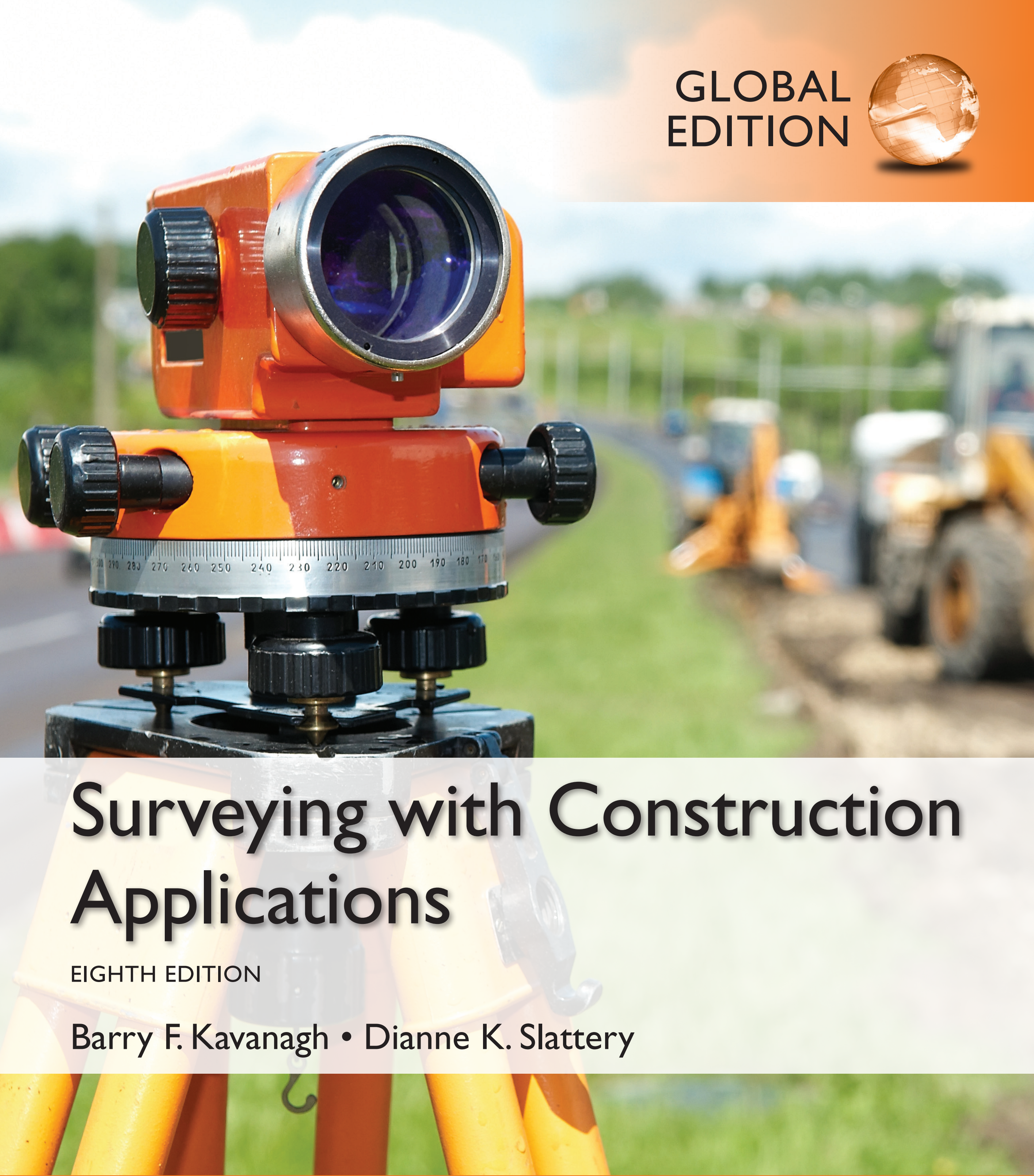

GLOBAL EDITION

Surveying with Construction Applications

EIGHTH EDITION

Barry F. Kavanagh • Dianne K. Slattery

ALWAYS LEARNING

PEARSON

EIGHTH EDITION

Surveying with Construction Applications

Global Edition

Barry F. Kavanagh, B.A., CET
Seneca College, Emeritus

Dianne K. Slattery, Ph.D., P.E.
Missouri State University

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CONTENTS

Field Note Index

PREFACE

Many technological advances have occurred in surveying since *Surveying with Construction Applications* was first published. This eighth edition is updated with the latest advances in instrumentation technology, field-data capture, and data-processing techniques. Although surveying is becoming much more efficient and automated, the need for a clear understanding of the principles underlying all forms of survey measurement remains unchanged.

NEW TO THIS EDITION

- General surveying principles and techniques, used in all branches of surveying, are presented in Part I, Chapters 1–11, while contemporary applications for the construction of most civil projects are covered in Chapters 12–19. With this organization, the text is useful not only for the student, but it can also be used as a handy reference for the graduate who may choose a career in civil/survey design or construction. The glossary has been expanded to include new terminology. Every effort has been made to remain on the leading edge of new developments in techniques and instrumentation, while maintaining complete coverage of traditional techniques and instrumentation.
- Chapter 2 is new, reflecting the need of modern high school graduates for the reinforcement of precalculus mathematics. In Chapter 2, students will have the opportunity to review techniques of units, conversions, areas, volumes, trigonometry, and geometry, which are all focused on the types of applications encountered in engineering and construction work.
- Chapter 3 follows with the fundamentals of distance measurement; Chapter 4 includes complete coverage of leveling practices and computations; and Chapter 5 presents an introduction to electronic distance measurement. Chapter 6 introduces the students to both theodolites and total stations, as well as common surveying practices with those instruments. Chapter 7 gives students a broad understanding of total station operations and applications. Chapter 8, "Traverse Surveys and Computations," introduces the students to the concepts of survey line directions in the form of bearings and azimuths; the analysis of closed surveys precision is accomplished using the techniques of latitudes and departures, which allow for precision determination and error balancing so that survey point coordinates can be determined and enclosed areas determined. Modern total stations (Chapter 7) have been programmed to accomplish all of the aforementioned activities, but it is here in Chapter 8 that students learn about the theories underlying total station applications.
- Chapter 9 covers satellite positioning, the modern technique of determining position. This chapter concentrates on America's Global Positioning System, but includes descriptions of the other systems now operating fully or partially around the Earth in Russia, China, Europe, Japan, and India. All these systems combined are known as

the Global Navigation Satellite System (GNSS). Chapter 10, "Geomatics," reflects the advances modern technology has made in the capture of positioning data on Earth-surface features, the processing of measurement technology, and the depiction of the surface features in the form of maps, plans, screen images, aerial photogrammetric images, and digital imaging taken from satellites and aircraft. Chapter 11 covers horizontal and vertical control, both at the national level and at the project level.

- Part II includes specific applications in engineering construction and begins with Chapter 12, an introduction to machine guidance and control. This new technology has recently made great advances in large-scale developments, such as highway and roads construction and airport construction. It involves creating three-dimensional data files for all existing ground surface features and all new-design surface features. Equipment operators (dozers, scrapers, loaders, and backhoes) can view the existing ground elevations, profiles, and cross-sections on in-cab computer monitors. They can also see the proposed elevations, and the like, for the project, and the current location of the cutting edge (blade, bucket, etc.) of their machine. Being able to see all of this from the cab, the operators don't need further help with line and grade directions.
- The remainder of Part II covers engineering projects: "Highway Curves" (Chapter 13), "Highway Construction Surveys" (Chapter 14), "Municipal Street Construction Surveys" (Chapter 15), "Pipeline and Tunnel Construction Surveys" (Chapter 16), "Culvert and Bridge Construction Surveys" (Chapter 17), and "Building Construction Surveys" (Chapter 18). Chapter 19, "Quantity and Final Surveys," introduces the student to the types of computations and records keeping that surveyors must do to provide data for the processing of interim and final payments to the contractors.
- To help streamline the text, some of the previous edition's appendices have been transferred to the Instructor's Manual (see below).
- Finally, this edition introduces coauthor Dianne K. Slattery, a professor in the Department of Technology and Construction Management at Missouri State University in Springfield, Missouri. Dr. Slattery has wide academic and practical experience in civil engineering and in engineering surveying, and has used previous editions of this text to teach undergraduate courses in Construction Surveying for more than 15 years.

SUPPLEMENTS

The available Instructor's Manual includes solutions for all end-of-chapter problems; a typical evaluation scheme; subject outlines (two terms or two-semester programs); term assignments, sample instruction class handouts for instrument use, and so on; and mid-term and final tests. Also included is a PowerPoint presentation that can be used as an aid in presenting text material and as a source for overhead transparencies. In addition, former text appendices are now also included in the Instructors Manual, including Steel Tape Corrections, Stadia Techniques and Calculations, Early Surveying, and Surveying and Mapping Web sites.

To access supplementary materials online, instructors need to request an instructor access code. Go to www.pearsonglobaleditions.com/kavanagh to register for an instructor access code. Within 48 hours of registering, you will receive a confirming e-mail including an instructor access code. Once you have received your code, locate your text in the online catalog and click on the Instructor Resources button on the left side of the catalog product page. Select a supplement, and a login page will appear. Once you have logged in, you can access instructor material for all Pearson textbooks. If you have any difficulties accessing the site or downloading a supplement, please contact Customer Service at http://247pearsoned.custhelp.com/.

Technology continues to expand; improvements to field equipment, data-processing techniques, and construction practices in general will inevitably continue. Surveyors must keep up with these dynamic events. We hope that students, by using this text, will be completely up to date in this subject area and will be readily able to cope with the technological changes that continue to occur. Comments and suggestions about the text are welcomed and can be e-mailed to us at *barry.kavanagh@cogeco.ca* and *DianneSlattery@Missouristate.edu.*

Barry F. Kavanagh

Dianne K. Slattery

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ABBREVIATIONS

GENERAL

AASHTO	American Association of State Highway and Transportation Officials
ACSM	American Congress on Surveying and Mapping
Az	azimuth
Bg	bearing
BM	benchmark
	(TBM) temporary benchmark
BS	backsight (rod reading in leveling; line sighting in theodolite work)
C_L	correction due to erroneous length of tape
C_P	correction due to nonstandard tension
C_S	correction due to effects of sag
C_T	corrections due to nonstandard temperature
CAD	computer assisted drafting (or design)
CADD	computer assisted drafting and design
cc	cut cross
CIG	Canadian Institute of Geomatics
conc.mon.	concrete monument
c + r	error in line of sight due to combined effects of curvature and refraction
Deg	degree
Dep	departure
DoD	Department of Defense
EDM	electronic distance measurement
EG	existing ground
Elev	elevation
FS	foresight (rod reading in leveling; line sighting in theodolite work)
GIS	geographic information system
GPS	global positioning system
HARN	high accuracy reference network
HI	height of instrument above a datum
hi	height of instrument above the instrument station
HOT	hub on tangent
IB	iron bar
Inst	instrument
IP	iron pipe
IS	intermediate sight, used in leveling and total station activities (also IFS, intermediate foresight)
Lat	latitude
Long	longitude
Lt	left
Mon	monument
NSPS	National Society of Professional Surveyors
Occ	occupied station reference
OG	original ground
o/s	offset
ppm	parts per million
RAP	reference azimuth point
ROW	right of way
RP	reference point
RR	rod reading
Rt	right
TBM	temporary benchmark
TP	turning point
Twp	township
UTM	universal transverse Mercator projection
X-sect	cross section

IMPERIAL UNITS

ac	acre
bbl	barrel
cu ft	cubic foot
cu in.	cubic inch
cu yd	cubic yard
cwt	hundred weight
fbm	foot board measure
ft	foot or feet
gal	gallon(s)
in.	inch(es)
lb	pound
lf	linear foot (feet)
mi	mile(s)
mph	miles per hour
psi	pounds per square inch
sq ft	square foot (feet)
sq in.	square inch(es)
sq yd	square yard(s)
mf bm	thousand foot board measure
m gal	thousand gallons
yd	yard(s)

METRIC UNITS

C	Celsius
cm	centimeter
ha	hectare
kg	kilogram(s)
km	kilometer(s)
kN	kilonewton(s)
kPa	kilopascal(s)
L	liter(s)
m	meter(s)
m^2	square meter
m^3	cubic meter
mm	millimeter(s)
t	tonne

SYMBOLS

BL	baseline
℄	centerline
SL	street line
ΔN	change in northing
ΔE	change in easting
Δλ″	change in longitude (seconds)
Δhi	difference in height between transit and EDM
ΔR	difference in height between reflector and target
ϕ, λ	latitude, longitude
$\bar{\Lambda}$	instrument
	occupied station (instrument)
	reference sighting station
※	point of intersection
=	is equal to
≠	is not equal to
>	is greater than
<	is less than
≈	is approximately equal to
Σ	the sum of

THE GREEK ALPHABET

Name	Uppercase	Lowercase
alpha	Α	α
beta	Β	β
gamma	Γ	γ
delta	Δ	δ
epsilon	Ε	ϵ
zeta	Ζ	ζ
eta	Η	η
theta	Θ	θ
iota	Ι	ι
kappa	Κ	κ
lambda	Λ	λ
mu	Μ	μ
nu	Ν	ν
xi	Ξ	ξ
omnicron	Ο	ο
pi	Π	π
rho	Ρ	ρ
sigma	Σ	σ
tau	Τ	τ
upsilon	Υ	υ
phi	Φ	ϕ
chi	Χ	χ
psi	Ψ	ψ
omega	Ω	ω

CONVERSIONS

LENGTH

1 ft = 0.3048 m exactly
1 in. = 2.54 cm = 25.4 mm
1 m = 10 decimeters = 100 cm = 1,000 mm
1 m = 39.37 in. = 3.2808 ft
1 mi = 5,280 ft = 1,609 m = 1.609 km
1 km = 1,000 m = 0.62137 mi.
1 nautical mi = 6,076.1 ft = 1852 m = 1.852 km
1 vara = about 33 in. in Mexico and California and 33$^1/_3$ in. in Texas
1 rod = 16.5 ft
1 chain = 66 ft = 4 rods
1 U.S. survey foot = 0.30480061 m (original ratio of 1,200/3,937)

AREA

1 acre = 43,560 sq. ft = 4,047 sq. m = 10 chains squared [i.e., 10(66 ft × 66 ft)]
1 ha (hectare) = 10,000 sq. m = 2.47 acres
1 sq. km = 247.1 acres
1 sq. ft = 0.09290 sq. m
1 sq. in. = 6.452 sq. cm

VOLUME

1 cu. m = 35.31 cu. ft
1 cu. yd = 27 cu. ft = 0.7646 cu. m
1 gal (U.S.) = 3.785 litres
1 gal (Imperial) = 4,546 litres
1 cu. ft = 7.481 gal. (U.S.) = 28.32 litres
1 liter = 0.001 cu. m

FORCE

1 lb weight = 16 oz. = 4.418 N (newtons) = 0.4536 kg weight
1 N = 100,000 dynes = 0.2248 lbs. weight = 0.1020 kg weight
1 kg weight = 9.807 N

PRESSURE

1 atmosphere = 760 mm Hg. = 14.7 lb/sq. in.
1 atmosphere = 101,300 N/sq. m (pascals) = 101 kilopascals
1 atmosphere = 1.013 bars = 760 torrs

ANGLES

1 revolution = 360 degrees
1 degree = 60 minutes
1 minute = 60 seconds
1 revolution = 400 grad, also known as grade and as gon
1 right angle = 90 degrees = 100.0000 grad (gon)
1 revolution = 2 pi radians
1 radian = 57.29578 degrees
1 degree = 0.017453 radians

PART

I

SURVEYING PRINCIPLES

Part I, which includes Chapters 1–11, introduces you to traditional and state-of-the-art techniques in data collection, layout, and presentation of field data. Chapter 1 covers surveying fundamentals. Elevation determination is covered in the chapters on leveling (Chapter 4), total stations (Chapter 7), and satellite positioning (Chapter 9). Distance measurements are covered, using both conventional taping techniques (Chapter 3), and electronic distance measurement (EDM) techniques (Chapter 5). Data presentation is covered in Chapters 7 and 10. Angle measurements and geometric analysis of field measurements are covered in Chapters 6–8. Horizontal positioning is covered in Chapters 9 and 10, and control for both data-gathering and layout surveys is covered in Chapter 11.

Although most distance measurements are now done with EDM techniques, many applications still exist for steel taping on the short-distance measurements often found in construction layouts. Techniques for taping corrections can be found in Chapter 3 and in the online Instructors Manual (see the Preface for access to the Instructors Manual).

CHAPTER ONE

SURVEYING FUNDAMENTALS

1.1 SURVEYING DEFINED

Surveying is the art and science of taking field measurements on or near the surface of the Earth. Survey field measurements include horizontal and slope distances, vertical distances, and horizontal and vertical angles. In addition to measuring distances and angles, surveyors can measure position as given by the northing, easting, and elevation of a survey station by using satellite-positioning and remote-sensing techniques. In addition to taking measurements in the field, the surveyor can derive related distances and directions through geometric and trigonometric analysis.

Once a survey station has been located by angle and distance, or by positioning techniques, the surveyor then attaches to that survey station (in handwritten or electronic field notes) a suitable identifier or attribute that describes the nature of the survey station. In Chapter 10, you will see that attribute data for a survey station can be expanded from a simple descriptive label to include a wide variety of related information that can be tagged specifically to that survey station.

Since the 1980s, the term **geomatics** has come into popular usage to describe the computerization and digitization of data collection, data processing, data analysis, and data output. Geomatics not only includes traditional surveying as its cornerstone but also reflects the now-broadened scope of measurement science and information technology. Figure 10.1 shows a digital surveying data model. This illustration gives you a sense of the diversity of the integrated scientific activities now covered by the term *geomatics*.

The vast majority of engineering and construction projects are so limited in geographic size that the surface of the Earth is considered to be a plane for all *X* (easterly) and *Y* (northerly) dimensions. *Z* dimension (height) is referred to a datum, usually mean sea level. Surveys that ignore the curvature of the Earth for horizontal dimensions are called **plane surveys**. Surveys that cover a large geographic area—for example, state or provincial boundary surveys—must have corrections made to the field measurements so that these measurements reflect the curved (ellipsoidal) shape of the Earth. These surveys are called **geodetic surveys**. The *Z* dimensions (**orthometric heights**) in geodetic surveys are also referenced to a datum—usually mean sea level.

In the past, geodetic surveys were very precise surveys of great magnitude, for example, national boundaries and control networks. Modern surveys (data gathering, control, and layout) utilizing satellite-positioning systems are geodetic surveys based on the ellipsoidal shape of the Earth and referenced to the geodetic reference system (GRS80) ellipsoid. Such survey measurements must be translated mathematically from ellipsoidal coordinates and ellipsoidal heights to plane grid coordinates and to orthometric heights (referenced to mean sea level) before being used in leveling and other local surveying projects.

Engineering or construction surveys that span long distances (e.g., highways, railroads) are treated as plane surveys, with corrections for the Earth's curvature being applied at regular intervals (e.g., at 1-mi intervals or at township boundaries). **Engineering surveying** is defined as those activities involved in the planning and execution of surveys for the location, design, construction, maintenance, and operation of civil and other engineered projects.* Such activities include the following:

1. Preparation of surveying and related mapping specifications.
2. Execution of photogrammetric and field surveys for the collection of required data, including topographic and hydrographic data.
3. Calculation, reduction, and plotting (manual and computer-aided) of survey data for use in engineering design.
4. Design and provision of horizontal and vertical control survey networks.
5. Provision of line and grade and other layout work for construction and mining activities.
6. Execution and certification of quality control measurements during construction.
7. Monitoring of ground and structural stability, including alignment observations, settlement levels, and related reports and certifications.
8. Measurement of material and other quantities for inventory, economic assessment, and cost accounting purposes.
9. Execution of as-built surveys and preparation of related maps, plans, and profiles upon completion of the project.
10. Analysis of errors and tolerances associated with the measurement, field layout, and mapping or other plots of survey measurements required in support of engineered projects.

Engineering surveying does not include surveys for the retracement of existing land ownership boundaries or the creation of new boundaries. These activities are reserved for licensed property surveyors—also known as professional land surveyors or cadastral surveyors.

1.2 SURVEYING: GENERAL BACKGROUND

Surveys are usually performed for one of two reasons. First, surveys are made to collect data, which can then be plotted to scale on a plan or map (these surveys are called **preliminary surveys** or **preengineering surveys**); second, field surveys are made to lay out dimensions taken from a design plan and thus define precisely, in the field, the location of the proposed construction works. The layouts of proposed property lines and corners as required in land division are called **layout surveys**; the layouts of proposed construction

*Adapted from the definition of *engineering surveying* as given by the American Society of Civil Engineers (ASCE) in their *Journal of Surveying Engineering* in 1987.

features are called **construction surveys**. Preliminary and construction surveys for the same area must have this one characteristic in common: Measurements for both surveys must be referenced to a common base for *X*, *Y*, and *Z* dimensions. The establishment of a base for horizontal and vertical measurements is known as **control survey**.

1.3 CONTROL SURVEYS

Control surveys establish reference points and reference lines for preliminary and construction surveys. Vertical reference points, called benchmarks, are established using leveling surveys (Chapter 4) or satellite-positioning surveys (Chapter 9). Horizontal control surveys (Chapter 11) use any of a variety of measuring and positioning techniques capable of providing appropriately precise results; such surveys can be tied into (1) state or provincial coordinate grids, (2) property lines, (3) roadway centerlines, and (4) arbitrarily placed baselines or grids. When using positioning satellites to establish or reestablish ground positions, the always-available satellite systems themselves can be considered as a control net—thus greatly reducing the need for numerous on-the-ground reference stations. At present, the only fully deployed satellite-positioning systems are the United States' Global Positioning System (GPS) and the Russian Global Navigation Satellite System (GLONASS). Other countries plan to have positioning systems deployed within the next 5 to 10 years—for example, Europe's Galileo System, China's Compass System, Japan's system, and an Indian positioning system.

1.4 PRELIMINARY SURVEYS

Preliminary surveys (also known as preengineering surveys, location surveys, or data-gathering surveys) are used to collect measurements that locate the position of natural features, such as trees, rivers, hills, valleys, and the like, and the position of built features, such as roads, structures, pipelines, and so forth. Measured tie-ins can be accomplished by any of the following techniques.

1.4.1 Rectangular Tie-Ins

The rectangular tie-in (also known as the right-angle offset tie) was once one of the most widely used field location techniques for preelectronic surveys. This technique, when used to locate point P in Figure 1.1(a) to baseline AB, requires distance AC (or BC), where C is on AB at 90° to point P, and it also requires measurement CP.

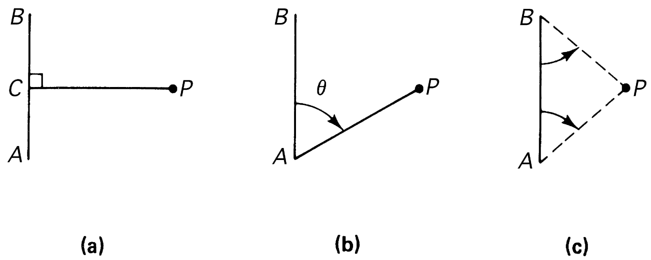


FIGURE 1.1 Location ties.

1.4.2 Polar Tie-Ins

Polar tie-ins (also known as the angle/distance technique) are now the most (refer also to Section 1.4.4) widely used location technique (Chapters 6 and 7). Here, point P is located from point A on baseline AB by measuring angle θ and distance AP [Figure 1.1(b)].

1.4.3 Intersection Tie-Ins

This technique is useful in specialized location surveys. Point P in Figure 1.1(c) is located to baseline AB either by measuring angles from A and B to P or by swinging out arc lengths AP and BP until they intersect. The angle intersection technique is useful for near-shore marine survey locations using theodolites or total stations set up on shore control points. The distance arc intersection technique is an effective method for replacing "lost" survey points from preestablished reference ties.

1.4.4 Positioning Tie-Ins

The second most widely used technique for locating topographic features utilizes direct positioning techniques common to total station surveys and ground-scanning techniques (Chapter 7), satellite-positioning techniques (Chapter 9), and remote-sensing techniques (Chapter 10).

1.5 SURVEYING INSTRUMENTS

The instruments most commonly used in field surveying are (1) level and rod, (2) steel tapes, (3) theodolite, (4) total station, and (5) satellite-positioning receiver. The level and rod are used to determine differences in elevation and elevations in a wide variety of surveying, mapping, and engineering applications. Levels and rods are discussed in Chapter 4. Steel tapes are relatively precise measuring instruments and are used mostly for short measurements in both preliminary and layout surveys. Steel tapes and their usage are discussed in detail in Chapter 3.

Theodolites (also called transits—short for transiting theodolites) are instruments designed for use in measuring horizontal and vertical angles and for establishing linear and curved alignments in the field. During the last 60 years, the theodolite has evolved through four distinct phases:

1. An open-faced, vernier-equipped (for angle determination) theodolite was commonly called a transit. The metallic horizontal and vertical circles were divided into half-degree (30′) or third-degree (20′) of arc. The accompanying 30′ or 20′ vernier scales allowed the surveyor to read the angle to the closest 1′ or 30″ of arc. A plumb bob was used to center the transit over the station mark. See Figures G.8 and G.9 (see the online Instructors Manual). Vernier transits are discussed in detail in Section G.3 (see the online Instructors Manual).
2. In the 1950s, the vernier transit gave way to the optical theodolite. This instrument came equipped with optical glass scales, permitting direct digital readouts or micrometer-assisted readouts. An optical plummet was used to center the instrument over the station mark. See Figure 6.4.

3. Electronic theodolites first appeared in the 1960s. These instruments used photoelectric sensors capable of sensing vertical and horizontal angles and displaying horizontal and vertical angles in degrees, minutes, and seconds. Optical plummets (and later, laser plummets) are used to center the instrument over the station mark (Figure 1.7). Optical and electronic theodolites are discussed in detail in Chapter 6.
4. The total station appeared in the 1980s. This instrument combines electronic distance measurement (EDM), which was developed in the 1950s, with an electronic theodolite. In addition to electronic distance- and angle-measuring capabilities, this instrument is equipped with a central processor, which enables the computation of horizontal and vertical positions. The central processor also monitors instrument status and helps the surveyor perform a wide variety of surveying applications. All data can be captured into electronic field books or into onboard storage as the data are received. See Figure 1.6. Total stations are described in detail in Chapters 6 and 7.

   Satellite-positioning system receivers (Figures 9.2–9.4) capture signals transmitted by four or more positioning satellites to determine position coordinates (e.g., northing, easting, and elevation) of a survey station. Satellite positioning is discussed in Chapter 9.

   Positions of ground points and surfaces can also be collected using various remote-sensing techniques (e.g., panchromatic, multispectral, lidar, and radar) utilizing ground stations as well as satellite and airborne platforms (Chapter 10).

1.6 CONSTRUCTION SURVEYS

Construction surveys provide the horizontal location and the height above sea level (also known as the provision of **line and grade**) for all component of a wide variety of construction projects—for example, highways, streets, pipelines, bridges, buildings, and site grading. Construction layout marks the horizontal location (line) as well as the vertical location or elevation (grade) for the proposed work. The builder can measure from the surveyor's markers to the exact location of each component of the facility to be constructed. Layout markers can be wood stakes, steel bars, nails with washers, spikes, chiseled marks in concrete, and so forth. Modern layout techniques also permit the contractor to position construction equipment for line and grade using machine guidance techniques involving lasers, total stations, and satellite-positioning receivers (Chapter 12, Sections 12.3–12.6). When commencing a construction survey, it is important that the surveyor use the same control survey points as those used for the preliminary survey on which the construction design was based.

1.7 DISTANCE MEASUREMENT

Distances between two points can be **horizontal**, slope, or **vertical** and are recorded in feet or in meters (Figure 1.2).

Vertical distances can be measured with a tape, as in construction work. However, they are more usually measured with a surveyor's level and rod (Figures 1.3 and 1.4) or with a total station (Figure 1.6).

Horizontal and slope distances can be measured with a fiberglass or steel tape (Figure 1.5) or with an electronic distance-measuring device (Figure 1.6). When surveying, the horizontal distance is always required for plan-plotting purposes. A distance measured

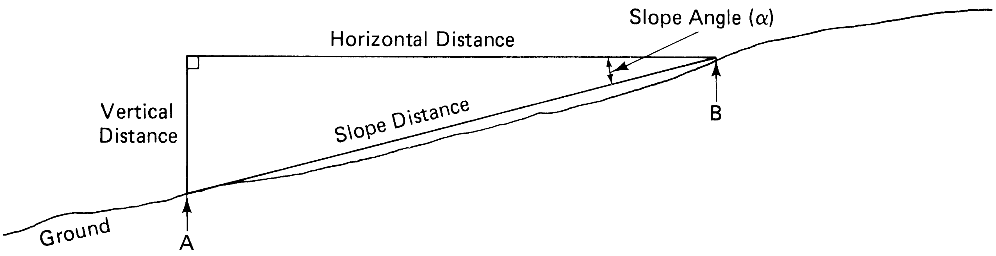


FIGURE 1.2 Distance measurement.

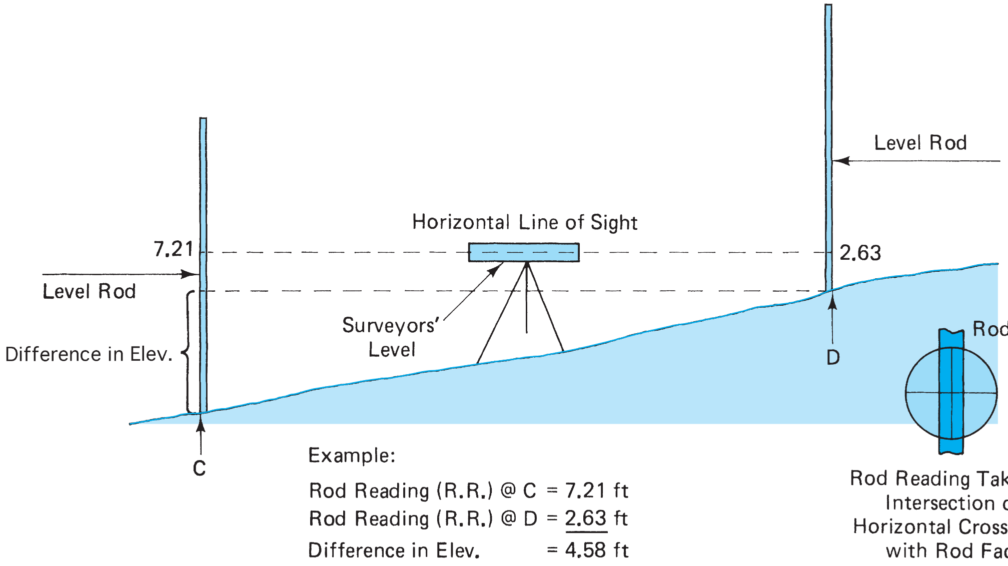


FIGURE 1.3 Leveling technique.

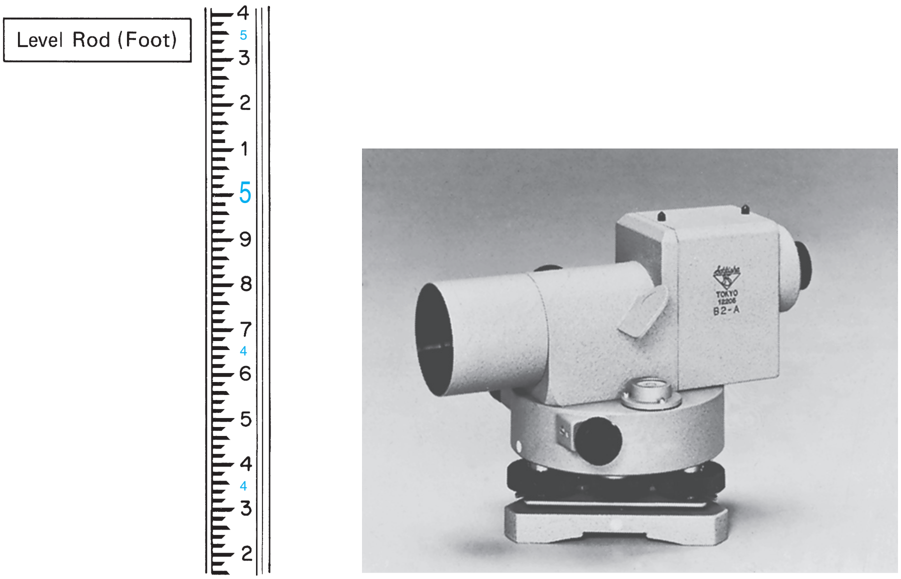


FIGURE 1.4 Level and rod. (Courtesy of SOKKIA Corp.)

FIGURE 1.5 Preparing to measure to a stake tack, using a plumb bob and steel tape.

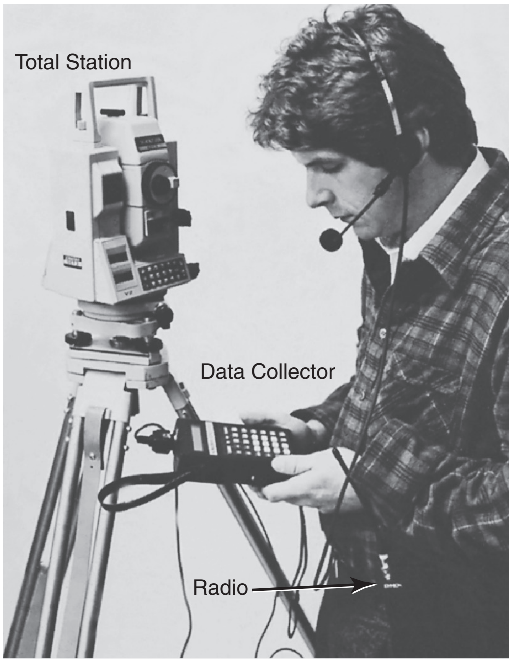


FIGURE 1.6 Sokkia total station.

with a steel tape on slope can be trigonometrically converted to its horizontal equivalent by using either the slope angle or the difference in elevation (vertical distance) between the two points.

1.8 ANGLE MEASUREMENT

Horizontal and vertical angles can be measured with a theodolite or total station. Theodolites are manufactured to read angles to the closest 1′, 20″, 10″, 6″, or 1″. Figure 1.7 shows a 20″ electronic theodolite. Slope angles can also be measured with a clinometer (Chapter 3); the angle measurement precision of that instrument is typically 10′.

1.9 POSITION MEASUREMENT

The position of a natural or built entity can be determined by using a satellite-positioning system receiver, which is simultaneously tracking four or more positioning satellites. The position can be expressed in geographic or grid coordinates, along with ellipsoidal or orthometric elevations (in feet or meters).

Position can also be recorded using airborne and satellite imagery. Such imagery includes aerial photography, lidar imaging, radar imaging, and spectral scanning (Chapter 10).

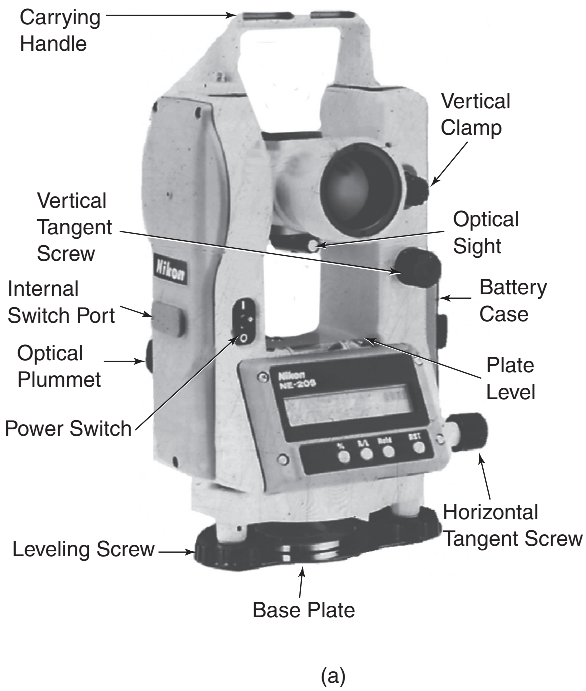
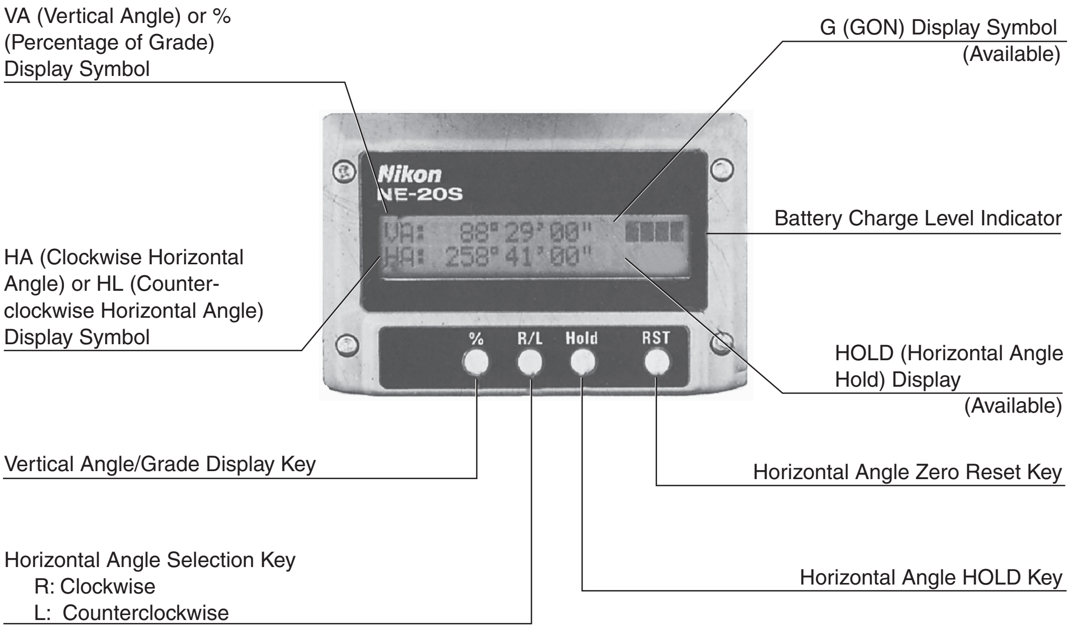


FIGURE 1.7 Nikon NE-20S electronic digital theodolite. (a) Theodolite; (b) operation keys and display. (Courtesy of Nikon Instruments, Inc.)



(b)

FIGURE 1.7 (*Continued*)

1.10 UNITS OF MEASUREMENT

Although the foot system of measurement has been in use in the United States from colonial days until the present, the metric system is in use in most other countries. In the United States, the Metric Conversion Act of 1975 made conversion to the metric system largely voluntary, but subsequent amendments and government actions have now made use of the metric system mandatory for all federal agencies as of September 1992. By January 1994, the metric system was required in the design of many federal facilities. Many states' departments of transportation have also commenced the switch to the metric system for field work and highway design. Although the enthusiasm for metric use in the United States by many surveyors seems to have waned in recent years, both metric units and English units are used in this text because both units are now in wide use.

The complete changeover to the metric system will take many years, perhaps several generations. The impact of all this on the American surveyor is that, from now on, most surveyors will have to be proficient in both the foot and the metric systems. Additional equipment costs in this dual system are limited mostly to measuring tapes and leveling rods.

System International (SI) units are a modernization (1960) of the long-used metric units. This modernization included a redefinition of the meter (international spelling "metre") and the addition of some new units.).

Table 1.1 describes and contrasts metric and foot units. Degrees, minutes, and seconds are used almost exclusively in both metric and foot systems; however, in some European countries, the circle has also been graduated into 400 gon (also called grad). In that system, angles are expressed to four decimals (e.g., a right angle = 100.0000 gon).

TABLE 1.1 Measurement definitions and equivalencies

Linear Measurement	Foot Units
1 mi = 5,280 ft	1 ft = 12 in.
= 1,760 yd	1 yd = 3 ft
= 320 rods	1 rod = 16½ ft
= 80 chains	1 chain = 66 ft
1 ac = 43,560 sq. ft = 10 square chains	1 chain = 100 links

Linear Measurement		Metric (SI) Units
1 km	=	1,000 m
1 m	=	100 cm
1 cm	=	10 mm
1 dm	=	10 cm
1 ha	=	10,000 m^2
1 sq. km	=	1,000,000 m^2 or 100 ha

Foot to Metric Conversion	
1 ft = 0.3048 m (exactly)	1 in. = 25.4 mm (exactly)*
1 km = 0.62137 mi	
1 ha = 2.471 ac	
1 sq. km = 247.1 ac	

Angular Measurements	
1 revolution = 360°	1 revolution = 400.0000 gon†
1 degree = 60′ (minutes)	
1 minute = 60″ (seconds)	

*Prior to 1959, the United States used the relationship 1 m = 39.37 in., which resulted in a U.S. survey foot of 0.3048006 m.

†Used in some European countries.

1.11 STATIONING

While surveying, measurements are often taken along a baseline and at right angles to that baseline. Distances along a baseline are referred to as **stations** or **chainages**, and distances at right angles to the baseline (offset distances) are simple dimensions. The beginning of the survey baseline—the zero end—is denoted as 0 + 00; a point 100 ft (m) from the zero end is denoted as 1 + 00; a point 156.73 ft (m) from the zero end is 1 + 56.73; and so on.

In the preceding paragraph, the full stations are at 100-ft (m) intervals, and the half stations are at even 50-ft (m) intervals. Twenty-meter intervals are often used as the key partial station in the metric system for preliminary and construction surveys. With the ongoing changeover to metric units, most municipalities have kept the 100-unit station (i.e., 1 + 00 = 100 m), whereas highway agencies have adopted the 1,000-unit station (i.e., 1 + 000 = 1,000 m).

Figure 1.8 shows a school building tied in to the centerline (℄) of Regent St. The figure also shows the ℄ (used here as a baseline) distances as stations, and the offset distances as simple dimensions.

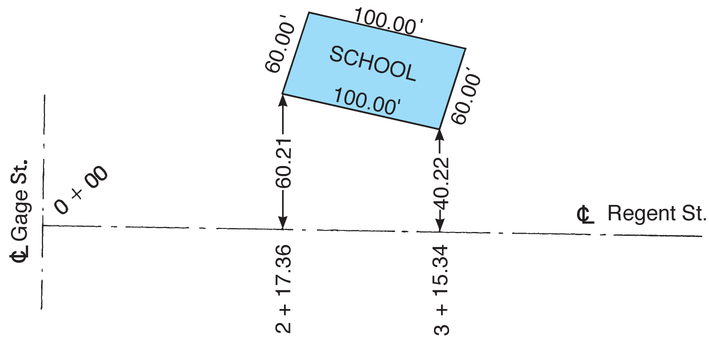


FIGURE 1.8 Baseline stations and offset distances, showing the location of the school on Regent St.

1.12 TYPES OF CONSTRUCTION PROJECTS

The first part of this text covers the surveying techniques common to most surveying endeavors. The second part of the text is devoted to construction surveying applications—an area that accounts for much surveying activity. Listed below are the types of construction projects that depend a great deal on the construction surveyor or engineering surveyor for the successful completion of the project:

1. Streets and highways
2. Drainage ditches
3. Intersections and interchanges
4. Sidewalks
5. High- and low-rise buildings
6. Bridges and culverts
7. Dams and weirs
8. River channelization
9. Sanitary landfills
10. Mining—tunnels, shafts
11. Gravel pits, quarries
12. Storm and sanitary sewers
13. Water and fuel pipelines
14. Piers and docks
15. Canals
16. Railroads
17. Airports
18. Reservoirs
19. Site grading, landscaping

20. Parks, formal walkways
21. Heavy equipment locations (millwright)
22. Electricity transmission lines.

1.13 RANDOM AND SYSTEMATIC ERRORS

An **error** is the difference between a measured, or observed, value and the "true" value. No measurement can be performed perfectly (except for counting), so every measurement must contain some error. Errors can be minimized to an acceptable level by the use of skilled techniques and appropriately precise equipment. For the purposes of calculating errors, the "true" value of a dimension is determined statistically after repeated measurements have been taken.

Systematic errors are defined as those errors for which the magnitude and the algebraic sign can be determined. The fact that these errors can be determined allows the surveyor to eliminate them from the measurements and thus further improve accuracy. An example of a systematic error is the effect of temperature on a steel tape. If the temperature is quite warm, the steel expands, and thus the tape is longer than normal. For example, at 83°F, a 100-ft steel tape can expand to 100.01 ft, a systematic error of 0.01 ft. Knowing this error, the surveyor can simply subtract 0.01 ft each time the full tape is used at that temperature.

Random errors are associated with the skill and vigilance of the surveyor. Random errors (also known as accidental errors) are introduced into each measurement mainly because no human can perform perfectly. Random errors can be illustrated by the following example. Let's say that point B is to be located a distance of 109.55 ft from point A. If the tape is only 100.00 ft long, an intermediate point must first be set at 100.00 ft, and then 9.55 ft must be measured from the intermediate point. Random errors occur as the surveyor is marking out 100.00 ft. The actual mark may be off a bit; that is, the mark may actually be made at 99.99 or 99.98, and so on. When the final 9.55 ft are measured out, two more opportunities for error exist: The lead surveyor will have the same opportunity for error as existed at the 100.00 mark, and the rear surveyor may introduce a random error by inadvertently holding something other than 0.00 ft (e.g., 0.01) on the intermediate mark.

This example illustrates two important characteristics of random errors. First, the magnitude of the random error is unknown. Second, because the surveyor is estimating too high (or too far right) on one occasion and probably too low (or too far left) on the next occasion, some random errors tend to cancel out over the long run.

A word of caution: Large random errors, possibly due to sloppy work, also tend to cancel out. Thus, sloppy work can give the appearance of accurate work—even when highly inaccurate.

1.14 ACCURACY AND PRECISION

Accuracy is the relationship between the value of a measurement and the "true" value of the dimension being measured; the greater the accuracy, the smaller the error. **Precision** describes the degree of refinement with which the measurement is made. For example, a distance measured four times with a steel tape by skilled personnel will be more precise than the same distance measured twice by unskilled personnel using a fiberglass tape. Figure 1.9 illustrates the difference between accuracy and precision by showing the results of target shooting using both a high-precision rifle and a low-precision shotgun.

The **accuracy ratio** of a measurement or a series of measurements is the ratio of the error of closure to the distance measured. The error of closure is the difference between the measured location and its theoretically correct location. Because relevant systematic errors and mistakes can and should be eliminated from all survey measurements, the error of closure will normally be composed of random errors.

To illustrate, a distance is measured and found to be 196.33 ft. The distance was previously known to be 196.28 ft. The error is 0.05 ft in a distance of 196.28 ft:

$$\text{Accuracy ratio} = \frac{0.05}{196.28} = \frac{1}{3{,}926} \approx \frac{1}{3{,}900}.$$

The accuracy ratio is expressed as a fraction whose numerator is 1 and whose denominator is rounded to the closest 100 units. Many engineering surveys are specified at 1/3,000 and 1/5,000 levels of accuracy; property surveys used to be specified at 1/5,000 and 1/7,500 levels of accuracy. With polar layouts now being used more often in total station surveys, the coordinated control stations needed for this type of layout must be established using techniques giving higher orders of accuracy (e.g., 1/10,000, 1/15,000, and the like). Sometimes the accuracy ratio, or error ratio, is expressed in parts per million (ppm). One ppm is simply the ratio of 1/1,000,000; 50 ppm is 50/1,000,000, or 1/20,000. See Table 3.1 and Tables 11.2–11.5 for more current survey specifications and standards.

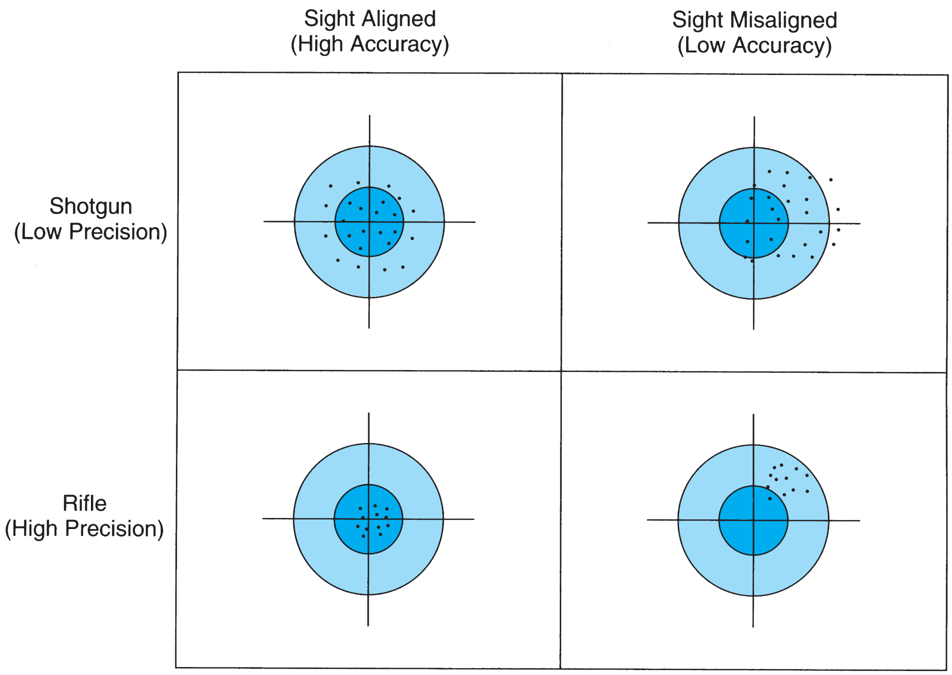


FIGURE 1.9 An illustration of the difference between accuracy and precision.

1.15 MISTAKES

Mistakes are blunders made by survey personnel. Examples of mistakes are transposing figures (recording a value of 86 as 68), miscounting the number of full tape lengths in a long measurement, and measuring to or from the wrong point. You should be aware that mistakes will occur! Mistakes must be discovered and eliminated, preferably by the people who made them. *All survey measurements are suspect until they have been verified.* Verification may be as simple as repeating the measurement, or verification may result from geometric or trigonometric analysis of related measurements. As a rule, all measurements are immediately repeated. This immediate repetition enables the surveyor to eliminate most mistakes and at the same time to improve the precision of the measurement.

1.16 FIELD NOTES

One of the most important aspects of surveying is the taking of neat, legible, and complete field notes. The notes will be used to plot scale drawings of the area surveyed and also to provide a permanent record of the survey proceedings. Modern surveys, employing electronic *data collectors*, automatically store point-positioning angles, distances, and attributes, which will later be transferred to the computer. Surveyors have discovered that some handwritten field notes are also valuable for these modern surveys. (See also Section 7.4.)

An experienced surveyor's notes should be complete, without redundancies; be arranged to aid comprehension; and be neat and legible to ensure that the correct information is conveyed. Sketches are used to illustrate the survey and thus help remove possible ambiguities.

Handwritten field notes are placed in bound field books or in loose-leaf binders. Loose-leaf notes are preferred for small projects because they can be filed alphabetically by project name or in order by number. Bound books are advantageous on large projects, such as highway construction or other heavy construction operations, where the data can readily fill one or more field books.

1.16.1 Requirements for Bound Books

Bound field books should include the following information:

1. Name, address, and phone number should be in ink on the outside cover.
2. Pages are numbered throughout.
3. Space is reserved at the front of the field book for a title, an index, and a diary.
4. Each project must show the date, title, surveyors' names, and instrument numbers.

1.16.2 Requirements for Loose-Leaf Books

Loose-leaf field books should include the following information:

1. Name, address, and phone number should be in ink on the binder.
2. Each page must be titled and dated, and must be identified by project number, surveyors' names, and instrument numbers.

1.16.3 Requirements for All Field Notes

All field notes, whether bound into books or organized into loose-leaf binders, should follow this checklist:

1. Entries should be in pencil, written with 2H–4H lead (lead softer than 2H will cause unsightly smears on the notes).
2. All entries are neatly printed. Uppercase letters can be used throughout, or they can be reserved for emphasis.
3. All arithmetic computations must be checked and signed.
4. Although sketches are not scale drawings, they are drawn roughly to scale to help order the inclusion of details.
5. Sketched details are arranged on the page such that the north arrow is oriented toward the top of the page.
6. Sketches are not freehand; straightedges and curve templates are used for all line work.
7. Do not crowd information on the page. Crowded information is one of the chief causes of poor field notes.
8. Mistakes in the entry of measured data are to be carefully lined out, not erased.
9. Mistakes in entries other than measured data (e.g., descriptions, sums, or products of measured data) may be erased and reentered neatly.
10. If notes are copied, they must be clearly labeled as such so that they are not thought to be field notes.
11. Lettering on sketches is to be read from the bottom of the page or from the right side; any other position is upside down.
12. Note keepers verify all given data by repeating the data aloud as they enter the data in their notes; the surveyor who originally gave the data to the note keeper listens and responds to the verification callout.
13. If the data on an entire page are to be voided, the word VOID, together with a diagonal line, is placed on the page. A reference page number is shown for the new location of the relevant data.

REVIEW QUESTIONS

1.1. Describe four different procedures used to locate a physical feature in the field so that it can be plotted later in its correct position on a scaled plan.

1.2. Describe how a very precise measurement can be inaccurate.

1.3. How do plane surveys and geodetic surveys differ?

1.4. How can you ensure that a survey measurement is free of mistakes?

1.5. Illustrate the reduction of a measured slope distance to the horizontal equivalent distance.

1.6. Describe the term *error*. How does this term differ from *mistake*?

1.7. What is the difference between a layout survey and a preliminary survey?

1.8. If a 100-ft steel tape were broken and then poorly repaired, resulting in a tape that was only 99.00 ft long, that tape would be

a. inaccurate
b. imprecise
c. both inaccurate and imprecise

1.9. If the poorly repaired tape from Question 1.8 were used to measure a rectangular plot whose the dimensions were 281.00 ft by 178.50 ft, what would the actual dimensions of the plot be?

CHAPTER
TWO

SURVEYING MATHEMATICS

Construction layout puts the surveyor in the unique role of "translating" the project visualized by the designer and shown on the architectural or civil drawings for the contractor who will build the project. This role requires the surveyor to be able to read and interpret the drawings and make the necessary computations to locate the project correctly on its site, and to accurately communicate those interpretations to the contractor. Construction surveyors must be both fast and accurate in their computations, and they should have a mastery of such things as the key conversion factors used in converting between systems of units, formulas for finding area and volume of basic geometric shapes, and procedures for working with rectangular coordinates and angles.

The mathematical tools required to make surveying computations are primarily algebra, geometry, and trigonometry. While modern surveying increasingly uses electronic data collection and computers to speed up the manipulation of data, it remains essential for the surveying student to have a thorough understanding of the mathematical basis for the software functions and to be able to carry out computations on handheld calculators in the field. This chapter will serve as a basic review of commonly used surveying mathematics.

2.1 UNIT CONVERSIONS

As noted in Chapter 1, construction surveyors must be able to work with both metric and English units and to convert between them. Currently, federal and some state agencies require plans to be drawn in metric units, while other states and most local agencies continue to work in English units. In interpreting construction drawings, the surveyor will also find that while architects use English units of feet, inches, and fractional inches, civil engineers who produce drawings for site improvements, parking lots, and entrance roads prefer to work in survey feet (feet, and tenths and hundredths of a foot). Parts of a construction drawing such as shop drawings, standard details, or the storm water management plan might use metric units, so that it is possible for a single set of plans to include three systems of linear measurement. Surveyors' tapes, meanwhile, are divided into feet, and tenths and hundredths of a foot, while carpenters tend to use tapes graduated into feet and inches. Land descriptions and property surveys may include additional units such as the chain and rod that reference historical surveying instruments such as a Gunter's chain that are not used in modern surveying. All of this makes unit conversion a necessity for nearly any construction project.

Angular measurements in the United States and Canada divide a complete revolution (circle) into 360 *degrees* (360°). Each degree contains 60′ (*minutes*) and each minute contains 60″ (*seconds*). Notice that the symbols for minutes and seconds are the same as those used for the unrelated units of feet and inches. Some handheld scientific calculators include a function key to convert angular measure stated in degrees, minutes, and seconds to decimal degrees for use in trigonometric functions, but surveyors should be able to make the conversion by hand if necessary. *Radians* are also used to describe angular measure. There are 2π radians in one complete revolution (360°), so 1 radian measures $180°/\pi$, about 57°. Table 2.1 gives the equivalencies for metric and English units and angular measures.

The basic principle of converting units properly is to set up an equation in such a way that the given units cancel to yield the desired units in the numerator. Some conversions require intermediate steps, such as converting inches to feet before converting to metric. Construction layout may require conversions in either direction (i.e., from English (either feet and inches or decimal feet) to metric, or from metric to English).

TABLE 2.1 Measurement definitions and equivalencies

Linear Measurement		**Foot Units**
1 mi = 5,280 ft		1 ft = 12 in.
= 1,760 yd		1 yd = 3 ft
= 320 rods		1 rod = $16\frac{1}{2}$ ft
= 80 chains		1 chain = 66 ft
1 ac = 43,560 sq. ft = 10 square chains		1 chain = 100 links
Linear Measurement		**Metric (SI) Units**
1 km	=	1,000 m
1 m	=	100 cm
1 cm	=	10 mm
1 dm	=	10 cm
1 ha	=	10,000 m^2
1 sq. km	=	1,000,000 m^2 = 100 ha
	Foot to Metric Conversion	
1 ft = 0.3048 m (exactly)		1 in. = 25.4 mm (exactly)*
1 ha = 10,000 sq. m		1 m = 3.2808 ft
1 ha = 2.471 ac		1 km = 0.62137 mi
1 sq. km = 247.1 ac		
	Angular Measurements	
1 revolution = 360°		1 revolution = 400.0000 gon†
1 degree = 60′ (minutes)		1 revolution = 2π radians
1 minute = 60″ (seconds)		1 radian = $180°/\pi$

*Prior to 1959, the United States used the relationship 1 m = 39.37 in., which resulted in a U.S. survey foot of 0.3048006 m (used mainly in older land and property surveys).

†The gradian, or grad (abbreviated gon) is used in some European countries.

Example 2.1

Convert 10′-1½″ (read 10 feet, one and one-half inches) to the metric equivalent.

First, convert the given length to decimal feet, working first with the fractional inches. Note that 1½″ can be expressed as 1.5 in. Set up the conversion equation such that inches cancel and the result is in feet.

$$1.5\ \cancel{\text{inches}} \times (1\ \text{ft}/12\ \cancel{\text{inches}}) = 0.125\ \text{ft.}$$

Answer: 10′-1½″ = 10.125′.

Next, convert the result to metric.
From Table 2.1, 1 ft = 0.3048 m.

$$10.125\ \cancel{\text{ft}} \times (0.3048\ \text{m}/1\ \cancel{\text{ft}}) = 3.086\ \text{m}$$

Answer: 10′-1½″ = 3.086 m

The conversion may be worked in reverse (i.e., convert 3.086 m to the English equivalent in feet and inches):

$$3.086\ \cancel{\text{m}} \times (1\ \text{ft}/0.3048\ \cancel{\text{m}}) = 10.125\ \text{ft}$$

Subtract the whole feet (10) before converting the decimal feet to inches and fractional inches.

$$10.125\ \text{ft} - 10\ \text{ft} = 0.125\ \text{ft}$$

$$0.125\ \cancel{\text{ft}} \times (12\ \text{in.}/1\ \cancel{\text{ft}}) = 1.5\ \text{in.} = 1^1/_2\ \text{in.}$$

Answer: 3.086 m = 10′-1½″

The area of a closed figure is expressed in units of square measure. In surveying, small areas such as those of individual building lots can be expressed in units of square feet or square meters. The area of larger tracts of land is usually expressed in acres (English units) or hectares (metric units). Very large areas are typically expressed in square miles (English units) or hectares or square kilometers (metric units). When converting between units of square measure, make sure that the units remain consistent.

In the United States, construction surveyors often have to set stakes to indicate to heavy equipment operators the amount of earth to be moved (cuts and fills) in feet and inches. The relationship between 0.01 feet and ⅛ of an inch shown below is simple and surveyors soon find that they can make the foot–inch conversions in their heads (Table 2.2).

TABLE 2.2 Decimal foot–inch conversion

	$1'' = {}^1/_{12}{}' = 0.083'$	
1″ = 0.08(3)′	7″ = 0.58′	$^1/_8{}'' = 0.01'$
2″ = 0.17′	8″ = 0.67′	$^1/_4{}'' = 0.02'$
3″ = 0.25′	9″ = 0.75′	$^1/_2{}'' = 0.04'$
4″ = 0.33′	10″ = 0.83′	$^3/_4{}'' = 0.06'$
5″ = 0.42′	11″ = 0.92′	
6″ = 0.50′	12″ = 1.00′	

Example 2.2

Convert 1 square mile to square feet.

A mile is equal to 5,280 ft. Set up a conversion equation, remembering to square the conversion factor of feet per mile in order to cancel the units of square miles and obtain the answer in square feet.

$$1 \text{ sq. mi} \times (5{,}280 \text{ ft}/1 \text{ mi})^2 = 27{,}878{,}400 \text{ ft}^2$$

Example 2.3

How many acres are in a square mile?

$$1 \cancel{\text{sq. mi}} \times (5{,}280 \cancel{\text{ft}}/1 \cancel{\text{mile}})^2 \times (1 \text{ acre}/43{,}560 \cancel{\text{ft}^2}) = 640 \text{ ac}$$

Example 2.4

How many hectares are in 1 square mile?

There are several ways to proceed in this conversion. One is to convert square miles to square feet, then convert square feet to square meters, and finally convert square meters to hectares.

$$1 \text{ sq. mi.} \times [5{,}280 \text{ ft.}/1 \text{ mi.}]^2 \times [0.3048 \cancel{\text{m}}/1\cancel{\text{ft}}]^2 \times (1 \text{ ha}/10{,}000 \cancel{\text{sq. m}})$$
$$= 1 \cancel{\text{sq. mi}} \times 27{,}878{,}400 \cancel{\text{ft}^2}/\cancel{\text{sq. mi}} \times 0.0929 \cancel{\text{m}^2}/\cancel{\text{ft}^2} \times 1 \text{ ha}/10{,}000 \cancel{\text{sq. m}}$$

Answer: 259.00 ha

Or, using the results of Example 2.3,

$$1 \cancel{\text{sq. mi}} \times (640 \cancel{\text{ac}}/\cancel{\text{sq. mi}}) \times \text{ha}/2.471 \cancel{\text{ac}}$$

Answer: 259.00 ha

Example 2.5

Convert 36°46′28″ to decimal degrees.

To make this conversion, the minutes and seconds must be converted to degrees.

$$36° + (46 \cancel{\text{min}} \times 1 \text{ degree}/60 \cancel{\text{min}}) + (28 \cancel{\text{sec}} \times 1 \cancel{\text{min}}/60 \cancel{\text{sec}} \times 1 \text{ degree}/60 \cancel{\text{min}})$$
$$= 36° + 0.7667° + 0.0078° = 36.7744°$$

Working the problem in reverse, convert 36.7744° to degrees, minutes, and seconds.

First, subtract the whole degrees and work with the decimal remainder:

$$0.7744 \text{ degrees} \times 60 \text{ min}/1 \text{ degree} = 46.4640 \text{ minutes}$$

Subtract the number of whole minutes (46′) and work with the decimal remainder:

$$0.4640 \text{ minutes} \times 60 \text{ s/min} = 27.84 \text{ seconds (round to } 28''\text{)}$$

Answer: 36.7744° = 36°46′28″

2.2 LINES AND ANGLES

Two-dimensional construction drawings (plans) use lines to represent physical elements such as the edges of buildings or pavements, as well as abstract elements such as right-of-way lines, property boundaries or building setback lines (Figure 2.1).

A **line** is defined by two points, and a straight line forms the shortest distance between those points. Lines in a two-dimensional plane may be parallel and never intersect, or non-parallel and intersect at a single point [Figure 2.2(a), (b)]. Two intersecting lines form equal opposite angles and parallel lines cut by an intersecting line form equal angles [Figure 2.2(c)].

Angles may be acute ($<90°$), obtuse ($>90°$), right (90°), or straight (180°). The measure of the acute angle (1) and obtuse angle (2) on the same side of the intersecting line in Figure 2.2(b) add to exactly 180°, forming a straight angle. These angles are called *supplementary*.

In the special case of *perpendicular* lines, all opposite angles formed by the intersecting lines measure 90°. A perpendicular line forms the shortest distance between a point and a line [Figure 2.2(d)]. Two angles that add to form a 90° angle are called *complementary* angles.

2.3 POLYGONS

A closed figure formed by three or more intersecting lines is a **polygon**. If the sides do not intersect each other, the closed figure is a simple polygon. In construction surveying, simple polygons may represent such things as the shape of buildings, the edges of concrete structures, or the boundaries of a building lot (Figure 2.3). A complex polygon has sides that cross each other, and may represent such things as project **control networks**. When used alone, the term "polygon" is understood to refer to a simple polygon.

The *perimeter* of any polygon is the sum of the length of all the sides. Perimeter and area relationships for four-sided figures (*quadrilaterals*) that include parallel sides (*parallelograms*), such as squares, rectangles, and trapezoids, as given in Figure 2.4.

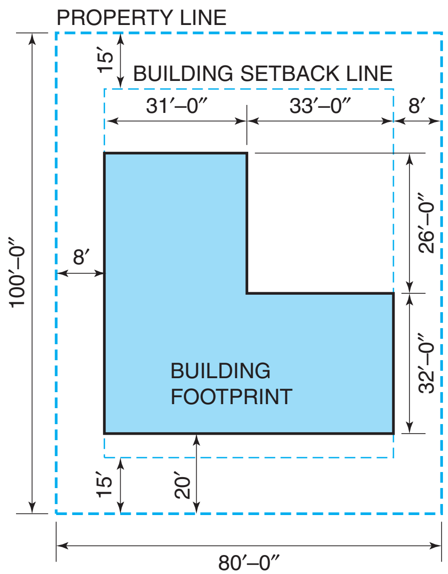


FIGURE 2.1 Lines in construction drawings.

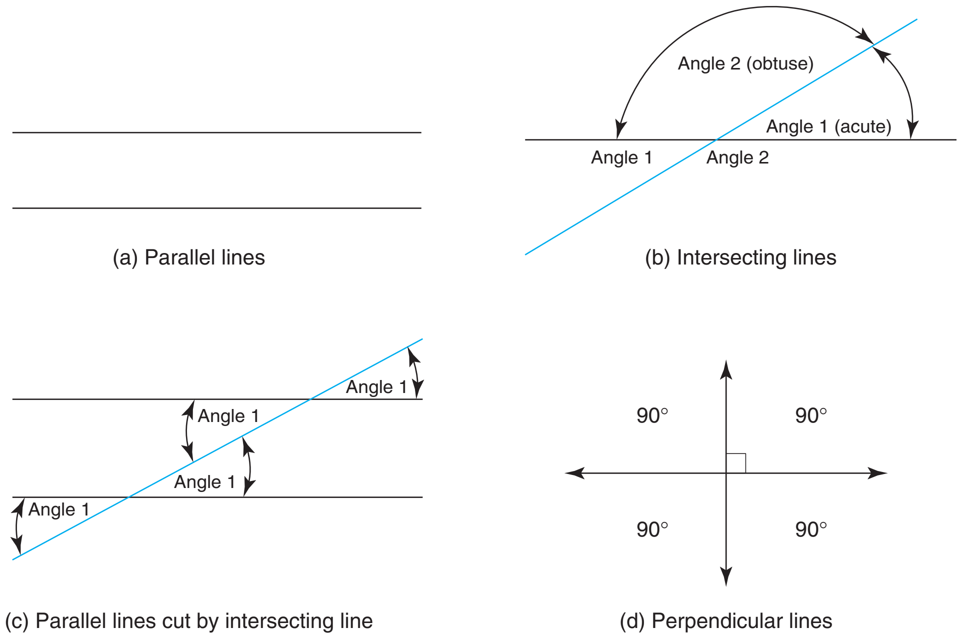


FIGURE 2.2 Angles.

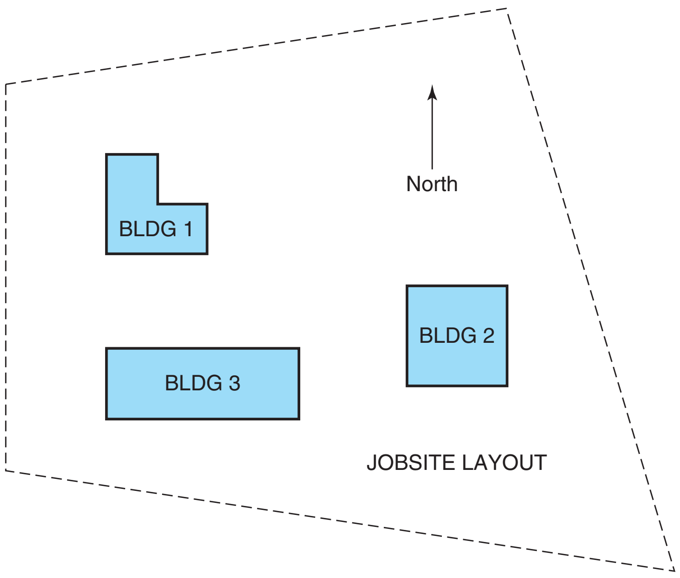


FIGURE 2.3 Simple polygons.

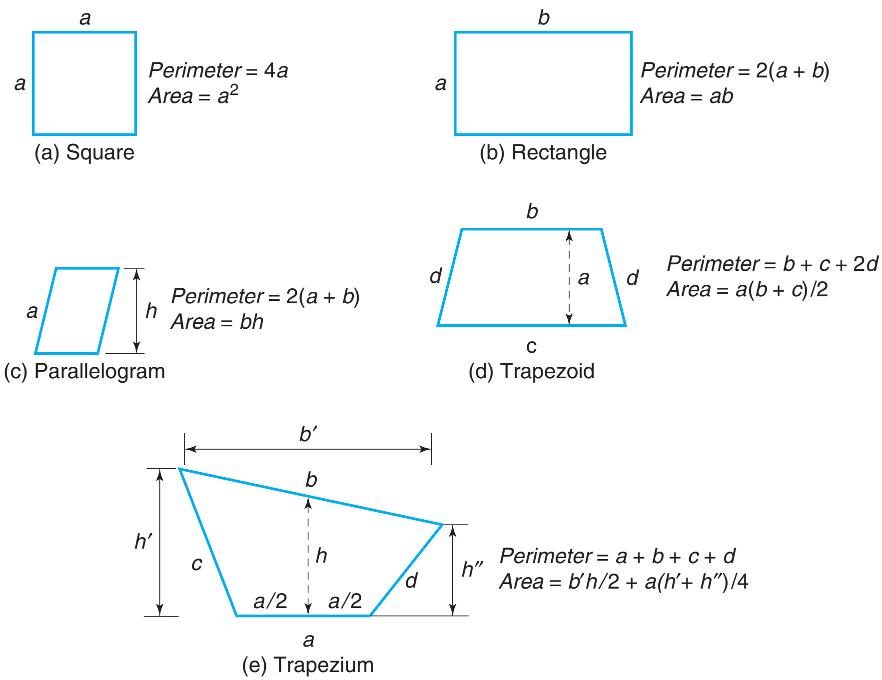


FIGURE 2.4 Perimeter and area relationships for simple polygons.

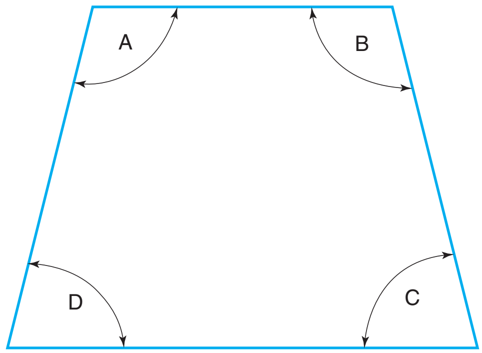


FIGURE 2.5 Interior angles.

The angles inside a simple polygon are called *interior angles*. A useful relationship to remember is that the sum (Σ) of the measures of the interior angles of a polygon (Figure 2.5) is related to the number of sides (n) by the following relationship:

$$\sum_{\text{interior angles}} = (n - 2)180° \quad \textbf{(2.1)}$$

Similarly, the sum of the *exterior angles* of a polygon (Figure 2.6) is related to the number of sides (n) by:

$$\sum_{\text{exterior angles}} = (n + 2)180° \quad \textbf{(2.2)}$$

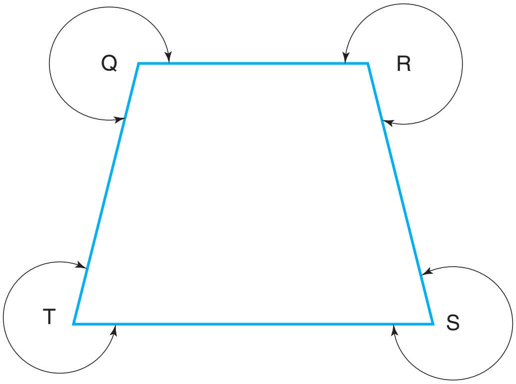


FIGURE 2.6 Exterior angles.

Example 2.6

A survey crew is measuring the interior angles of an arena, the shape of which is represented in Figure 2.7. What should the sum of the interior angles be if there were no errors?

The figure has 8 sides. The sum of the interior angles should be

$$(8 - 2)180° = 1{,}080°$$

Answer: $\sum_{\text{interior angles}} = 1{,}080°$

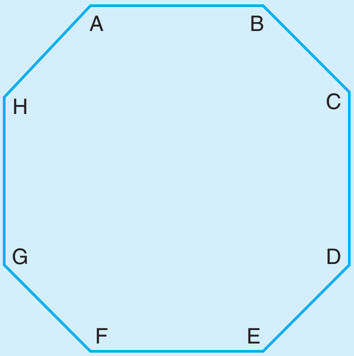


FIGURE 2.7 Sketch for Example 2.6.

Formulas for the area of a variety of common polygon shapes are presented in Chapter 19, Quantity and Final Surveys. However, any polygon can be divided into a number of triangles and the area is the sum of the areas of the triangles composing the polygon.

2.3.1 Triangles

A polygon with exactly three sides, the triangle, is central to many surveying computations and deserves special attention. The sum of the measures of the three interior angles of a triangle is 180°. An *isosceles* triangle has two angles of equal measure and two sides

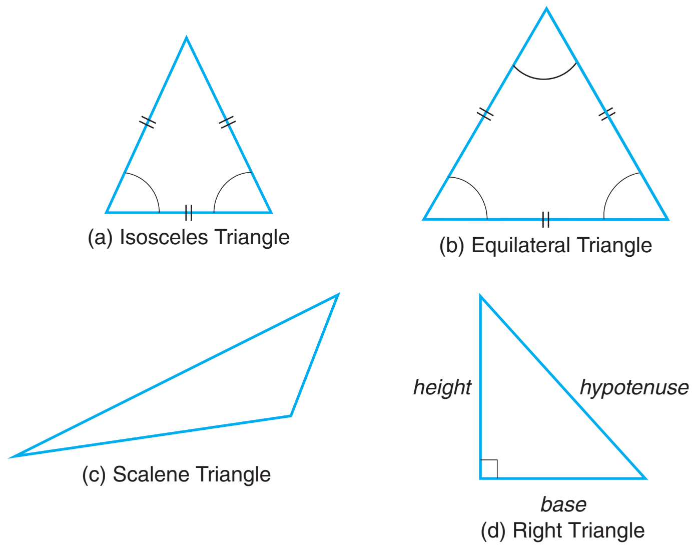


FIGURE 2.8 Triangles.

of equal length [Figure 2.8(a)]. An *equilateral* triangle has three angles of equal measure ($180°/3 = 60°$) and three sides of equal length [Figure 2.8(c)]. A *right triangle*, perhaps the most frequently used in construction surveying, has one angle equal to 90°. The side opposite the 90° angle is termed the *hypotenuse*, and the other two sides (or legs) are the *base* (b) and the altitude or *height* (h) [Figure 2.8(d)].

Two triangles are termed *congruent* if three of their corresponding parts (sides or angles) are equal (Figure 2.9). Two triangles are termed *similar* if two of their corresponding

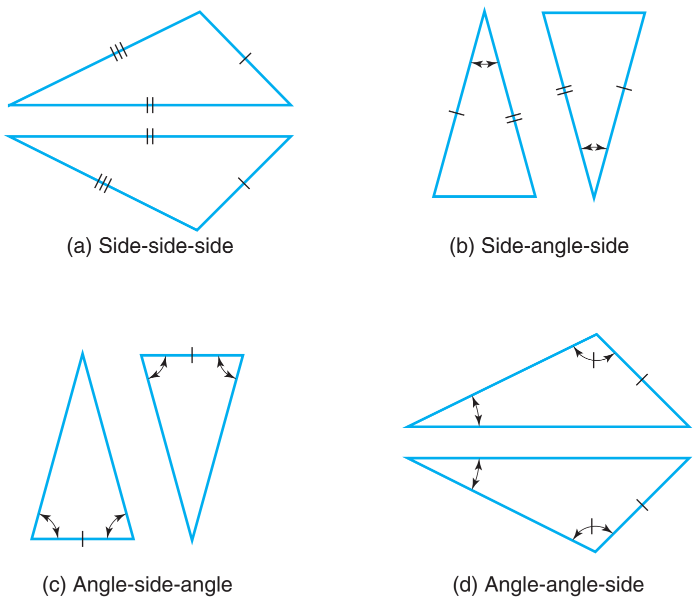


FIGURE 2.9 Congruent triangles.

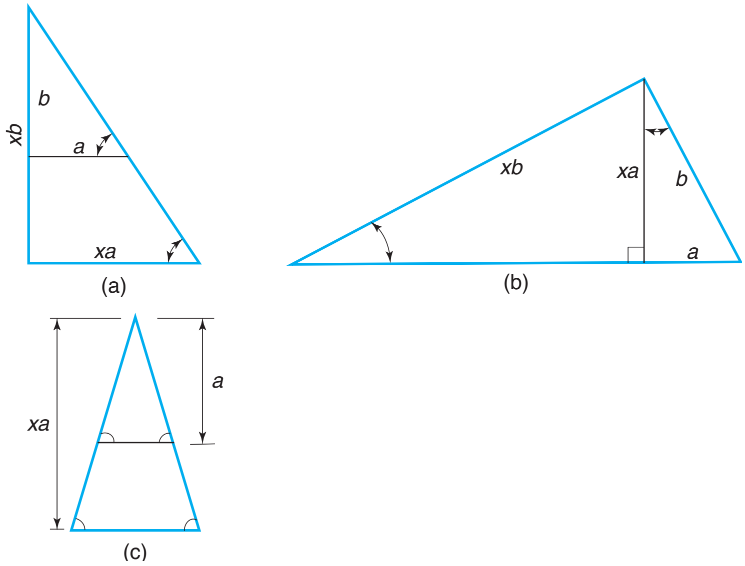


FIGURE 2.10 Similar triangles.

angles are equal (Figure 2.10). The lengths of the sides of similar triangles are proportional, so that the unknown lengths of corresponding sides can be found by *ratio and proportion*.

Example 2.7

Given a pair of isosceles triangles [Figure 2.10(c)] with base angles equal to 35°, if the height of the larger triangle is 15.27′, what is the height of the smaller triangle?

The two triangles are similar: the bases are parallel, so the base angles of the smaller triangle are also 35°. The height of the smaller triangle can be found by the following ratio:

$$\frac{\text{Height}_{\text{small}}}{\text{Height}_{\text{large}}} = \frac{\text{Base}_{\text{small}}}{\text{Base}_{\text{large}}}$$

Inserting the known measures and solving for the unknown height of the small triangle:

$$\frac{H_{\text{small}}}{15.27'} = \frac{8.35'}{12.50'}$$

Solve for H_{small} by cross-multiplication.

$$H_{\text{small}} = 15.27' \times 8.35'/12.50' = 10.20'$$

Answer: $H_{\text{small}} = 10.20'$

The *Pythagorean theorem*, which states that the square of the hypotenuse (c) of a right triangle equals the square of the length of leg a plus the square of the length of leg b, allows computation of the length of one side if the other two are known. Mathematically, the relationship is given by:

$$a^2 + b^2 = c^2 \tag{2.3}$$

This relationship can be rearranged to solve for the length of one side if the length of the hypotenuse and the remaining side are known:

$$a = \sqrt{c^2 - b^2};\ b = \sqrt{c^2 - a^2} \tag{2.4}$$

Example 2.7

A survey crew is laying out a rectangular building that measures 33′-4″ by 46′-10″. Verify that the building is square by checking the length of the diagonals, which should be equal.

First convert the length of the sides to decimal feet. The diagonal of the rectangle is the hypotenuse of the triangle (Figure 2.11). The length of the diagonals is:

$$\sqrt{33.33^2 + 46.83^2} = 57.48'$$

Answer: Diagonals = 57.48′

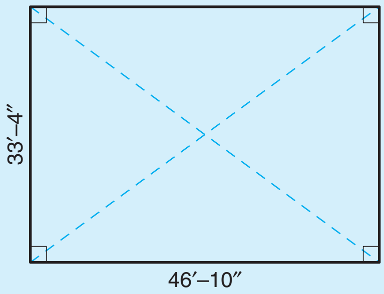


FIGURE 2.11 Sketch for Example 2.7.

Special forms of the right triangle are the isosceles right triangle, with angles measuring 45°-45°-90°, which has sides with lengths in the ratio of 1-1-$\sqrt{2}$ [Figure 2.12(a)], and the 30°-60°-90° right triangle with sides in the ratio of 1-$\sqrt{3}$-2 [Figure 2.12(b)]. A third special form is the right triangle that has sides with lengths in the ratio of 3-4-5 or any multiple of this ratio (e.g., 6-8-10, 30-40-50, 15-20-25). This easily remembered relationship allows the field layout of a right angle using a measuring tape [Figure 2.12(c)].

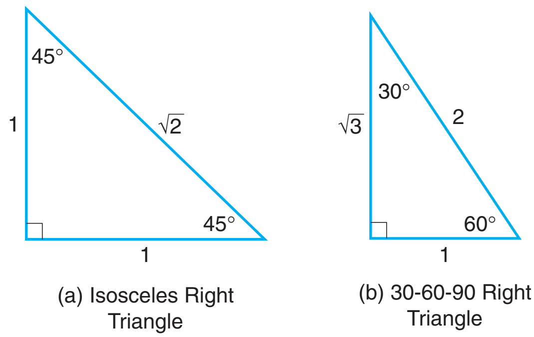
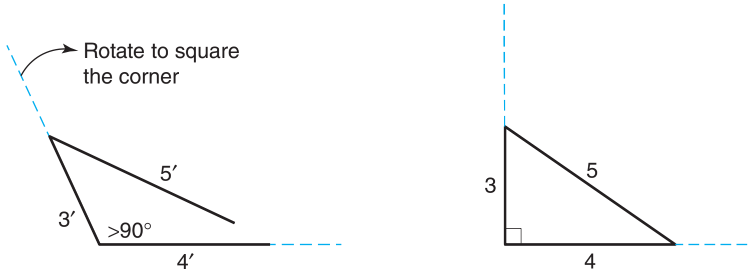


FIGURE 2.12 Right triangles.



(c) Laying out a right angle using 3-4-5 right triangle

FIGURE 2.12 (*Continued*)

2.3.2 Trigonometry

Trigonometry is the study of triangles and the relationships between angles and sides. The trigonometric functions can be defined for right triangles in terms of the angles A, B, and C and the sides opposite those angles (Figure 2.13). Since the advent of handheld calculators, only three relationships are typically used: the sine, cosine, and tangent functions.

$$\sin A = \text{opposite}/\text{hypotenuse} = a/c \tag{2.5}$$

$$\cos A = \text{adjacent}/\text{hypotenuse} = b/c \tag{2.6}$$

$$\tan A = \text{opposite}/\text{adjacent} = a/b \tag{2.7}$$

The trigonometric functions allow computation of the length of any side of a right triangle if the length of another side and the value of one angle are known.

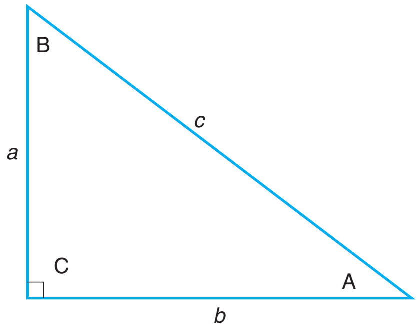


FIGURE 2.13 Labeling angles and sides of a right triangle.

Example 2.8

A 30-60-90 right triangle has a base (b) of 32.51′ [Figure 2.12(b)]. What is the length of the hypotenuse (c) and the altitude (a) of this triangle?

The hypotenuse length s is found by

$$\sin 30^\circ = \frac{32.51'}{c}$$

$$c = \frac{32.51'}{\sin 30^\circ}$$

Using a scientific calculator, the value of sin 30° is 0.5

$$c = \frac{32.51''}{0.5} = 65.02'$$

The length of the altitude *a* is found by

$$\tan 30° = \frac{32.51'}{a}$$

$$a = \frac{(32.51')}{\tan 30°}$$

$$a = \frac{32.51}{0.5774} = 56.31'$$

Answer: $c = 65.02'$, $a = 56.31'$

If the length of any two sides is known, the value of the angle *A* can be calculated by the inverse function, denoted as $\sin^{-1}$ or arcsin (hypotenuse and opposite side known), $\cos^{-1}$ or arccos (hypotenuse and adjacent side known), and $\tan^{-1}$ or arctan (opposite and adjacent sides known).

Example 2.9

A right triangle has sides that are 3′, 4′, and 5′ long (Figure 2.14). What is the measure of the angle *A* opposite the short side, and the angle *B* opposite the long side?

The angle opposite the short side is found by

$$\sin B = 3/5$$

$$\sin^{-1}(3/5) = 36.87°$$

The angle opposite the long side is the complement of this angle (90 − 36.87 = 53.13°). This can be verified by

$$\sin A = 4/5$$

$$\sin^{-1}(4/5) = 53.13°$$

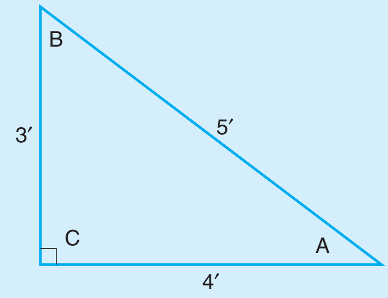


FIGURE 2.14 Sketch for Example 2.9.

Triangles that do not contain a right angle are called *oblique*. The trigonometric functions do not apply for oblique triangles, but other relationships can be defined in terms of known angles or sides. The two most useful are the *Law of Sines* and the *Law of Cosines*. The labeling convention for oblique triangles is that the angles are represented by capital letters *A*, *B*, and *C*, and the sides of the triangle lying opposite those angles are represented by lower case letters *a*, *b*, and *c*. The most convenient location of angles corresponding to *A*, *B*, or *C* or sides *a*, *b*, or *c* can be selected for solving for the desired unknown angle or side.

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \tag{2.8}$$

Example 2.10

A surveyor needs to determine the distance across an eight-lane highway as shown in Figure 2.15. Points A and C on the near side of the highway are selected for convenience. Occupying point A, the surveyor sights on point B on the far side and finds the angle BAC. The distance from A to C can be measured. Moving to point C on the near side of the highway, point A is sighted, and the angle ACB is found. Knowing the length of one side and two angles, the surveyor can compute the angle at B and find the unknown distance AB using the law of sines.

We label the given information as follows:

$A = 63°14'15''$
$a =$ unknown
$B = 180° - (63°14'15'' + 41°40'40'') = 75°05'05''$
$b = 421.42'$
$C = 41°40'40''$
$c =$ desired distance

Using Equation (2.8),

$$\frac{c}{\sin 41°40'40''} = \frac{421.42'}{\sin 75°05'05''}$$

$$c = \frac{(0.6649)(421.42')}{0.9663}$$

Answer: $c = 289.99'$

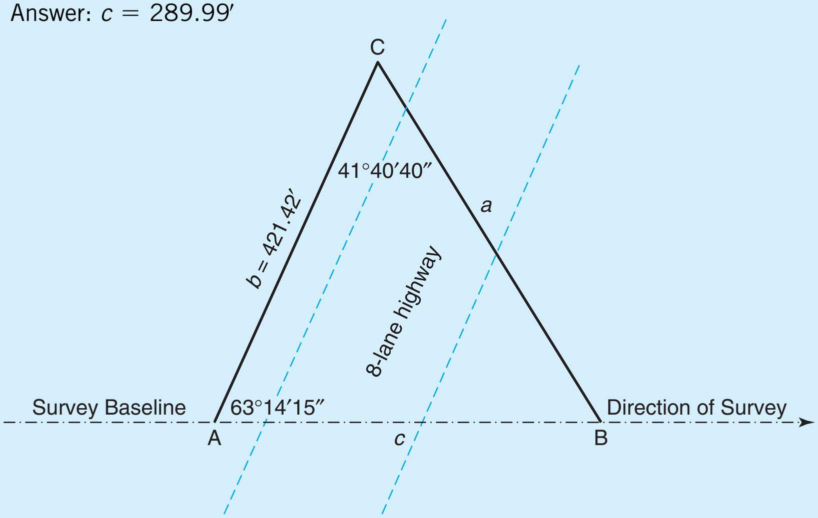


FIGURE 2.15 Sketch for Example 2.10.

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \textbf{(2.9)}$$

These formulas can be rearranged to take advantage of the known angles or sides. Table 2.3 contains many useful relationships. Additional trigonometric identities are found in Appendix 2.

Example 2.11

Given the building shown in Figure 2.16, find the angle from line KL to a point Q that lies outside the building, 10′ from point K and 14′ from point L.

To physically find Q, the surveyor can use a pair of tapes, pull a distance of 10′ from point K and 14′ from point L, and find where the two tapes intersect. However, to compute the angle LKQ for layout by angle and distance from point K, observe that the given information includes only the length of the 3 sides of the triangle. From Table 2.3, select a form of the Law of Cosines that allows computation of an unknown angle given the length of 3 sides, where angle A represents angle LKQ:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Inserting the values that correspond to a, b, and c:

$$\cos A = (10^2 + 20^2 - 14^2)/2(10)(20)$$

$$\cos A = 0.76$$

$$A = \cos^{-1}(0.76) = 40.53458° = 40°32'08''$$

Answer: $A = 40°32'08''$

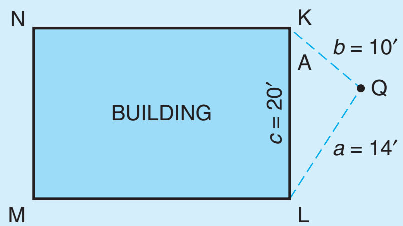


FIGURE 2.16 Sketch for Example 2.11.

TABLE 2.3

Given	Required	Formulas	
A, B, a	C, b, c	$c = 180 - (A + B);\ b = \dfrac{a}{\sin A}\sin B;\ c = \dfrac{a}{\sin A}\sin C$	
A, b, c	a	$a^2 = b^2 + c^2 - 2bc\cos A$	**(2.10)**
a, b, c	A	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**(2.11)**
a, b, c	A	$\cos\dfrac{1}{2}A = \sqrt{\dfrac{s(s-a)}{bc}}$ where $s = \dfrac{a+b+c}{2}$	**(2.12)**

Note that the values of angles KQL and QLK of this oblique triangle can be found by using the Law of Sines once the value of one angle is known.

Example 2.12

A surveyor needs to know the distance between points C and B in Figure 2.17. A building obscures the view directly between the two points. Setting up at the end of the building at point A, the surveyor measures the two distances AC and AB, and the interior angle at A. What is the desired distance, CB?

Label the triangle with side CB = *a*, side AC = *b* = 566.81′, and side AB = *c* = 591.13′ and the interior angle at A is 80° 30′. Select Equation (2.10) from Table 2.3 to solve for the unknown side *a*.

$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

$$a = \sqrt{(566.81)^2 + (591.13)^2 - 2(566.81)(591.13)\cos(80.5°)}$$

$$a = 748.40'$$

Answer: Distance CB = 748.40′

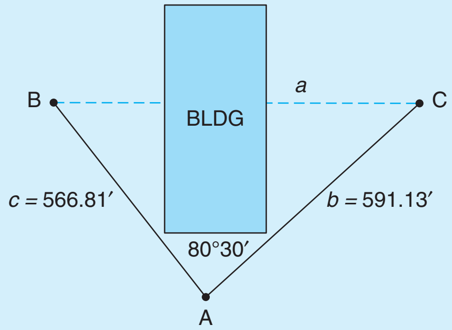


FIGURE 2.17 Sketch for Example 2.12.

The **area** of a right triangle is base times height divided by two, or

$$\text{Area} = \frac{1}{2}bh \qquad \textbf{(2.12)}$$

The area of an oblique triangle may be found if an equivalent base (b') can be found that is perpendicular to an equivalent height (h') (Figure 2.18), in which case

$$\text{Area} = \frac{1}{2}b'h' \qquad \textbf{(2.13)}$$

The area of an oblique triangle may also be found using the relationships shown in Table 2.4:

TABLE 2.4

a, *b*, *c*	Area	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$	**(2.14)**
		where $s = \frac{1}{2}(a+b+c)$	**(2.15)**
C, *a*, *b*	Area	$\text{Area} = \frac{1}{2}ab\sin C$	**(2.16)**
		where *C* is the included angle between *a* and *b*	

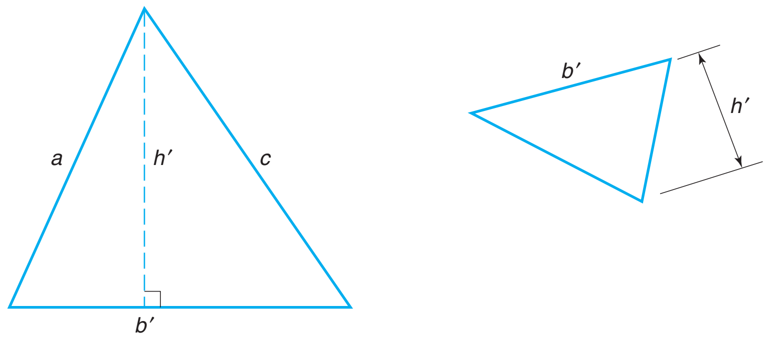


FIGURE 2.18 Equivalent height of an oblique triangle.

Example 2.13

What is the area of the triangle ABC of Example 2.12?

For this triangle, $a = 748.40'$, $b = 566.81'$, and $c = 591.13'$

$$\text{Therefore } s = \frac{1}{2}(748.40 + 566.81 + 591.13) = 953.17'$$

From Table 2.4, Equation (2.14),

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area} = \sqrt{971.21(971.21 - 784.48)(971.21 - 566.81)(971.21 - 591.13)}$$

$$\text{Area} = \sqrt{953.17(204.77)(386.36)(362.04)} = 165{,}231 \text{ sq. ft}$$

Alternatively, use Equation (2.15) because the length of two sides and the included angle is known:

$$Area = \frac{1}{2}ab \sin C$$

$$\text{Area} = (0.5)(566.81)(591.13)\sin(80.5°)$$

$$\text{Area} = 165{,}231 \text{ sq. ft}$$

2.4 CIRCLES

Many construction surveying problems in both building and highway construction involve curves, which may be all or a portion of a complete circle. A circle is a curved line that lies equidistant from a single point that lies at the center of the circle (Figure 2.19). The distance from the center to the circle is the *radius*, R; the line that intersects the circle at two points and passes through the center is the *diameter* of the circle (D), which equals twice the radius.

The perimeter of a circle is termed its *circumference*, which is related to the diameter of the circle by the constant π (*pi*), a dimensionless number with a value of approximately 3.1416.

$$\text{Circumference} = \pi D = 2\pi R \qquad \textbf{(2.17)}$$

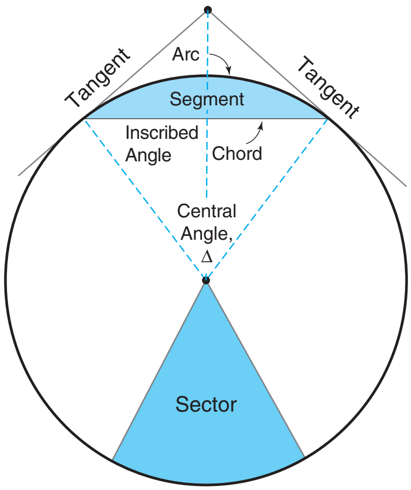


FIGURE 2.19 Parts of a circle.

The *area* of a circle is also related to π:

$$\text{Area} = \pi R^2 = \pi D^2/4 \tag{2.18}$$

A line with its two endpoints on the circle is a *chord*. The two ends of a chord *intercept* an *arc* of the circle. The radii that intercept the two ends of a chord *subtend* an angle at the center of the circle called the *central angle*, which is represented by the Greek letter Δ (delta). The length of the arc (l) or the related central angle Δ can be found by ratio and proportion:

$$\frac{l}{2\pi R} = \frac{\Delta}{360^\circ}$$

Or,

$$l = \frac{\pi R \Delta}{180^\circ} \tag{2.19}$$

Example 2.14

How long is the arc formed by a central angle of 30° for a circle with a radius of 20 feet?

Using Equation (2.19):

$$l = \pi(20')(30^\circ)/180^\circ$$

$$l = 10.47'$$

Answer: The arc length is 10.47′

A **tangent** line intersects the circle at one point, and is perpendicular to the radius at that point. Two tangents drawn from a single point outside the circle will have equal length and form equal angles between the tangent and the chord. A line from the single point to the center of the circle will bisect the intercepted arc, the chord, and the central angle.

The portion of the circle that lies within an inscribed arc is a *sector*. The portion of the sector that lies between the chord and the circle is a *segment*. The angles formed by the chord and the radii are *inscribed angles*. The area of a sector is related to the area of the entire circle by the ratio of the central angle to 360°.

$$\frac{\text{Area}_{\text{sector}}}{\pi R^2} = \frac{\Delta}{360^\circ}$$

or,

$$\text{Area}_{\text{sector}} = \pi R^2 \Delta / 360^\circ \qquad \textbf{(2.20)}$$

The area of a segment is the area of a sector minus the area of the triangle formed by the chord and the two radii that bound the sector. Using Equation (2.16), the area of this triangle is equal to ½ R^2 sin Δ. The area of the segment is therefore

$$\text{Area}_{\text{segment}} = \pi R^2 \Delta / 360^\circ - \frac{1}{2} R^2 \sin \Delta \qquad \textbf{(2.21)}$$

Example 2.15

A subway tunnel is shaped as shown in Figure 2.20, with a radius of 150 feet. What is the cross-sectional area of this tunnel?

The area of the shape can be found by noting that the shape is a circle with a segment removed. The chord that forms the base of the tunnel has a length equal to the radius of the circle. The triangle formed by the chord and two radii creates an equilateral triangle, and the central angle Δ is therefore 60°.

$$\text{Area} = \text{Area}_{\text{circle}} - \text{Area}_{\text{segment}}$$

$$\text{Area} = \pi(150)^2 - \left[\pi(150)^2 \frac{60}{360} - 0.5(150)^2 \sin 60^\circ\right] \qquad \textbf{(2.21)}$$

$$\text{Area} = 70{,}682.8 - 2{,}038.2$$

$$\text{Area} = 68{,}647.6 \text{ sq. ft}$$

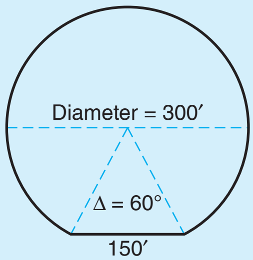


FIGURE 2.20 Sketch for Example 2.15.

2.5 RECTANGULAR COORDINATES

To lay out a building project, both the length and direction of lines may be of interest. Surveyors typically reference the location of a line to a coordinate axis in a two-dimensional or *x,y* plane that considers the positive *y* direction to represent North, and the positive *x*

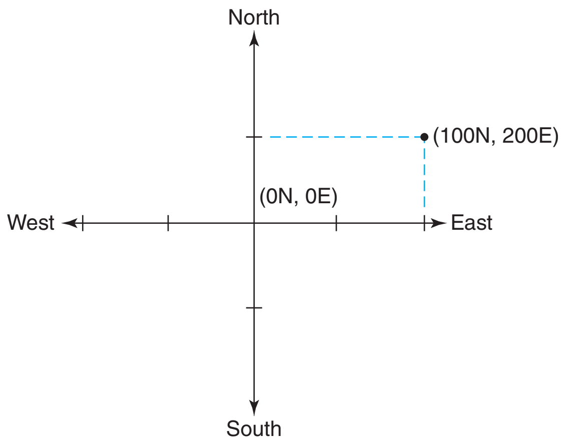


FIGURE 2.21 Rectangular coordinates.

direction to represent East. Coordinate pairs are denoted as (N, E) rather than (x,y), and refer to the location of a point by its North coordinate (*northing*) and its East coordinate (*easting*). A point located 100 units north and 200 units east of the origin would be denoted as (100N, 200E) (Figure 2.21).

Construction drawings show dimensions related to the facility to be built, but during construction the surveyor must compute information not shown on the plans, such as the distance from known reference points to key points such as building corners, location of underground utilities, and radius points of curved lines, as well as computed lengths such as diagonal distances used for squaring up buildings. Scaling distances from the plans does not provide sufficient precision, so surveyors must compute the required distances based on the (N, E) coordinates assigned to reference points.

Rectangular coordinates are used to compute the direction and distance between points on a construction site. The difference of the northings and the difference of the eastings define the altitude and base, respectively, of a right triangle. The Pythagorean theorem can be used to compute the length of the hypotenuse if the coordinates of its two endpoints are known. The process of finding the distance between two points using their rectangular coordinates is called *inversing the distance.*

$$\text{Distance}_{2-1} = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2} \tag{2.22}$$

where E_2 is the east coordinate of the second point and E_1 is the east coordinate of the first point of the pair of endpoints; N_2 is the north coordinate of the second point and N_1 is the north coordinate of the first point of the pair of endpoints.

Example 2.16

How long is the length of the diagonal line *ac* of Figure 2.1 if the coordinates of *a* are (1027N, 1018E) and the coordinates of *c* are (1049N, 1060E)?

Because we are examining the line *ac*, we are considering *a* to be the first point and *c* to be the second point on the line. We therefore use the coordinates of *c* for N_2 and E_2 in Equation (2.22).

$$N_2 - N_1 = (1049 - 1027) = 22 \text{ units}$$
$$E_2 - E_1 = (1060 - 1018) = 42 \text{ units}$$
$$\text{Distance} = \sqrt{(22^2 + 42^2)} = 47.41 \text{ units}$$

Answer: Length of line ac = 47.41 units

Knowing the coordinates of a pair of endpoints of a line also allows computation of the angle formed by line and either the x or y axis. Because surveyors reference the orientation of lines to North, the angle of interest is typically that formed between a line and the y axis, or North direction. Using trigonometry, the angle formed between a line and North typically is

$$\theta = \tan^{-1}\left(\frac{E_2 - E_1}{N_2 - N_1}\right) \tag{2.23}$$

Example 2.17

What is the direction (with reference to North) of the line formed by line ac of Figure 2.1 if the coordinates of a are (1027N, 1018E) and the coordinates of c are (1049N, 1060E)?

Using Equation (2.23), the angle formed by the y axis (North direction) and the line ac is found by

$$\theta = \tan^{-1}\left(\frac{(1060 - 1018)}{(1049 - 1027)}\right)$$
$$\theta = \tan^{-1}\left(\frac{42}{22}\right)$$

Answer: $\theta = 62°21'14''$

The use of coordinate geometry in construction surveying is extensive. Software to perform coordinate geometry, or COGO, calculations and electronic equipment to capitalize on the use of COGO have greatly reduced the time required to do construction layout.

PROBLEMS

2.1. A rectangular plot measures 452.8 ft by 134.8 ft. Compute:

- **a.** the dimensions of the plot in chains,
- **b.** the dimensions of the plot in miles,
- **c.** the area of the plot in acres,
- **d.** the area of the plot in hectares
- **e.** the length of a diagonal line in yards
- **f.** the two angles at a corner in degrees (D)
- **g.** the two angles at a corner in minutes (M)
- **h.** the two angles at a corner in seconds (S)
- **i.** the two angles at a corner in DMS
- **j.** the two angles at a corner in radians
- **k.** the two angles at a corner in gon

2.2. The three interior angles of a 4-sided figure (Figure 2.22) are given as A = 62°40′30″, B = 107°15′45″, and D = 85°48′30″. If the total observational error is −56″, what is the observed value of angle C?

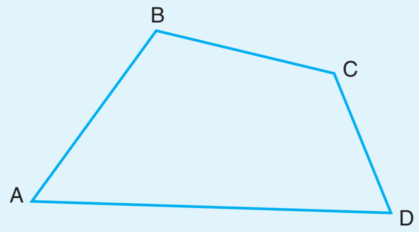


FIGURE 2.22 Sketch for Problem 2.2.

2.3. Find the area of the given right triangle ABC (Figure 2.23), when:

a. A = 65° and a = 21.8′
b. A = 65° and c = 21.8′

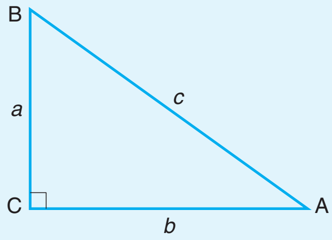


FIGURE 2.23 Sketch for Problem 2.3.

2.4. Find the length of the unknown sides in Figure 2.24.

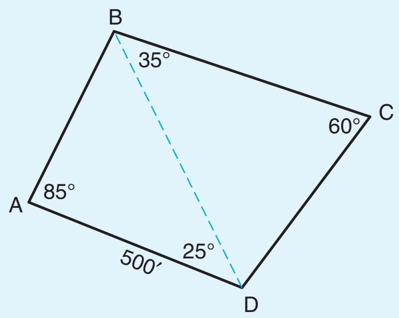


FIGURE 2.24 Sketch for Problem 2.4.

2.5. Find the value of the unknown angle(s) and side(s) in Figure 2.25.

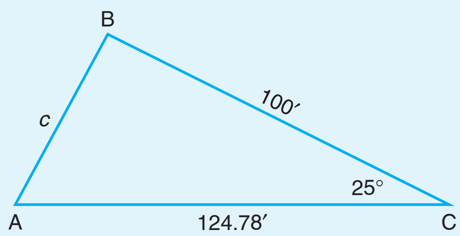


FIGURE 2.25 Sketch for Problem 2.5.

2.6. Find the value of the unknown angle(s) in Figure 2.26.

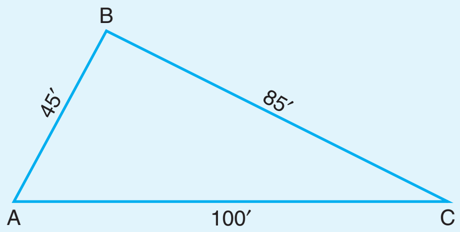


FIGURE 2.26 Sketch for Problem 2.6.

2.7. Find the area of the following:

a. Figure 2.27

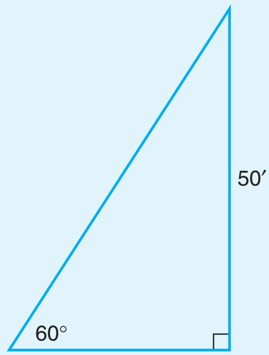


FIGURE 2.27 Sketch for Problem 2.7a.

b. Figure 2.28

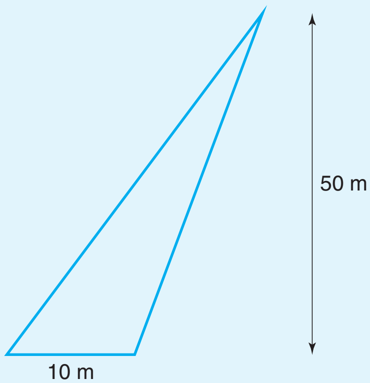


FIGURE 2.28 Sketch for Problem 2.7b.

c. Figure 2.29

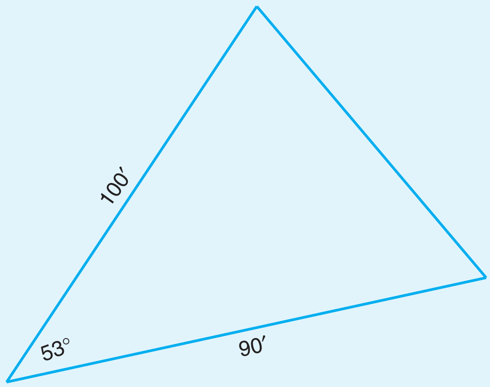


FIGURE 2.29 Sketch for Problem 2.7c.

d. Figure 2.30

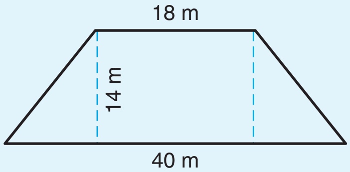


FIGURE 2.30 Sketch for Problem 2.7d.

e. Figure 2.31

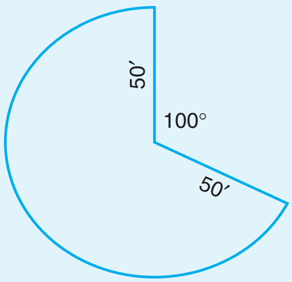


FIGURE 2.31 Sketch for Problem 2.7e.

2.8. The corners of a building lot ABCDE have rectangular coordinates (N, E) as shown in Figure 2.32.

a. What is the area of the lot in sf?
b. What is the distance from A to C?
c. What is the angle B-A-C?
d. Is AC parallel to ED? Explain.

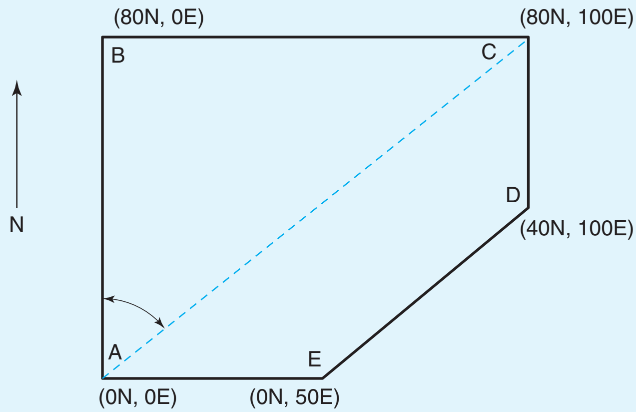


FIGURE 2.32 Sketch for Problem 2.8.

CHAPTER THREE

TAPE MEASUREMENTS

3.1 BACKGROUND

Historically, distances have been directly measured by applying an instrument of known length against the distance between two or more ground points. As noted in the Instructors Manual, *Early Surveying*, early Egyptians (3000 B.C.) used ropes for distance measurements, having knots tied at convenient points on the rope to aid in the measurement process. Much later, in the 1500s, Edmund Gunter invented a 66-ft chain, comprising 100 links. The Gunter's chain has significance for North American surveyors because this was the instrument originally used to lay out most of the continent's townships in the 1700s and 1800s. In the early 1900s, various types of reel-mounted tapes came into use. These tapes were made of cloth, copper wire-reinforced cloth, fiberglass, and steel; all precise measurements were made with steel tapes. In the second half of the twentieth century, electronic distance measurement (EDM) instruments came into wide use, especially as integrated components of total stations.

An example of calculated measurement occurs when the desired measurement (perhaps over water) is one side of a triangle whose other side(s) and angles have been measured, or when the slope distance and slope angle have been measured between two points and the required horizontal distance is required.

3.1.1 Pacing

Pacing is a useful method of approximate measure. Surveyors can determine the length of pace that, for them, can be comfortably repeated (for convenience, some surveyors develop a 3-ft stride). Pacing is particularly useful when looking for survey markers in the field. The plan distance from a found marker to another marker can be paced off to aid in locating the second marker. Another important use of pacing is for a rough check of all key points in construction layouts. Distances typically can be paced with an accuracy ratio of 1:100.

3.1.2 Odometer

Automobile odometer readings can be used to measure from one fence line to another when they intersect the road right-of-way. These readings are precise enough to differentiate rural fence lines and can thus assist in identifying platted property lines. This information is useful when collecting information to begin a survey. Odometers are also used on measuring wheels that are simply rolled along the ground on the desired route; this approximate technique is employed where low-order precision is acceptable (on the order of 1:100 to 1:200). For example, surveyors from the assessor's office often check property frontages this way, and police officers sometimes use this technique when preparing sketches of automobile accident scenes.

3.1.3 Electronic Distance Measurement (EDM)

Most EDM instruments function by sending a light wave along the path to be measured from the instrument station, and then the instrument measures the phase differences between the transmitted light wave and the light wave as it is reflected back to its source from a reflecting prism at the second point. Pulse laser EDMs operate by measuring the time for a laser pulse to be transmitted to a reflector and then returned to the EDM; with the velocity of light programmed into the EDM, the distance to the reflector and back is quickly determined. A more detailed discussion of this topic is presented in Chapters 5 and 7.

3.1.4 Distances Derived from the Analysis of Position Coordinates

Spatial point positioning can be determined by using satellite-positioning techniques and by using various scanning techniques (from satellite, aerial, and ground platforms). Once the spatial coordinates of ground points are known, it becomes a simple matter, using trigonometric and/or geometric relationships, to compute the horizontal distances (and directions) between them. Also, when the ground coordinates of a key survey point marker are known, the surveyor can use a satellite-positioning receiver to navigate to, and thus locate the marker (which may be buried).

3.2 GUNTER'S CHAIN

The measuring device in popular use during settlement of North America (eighteenth and nineteenth centuries) was the Gunter's chain; it was 66-ft long and subdivided into 100 links. It has more than historical interest to surveyors because many property descriptions still on file include dimensions in chains and links. This chain, named after its inventor (Edmund Gunter, 1581–1626), was uniquely suited for work in English units:

$$\begin{aligned} 1 \text{ chain} &= 100 \text{ links} = 66 \text{ feet} \\ 1 \text{ rod} &= 25 \text{ links} \\ 4 \text{ rods} &= 1 \text{ chain} \\ 80 \text{ chains} &= 1 \text{ mile} \\ 10 \text{ sq. chains} &= 1 \text{ ac} \left(10 \times 66^2 = 43{,}560 \text{ sq. ft}\right) \end{aligned}$$

Because Gunter's chain was used in many of the original surveys of North America, most of the continent's early legal plans and records contain dimensions in chains and links. Present-day surveyors occasionally must use these old plans and then must make conversions to feet or meters.

Example 3.1

An old plan shows a dimension of 5 chains, 32 links. Convert this value to (a) feet and (b) meters.

SOLUTION

(a) $5.32 \times 66 = 351.12$ ft

(b) $5.32 \times 66 \times 0.3048 = 107.02$ m (1 m = 0.3048 ft)

3.3 TAPES

Woven tapes made of linen, Dacron, and the like can have fine copper strands interwoven to provide strength and to limit deformation due to long use and moisture. See Figure 3.1. Measurements taken near electric stations should be made with dry nonmetallic or fiberglass tapes. Fiberglass tapes have now come into widespread use.

All tapes come in various lengths, the 100-ft and 30-m tapes being the most popular, and are used for many types of measurements where high precision is not required. All woven tapes should be checked periodically (e.g., against a steel tape) to verify their precision.

Many tapes are now manufactured with foot units on one side and metric units on the reverse side. Foot-unit tapes are graduated in feet (0.10 ft and 0.05 ft) or in feet, inches, and $^1/_4$ in. Metric tapes are graduated in meters, centimeters (0.01 m), and $^1/_2$ cm (0.005 m).

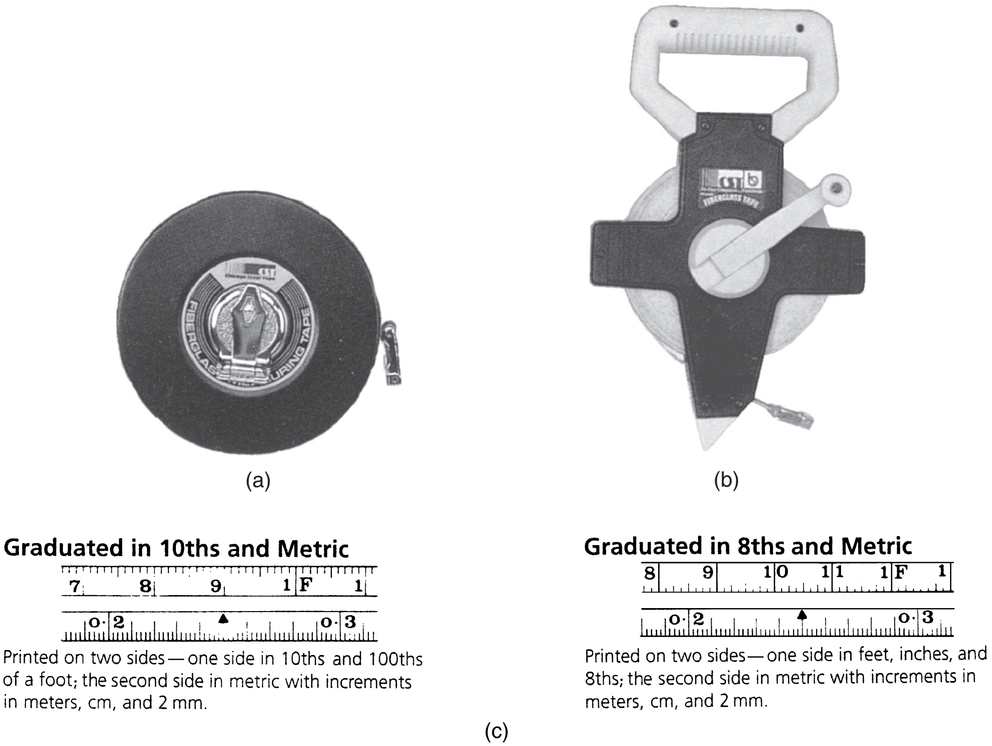
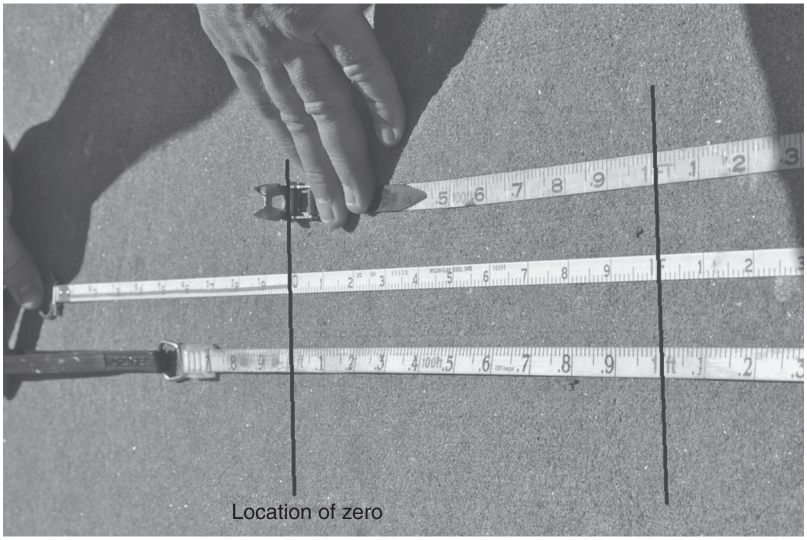


FIGURE 3.1 Fiberglass tapes. (a) Closed case; (b) open reel; (c) tape graduations. (Courtesy of CST/Berger, Illinois)



(d)

FIGURE 3.1 (*Continued*)

3.4 STEEL TAPES

3.4.1 General Background

Not so many years ago, most precise measurements were made using steel tapes. Although EDM is now favored because of its high precision and the quickness of repeated measurements, even over rough ground and very long distances, there are some drawbacks when EDM is used for single distances, particularly in the short-distance situations that regularly crop up in engineering and construction layout applications. The problems with EDM in short-distance situations are twofold: (1) the time involved in setting up the EDM and the prism, and (2) the unreliable accuracies in some short-distance situations. For example, if a single-check distance is required in a construction layout, and if the distance is short (especially distances less than one tape length), it is much quicker to obtain the distance through taping. Also, because most EDMs now have stated accuracies in the range of $\pm(5\,\text{mm} + 5\,\text{ppm})$ to $\pm(2\,\text{mm} + 2\,\text{ppm})$, the 5-mm to 2-mm errors occur regardless of the length of distance measured. These errors have little impact on accuracy over long distances, but can severely impact the measurement of short distances. For example, at a distance of 10.000 m, an error of 0.005 m limits accuracy to 1:2,000. There is possibility for additional errors when centering the EDM instrument over the point (e.g., an additional 0.001–0.002 m for well-adjusted laser plummets), and when centering the prism over the target point, either by using tribrach-mounted prisms or by using prism-pole assemblies. The errors here can range from 2 mm for well-adjusted optical/laser plummets to several millimeters for well-adjusted prism-pole circular bubble levels. When laser or optical plummets, and/or prism-pole levels, are poorly adjusted, serious errors can occur in distance measurements—errors that do diminish in relative severity as the distance measured increases. For these reasons, the steel tape remains a valuable tool for the engineering surveyor.

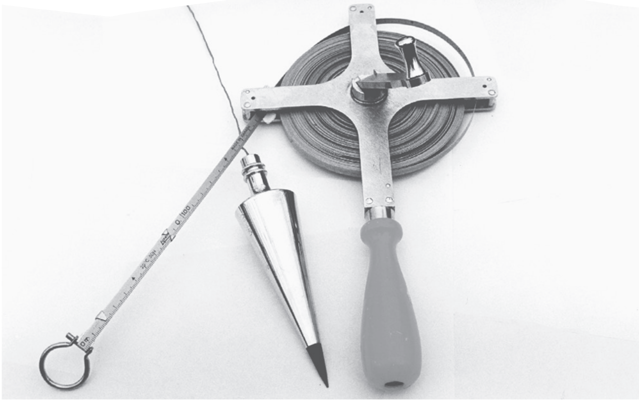


FIGURE 3.2 Steel tape and plumb bob.

Steel tapes (Figure 3.2) are manufactured in both foot and metric units and come in a variety of lengths, graduations, and unit weights. Commonly used foot-unit tapes are of 100-, 200-, and 300-ft lengths, with the 100-ft length being the most widely used. Commonly used metric-unit tapes are of 20-, 30-, 50-, and 100-m lengths. The steel tape of 30-m length is the most widely used because it closely resembles the 100-ft length tape in field characteristics.

Generally, lightweight tapes are graduated throughout and are used on the reel; heavier tapes are designed for use off the reel (drag tapes) and do not have continuous small-interval markings. Drag tapes are popular in route surveys (highways, railways, transmission lines, etc.), whereas lightweight tapes are more popular in building and municipal works.

Invar tapes are composed of 35 percent nickel and 65 percent steel. This alloy has a very low coefficient of thermal expansion, which made this tape useful for pre-EDM precise distance measurement. Steel tapes are occasionally referred to as chains, a throwback to early surveying practice, and to distinguish them from the 25-ft carpenter's tape.

3.4.2 Types of Readouts

Steel tapes are normally graduated in one of three ways; consider a distance of 38.82 ft (m):

1. The tape is graduated throughout in feet and hundredths (0.01) of a foot, or in meters and millimeters (see Figures 3.2 and 3.3). The distance (38.82 ft) is read directly from the steel tape.
2. The cut tape is marked throughout in feet, with the first and last foot graduated in tenths and hundredths of a foot [Figure 3.3(a)]. The metric cut tape is marked throughout in meters and decimeters, with the first and last decimeters graduated in millimeters. A measurement is made with the cut tape by one surveyor holding the even-foot mark (39 ft in this example). This arrangement allows the other surveyor to read a distance on the first foot (decimeter), which is graduated in hundredths of a foot (millimeters). For example, the distance from A to B in Figure 3.3(a) is determined by holding 39 ft at B and reading 0.18 ft at A.

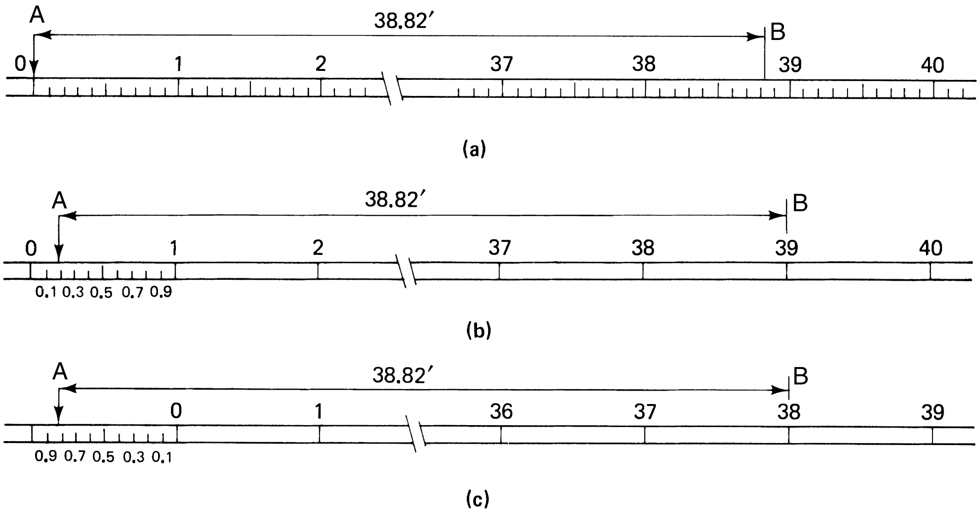


FIGURE 3.3 Various steel tape markings (hundredth marks not shown). (a) Fully graduated tape; (b) cut tape; (c) add tape.

Distance AB = 38.82 ft (i.e., 39 ft cut 0.18). Because each measurement involves this type of mental subtraction, care and attention are required from the surveyor to avoid unwelcome blunders.

3. The add tape is also marked throughout in feet, with the last foot graduated in hundredths of a foot. An additional foot, graduated in hundredths, is included prior to the zero mark. For metric tapes, the last decimeter and the extra before-the-zero decimeter are graduated in millimeters. The distance AB in Figure 3.3(b) is determined by holding 38 ft at B and reading 0.82 ft at A. Distance AB is 38.82 ft (i.e., 38 ft add 0.82).

As noted, cut tapes have the disadvantage of creating possibilities of subtraction mistakes; the add tapes have the disadvantage of forcing the surveyor to adopt awkward measuring stances when measuring from the zero mark. The full meter add tape is the most difficult to use correctly because the surveyor must fully extend his or her left (right) arm (which is holding the end of the tape) to position the zero mark on the tape over the ground point. The problems associated with both add and cut tapes can be eliminated if, instead, the surveyor uses tapes graduated throughout. These tapes are available in both drag- and reel-type tapes.

3.5 TAPING ACCESSORIES AND THEIR USE

3.5.1 Plumb Bob

Plumb bobs are normally made of brass and weigh from 8 to 18 oz, with 10 and 12 oz being the most common. Plumb bobs are used in taping to transfer from tape to ground (or vice versa) when the tape is being held off the ground to maintain its horizontal