treated in a similar way. The **average acceleration vector**, over a time interval $\Delta t = t_2 - t_1$ is defined as

$$\text{average acceleration} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1}, \qquad \textbf{(3–10)}$$

where $\Delta\vec{\mathbf{v}}$ is the change in the instantaneous velocity vector during that time interval: $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$. Note that $\vec{\mathbf{v}}_2$ in many cases, such as in Fig. 3–18a, may not be in the same direction as $\vec{\mathbf{v}}_1$. Hence the average acceleration vector may be in a different direction from either $\vec{\mathbf{v}}_1$ or $\vec{\mathbf{v}}_2$ (Fig. 3–18b). Furthermore, $\vec{\mathbf{v}}_2$ and $\vec{\mathbf{v}}_1$ may have the same magnitude but different directions, and the difference of two such vectors will not be zero. Hence acceleration can result from either a change in the magnitude of the velocity, or from a change in direction of the velocity, or from a change in both.

FIGURE 3–18 (a) Velocity vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ at instants t_1 and t_2 for a particle at points P_1 and P_2, as in Fig. 3–16. (b) The direction of the average acceleration is in the direction of $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$.



The **instantaneous acceleration vector** is defined as the limit of the average acceleration vector as the time interval Δt is allowed to approach zero:

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}, \qquad \textbf{(3–11)}$$

and is thus the derivative of $\vec{\mathbf{v}}$ with respect to t.

We can write $\vec{\mathbf{a}}$ using components:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}} + \frac{dv_z}{dt}\hat{\mathbf{k}}$$

$$= a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}, \qquad \textbf{(3–12a)}$$

where $a_x = dv_x/dt$, etc. Because $v_x = dx/dt$, then $a_x = dv_x/dt = d^2x/dt^2$, as we saw in Section 2–4. Thus we can also write the acceleration as

$$\vec{\mathbf{a}} = \frac{d^2x}{dt^2}\hat{\mathbf{i}} + \frac{d^2y}{dt^2}\hat{\mathbf{j}} + \frac{d^2z}{dt^2}\hat{\mathbf{k}}. \qquad \textbf{(3–12b)}$$

The instantaneous acceleration will be nonzero not only when the magnitude of the velocity changes, but also if its direction changes. For example, a person riding in a car traveling at constant speed around a curve, or a child riding on a merry-go-round, will both experience an acceleration because of a change in the direction of the velocity, even though the speed may be constant. (More on this in Chapter 5.)

In general, we will use the terms "velocity" and "acceleration" to mean the instantaneous values. If we want to discuss average values, we will use the word "average."

EXAMPLE 3–5 **Position given as a function of time.** The position of a particle as a function of time is given by

$$\vec{\mathbf{r}} = \left[(5.0\,\text{m/s})t + \left(6.0\,\text{m/s}^2\right)t^2\right]\hat{\mathbf{i}} + \left[(7.0\,\text{m}) - \left(3.0\,\text{m/s}^3\right)t^3\right]\hat{\mathbf{j}},$$

where r is in meters and t is in seconds. (a) What is the particle's displacement between $t_1 = 2.0\,\text{s}$ and $t_2 = 3.0\,\text{s}$? (b) Determine the particle's instantaneous velocity and acceleration as a function of time. (c) Evaluate $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ at $t = 3.0\,\text{s}$.

APPROACH For (a), we find $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$, inserting $t_1 = 2.0\,\text{s}$ for finding $\vec{\mathbf{r}}_1$, and $t_2 = 3.0\,\text{s}$ for $\vec{\mathbf{r}}_2$. For (b), we take derivatives (Eqs. 3–9 and 3–12), and for (c) we substitute $t = 3.0\,\text{s}$ into our results in (b).

SOLUTION (a) We insert $t_1 = 2.0\,\text{s}$ into the given equation for $\vec{\mathbf{r}}$:

$$\vec{\mathbf{r}}_1 = \left[(5.0\,\text{m/s})(2.0\,\text{s}) + \left(6.0\,\text{m/s}^2\right)(2.0\,\text{s})^2\right]\hat{\mathbf{i}} + \left[(7.0\,\text{m}) - \left(3.0\,\text{m/s}^3\right)(2.0\,\text{s})^3\right]\hat{\mathbf{j}}$$
$$= (34\,\text{m})\hat{\mathbf{i}} - (17\,\text{m})\hat{\mathbf{j}}.$$

Similarly, at $t_2 = 3.0\,\text{s}$,

$$\vec{\mathbf{r}}_2 = (15\,\text{m} + 54\,\text{m})\hat{\mathbf{i}} + (7.0\,\text{m} - 81\,\text{m})\hat{\mathbf{j}} = (69\,\text{m})\hat{\mathbf{i}} - (74\,\text{m})\hat{\mathbf{j}}.$$

Thus

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = (69\,\text{m} - 34\,\text{m})\hat{\mathbf{i}} + (-74\,\text{m} + 17\,\text{m})\hat{\mathbf{j}} = (35\,\text{m})\hat{\mathbf{i}} - (57\,\text{m})\hat{\mathbf{j}}.$$

That is, $\Delta x = 35\,\text{m}$, and $\Delta y = -57\,\text{m}$.

(b) To find velocity, we take the derivative of the given $\vec{\mathbf{r}}$ with respect to time, noting (Appendix B–2) that $d\left(t^2\right)/dt = 2t$, and $d\left(t^3\right)/dt = 3t^2$:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \left[5.0\,\text{m/s} + \left(12\,\text{m/s}^2\right)t\right]\hat{\mathbf{i}} + \left[0 - \left(9.0\,\text{m/s}^3\right)t^2\right]\hat{\mathbf{j}}.$$

The acceleration is (keeping only two significant figures):

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \left(12\,\text{m/s}^2\right)\hat{\mathbf{i}} - \left(18\,\text{m/s}^3\right)t\,\hat{\mathbf{j}}.$$

Thus $a_x = 12\,\text{m/s}^2$ is constant; but $a_y = -\left(18\,\text{m/s}^3\right)t$ depends linearly on time, increasing in magnitude with time in the negative y direction.

(c) We substitute $t = 3.0\,\text{s}$ into the equations we just derived for $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$:

$$\vec{\mathbf{v}} = (5.0\,\text{m/s} + 36\,\text{m/s})\hat{\mathbf{i}} - (81\,\text{m/s})\hat{\mathbf{j}} = (41\,\text{m/s})\hat{\mathbf{i}} - (81\,\text{m/s})\hat{\mathbf{j}}$$
$$\vec{\mathbf{a}} = \left(12\,\text{m/s}^2\right)\hat{\mathbf{i}} - \left(54\,\text{m/s}^2\right)\hat{\mathbf{j}}.$$

Their magnitudes at $t = 3.0\,\text{s}$ are $v = \sqrt{(41\,\text{m/s})^2 + (81\,\text{m/s})^2} = 91\,\text{m/s}$, and $a = \sqrt{\left(12\,\text{m/s}^2\right)^2 + \left(54\,\text{m/s}^2\right)^2} = 55\,\text{m/s}^2$.

Constant Acceleration

In Chapter 2 we studied the important case of one-dimensional motion for which the acceleration is constant. In two or three dimensions, if the acceleration vector, $\vec{\mathbf{a}}$, is constant in magnitude and direction, then $a_x = \text{constant}$, $a_y = \text{constant}$, $a_z = \text{constant}$. The average acceleration in this case is equal to the instantaneous acceleration at any moment. The equations we derived in Chapter 2 for one dimension, Eqs. 2–12a, b, and c, apply separately to each perpendicular component of two- or three-dimensional motion.

In two dimensions we let $\vec{\mathbf{v}}_0 = v_{x0}\hat{\mathbf{i}} + v_{y0}\hat{\mathbf{j}}$ be the initial velocity, and we apply Eqs. 3–6a, 3–9, and 3–12b for the position vector $\vec{\mathbf{r}}$, velocity $\vec{\mathbf{v}}$, and acceleration $\vec{\mathbf{a}}$. We can then write Eqs. 2–12a, b, and c for two dimensions as shown in Table 3–1.

TABLE 3–1 Kinematic Equations for Constant Acceleration in 2 Dimensions

x component (horizontal)		*y* component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–12a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(Eq. 2–12b)	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$	(Eq. 2–12c)	$v_y^2 = v_{y0}^2 + 2a_y (y - y_0)$

The first two of the equations in Table 3–1 can be written more formally in vector notation.

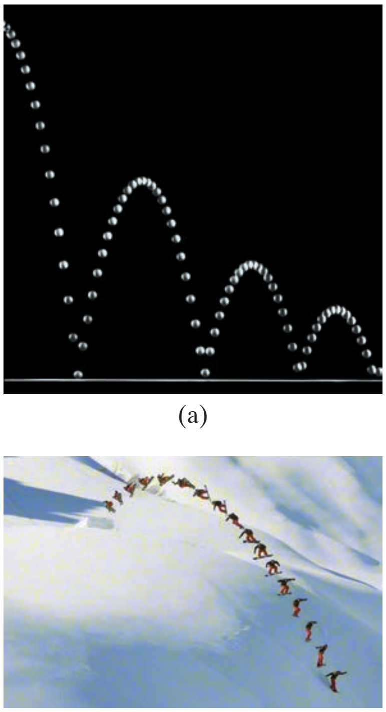
$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t \qquad [\vec{\mathbf{a}} = \text{constant}] \quad \textbf{(3–13a)}$$

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2. \qquad [\vec{\mathbf{a}} = \text{constant}] \quad \textbf{(3–13b)}$$

Here, $\vec{\mathbf{r}}$ is the position vector at any time, and $\vec{\mathbf{r}}_0$ is the position vector at $t = 0$. These equations are the vector equivalent of Eqs. 2–12a and b. In practical situations, we usually use the component form given in Table 3–1.

3–7 Projectile Motion

In Chapter 2, we studied one-dimensional motion of an object in terms of displacement, velocity, and acceleration, including purely vertical motion of a falling object undergoing acceleration due to gravity. Now we examine the more general translational motion of objects moving through the air in two dimensions near the Earth's surface, such as a golf ball, a thrown or batted baseball, kicked footballs, and speeding bullets. These are all examples of **projectile motion** (see Fig. 3–19), which we can describe as taking place in two dimensions if there is no wind.



(a)

(b)

FIGURE 3–19 Photographs of (a) a bouncing ball and (b) a snowboarder, each showing the characteristic "parabolic" path of projectile motion.

Although air resistance is often important, in many cases its effect can be ignored, and we will ignore it in the following analysis. We will not be concerned now with the process by which the object is thrown or projected. We consider only its motion *after* it has been projected, and *before* it lands or is caught—that is, we analyze our projected object only when it is moving freely through the air under the action of gravity alone. Then the acceleration of the object is that due to gravity, which acts downward with magnitude $g = 9.80\,\text{m/s}^2$, and we assume it is constant.†

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time $t = 0$ at the origin of an *xy* coordinate system (so $x_0 = y_0 = 0$).

†This restricts us to objects whose distance traveled and maximum height above the Earth are small compared to the Earth's radius (6400 km).

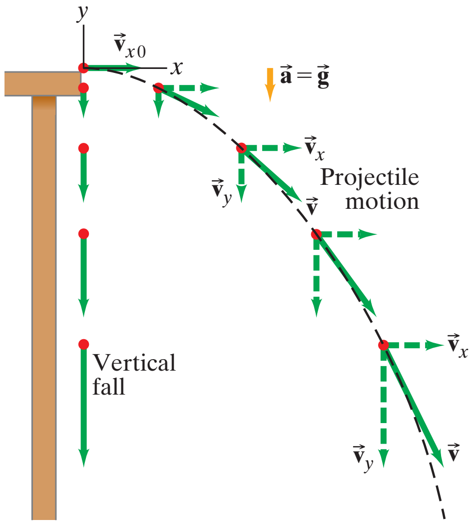


FIGURE 3–20 Projectile motion of a small ball projected horizontally with initial velocity $\vec{\mathbf{v}} = \vec{\mathbf{v}}_{x0}$. The dashed black line represents the path of the object. The velocity vector $\vec{\mathbf{v}}$ is in the direction of motion at each point, and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting from rest at the same place and time is shown at the left for comparison; v_y is the same at each instant for the falling object and the projectile.)

Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal (x) direction, $\vec{\mathbf{v}}_{x0}$. See Fig. 3–20, where an object falling vertically is also shown for comparison. The velocity vector $\vec{\mathbf{v}}$ at each instant points in the direction of the ball's motion at that instant and is thus always tangent to the path. Like Galileo, we treat the horizontal and vertical components of the velocity and acceleration separately, and we apply the kinematic equations (Eqs. 2–12) to the x and y components of the motion.

First we examine the vertical (y) component of the motion. At the instant the ball leaves the table's top $(t = 0)$, it has only an x component of velocity. Once the ball leaves the table (at $t = 0$), it experiences a vertically downward acceleration g, the acceleration due to gravity. Thus v_y is initially zero $(v_{y0} = 0)$ but increases continually in the downward direction (until the ball hits the ground). Let us take y to be positive upward. Then the acceleration due to gravity is in the $-y$ direction, so $a_y = -g$. From Eq. 2–12a (using y in place of x) we can write $v_y = v_{y0} + a_y t = -gt$ since we set $v_{y0} = 0$. The vertical displacement is given by Eq. 2–12b written in terms of y: $y = y_0 + v_{y0} + \frac{1}{2}a_y t^2$. Given $y_0 = 0$, $v_{y0} = 0$, and $a_y = -g$, then $y = -\frac{1}{2}gt^2$.

In the horizontal direction, the acceleration is zero (ignoring air resistance). With $a_x = 0$, the horizontal component of velocity, v_x, remains constant, equal to its initial value, v_{x0}, and thus has the same magnitude at each point on the path. The horizontal displacement (with $a_x = 0$) is given by $x = v_{x0}t + \frac{1}{2}a_x t^2 = v_{x0}t$.

The two vector components, $\vec{\mathbf{v}}_x$ and $\vec{\mathbf{v}}_y$, can be added vectorially at any instant to obtain the velocity $\vec{\mathbf{v}}$ at that time (each point on the path), as shown in Fig. 3–20.

One result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. The vertical motions are the same in both cases, as shown in Fig. 3–20. Figure 3–21 is a multiple-exposure photograph of an experiment that confirms this.

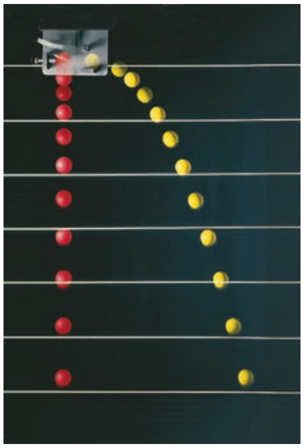


FIGURE 3–21 Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

If an object is projected at an upward angle, as in Fig. 3–22, the analysis is similar, but now there is an initial vertical component of velocity, v_{y0}. Because of the downward acceleration of gravity, the upward component of velocity v_y gradually decreases with time until the object reaches the highest point on its path, at which point $v_y = 0$. The object then moves downward (Fig. 3–22) and v_y increases in the downward direction (becoming more negative). As before, v_x remains constant.

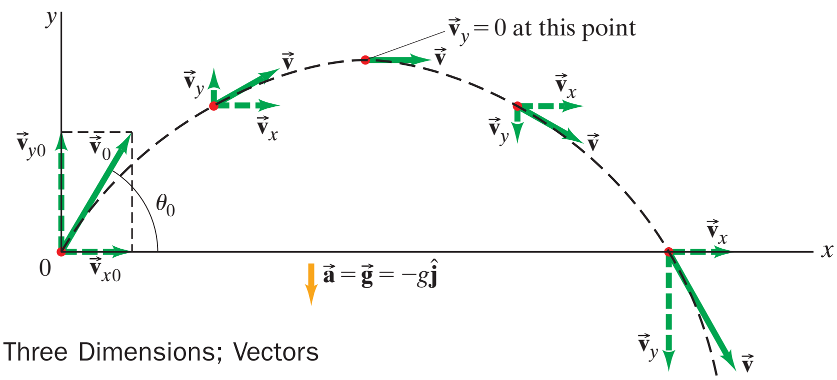


FIGURE 3–22 Path of a projectile fired with initial velocity $\vec{\mathbf{v}}_0$ at angle θ_0 to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The acceleration $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$ is downward. That is, $\vec{\mathbf{a}} = \vec{\mathbf{g}} = -g\hat{\mathbf{j}}$ where $\hat{\mathbf{j}}$ is the unit vector in the positive y direction. Not shown is where the projectile hits the ground (at that point projectile motion ceases).

CONCEPTUAL EXAMPLE 3–6 **Where does the apple land?** A child sits upright in a wagon that moves to the right at constant speed, Fig. 3–23. The child extends her hand and throws an apple straight up (from her own point of view, Fig. 3–23a), as the wagon moves forward at constant speed. Neglecting air resistance, will the apple land (*a*) behind the wagon, (*b*) in the wagon, or (*c*) in front of the wagon?

RESPONSE The child throws the apple straight up in her own reference frame with initial velocity $\vec{\mathbf{v}}_{y0}$ (Fig. 3–23a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the velocity of the wagon, $\vec{\mathbf{v}}_{x0}$. Thus, to a person on the ground, the apple follows the path of a projectile as shown in Fig. 3–23b. The apple experiences no horizontal acceleration, so $\vec{\mathbf{v}}_{x0}$ stays constant, equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal velocity. When the apple comes down, it will drop right into the outstretched hand of the child. The answer is (*b*).

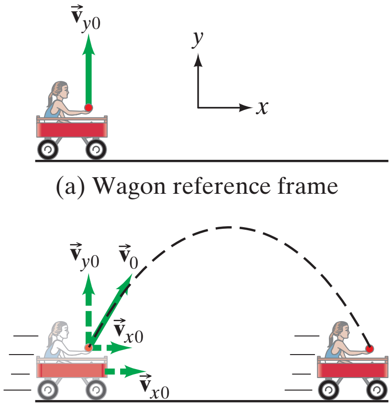


FIGURE 3–23 Example 3–6.

EXERCISE C Return to the Chapter-Opening Question, page 76, and answer it again now. Try to explain why you may have answered differently the first time. Describe the role of the helicopter in this example of projectile motion.

3–8 Solving Problems Involving Projectile Motion

We can simplify Eqs. 2–12 (Table 3–1) for the case of projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes y is positive upward, so $a_y = -g = -9.80\,\text{m/s}^2$. If θ is chosen relative to the $+x$ axis, as in Fig. 3–22, then

$$v_{x0} = v_0 \cos\theta_0, \qquad \text{and} \qquad v_{y0} = v_0 \sin\theta_0.$$

In doing Problems involving projectile motion, we must consider a time interval for which our chosen object is in the air, influenced only by gravity. We do not consider the throwing (or projecting) process, nor the time after the object lands or is caught, because then other influences act on the object, and we can no longer set $\vec{\mathbf{a}} = \vec{\mathbf{g}}$.

PROBLEM SOLVING
Choice of time interval

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\,\text{m/s}^2$)

Horizontal Motion ($a_x = 0$, $v_x =$ constant)		Vertical Motion† ($a_y = -g =$ constant)
$v_x = v_{x0}$	(Eq. 2–12a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	(Eq. 2–12b)	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	(Eq. 2–12c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

†If y is taken positive downward, the minus (−) signs in front of g become plus (+) signs.

PROBLEM SOLVING — Projectile Motion

Our approach to solving Problems in Section 2–6 also applies here. Solving projectile motion Problems can require creativity, and cannot be done just by following rules. You must avoid just plugging numbers into equations that seem to "work."

1. As always, **read** carefully; **choose** the **object** (or objects) you are going to analyze.
2. **Draw** a careful **diagram** showing what is happening.
3. **Choose** an origin and an xy **coordinate system**.
4. Decide on the **time interval**, which for projectile motion can only include motion under the effect of gravity alone, not throwing or landing. The time interval must be the same for the x and y analyses. The x and y motions are connected by the common time, t.
5. **Examine** the horizontal (x) and vertical (y) **motions** separately. If you are given the initial velocity, you may want to resolve it into its x and y components.
6. List the **known** and **unknown** quantities, choosing $a_x = 0$ and $a_y = -g$ or $+g$, where $g = 9.80\,\text{m/s}^2$, depending on choice of y positive up or down. Recall: v_x never changes throughout the trajectory, and $v_y = 0$ at the highest point of any trajectory that returns downward. The velocity just before landing is generally not zero.
7. Think for a minute before jumping into the equations. **Apply** the **relevant equations** (Table 3–2), combining them if necessary. You may need to combine components of a vector to get magnitude and direction (Eqs. 3–3).

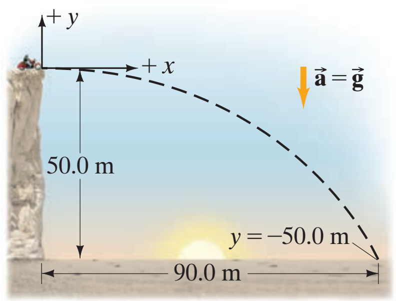


FIGURE 3–24 Example 3–7.

Known	Unknown
$x_0 = y_0 = 0$	v_{x0}
$x = 90.0\,\text{m}$	t
$y = -50.0\,\text{m}$	
$a_x = 0$	
$a_y = -g = -9.80\,\text{m/s}^2$	
$v_{y0} = 0$	

EXAMPLE 3–7 **Driving off a cliff.** A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the clifftop to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.

APPROACH We explicitly follow the steps of the Problem Solving Strategy on the previous page.

SOLUTION

1. and **2. Read, choose the object, and draw a diagram**. Our object is the motorcycle and driver, taken as a single unit. The diagram is shown in Fig. 3–24.
3. **Choose a coordinate system**. We choose the y direction to be positive upward, with the top of the cliff as $y_0 = 0$. The x direction is horizontal with $x_0 = 0$ at the point where the motorcycle leaves the cliff.
4. **Choose a time interval**. We choose our time interval to begin $(t = 0)$ just as the motorcycle leaves the clifftop at position $x_0 = 0$, $y_0 = 0$. Our time interval ends just before the motorcycle touches the ground below.
5. **Examine *x* and *y* motions**. In the horizontal (x) direction, the acceleration $a_x = 0$, so the velocity is constant. The value of x when the motorcycle reaches the ground is $x = +90.0\,\text{m}$. In the vertical direction, the acceleration is the acceleration due to gravity, $a_y = -g = -9.80\,\text{m/s}^2$. The value of y when the motorcycle reaches the ground is $y = -50.0\,\text{m}$. The initial velocity is horizontal and is our unknown, v_{x0}; the initial vertical velocity is zero, $v_{y0} = 0$.
6. **List knowns and unknowns**. See the Table in the margin. Note that in addition to not knowing the initial horizontal velocity v_{x0} (which stays constant until landing), we also do not know the time t when the motorcycle reaches the ground.
7. **Apply relevant equations**. The motorcycle maintains constant v_x as long as it is in the air. The time it stays in the air is determined by the y motion—when it reaches the ground. So we first find the time using the y motion, and then use this time value in the x equations. To find out how long it takes the motorcycle to reach the ground below, we use Eq. 2–12b (Table 3–2) for the vertical (y) direction with $y_0 = 0$ and $v_{y0} = 0$:

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2$$
$$= 0 + 0 + \tfrac{1}{2}(-g)t^2$$

or

$$y = -\tfrac{1}{2}gt^2.$$

We solve for t and set $y = -50.0\,\text{m}$:

$$t = \sqrt{\frac{2y}{-g}} = \sqrt{\frac{2(-50.0\,\text{m})}{-9.80\,\text{m/s}^2}} = 3.19\,\text{s}.$$

To calculate the needed initial velocity, v_{x0}, we again use Eq. 2–12b, but this time for the horizontal (x) direction, with $a_x = 0$ and $x_0 = 0$:

$$x = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2$$
$$= 0 + v_{x0}t + 0$$

or

$$x = v_{x0}t.$$

So the motorcycle needs to leave the clifftop with a speed

$$v_{x0} = \frac{x}{t} = \frac{90.0\,\text{m}}{3.19\,\text{s}} = 28.2\,\text{m/s},$$

which is about 100 km/h (roughly 60 mi/h).

NOTE In the time interval of the projectile motion, the only acceleration is g in the negative y direction. The acceleration in the x direction is zero.

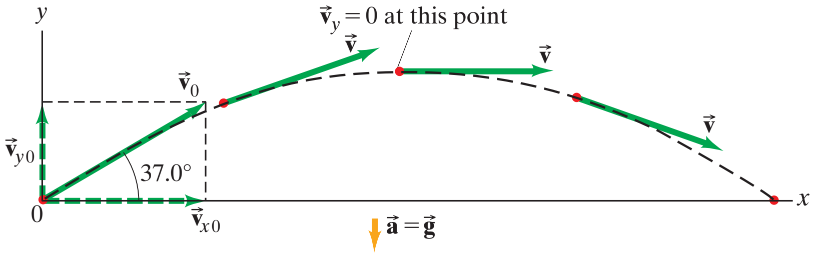


FIGURE 3–25 Example 3–8.

PHYSICS APPLIED
Sports

EXAMPLE 3–8 **A kicked football.** A football is kicked at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown in Fig. 3–25. Calculate (*a*) the maximum height, (*b*) the time of travel before the football hits the ground, and (*c*) how far away it hits the ground. Assume the ball leaves the foot at ground level, and ignore air resistance, wind, and rotation of the ball.

APPROACH This may seem difficult at first because there are so many questions. But we can deal with them one at a time. We take the *y* direction as positive upward, and treat the *x* and *y* motions separately. The total time in the air is again determined by the *y* motion. The *x* motion occurs at constant velocity. The *y* component of velocity varies, being positive (upward) initially, decreasing to zero at the highest point, and then becoming negative as the football falls.

SOLUTION We resolve the initial velocity into its components (Fig. 3–25):

$$v_{x0} = v_0 \cos 37.0° = (20.0\,\text{m/s})(0.799) = 16.0\,\text{m/s}$$
$$v_{y0} = v_0 \sin 37.0° = (20.0\,\text{m/s})(0.602) = 12.0\,\text{m/s}.$$

(*a*) To find the maximum height, we consider a time interval that begins just after the football loses contact with the foot until the ball reaches its maximum height. During this time interval, the acceleration is *g* downward. At the maximum height, the velocity is horizontal (Fig. 3–25), so $v_y = 0$; and this occurs at a time given by $v_y = v_{y0} - gt$ with $v_y = 0$ (see Eq. 2–12a in Table 3–2). Thus

$$t = \frac{v_{y0}}{g} = \frac{(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 1.224\,\text{s} \approx 1.22\,\text{s}.$$

From Eq. 2–12b, with $y_0 = 0$, we can solve for *y* at this time $t = v_{y0}/g$:

$$y = v_{y0}t - \tfrac{1}{2}gt^2 = \frac{v_{y0}^2}{g} - \frac{1}{2}\frac{v_{y0}^2}{g} = \frac{v_{y0}^2}{2g} = \frac{(12.0\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = 7.35\,\text{m}.$$

The maximum height is 7.35 m. [Solving Eq. 2–12c for *y* gives the same result.]

(*b*) To find the time it takes for the ball to return to the ground, we consider a different time interval, starting at the moment the ball leaves the foot $(t = 0,\ y_0 = 0)$ and ending just before the ball touches the ground $(y = 0$ again). We can use Eq. 2–12b with $y_0 = 0$ and also set $y = 0$ (ground level):

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$
$$0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2.$$

This equation can be easily factored:

$$t\left(\tfrac{1}{2}gt - v_{y0}\right) = 0.$$

There are two solutions, $t = 0$ (which corresponds to the initial point, y_0), and

$$t = \frac{2v_{y0}}{g} = \frac{2(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 2.45\,\text{s},$$

which is the total travel time of the football.

(*c*) The total distance traveled in the *x* direction is found by applying Eq. 2–12b with $x_0 = 0$, $a_x = 0$, $v_{x0} = 16.0\,\text{m/s}$, and $t = 2.45\,\text{s}$:

$$x = v_{x0}t = (16.0\,\text{m/s})(2.45\,\text{s}) = 39.2\,\text{m}.$$

NOTE In (*b*) the time needed for the whole trip, $t = 2v_{y0}/g = 2.45\,\text{s}$, is double the time to reach the highest point, calculated in (*a*). That is, the time to go up equals the time to come back down to the same level (ignoring air resistance).

EXERCISE D In Example 3–8, what is (*a*) the velocity vector at the maximum height, and (*b*) the acceleration vector at maximum height?

In Example 3–8 we treated the football as if it were a particle, ignoring its rotation. We also ignored air resistance. Because air resistance is significant on a football, our results are only estimates (mainly overestimates). We also ignored any wind. If there happened to be a cross-wind, we would need a third axis for such 3-dimensional motion.

EXERCISE E Two balls are thrown in the air at different angles, but each reaches the same height. Which ball remains in the air longer: the one thrown at the steeper angle or the one thrown at a shallower angle?

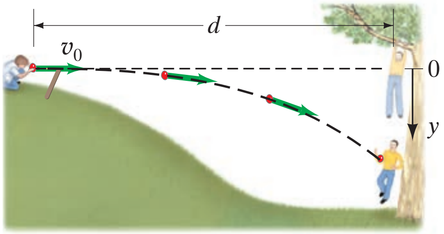


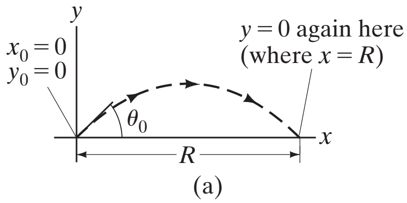
FIGURE 3–26 Example 3–9.

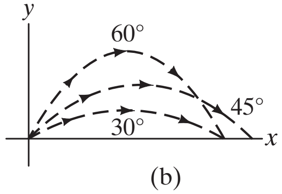
CONCEPTUAL EXAMPLE 3–9 **The wrong strategy.** A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away, Fig. 3–26. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.

RESPONSE Both the water balloon and the boy in the tree start falling at the same instant, and in a time t they each fall the same vertical distance $y = \frac{1}{2}gt^2$, much like Fig. 3–21. In the time it takes the water balloon to travel the horizontal distance d, the balloon will have the same y position as the falling boy. Splat. If the boy had stayed in the tree, he would have avoided the humiliation.

Level Horizontal Range

FIGURE 3–27 (a) The range R of a projectile. (b) There are generally two angles θ_0 that will give the same range. If one angle is θ_{01}, the other is $\theta_{02} = 90° - \theta_{01}$. Also see Example 3–10.





The total distance the football traveled in Example 3–8 is called the horizontal **range** R. We now derive a formula for the range, which applies to a projectile that lands at the same level it started $(=y_0)$: that is, $y(\text{final}) = y_0$ (see Fig. 3–27a). Looking back at Example 3–8 part (*c*), we see that $x = R = v_{x0}t$ where (from part *b*) $t = 2v_{y0}/g$. Thus

$$R = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}, \qquad [y = y_0]$$

where $v_{x0} = v_0\cos\theta_0$ and $v_{y0} = v_0\sin\theta_0$. This can be rewritten, using the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$ (Appendix A or inside the rear cover):

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \qquad [\text{only if } y(\text{final}) = y_0]$$

Note that the *maximum* range, for a given initial velocity v_0, is obtained when $\sin 2\theta$ takes on its maximum value of 1.0, which occurs for $2\theta_0 = 90°$; so

$$\theta_0 = 45° \text{ for maximum range, and } R_{max} = v_0^2/g.$$

The maximum range increases by the square of v_0, so doubling the initial velocity of a projectile increases its maximum range by a factor of 4.

When air resistance is important, the range is less for a given v_0, and the maximum range is obtained at an angle smaller than 45°.

EXAMPLE 3–10 **Range of a cannon ball.** Suppose one of Napoleon's cannons had a muzzle speed, v_0, of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?

APPROACH We use the equation just derived for the range, $R = v_0^2 \sin 2\theta_0/g$, with $R = 320$ m.

SOLUTION We solve for $\sin 2\theta_0$ in the range formula:

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(320\,\text{m})(9.80\,\text{m/s}^2)}{(60.0\,\text{m/s})^2} = 0.871.$$

We want to solve for an angle θ_0 that is between 0° and 90°, which means $2\theta_0$ in this equation can be as large as 180°. Then $\sin^{-1}(0.871) = 2\theta_0 = 60.6°$ is a solution, so the cannon should be aimed at $\theta_0 = 30.3°$. But $2\theta_0 = 180° - 60.6° = 119.4°$ is also a solution (see Appendix A–9), so θ_0 can also be $\theta_0 = 59.7°$. In general we have two solutions (see Fig. 3–27b), which in the present case are given by

$$\theta_0 = 30.3° \quad \text{or} \quad 59.7°.$$

Either angle gives the same range. Only when $\sin 2\theta_0 = 1$ (so $\theta_0 = 45°$) is there a single solution (that is, both solutions are the same).

The level range formula applies only if takeoff and landing are at the same height $(y = y_0)$. Example 3–11 below considers a case where they are not equal heights $(y \neq y_0)$.

EXAMPLE 3–11 **A punt.** Suppose the football in Example 3–8 was punted and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0$, $y_0 = 0$.

PHYSICS APPLIED
Sports

APPROACH The x and y motions are again treated separately. But we cannot use the range formula as in Example 3–10 because the range formula is valid only if $y(\text{final}) = y_0$, which is not the case here. Now we have $y_0 = 0$, and the football hits the ground where $y = -1.00$ m (see Fig. 3–28). We choose our time interval to start when the ball leaves his foot $(t = 0,\ y_0 = 0,\ x_0 = 0)$ and end just before the ball hits the ground $(y = -1.00\text{ m})$. We can get x from Eq. 2–12b, $x = v_{x0}t$, and we saw that $v_{x0} = 16.0$ m/s in Example 3–8. But first we must find t, the time at which the ball hits the ground, which we obtain from the y motion.

PROBLEM SOLVING
Do not use any formula unless you are sure its range of validity fits the problem; the range formula does not apply here because $y \neq y_0$

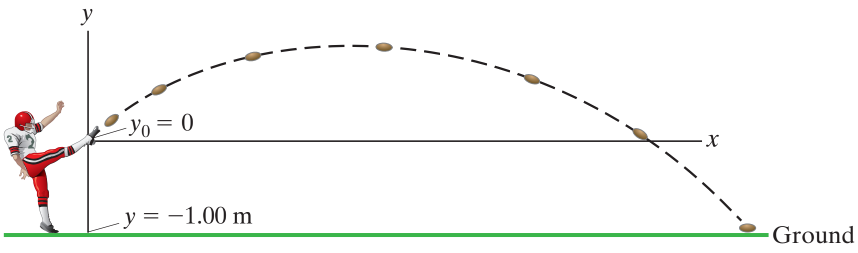


FIGURE 3–28 Example 3–11: the football leaves the punter's foot at $y = 0$, and reaches the ground where $y = -1.00$ m.

SOLUTION To find t with $y = -1.00$ m and $v_{y0} = 12.0$ m/s (see Example 3–8), we use the equation

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2,$$

and obtain

$$-1.00\text{ m} = 0 + (12.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$$

We rearrange this equation into standard form $(ax^2 + bx + c = 0)$ so we can use the quadratic formula:

$$(4.90\text{ m/s}^2)t^2 - (12.0\text{ m/s})t - (1.00\text{ m}) = 0.$$

The quadratic formula (Appendix A–1) gives

$$t = \frac{12.0\text{ m/s} \pm \sqrt{(-12.0\text{ m/s})^2 - 4(4.90\text{ m/s}^2)(-1.00\text{ m})}}{2(4.90\text{ m/s}^2)}$$

$$= 2.53\text{ s} \quad \text{or} \quad -0.081\text{ s}.$$

The second solution would correspond to a time prior to our chosen time interval that begins at the kick $(t = 0)$, so it doesn't apply. With $t = 2.53$ s for the time at which the ball touches the ground, the horizontal distance the ball traveled is (using $v_{x0} = 16.0$ m/s from Example 3–8):

$$x = v_{x0}t = (16.0\text{ m/s})(2.53\text{ s}) = 40.5\text{ m}.$$

Our assumption in Example 3–8 that the ball leaves the foot at ground level gives a result (39.2 m) that is an underestimate of about 1.3 m in the distance our punt traveled. (But Example 3–8 would apply for a kickoff or field goal in American football.)

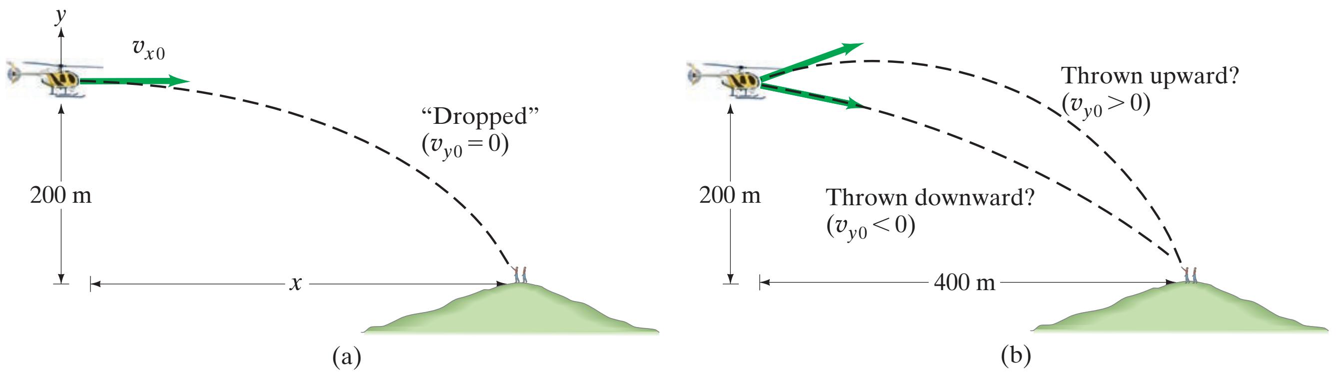


FIGURE 3–29 Example 3–12.

PHYSICS APPLIED

Reaching a target from a moving helicopter

EXAMPLE 3–12 Rescue helicopter drops supplies. A rescue helicopter wants to drop a package of supplies to isolated mountain climbers on a rocky ridge 200 m below. If the helicopter is traveling horizontally with a speed of 70 m/s (250 km/h), (*a*) how far in advance of the recipients (horizontal distance) must the package be dropped (Fig. 3–29a)? (*b*) Suppose, instead, that the helicopter releases the package a horizontal distance of 400 m in advance of the mountain climbers. What vertical velocity should the package be given (up or down) so that it arrives precisely at the climbers' position (Fig. 3–29b)? (*c*) With what speed does the package land in the latter case?

APPROACH We choose the origin of our *xy* coordinate system at the initial position of the helicopter, taking $+y$ upward, and use the kinematic equations (Table 3–2).

SOLUTION (*a*) We can find the time to reach the climbers using the vertical distance of 200 m. The package is "dropped" so initially it has the velocity of the helicopter, $v_{x0} = 70\ \text{m/s}$, $v_{y0} = 0$. Then, since $y = -\frac{1}{2}gt^2$, we have

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-200\ \text{m})}{9.80\ \text{m/s}^2}} = 6.39\ \text{s}.$$

The horizontal motion of the falling package is at constant speed of 70 m/s. So

$$x = v_{x0}t = (70\ \text{m/s})(6.39\ \text{s}) = 447\ \text{m} \approx 450\ \text{m},$$

assuming the given numbers were good to two significant figures.

(*b*) We are given $x = 400\ \text{m}$, $v_{x0} = 70\ \text{m/s}$, $y = -200\ \text{m}$, and we want to find v_{y0} (see Fig. 3–29b). Like most problems, this one can be approached in various ways. Instead of searching for a formula or two, let's try to reason it out in a simple way, based on what we did in part (*a*). If we know t, perhaps we can get v_{y0}. Since the horizontal motion of the package is at constant speed (once it is released we don't care what the helicopter does), we have $x = v_{x0}t$, so

$$t = \frac{x}{v_{x0}} = \frac{400\ \text{m}}{70\ \text{m/s}} = 5.71\ \text{s}.$$

Now let's try to use the vertical motion to get v_{y0}: $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$. Since $y_0 = 0$ and $y = -200\ \text{m}$, we can solve for v_{y0}:

$$v_{y0} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{-200\ \text{m} + \frac{1}{2}(9.80\ \text{m/s}^2)(5.71\ \text{s})^2}{5.71\ \text{s}} = -7.0\ \text{m/s}.$$

Thus, in order to arrive at precisely the mountain climbers' position, the package must be thrown *downward* from the helicopter with a speed of 7.0 m/s.

(*c*) We want to know v of the package at $t = 5.71\ \text{s}$. The components are:

$$v_x = v_{x0} = 70\ \text{m/s}$$
$$v_y = v_{y0} - gt = -7.0\ \text{m/s} - (9.80\ \text{m/s}^2)(5.71\ \text{s}) = -63\ \text{m/s}.$$

So $v = \sqrt{(70\ \text{m/s})^2 + (-63\ \text{m/s})^2} = 94\ \text{m/s}$. (Better not to release the package from such an altitude, or use a parachute.)

Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a *parabola*, if we can ignore air resistance and can assume that only gravity is acting with $\vec{\mathbf{g}}$ constant. To do so, we need to find y as a function of x by eliminating t between the two equations for horizontal and vertical motion (Eq. 2–12b in Table 3–2), and for simplicity we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$

$$y = v_{y0}t - \tfrac{1}{2}gt^2.$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second equation to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2. \qquad \textbf{(3–14)}$$

We see that y as a function of x has the form

$$y = Ax - Bx^2,$$

where A and B are constants for any specific projectile motion. This is the well-known equation for a parabola. See Figs. 3–19 and 3–30.

The idea that projectile motion is parabolic was at the forefront of physics research in Galileo's day. Today we discuss it in Chapter 3 of introductory physics!

FIGURE 3–30 Examples of projectile motion: (a) a boy leaping, (b) glowing lava from the volcano Stromboli.

3–9 Relative Velocity

We now consider how observations made in different frames of reference are related to each other. For example, consider two trains approaching one another, each with a speed of 80 km/h with respect to the Earth. Observers on the Earth beside the train tracks will measure 80 km/h for the speed of each of the trains. Observers on either one of the trains (different frames of reference) will measure a speed of 160 km/h for the other train approaching them.

Similarly, when one car traveling 90 km/h passes a second car traveling in the same direction at 75 km/h, the first car has a speed relative to the second car of 90 km/h − 75 km/h = 15 km/h.

When the velocities are along the same line, simple addition or subtraction is sufficient to obtain the **relative velocity**. But if they are not along the same line, we must make use of vector addition. We emphasize, as mentioned in Section 2–1, that when specifying a velocity, it is important to specify what the reference frame is.

When determining relative velocity, it is easy to make a mistake by adding or subtracting the wrong velocities. It is important, therefore, to draw a diagram and use a careful labeling process. Each velocity is labeled by *two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity*. For example, suppose a boat heads directly across a river, as shown in Fig. 3–31. We let $\vec{\mathbf{v}}_{BW}$ be the velocity of the **B**oat with respect to the **W**ater. (This is also what the boat's velocity would be relative to the shore if the water were still.) Similarly, $\vec{\mathbf{v}}_{BS}$ is the velocity of the **B**oat with respect to the **S**hore, and $\vec{\mathbf{v}}_{WS}$ is the velocity of the **W**ater with respect to the **S**hore (this is the river current). Note that $\vec{\mathbf{v}}_{BW}$ is what the boat's motor produces (against the water), whereas $\vec{\mathbf{v}}_{BS}$ is equal to $\vec{\mathbf{v}}_{BW}$ plus the effect of the current, $\vec{\mathbf{v}}_{WS}$. Therefore, the velocity of the boat relative to the shore is (see vector diagram, Fig. 3–31)

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}. \qquad \textbf{(3–15)}$$

By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3–15 are the same; also, the outer subscripts on the right of Eq. 3–15 (the B and the S) are the same as the two subscripts for the sum vector on the left, $\vec{\mathbf{v}}_{BS}$.

FIGURE 3–31 A boat heads north directly across a river which flows west. Velocity vectors are shown as green arrows:

$\vec{\mathbf{v}}_{BS}$ = velocity of **B**oat with respect to the **S**hore,

$\vec{\mathbf{v}}_{BW}$ = velocity of **B**oat with respect to the **W**ater,

$\vec{\mathbf{v}}_{WS}$ = velocity of **W**ater with respect to the **S**hore (river current).

As it crosses the river, the boat is dragged downstream by the current ($\vec{\mathbf{v}}_{WS}$).

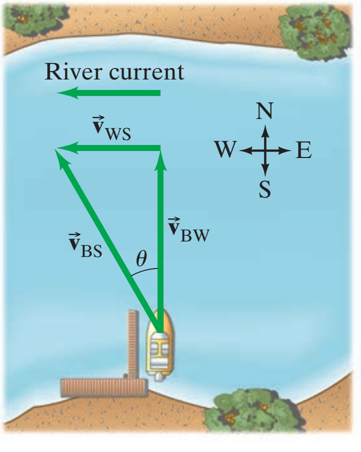


FIGURE 3–32 Derivation of relative velocity equation (Eq. 3–15), in this case for a person walking along the corridor in a train. We are looking down on the train and two reference frames are shown: xy on the Earth and $x'y'$ fixed on the train. We have:

$$\vec{\mathbf{r}}_{PT} = \text{position vector of person (P) relative to train (T)},$$
$$\vec{\mathbf{r}}_{PE} = \text{position vector of person (P) relative to Earth (E)},$$
$$\vec{\mathbf{r}}_{TE} = \text{position vector of train's coordinate system (T) relative to Earth (E)}.$$

From the diagram we see that

$$\vec{\mathbf{r}}_{PE} = \vec{\mathbf{r}}_{PT} + \vec{\mathbf{r}}_{TE}.$$

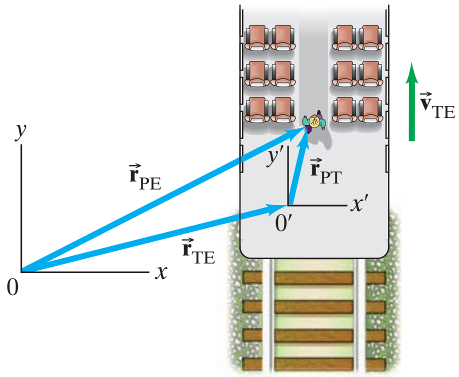
We take the derivative with respect to time to obtain

$$\frac{d}{dt}(\vec{\mathbf{r}}_{PE}) = \frac{d}{dt}(\vec{\mathbf{r}}_{PT}) + \frac{d}{dt}(\vec{\mathbf{r}}_{TE}),$$

or, since $d\vec{\mathbf{r}}/dt = \vec{\mathbf{v}}$,

$$\vec{\mathbf{v}}_{PE} = \vec{\mathbf{v}}_{PT} + \vec{\mathbf{v}}_{TE}.$$

This is the equivalent of Eq. 3–15 for the present situation (check the subscripts!).



By following this convention (first subscript for the object, second for the reference frame), you can write down the correct equation relating velocities in different reference frames.[†]

Figure 3–32 gives a derivation of Eq. 3–15, $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$, but using different subscripts.

Equation 3–15 is valid in general and can be extended to three or more velocities. For example, if a fisherman on a boat walks with a velocity $\vec{\mathbf{v}}_{FB}$ relative to the boat, his velocity relative to the shore is $\vec{\mathbf{v}}_{FS} = \vec{\mathbf{v}}_{FB} + \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$. The equations involving relative velocity will be correct when adjacent inner subscripts are identical and when the outermost ones correspond exactly to the two on the velocity on the left of the equation. But this works only with plus signs (on the right), not minus signs.

It is often useful to remember that for any two objects or reference frames, A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\vec{\mathbf{v}}_{BA} = -\vec{\mathbf{v}}_{AB}. \quad \textbf{(3–16)}$$

For example, if a train is traveling 100 km/h relative to the Earth in a certain direction, objects on the Earth (such as trees) appear to an observer on the train to be traveling 100 km/h in the opposite direction.

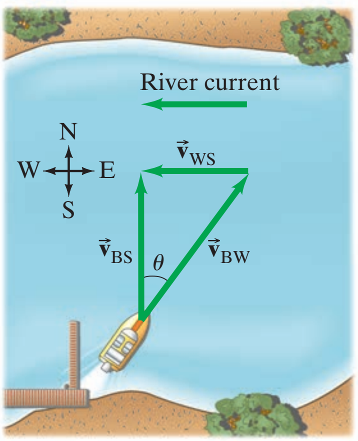


FIGURE 3–33 Example 3–13. A boat, in order to go directly across a moving river current, must head upstream.

EXAMPLE 3–13 Heading upstream. A boat's speed in still water is $v_{BW} = 1.85\,\text{m/s}$. If the boat is to travel north directly across a river whose westward current has speed $v_{WS} = 1.20\,\text{m/s}$, at what upstream angle must the boat head? See Fig. 3–33.

APPROACH If the boat heads straight across the river, the current will drag the boat downstream (westward). To overcome the river's current, the boat must have an upstream (eastward) component of velocity as well as a cross-stream (northward) component. Figure 3–33 has been drawn with $\vec{\mathbf{v}}_{BS}$, the velocity of the **B**oat relative to the **S**hore, pointing directly across the river because this is where the boat is supposed to go. Note that $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$.

SOLUTION Vector $\vec{\mathbf{v}}_{BW}$ points upstream at angle θ as shown. From the diagram,

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\,\text{m/s}}{1.85\,\text{m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a 40.4° angle.

[†]We thus can see, for example, that the equation $\vec{\mathbf{v}}_{BW} = \vec{\mathbf{v}}_{BS} + \vec{\mathbf{v}}_{WS}$ is wrong: the inner subscripts are not the same, and the outer ones on the right do not correspond to the subscripts on the left.

EXAMPLE 3–14 **Heading across the river.** The same boat $(v_{BW} = 1.85\ \mathrm{m/s})$ now heads directly across the river whose current is still 1.20 m/s. (*a*) What is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) If the river is 110 m wide, how long will it take to cross and how far downstream will the boat be then?

APPROACH The boat now heads directly across the river and is pulled downstream by the current, as shown in Fig. 3–34. The boat's velocity with respect to the shore, $\vec{\mathbf{v}}_{BS}$, is the sum of the boat's velocity with respect to the water, $\vec{\mathbf{v}}_{BW}$, plus the velocity of the water with respect to the shore, $\vec{\mathbf{v}}_{WS}$ (= the river's current). Just as before,

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}.$$

SOLUTION (*a*) Since $\vec{\mathbf{v}}_{BW}$ is perpendicular to $\vec{\mathbf{v}}_{WS}$, we can get v_{BS} using the theorem of Pythagoras:

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(1.85\ \mathrm{m/s})^2 + (1.20\ \mathrm{m/s})^2} = 2.21\ \mathrm{m/s}.$$

We can obtain the angle (note how θ is defined in Fig. 3–34) from:

$$\tan\theta = v_{WS}/v_{BW} = (1.20\ \mathrm{m/s})/(1.85\ \mathrm{m/s}) = 0.6486.$$

A calculator with a key such as INV TAN or ARC TAN or TAN^{-1} gives $\theta = \tan^{-1}(0.6486) = 33.0°$. Note that this angle is not equal to the angle calculated in Example 3–13.

(*b*) To find the travel time for the boat to cross the river, recall the river's width $D = 110\ \mathrm{m}$, and use the velocity component that points directly across the river, $v_{BW} = D/t$. Solving for t, we get $t = 110\ \mathrm{m}/(1.85\ \mathrm{m/s}) = 59.5\ \mathrm{s}$. Also, the boat will have been carried downstream, in this time, a distance

$$d = v_{WS}t = (1.20\ \mathrm{m/s})(59.5\ \mathrm{s}) = 71.4\ \mathrm{m} \approx 71\ \mathrm{m}.$$

NOTE There is no acceleration in this Example, so the motion involves only constant velocities (of the boat or of the river).

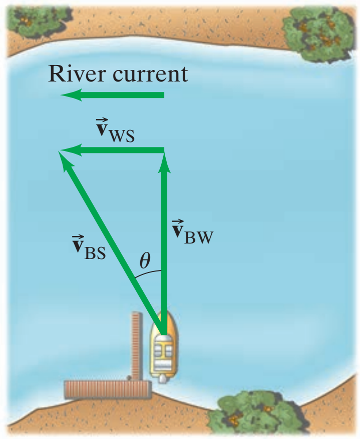


FIGURE 3–34 Example 3–14. A boat heading directly across a river whose current moves at 1.20 m/s.

EXAMPLE 3–15 **Car velocities at 90°.** Two cars approach a street corner at right angles to each other with the same speed of 40.0 km/h (= 11.11 m/s) relative to the ground, as shown in Fig. 3–35a. What is the relative velocity of car 1 as seen by car 2?

APPROACH Figure 3–35a shows the situation in a reference frame fixed to the Earth. But we want to view the situation from a reference frame in which car 2 is at rest, and this is shown in Fig. 3–35b. In this reference frame (the world as seen by the driver of car 2), the Earth moves toward car 2 with velocity $\vec{\mathbf{v}}_{E2}$ (speed of 40.0 km/h), which is of course equal and opposite to $\vec{\mathbf{v}}_{2E}$, the velocity of car 2 with respect to the Earth (Eq. 3–16):

$$\vec{\mathbf{v}}_{2E} = -\vec{\mathbf{v}}_{E2}.$$

Then the velocity of car 1 as seen by car 2 is (see Eq. 3–15)

$$\vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_{1E} + \vec{\mathbf{v}}_{E2}.$$

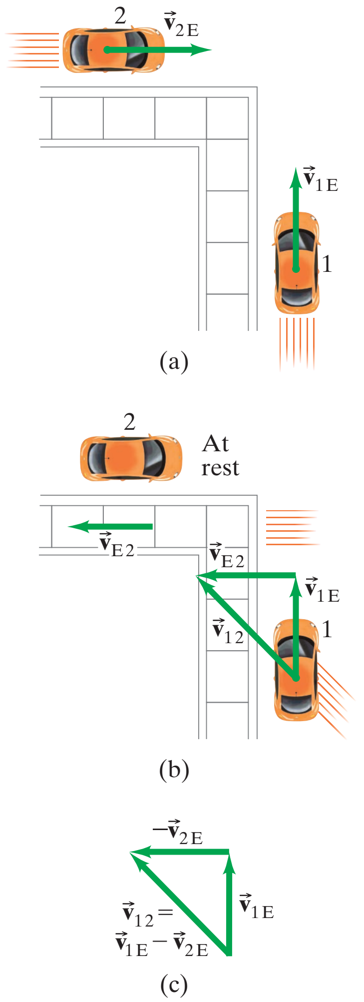
SOLUTION Because $\vec{\mathbf{v}}_{E2} = -\vec{\mathbf{v}}_{2E}$, then

$$\vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_{1E} - \vec{\mathbf{v}}_{2E}.$$

That is, the velocity of car 1 as seen by car 2 is the difference of their velocities, $\vec{\mathbf{v}}_{1E} - \vec{\mathbf{v}}_{2E}$, both measured relative to the Earth; see Fig. 3–35c. Since the magnitudes of $\vec{\mathbf{v}}_{1E}$, $\vec{\mathbf{v}}_{2E}$, and $\vec{\mathbf{v}}_{E2}$ are equal (40.0 km/h = 11.11 m/s), we see (Fig. 3–35b) that $\vec{\mathbf{v}}_{12}$ points at a 45° angle toward car 2; the speed is

$$v_{12} = \sqrt{(11.11\ \mathrm{m/s})^2 + (11.11\ \mathrm{m/s})^2} = 15.7\ \mathrm{m/s}\ (= 56.6\ \mathrm{km/h}).$$

FIGURE 3–35 Example 3–15.



Summary

A quantity such as velocity, that has both a magnitude and a direction, is called a **vector**. A quantity such as mass, that has only a magnitude, is called a **scalar**. On diagrams, vectors are represented by arrows.

Addition of vectors can be done graphically by placing the tail of each successive arrow at the tip of the previous one. The sum, or **resultant vector**, is the arrow drawn from the tail of the first vector to the tip of the last vector. Two vectors can also be added using the *parallelogram* method.

Vectors can be added more accurately by adding their **components** along chosen axes with the aid of trigonometric functions. A vector of magnitude V making an angle θ with the $+x$ axis has components

$$V_x = V\cos\theta, \qquad V_y = V\sin\theta. \tag{3–2}$$

Given the components, we can find a vector's magnitude and direction from

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}. \tag{3–3}$$

It is often helpful to express a vector in terms of its components along chosen axes using **unit vectors**, which are vectors of unit length along the chosen coordinate axes; for Cartesian coordinates the unit vectors along the x, y, and z axes are called $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

The general definitions for the **instantaneous velocity**, $\vec{\mathbf{v}}$, and **acceleration**, $\vec{\mathbf{a}}$, of a particle (in one, two, or three dimensions) are

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} \tag{3–8}$$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt}, \tag{3–11}$$

where $\vec{\mathbf{r}}$ is the position vector of the particle. The kinematic equations for motion with constant acceleration can be written for each of the x, y, and z components of the motion and have the same form as for one-dimensional motion (Eqs. 2–12). Or they can be written in the more general vector form:

$$\begin{aligned} \vec{\mathbf{v}} &= \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t \\ \vec{\mathbf{r}} &= \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}\,t^2. \end{aligned} \tag{3–13}$$

Projectile motion is the motion of an object in the air near the Earth's surface under the effect of gravity alone. It can be analyzed as two separate motions if air resistance can be ignored. The horizontal component of motion is at constant velocity, whereas the vertical component is at constant acceleration, $\vec{\mathbf{g}}$, just as for an object falling vertically under the action of gravity.

The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the **relative velocity** of the two reference frames, are known.

Questions

1. One car travels due east at 40 km/h, and a second car travels north at 40 km/h. Are their velocities equal? Explain.
2. Can you conclude that a car is not accelerating if its speedometer indicates a steady 60 km/h? Explain.
3. Give several examples of an object's motion in which a great distance is traveled but the displacement is zero.
4. Can the displacement vector for a particle moving in two dimensions be longer than the length of path traveled by the particle over the same time interval? Can it be less? Discuss.
5. During baseball practice, a player hits a very high fly ball and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball? Explain.
6. If $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, is V necessarily greater than V_1 and/or V_2? Discuss.
7. Two vectors have lengths $V_1 = 4.5$ km and $V_2 = 5.0$ km. What are the maximum and minimum magnitudes of their vector sum?
8. Can two vectors, of unequal magnitude, add up to give the zero vector? Can *three* unequal vectors? Under what conditions?
9. Can the magnitude of a vector ever (*a*) equal, or (*b*) be less than, one of its components?
10. Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?
11. How could you determine the speed a slingshot imparts to a rock, using only a meter stick, a rock, and the slingshot?
12. In archery, should the arrow be aimed directly at the target? How should your angle of aim depend on the distance to the target?
13. Where in Fig. 3–22 is (*a*) $\vec{\mathbf{v}} = 0$, (*b*) $v_y = 0$, (*c*) $v_x = 0$?
14. It was reported in World War I that a pilot flying at an altitude of 2 km caught in his bare hands a bullet fired at the plane! Using the fact that a bullet slows down considerably due to air resistance, explain how this incident occurred.
15. You are on the street trying to hit a friend in his dorm window with a water balloon. He has a similar idea and is aiming at you with *his* water balloon. You aim straight at each other and throw at the same instant. Do the water balloons hit each other? Explain why or why not.
16. A projectile is launched at an upward angle of 30° to the horizontal with a speed of 30 m/s. How does the horizontal component of its velocity 1.0 s after launch compare with its horizontal component of velocity 2.0 s after launch, ignoring air resistance? Explain.
17. A projectile has the least speed at what point in its path?
18. Two cannonballs, A and B, are fired from the ground with identical initial speeds, but with θ_A larger than θ_B. (*a*) Which cannonball reaches a higher elevation? (*b*) Which stays longer in the air? (*c*) Which travels farther? Explain.
19. A person sitting in an enclosed train car, moving at constant velocity, throws a ball straight up into the air in her reference frame. (*a*) Where does the ball land? What is your answer if the car (*b*) accelerates, (*c*) decelerates, (*d*) rounds a curve, (*e*) moves with constant velocity but is open to the air?
20. If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?
21. Two rowers, who can row at the same speed in still water, set off across a river at the same time. One heads straight across and is pulled downstream somewhat by the current. The other one heads upstream at an angle so as to arrive at a point opposite the starting point. Which rower reaches the opposite side first? Explain.
22. If you stand motionless under an umbrella in a rainstorm where the drops fall vertically, you remain relatively dry. However, if you start running, the rain begins to hit your legs even if they remain under the umbrella. Why?

MisConceptual Questions

1. You are adding vectors of length 20 and 40 units. Which of the following choices is a possible resultant magnitude?
(*a*) 0. (*b*) 18. (*c*) 37. (*d*) 64. (*e*) 100.

2. The magnitude of a component of a vector must be
(*a*) less than or equal to the magnitude of the vector.
(*b*) equal to the magnitude of the vector.
(*c*) greater than or equal to the magnitude of the vector.
(*d*) less than, equal to, or greater than the magnitude of the vector.

3. You are in the middle of a large field. You walk in a straight line for 100 m, then turn left and walk 100 m more in a straight line before stopping. When you stop, you are 100 m from your starting point. By how many degrees did you turn?
(*a*) 90°. (*b*) 120°. (*c*) 30°. (*d*) 180°.
(*e*) This is impossible. You cannot walk 200 m and be only 100 m away from where you started.

4. Which of the following equations correctly expresses the relation between vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$, shown in Fig. 3–36?
(*a*) $\vec{\mathbf{A}} = \vec{\mathbf{B}} + \vec{\mathbf{C}}$.
(*b*) $\vec{\mathbf{B}} = \vec{\mathbf{A}} + \vec{\mathbf{C}}$.
(*c*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.
(*d*) $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$.

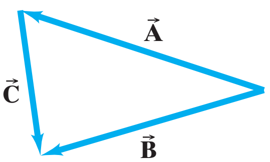
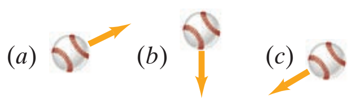


FIGURE 3–36 MisConceptual Question 4.

5. A car is driven at a constant speed of 10.0 m/s around a circle of radius 20.0 m. As it goes $\frac{1}{4}$ of the way around,
(*a*) the magnitude of the average velocity is 0.
(*b*) the magnitude of the average velocity is 10 m/s.
(*c*) the magnitude of the average velocity is between 0 and 10 m/s.
(*d*) the magnitude of the average velocity is greater than 10 m/s.

6. A bullet fired horizontally from a rifle begins to fall
(*a*) as soon as it leaves the barrel.
(*b*) after air friction reduces its speed.
(*c*) not at all if air resistance is ignored.

7. A baseball player hits a ball that soars high into the air. After the ball has left the bat, and while it is traveling upward (at point P in Fig. 3–37), what is the direction of acceleration? Ignore air resistance.



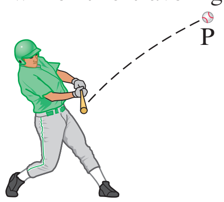


FIGURE 3–37 MisConceptual Question 7.

8. One ball is dropped vertically from a window. At the same instant, a second ball is thrown horizontally from the same window. Which ball has the greater speed at ground level?
(*a*) The dropped ball.
(*b*) The thrown ball.
(*c*) Neither—they both have the same speed on impact.
(*d*) It depends on how hard the ball was thrown.

9. Two balls having different speeds roll off the edge of a horizontal table at the same time. Which hits the floor sooner?
(*a*) The faster one.
(*b*) The slower one.
(*c*) Both the same.

10. You are riding in an enclosed train car moving at 90 km/h. If you throw a baseball straight up, where will the baseball land?
(*a*) In front of you.
(*b*) Behind you.
(*c*) In your hand.
(*d*) Can't decide from the given information.

11. Which of the three kicks in Fig. 3–38 is in the air for the longest time? They all reach the same maximum height h. Ignore air resistance.
(*a*), (*b*), (*c*), or
(*d*) all the same time.

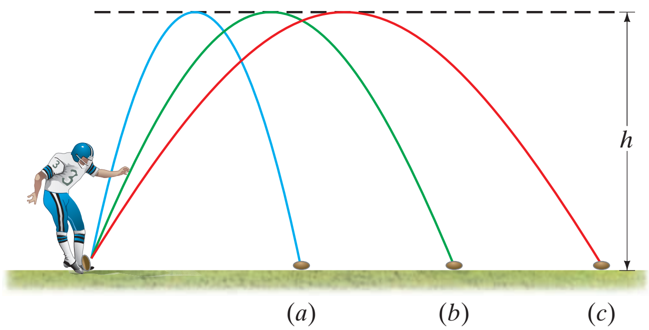


FIGURE 3–38 MisConceptual Question 11.

12. A baseball is hit high and far. Which of the following statements is true? At the highest point,
(*a*) the magnitude of the acceleration is zero.
(*b*) the magnitude of the velocity is zero.
(*c*) the magnitude of the velocity is the slowest.
(*d*) more than one of the above is true.
(*e*) none of the above are true.

13. A hunter is aiming horizontally at a monkey who is sitting in a tree. The monkey is so terrified when it sees the gun that it falls off the tree. At that very instant, the hunter pulls the trigger. What will happen?
(*a*) The bullet will miss the monkey because the monkey falls down while the bullet speeds straight forward.
(*b*) The bullet will hit the monkey because both the monkey and the bullet are falling downward at the same rate due to gravity.
(*c*) The bullet will miss the monkey because although both the monkey and the bullet are falling downward due to gravity, the monkey is falling faster.
(*d*) It depends on how far the hunter is from the monkey.

14. Which statements are *not* valid for a projectile? Take up as positive and ignore air resistance.
(*a*) The projectile has the same x velocity at any point on its path.
(*b*) The acceleration of the projectile is positive and decreasing when the projectile is moving upwards, zero at the top, and increasingly negative as the projectile descends.
(*c*) The acceleration of the projectile has a constant negative value.
(*d*) The y component of the velocity of the projectile is zero at the highest point of the projectile's path.
(*e*) The velocity at the highest point is zero.

15. A car travels 10 m/s east. Another car travels 10 m/s north. The relative speed of the first car with respect to the second is
(*a*) less than 20 m/s.
(*b*) exactly 20 m/s.
(*c*) more than 20 m/s.

Problems

3–2 to 3–5 Vector Addition; Unit Vectors

1. (I) A car is driven 245 km west and then 118 km southwest (45.0°). What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.

2. (I) A delivery truck travels 21 blocks north, 16 blocks east, and 26 blocks south. What is its final displacement from the origin? Assume the blocks are equal length.

3. (I) If $V_x = 9.40$ units and $V_y = -6.80$ units, determine the magnitude and direction of $\vec{\mathbf{V}}$.

4. (II) Graphically determine the resultant of the following three vector displacements: (1) 24 m, 36° north of east; (2) 18 m, 37° east of north; and (3) 26 m, 33° west of south.

5. (II) $\vec{\mathbf{V}}$ is a vector 21.8 units in magnitude and points at an angle of 23.4° above the negative x axis. (*a*) Sketch this vector. (*b*) Calculate V_x and V_y. (*c*) Use V_x and V_y to obtain (again) the magnitude and direction of $\vec{\mathbf{V}}$. [*Note*: Part (*c*) is a good way to check if you've resolved your vector correctly.]

6. (II) Vector $\vec{\mathbf{V}}_1$ is 6.2 units long and points along the negative x axis. Vector $\vec{\mathbf{V}}_2$ is 8.1 units long and points at +55° to the positive x axis. (*a*) What are the x and y components of each vector? (*b*) Determine the sum $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ (magnitude and angle).

7. (II) Figure 3–39 shows two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, whose magnitudes are $A = 6.8$ units and $B = 5.5$ units. Determine $\vec{\mathbf{C}}$ if (*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, (*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, (*c*) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}}$. Give the magnitude and direction for each.

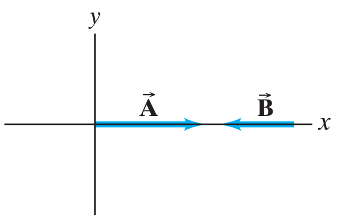


FIGURE 3–39 Problem 7.

8. (II) An airplane is traveling 815 km/h in a direction 41.5° west of north (Fig. 3–40). (*a*) Find the components of the velocity vector in the northerly and westerly directions. (*b*) How far north and how far west has the plane traveled after 1.75 h?

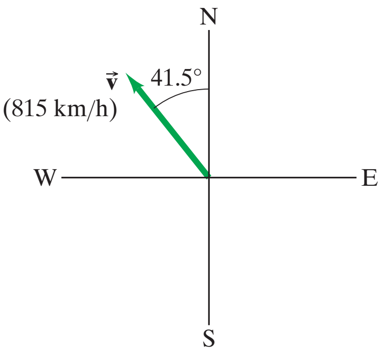


FIGURE 3–40 Problem 8.

9. (II) The summit of a mountain, 2450 m above base camp, is measured on a map to be 4580 m horizontally from the camp in a direction 38.4° west of north. What are the components of the displacement vector from camp to summit? What is its magnitude? Choose the x axis east, y axis north, and z axis up.

10. (II) Three vectors are shown in Fig. 3–41. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (*a*) components, (*b*) magnitude and angle with the $+x$ axis.

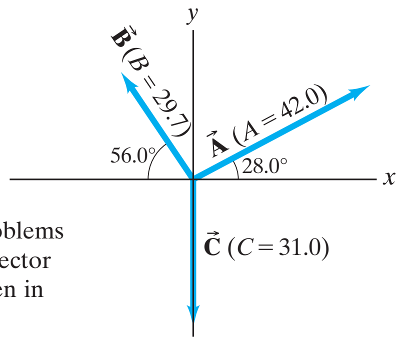


FIGURE 3–41 Problems 10, 11, 12, and 13. Vector magnitudes are given in arbitrary units.

11. (II) (*a*) Given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Fig. 3–41, determine $\vec{\mathbf{B}} - \vec{\mathbf{A}}$. (*b*) Determine $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ without using your answer in (*a*). (*c*) Compare your results and see if they are opposite.

12. (II) Determine the vector $\vec{\mathbf{A}} - \vec{\mathbf{C}}$, given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{C}}$ in Fig. 3–41.

13. (II) For the vectors shown in Fig. 3–41, determine (*a*) $\vec{\mathbf{B}} - 3\vec{\mathbf{A}}$, (*b*) $2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}}$, and (*c*) $\vec{\mathbf{C}} - \vec{\mathbf{A}} - \vec{\mathbf{B}}$.

14. (II) Let $\vec{\mathbf{V}}_1 = -6.0\,\hat{\mathbf{i}} + 8.0\,\hat{\mathbf{j}}$ and $\vec{\mathbf{V}}_2 = -4.5\,\hat{\mathbf{i}} - 5.0\,\hat{\mathbf{j}}$. Determine the magnitude and direction of (*a*) $\vec{\mathbf{V}}_1$, (*b*) $\vec{\mathbf{V}}_2$, (*c*) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, and (*d*) $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$.

15. (II) (*a*) Determine the magnitude and direction of the sum of the three vectors $\vec{\mathbf{V}}_1 = 4.0\,\hat{\mathbf{i}} - 8.0\,\hat{\mathbf{j}}$, $\vec{\mathbf{V}}_2 = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, and $\vec{\mathbf{V}}_3 = -2.0\,\hat{\mathbf{i}} + 4.0\,\hat{\mathbf{j}}$. (*b*) Determine $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3$.

16. (II) Suppose a vector $\vec{\mathbf{V}}$ makes an angle ϕ with respect to the y axis. What could be the x and y components of the vector $\vec{\mathbf{V}}$?

17. (II) Two vectors, $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$, add to a resultant $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$. Describe $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ if (*a*) $V_R = V_1 + V_2$, (*b*) $V_R^2 = V_1^2 + V_2^2$, (*c*) $V_1 + V_2 = V_1 - V_2$.

18. (III) You are given a vector in the xy plane that has a magnitude of 95.0 units and a y component of −60.0 units. (*a*) What are the two possibilities for its x component? (*b*) Assuming the x component is known to be positive, specify the vector which, if you add it to the original one, would give a resultant vector that is 80.0 units long and points entirely in the $-x$ direction.

3–6 Vector Kinematics

19. (I) The position of a particular particle as a function of time is given by $\vec{\mathbf{r}} = \left(9.60\,t\,\hat{\mathbf{i}} + 6.45\,\hat{\mathbf{j}} - 1.50\,t^2\,\hat{\mathbf{k}}\right)\mathrm{m}$. Determine the particle's velocity and acceleration as a function of time.

20. (I) What was the average velocity of the particle in Problem 19 between $t = 1.00$ s and $t = 3.00$ s? What is the magnitude of the instantaneous velocity at $t = 2.00$ s?

21. (II) A car is moving with speed 16.0 m/s due south at one moment and 25.7 m/s due east 8.00 s later. Over this time interval, determine the magnitude and direction of (*a*) its average velocity, (*b*) its average acceleration. (*c*) What is its average speed? [*Hint*: Can you determine all these from the information given?]

22. (II) At $t = 0$, a particle starts from rest at $x = 0$, $y = 0$, and moves in the xy plane with an acceleration $\vec{\mathbf{a}} = (4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}})\,\mathrm{m/s^2}$. Determine (*a*) the *x* and *y* components of velocity, (*b*) the speed of the particle, and (*c*) the position of the particle, all as a function of time. (*d*) Evaluate all the above at $t = 2.0\,\mathrm{s}$.

23. (II) (*a*) A skier is accelerating down a 30.0° hill at $1.80\,\mathrm{m/s^2}$ (Fig. 3–42). What is the vertical component of her acceleration? (*b*) How long will it take her to reach the bottom of the hill, assuming she starts from rest and accelerates uniformly, if the elevation change is 125 m?

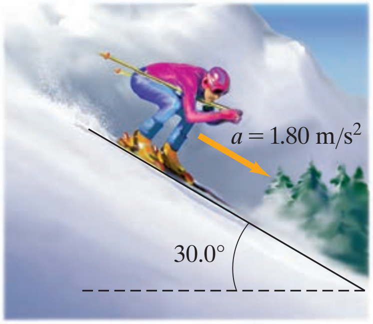


FIGURE 3–42 Problem 23.

24. (II) A hiker follows a winding trail for 5.5 hours while climbing a mountain. The distance along the trail is 11.5 km and the summit is 850 m above and 8.0 km due north of the starting point. What are the average speed and the magnitude and direction of the average velocity vector?

25. (II) An ant walks on a piece of graph paper straight along the *x* axis a distance of 10.0 cm in 2.40 s. It then turns left 40.0° and walks in a straight line another 10.0 cm in 1.80 s. Finally, it turns another 70.0° to the left and walks another 10.0 cm in 1.55 s. Determine (*a*) the *x* and *y* components of the ant's average velocity, and (*b*) its magnitude and direction.

26. (II) Suppose the position of an object is given by $\vec{\mathbf{r}} = (3.0t^2\,\hat{\mathbf{i}} - 6.0t^3\,\hat{\mathbf{j}})\,\mathrm{m}$. (*a*) Determine its velocity $\vec{\mathbf{v}}$ and acceleration $\vec{\mathbf{a}}$, as a function of time. (*b*) Determine $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$ at time $t = 3.5\,\mathrm{s}$.

27. (II) A particle's position as a function of time *t* is given by $\vec{\mathbf{r}} = (5.0t + 6.0t^2)\,\mathrm{m}\,\hat{\mathbf{i}} + (7.0 - 3.0t^3)\,\mathrm{m}\,\hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$, find the magnitude and direction of the particle's displacement vector $\Delta\vec{\mathbf{r}}$ relative to the point $\vec{\mathbf{r}}_0 = (2.0\,\hat{\mathbf{i}} + 7.0\,\hat{\mathbf{j}})\,\mathrm{m}$.

28. (II) On mountainous downhill roads, an escape lane is sometimes placed to the side of the road for trucks whose brakes might fail. Assuming a constant upward slope of 26°, calculate the horizontal and vertical components of the acceleration of a truck that slowed from 110 km/h to rest in 7.0 s. See Fig. 3–43.

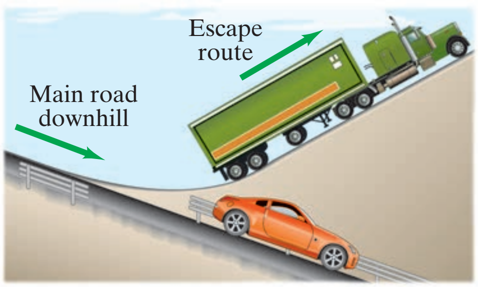


FIGURE 3–43 Problem 28.

29. (II) A light plane is headed due south with a speed relative to still air of 185 km/h. After 1.25 h, the pilot notices that they have covered only 135 km and their direction is not south but 15.0° east of south. What is the wind velocity?

30. (III) An object, which is at the origin at time $t = 0$, has initial velocity $\vec{\mathbf{v}}_0 = (-14.0\,\hat{\mathbf{i}} - 7.0\,\hat{\mathbf{j}})\,\mathrm{m/s}$ and constant acceleration $\vec{\mathbf{a}} = (6.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}})\,\mathrm{m/s^2}$. Find the position $\vec{\mathbf{r}}$ where the object comes to rest (momentarily).

31. (III) A particle starts from the origin at $t = 0$ with an initial velocity of 5.0 m/s along the positive *x* axis. If the acceleration is $(-3.0\,\hat{\mathbf{i}} + 4.5\,\hat{\mathbf{j}})\,\mathrm{m/s^2}$, determine the velocity and position of the particle at the moment it reaches its maximum *x* coordinate.

3–7 and 3–8 Projectile Motion (neglect air resistance)

32. (I) A tiger leaps horizontally from a 7.5-m-high rock with a speed of 3.0 m/s. How far from the base of the rock will she land?

33. (I) A diver running 2.5 m/s dives out horizontally from the edge of a vertical cliff and 3.5 s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?

34. (II) A ball is thrown horizontally from the roof of a building 7.5 m tall and lands 9.5 m from the base. What was the ball's initial speed?

35. (II) A ball thrown horizontally at 10.8 m/s from the roof of a building lands 21.0 m from the base of the building. How high is the building?

36. (II) A football is kicked at ground level with a speed of 18.0 m/s at an angle of 31.0° to the horizontal. How much later does it hit the ground?

37. (II) A fire hose held near the ground shoots water at a speed of 6.5 m/s. At what angle(s) should the nozzle point in order that the water land 2.5 m away (Fig. 3–44)? Why are there two different angles? Sketch the two trajectories.

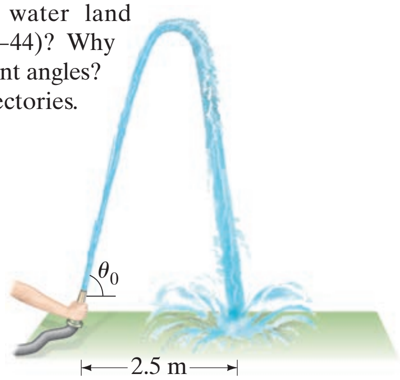


FIGURE 3–44 Problem 37.

38. (II) You buy a plastic dart gun, and being a clever physics student you decide to do a quick calculation to find its maximum horizontal range. You shoot the gun straight up, and it takes 3.4 s for the dart to land back at the barrel. What is the maximum horizontal range of your gun?

39. (II) A projectile is fired with an initial speed of 38.8 m/s at an angle of 42.2° above the horizontal on a long flat firing range. Determine (*a*) the maximum height reached by the projectile, (*b*) the total time in the air, (*c*) the total horizontal distance covered (that is, the range), and (*d*) the speed of the projectile 1.50 s after firing.

40. (II) A diver leaves the end of a diving board that is 2.0 m above the surface of the water. Her dive takes her 1.2 m above the board, and then into the water a horizontal distance of 2.2 m from the end of the board. At what speed and angle did she leave the board?

41. (II) The maximum range of a projectile is found to be 95 m. If the projectile strikes the ground a distance of 68 m away, what was the angle of launch?

42. (II) A projectile is shot from the edge of a cliff 125 m above ground level with an initial speed of 62.0 m/s at an angle of 35.0° with the horizontal, as shown in Fig. 3–45. (*a*) Determine the time taken by the projectile to hit point P at ground level. (*b*) Determine the distance X of point P from the base of the vertical cliff. At the instant just before the projectile hits point P, find (*c*) the horizontal and the vertical components of its velocity, (*d*) the magnitude of the velocity, and (*e*) the angle made by the velocity vector with the horizontal. (*f*) Find the maximum height above the cliff top reached by the projectile.

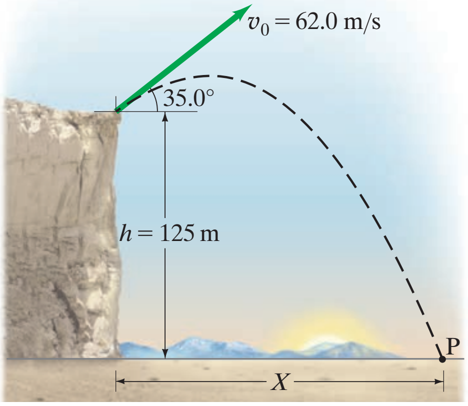


FIGURE 3–45 Problem 42.

43. (II) An athlete performing a long jump leaves the ground at a 27.0° angle and lands 7.80 m away. (*a*) What was the takeoff speed? (*b*) If this speed were increased by just 5.0%, how much longer would the jump be?

44. (II) In Example 3–11 we chose the x axis to the right and y axis up. Redo this problem by defining the x axis to the left and y axis down, and show that the conclusion remains the same—the football lands on the ground 40.5 m to the right of where it departed the punter's foot.

45. (II) A stunt driver wants to make his car jump over 8 cars parked side by side below a horizontal ramp (Fig. 3–46). (*a*) With what minimum speed must he drive off the horizontal ramp? The vertical height of the ramp is 1.5 m above the car roofs and the horizontal distance he must clear is 22 m. (*b*) If the ramp is now tilted upward, so that "takeoff angle" is 9.0° above the horizontal, what is the new minimum speed?

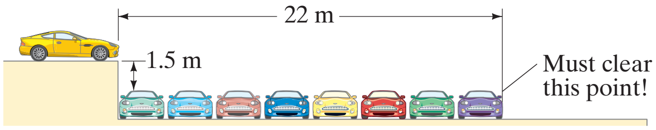


FIGURE 3–46 Problem 45.

46. (II) A baseball is hit with a speed of 27.0 m/s at an angle of 45.0°. It lands on the flat roof of a 12.5-m-tall nearby building. If the ball was hit when it was 1.2 m above the ground, what horizontal distance does it travel before it lands on the building?

47. (II) A rescue plane wants to drop supplies to isolated mountain climbers on a rocky ridge 265 m below. If the plane is traveling horizontally with a speed of 125 km/h, how far in advance of the recipients (horizontal distance) must the goods be dropped?

48. (II) Show that the time required for a projectile to reach its highest point is equal to the time for it to return to its original height if air resistance is negligible.

49. (II) Exactly 3.0 s after a projectile is fired into the air from the ground, it is observed to have a velocity $\vec{\mathbf{v}} = (7.8\,\hat{\mathbf{i}} + 5.2\,\hat{\mathbf{j}})\,\mathrm{m/s}$, where the x axis is horizontal and the y axis is positive upward. Determine (*a*) the horizontal range of the projectile, (*b*) its maximum height above the ground, and (*c*) its speed and angle of motion just before it strikes the ground.

50. (II) At what projection angle will the range of a projectile equal its maximum height?

51. (II) A ball is thrown horizontally from the top of a cliff with initial speed v_0 (at $t = 0$). At any moment, its direction of motion makes an angle θ to the horizontal (Fig. 3–47). Derive a formula for θ as a function of time, t, as the ball follows a projectile's path.

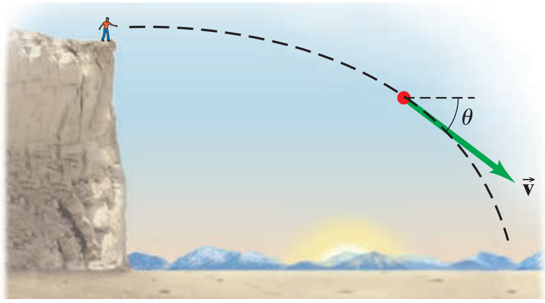


FIGURE 3–47 Problem 51.

52. (II) Romeo is throwing pebbles gently up to Juliet's window, and he wants the pebbles to hit the window with only a horizontal component of velocity. He is standing at the edge of a rose garden 8.0 m below her window and 8.5 m from the base of the wall (Fig. 3–48). How fast are the pebbles going when they hit her window?

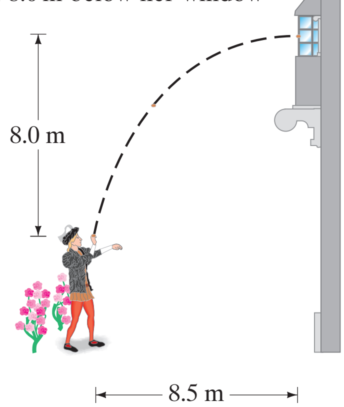


FIGURE 3–48 Problem 52.

53. (II) (*a*) A long jumper leaves the ground at 45° above the horizontal and lands 8.0 m away. What is her "takeoff" speed v_0? (*b*) Now she is out on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5 m vertically below. If she jumps from the edge of the left bank at 45° with the speed calculated in (*a*), how long, or short, of the opposite bank will she land (Fig. 3–49)?

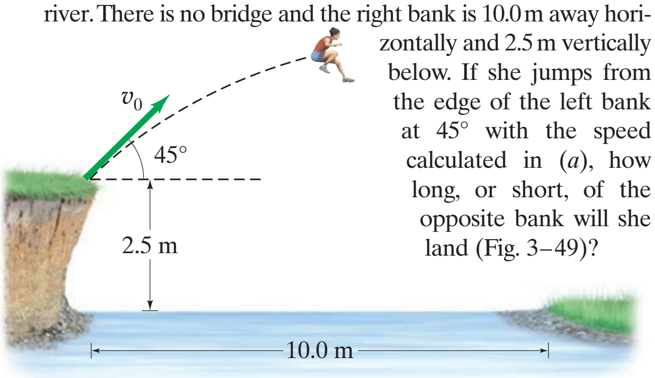


FIGURE 3–49 Problem 53.

54. (II) Show that the speed with which a projectile leaves the ground is equal to its speed just before it strikes the ground at the end of its journey, assuming the firing level equals the landing level.

55. (II) If a ball is kicked from ground level at 15.0 m/s, there are two launch angles that will make the ball land 20.0 m away. (*a*) What are the two angles? (*b*) What maximum height does the ball reach in each case? (*c*) How long is the ball in the air for each case? Ignore air resistance.

56. (II) At serve, a tennis player aims to hit the ball horizontally. What minimum speed is required for the ball to clear the 0.90-m-high net about 15.0 m from the server if the ball is "launched" from a height of 2.30 m? Where will the ball land if it just clears the net (and will it be "good" in the sense that it lands within 7.0 m of the net)? How long will it be in the air? See Fig. 3–50.

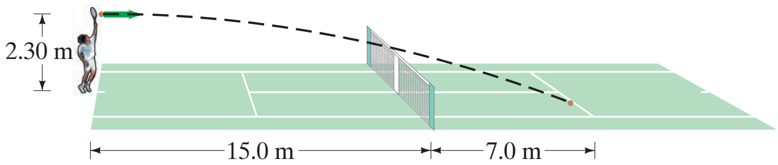


FIGURE 3–50 Problem 56.

57. (II) An Olympic long jumper is capable of jumping 8.0 m. Assuming his horizontal speed is 9.1 m/s as he leaves the ground, how long is he in the air and how high does he go? Assume that he lands standing upright—that is, the same way he left the ground.

58. (III) Revisit Example 3–9, and assume that the boy with the slingshot is *below* the boy in the tree (Fig. 3–51) and so aims *upward*, directly at the boy in the tree. Show that again the boy in the tree makes the wrong move by letting go at the moment the water balloon is shot.

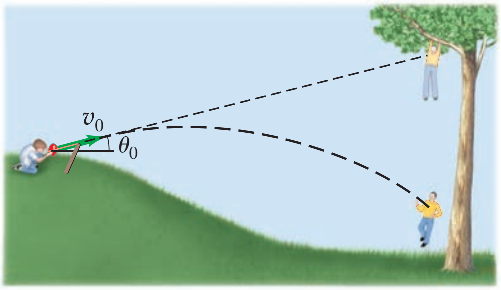
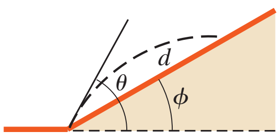


FIGURE 3–51 Problem 58.

59. (III) Suppose the kick in Example 3–8 is attempted 36.0 m from the goalposts, whose crossbar is 3.05 m above the ground. If the football is directed perfectly between the goalposts, will it pass over the bar and be a field goal? Show why or why not. If not, from what horizontal distance must this kick be made if it is to score?

60. (III) A person stands at the base of a hill that is a straight incline making an angle ϕ with the horizontal (Fig. 3–52). For a given initial speed v_0, at what angle θ (to the horizontal) should objects be thrown so that the distance d they land up the hill is as large as possible?

FIGURE 3–52 Problem 60. Given ϕ and v_0, determine θ to make d maximum.



61. (III) Derive a formula for the horizontal range R of a projectile when it lands at a height h above its initial point. (For $h < 0$, it lands a distance $-h$ below the starting point.) Assume it is projected at an angle θ_0 with initial speed v_0.

3–9 Relative Velocity

62. (I) A person going for a morning jog on the deck of a cruise ship is running toward the bow (front) of the ship at 2.5 m/s while the ship is moving ahead at 8.8 m/s. What is the velocity of the jogger relative to the water? Later, the jogger is moving toward the stern (rear) of the ship. What is the jogger's velocity relative to the water now?

63. (I) Huck Finn walks at a speed of 0.70 m/s across his raft (that is, he walks perpendicular to the raft's motion relative to the shore). The heavy raft is traveling down the Mississippi River at a speed of 1.50 m/s relative to the river bank (Fig. 3–53). What is Huck's velocity (speed and direction) relative to the river bank?

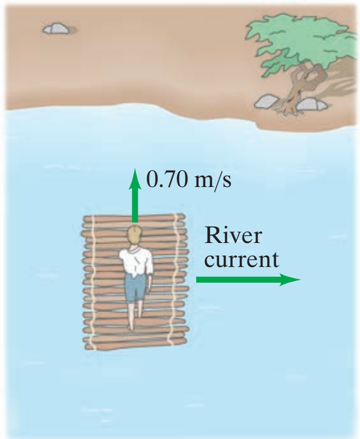


FIGURE 3–53 Problem 63.

64. (II) Determine the speed of the boat with respect to the shore in Example 3–13.

65. (II) A motorboat whose speed in still water is 4.30 m/s must aim upstream at an angle of 23.5° (with respect to a line perpendicular to the shore) in order to travel directly across the stream. (*a*) What is the speed of the current? (*b*) What is the resultant speed of the boat with respect to the shore? (See Fig. 3–33.)

66. (II) A passenger on a boat moving at 1.70 m/s on a still lake walks up a flight of stairs at a speed of 0.60 m/s, Fig. 3–54. The stairs are angled at 45° pointing in the direction of motion as shown. Write the vector velocity of the passenger relative to the water.

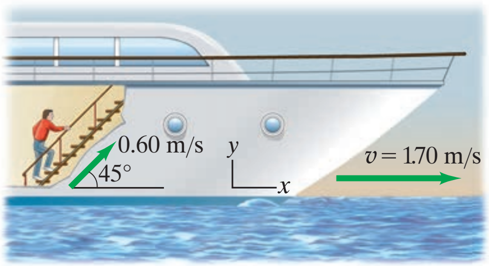


FIGURE 3–54 Problem 66.

67. (II) An airplane is heading due south at a speed of 688 km/h. If a wind begins blowing from the southwest at a speed of 85.0 km/h (average), calculate (*a*) the velocity (magnitude and direction) of the plane, relative to the ground, and (*b*) how far from its intended position it will be after 11.0 min if the pilot takes no corrective action. [*Hint*: First draw a diagram.]

68. (II) In what direction should the pilot aim the plane in Problem 67 so that it will fly due south?

69. (II) Raindrops make an angle θ with the vertical when viewed through a moving train window (Fig. 3–55). If the speed of the train is v_T, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?

FIGURE 3–55 Problem 69.

70. (II) A boat, whose speed in still water is 2.80 m/s, must cross a 265-m-wide river and arrive at a point 118 m upstream from where it starts (Fig. 3–56). To do so, the pilot must head the boat at a 45.0° upstream angle. What is the speed of the river's current?

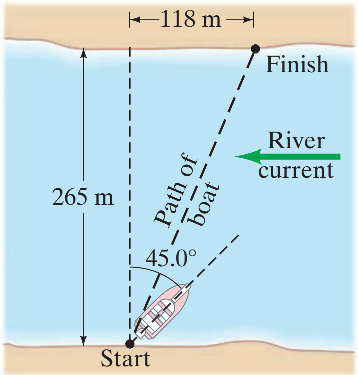


FIGURE 3–56 Problem 70.

71. (II) A swimmer is capable of swimming 0.60 m/s in still water. (*a*) If she aims her body directly across a 55-m-wide river whose current is 0.50 m/s, how far downstream (from a point opposite her starting point) will she land? (*b*) How long will it take her to reach the other side?

72. (II) (*a*) At what upstream angle must the swimmer in Problem 71 aim, if she is to arrive at a point directly across the stream? (*b*) How long will it take her?

73. (II) Two cars approach a street corner at right angles to each other (Fig. 3–57). Car 1 travels at a speed relative to Earth $v_{1E} = 35$ km/h, and car 2 at $v_{2E} = 55$ km/h. What is the relative velocity of car 1 as seen by car 2? What is the velocity of car 2 relative to car 1?

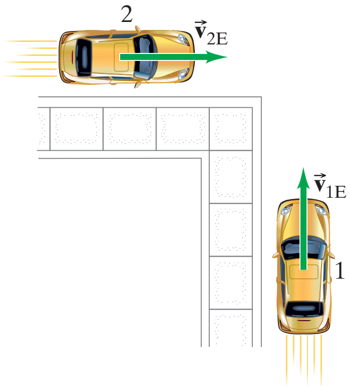


FIGURE 3–57 Problem 73.

74. (III) An airplane, whose air speed is 560 km/h, is supposed to fly in a straight path 38.0° N of E. But a steady 82 km/h wind is blowing from the north. In what direction should the plane head? [*Hint*: Use the law of sines, Appendix A–9.]

75. (III) Point B is located across the river from point A and at a 42.0° angle upstream from A. A boat travels from point A to point B, sailing at 18.0 km/h relative to the water. The current in the river flows at 5.60 km/h. The river is 1.45 km wide. (*a*) At what angle should the boat head? (*b*) What will be the boat's speed relative to the ground? (*c*) How much time does the trip from point A to point B take?

General Problems

76. A plumber steps out of his truck, walks 55 m east and 38 m south, and then takes an elevator 12 m into the subbasement of a building where a bad leak is occurring. What is the displacement of the plumber relative to his truck? Give your answer in components. Also give the magnitude and angles, with respect to the x axis, in the vertical and horizontal planes. Assume x is east, y is north, and z is up.

77. *Apollo* astronauts took a "nine iron" to the Moon and hit a golf ball about 180 m. Assuming that the swing, launch angle, and so on, were the same as on Earth where the same astronaut could hit it only 32 m, estimate the acceleration due to gravity on the surface of the Moon. (We neglect air resistance in both cases, but on the Moon there is none.)

78. A car moving at 95 km/h passes a 1.40-km-long train traveling in the same direction on a track that is parallel to the road. If the speed of the train is 75 km/h, how long does it take the car to pass the train, and how far will the car have traveled in this time? What are the results if the car and train are instead traveling in opposite directions?

79. When Babe Ruth hit a homer over the 8.0-m-high right-field fence 98 m from home plate, roughly what was the minimum speed of the *baseball* when it left the bat? Assume the ball was hit 1.0 m above the ground and its path initially made a 36° angle with the ground.

80. A child runs down a 12° hill and then suddenly jumps upward at a 15° angle above horizontal and lands 1.3 m down the hill as measured along the hill. What was the child's initial speed at the jump?

81. Here is something to try at a sporting event. Show that the maximum height h attained by an object projected into the air, such as a baseball, football, or soccer ball, is approximately given by

$$h \approx 1.2\, t^2\ \text{m},$$

where t is the total time of flight for the object in seconds. Assume that the object returns to the same level as that from which it was launched, as in Fig. 3–58. For example, if you count seconds and find that a baseball was in the air for $t = 5.0$ s, the maximum height attained was $h = 1.2 \times (5.0)^2 = 30$ m. The fun of this relation is that h can be determined without knowledge of the launch speed v_0 or launch angle θ_0. Why is that exactly? See Section 3–8.

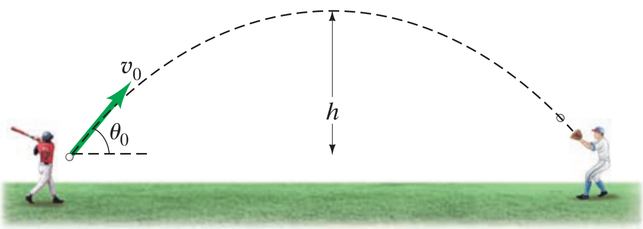


FIGURE 3–58 Problem 81.

82. The cliff divers of Acapulco push off horizontally from rock platforms about 35 m above the water, but they must clear rocky outcrops at water level that extend out into the water 5.0 m from the base of the cliff directly under their launch point. See Fig. 3–59. What minimum pushoff speed is necessary to clear the rocks? How long are they in the air?

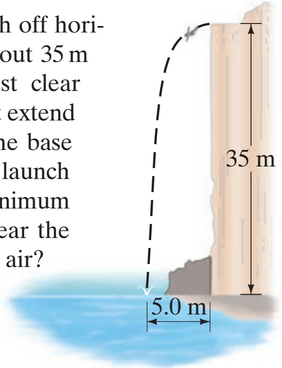


FIGURE 3–59 Problem 82.

83. A grasshopper hops along a level road. On each hop, the grasshopper launches itself at angle $\theta_0 = 45°$ and achieves a range $R = 0.80$ m. What is the average horizontal speed of the grasshopper as it hops along the road? Assume that the time spent on the ground between hops is negligible.

84. Spymaster Andrea, flying a constant 208 km/h horizontally in a low-flying helicopter, wants to drop secret documents into her contact's open car which is traveling 135 km/h on a level highway 78.0 m below. At what angle (with the horizontal) should the car be in her sights when the packet is released (Fig. 3–60)?

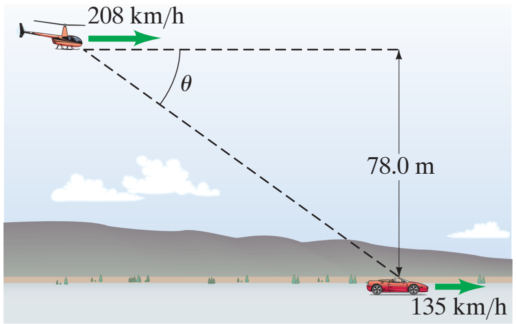


FIGURE 3–60 Problem 84.

85. A basketball leaves a player's hands at a height of 2.10 m above the floor. The basket is 3.05 m above the floor. The player likes to shoot the ball at a 38.0° angle. If the shot is made from a horizontal distance of 11.00 m and must be accurate to ± 0.22 m (horizontally), what is the range of initial speeds allowed to make the basket?

86. Extreme-sports enthusiasts have been known to jump off the top of El Capitan, a sheer granite cliff of height 910 m in Yosemite National Park. Assume a jumper runs horizontally off the top of El Capitan with speed 4.0 m/s and enjoys a free fall until she is 150 m above the valley floor, at which point she opens her parachute (Fig. 3–61). (*a*) How long is the jumper in free fall? Ignore air resistance. (*b*) It is important to be as far away from the cliff as possible before opening the parachute. How far from the cliff is this jumper when she opens her chute?

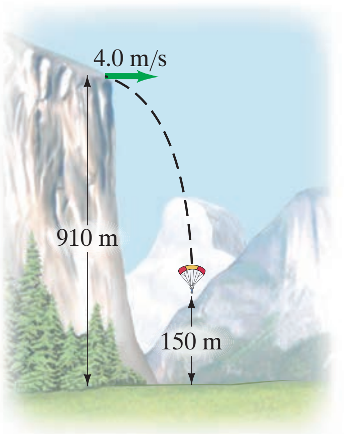


FIGURE 3–61 Problem 86.

87. If a baseball pitch leaves the pitcher's hand horizontally at a velocity of 130 km/h, by what % will the pull of gravity change the magnitude of the velocity when the ball reaches the batter, 18 m away? For this estimate, ignore air resistance and spin on the ball.

88. A projectile is launched from ground level to the top of a cliff which is 195 m away and 135 m high (see Fig. 3–62). If the projectile lands on top of the cliff 6.6 s after it is fired, find the initial velocity of the projectile (magnitude and direction). Neglect air resistance.

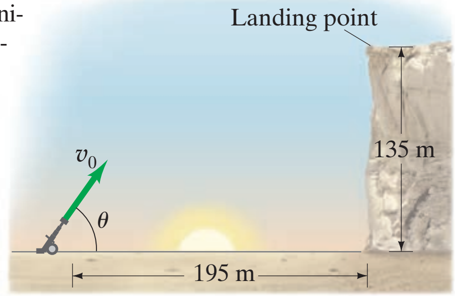


FIGURE 3–62 Problem 88.

89. A diver leaves the end of a 3.5-m-high diving board and strikes the water 1.3 s later, 3.0 m beyond the end of the board. Considering the diver as a particle, determine: (*a*) her initial velocity, $\vec{\mathbf{v}}_0$; (*b*) the maximum height reached; and (*c*) the velocity $\vec{\mathbf{v}}_f$ with which she enters the water.

90. A basketball is shot from an initial height of 2.40 m (Fig. 3–63) with an initial speed $v_0 = 12$ m/s directed at an angle $\theta_0 = 35°$ above the horizontal. (*a*) How far from the basket was the player if he made a basket? (*b*) At what angle to the horizontal did the ball enter the basket?

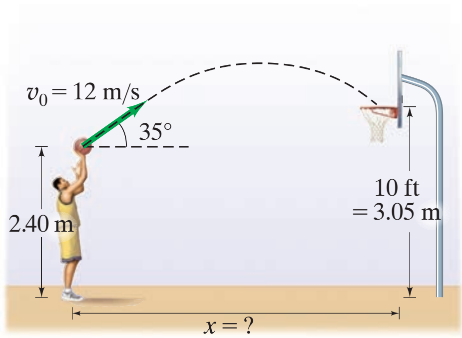


FIGURE 3–63 Problem 90.

91. A hunter aims directly at a target (on the same level) 38.0 m away. (*a*) If the arrow leaves the bow at a speed of 21.3 m/s, by how much will it miss the target? (*b*) At what angle should the bow be aimed so the target will be hit?

92. A person in the passenger basket of a hot-air balloon throws a ball horizontally outward from the basket with speed 12.0 m/s (Fig. 3–64). What initial velocity (magnitude and direction) does the ball have relative to a person standing on the ground (*a*) if the hot-air balloon is rising at 3.0 m/s relative to the ground during this throw, and (*b*) if the hot-air balloon is descending at 3.0 m/s relative to the ground?

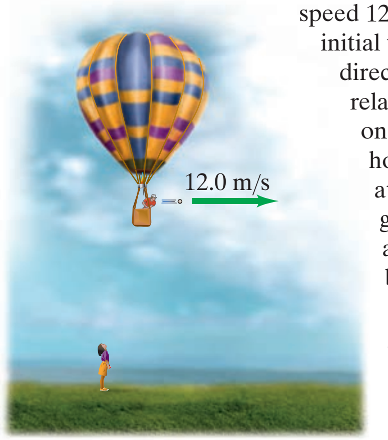


FIGURE 3–64 Problem 92.

93. A rock is kicked horizontally at 15 m/s from a hill with a 45° slope (Fig. 3–65). How long does it take for the rock to hit the ground?

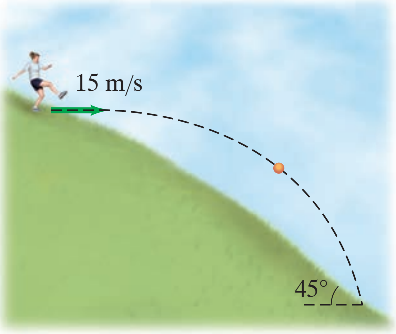


FIGURE 3–65 Problem 93.

94. A batter hits a fly ball which leaves the bat 0.90 m above the ground at an angle of 64° with an initial speed of 28 m/s heading toward centerfield. Ignore air resistance. (*a*) How far from home plate would the ball land if not caught? (*b*) The ball is caught by the centerfielder who, starting at a distance of 105 m from home plate just as the ball was hit, runs straight toward home plate at a constant speed and makes the catch at ground level. Find his speed.

95. A ball is shot from the top of a building with an initial velocity of 24 m/s at an angle $\theta = 42°$ above the horizontal. (*a*) What are the horizontal and vertical components of the initial velocity? (*b*) If a nearby building is the same height and 65 m away, how far below the top of the building will the ball strike the nearby building?

96. The speed of a boat in still water is v. The boat is to make a round trip in a river whose current travels at speed u. Derive a formula for the time needed to make a round trip of total distance D if the boat makes the round trip by moving (*a*) upstream and back downstream, and (*b*) directly across the river and back. We must assume $u < v$; why?

97. At $t = 0$ a batter hits a baseball with an initial speed of 28 m/s at a 55° angle to the horizontal. An outfielder is 85 m from the batter at $t = 0$ and, as seen from home plate, the line of sight to the outfielder makes a horizontal angle of 22° with the plane in which the ball moves (see Fig. 3–66). What speed and direction must the fielder take to catch the ball at the same height from which it was struck? Give the angle with respect to the outfielder's line of sight to home plate.

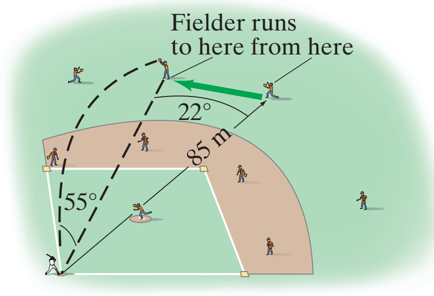


FIGURE 3–66 Problem 97.

98. A child, who is 45 m from the bank of a river, is being carried helplessly downstream by the river's swift current of 1.0 m/s. As the child passes a lifeguard on the river's bank, the lifeguard starts swimming in a straight line (Fig. 3–67) until she reaches the child at a point downstream. If the lifeguard can swim at a speed of 2.0 m/s relative to the water, how long does it take her to reach the child? How far downstream does the lifeguard intercept the child?

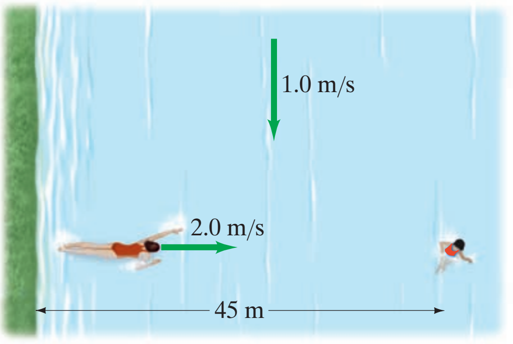


FIGURE 3–67 Problem 98.

99. A particle has a velocity of $\vec{\mathbf{v}} = \left(-3.0\,\hat{\mathbf{i}} + 4.5t\,\hat{\mathbf{j}}\right)$ m/s. The particle starts at $\vec{\mathbf{r}} = \left(2.5\,\hat{\mathbf{i}} - 3.1\,\hat{\mathbf{j}}\right)$ m at $t = 0$. Give the position and acceleration as a function of time. What is the shape of the resulting path?

100. In hot pursuit, Agent Logan of the FBI must get directly across a 1300-m-wide river in minimum time. The river's current is 0.80 m/s, he can row a boat at 1.60 m/s, and he can run 3.00 m/s. Describe the path he should take (rowing plus running along the shore) for the minimum crossing time, and determine the minimum time.

101. You are driving south on a highway at 12 m/s (approximately 25 mi/h) in a snowstorm. When you last stopped, you noticed that the snow was coming down vertically, but it is passing the windows of the moving car at an angle of 9.0° to the horizontal. Estimate the speed of the vertically falling snowflakes relative to the ground. [*Hint*: Construct a relative velocity diagram similar to Fig. 3–33 or 3–34. Be careful about which angle is the angle given.]

102. A boat is traveling where there is a current of 0.20 m/s east (Fig. 3–68). To avoid some offshore rocks, the boat must clear a buoy that is NNE (22.5°) and 2.8 km away. The boat's speed through still water is 2.1 m/s. If the boat wants to pass the right side of the buoy by 0.15 km, at what angle should the boat head?

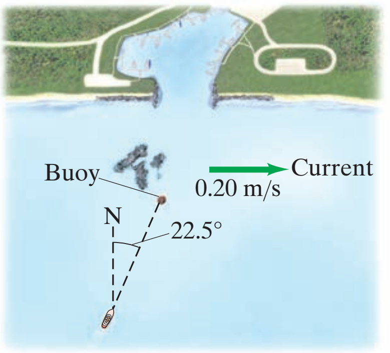
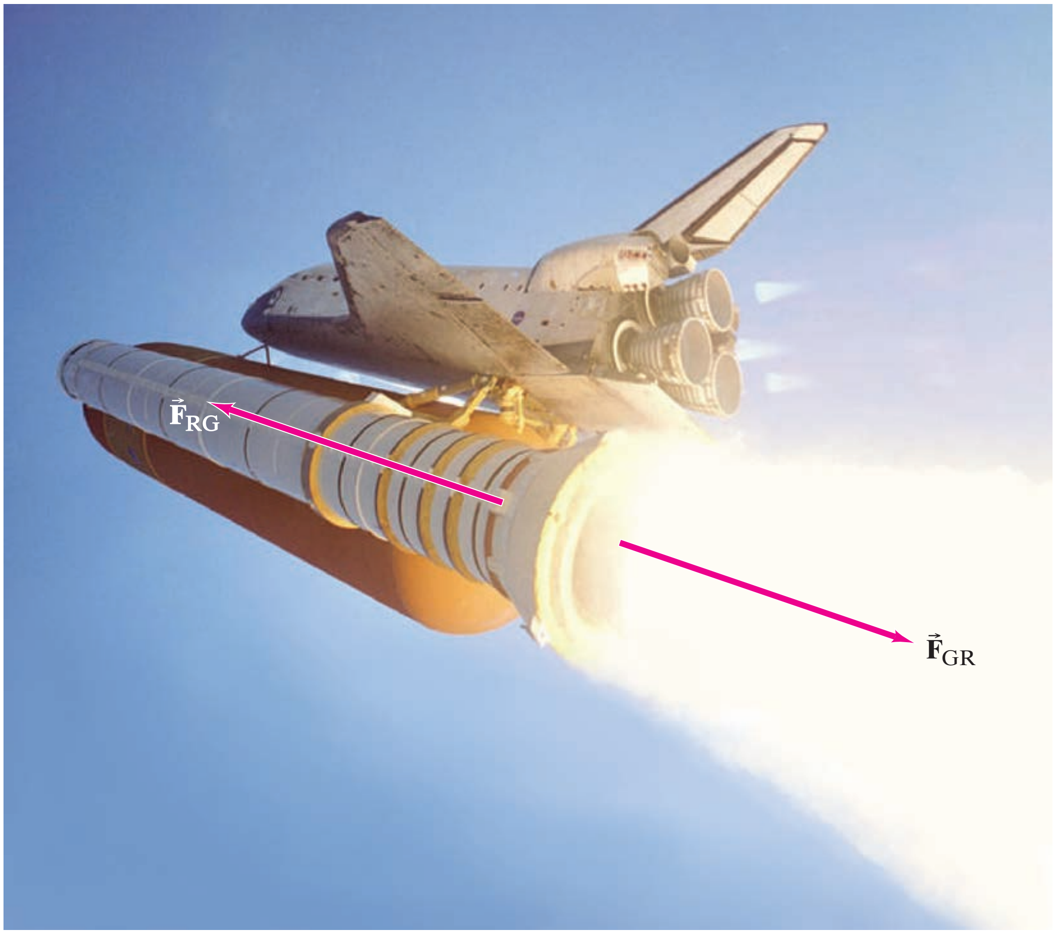


FIGURE 3–68 Problem 102.

ANSWERS TO EXERCISES

A: $3.0\sqrt{2} \approx 4.2$ units.
B: (*a*).
C: (*d*). Helicopter provides the initial velocity of the box.
D: (*a*) $v = v_{x0} = 16.0$ m/s, horizontal; (*b*) 9.80 m/s^2 down.
E: Both balls reach the same height, so are in the air for the same length of time.



A space shuttle is carried out into space by powerful rockets. They are accelerating, increasing in speed rapidly. To do so, a force must be exerted on them according to Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. What exerts this force? The **r**ocket engines exert a force on the **g**ases they push out (expel) from the rear of the rockets (labeled $\vec{\mathbf{F}}_{GR}$). According to Newton's third law, these ejected gases exert an equal and opposite force on the rockets in the forward direction. This "reaction" force exerted *on* the **r**ockets *by* the **g**ases, labeled $\vec{\mathbf{F}}_{RG}$, is the *net* force on the rockets and accelerates the rockets. (Any other forces, such as gravity, are assumed small in comparison.)

CHAPTER 4

Dynamics: Newton's Laws of Motion

CHAPTER-OPENING QUESTIONS—Guess now!

1. A 150-kg football player collides head-on with a 75-kg running back. During the collision, the heavier player exerts a force of magnitude F_A on the smaller player. If the smaller player exerts a force F_B back on the heavier player, which response is most accurate?

(a) $F_B = F_A$.
(b) $F_B < F_A$.
(c) $F_B > F_A$.
(d) $F_B = 0$.
(e) We need more information.

2. A line by the poet T. S. Eliot (from *Murder in the Cathedral*) has the women of Canterbury say "the earth presses up against our feet." What force is this?

(a) Gravity.
(b) The normal force.
(c) A friction force.
(d) Centrifugal force.
(e) No force—they are being poetic.

CONTENTS

We have discussed how motion is *described* in terms of velocity and acceleration. Now we deal with the question of *why* objects move as they do: What makes an object at rest begin to move? What causes an object to accelerate or decelerate? What is involved when an object moves in a curved path? We can answer in each case that a force is required. In this Chapter,† we will investigate the connection between force and motion, which is the subject called **dynamics**.

4–1 Force

FIGURE 4–1 A force exerted on a grocery cart—in this case exerted by a person.

Intuitively, we experience **force** as any kind of a push or a pull on an object. When you push a stalled car or a grocery cart (Fig. 4–1), you are exerting a force on it. When a motor lifts an elevator, or a hammer hits a nail, or the wind blows the leaves of a tree, a force is being exerted. We often call these *contact forces* because the force is exerted when one object comes in contact with another object. On the other hand, we say that an object falls because of the *force of gravity* (which is not a contact force).

If an object is at rest, to start it moving requires force—that is, a force is needed to accelerate an object from zero velocity to a nonzero velocity. For an object already moving, if you want to change its velocity—either in direction or in magnitude—a force is required. In other words, to accelerate an object, a force is always required. In Section 4–4 we discuss the precise relation between acceleration and net force, which is Newton's second law.

One way to measure the magnitude (or strength) of a force is to use a spring scale (Fig. 4–2). Normally, such a spring scale is used to find the weight of an object; by weight we mean the force of gravity acting on the object (Section 4–6). The spring scale, once calibrated, can be used to measure other kinds of forces as well, such as the pulling force shown in Fig. 4–2.

A force exerted in a different direction has a different effect. Force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition discussed in Chapter 3. We can represent any force on a diagram by an arrow, just as we did with velocity. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force. Force vectors in this book are drawn as red in color, velocity is green.

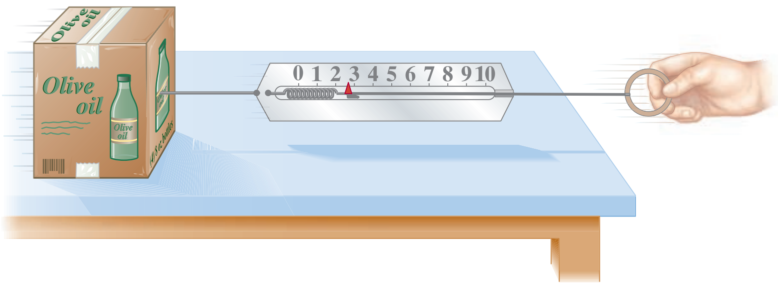


FIGURE 4–2 A spring scale used to measure a force.

4–2 Newton's First Law of Motion

What is the relationship between force and motion? The ancient Greeks, including Aristotle (384–322 B.C.), believed that a force was required to keep an object moving along a horizontal plane. To Aristotle, the natural state of an object was at rest, and a force was thought necessary to keep an object in motion. Aristotle also argued that the greater the force on the object, the greater its speed.

Some 2000 years later, in the early 1600s, Galileo disagreed. Galileo maintained that it is just as natural for an object to be in motion with a constant velocity as it is for it to be at rest. See Section 1–1, page 24.

†We treat everyday objects in motion here. When velocities are extremely high, close to the speed of light $(3.0 \times 10^8\ \mathrm{m/s})$, we use the theory of relativity (Chapter 36), and in the submicroscopic world of atoms and molecules we use quantum theory (Chapter 37 *ff*).

To understand Galileo's idea, consider the following observations involving motion along a horizontal plane. To push an object with a rough surface along a tabletop at constant speed requires a certain amount of force. To push an equally heavy object with a very smooth surface across the table at the same speed will require less force. If a layer of oil or other lubricant is placed between the surface of the object and the table, then almost no force is required to keep the object moving. Notice that in each successive step, less force is required. As the next step, we imagine there is no friction at all, that the object does not rub against the table—or there is a perfect lubricant between the object and the table—and theorize that once started, the object would move across the table at constant speed with *no* force applied. A steel ball bearing rolling on a hard horizontal surface approaches this situation. So does a puck on an air table, in which a thin layer of air reduces friction almost to zero.

It was Galileo's genius to imagine such an idealized world—in this case, one where there is no friction—and to see that it could lead to a more accurate and richer understanding of the real world. This idealization led him to his remarkable conclusion that if no force is applied to a moving object, it will continue to move with constant speed in a straight line. An object slows down only if a force is exerted on it. Galileo thus interpreted friction as a force akin to ordinary pushes and pulls.

To push an object across a table at constant speed requires a force from your hand that can balance the force of friction (Fig. 4–3). When the object moves at constant speed, your pushing force is equal in magnitude to the friction force; but these two forces are in opposite directions, so the *net* force on the object (the vector sum of the two forces) is zero. This is consistent with Galileo's viewpoint: the object moves with constant velocity when no *net* force is exerted on it.

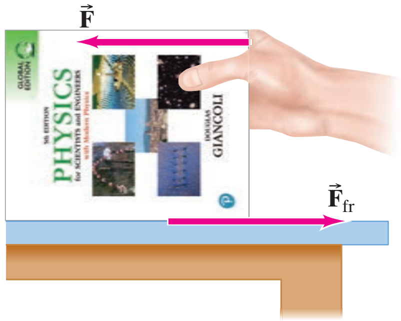


FIGURE 4–3 $\vec{\mathbf{F}}$ represents the force applied by the person and $\vec{\mathbf{F}}_{fr}$ represents the force of friction.

Upon this foundation laid by Galileo, Isaac Newton (Fig. 4–4) built his great theory of motion. Newton's analysis of motion is summarized in his famous "three laws of motion." In his great work, the *Principia* (published in 1687), Newton readily acknowledged his debt to Galileo. In fact, **Newton's first law of motion** is close to Galileo's conclusions. It states that

Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it.

NEWTON'S FIRST LAW OF MOTION

The tendency of an object to maintain its state of rest or of uniform velocity in a straight line is called **inertia**. Newton's first law is thus often called the **law of inertia**.

CONCEPTUAL EXAMPLE 4–1 **Newton's first law.** A school bus comes to a sudden stop, and all of the backpacks on the floor start to slide forward. What force causes them to do that?

RESPONSE It isn't "force" that does it. By Newton's first law, the backpacks continue their state of motion, maintaining their velocity. The backpacks slow down if a force is applied, such as friction with the floor.

FIGURE 4–4
Isaac Newton (1642–1727). Besides developing mechanics, including his three great laws of motion and the law of universal gravitation, he also tried to understand the nature of light.

Inertial Reference Frames

Newton's first law does not hold in every reference frame. For example, if your reference frame is an accelerating car, an object such as a cup resting on the dashboard may begin to move toward you (it stayed at rest as long as the car's velocity remained constant). The cup accelerated toward you, but neither you nor anything else exerted a force on it in that direction. Similarly, in the reference frame of the decelerating bus in Example 4–1, there was no force pushing the backpacks forward. In accelerating reference frames, Newton's first law does not hold. Physics is easier in reference frames in which Newton's first law *does* hold, and they are called **inertial reference frames** (the law of inertia is valid in them). For most purposes, we usually make the approximation that a reference frame fixed on the Earth is an inertial frame. This is not precisely true, due to the Earth's rotation, but usually it is close enough.

Any reference frame that moves with constant velocity (say, a car or an airplane) relative to an inertial frame is also an inertial reference frame. Reference frames where the law of inertia does *not* hold, such as the accelerating reference frames discussed above, are called **noninertial** reference frames. How can we be sure a reference frame is inertial or not? By checking to see if Newton's first law holds. Thus Newton's first law serves as the definition of inertial reference frames.

4–3 Mass

Newton's second law, which we come to in the next Section, makes use of the concept of mass. Newton used the term *mass* as a synonym for "quantity of matter." This intuitive notion of the mass of an object is not very precise because the concept "quantity of matter" is not very well defined. More precisely, we can say that **mass** is a *measure of the inertia* of an object. The more mass an object has, the greater the force needed to give it a particular acceleration. It is harder to start it moving from rest, or to stop it when it is moving, or to change its velocity sideways out of a straight-line path. A truck has much more inertia than a baseball moving at the same speed, and a much greater force is needed to change the truck's velocity at the same rate as the ball's. The truck therefore has much more mass.

To quantify the concept of mass, we must define a standard. In SI units, the unit of mass is the **kilogram** (kg) as we discussed in Chapter 1, Section 1–4.

CAUTION
Distinguish mass from weight

The terms *mass* and *weight* are often confused with one another, but it is important to distinguish between them. Mass is a property of an object itself (a measure of an object's inertia, or its "quantity of matter"). Weight, on the other hand, is a force, the pull of gravity acting on an object. To see the difference, suppose we take an object to the Moon. The object will weigh only about one-sixth as much as it did on Earth, since the force of gravity is weaker. But its mass will be the same. It will have the same amount of matter as on Earth, and will have just as much inertia. It will be just as hard to start it moving on the Moon as on Earth. (More on weight in Section 4–6.)

4–4 Newton's Second Law of Motion

Newton's first law states that if no net force is acting on an object at rest, the object remains at rest; or if the object is moving, it continues moving with constant speed in a straight line. But what happens if a net force *is* exerted on an object? Newton perceived that the object's velocity will change (Fig. 4–5). A net force exerted on an object may make its velocity increase. Or, if the net force is in a direction opposite to the motion, that force will reduce the object's velocity. If the net force acts sideways on a moving object, the *direction* of the object's velocity changes. That change in the *direction* of the velocity is also an acceleration. So a sideways net force on an object also causes acceleration. In general, we can say that *a net force causes acceleration*.

FIGURE 4–5 The bobsled accelerates because the team exerts a force.

What precisely is the relationship between acceleration and force? Everyday experience can suggest an answer. Consider the force required to push a cart when friction is small enough to ignore. (If there is friction, consider the *net* force, which is the force you exert minus the force of friction.) If you push the cart horizontally with a gentle but constant force for a certain period of time, you will make the cart accelerate from rest up to some speed, say 3 km/h. If you push with twice the force, the cart will reach 3 km/h in half the time. The acceleration will be twice as great. If you triple the force, the acceleration is tripled, and so on. Thus, the acceleration of an object is directly proportional to the net applied force. But the acceleration depends on the mass of the object as well. If you push an empty grocery cart with the same force as you push one that is filled with groceries, you will find that the full cart has a smaller acceleration. The greater the mass, the less the acceleration for the same net force. The mathematical relation, as Newton argued, is that the acceleration of an object is inversely proportional to its mass. These relationships are found to hold in general and can be summarized as follows:

> **The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.**

NEWTON'S SECOND LAW OF MOTION

This is **Newton's second law of motion**.

Newton's second law can be written as an equation:

$$\vec{\mathbf{a}} = \frac{\Sigma\vec{\mathbf{F}}}{m},$$

where $\vec{\mathbf{a}}$ stands for acceleration, m for the mass, and $\Sigma\vec{\mathbf{F}}$ for the *net force* on the object. The symbol Σ (Greek "sigma") stands for "sum of"; $\vec{\mathbf{F}}$ stands for force, so $\Sigma\vec{\mathbf{F}}$ means the *vector sum of all forces* acting on the object, which we define as the **net force**.

We rearrange this equation to obtain the familiar statement of Newton's second law:

NEWTON'S SECOND LAW OF MOTION

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \tag{4–1a}$$

Newton's second law relates the description of motion (acceleration) to the cause of motion (force). It is one of the most fundamental relationships in physics. From Newton's second law we can make a more precise definition of **force** as *an action capable of accelerating an object.*

Every force $\vec{\mathbf{F}}$ is a vector, with magnitude and direction. Equation 4–1a is a vector equation valid in any inertial reference frame. It can be written in component form in rectangular coordinates as

$$\Sigma F_x = ma_x, \qquad \Sigma F_y = ma_y, \qquad \Sigma F_z = ma_z, \tag{4–1b}$$

where

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} + F_z\hat{\mathbf{k}}.$$

The component of acceleration in each direction is affected only by the component of the net force in that direction. (See Section 3–5 for unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$.)

If the motion is all along a line (one-dimensional), we can leave out the subscripts and simply write $\Sigma F = ma$. Again, a is the acceleration of an object of mass m, and ΣF includes all the forces acting on that object, and *only* forces acting on that object. Sometimes the net force ΣF is written as F_{net}, so $F_{net} = ma$.

In SI units, with the mass in kilograms, the unit of force is called the **newton** (N). One newton is the force required to impart an acceleration of $1\,m/s^2$ to a mass of 1 kg. Thus $1\,N = 1\,kg \cdot m/s^2$.

In cgs units, the unit of mass is the **gram** (g), which is $\frac{1}{1000}$ of a kilogram.[†] The unit of force is the **dyne**, which is defined as the net force needed to impart an acceleration of $1\,cm/s^2$ to a mass of 1 g. Thus $1\,dyne = 1\,g \cdot cm/s^2$. Because $1\,g = 10^{-3}\,kg$ and $1\,cm = 10^{-2}\,m$, then $1\,dyne = 10^{-5}\,N$.

In the British system, which we rarely use, the unit of force is the **pound** (abbreviated lb),[‡] where $1\,lb = 4.44822\,N \approx 4.45\,N$. The unit of mass is the **slug**, which is defined as that mass which will undergo an acceleration of $1\,ft/s^2$ when a force of 1 lb is applied to it. Thus $1\,lb = 1\,slug \cdot ft/s^2$. Table 4–1 summarizes the units in the different systems.

TABLE 4–1 Units for Mass and Force

System	Mass	Force
SI	kilogram (kg)	newton (N) $(= kg \cdot m/s^2)$
cgs	gram (g)	dyne $(= g \cdot cm/s^2)$
British	slug	pound (lb)

Conversion factors: $1\,dyne = 10^{-5}\,N$; $1\,lb \approx 4.45\,N$; $1\,slug \approx 14.6\,kg$.

PROBLEM SOLVING
Use a consistent set of units

It is very important that only one set of units be used in a given calculation or problem, with the SI being what we almost always use. If the force is given in, say, newtons, and the mass in grams, then before attempting to solve for the acceleration in SI units, we must change the mass to kilograms. For example, if the force is given as 2.0 N along the x axis and the mass is 500 g, we change the latter to 0.50 kg, and the acceleration will then automatically come out in m/s^2 when Newton's second law is used:

$$a_x = \frac{\Sigma F_x}{m} = \frac{2.0\,N}{0.50\,kg} = \frac{2.0\,kg \cdot m/s^2}{0.50\,kg} = 4.0\,m/s^2,$$

where we set $1\,N = 1\,kg \cdot m/s^2$.

[†]Be careful not to confuse g for gram with g for the acceleration due to gravity. The latter is always italicized (or boldface when shown as a vector).

[‡]The abbreviation lb for pound comes from the Latin (language of the ancient Romans) word "libra."

EXAMPLE 4–2 ESTIMATE **Force to accelerate a fast car.** Estimate the net force needed to accelerate (*a*) a 1000-kg car at $\frac{1}{2}g$; (*b*) a 200-gram apple at the same rate.

APPROACH We use Newton's second law to find the net force needed for each object; we are given the mass and the acceleration. This is an estimate (the $\frac{1}{2}$ is not said to be precise) so we round off to one significant figure.

SOLUTION (*a*) The car's acceleration is $a = \frac{1}{2}g = \frac{1}{2}(9.8\,\text{m/s}^2) \approx 5\,\text{m/s}^2$. We use Newton's second law to get the net force needed to achieve this acceleration:

$$\Sigma F = ma \approx (1000\,\text{kg})(5\,\text{m/s}^2) = 5000\,\text{N}.$$

(If you are used to British units, to get an idea of what a 5000-N force is, you can divide by 4.45 N/lb and get a force of about 1000 lb.)

(*b*) For the apple, $m = 200\,\text{g} = 0.2\,\text{kg}$, so

$$\Sigma F = ma \approx (0.2\,\text{kg})(5\,\text{m/s}^2) = 1\,\text{N}.$$

EXAMPLE 4–3 **Force to stop a car.** What average net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

APPROACH We use Newton's second law, $\Sigma F = ma$, to determine the force, but first we need to calculate the acceleration a. We assume the acceleration is constant so that we can use the kinematic equations, Eqs. 2–12, to calculate it.

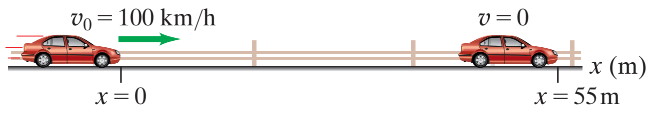


FIGURE 4–6 Example 4–3.

SOLUTION We assume the motion is along the $+x$ axis (Fig. 4–6). We are given the initial velocity $v_0 = 100\,\text{km/h} = 27.8\,\text{m/s}$ (Section 1–5), the final velocity $v = 0$, and the distance traveled $x - x_0 = 55\,\text{m}$. From Eq. 2–12c, we have

$$v^2 = v_0^2 + 2a(x - x_0),$$

so

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (27.8\,\text{m/s})^2}{2(55\,\text{m})} = -7.0\,\text{m/s}^2.$$

The net force required is then

$$\Sigma F = ma = (1500\,\text{kg})(-7.0\,\text{m/s}^2) = -1.1 \times 10^4\,\text{N},$$

or 11,000 N. The force must be exerted in the direction *opposite* to the initial velocity, which is what the negative sign means.

NOTE If the acceleration is not precisely constant, then we are determining an "average" acceleration and we obtain an "average" net force.

Newton's second law, like the first law, is valid only in inertial reference frames (Section 4–2). In the noninertial reference frame of a car that begins accelerating, a cup on the dashboard starts sliding—it accelerates—even though the net force on it is zero. Thus $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ does not work in such an accelerating reference frame $(\Sigma\vec{\mathbf{F}} = 0$, but $\vec{\mathbf{a}} \neq 0$ in this noninertial frame$)$.

EXERCISE A Suppose you watch a cup slide on the (smooth) dashboard of an accelerating car as we just discussed, but this time from an inertial reference frame outside the car, on the street. From your inertial frame, Newton's laws are valid. What force pushes the cup off the dashboard?

*Precise Definition of Mass

As mentioned in Section 4–3, we can quantify the concept of mass using its definition as a measure of inertia. How to do this is evident from Eq. 4–1a, where we see that the acceleration of an object is inversely proportional to its mass.

If the same net force ΣF acts to accelerate each of two masses, m_1 and m_2, then the ratio of their masses can be defined as the inverse ratio of their accelerations:

$$\frac{m_2}{m_1} = \frac{a_1}{a_2}.$$

If one of the masses is known (it could be the standard kilogram) and the two accelerations are precisely measured, then the unknown mass is obtained from this definition. For example, if $m_1 = 1.00\,\text{kg}$, and for a particular force $a_1 = 3.00\,\text{m/s}^2$ and $a_2 = 2.00\,\text{m/s}^2$, then $m_2 = 1.50\,\text{kg}$.

4–5 Newton's Third Law of Motion

Newton's second law of motion describes quantitatively how forces affect motion. But where do forces come from? Observations suggest that a force exerted on any object is always exerted *by another object*. A horse pulls a wagon, a person pushes a grocery cart, a hammer pushes on a nail, a magnet attracts a paper clip. In each of these examples, a force is exerted *on* one object, and that force is exerted *by* another object. The force exerted *on* the nail is exerted *by* the hammer.

But Newton realized that things are not so one-sided. True, the hammer exerts a force on the nail (Fig. 4–7). But the nail evidently exerts a force back on the hammer as well, for the hammer's speed is rapidly reduced to zero upon contact. Only a strong force could cause such a rapid deceleration of the hammer. Thus, said Newton, the two objects must be treated on an equal basis. The hammer exerts a force on the nail, and the nail exerts a force back on the hammer. This is the essence of **Newton's third law of motion**:

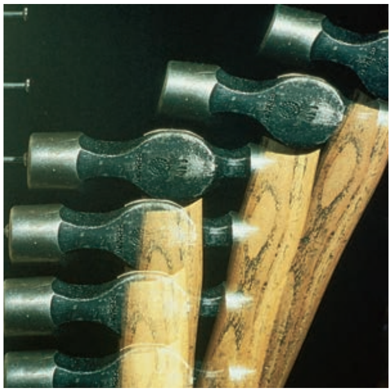


FIGURE 4–7 A hammer striking a nail. The hammer exerts a force on the nail and the nail exerts a force back on the hammer. The latter force decelerates the hammer and brings it to rest.

NEWTON'S THIRD LAW OF MOTION

> **Whenever one object exerts a force on a second object, the second object exerts an equal force in the opposite direction on the first.**

This law is sometimes stated as "to every action there is an equal and opposite reaction." But to avoid confusion, it is very important to remember that the "action" force and the "reaction" force are acting on *different* objects.

CAUTION

Action and reaction forces act on different objects

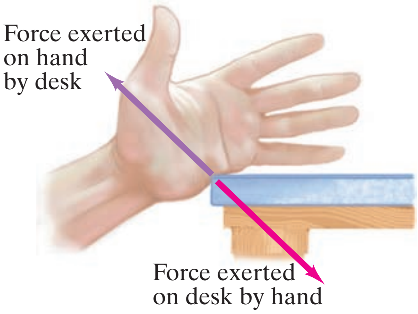


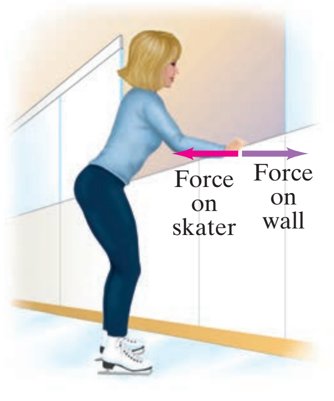
FIGURE 4–8 If your hand pushes against the edge of a desk (the force vector is shown in red), the desk pushes back against your hand (this force vector is shown in a different color, violet, to remind us that this force acts on a different object).

As evidence for the validity of Newton's third law, look at your hand when you push against the edge of a desk, Fig. 4–8. Your hand's shape is distorted, clear evidence that a force is being exerted on it. You can *see* the edge of the desk pressing into your hand. You can even *feel* the desk exerting a force on your hand; it hurts! The harder you push against the desk, the harder the desk pushes back on your hand. (You only feel forces exerted *on* you; when you exert a force on another object, what you feel is that object pushing back on you.)

The force the desk exerts on your hand has the same magnitude as the force your hand exerts on the desk. This is true not only if the desk is at rest but is true even if the desk is accelerating due to the force your hand exerts.

As another demonstration of Newton's third law, consider the ice skater in Fig. 4–9. There is very little friction between her skates and the ice, so she will move freely if a force is exerted on her. She pushes against the wall; and then *she* starts moving backward. The force she exerts on the wall cannot make *her* start moving, because that force acts on the wall. Something had to exert a force *on her* to start her moving, and that force could only have been exerted by the wall. The wall pushes on her with a force, by Newton's third law, equal and opposite to the force she exerts on the wall.

FIGURE 4–9 An example of Newton's third law: when an ice skater pushes against the wall, the wall pushes back and this force causes her to accelerate away.



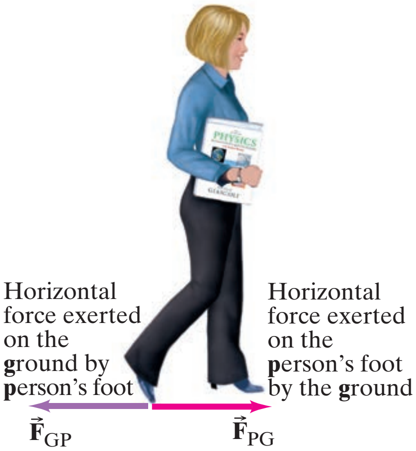
When a person throws a package out of a small boat (initially at rest), the boat starts moving in the opposite direction. The person exerts a force on the package. The package exerts an equal and opposite force back on the person, and this force propels the person (and the boat) backward slightly.

Rocket propulsion also is explained using Newton's third law (Fig. 4–10). A common misconception is that rockets accelerate because the gases rushing out the back of the engine push against the ground or the atmosphere. Not true. What happens, instead, is that a rocket exerts a strong force on the gases, expelling them; and the gases exert an equal and opposite force *on the rocket*. It is this latter force that propels the rocket forward—the force exerted *on* the rocket *by* the gases (see Chapter-Opening Photo, page 107). Thus, a space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate. When the rocket pushes on the gases in one direction, the gases push back on the rocket in the opposite direction. Jet aircraft too accelerate because the gases they thrust out backwards exert a forward force on the engines (Newton's third law).

FIGURE 4–10 Another example of Newton's third law: the launch of a rocket. The rocket engine pushes the gases downward, and the gases exert an equal and opposite force upward on the rocket, accelerating it upward. (A rocket does *not* accelerate as a result of its expelled gases pushing against the ground.)

Consider how we walk. A person begins walking by pushing with the foot backward against the ground. The ground then exerts an equal and opposite force forward on the person (Fig. 4–11), and it is this force, *on* the person, that moves the person forward. (If you doubt this, try walking normally where there is no friction, such as on very smooth slippery ice.) In a similar way, a bird flies forward by exerting a backward force on the air, but it is the air pushing forward (Newton's third law) on the bird's wings that propels the bird forward.

FIGURE 4–11 We can walk forward because, when one foot pushes backward against the ground, the ground pushes forward on that foot (Newton's third law). The two forces shown *act on different objects*.



CONCEPTUAL EXAMPLE 4–4 **What exerts the force to move a car?** What makes a car go forward?

RESPONSE A common answer is that the engine makes the car move forward. But it is not so simple. The engine makes the wheels and tires go around. But if the tires are on slick ice or wet mud, they just spin. Friction is needed. On firm ground, the tires push backward against the ground because of friction between the tires and the ground. By Newton's third law, the ground pushes on the tires in the opposite direction, accelerating the car forward.

We tend to associate forces with active objects such as humans, animals, engines, or a moving object like a hammer. It is often difficult to see how an inanimate object at rest, such as a wall or a desk (Fig. 4–8), or the wall of an ice rink (Fig. 4–9), can exert a force. The explanation is that every material, no matter how hard, is elastic (springy) at least to some degree. A stretched rubber band can exert a force on a wad of paper and accelerate it to fly across the room. Other materials may not stretch as readily as rubber, but they do stretch or compress when a force is applied to them. And just as a stretched rubber band exerts a force, so does a stretched (or compressed) wall, desk, or car fender.

CAUTION
Distinguish ***on*** *what object a force is exerted, and* ***by*** *what object*

From the examples discussed above, we can see how important it is to remember *on* what object a given force is exerted and *by* what object that force is exerted. A force influences the motion of an object only when it is applied *on* that object. A force exerted *by* an object does not influence that same object; it only influences the other object *on* which it is exerted. Thus, to avoid confusion, the two prepositions *on* and *by* must always be used—and used with care.

One way to keep clear which force acts on which object is to use double subscripts. For example, the force exerted *on* the **P**erson *by* the **G**round as the person walks in Fig. 4–11 can be labeled $\vec{\mathbf{F}}_{PG}$. And the force exerted on the ground by the person is $\vec{\mathbf{F}}_{GP}$. By Newton's third law

NEWTON'S THIRD LAW OF MOTION

$$\vec{\mathbf{F}}_{GP} = -\vec{\mathbf{F}}_{PG}. \qquad (4\text{–}2)$$

$\vec{\mathbf{F}}_{GP}$ and $\vec{\mathbf{F}}_{PG}$ have the same magnitude (Newton's third law), and the minus sign reminds us that these two forces are in opposite directions.

Note carefully that the two forces shown in Fig. 4–11 act on different objects—to emphasize this we used slightly different colors for the vector arrows representing these forces. These two forces would never appear together in a sum of forces in Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Why not? Because they act on different objects: $\vec{\mathbf{a}}$ is the acceleration of one particular object, and $\Sigma\vec{\mathbf{F}}$ must include *only* the forces on that *one* object.

CAUTION

The 2 forces in Newton's third law act on different bodies. Only one can be included in $\Sigma F = ma$ for an object

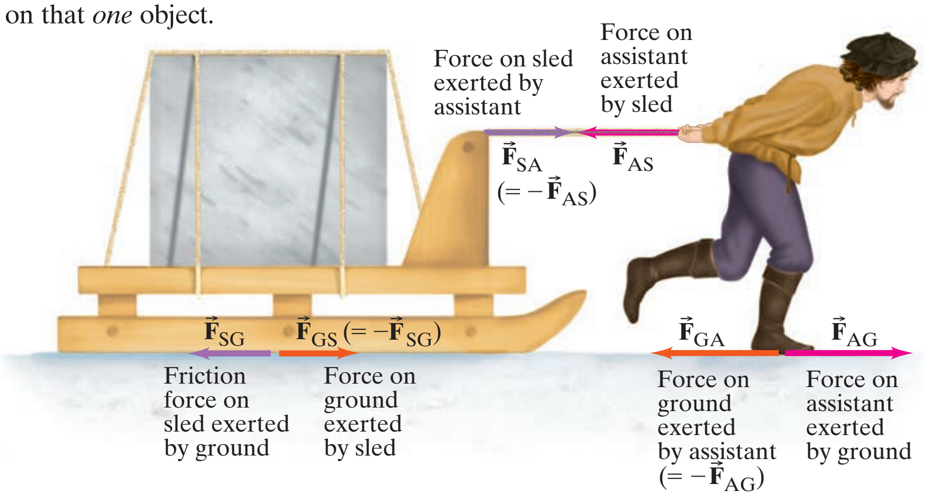


FIGURE 4–12 Example 4–5, showing only horizontal forces. Michelangelo has selected a fine block of marble for his next sculpture. Shown here is his assistant pulling it on a sled away from the quarry. Forces on the assistant are shown as red (magenta) arrows. Forces on the sled are purple arrows. Forces acting on the ground are orange arrows. Action–reaction forces that are equal and opposite are labeled by the same subscripts but reversed (such as $\vec{\mathbf{F}}_{GA}$ and $\vec{\mathbf{F}}_{AG}$) and are of different colors because they act on different objects.

CONCEPTUAL EXAMPLE 4–5 **Third law clarification.** Michelangelo's assistant has been assigned the task of moving a block of marble using a sled (Fig. 4–12). He says to his boss, "When I exert a forward force on the sled, the sled exerts an equal and opposite force backward. So how can I ever start it moving? No matter how hard I pull, the backward reaction force always equals my forward force, so the net force must be zero. I'll never be able to move this load." Is he correct?

RESPONSE No. Although it is true that the action and reaction forces are equal in magnitude, the assistant has forgotten that they are exerted on different objects. The forward ("action") force is exerted by the assistant on the sled (Fig. 4–12), whereas the backward "reaction" force is exerted by the sled on the assistant. To determine if the *assistant* moves or not, we must consider only the forces *on the assistant* and then apply $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, where $\Sigma\vec{\mathbf{F}}$ is the net force *on the assistant*, $\vec{\mathbf{a}}$ is the acceleration of the assistant, and m is the assistant's mass. There are two forces on the assistant that affect his forward motion; they are shown as bright red (magenta) arrows in Figs. 4–12 and 4–13: they are (1) the horizontal force $\vec{\mathbf{F}}_{AG}$ exerted on the assistant by the ground (the harder he pushes backward against the ground, the harder the ground pushes forward on him—Newton's third law), and (2) the force $\vec{\mathbf{F}}_{AS}$ exerted on the assistant by the sled, pulling backward on him; see Fig. 4–13. If he pushes hard enough on the ground, the force on him exerted by the ground, $\vec{\mathbf{F}}_{AG}$, will be larger than the sled pulling back, $\vec{\mathbf{F}}_{AS}$, and the assistant accelerates forward (Newton's second law). The sled, on the other hand, accelerates forward when the force on *it* exerted by the assistant is greater than the frictional force exerted backward on it by the ground (that is, when $\vec{\mathbf{F}}_{SA}$ has greater magnitude than $\vec{\mathbf{F}}_{SG}$ in Fig. 4–12).

PROBLEM SOLVING

A study of Newton's second and third laws

Using double subscripts to clarify Newton's third law can become cumbersome, and we won't usually use them in this way. We will usually use a single subscript referring to what exerts the force on the object being discussed. Nevertheless, if there is any confusion in your mind about a given force, go ahead and use two subscripts to identify *on* what object and *by* what object the force is exerted.

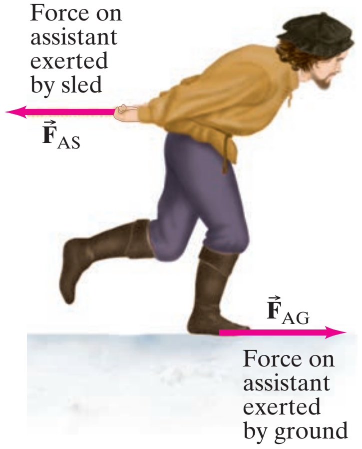


FIGURE 4–13 Example 4–5. The horizontal forces on the assistant.

EXERCISE B A tennis ball collides head-on with a more massive baseball. (i) Which ball experiences the greater force of impact? (ii) Which experiences the greater acceleration during the impact? (iii) Which of Newton's laws are useful to obtain the correct answers?

EXERCISE C If you push on a heavy desk, does it always push back on you? (*a*) No. (*b*) Yes. (*c*) Not unless someone else also pushes on it. (*d*) Yes, if it is out in space. (*e*) A desk never pushes.

EXERCISE D Return to the first Chapter-Opening Question, page 107, and answer it again now. Try to explain why you may have answered differently the first time.

4–6 Weight—the Force of Gravity; and the Normal Force

As we saw in Chapter 2, Galileo claimed that all objects dropped near the surface of the Earth would fall with the same acceleration, $\vec{\mathbf{g}}$, if air resistance was negligible. The force that causes this acceleration is called the *force of gravity* or *gravitational force*. What exerts the gravitational force on an object? It is the Earth, as we will discuss in Chapter 5, and the force acts vertically[†] downward, toward the center of the Earth. Let us apply Newton's second law to an object of mass m falling freely due to gravity. For the acceleration, $\vec{\mathbf{a}}$, we use the downward acceleration due to gravity, $\vec{\mathbf{g}}$. Thus, the **gravitational force** on an object, $\vec{\mathbf{F}}_G$, can be written as

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \tag{4–3}$$

The direction of this force is down toward the center of the Earth. The magnitude of the force of gravity on an object, mg, is commonly called the object's **weight**.

In SI units, $g = 9.80\,\mathrm{m/s^2} = 9.80\,\mathrm{N/kg}$,[‡] so the weight of a 1.00-kg mass on Earth is $1.00\,\mathrm{kg} \times 9.80\,\mathrm{m/s^2} = 9.80\,\mathrm{N}$. We will mainly be concerned with the weight of objects on Earth, but we note that on the Moon, on other planets, or in space, the weight of a given mass will be different than it is on Earth. For example, on the Moon the acceleration due to gravity is about one-sixth what it is on Earth, and a 1.0-kg mass weighs only 1.6 N. Although we will not use British units, we note that for practical purposes on the Earth, a mass of 1.0 kg weighs about 2.2 lb. (On the Moon, 1 kg weighs only about 0.4 lb.)

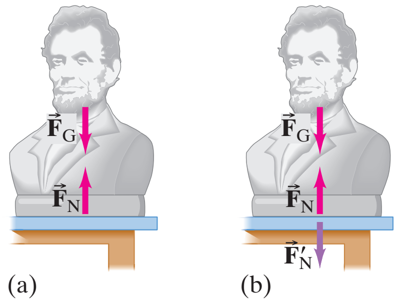


FIGURE 4–14 (a) The net force on an object at rest is zero according to Newton's second law. Therefore the downward force of gravity $(\vec{\mathbf{F}}_G)$ on an object at rest must be balanced by an upward force (the normal force $\vec{\mathbf{F}}_N$) exerted by the table in this case. (b) $\vec{\mathbf{F}}'_N$ is the force exerted on the table by the statue and is the reaction force to $\vec{\mathbf{F}}_N$ by Newton's third law. ($\vec{\mathbf{F}}'_N$ is shown in a different color to remind us it acts on a different object.) The reaction force to $\vec{\mathbf{F}}_G$ is not shown.

CAUTION

Weight and normal force are ***not*** *action–reaction pairs*

The force of gravity acts on an object when it is falling. When an object is at rest on the Earth, the gravitational force on it does not disappear, as we know if we weigh it on a spring scale. The same force, given by Eq. 4–3, continues to act. Why, then, doesn't the object move? From Newton's second law, the net force on an object that remains at rest is zero. There must be another force on the object to balance the gravitational force. For an object resting on a table, the table exerts this upward force; see Fig. 4–14a. The table is compressed slightly beneath the object, and due to its elasticity, it pushes up on the object as shown. The force exerted by the table is often called a **contact force**, since it occurs when two objects are in contact. (The force of your hand pushing on a cart is also a contact force.) When a contact force acts *perpendicular* to the common surface of contact, it is referred to as the **normal force** ("normal" means perpendicular); hence it is labeled $\vec{\mathbf{F}}_N$ in Fig. 4–14a.

The two forces shown in Fig. 4–14a are both acting on the statue, which remains at rest, so the vector sum of these two forces must be zero (Newton's second law). Hence $\vec{\mathbf{F}}_G$ and $\vec{\mathbf{F}}_N$ must be of equal magnitude and in opposite directions. But they are *not* the equal and opposite forces spoken of in Newton's third law. The action and reaction forces of Newton's third law act on *different objects*, whereas the two forces shown in Fig. 4–14a act on the *same* object. For each of the forces shown in Fig. 4–14a, we can ask, "What is the reaction force?" The upward force $\vec{\mathbf{F}}_N$ on the statue is exerted by the table. The reaction to this force is a force exerted by the statue downward on the table. It is shown in Fig. 4–14b, where it is labeled $\vec{\mathbf{F}}'_N$. This force, $\vec{\mathbf{F}}'_N$, exerted on the table by the statue, is the reaction force to $\vec{\mathbf{F}}_N$ in accord with Newton's third law. What about the other force on the statue, the force of gravity $\vec{\mathbf{F}}_G$ exerted by the Earth? Can you guess what the reaction is to this force? We will see in Chapter 5 that the reaction force is also a gravitational force, exerted on the Earth by the statue.

[†]The concept of "vertical" is tied to gravity. The best definition of *vertical* is that it is the direction in which objects fall. A surface that is "horizontal," on the other hand, is a surface on which a round object won't start rolling: gravity has no effect. Horizontal is perpendicular to vertical.

[‡]Since $1\,\mathrm{N} = 1\,\mathrm{kg\cdot m/s^2}$ (Section 4–4), then $1\,\mathrm{m/s^2} = 1\,\mathrm{N/kg}$.

EXERCISE E Return to the second Chapter-Opening Question, page 107, and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 4–6 **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 4–15a). (*a*) Determine the weight of the box and the normal force exerted on it by the table. (*b*) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 4–15b. Again determine the normal force exerted on the box by the table. (*c*) If your friend instead pulls upward on the box with a force of 40.0 N (Fig. 4–15c), what now is the normal force exerted on the box by the table?

APPROACH The box is at rest on the table, so the net force on the box in each case is zero (Newton's first or second law). The weight of the box has magnitude mg in all three cases.

SOLUTION (*a*) The weight of the box is $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$, and this force acts downward. The only other force on the box is the normal force $\vec{\mathbf{F}}_N$ exerted upward on it by the table, as shown in Fig. 4–15a. We choose the upward direction as the positive y direction; then the net force ΣF_y on the box is $\Sigma F_y = F_N - mg$; the minus sign means mg acts in the negative y direction (m and g are magnitudes). The box is at rest, so the net force on it must be zero (Newton's second law, $\Sigma F_y = ma_y$, and $a_y = 0$). Thus

$$\begin{aligned} \Sigma F_y &= ma_y \\ F_N - mg &= 0, \end{aligned}$$

so we have

$$F_N = mg.$$

The normal force on the box, exerted by the table, is 98.0 N upward, and has magnitude equal to the box's weight.

(*b*) Your friend is pushing down on the box with a force of 40.0 N. So instead of only two forces acting on the box, now there are three forces acting on the box, as shown in Fig. 4–15b. The weight of the box is still $mg = 98.0\,\text{N}$. The net force is $\Sigma F_y = F_N - mg - 40.0\,\text{N}$, and is equal to zero because the box remains at rest $(a = 0)$. Newton's second law gives

$$\Sigma F_y = F_N - mg - 40.0\,\text{N} = 0.$$

We solve this equation for the normal force:

$$F_N = mg + 40.0\,\text{N} = 98.0\,\text{N} + 40.0\,\text{N} = 138.0\,\text{N},$$

which is greater than in (*a*). The table pushes back with more force when a person pushes down on the box. The normal force is not always equal to the weight!

(*c*) The box's weight is still 98.0 N and acts downward. The force exerted by your friend and the normal force both act upward (positive direction), as shown in Fig. 4–15c. The box doesn't move since your friend's upward force is less than the weight. The net force, again set to zero in Newton's second law because $a = 0$, is

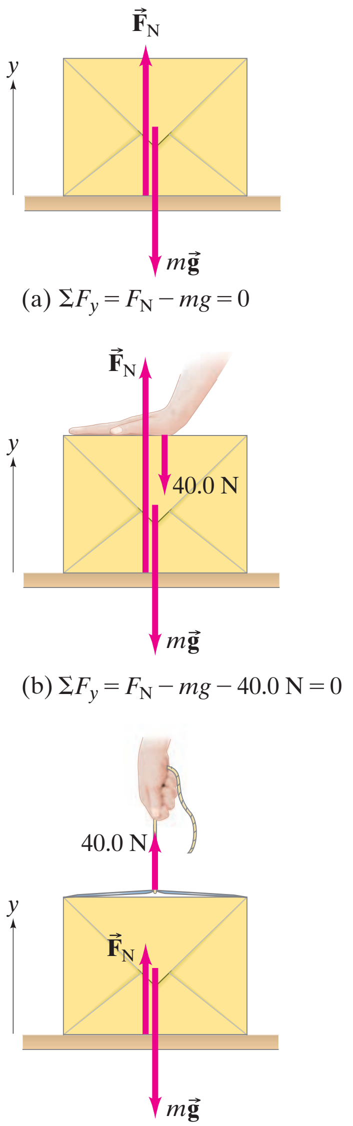
$$\Sigma F_y = F_N - mg + 40.0\,\text{N} = 0,$$

so

$$F_N = mg - 40.0\,\text{N} = 98.0\,\text{N} - 40.0\,\text{N} = 58.0\,\text{N}.$$

The table does not push against the full weight of the box because of the upward force exerted by your friend.

NOTE The weight of the box $(= mg)$ does not change as a result of your friend's push or pull. Only the normal force is affected.



(a) $\Sigma F_y = F_N - mg = 0$

(b) $\Sigma F_y = F_N - mg - 40.0\,\text{N} = 0$

(c) $\Sigma F_y = F_N - mg + 40.0\,\text{N} = 0$

FIGURE 4–15 Example 4–6. (a) A 10-kg gift box is at rest on a table. (b) A person pushes down on the box with a force of 40.0 N. (c) A person pulls upward on the box with a force of 40.0 N. The forces are all assumed to act along a line; they are shown slightly displaced in order to be distinguishable. Only forces acting on the box are shown.

CAUTION
The normal force F_N is not always equal to the weight

Recall that the normal force is elastic in origin (the table in Fig. 4–15 sags slightly under the weight of the box). The normal force in Example 4–6 is vertical, perpendicular to the horizontal table. The normal force is not always vertical, however. When you push against a wall, for example, the normal force with which the wall pushes back on you is horizontal (see Fig. 4–9). If an object is on a plane inclined at an angle to the horizontal, such as a skier or car on a hill, the normal force acts perpendicular to the plane and so is not vertical.

CAUTION
The normal force, $\vec{\mathbf{F}}_N$, is not necessarily vertical

EXAMPLE 4–7 **Accelerating the box.** What happens when a person pulls upward on the box in Example 4–6*c* with a force equal to, or greater than, the box's weight? For example, let $F_P = 100.0\,\mathrm{N}$ (Fig. 4–16) rather than the 40.0 N shown in Fig. 4–15c.

APPROACH We can start just as in Example 4–6, but be ready for a surprise.

SOLUTION The net force on the box is

$$\begin{aligned}\Sigma F_y &= F_N - mg + F_P \\ &= F_N - 98.0\,\mathrm{N} + 100.0\,\mathrm{N},\end{aligned}$$

and if we set this equal to zero (thinking the acceleration might be zero), we would get $F_N = -2.0\,\mathrm{N}$. This is nonsense, since the negative sign implies F_N points downward, and the table surely cannot *pull* down on the box (unless there's glue on the table). The least F_N can be is zero, which it will be in this case, $F_N = 0$. What really happens here is that the box leaves the table and accelerates upward ($a \neq 0$, see gold vector in Fig. 4–16) because the net force is not zero. The net force (setting the normal force $F_N = 0$) is

$$\begin{aligned}\Sigma F_y = F_P - mg &= 100.0\,\mathrm{N} - 98.0\,\mathrm{N} \\ &= 2.0\,\mathrm{N}\end{aligned}$$

upward. See Fig. 4–16. We apply Newton's second law and see that the box moves upward with an acceleration

$$\begin{aligned}a_y = \frac{\Sigma F_y}{m} &= \frac{2.0\,\mathrm{N}}{10.0\,\mathrm{kg}} \\ &= 0.20\,\mathrm{m/s^2}.\end{aligned}$$

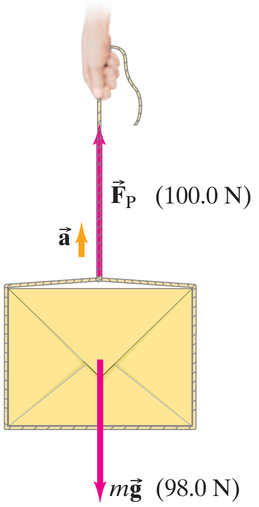


FIGURE 4–16 Example 4–7. The box accelerates upward because $F_P > mg$.

EXAMPLE 4–8 **Apparent weight loss.** A 65-kg woman descends in an elevator that briefly accelerates at $0.20\,g$ downward. She stands on a scale that reads in kg. (*a*) During this acceleration, what is her weight and what does the scale read? (*b*) What does the scale read when the elevator descends at a constant speed of 2.0 m/s?

APPROACH Figure 4–17 shows all the forces that act on the woman (and *only* those that act on her). The direction of the acceleration is downward, so we choose the positive direction as down (this is the opposite choice from Examples 4–6 and 4–7).

SOLUTION (*a*) From Newton's second law,

$$\begin{aligned}\Sigma F &= ma \\ mg - F_N &= m(0.20\,g).\end{aligned}$$

We solve for F_N:

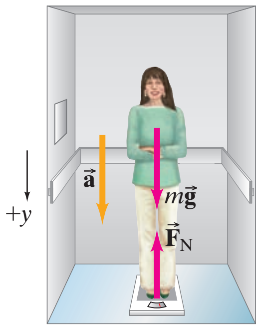
$$\begin{aligned}F_N &= mg - 0.20\,mg \\ &= 0.80\,mg,\end{aligned}$$

and it acts upward. The normal force $\vec{\mathbf{F}}_N$ is the force the scale exerts on the person, and it is equal and opposite to the force she exerts on the scale: $F'_N = 0.80\,mg$ downward. Her weight (force of gravity on her) is still $mg = (65\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 640\,\mathrm{N}$. But the scale, needing to exert a force of only $0.80\,mg$, will give a reading of $0.80\,m = 52\,\mathrm{kg}$.

(*b*) Now there is no acceleration, $a = 0$, so by Newton's second law, $mg - F_N = 0$ and $F_N = mg$. The scale reads her true mass of 65 kg.

NOTE The scale in (*a*) gives a reading of 52 kg (as an "apparent mass"), but her mass doesn't change as a result of the acceleration: it stays at 65 kg.

FIGURE 4–17 Example 4–8. The acceleration vector is shown in gold to distinguish it from the red force vectors.



4–7 Solving Problems with Newton's Laws: Free-Body Diagrams

Newton's second law tells us that the acceleration of an object is proportional to the *net force* acting on the object. The **net force**, as mentioned earlier, is the *vector sum* of all forces acting on the object. Indeed, extensive experiments have shown that forces do add together as vectors precisely according to the rules we developed in Chapter 3. For example, in Fig. 4–18, two forces of equal magnitude (100 N each) are shown acting on an object at right angles to each other. Intuitively, or from *symmetry*, we can see that the object will start moving at a 45° angle and thus the net force acts at a 45° angle. This is just what the rules of vector addition give. From the theorem of Pythagoras, the magnitude of the resultant force is $F_R = \sqrt{(100\text{ N})^2 + (100\text{ N})^2} = 141\text{ N}$.

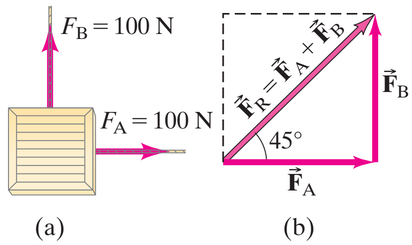


FIGURE 4–18 (a) Two horizontal forces, $\vec{F}_A$ and $\vec{F}_B$, exerted by workers A and B, act on a crate (we are looking down from above). (b) The sum, or resultant, of $\vec{F}_A$ and $\vec{F}_B$ is $\vec{F}_R$.

EXAMPLE 4–9 **Adding force vectors.** Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 4–19a.

APPROACH We add force vectors like any other vectors as described in Chapter 3. The first step is to choose an *xy* coordinate system (see Fig. 4–19a), and then resolve vectors into their components.

SOLUTION The two force vectors are shown resolved into components in Fig. 4–19b. We add the forces using the method of components. The components of $\vec{F}_A$ are

$$F_{Ax} = F_A \cos 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N},$$
$$F_{Ay} = F_A \sin 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N}.$$

The components of $\vec{F}_B$ are

$$F_{Bx} = +F_B \cos 37.0° = +(30.0\text{ N})(0.799) = +24.0\text{ N},$$
$$F_{By} = -F_B \sin 37.0° = -(30.0\text{ N})(0.602) = -18.1\text{ N}.$$

F_{By} is negative because it points along the negative *y* axis. The components of the resultant force are (see Fig. 4–19c)

$$F_{Rx} = F_{Ax} + F_{Bx} = 28.3\text{ N} + 24.0\text{ N} = 52.3\text{ N},$$
$$F_{Ry} = F_{Ay} + F_{By} = 28.3\text{ N} - 18.1\text{ N} = 10.2\text{ N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem,

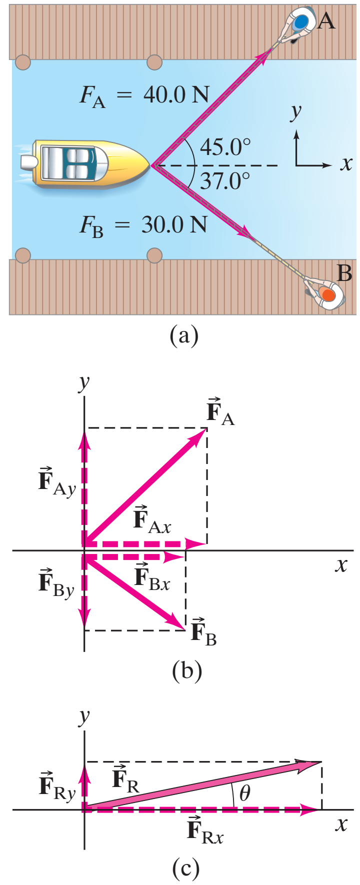
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2}\text{ N} = 53.3\text{ N}.$$

The only remaining question is the angle θ that the net force $\vec{F}_R$ makes with the *x* axis, Fig. 4–19c. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\text{ N}}{52.3\text{ N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0°$. The net force on the boat has magnitude 53.3 N and acts at an 11.0° angle to the *x* axis.

FIGURE 4–19 Example 4–9: Two force vectors act on a boat.



When solving problems involving Newton's laws and force, it is very important to draw a diagram showing all the forces acting *on* each object involved. Such a diagram is called a **free-body diagram**, or **force diagram**: choose one object, and draw an arrow to represent each force acting on it. Include *every* force acting on that object. Do not show forces that the chosen object exerts on *other* objects. To help you identify each and every force that is exerted on your chosen object, ask yourself what other objects could exert a force on it. If your problem involves more than one object, a separate free-body diagram is needed for each object. For now, the likely forces that could be acting are *gravity* and *contact forces* (one object pushing or pulling another, normal force, friction). Later we will consider other types of force such as buoyancy, fluid pressure, and electric and magnetic forces.

PROBLEM SOLVING
Free-body diagram

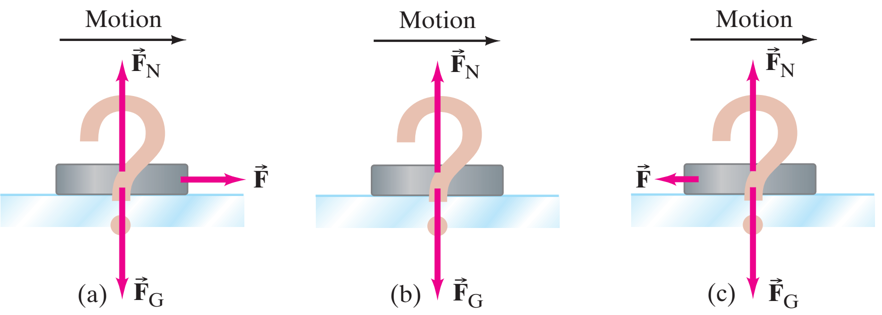


FIGURE 4–20 Example 4–10. Which is the correct free-body diagram for a hockey puck sliding across frictionless ice?

CONCEPTUAL EXAMPLE 4–10 **The hockey puck.** A hockey puck is sliding at constant velocity across a flat horizontal ice surface that is assumed to be frictionless. Which of the sketches in Fig. 4–20 is the correct free-body diagram for this puck? What would your answer be if the puck slowed down?

RESPONSE Did you choose (*a*)? If so, can you answer the question: what exerts the horizontal force labeled $\vec{\mathbf{F}}$ on the puck? If you say that it is the force needed to maintain the motion, ask yourself: what exerts this force? Remember that another object must exert any force—and there simply isn't any possibility here. Therefore, (*a*) is wrong. Besides, the force $\vec{\mathbf{F}}$ in Fig. 4–20a would give rise to an acceleration by Newton's second law. It is (*b*) that is correct. No net force acts on the puck, and the puck slides at constant velocity across the ice.

In the real world, where even smooth ice exerts at least a tiny friction force, (*c*) is the correct answer. The tiny friction force is in the direction opposite to the motion, and the puck's velocity decreases, even if very slowly.

PROBLEM SOLVING Newton's Laws; Free-Body Diagrams

1. **Draw a sketch** of the situation, after carefully reading the problem at least twice.

2. Consider only one object (at a time), and draw a **free-body diagram** for that object, showing *all* the forces acting *on* that object. Include any unknown forces that you have to solve for. Do not show any forces that the chosen object exerts on other objects.

   Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force acting on the object, including forces you must solve for, according to its source (gravity, person, friction, and so on).

   If several objects are involved, draw a free-body diagram for each object *separately*. For each object, show all the forces acting *on that object* (and *only* forces acting on that object). For each (and every) force, you must be clear about: *on* what object that force acts, and *by* what object that force is exerted. Only forces acting *on* a given object can be included in $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for that object.

3. Newton's second law involves vectors, and it is usually important to **resolve vectors** into components. **Choose** *x* and *y* **axes** in a way that simplifies the calculation. For example, it often saves work if you choose one coordinate axis to be in the direction of the acceleration (if known).

4. For each object, **apply Newton's second law** to the *x* and *y* components separately. That is, the *x* component of the net force on that object is related to the *x* component of that object's acceleration: $\Sigma F_x = ma_x$, and similarly for the *y* direction.

5. **Solve** the equation or equations for the unknown(s). Put in numerical values only at the end, and keep track of units.

This Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do that will start you thinking and getting involved in the problem at hand.

CAUTION
Treating an object as a particle

When we are concerned only about *translational motion*, all the forces on a given object can be drawn as acting at the center of the object, thus treating the object as a **point particle**. However, for problems involving rotation or statics, the place *where* each force acts is also important, as we shall see in Chapters 10, 11, and 12.

In the Examples in this Section, we assume that all surfaces are very smooth so that friction can be ignored. (Friction, and Examples using it, are discussed in Chapter 5.)

EXAMPLE 4–11 **Pulling the mystery box.** Suppose a friend asks to examine the 10.0-kg box you were given (Example 4–6, Fig. 4–15), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached cord, as shown in Fig. 4–21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0\,\text{N}$, and it is exerted at a 30.0° angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.

APPROACH We follow the Problem Solving Strategy on the previous page.

SOLUTION

1. **Draw a sketch**: The situation is shown in Fig. 4–21a; it shows the box and the force applied by the person, F_P.
2. **Free-body diagram**: Figure 4–21b shows the free-body diagram of the box. To draw it correctly, we show *all* the forces acting on the box and *only* the forces acting on the box. They are: the force of gravity $m\vec{\mathbf{g}}$; the normal force exerted by the table $\vec{\mathbf{F}}_N$; and the force exerted by the person $\vec{\mathbf{F}}_P$. We are interested only in translational motion, so we can show the three forces acting at a point, Fig. 4–21c.
3. **Choose axes and resolve vectors**: We expect the motion to be horizontal, so we choose the x axis horizontal and the y axis vertical. The pull of 40.0 N has components

$$F_{Px} = (40.0\,\text{N})(\cos 30.0°) = (40.0\,\text{N})(0.866) = 34.6\,\text{N},$$
$$F_{Py} = (40.0\,\text{N})(\sin 30.0°) = (40.0\,\text{N})(0.500) = 20.0\,\text{N}.$$

In the horizontal (x) direction, $\vec{\mathbf{F}}_N$ and $m\vec{\mathbf{g}}$ have zero components. Thus the horizontal component of the net force is F_{Px}.

4. (*a*) **Apply Newton's second law** to determine the x component of the acceleration:

$$F_{Px} = ma_x.$$

5. (*a*) **Solve**:

$$a_x = F_{Px}/m = 34.6\,\text{N}/10.0\,\text{kg} = 3.46\,\text{m/s}^2.$$

The acceleration of the box is $3.46\,\text{m/s}^2$ to the right.

(*b*) Next we want to find F_N.

4′. (*b*) **Apply Newton's second law** to the vertical (y) direction, with upward as positive:

$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

5′. (*b*) **Solve**: We have $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$ and, from step 3 above, $F_{Py} = 20.0\,\text{N}$. Furthermore, since $F_{Py} < mg$, the box does not move vertically, so $a_y = 0$. Thus

$$F_N - 98.0\,\text{N} + 20.0\,\text{N} = 0,$$

so $F_N = 78.0\,\text{N}$.

NOTE F_N is less than mg: the table does not push against the full weight of the box because part of the pull exerted by the person is in the upward direction.

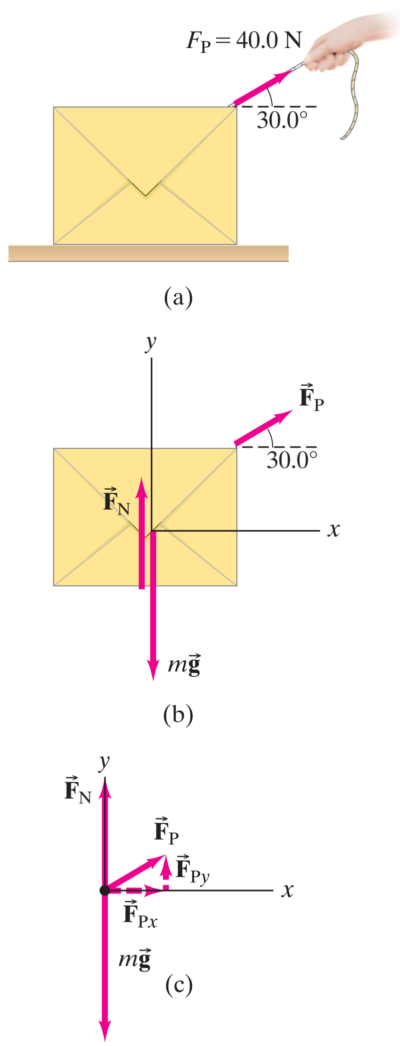


FIGURE 4–21 (a) Pulling the box, Example 4–11; (b) is the free-body diagram for the box, and (c) is the free-body diagram considering all the forces to act at a point (translational motion only, which is what we have here).

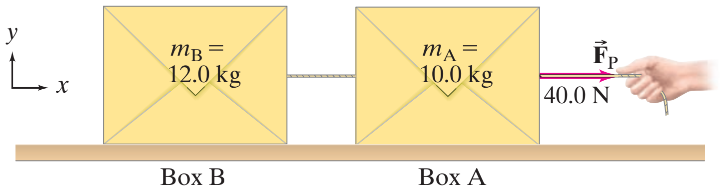
EXERCISE F If the force $\vec{\mathbf{F}}_P$ (Fig. 4–21) exerted by the person is doubled in magnitude, what now will be the normal force?

EXERCISE G A 10.0-kg box is dragged on a horizontal frictionless surface by a horizontal force of 10.0 N. If the applied force is doubled, the normal force on the box will (*a*) increase; (*b*) remain the same; (*c*) decrease.

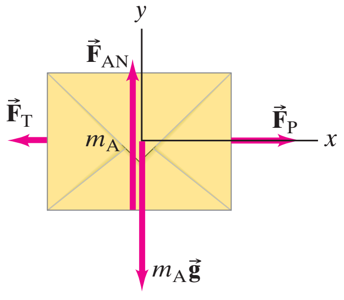
Tension in a Flexible Cord

When a flexible cord pulls on an object, the cord is said to be under **tension**, and the force it exerts on the object is the tension F_T. If the cord has negligible mass, the force exerted at one end is transmitted undiminished to each adjacent piece of cord along the entire length to the other end. Why? Because $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = 0$ for the cord if the cord's mass m is zero (or negligible) no matter what $\vec{\mathbf{a}}$ is. Hence the forces pulling on the cord at its two ends must add up to zero (F_T and $-F_T$). Note that flexible cords and strings can only pull. They can't push because they bend.

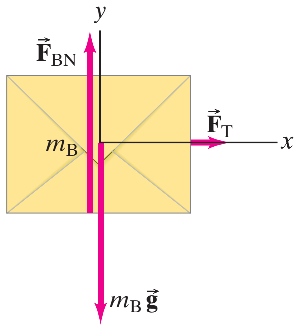
PROBLEM SOLVING
Cords can pull but can't push; tension exists throughout a taut cord



(a)



(b)



(c)

FIGURE 4–22 Example 4–12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0$ N. (b) Free-body diagram for box A. (c) Free-body diagram for box B.

Our next Example involves two boxes connected by a cord. We can refer to this group of objects as a system. A **system** is any group of one or more objects we choose to consider and study.

EXAMPLE 4–12 Two boxes connected by a cord. Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied to the 10.0-kg box by a person pulling on a cord, as shown in Fig. 4–22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord connecting the boxes.

APPROACH We streamline our approach by not listing each step. We have two boxes so we draw a free-body diagram for each. To draw them correctly, we must consider the forces on *each* box by itself, so that Newton's second law can be applied to each. The person exerts a force F_P on box A. Box A exerts a force F_T on the connecting cord, and the cord exerts an opposite but equal magnitude force F_T back on box A (Newton's third law). The two horizontal forces on box A are shown in Fig. 4–22b, along with the force of gravity $m_A\vec{g}$ downward and the normal force $\vec{F}_{AN}$ exerted upward by the table. The cord is light, so we neglect its mass. The tension at each end of the cord is thus the same. Hence the cord exerts a force F_T on the second box. Figure 4–22c shows the forces on box B, which are $\vec{F}_T$, $m_B\vec{g}$, and the normal force $\vec{F}_{BN}$. There will be only horizontal motion. We take the positive x axis to the right.

SOLUTION (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_A = a_B = a$. We now have 2 unknowns, F_T and a, and the two equations above to solve for them. We are given $m_A = 10.0$ kg and $m_B = 12.0$ kg. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\text{ N}}{22.0\text{ kg}} = 1.82\text{ m/s}^2.$$

This is what we sought.

(*b*) From the equation for box B above $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\text{ kg})(1.82\text{ m/s}^2) = 21.8\text{ N}.$$

Thus, $F_T < F_P\ (= 40.0\text{ N})$, as we expect, since F_T acts to accelerate only m_B.

Alternate Solution to (a) We would have obtained the same result had we considered a single system, of mass $m_A + m_B$, acted on by a net horizontal force equal to F_P. (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

NOTE It might be tempting to say that the force the person exerts, F_P, acts not only on box A but also on box B. It doesn't. F_P acts only on box A. It affects box B via the tension in the cord, F_T, which acts on box B and accelerates it. (You could look at it this way: $F_T < F_P$ because F_P accelerates *both* boxes whereas F_T only accelerates box B.)

CAUTION

For any object, use only the forces on that object in calculating $\Sigma F = ma$

EXAMPLE 4–13 **Elevator and counterweight (Atwood machine).** A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are made similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\,\text{kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\,\text{kg}$. For the latter case $(m_E = 1150\,\text{kg})$, calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

PHYSICS APPLIED

Elevator (as Atwood machine)

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{\mathbf{F}}_T$. Figures 4–23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable is massless and doesn't stretch). For the counterweight, $m_C g = (1000\,\text{kg})(9.80\,\text{m/s}^2) = 9800\,\text{N}$, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150\,\text{kg})(9.80\,\text{m/s}^2) = 11{,}300\,\text{N}$, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

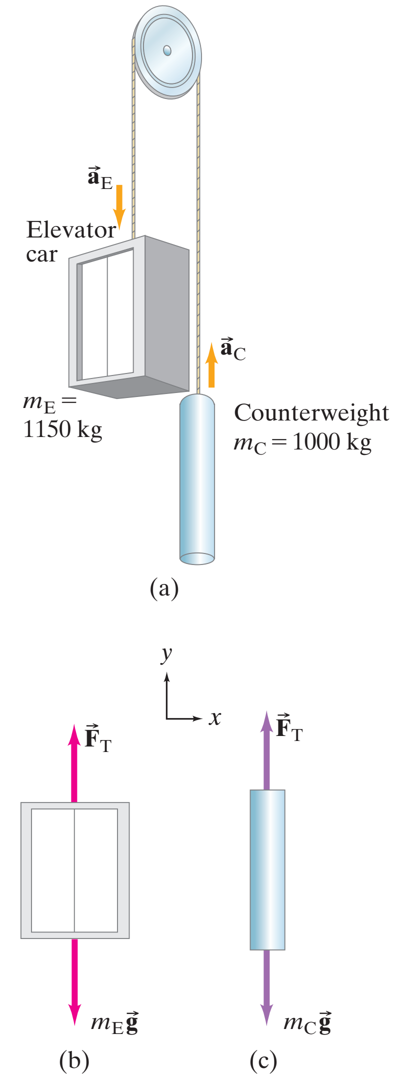


FIGURE 4–23 Example 4–13. (a) Atwood machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}g = \frac{1150\,\text{kg} - 1000\,\text{kg}}{1150\,\text{kg} + 1000\,\text{kg}}g = 0.070\,g = 0.68\,\text{m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070\,g = 0.68\,\text{m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations at the start of our solution, setting $a = 0.070\,g = 0.68\,\text{m/s}^2$:

$$F_T = m_E g - m_E a = m_E(g - a)$$
$$= 1150\,\text{kg}\,(9.80\,\text{m/s}^2 - 0.68\,\text{m/s}^2) = 10{,}500\,\text{N},$$

or

$$F_T = m_C g + m_C a = m_C(g + a)$$
$$= 1000\,\text{kg}\,(9.80\,\text{m/s}^2 + 0.68\,\text{m/s}^2) = 10{,}500\,\text{N},$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal $(m_E = m_C)$, then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass $(m_E \neq 0)$ would be predicted by our equation to accelerate at $a = g$, again as expected.

PROBLEM SOLVING

Check your result by seeing if it works in situations where the answer is easily guessed

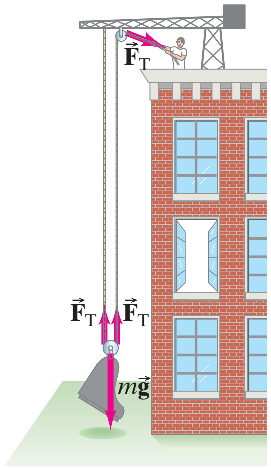


FIGURE 4–24 Example 4–14.

CONCEPTUAL EXAMPLE 4–14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 1600-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore the mass of the rope and pulleys. First notice the forces acting on the lower pulley at the piano. The weight of the piano $(= mg)$ pulls down on the pulley. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m), choosing the upward direction as positive:

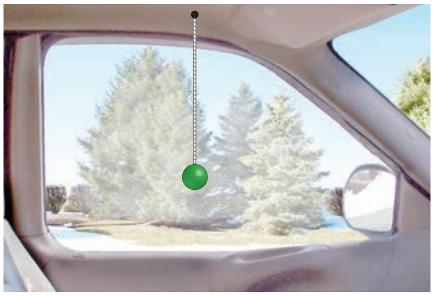
$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The piano mover can exert a force equal to half the piano's weight.

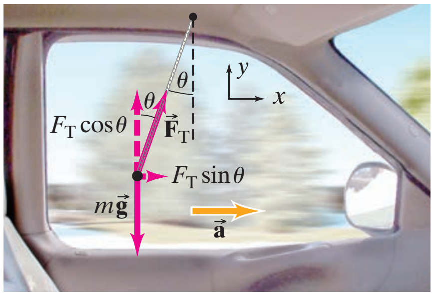
NOTE We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

PHYSICS APPLIED
Accelerometer

FIGURE 4–25 Example 4–15.



(a)



(b)

EXAMPLE 4–15 **Accelerometer.** A small mass m hangs from a thin string and can swing like a pendulum. You attach it above the window of your car as shown in Fig. 4–25a. When the car is at rest, the string hangs vertically. What angle θ does the string make (*a*) when the car accelerates at a constant $a = 1.20\ \text{m/s}^2$, and (*b*) when the car moves at constant velocity, $v = 90\ \text{km/h}$?

APPROACH The free-body diagram of Fig. 4–25b shows the pendulum at some angle θ relative to the vertical, and the forces on it: $m\vec{g}$ downward, and the tension $\vec{F}_T$ in the cord (including its components). These forces do not add up to zero if $\theta \neq 0$; and since we have an acceleration a, we expect $\theta \neq 0$.

SOLUTION (*a*) The acceleration $a = 1.20\ \text{m/s}^2$ is horizontal $(= a_x)$, and the only horizontal force is the x component of $\vec{F}_T$: $F_T \sin\theta$ (Fig. 4–25b). Then from Newton's second law,

$$ma = F_T \sin\theta.$$

The vertical component of Newton's second law gives, since $a_y = 0$,

$$0 = F_T \cos\theta - mg.$$

So

$$mg = F_T \cos\theta.$$

Dividing the two equations involving F_T, we obtain

$$\tan\theta = \frac{F_T \sin\theta}{F_T \cos\theta} = \frac{ma}{mg} = \frac{a}{g}$$

or

$$\tan\theta = \frac{1.20\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.122,$$

so

$$\theta = 7.0°.$$

(*b*) The velocity is constant, so $a = 0$ and therefore $\tan\theta = 0$. Hence the pendulum hangs vertically $(\theta = 0°)$.

NOTE This simple device is an **accelerometer**—it can be used to determine acceleration, by measuring the angle θ.

Inclines

Now we consider what happens when an object slides down an incline, such as a hill or ramp. Such problems are interesting because gravity is the accelerating force, yet the acceleration is not vertical. Solving problems is usually easier if we choose the xy coordinate system so the x axis points along the incline (the direction of motion) and the y axis is perpendicular to the incline, as shown in Fig. 4–26a. Note also that the normal force is not vertical, but is perpendicular to the sloping surface of the plane, along the y axis in Fig. 4–26b.

PROBLEM SOLVING
Good choice of coordinate system simplifies the calculation

EXAMPLE 4–16 **Box slides down an incline.** A box of mass m is placed on a smooth (frictionless) incline that makes an angle θ with the horizontal, as shown in Fig. 4–26. (*a*) Determine the normal force on the box. (*b*) Determine the box's acceleration. (*c*) Evaluate for a mass $m = 10\,\text{kg}$ and an incline of $\theta = 30°$.

APPROACH We expect the motion to be along the incline, so we choose the x axis along the slope, positive down the slope (the direction of motion). The y axis is perpendicular to the incline, positive upwards. The free-body diagram is shown in Fig. 4–26b. The forces on the box are its weight $m\vec{\mathbf{g}}$ vertically downward, which is shown resolved into its components parallel and perpendicular to the incline, and the normal force $\vec{\mathbf{F}}_N$ along the $+y$ axis. The components of $m\vec{\mathbf{g}}$ along the x and y axes are found using the definitions of sine ("side opposite") and cosine ("side adjacent"): $(mg)_x = mg\sin\theta$, and $(mg)_y = mg\cos\theta$. (The angle θ of the plane equals the angle between $m\vec{\mathbf{g}}$ and its y component because the left sides of the two angles are perpendicular to each other and so are the right sides—see Appendix A–6, Fig. A–2.)

SOLUTION (*a*) There is no motion in the y direction, so $a_y = 0$. Applying Newton's second law we have

$$F_y = ma_y$$
$$F_N - mg\cos\theta = 0,$$

where F_N and the y component of gravity $(mg\cos\theta)$ are all the forces acting on the box in the y direction. Thus the normal force is given by

$$F_N = mg\cos\theta.$$

Note carefully that unless $\theta = 0°$, F_N has magnitude less than the weight mg.
(*b*) In the x direction the only force acting is the x component of $m\vec{\mathbf{g}}$, which is $mg\sin\theta$. The acceleration a is in the x direction so

$$F_x = ma_x$$
$$mg\sin\theta = ma,$$

and we see that the acceleration down the plane is

$$a = g\sin\theta.$$

Thus the acceleration along an incline is always less than g, except at $\theta = 90°$, for which $\sin\theta = 1$ and $a = g$. This makes sense since $\theta = 90°$ is pure vertical fall. For $\theta = 0°$, $a = 0$, which makes sense because $\theta = 0°$ means the plane is horizontal so gravity causes no acceleration. Note too that the *acceleration does not depend on the mass* (m).
(*c*) For $\theta = 30°$, $\cos\theta = 0.866$ and $\sin\theta = 0.500$, so

$$F_N = 0.866\,mg = 85\,\text{N},$$

and

$$a = 0.500\,g = 4.9\,\text{m/s}^2.$$

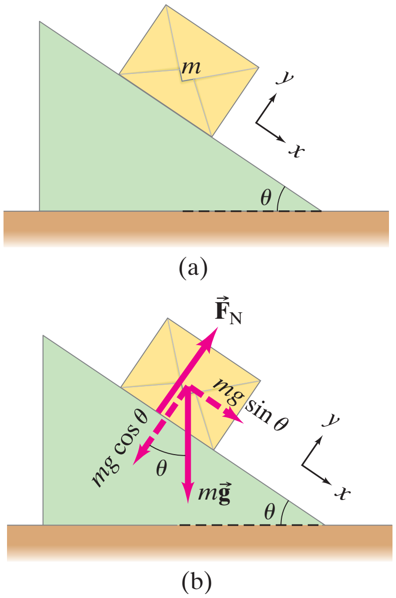


FIGURE 4–26 Example 4–16. (a) Box sliding on inclined plane. (b) Free-body diagram of box.

EXERCISE H Is the normal force always perpendicular to an inclined plane? Is it always vertical?

We will discuss more Examples of motion on an incline in the next Chapter, where friction will be included.

4–8 Problem Solving—A General Approach

A basic part of a physics course is solving problems effectively. The approach discussed here, though emphasizing Newton's laws, can be applied generally for other topics discussed throughout this book.

PROBLEM SOLVING

In General

1. **Read** and reread written problems carefully. A common error is to skip a word or two when reading, which can completely change the meaning of a problem.

2. **Draw** an accurate picture or diagram of the situation. (This is probably the most overlooked, yet most crucial, part of solving a problem.) Use arrows to represent vectors such as velocity or force, and label the vectors with appropriate symbols. When dealing with forces and applying Newton's laws, make sure to include all forces on a given object, including unknown ones, and make clear what forces act on what object (otherwise you may make an error in determining the *net force* on a particular object).

3. A separate **free-body diagram** needs to be drawn for each object involved, and it must show *all* the forces acting on a given object (and only on that object). Do not show forces that act on other objects.

4. Choose a convenient *xy* **coordinate system** (one that makes your calculations easier, such as one axis in the direction of the acceleration). Vectors are to be resolved into components along the coordinate axes. When using Newton's second law, apply $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ separately to x and y components, remembering that x direction forces are related to a_x, and similarly for y. If more than one object is involved, you can choose different (convenient) coordinate systems for each.

5. List the knowns and the unknowns (what you are trying to determine), and decide what you need in order to find the unknowns. For problems in the present Chapter, we use Newton's laws. More generally, it may help to see if one or more **relationships** (or **equations**) relate the unknowns to the knowns. But be sure each relationship is applicable in the given case. It is very important to know the limitations of each formula or relationship—when it is valid and when not. In this book, the more general equations have been given numbers, but even these can have a limited range of validity (often stated in brackets to the right of the equation).

6. Try to solve the problem approximately, to see if it is doable (to check if enough information has been given) and reasonable. Use your intuition, and make **rough calculations**—see "Order of Magnitude Estimating" in Section 1–6. A rough calculation, or a reasonable guess about what the range of final answers might be, is very useful. And a rough calculation can be checked against the final answer to catch errors in calculation, such as in a decimal point or the powers of 10.

7. **Solve** the problem, which may include algebraic manipulation of equations and/or numerical calculations. Recall the mathematical rule that you need as many independent equations as you have unknowns; if you have three unknowns, for example, then you need three independent equations. It is usually best to work out the algebra symbolically before putting in the numbers. Why? Because (*a*) you can then solve a whole class of similar problems with different numerical values; (*b*) you can check your result for cases already understood (say, $\theta = 0°$ or 90°); (*c*) there may be cancellations or other simplifications; (*d*) there is usually less chance for numerical error; and (*e*) you may gain better insight into the problem.

8. Be sure to keep track of **units**, for they can serve as a check (they must balance on both sides of any equation).

9. Again consider if your answer is **reasonable**. The use of dimensional analysis, described in Section 1–7, can also serve as a check for many problems.

Summary

Newton's three laws of motion are the basic classical laws describing motion.

Newton's first law (the **law of inertia**) states that if the net force on an object is zero, an object originally at rest remains at rest, and an object in motion remains in motion in a straight line with constant velocity.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}. \quad \text{(4–1a)}$$

Newton's second law is one of the most important and fundamental laws in classical physics.

Newton's third law states that whenever one object exerts a force on a second object, the second object always exerts a force on the first object which is equal in magnitude but opposite in direction:

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}, \quad \text{(4–2)}$$

where $\vec{\mathbf{F}}_{BA}$ is the force on object B exerted by object A. This is true even if objects are moving and accelerating, and/or have different masses.

The tendency of an object to resist a change in its motion is called **inertia**. **Mass** is a measure of the inertia of an object.

Weight refers to the **gravitational force** on an object, and is equal to the product of the object's mass m and the acceleration of gravity $\vec{\mathbf{g}}$:

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \quad \text{(4–3)}$$

Force, which is a vector, can be considered as a push or pull; or, from Newton's second law, force can be defined as an action capable of giving rise to acceleration. The **net force** on an object is the vector sum of all forces acting on that object.

For solving problems involving the forces on one or more objects, it is essential to draw a **free-body diagram** for *each* object, showing all the forces acting on only that object. Newton's second law can be applied to the vector components for each object.

Questions

1. Why does a child in a wagon seem to fall backward when you give the wagon a sharp pull forward?
2. A box rests on a frictionless part of a truck bed. The truck driver starts the truck and accelerates forward. The box immediately starts to slide toward the rear of the truck bed. Discuss the motion of the box, in terms of Newton's laws, as seen (*a*) by Francesca standing on the ground beside the truck, and (*b*) by Phil who is riding on the truck (Fig. 4–27).

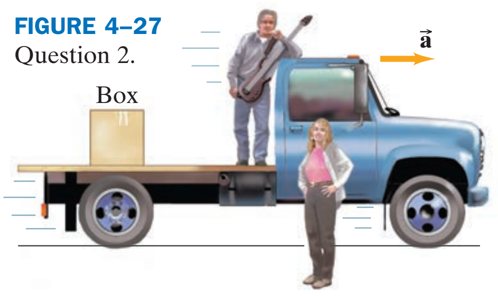


FIGURE 4–27 Question 2.

3. If an object is moving, is it possible for the net force acting on it to be zero? Explain.
4. If the acceleration of an object is zero, are no forces acting on it? Explain.
5. Only one force acts on an object. Can the object have zero acceleration? Can it have zero velocity? Explain.
6. When a golf ball is dropped to the pavement, it bounces back up. (*a*) Is a force needed to make it bounce back up? (*b*) If so, what exerts the force?
7. If you walk on a log floating on a lake, why does the log move in the opposite direction?
8. (*a*) Why do you push down harder on the pedals of a bicycle when first starting out than when moving at constant speed? (*b*) Why do you need to pedal at all when cycling at constant speed?
9. A stone hangs by a fine thread from the ceiling, and a section of the same thread dangles from the bottom of the stone (Fig. 4–28). If a person gives a sharp pull on the dangling thread, where is the thread likely to break: below the stone or above it? What if the person gives a slow and steady pull? Explain your answers.

FIGURE 4–28 Question 9.

10. The force of gravity on a 2-kg rock is twice as great as that on a 1-kg rock. Why then doesn't the heavier rock fall faster?
11. You pull a box with a constant force across a frictionless table using an attached rope held horizontally. If you now pull the rope with the same force at an angle to the horizontal (with the box remaining flat on the table), does the acceleration of the box increase, decrease, or remain the same? Explain.
12. When an object falls freely under the influence of gravity there is a net force mg exerted on it by the Earth. Yet by Newton's third law the object exerts an equal and opposite force on the Earth. Does the Earth move? Explain.
13. Compare the effort (or force) needed to lift a 10-kg object when you are on the Moon with the force needed to lift it on Earth. Compare the force needed to throw a 2-kg object horizontally with a given speed on the Moon and on Earth.

14. According to Newton's third law, each team in a tug of war (Fig. 4–29) pulls with equal force on the other team. What, then, determines which team will win?

FIGURE 4–29 Question 14. A tug of war. Describe the forces on each of the teams and on the rope.

15. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?

16. Whiplash sometimes results from an automobile accident when the victim's car is struck violently from the rear. Explain why the head of the victim seems to be thrown backward in this situation. Is it really?

17. Mary exerts an upward force of 40 N to hold a bag of groceries. Describe the "reaction" force (Newton's third law) by stating (*a*) its magnitude, (*b*) its direction, (*c*) *on* what object it is exerted, and (*d*) *by* what object it is exerted.

18. A father and his young daughter are ice skating. They face each other at rest and then push each other so they begin moving in opposite directions. Which one has the greater speed? Explain.

19. What would your bathroom scale read if you weighed yourself on an inclined plane? Assume the mechanism functions properly, even at an angle.

20. Which of the following objects weighs about 1 N: (*a*) an apple, (*b*) a mosquito, (*c*) this book, (*d*) you?

21. Why might your foot hurt if you kick a heavy desk or a wall?

22. When you are running and want to stop quickly, you must decelerate quickly. (*a*) What is the origin of the force that causes you to stop? (*b*) Estimate (using your own experience) the maximum rate of deceleration of a person running at top speed to come to rest.

23. Suppose that you are standing on top of a cardboard carton that just barely supports you. What would happen to it if you jumped up into the air? It would: (*a*) collapse; (*b*) be unaffected; (*c*) spring upward a bit; (*d*) move sideways. Explain your answer.

MisConceptual Questions

1. A truck is traveling horizontally to the right (Fig. 4–30). When the truck starts to slow down, the crate on the frictionless truck bed starts to slide. In what direction could the net force be on the crate?
(*a*) No direction. The net force is zero.
(*b*) Straight down (gravity).
(*c*) Straight up (the normal force).
(*d*) Horizontal and to the right.
(*e*) Horizontal and to the left.

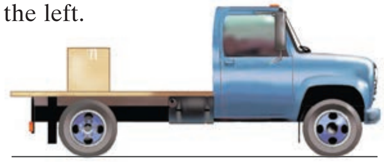


FIGURE 4–30 MisConceptual Question 1.

2. George, in the foreground of Fig. 4–31, is able to move the large truck because
(*a*) he is stronger than the truck.
(*b*) he is heavier in some respects than the truck.
(*c*) the truck offers no resistance because its brakes are off.
(*d*) the ground exerts a greater friction force on George than it does on the truck.
(*e*) he exerts a greater force on the truck than the truck exerts back on him.

FIGURE 4–31 MisConceptual Question 2.

3. A bear sling, Fig. 4–32, is used in some national parks for placing backpackers' food out of the reach of bears. As the backpacker raises the pack by pulling down on the rope, the force *F* needed:
(*a*) decreases as the pack rises until the rope is straight across.
(*b*) doesn't change.
(*c*) increases until the rope is straight.
(*d*) increases but the rope always sags where the pack hangs.

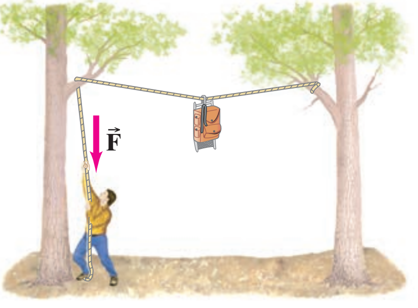


FIGURE 4–32 MisConceptual Question 3.

4. What causes the boat in Fig. 4–33 to move forward?
(*a*) The force the man exerts on the paddle.
(*b*) The force the paddle exerts on the water.
(*c*) The force the water exerts on the paddle.
(*d*) The motion of the water itself.

FIGURE 4–33 MisConceptual Question 4.

5. You are trying to push your stalled car. Although you apply a horizontal force of 400 N on the car, it doesn't budge, and neither do you. What other force(s) must also have a magnitude of 400 N?
(*a*) The force exerted by the car on you.
(*b*) The friction force exerted by the car on the road.
(*c*) The normal force exerted by the road on you.
(*d*) The friction force exerted by the road on you.

6. When a skier skis down a hill, the normal force exerted on the skier by the hill is
(*a*) equal to the weight of the skier.
(*b*) greater than the weight of the skier.
(*c*) less than the weight of the skier.

7. A golf ball is hit with a golf club. While the ball flies through the air, which forces act on the ball? Neglect air resistance.
(*a*) The force of the golf club acting on the ball.
(*b*) The force of gravity acting on the ball.
(*c*) The force of the ball moving forward through the air.
(*d*) All of the above.
(*e*) Both (*a*) and (*b*).

8. Suppose an object is accelerated by a force of 100 N. Suddenly a second force of 100 N in the opposite direction is exerted on the object, so that the forces cancel. The object
(*a*) quickly stops.
(*b*) decelerates gradually to rest.
(*c*) continues at the velocity it had before the second force was applied.
(*d*) is gradually brought to rest and then accelerates in the direction of the second force.

9. You are pushing a heavy box across a rough floor. When you are initially pushing the box and it is accelerating,
(*a*) you exert a force on the box, but the box does not exert a force on you.
(*b*) the box is so heavy it exerts a force on you, but you do not exert a force on the box.
(*c*) the force you exert on the box is greater than the force of the box pushing back on you.
(*d*) the force you exert on the box is equal to the force of the box pushing back on you.
(*e*) the force that the box exerts on you is greater than the force you exert on the box.

10. Two 5-newton boxes are attached to opposite ends of a spring scale and suspended from pulleys as shown in Fig. 4–34. What is the reading on the scale?
(*a*) 0 N.
(*b*) Between 0 and 5 N.
(*c*) 5 N.
(*d*) Between 5 and 10 N.
(*e*) 10 N.

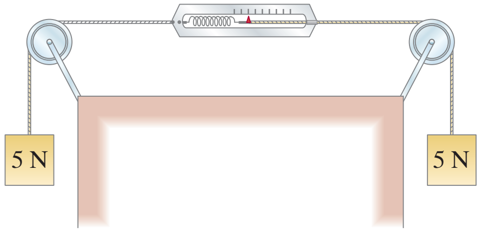


FIGURE 4–34 MisConceptual Question 10.

11. Two tug of war opponents each pull with a force of 500 N on opposite ends of a rope. Assume the rope is massless. What is the tension in the rope?
(*a*) 0 N.
(*b*) 250 N.
(*c*) 500 N.
(*d*) 1000 N.
(*e*) Impossible to tell.

12. The normal force on an extreme skier descending a very steep slope (Fig. 4–35) can be zero if
(*a*) his speed is great enough.
(*b*) he leaves the slope (no longer touches the snow).
(*c*) the slope is greater than 75°.
(*d*) the slope is vertical (90°).

FIGURE 4–35 MisConceptual Question 12.

13. To pull an old stump out of the ground, you and a friend tie two ropes to the stump. You pull on it with a force of 500 N to the north while your friend pulls with a force of 450 N to the northwest. The total force exerted by the two ropes is
(*a*) less than 950 N.
(*b*) exactly 950 N.
(*c*) more than 950 N.

Problems

4–4 to 4–6 Newton's Laws, Gravitational Force, Normal Force [Assume no friction.]

1. (I) What net force is needed to accelerate a 45-kg sled at $1.4\ \mathrm{m/s^2}$ on horizontal frictionless ice?

2. (I) What is the weight of a 74-kg astronaut (*a*) on Earth, (*b*) on the Moon $(g = 1.7\ \mathrm{m/s^2})$, (*c*) on Mars $(g = 3.7\ \mathrm{m/s^2})$, (*d*) in outer space traveling with constant velocity?

3. (I) How much tension must a rope withstand if it is used to accelerate a 1210-kg car horizontally along a frictionless surface at $1.35\ \mathrm{m/s^2}$?

4. (I) A net force of 215 N accelerates a bike and rider at $2.30\ \mathrm{m/s^2}$. What is the mass of the bike and rider together?

5. (II) What average force is required to stop a 950-kg car in 8.0 s if the car is traveling at 95 km/h?

6. (II) According to a simplified model of a *mammalian heart*, at each pulse approximately 20 g of blood is accelerated from 0.25 m/s to 0.35 m/s during a period of 0.10 s. What is the magnitude of the force exerted by the heart muscle?

7. (II) Superman must stop a 120-km/h train in 150 m to keep it from hitting a stalled car on the tracks. If the train's mass is 3.6×10^5 kg, how much force must he exert? Compare to the weight of the train (give as %). How much force does the train exert on Superman?

8. (II) A person has a reasonable chance of surviving an *automobile crash* if the deceleration is no more than 30 g's. Calculate the force on a 65-kg person accelerating at this rate. What distance is traveled if brought to rest at this rate from 85 km/h?

9. (II) Estimate the average force exerted by a shot-putter on a 7.0-kg shot if the shot is moved through a distance of 2.8 m and is released with a speed of 13 m/s.

10. (II) A 0.140-kg baseball traveling 35.0 m/s strikes the catcher's mitt, which, in bringing the ball to rest, recoils backward 11.0 cm. What was the average force applied by the ball on the glove?

11. (II) A fisherman yanks a fish vertically out of the water with an acceleration of $2.5\ \mathrm{m/s^2}$ using very light fishing line that has a breaking strength of 18 N (≈ 4 lb). The fisherman unfortunately loses the fish as the line snaps. What can you say about the mass of the fish?

12. (II) How much tension must a cable withstand if it is used to accelerate a 1400-kg car vertically upward at $0.70\ \mathrm{m/s^2}$?

13. (II) A 20.0-kg box rests on a table. (*a*) What is the weight of the box and the normal force acting on it? (*b*) A 10.0-kg box is placed on top of the 20.0-kg box, as shown in Fig. 4–36. Determine the normal force that the table exerts on the 20.0-kg box and the normal force that the 20.0-kg box exerts on the 10.0-kg box.

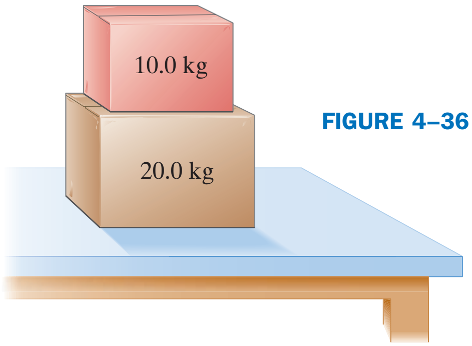


FIGURE 4–36 Problem 13.

14. (II) A particular race car can cover a quarter-mile track (402 m) in 6.40 s starting from a standstill. Assuming the acceleration is constant, how many horizontal "g's" does the driver experience? If the combined mass of the driver and race car is 535 kg, what horizontal force must the road exert on the tires? Ignore air resistance.

15. (II) A 14.0-kg bucket is lowered vertically by a rope in which there is 132 N of tension at a given instant. What is the acceleration of the bucket? Is it up or down?

16. (II) A 75-kg petty thief wants to escape from a third-story jail window. Unfortunately, a makeshift rope made of sheets tied together can support a mass of only 62 kg. How might the thief use this "rope" to escape? Give a quantitative answer.

17. (II) A woman stands on a bathroom scale in a motionless elevator. When the elevator begins to move, the scale briefly reads only 0.75 of her regular weight. Calculate the acceleration of the elevator, and find the direction of acceleration.

18. (II) Can cars "stop on a dime"? Calculate the acceleration of a 1400-kg car if it can stop from 35 km/h on a dime (diameter = 1.7 cm). How many g's is this? What is the force felt by the 68-kg occupant of the car?

19. (II) The cable supporting a 2375-kg elevator has a maximum strength of 24,950 N. What maximum upward acceleration can it give the elevator without breaking?

20. (II) Using focused laser light, *optical tweezers* can apply a force of about 10 pN (piconewtons) to a 1.0-μm-diameter polystyrene bead, which has a density about equal to that of water: a volume of $1.0\ \mathrm{cm^3}$ has a mass of about 1.0 g. Estimate the bead's acceleration in g's.

21. (II) A *rocket* with a mass of 2.45×10^6 kg is launched by exerting a vertical force of 3.55×10^7 N on the gases it expels. Determine (*a*) the acceleration of the rocket, (*b*) its velocity after 8.0 s, and (*c*) how long it takes to reach an altitude of 9500 m. Assume g remains constant, and ignore the mass of gas expelled (not realistic).

22. (II) (*a*) What is the acceleration of two falling *sky divers* (total mass = 148 kg including parachute) when the upward force of air resistance is equal to one-fourth of their weight? (*b*) After opening the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute?

23. (II) An elevator (mass 4850 kg, with people) is to be designed so that the maximum acceleration is $0.0640g$. What are the maximum and minimum forces the motor should exert on the supporting cable?

24. (II) An exceptional standing jump would raise a person 0.80 m off the ground. To do this, what force must a 68-kg person exert against the ground? Assume the person crouches a distance of 0.20 m prior to jumping, and thus the upward force has this distance to act over before he leaves the ground.

25. (II) High-speed *elevators* function under two limitations: (1) the maximum magnitude of vertical acceleration that a typical human body can experience without discomfort is about $1.2\ \mathrm{m/s^2}$, and (2) the typical maximum speed attainable is about 9.0 m/s. You board an elevator on a skyscraper's ground floor and are transported 180 m above the ground level in three steps: acceleration of magnitude $1.2\ \mathrm{m/s^2}$ from rest to 9.0 m/s, followed by constant upward velocity of 9.0 m/s, then deceleration of magnitude $1.2\ \mathrm{m/s^2}$ from 9.0 m/s to rest. (*a*) Determine the elapsed time for each of these 3 stages. (*b*) Determine the change in the magnitude of the normal force, expressed as a % of your normal weight during each stage. (*c*) What fraction of the total transport time does the normal force not equal the person's weight?

26. (III) The 100-m dash can be run by the best sprinters in 10.0 s. A 66-kg sprinter accelerates uniformly for the first 45 m to reach top speed, which he maintains for the remaining 55 m. (*a*) What is the average horizontal component of force exerted on his feet by the ground during acceleration? (*b*) What is the speed of the sprinter over the last 55 m of the race (that is, his top speed)?

27. (III) A person jumps from the roof of a house 2.8 m high. When he strikes the ground below, he bends his knees so that his torso decelerates over an approximate distance of 0.70 m. If the mass of his torso (excluding legs) is 42 kg, find (*a*) his velocity just before his feet strike the ground, and (*b*) the average force exerted on his torso by his legs during deceleration.

4–7 Newton's Laws and Vectors [Ignore friction.]

28. (I) A box weighing 66.0 N rests on a table. A rope tied to the box runs vertically upward over a pulley and a weight is hung from the other end (Fig. 4–37). Determine the force that the table exerts on the box if the weight hanging on the other side of the pulley weighs (*a*) 30.0 N, (*b*) 60.0 N, and (*c*) 90.0 N.

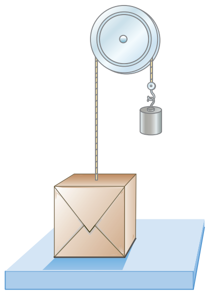


FIGURE 4–37 Problem 28.

29. (I) Draw the free-body diagram for a basketball player (*a*) just before leaving the ground on a jump, and (*b*) while in the air. See Fig. 4–38.

FIGURE 4–38 Problem 29.

30. (I) A 650-N force acts in a northwesterly direction. A second 650-N force must be exerted in what direction so that the resultant of the two forces points westward? Illustrate your answer with a vector diagram.

31. (I) Sketch the free-body diagram of a baseball (*a*) at the moment it is hit by the bat, and again (*b*) after it has left the bat and is flying toward the outfield. Ignore air resistance.

32. (II) Arlene is to walk across a "high wire" strung horizontally between two buildings 10.0 m apart. The sag in the rope when she is at the midpoint is 10.0°, as shown in Fig. 4–39. If her mass is 50.0 kg, what is the tension in the rope at this point?

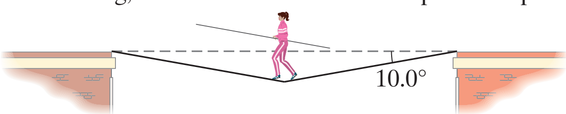


FIGURE 4–39 Problem 32.

33. (II) A window washer pulls herself upward using the bucket–pulley apparatus shown in Fig. 4–40. (*a*) How hard must she pull downward to raise herself slowly at constant speed? (*b*) If she increases this force by 15%, what will her acceleration be? The mass of the person plus the bucket is 78 kg.

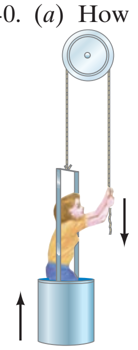


FIGURE 4–40 Problem 33.

34. (II) One 3.2-kg paint bucket is hanging by a massless cord from another 3.2-kg paint bucket, also hanging by a massless cord, as shown in Fig. 4–41. (*a*) If the buckets are at rest, what is the tension in each cord? (*b*) If the two buckets are pulled upward with an acceleration of $1.45\ \mathrm{m/s^2}$ by the upper cord, calculate the tension in each cord.

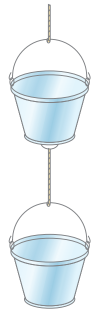


FIGURE 4–41 Problems 34 and 35.

35. (II) The cords accelerating the buckets in Problem 34b, Fig. 4–41, each have a weight of 2.0 N. Determine the tension in each cord at the three points of attachment.

36. (II) Two large snowcats are towing a housing unit north, as shown in Fig. 4–42. The sum of the forces $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ exerted on the unit by the horizontal cables is north, parallel to the line L, and $F_A = 4200$ N. Determine F_B and the magnitude of $\vec{\mathbf{F}}_A + \vec{\mathbf{F}}_B$.

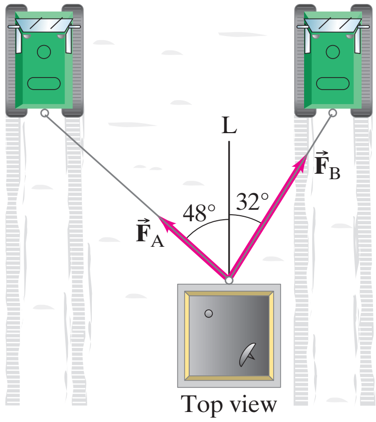


FIGURE 4–42 Problem 36.

37. (II) A train locomotive is pulling two cars of the same mass behind it, Fig. 4–43. Determine the ratio of the tension in the coupling (think of it as a cord) between the locomotive and the first car (F_{T1}) to that between the first car and the second car (F_{T2}), for any nonzero acceleration of the train.

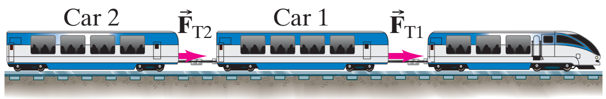


FIGURE 4–43 Problem 37.

38. (II) The two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ shown in Fig. 4–44a and b (looking down) act on an 18.5-kg object on a frictionless tabletop. If $F_1 = 10.4$ N and $F_2 = 16.2$ N, find the net force on the object and its acceleration for (a) and (b).

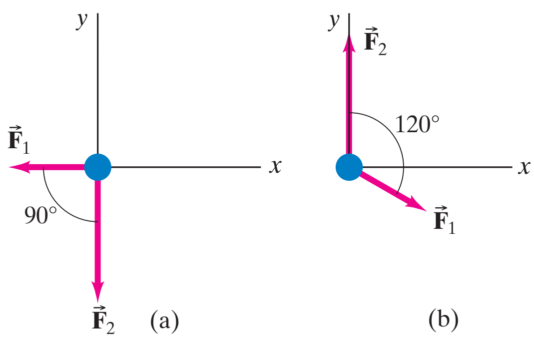


FIGURE 4–44 Problem 38.

39. (II) A skateboarder, with an initial speed of 2.0 m/s, rolls virtually friction free down a straight incline of length 18 m in 3.3 s. At what angle θ is the incline oriented above the horizontal?

40. (II) At the instant a race began, a 65-kg sprinter exerted a force of 720 N on the starting block at a 22° angle with respect to the ground. (*a*) What was the horizontal acceleration of the sprinter? (*b*) If the force was exerted for 0.32 s, with what speed did the sprinter leave the starting block?

41. (II) A mass m is at rest on a horizontal frictionless surface at $t = 0$. Then a constant force F_0 acts on it for a time t_0. Suddenly the force doubles to $2F_0$ and remains constant until $t = 2t_0$. Determine the total distance traveled from $t = 0$ to $t = 2t_0$.

42. (II) A 3.0-kg object has the following two forces acting on it:

$$\vec{\mathbf{F}}_1 = (16\hat{\mathbf{i}} + 12\hat{\mathbf{j}})\,\mathrm{N}$$
$$\vec{\mathbf{F}}_2 = (-10\hat{\mathbf{i}} + 22\hat{\mathbf{j}})\,\mathrm{N}$$

If the object is initially at rest, determine its velocity $\vec{\mathbf{v}}$ at $t = 4.0$ s.

43. (II) A 27-kg chandelier hangs from a ceiling on a vertical 3.4-m-long wire. (*a*) What horizontal force would be necessary to displace its position 0.15 m to one side? (*b*) What will be the tension in the wire?

44. (II) Redo Example 4–13 but (*a*) set up the equations so that the direction of the acceleration $\vec{\mathbf{a}}$ of each object is in the direction of motion of that object. (In Example 4–13, we took $\vec{\mathbf{a}}$ as positive upward for both masses.) (*b*) Solve the equations to obtain the same answers as in Example 4–13.

45. (II) The block shown in Fig. 4–45 has mass $m = 7.0$ kg and lies on a fixed smooth frictionless plane tilted at an angle $\theta = 22.0°$ to the horizontal. (*a*) Determine the acceleration of the block as it slides down the plane. (*b*) If the block starts from rest 12.0 m up the plane from its base, what will be the block's speed when it reaches the bottom of the incline?

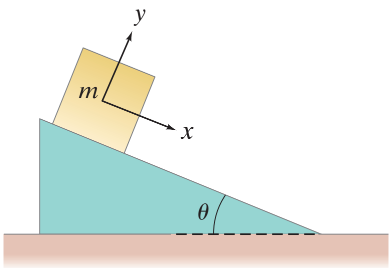


FIGURE 4–45 Block on inclined plane. Problems 45 and 46.

46. (II) A block is given an initial speed of 5.2 m/s up the 22.0° plane shown in Fig. 4–45. (*a*) How far up the plane will it go? (*b*) How much time elapses before it returns to its starting point? Ignore friction.

47. (II) An object is hanging by a string from your rearview mirror. While you are accelerating at a constant rate from rest to 28 m/s in 5.0 s, what angle θ does the string make with the vertical? See Fig. 4–46.

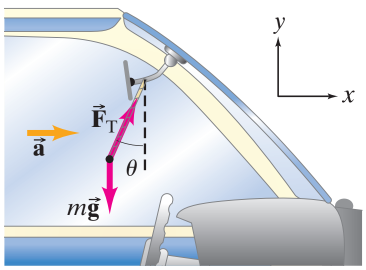


FIGURE 4–46 Problem 47.

48. (II) A 2.0-kg purse is dropped from the top of the Leaning Tower of Pisa and falls 55 m before reaching the ground with a speed of 27 m/s. What was the average force of air resistance?

49. (II) Bob traverses a chasm by stringing a rope between a tree on one side of the chasm and a tree on the opposite side, 25 m away, Fig. 4–47. Assume the rope can provide a tension force of up to 31 kN before breaking, and use a "safety factor" of 10 (that is, the rope should only be required to undergo a tension force of 3.1 kN). (*a*) If Bob's mass is 72.0 kg, determine the distance x that the rope must sag at a point halfway across if it is to be within its recommended safety range. (*b*) If the rope sags by only one-fourth the distance found in (*a*), determine the tension force in the rope. Will the rope break?

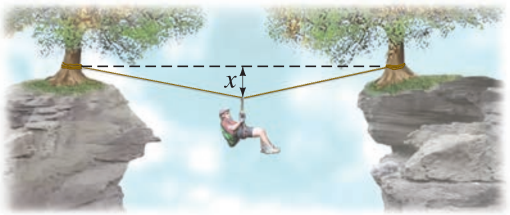


FIGURE 4–47 Problem 49.

50. (II) As shown in Fig. 4–48, five balls (masses 2.00, 2.05, 2.10, 2.15, 2.20 kg) hang from a crossbar. Each mass is supported by "5-lb test" fishing line which will break when its tension force exceeds 22.2 N (= 5.00 lb). When this device is placed in an elevator, which accelerates upward, only the lines attached to the 2.05 and 2.00 kg masses do not break. Within what range is the elevator's acceleration?

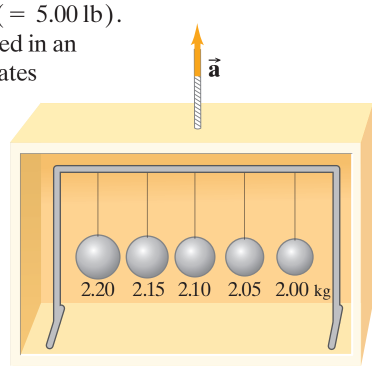


FIGURE 4–48 Problem 50.

51. (II) A high-speed 14-car Italian train has a mass of 640 metric tons (640,000 kg). It can exert a maximum force of 400 kN horizontally against the tracks, whereas at maximum constant velocity (300 km/h), it exerts a force of about 150 kN. Calculate (*a*) its maximum acceleration, and (*b*) estimate the total force of friction and air resistance at top speed.

52. (II) A 450-kg piano is being unloaded from a truck by rolling it down a ramp inclined at 15°. There is negligible friction and the ramp is 4.0 m long. Two workers slow the rate at which the piano moves by pushing with a combined force of 1020 N parallel to the ramp. If the piano starts from rest, how fast is it moving at the bottom?

53. (II) Uphill escape ramps are sometimes provided to the side of steep downhill highways for trucks with overheated brakes. For a simple 11° upward ramp, what length would be needed to stop a runaway truck traveling 140 km/h? Note the large size of your calculated length. (If sand is used for the bed of the ramp, its length can be reduced by a factor of about 2.)

54. (II) A child on a sled reaches the bottom of a hill with a velocity of 10.0 m/s and travels 25.0 m along a horizontal straightaway to a stop. If the child and sled together have a mass of 60.0 kg, what is the average retarding force on the sled on the horizontal straightaway?

55. (II) Figure 4–49 shows a block (mass m_A) on a smooth horizontal surface, connected by a thin cord that passes over a pulley to a second block (m_B), which hangs vertically. (*a*) Draw a free-body diagram for each block, showing the force of gravity on each, the force (tension) exerted by the cord, and any normal force. (*b*) Apply Newton's second law to find formulas for the acceleration of the system and for the tension in the cord. Ignore friction and the masses of the pulley and cord.

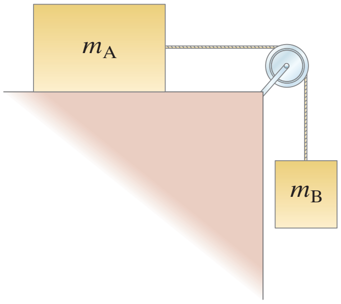


FIGURE 4–49 Problems 55, 56, and 57. Mass m_A rests on a smooth horizontal surface; m_B hangs vertically.

56. (II) (*a*) If $m_A = 14.0\,\text{kg}$ and $m_B = 5.0\,\text{kg}$ in Fig. 4–49, determine the acceleration of each block. (*b*) If initially m_A is at rest 1.250 m from the edge of the table, how long does it take to reach the edge of the table if the system is allowed to move freely? (*c*) If $m_B = 1.0\,\text{kg}$, how large must m_A be if the acceleration of the system is to be kept at $\frac{1}{100}g$?

57. (III) Determine a formula for the acceleration of the system shown in Fig. 4–49 (see Problem 55) if the cord has a non-negligible mass m_C. Specify in terms of ℓ_A and ℓ_B, the lengths of cord from the respective masses to the pulley. (The total cord length is $\ell = \ell_A + \ell_B$.)

58. (III) Suppose the pulley in Fig. 4–50 is suspended by a cord C. Determine the tension in this cord after the masses are released and before one hits the ground. Ignore the mass of the pulley and cords.

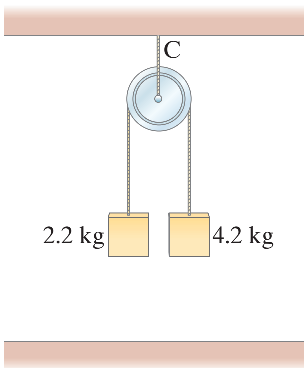
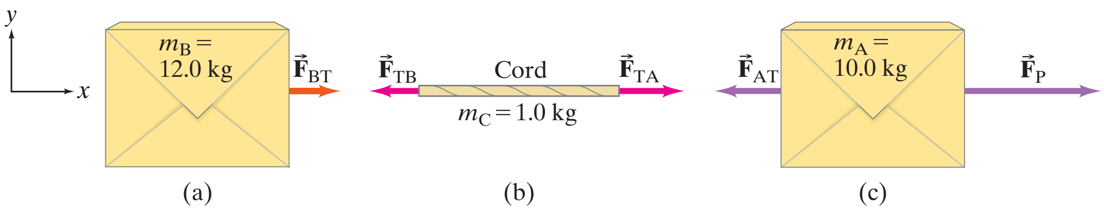


FIGURE 4–50 Problem 58.

59. (III) Suppose two boxes on a frictionless table are connected by a heavy cord of mass 1.0 kg. Calculate the acceleration of each box and the tension at each end of the cord, using the free-body diagrams shown in Fig. 4–51. Assume $F_P = 40.0\,\text{N}$, and ignore sagging of the cord. Compare your results to Example 4–12 and Fig. 4–22.

FIGURE 4–51 Problem 59. Free-body diagrams for each of the objects of the system shown in Fig. 4–22a. Vertical forces, $\vec{\mathbf{F}}_N$ and $\vec{\mathbf{F}}_G$, are not shown.



60. (III) Three blocks on a frictionless horizontal surface are in contact with each other as shown in Fig. 4–52. A force $\vec{\mathbf{F}}$ is applied to block A (mass m_A). (*a*) Draw a free-body diagram for each block. Determine (*b*) the acceleration of the system (in terms of m_A, m_B, and m_C), (*c*) the net force on each block, and (*d*) the force of contact that each block exerts on its neighbor. (*e*) If $m_A = m_B = m_C = 10.0\,\text{kg}$ and $F = 96.0\,\text{N}$, give numerical answers to (*b*), (*c*), and (*d*). Explain how your answers make sense intuitively.

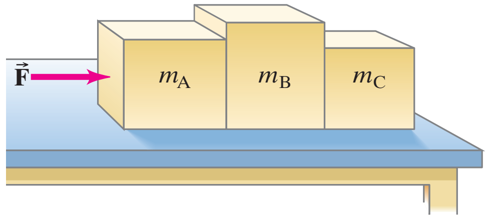


FIGURE 4–52 Problem 60.

61. (III) A 2.5-kg block is placed on a frictionless table. The block is connected by massless ropes over massless pulleys to a 5.0-kg block on the right, and a 3.0-kg block on the left, as shown in Fig. 4–53. Find the acceleration of the block on the table.

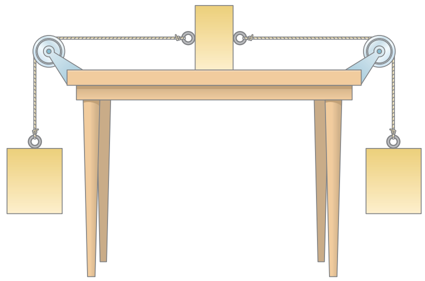


FIGURE 4–53 Problem 61.

62. (III) A small block of mass m rests on the sloping side of a triangular block of mass M which itself rests on a horizontal table as shown in Fig. 4–54. Assuming all surfaces are frictionless, determine the magnitude of the force $\vec{\mathbf{F}}$ that must be applied to M so that m remains in a fixed position relative to M (that is, m doesn't move on the incline). [*Hint*: Take x and y axes horizontal and vertical.]

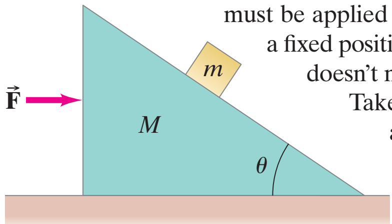


FIGURE 4–54 Problem 62.

63. (III) The double Atwood machine shown in Fig. 4–55 has frictionless, massless pulleys and cords. Determine (*a*) the acceleration of masses m_A, m_B, and m_C, and (*b*) the tensions F_{TA} and F_{TC} in the cords.

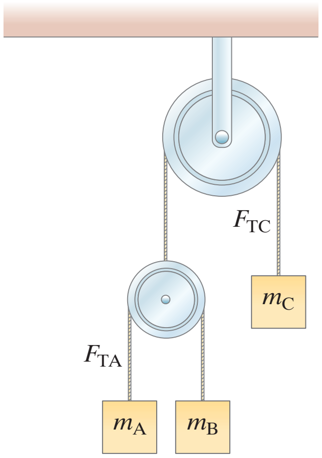


FIGURE 4–55 Problem 63.

64. (III) Determine a formula for the magnitude of the force $\vec{\mathbf{F}}$ exerted on the large block (m_C) in Fig. 4–56 so that the mass m_A does not move relative to m_C. Ignore all friction. Assume m_B does not make contact with m_C.

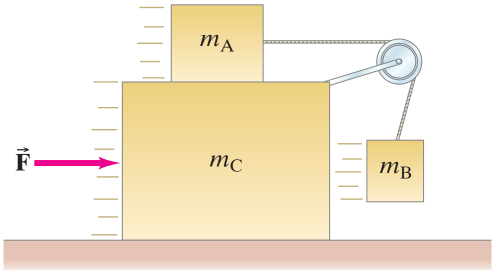


FIGURE 4–56 Problem 64.

65. (III) The two masses shown in Fig. 4–57 are each initially 1.8 m above the ground, and the massless frictionless pulley is 4.8 m above the ground. What maximum height does the lighter object reach after the system is released? [*Hint*: First determine the acceleration of the lighter mass and then its velocity at the moment the heavier one hits the ground. This is its "launch" speed. Assume the mass doesn't hit the pulley or the ceiling. Ignore the mass of the cord.]

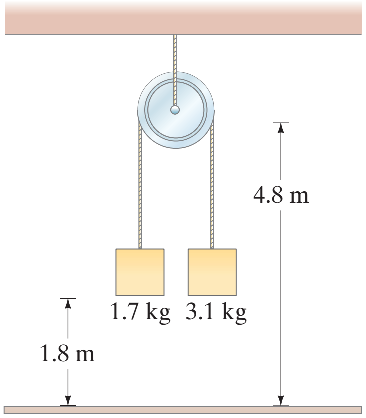


FIGURE 4–57 Problem 65.

66. (III) A particle of mass m, initially at rest at $x = 0$, is accelerated by a force that increases in time as $F = Ct^2$. Determine its velocity v and position x as a function of time.

General Problems

67. A crane's trolley at point P in Fig. 4–58 moves for a few seconds to the right with constant acceleration, and the 780-kg load hangs on a light cable at a 5.0° angle to the vertical as shown. What is the acceleration of the trolley and load?

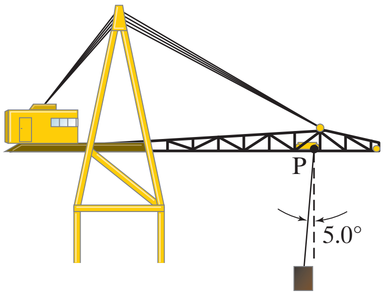


FIGURE 4–58 Problem 67.

68. A 75.0-kg person stands on a scale in an elevator. What does the scale read (in N and in kg) when (*a*) the elevator is at rest, (*b*) the elevator is climbing at a constant speed of 2.0 m/s, (*c*) the elevator is descending at 2.0 m/s, (*d*) the elevator is accelerating upward at 2.0 m/s^2, (*e*) the elevator is accelerating downward at 2.0 m/s^2?

69. A *city planner* is working on the redesign of a hilly portion of a city. An important consideration is how steep the roads can be so that even low-powered cars can get up the hills without slowing down. A particular small car, with a mass of 920 kg, can accelerate on a level road from rest to 21 m/s (75 km/h) in 14.5 s. Using these data, calculate the maximum steepness of a hill.

70. If a bicyclist of mass 65 kg (including the *bicycle*) can coast down a 6.5° hill at a steady speed of 6.0 km/h because of air resistance, how much force must be applied to climb the hill at the same speed (and the same air resistance)?

71. Tom's *hang glider* supports his weight using the six ropes shown in Fig. 4–59. Each rope is designed to support an equal fraction of Tom's weight. Tom's mass is 78.0 kg. What is the tension in each of the support ropes?

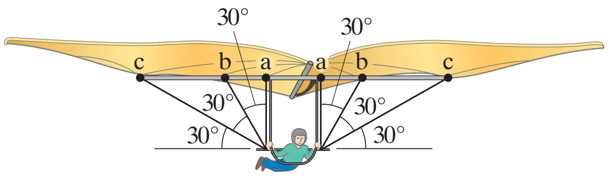


FIGURE 4–59 Problem 71.

72. A wet bar of soap $(m = 150\text{ g})$ slides freely down a ramp 3.0 m long inclined at 8.5°. How long does it take to reach the bottom? How would this change if the soap's mass were 250 g?

73. A parent pulls his child in a wagon across a floor. The child and wagon have a combined mass of 35 kg. The wagon handle is inclined at 55° to the horizontal, and the child and wagon (whose wheels are nearly frictionless) are accelerating at 0.42 m/s^2. With what force is the parent pulling on the wagon handle?

74. Two equal masses in contact on a frictionless surface are acted on by the forces F_1 and F_2 as shown in Fig. 4–60. Determine the magnitude of the contact force exerted on each mass by the other when (*a*) $F_2 = F_1$; (*b*) $F_2 = \frac{1}{2}F_1$; (*c*) $F_2 = 0$. Express your answer in terms of m and F_1.

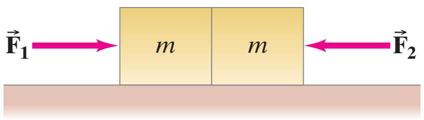


FIGURE 4–60 Problem 74.

75. Andrea dangles her watch from a thin piece of string while the jetliner she is in accelerates for takeoff, which takes about 21 s. Estimate the takeoff speed of the aircraft if the string makes an angle of 24° with respect to the vertical, Fig. 4–61.

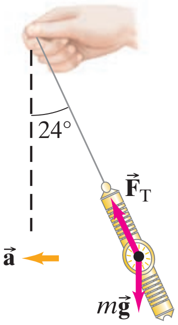


FIGURE 4–61 Problem 75.

76. A block (mass m_A) lying on a fixed frictionless inclined plane is connected to a mass m_B by a cord passing over a pulley, as shown in Fig. 4–62. (*a*) Determine a formula for the acceleration of the system in terms of m_A, m_B, θ, and g. (*b*) What conditions apply to masses m_A and m_B for the acceleration to be in one direction (say, m_A down the plane), or in the opposite direction? Ignore the mass of the cord and pulley.

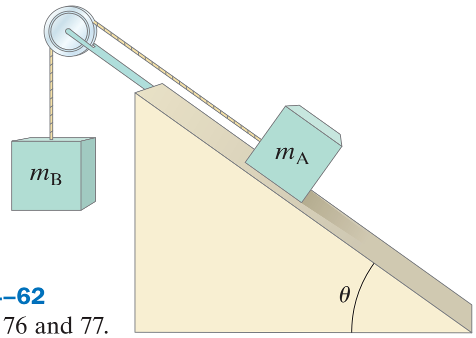


FIGURE 4–62 Problems 76 and 77.

77. (*a*) In Fig. 4–62, if $m_A = m_B = 1.00$ kg and $\theta = 38.0°$, what will be the acceleration of the system? (*b*) If $m_A = 1.00$ kg and the system remains at rest, what must the mass m_B be? (*c*) Calculate the tension in the cord for (*a*) and (*b*).

78. The masses m_A and m_B slide on the smooth (frictionless) inclines fixed as shown in Fig. 4–63. (*a*) Determine a formula for the acceleration of the system in terms of m_A, m_B, θ_A, θ_B, and g. (*b*) If $\theta_A = 34°$, $\theta_B = 21°$, and $m_A = 5.0$ kg, what value of m_B would keep the system at rest? What would be the tension in the cord (negligible mass) in this case? (*c*) What ratio, m_A/m_B, would allow the masses to move at constant speed along their ramps in either direction?

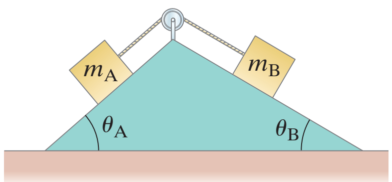


FIGURE 4–63 Problem 78.

79. (*a*) What minimum force F is needed to lift a piano (mass M) using the pulley apparatus shown in Fig. 4–64? (*b*) Determine the tension in each section of rope: F_{T1}, F_{T2}, F_{T3}, and F_{T4}. Assume pulleys are massless and frictionless, and that ropes are massless.

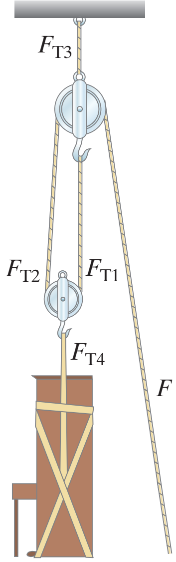


FIGURE 4–64 Problem 79.

80. In the *design of a supermarket*, there are to be several ramps connecting different parts of the store. Customers will have to push grocery carts up the ramps and it is desirable that this not be too difficult. The engineer has done a survey and found that almost no one complains if the force required is no more than 23 N. Ignoring friction, at what maximum angle θ should the ramps be built, assuming a full 25-kg cart?

81. A jet aircraft is accelerating at 3.8 m/s^2 as it climbs at an angle of 18° above the horizontal (Fig. 4–65). What is the total force that the cockpit seat exerts on the 75-kg pilot?

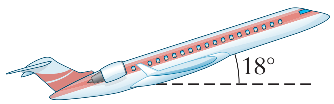


FIGURE 4–65 Problem 81.

82. A 6750-kg helicopter accelerates upward at 0.80 m/s^2 while lifting a 1080-kg frame at a construction site, Fig. 4–66. (*a*) What is the lift force exerted by the air on the helicopter rotors? (*b*) What is the tension in the cable (ignore its mass) which connects the frame to the helicopter? (*c*) What force does the cable exert on the helicopter?

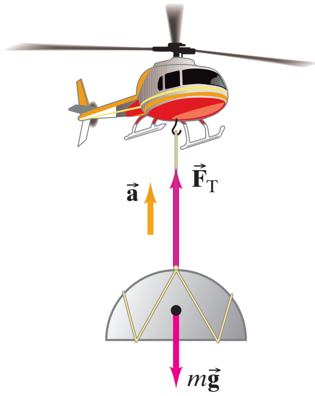


FIGURE 4–66 Problem 82.

83. An elevator in a tall building is allowed to reach a maximum speed of 3.5 m/s going down. What must the tension be in the cable to stop this elevator over a distance of 2.6 m if the elevator has a mass of 1650 kg including occupants?

84. A fisherman in a boat is using a "10-lb test" fishing line. This means that the line can exert a force of 45 N without breaking $(1\text{ lb} = 4.45\text{ N})$. (*a*) How heavy a fish can the fisherman land if he pulls the fish up vertically at constant speed? (*b*) If he accelerates the fish upward at 2.0 m/s^2, what maximum weight fish can he land? (*c*) Is it possible to land a 15-lb trout on 10-lb test line? Why or why not?

85. A "doomsday" *asteroid* with a mass of 2.0×10^{10} kg is hurtling through space. Unless the asteroid's speed is changed by about 0.20 cm/s, it will collide with Earth and cause tremendous damage. Researchers suggest that a small "space tug" sent to the asteroid's surface could exert a gentle constant force of 2.5 N. For how long must this force act?

86. Three mountain climbers who are roped together in a line are ascending an icefield inclined at 29° to the horizontal (Fig. 4–67). The last climber slips, pulling the second climber off his feet. The first climber is able to hold them both. If each climber has a mass of 75 kg, calculate the tension in each of the two sections of rope between the three climbers. Ignore friction between the ice and the fallen climbers.

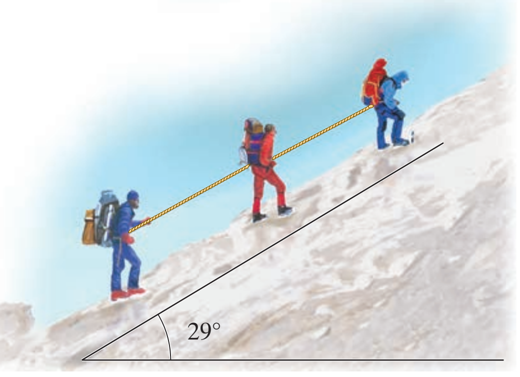


FIGURE 4–67 Problem 86.

87. A *bicyclist* can coast down a 5.0° hill at a constant speed of 6.0 km/h. If the force of air resistance is proportional to the speed v so that $F_{\text{air}} = cv$, calculate (*a*) the value of the constant c, and (*b*) the average force that must be applied (by the rider) in order to descend the hill at 18.0 km/h. The mass of the cyclist plus bicycle is 72.0 kg.

88. Consider the system shown in Fig. 4–68 with $m_A = 8.2$ kg and $m_B = 11.5$ kg. The angles $\theta_A = 59°$ and $\theta_B = 32°$. (*a*) In the absence of friction, what force $\vec{\mathbf{F}}$ would be required to pull the masses at a constant velocity up the fixed inclines? (*b*) The force $\vec{\mathbf{F}}$ is now removed. What are the magnitude and direction of the acceleration of the two blocks? (*c*) In the absence of $\vec{\mathbf{F}}$, what is the tension in the string?

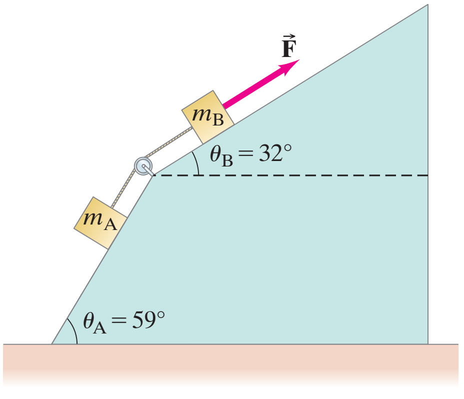


FIGURE 4–68 Problem 88.

89. A car starts rolling down a 1-in-4 hill (1-in-4 means that for each 4 m traveled along the sloping road, the elevation change is 1 m). How fast is it going when it reaches the bottom after traveling 55 m? Ignore friction.

90. An 18-kg child is riding in a child-restraint chair, securely fastened to the seat of a car (Fig. 4–69). Assume the car has speed 45 km/h when it hits a tree and is brought to rest in 0.20 s. Assuming constant deceleration during the collision, estimate the net horizontal force F that the straps of the restraint chair exert on the child to hold her in the chair.

FIGURE 4–69 Problem 90.

91. A 1.5-kg block rests on top of a 7.5-kg block (Fig. 4–70). The cord and pulley have negligible mass, and there is no significant friction anywhere. (*a*) What force F must be applied to the bottom block so the top block accelerates to the right at 2.2 m/s^2? (*b*) What is the tension in the connecting cord?

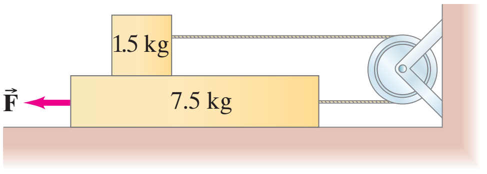


FIGURE 4–70 Problem 91.

92. You are driving home in your 860-kg car at 15 m/s. At a point 45 m from the beginning of an intersection, you see a green traffic light change to yellow, which you expect will last 4.0 s, and the distance to the far side of the intersection is 65 m (Fig. 4–71). (*a*) If you choose to accelerate, your car's engine will furnish a forward force of 1200 N. Will you make it completely through the intersection before the light turns red? (*b*) If you decide to panic stop, your brakes will provide a force of 2300 N. Will you stop before entering the intersection? Assume your car is 4 m long.

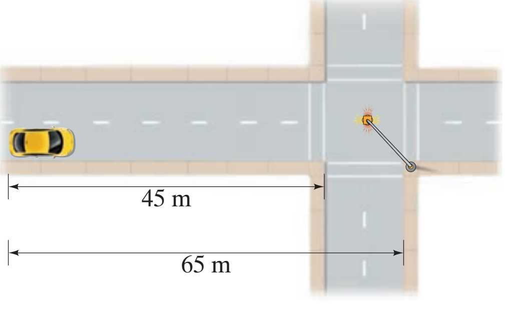


FIGURE 4–71 Problem 92.

93. A person jumps from a height of 5.0 m into a swimming pool entering feet first with her hands at her sides. She maintains this position upon entering the water and experiences a constant upward force equal to 350% of her weight due to the water itself. How far down does she go?

94. Two rock climbers, Paul and Jeanne, use safety ropes of similar length. Jeanne's rope is more elastic, called a *dynamic rope* by climbers. Paul has a *static rope*, not recommended for safety reasons. (*a*) Jeanne (Fig. 4–72) falls freely about 2.0 m and then the rope stops her over a distance of 1.0 m. Estimate how large a force (assume constant) she will feel from the rope. (Express the result in multiples of her weight.) (*b*) In a similar fall, Paul's rope stretches by only 30 cm. How many times his weight will the rope pull on him? Which climber is more likely to be hurt?

FIGURE 4–72 Problem 94.

95. (*a*) Finding her car stuck in the mud, a bright graduate of a good physics course ties a strong rope to the back bumper of the car, and the other end to a boulder, as shown in Fig. 4–73a. She pushes at the midpoint of the rope with her maximum effort, which she estimates to be a force $F_P \approx 300$ N. The car just begins to budge with the rope at an angle θ, which she estimates to be 5°. With what force is the rope pulling on the car? Neglect the mass of the rope. (*b*) What is the "mechanical advantage" of this technique (see Example 4–14)? (*c*) At what angle θ would this technique become counterproductive? [*Hint*: Consider the forces on a small segment of rope where $\vec{\mathbf{F}}_P$ acts, Fig. 4–73b.]

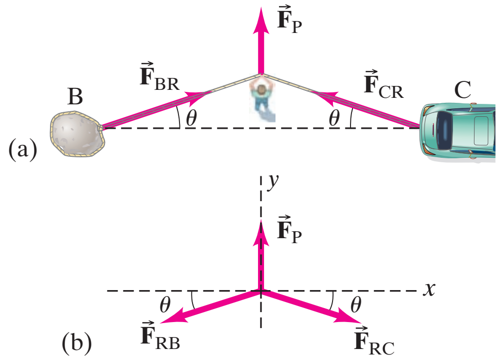


FIGURE 4–73 Problem 95. (a) Getting a car out of the mud, showing the forces on the boulder, on the car, and exerted by the person. (b) The free-body diagram: forces on a small segment of rope.

ANSWERS TO EXERCISES

A: No force is needed. The car accelerates out from under the cup, which tends to remain at rest as seen from the reference frame of the street. Think of Newton's first law (see Example 4–1).

B: (i) The same; (ii) the tennis ball; (iii) Newton's third law for part (i), second law for part (ii).

C: (*b*).

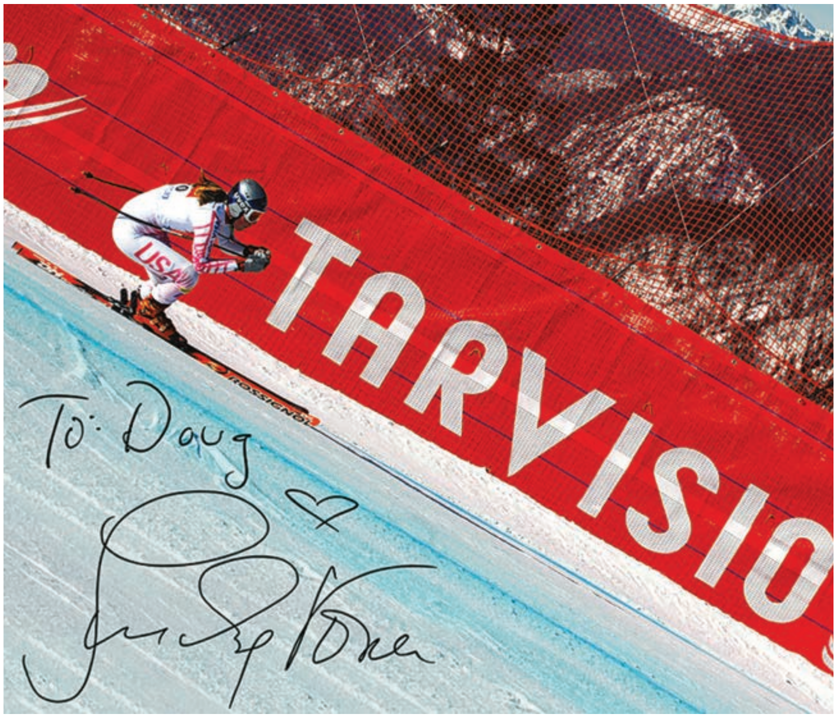
D: (*a*).

E: (*b*).

F: 58.0 N.

G: (*b*).

H: Yes; no.



Newton's laws are fundamental in physics. These photos show two situations of using Newton's laws which involve some new elements in addition to those discussed in the previous Chapter. The downhill skier illustrates *friction* on an incline; she is also retarded by air resistance, a velocity-dependent force.

The people on the rotating amusement park ride below illustrate the dynamics of circular motion.

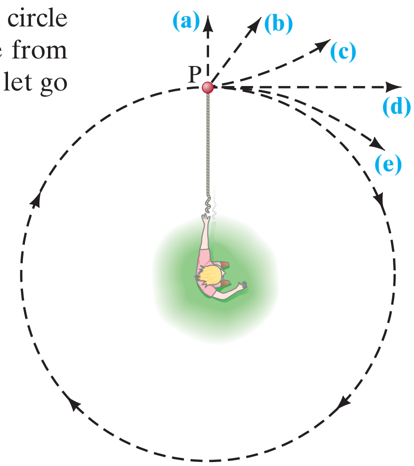
CHAPTER 5

Using Newton's Laws: Friction, Circular Motion, Drag Forces

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

You revolve a ball around you in a horizontal circle at constant speed on a string, as shown here from above. Which path will the ball follow if you let go of the string when the ball is at point P?



This chapter continues our study of Newton's laws and emphasizes their fundamental importance in physics. We will see how to apply Newton's laws to understand important situations including friction, and in circular motion, as well as with drag forces which are velocity-dependent (an optional-advanced Section).

5–1 Using Newton's Laws with Friction

Until now we have mostly ignored friction, but it must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 5–1. When we try to slide an object across a surface, these microscopic bumps impede the motion. Exactly what is happening at the microscopic level is not fully understood. One possibility is that the atoms on a bump of one surface may come so close to the atoms of the other surface that the atoms form a sort of "bond" or brief tiny weld between the two surfaces. Sliding an object across a surface is often jerky, perhaps due to the making and breaking of these bonds.

Even when a round object rolls across a surface, there is still some friction, called *rolling friction*, although it is generally much less than when an object slides across a surface.

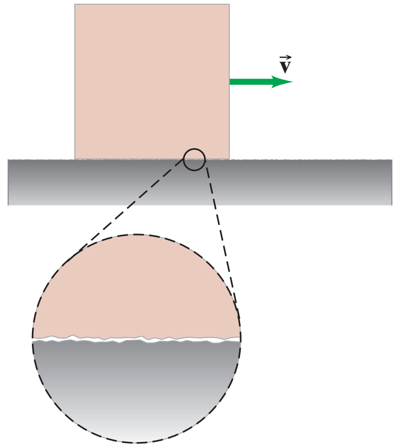
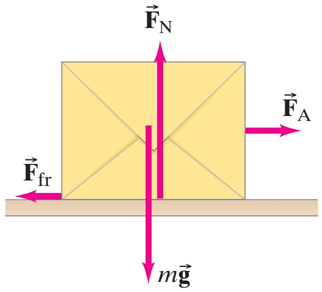


FIGURE 5–1 An object moving to the right on a table or floor. The two surfaces in contact are rough, at least on a microscopic scale.

We focus our attention now on sliding friction, which is usually called **kinetic friction** (*kinetic* is from the Greek for "moving"). When an object slides along a rough surface, the force of kinetic friction acts opposite to the direction of the object's velocity. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, experiments show that the friction force is approximately proportional to the *normal force* between the two surfaces, which is the force that either object exerts on the other and is perpendicular to their common surface of contact (see Fig. 5–2). The force of friction in many cases depends very little on the total surface area of contact; that is, the friction force on a book is roughly the same whether it is being slid across a table on its wide face or on its spine, assuming the surfaces have similar smoothness. We consider a simple model of friction in which we make this assumption that the friction force is independent of area. Then we write the proportionality between the magnitudes of the friction force F_{fr} and the normal force F_{N} as an equation by inserting a constant of proportionality, μ_{k}:

$$F_{\text{fr}} = \mu_{\text{k}} F_{\text{N}}. \qquad \text{[kinetic friction]}$$

FIGURE 5–2 When an object is pulled along a surface by an applied force $(\vec{\mathbf{F}}_{\text{A}})$, the force of friction $\vec{\mathbf{F}}_{\text{fr}}$ opposes the motion. The magnitude of $\vec{\mathbf{F}}_{\text{fr}}$ is proportional to the magnitude of the normal force (F_{N}).



This relation is not a fundamental law. It is an experimental relation between the magnitude of the friction force F_{fr}, which acts parallel to the two surfaces, and the magnitude of the normal force F_{N}, which acts perpendicular to the surfaces. It is *not* a vector equation since the two forces have different directions, perpendicular to one another. The term μ_{k} is called the *coefficient of kinetic friction*, and its value depends on the roughness of the two surfaces. Measured values for a variety of surfaces are given in Table 5–1. These are only approximate, however, since μ depends on whether the surfaces are wet or dry, on how much they have been sanded or rubbed, if any burrs remain, and other such factors. But μ_{k} (which has no units) is roughly independent of the sliding speed, as well as the area in contact.

TABLE 5–1 Coefficients of Friction†

Surfaces	Coefficient of Kinetic Friction, μ_{k}	Coefficient of Static Friction, μ_{s}
Wood on wood	0.2	0.4
Ice on ice	0.03	$\lesssim 0.1$
Metal on metal (lubricated)	0.07	0.1
Steel on steel (unlubricated)	0.6	0.7
Rubber on dry concrete	0.8	0.8
Rubber on wet concrete	0.5	0.7
Rubber on other solid surfaces	1	1–4
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	<0.01	<0.01

†Values are approximate and intended only as a guide.

What we have been discussing up to now is *kinetic friction*, when one object slides over another. There is also **static friction**, which refers to a force parallel to the two surfaces that can arise even when they are not moving. Suppose an object such as a desk is resting on a horizontal floor. If no horizontal force is exerted on the desk, there also is no friction force. But now suppose you try to push the desk, and it doesn't move. You are exerting a horizontal force, but the desk isn't moving, so there must be another force on the desk, in the opposite direction, keeping it from moving (the net force is zero on an object at rest). This is the force of *static friction* exerted by the floor on the desk. If you push with a greater force without moving the desk, the force of static friction in the opposite direction also has increased. If you push hard enough, the desk will eventually start to move, and kinetic friction takes over. At this point, you have exceeded the maximum force of static friction, which is given by $(F_{fr})_{max} = \mu_s F_N$, where μ_s is the *coefficient of static friction* (Table 5–1). Because the force of static friction can vary from zero to this maximum value, we write

$$F_{fr} \leq \mu_s F_N. \qquad \text{[static friction]}$$

You may have noticed that it is often easier to keep a heavy object sliding than it is to start it sliding in the first place. This is consistent with μ_s generally being greater than μ_k (see Table 5–1).

It seems that this simple relation for friction, $F_{fr} \leq \mu F_N$, was first established by the great "Renaissance man" Leonardo da Vinci (1452–1519).

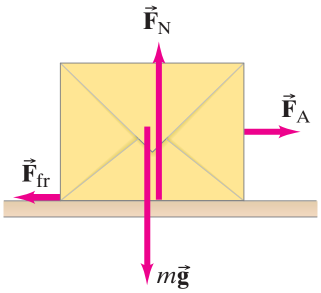


FIGURE 5–2 Repeated for Example 5-1.

EXAMPLE 5–1 **Friction: static and kinetic.** A 10.0-kg box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal external applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

APPROACH We don't know, right off, if we are dealing with static friction or kinetic friction, nor if the box remains at rest or accelerates. We need to draw a free-body diagram, and then determine in each case whether or not the box will move: the box starts moving if F_A is greater than the maximum static friction force. The forces on the box are shown in Fig. 5–2: gravity $m\vec{g}$; the normal force exerted upward by the floor $\vec{F}_N$; the horizontal applied force $\vec{F}_A$; and the friction force $\vec{F}_{fr}$.

SOLUTION The free-body diagram of the box is shown in Fig. 5–2. In the vertical direction there is no motion, so Newton's second law in the vertical direction gives $\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0$. Hence the normal force is

$$F_N = mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}.$$

(*a*) Because $F_A = 0$ in this first case, the box doesn't move, and $F_{fr} = 0$.

(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98.0\,\text{N}) = 39\,\text{N}.$$

When the applied force is $F_A = 10\,\text{N}$, the box will not move. Newton's second law gives $\Sigma F_x = F_A - F_{fr} = 0$, so $F_{fr} = 10\,\text{N}$.

(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{fr} = 20\,\text{N}$ to balance the applied force.

(*d*) The applied force of 38 N is still not quite large enough to move the box. So the friction force has now increased to 38 N to keep the box at rest.

(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

$$F_{fr} = \mu_k F_N = (0.30)(98.0\,\text{N}) = 29\,\text{N}.$$

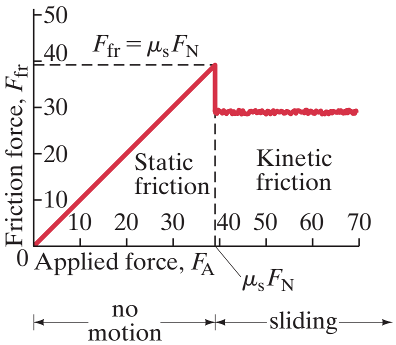
There is now a net (horizontal) force on the box of magnitude $F = 40\,\text{N} - 29\,\text{N} = 11\,\text{N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2$$

as long as the applied force is 40 N.

NOTE Figure 5–3 shows a graph that summarizes this Example.

FIGURE 5–3 Example 5–1. Magnitude of the force of friction as a function of the external force applied to an object initially at rest. As the applied force is increased in magnitude, the force of static friction increases in proportion until the applied force equals $\mu_s F_N$. If the applied force increases further, the object will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.



Friction can be a hindrance. It slows down moving objects and causes heating and binding of moving parts in machinery. Friction can be reduced by using lubricants such as oil. More effective in reducing friction between two surfaces is to maintain a layer of air or other gas between them. Devices using this concept, which is not practical for most situations, include air tracks and air tables in which the layer of air is maintained by forcing air through many tiny holes. Another technique to maintain the air layer is to suspend objects in air using magnetic fields ("magnetic levitation").

On the other hand, friction can be helpful. Our ability to walk (see Fig. 4–11) depends on friction between the soles of our shoes (or feet) and the ground. Walking involves static friction, not kinetic friction. The movement of a car, and also its stability, depend on friction. When friction is low, such as on ice, safe walking or driving becomes difficult.

CONCEPTUAL EXAMPLE 5–2 **A box against a wall.** You can hold a box against a rough wall (Fig. 5–4) and prevent it from slipping down by pressing hard horizontally. How does the application of a horizontal force keep an object from moving vertically?

RESPONSE This won't work well if the wall is slippery. You need friction. Even then, if you don't press hard enough, the box will slip. The horizontal force you apply produces a normal force on the box exerted by the wall (net force horizontally is zero since box doesn't move horizontally.) The force of gravity mg, acting downward on the box, can now be balanced by an upward static friction force whose maximum magnitude is proportional to the normal force. The harder you push, the greater F_N is and the greater F_{fr} can be. If you don't press hard enough, then $mg > \mu_s F_N$ and the box begins to slide down.

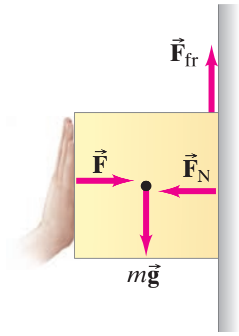


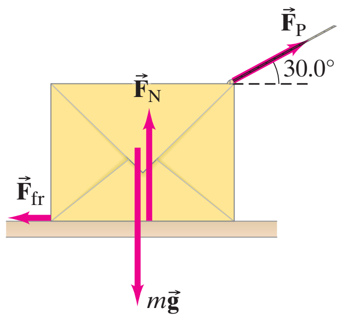
FIGURE 5–4 Example 5–2.

EXERCISE A If $\mu_s = 0.40$ and $mg = 20$ N, what minimum force F will keep the box from falling: (*a*) 100 N; (*b*) 80 N; (*c*) 50 N; (*d*) 20 N; (*e*) 8 N?

EXAMPLE 5–3 **Pulling against friction.** A 10.0-kg box is being pulled along a horizontal surface by a force F_P of 40.0 N applied at a 30.0° angle above horizontal. This is like Example 4–11 except now there is friction, and we assume a coefficient of kinetic friction of 0.30. Calculate the acceleration.

APPROACH The free-body diagram is shown in Fig. 5–5. It is much like that in Fig. 4–21, but with one more force, that of friction.

FIGURE 5–5 Example 5–3.



SOLUTION The calculation for the vertical (y) direction is just the same as in Example 4–11, $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$ and $F_{Py} = (40.0\,\text{N})(\sin 30.0°) = 20.0\,\text{N}$. With y positive upward and $a_y = 0$, we have

$$\begin{aligned} F_N - mg + F_{Py} &= ma_y \\ F_N - 98.0\,\text{N} + 20.0\,\text{N} &= 0, \end{aligned}$$

so the normal force is $F_N = 78.0\,\text{N}$. Now we apply Newton's second law for the horizontal (x) direction (positive to the right), and include the friction force:

$$F_{Px} - F_{fr} = ma_x.$$

The friction force is kinetic as long as $F_{fr} = \mu_k F_N$ is less than $F_{Px} = (40.0\,\text{N})\cos 30.0° = 34.6\,\text{N}$, which it is:

$$F_{fr} = \mu_k F_N = (0.30)(78.0\,\text{N}) = 23.4\,\text{N}.$$

Hence the box does accelerate:

$$a_x = \frac{F_{Px} - F_{fr}}{m} = \frac{34.6\,\text{N} - 23.4\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2.$$

In the absence of friction, as we saw in Example 4–11, the acceleration would be much greater than this.

NOTE Our final answer has only two significant figures because our least significant input value $(\mu_k = 0.30)$ has two.

EXERCISE B If $\mu_k F_N$ were greater than F_{Px}, what would you conclude?

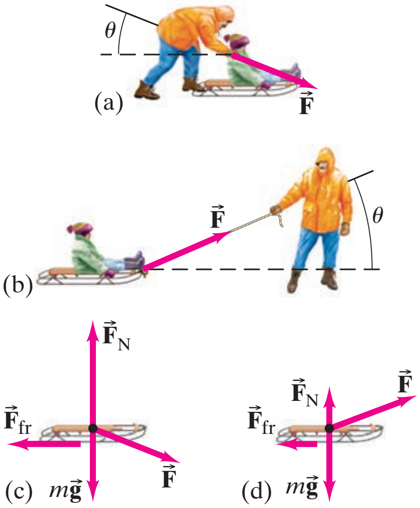
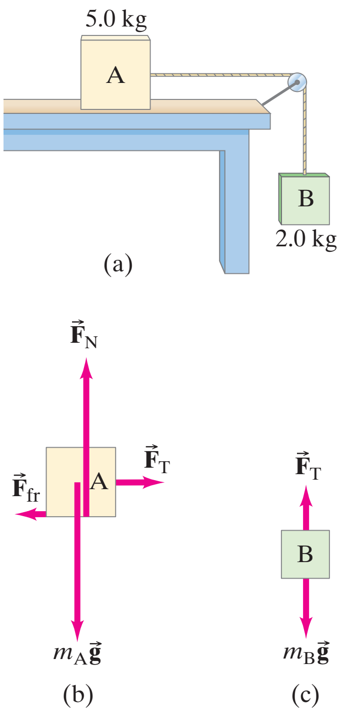


FIGURE 5–6 Example 5–4.

FIGURE 5–7 Example 5–5.



CONCEPTUAL EXAMPLE 5–4 **To push or to pull a sled?** Your little sister wants a ride on her sled. If you are on flat snow, will you exert less force if you push her or pull her? See Figs. 5–6a and b. Assume the same angle θ in each case.

RESPONSE Let us draw free-body diagrams for the sled–sister combination, as shown in Figs. 5–6c and d. They show, for the two cases: the force exerted by you, $\vec{F}$ (an unknown); by the snow, $\vec{F}_N$ and $\vec{F}_{fr}$; and gravity $m\vec{g}$. (*a*) If you push her, and $\theta > 0$, there is a vertically downward component to your force. Hence the upward normal force exerted by the ground (Fig. 5–6c) will be larger than mg (where m is the mass of sister plus sled). (*b*) If you pull her, your force has a vertically upward component, so the normal force F_N will be less than mg, Fig. 5–6d. Because the friction force is proportional to the normal force, F_{fr} will be less if you pull her. So you can exert less force if you pull her.

EXAMPLE 5–5 **Two boxes and a pulley.** In Fig. 5–7a, two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A and the table is 0.20. We ignore the mass of the cord and pulley and any friction in the pulley, which means we can assume that a force applied to one end of the cord will have the same magnitude at the other end. We wish to find the acceleration of the system, a, which will have the same magnitude for both boxes assuming the cord doesn't stretch. As box B moves down, box A moves to the right.

APPROACH The free-body diagrams for each box are shown in Figs. 5–7b and c. The forces on box A are the pulling force of the cord F_T, gravity $m_A g$, the normal force exerted by the table F_N, and a friction force exerted by the table F_{fr}. The forces on box B are gravity $m_B g$, and the cord pulling up, F_T.

SOLUTION Box A does not move vertically, so Newton's second law tells us the normal force just balances the weight,

$$F_N = m_A g = (5.0\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 49\,\text{N}.$$

In the horizontal direction, there are two forces on box A (Fig. 5–7b): F_T, the tension in the cord (whose value we don't know), and the force of friction

$$F_{fr} = \mu_k F_N = (0.20)(49\,\text{N}) = 9.8\,\text{N}.$$

The horizontal acceleration of box A is what we wish to find; we use Newton's second law in the x direction, $\Sigma F_{Ax} = m_A a_x$, which becomes (taking the positive direction to the right and setting $a_{Ax} = a$):

$$\Sigma F_{Ax} = F_T - F_{fr} = m_A a. \qquad \text{[box A]}$$

Next consider box B. The force of gravity $m_B g = (2.0\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 19.6\,\text{N}$ pulls downward; and the cord pulls upward with a force F_T. So we can write Newton's second law for box B (taking the downward direction as positive):

$$\Sigma F_{By} = m_B g - F_T = m_B a. \qquad \text{[box B]}$$

[Notice that if $a \neq 0$, then F_T is not equal to $m_B g$.]

We have two unknowns, a and F_T, and we also have two equations. We solve the box A equation for F_T:

$$F_T = F_{fr} + m_A a,$$

and substitute this into the box B equation:

$$m_B g - F_{fr} - m_A a = m_B a.$$

Now we solve for a and put in numerical values:

$$a = \frac{m_B g - F_{fr}}{m_A + m_B} = \frac{19.6\,\text{N} - 9.8\,\text{N}}{5.0\,\text{kg} + 2.0\,\text{kg}} = 1.4\,\text{m/s}^2,$$

which is the acceleration of box A to the right, and of box B down.

If we wish, we can calculate F_T using the third equation up from here:

$$F_T = F_{fr} + m_A a = 9.8\,\text{N} + (5.0\,\text{kg})\left(1.4\,\text{m/s}^2\right) = 17\,\text{N}.$$

NOTE Box B is not in free fall. It does not fall at $a = g$ because an additional force, F_T, is acting upward on it.

In Chapter 4 we examined motion on ramps and inclines, and saw that it is usually an advantage to choose the x axis along the plane, in the direction of acceleration. There we ignored friction, but now we take it into account.

EXAMPLE 5–6 **The skier.** The skier in Fig. 5–8a descends a 30° slope. If the coefficient of kinetic friction is 0.10, what is her acceleration when she is in contact with the snow?

APPROACH We choose the x axis along the slope, positive downslope in the direction of the skier's motion. The y axis is perpendicular to the surface. The forces acting on the skier (Fig. 5–8b) are gravity, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$, which points vertically downward (*not* perpendicular to the slope), and the two forces exerted on her skis by the snow—the normal force perpendicular to the snowy slope (*not* vertical), and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 5–8b, which is our free-body diagram for the skier.

SOLUTION We have to resolve only one vector into components, the weight $\vec{\mathbf{F}}_G$, and its components are shown as dashed lines in Fig. 5–8c. To be general, we use θ rather than 30° for now. We use the definitions of sine ("side opposite") and cosine ("side adjacent") to obtain the components:

$$F_{Gx} = mg\sin\theta,$$
$$F_{Gy} = -mg\cos\theta$$

where F_{Gy} is in the negative y direction. To calculate the skier's acceleration down the hill, a_x, we apply Newton's second law to the x direction:

$$\Sigma F_x = ma_x$$
$$mg\sin\theta - \mu_k F_N = ma_x$$

where the two forces are the x component of the gravity force ($+x$ direction) and the friction force ($-x$ direction). We want to find the value of a_x, but we don't yet know F_N in the last equation. Let's see if we can get F_N from the y component of Newton's second law:

$$\Sigma F_y = ma_y$$
$$F_N - mg\cos\theta = ma_y = 0$$

where we set $a_y = 0$ because there is no motion in the y direction (perpendicular to the slope) when the skier is in contact with the slope. Thus we can solve for F_N:

$$F_N = mg\cos\theta$$

and we can substitute this into our equation above for ma_x:

$$mg\sin\theta - \mu_k(mg\cos\theta) = ma_x.$$

There is an m in each term which can be canceled out. Thus (setting $\theta = 30°$ and $\mu_k = 0.10$):

$$a_x = g\sin 30° - \mu_k g\cos 30°$$
$$= 0.50g - (0.10)(0.866)g = 0.41g.$$

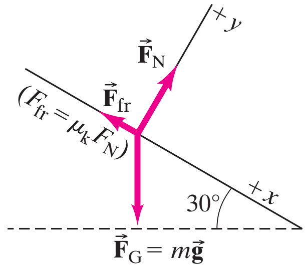
The skier's acceleration is 0.41 times the acceleration of gravity, which in numbers[†] is $a = (0.41)(9.8\,\text{m/s}^2) = 4.0\,\text{m/s}^2$.

NOTE The mass canceled out, so we have the useful conclusion that *the acceleration doesn't depend on the mass*. That such a cancellation sometimes occurs, and thus may give a useful conclusion as well as saving calculation, is a big advantage of working with the algebraic equations and putting in the numbers only at the end.

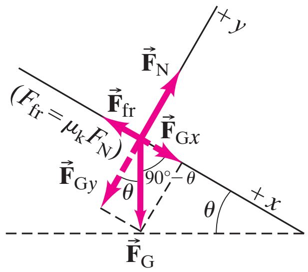
NOTE The friction force on high-speed alpine skiers is very small—it seems the heat produced under the skis by friction melts the snow so the skis float on tiny balls of water. Fast skiers are slowed more by air resistance, which they can reduce by going into a tuck to reduce their surface area. When going airborne (as over the top of a hill) they are slowed more (can't hold that tuck, especially when landing) than when in contact with the snow.

PHYSICS APPLIED
Skiing

(a)



(b)



(c)

FIGURE 5–8 Example 5–6. A skier descending a slope; $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$ is the force of gravity (weight) on the skier.

PROBLEM SOLVING
It is often helpful to put in numbers only at the end

PHYSICS APPLIED
Skiers are faster on snow than when airborne

[†]We used values rounded off to 2 significant figures to obtain $a = 4.0\,\text{m/s}^2$. If we kept all the extra digits in our calculator, we would find $a = 0.4134\,g \approx 4.1\,\text{m/s}^2$. This difference is within the expected precision (number of significant figures, Section 1–3).

CAUTION
Directions of gravity and the normal force

In problems involving a slope or "inclined plane," avoid making errors in the directions of the normal force and gravity. The normal force is *not* vertical: it is perpendicular to the slope or plane. And gravity is *not* perpendicular to the slope—gravity acts vertically downward toward the center of the Earth.

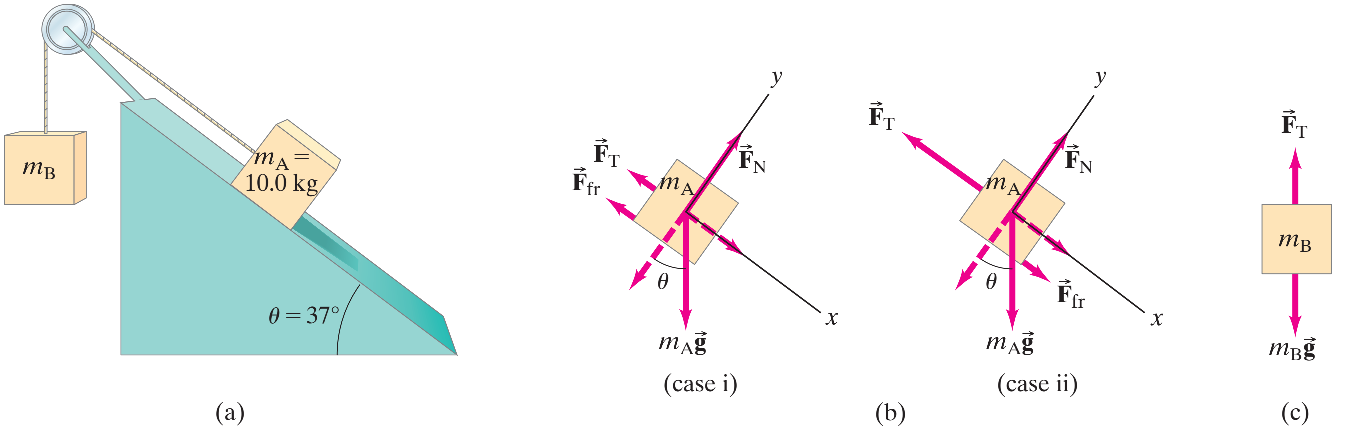


FIGURE 5–9 Example 5–7. Note choice of x and y axes.

EXAMPLE 5–7 A ramp, a pulley, and two boxes. A box of mass $m_A = 10.0\,\text{kg}$ rests on a surface inclined at $\theta = 37°$ to the horizontal. It is connected by a lightweight cord, which passes over a massless and frictionless pulley, to a second box of mass m_B, which hangs freely as shown in Fig. 5–9a. (*a*) If the coefficient of static friction is $\mu_s = 0.40$, determine what range of values for mass m_B will keep the system at rest. (*b*) If the coefficient of kinetic friction is $\mu_k = 0.30$, and $m_B = 10.0\,\text{kg}$, determine the acceleration of the system.

APPROACH Figure 5–9b shows two free-body diagrams for box m_A because the force of friction can be either up or down the slope, depending on which direction the box slides: (i) if $m_B = 0$ or is sufficiently small, m_A would tend to slide down the incline, so $\vec{\mathbf{F}}_{fr}$ would be directed up the incline; (ii) if m_B is large enough, m_A will tend to be pulled up the plane, so $\vec{\mathbf{F}}_{fr}$ would point down the plane. The tension force exerted by the cord is labeled $\vec{\mathbf{F}}_T$.

SOLUTION (*a*) For both cases (i) and (ii), Newton's second law for the y direction (perpendicular to the plane) is the same:

$$F_N - m_A g\cos\theta = m_A a_y = 0$$

since there is no y motion. So

$$F_N = m_A g\cos\theta.$$

Now for the x motion. We consider case (i) first for which $\Sigma F = ma$ gives

$$m_A g\sin\theta - F_T - F_{fr} = m_A a_x.$$

We want no acceleration, $a_x = 0$, and we solve for F_T since F_T is related to m_B (whose value we are seeking) by $F_T = m_B g$ (see Fig. 5–9c). Thus

$$m_A g\sin\theta - F_{fr} = F_T = m_B g.$$

We solve this for m_B and set F_{fr} at its maximum value $\mu_s F_N = \mu_s m_A g\cos\theta$ to find the minimum value that m_B can have to prevent motion $(a_x = 0)$:

$$\begin{aligned} m_B &= m_A\sin\theta - \mu_s m_A\cos\theta \\ &= (10.0\,\text{kg})(\sin 37° - 0.40\cos 37°) = 2.8\,\text{kg}. \end{aligned}$$

Thus if $m_B < 2.8\,\text{kg}$, then box A will slide down the incline.