


Pearson New International Edition

Physics for Scientists & Engineers with Modern Physics Vol 3

Douglas C. Giancoli

Fourth Edition

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Table of Contents

USE OF COLOR

Vectors

- A general vector
  - resultant vector (sum) is slightly thicker
  - components of any vector are dashed
- Displacement ($\vec{\mathbf{D}}, \vec{\mathbf{r}}$)
- Velocity ($\vec{\mathbf{v}}$)
- Acceleration ($\vec{\mathbf{a}}$)
- Force ($\vec{\mathbf{F}}$)
  - Force on second or
  - third object in same figure
- Momentum ($\vec{\mathbf{p}}$ or $m\vec{\mathbf{v}}$)
- Angular momentum ($\vec{\mathbf{L}}$)
- Angular velocity ($\vec{\boldsymbol{\omega}}$)
- Torque ($\vec{\boldsymbol{\tau}}$)
- Electric field ($\vec{\mathbf{E}}$)
- Magnetic field ($\vec{\mathbf{B}}$)

Electricity and magnetism

- Electric field lines
- Equipotential lines
- Magnetic field lines
- Electric charge (+) — + or +
- Electric charge (–) — – or –

Electric circuit symbols

- Wire, with switch S
- Resistor
- Capacitor
- Inductor
- Battery
- Ground

Optics

- Light rays
- Object
- Real image (dashed)
- Virtual image (dashed and paler)

Other

- Energy level (atom, etc.)
- Measurement lines — 1.0 m
- Path of a moving object
- Direction of motion or current

Fundamental Constants

Quantity	Symbol	Approximate Value	Current Best Value[†]
Speed of light in vacuum	c	3.00×10^8 m/s	2.99792458×10^8 m/s
Gravitational constant	G	6.67×10^{-11} N·m²/kg²	$6.6728(67) \times 10^{-11}$ N·m²/kg²
Avogadro's number	N_A	6.02×10^{23} mol^{-1}	$6.02214179(30) \times 10^{23}$ mol^{-1}
Gas constant	R	8.314 J/mol·K = 1.99 cal/mol·K = 0.0821 L·atm/mol·K	8.314472(15) J/mol·K
Boltzmann's constant	k	1.38×10^{-23} J/K	$1.3806504(24) \times 10^{-23}$ J/K
Charge on electron	e	1.60×10^{-19} C	$1.602176487(40) \times 10^{-19}$ C
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m²·K⁴	$5.670400(40) \times 10^{-8}$ W/m²·K⁴
Permittivity of free space	$\epsilon_0 = (1/c^2\mu_0)$	8.85×10^{-12} C²/N·m²	$8.854187817\ldots \times 10^{-12}$ C²/N·m²
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T·m/A	$1.2566370614\ldots \times 10^{-6}$ T·m/A
Planck's constant	h	6.63×10^{-34} J·s	$6.62606896(33) \times 10^{-34}$ J·s
Electron rest mass	m_e	9.11×10^{-31} kg = 0.000549 u = 0.511 MeV/c^2	$9.10938215(45) \times 10^{-31}$ kg = $5.4857990943(23) \times 10^{-4}$ u
Proton rest mass	m_p	1.6726×10^{-27} kg = 1.00728 u = 938.27 MeV/c^2	$1.672621637(83) \times 10^{-27}$ kg = 1.00727646677(10) u
Neutron rest mass	m_n	1.6749×10^{-27} kg = 1.008665 u = 939.57 MeV/c^2	$1.674927211(84) \times 10^{-27}$ kg = 1.00866491597(43) u
Atomic mass unit (1 u)		1.6605×10^{-27} kg = 931.49 MeV/c^2	$1.660538782(83) \times 10^{-27}$ kg = 931.494028(23) MeV/c^2

[†] CODATA (3/07), Peter J. Mohr and Barry N. Taylor, National Institute of Standards and Technology. Numbers in parentheses indicate one-standard-deviation experimental uncertainties in final digits. Values without parentheses are exact (i.e., defined quantities).

Other Useful Data

Joule equivalent (1 cal)	4.186 J
Absolute zero (0 K)	−273.15°C
Acceleration due to gravity at Earth's surface (avg.)	9.80 m/s² (= g)
Speed of sound in air (20°C)	343 m/s
Density of air (dry)	1.29 kg/m³
Earth: Mass	5.98×10^{24} kg
Radius (mean)	6.38×10^3 km
Moon: Mass	7.35×10^{22} kg
Radius (mean)	1.74×10^3 km
Sun: Mass	1.99×10^{30} kg
Radius (mean)	6.96×10^5 km
Earth–Sun distance (mean)	149.6×10^6 km
Earth–Moon distance (mean)	384×10^3 km

The Greek Alphabet

Alpha	Α	α	Nu	Ν	ν
Beta	Β	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	Ο	o
Delta	Δ	δ	Pi	Π	π
Epsilon	Ε	ϵ, ε	Rho	Ρ	ρ
Zeta	Ζ	ζ	Sigma	Σ	σ
Eta	Η	η	Tau	Τ	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	Ι	ι	Phi	Φ	ϕ, φ
Kappa	Κ	κ	Chi	Χ	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	Μ	μ	Omega	Ω	ω

Values of Some Numbers

$\pi = 3.1415927$	$\sqrt{2} = 1.4142136$	$\ln 2 = 0.6931472$	$\log_{10} e = 0.4342945$
$e = 2.7182818$	$\sqrt{3} = 1.7320508$	$\ln 10 = 2.3025851$	1 rad = 57.2957795°

Mathematical Signs and Symbols

$\propto$	is proportional to	$\leq$	is less than or equal to
$=$	is equal to	$\geq$	is greater than or equal to
$\approx$	is approximately equal to	$\sum$	sum of
$\neq$	is not equal to	$\bar{x}$	average value of x
$>$	is greater than	Δx	change in x
$\gg$	is much greater than	$\Delta x \to 0$	Δx approaches zero
$<$	is less than	$n!$	$n(n-1)(n-2)\ldots(1)$
$\ll$	is much less than		

Properties of Water

Density (4°C)	1.000×10^3 kg/m³
Heat of fusion (0°C)	333 kJ/kg (80 kcal/kg)
Heat of vaporization (100°C)	2260 kJ/kg (539 kcal/kg)
Specific heat (15°C)	4186 J/kg·C° (1.00 kcal/kg·C°)
Index of refraction	1.33

Unit Conversions (Equivalents)

Length

1 in. = 2.54 cm (defined)
1 cm = 0.3937 in.
1 ft = 30.48 cm
1 m = 39.37 in. = 3.281 ft
1 mi = 5280 ft = 1.609 km
1 km = 0.6214 mi
1 nautical mile (U.S.) = 1.151 mi = 6076 ft = 1.852 km
1 fermi = 1 femtometer (fm) = 10^{-15} m
1 angstrom (Å) = 10^{-10} m = 0.1 nm
1 light-year (ly) = 9.461×10^{15} m
1 parsec = 3.26 ly = 3.09×10^{16} m

Volume

1 liter (L) = 1000 mL = 1000 cm^3 = 1.0×10^{-3} m^3 = 1.057 qt (U.S.) = 61.02 $in.^3$
1 gal (U.S.) = 4 qt (U.S.) = 231 $in.^3$ = 3.785 L = 0.8327 gal (British)
1 quart (U.S.) = 2 pints (U.S.) = 946 mL
1 pint (British) = 1.20 pints (U.S.) = 568 mL
1 m^3 = 35.31 ft^3

Speed

1 mi/h = 1.4667 ft/s = 1.6093 km/h = 0.4470 m/s
1 km/h = 0.2778 m/s = 0.6214 mi/h
1 ft/s = 0.3048 m/s (exact) = 0.6818 mi/h = 1.0973 km/h
1 m/s = 3.281 ft/s = 3.600 km/h = 2.237 mi/h
1 knot = 1.151 mi/h = 0.5144 m/s

Angle

1 radian (rad) = 57.30° = 57°18′
1° = 0.01745 rad
1 rev/min (rpm) = 0.1047 rad/s

Time

1 day = 8.640×10^4 s
1 year = 3.156×10^7 s

Mass

1 atomic mass unit (u) = 1.6605×10^{-27} kg
1 kg = 0.06852 slug
[1 kg has a weight of 2.20 lb where g = 9.80 m/s^2.]

Force

1 lb = 4.44822 N
1 N = 10^5 dyne = 0.2248 lb

Energy and Work

1 J = 10^7 ergs = 0.7376 ft·lb
1 ft·lb = 1.356 J = 1.29×10^{-3} Btu = 3.24×10^{-4} kcal
1 kcal = 4.19×10^3 J = 3.97 Btu
1 eV = 1.6022×10^{-19} J
1 kWh = 3.600×10^6 J = 860 kcal
1 Btu = 1.055×10^3 J

Power

1 W = 1 J/s = 0.7376 ft·lb/s = 3.41 Btu/h
1 hp = 550 ft·lb/s = 746 W

Pressure

1 atm = 1.01325 bar = 1.01325×10^5 N/m^2
= 14.7 $lb/in.^2$ = 760 torr
1 $lb/in.^2$ = 6.895×10^3 N/m^2
1 Pa = 1 N/m^2 = 1.450×10^{-4} $lb/in.^2$

SI Derived Units and Their Abbreviations

Quantity	Unit	Abbreviation	In Terms of Base Units†
Force	newton	N	$kg \cdot m/s^2$
Energy and work	joule	J	$kg \cdot m^2/s^2$
Power	watt	W	$kg \cdot m^2/s^3$
Pressure	pascal	Pa	$kg/(m \cdot s^2)$
Frequency	hertz	Hz	s^{-1}
Electric charge	coulomb	C	$A \cdot s$
Electric potential	volt	V	$kg \cdot m^2/(A \cdot s^3)$
Electric resistance	ohm	Ω	$kg \cdot m^2/(A^2 \cdot s^3)$
Capacitance	farad	F	$A^2 \cdot s^4/(kg \cdot m^2)$
Magnetic field	tesla	T	$kg/(A \cdot s^2)$
Magnetic flux	weber	Wb	$kg \cdot m^2/(A \cdot s^2)$
Inductance	henry	H	$kg \cdot m^2/(s^2 \cdot A^2)$

† kg = kilogram (mass), m = meter (length), s = second (time), A = ampere (electric current).

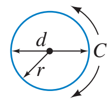
Metric (SI) Multipliers

Prefix	Abbreviation	Value
yotta	Y	10^{24}
zeta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deka	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

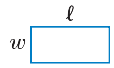
Useful Geometry Formulas—Areas, Volumes

Circumference of circle $C = \pi d = 2\pi r$

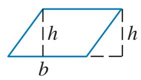
Area of circle $A = \pi r^2 = \dfrac{\pi d^2}{4}$



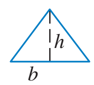
Area of rectangle $A = \ell w$

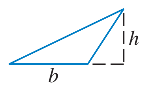


Area of parallelogram $A = bh$

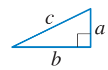


Area of triangle $A = \frac{1}{2} hb$

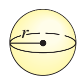




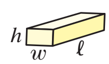
Right triangle (Pythagoras) $c^2 = a^2 + b^2$



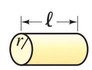
Sphere: surface area $A = 4\pi r^2$
volume $V = \frac{4}{3}\pi r^3$



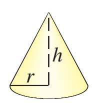
Rectangular solid:
volume $V = \ell w h$



Cylinder (right):
surface area $A = 2\pi r\ell + 2\pi r^2$
volume $V = \pi r^2 \ell$



Right circular cone:
surface area $A = \pi r^2 + \pi r\sqrt{r^2 + h^2}$
volume $V = \frac{1}{3}\pi r^2 h$



Exponents

$(a^n)(a^m) = a^{n+m}$ [Example: $(a^3)(a^2) = a^5$]
$(a^n)(b^n) = (ab)^n$ [Example: $(a^3)(b^3) = (ab)^3$]
$(a^n)^m = a^{nm}$ [Example: $(a^3)^2 = a^6$; Example: $(a^{\frac{1}{4}})^4 = a$]

$a^{-1} = \dfrac{1}{a}$ $\quad a^{-n} = \dfrac{1}{a^n}$ $\quad a^0 = 1$

$a^{\frac{1}{2}} = \sqrt{a}$ $\quad a^{\frac{1}{4}} = \sqrt{\sqrt{a}}$

$(a^n)(a^{-m}) = \dfrac{a^n}{a^m} = a^{n-m}$ [Ex.: $(a^5)(a^{-2}) = a^3$]

$\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$

Logarithms [Appendix A–7; Table A–1]

If $y = 10^x$, then $x = \log_{10} y = \log y$.
If $y = e^x$, then $x = \log_e y = \ln y$.
$\log(ab) = \log a + \log b$

$\log\left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Some Derivatives and Integrals†

$\dfrac{d}{dx} x^n = n x^{n-1}$ $\qquad \displaystyle\int \sin ax\, dx = -\frac{1}{a}\cos ax$

$\dfrac{d}{dx} \sin ax = a \cos ax$ $\qquad \displaystyle\int \cos ax\, dx = \frac{1}{a}\sin ax$

$\dfrac{d}{dx} \cos ax = -a \sin ax$ $\qquad \displaystyle\int \frac{1}{x}\, dx = \ln x$

$\displaystyle\int x^m\, dx = \frac{1}{m+1} x^{m+1}$ $\qquad \displaystyle\int e^{ax}\, dx = \frac{1}{a} e^{ax}$

† See Appendix B for more.

Quadratic Formula

Equation with unknown x, in the form

$$ax^2 + bx + c = 0,$$

has solutions

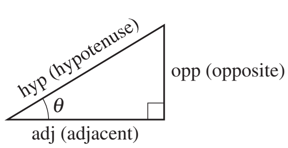
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2\cdot 1} x^2 \pm \frac{n(n-1)(n-2)}{3\cdot 2\cdot 1} x^3 + \cdots \quad [\text{for } x^2 < 1]$$

$$\approx 1 \pm nx \quad [\text{for } x \ll 1]$$

Trigonometric Formulas [Appendix A–9]



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$ (Pythagorean theorem)

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = (\cos^2\theta - \sin^2\theta) = (1 - 2\sin^2\theta) = (2\cos^2\theta - 1)$

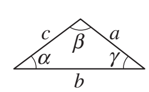
$\sin(180° - \theta) = \sin\theta$ $\qquad \cos(180° - \theta) = -\cos\theta$
$\sin(90° - \theta) = \cos\theta$
$\cos(90° - \theta) = \sin\theta$
$\sin\frac{1}{2}\theta = \sqrt{(1 - \cos\theta)/2}$ $\qquad \cos\frac{1}{2}\theta = \sqrt{(1 + \cos\theta)/2}$
$\sin\theta \approx \theta$ [for small $\theta \lesssim 0.2$ rad]

$\cos\theta \approx 1 - \dfrac{\theta^2}{2}$ [for small $\theta \lesssim 0.2$ rad]

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

For any triangle:
$c^2 = a^2 + b^2 - 2ab\cos\gamma$ (law of cosines)

$\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$ (law of sines)



Periodic Table of the Elements§

Symbol — **Cl** 17 — Atomic Number
Atomic Mass§ — 35.453
$3p^5$ — Electron Configuration (outer shells only)

Group I	Group II	Transition Elements										Group III	Group IV	Group V	Group VI	Group VII	Group VIII
H 1 1.00794 $1s^1$																	**He** 2 4.002602 $1s^2$
Li 3 6.941 $2s^1$	**Be** 4 9.012182 $2s^2$											**B** 5 10.811 $2p^1$	**C** 6 12.0107 $2p^2$	**N** 7 14.0067 $2p^3$	**O** 8 15.9994 $2p^4$	**F** 9 18.9984032 $2p^5$	**Ne** 10 20.1797 $2p^6$
Na 11 22.98976928 $3s^1$	**Mg** 12 24.3050 $3s^2$											**Al** 13 26.9815386 $3p^1$	**Si** 14 28.0855 $3p^2$	**P** 15 30.973762 $3p^3$	**S** 16 32.065 $3p^4$	**Cl** 17 35.453 $3p^5$	**Ar** 18 39.948 $3p^6$
K 19 39.0983 $4s^1$	**Ca** 20 40.078 $4s^2$	**Sc** 21 44.955912 $3d^14s^2$	**Ti** 22 47.867 $3d^24s^2$	**V** 23 50.9415 $3d^34s^2$	**Cr** 24 51.9961 $3d^54s^1$	**Mn** 25 54.938045 $3d^54s^2$	**Fe** 26 55.845 $3d^64s^2$	**Co** 27 58.933195 $3d^74s^2$	**Ni** 28 58.6934 $3d^84s^2$	**Cu** 29 63.546 $3d^{10}4s^1$	**Zn** 30 65.409 $3d^{10}4s^2$	**Ga** 31 69.723 $4p^1$	**Ge** 32 72.64 $4p^2$	**As** 33 74.92160 $4p^3$	**Se** 34 78.96 $4p^4$	**Br** 35 79.904 $4p^5$	**Kr** 36 83.798 $4p^6$
Rb 37 85.4678 $5s^1$	**Sr** 38 87.62 $5s^2$	**Y** 39 88.90585 $4d^15s^2$	**Zr** 40 91.224 $4d^25s^2$	**Nb** 41 92.90638 $4d^45s^1$	**Mo** 42 95.94 $4d^55s^1$	**Tc** 43 (98) $4d^55s^2$	**Ru** 44 101.07 $4d^75s^1$	**Rh** 45 102.90550 $4d^85s^1$	**Pd** 46 106.42 $4d^{10}5s^0$	**Ag** 47 107.8682 $4d^{10}5s^1$	**Cd** 48 112.411 $4d^{10}5s^2$	**In** 49 114.818 $5p^1$	**Sn** 50 118.710 $5p^2$	**Sb** 51 121.760 $5p^3$	**Te** 52 127.60 $5p^4$	**I** 53 126.90447 $5p^5$	**Xe** 54 131.293 $5p^6$
Cs 55 132.9054519 $6s^1$	**Ba** 56 137.327 $6s^2$	57–71†	**Hf** 72 178.49 $5d^26s^2$	**Ta** 73 180.94788 $5d^36s^2$	**W** 74 183.84 $5d^46s^2$	**Re** 75 186.207 $5d^56s^2$	**Os** 76 190.23 $5d^66s^2$	**Ir** 77 192.217 $5d^76s^2$	**Pt** 78 195.084 $5d^96s^1$	**Au** 79 196.966569 $5d^{10}6s^1$	**Hg** 80 200.59 $5d^{10}6s^2$	**Tl** 81 204.3833 $6p^1$	**Pb** 82 207.2 $6p^2$	**Bi** 83 208.98040 $6p^3$	**Po** 84 (209) $6p^4$	**At** 85 (210) $6p^5$	**Rn** 86 (222) $6p^6$
Fr 87 (223) $7s^1$	**Ra** 88 (226) $7s^2$	89–103‡	**Rf** 104 (267) $6d^27s^2$	**Db** 105 (268) $6d^37s^2$	**Sg** 106 (271) $6d^47s^2$	**Bh** 107 (272) $6d^57s^2$	**Hs** 108 (277) $6d^67s^2$	**Mt** 109 (276) $6d^77s^2$	**Ds** 110 (281) $6d^97s^1$	**Rg** 111 (280) $6d^{10}7s^1$	112 (285) $6d^{10}7s^2$						

†**Lanthanide Series**	**La** 57 138.90547 $5d^16s^2$	**Ce** 58 140.116 $4f^15d^16s^2$	**Pr** 59 140.90765 $4f^35d^06s^2$	**Nd** 60 144.242 $4f^45d^06s^2$	**Pm** 61 (145) $4f^55d^06s^2$	**Sm** 62 150.36 $4f^65d^06s^2$	**Eu** 63 151.964 $4f^75d^06s^2$	**Gd** 64 157.25 $4f^75d^16s^2$	**Tb** 65 158.92535 $4f^95d^06s^2$	**Dy** 66 162.500 $4f^{10}5d^06s^2$	**Ho** 67 164.93032 $4f^{11}5d^06s^2$	**Er** 68 167.259 $4f^{12}5d^06s^2$	**Tm** 69 168.93421 $4f^{13}5d^06s^2$	**Yb** 70 173.04 $4f^{14}5d^06s^2$	**Lu** 71 174.967 $4f^{14}5d^16s^2$
‡**Actinide Series**	**Ac** 89 (227) $6d^17s^2$	**Th** 90 232.03806 $6d^27s^2$	**Pa** 91 231.03588 $5f^26d^17s^2$	**U** 92 238.0289 $5f^36d^17s^2$	**Np** 93 (237) $5f^46d^17s^2$	**Pu** 94 (244) $5f^66d^07s^2$	**Am** 95 (243) $5f^76d^07s^2$	**Cm** 96 (247) $5f^76d^17s^2$	**Bk** 97 (247) $5f^96d^07s^2$	**Cf** 98 (251) $5f^{10}6d^07s^2$	**Es** 99 (252) $5f^{11}6d^07s^2$	**Fm** 100 (257) $5f^{12}6d^07s^2$	**Md** 101 (258) $5f^{13}6d^07s^2$	**No** 102 (259) $5f^{14}6d^07s^2$	**Lr** 103 (262) $5f^{14}6d^17s^2$

§ Atomic mass values averaged over isotopes in percentages they occur on Earth's surface. For many unstable elements, mass of the longest-lived known isotope is given in parentheses. 2006 revisions. Preliminary evidence (unconfirmed) has been reported for elements 113, 114, 115, 116 and 118.

Preface

Preface

I was motivated from the beginning to write a textbook different from others that present physics as a sequence of facts, like a Sears catalog: "here are the facts and you better learn them." Instead of that approach in which topics are begun formally and dogmatically, I have sought to begin each topic with concrete observations and experiences students can relate to: start with specifics and only then go to the great generalizations and the more formal aspects of a topic, showing *why* we believe what we believe. This approach reflects how science is actually practiced.

Why a Fourth Edition?

Two recent trends in physics texbooks are disturbing: (1) their revision cycles have become short—they are being revised every 3 or 4 years; (2) the books are getting larger, some over 1500 pages. I don't see how either trend can be of benefit to students. My response: (1) It has been 8 years since the previous edition of this book. (2) This book makes use of physics education research, although it avoids the detail a Professor may need to say in class but in a book shuts down the reader. And this book still remains among the shortest.

This new edition introduces some important new pedagogic tools. It contains new physics (such as in cosmology) and many new appealing applications (list on previous page). Pages and page breaks have been carefully formatted to make the physics easier to follow: no turning a page in the middle of a derivation or Example. Great efforts were made to make the book attractive so students will want to *read* it.

Some of the new features are listed below.

What's New

Chapter-Opening Questions: Each Chapter begins with a multiple-choice question, whose responses include common misconceptions. Students are asked to answer before starting the Chapter, to get them involved in the material and to get any preconceived notions out on the table. The issues reappear later in the Chapter, usually as Exercises, after the material has been covered. The Chapter-Opening Questions also show students the power and usefulness of Physics.

APPROACH paragraph in worked-out numerical Examples: A short introductory paragraph before the Solution, outlining an approach and the steps we can take to get started. Brief NOTES after the Solution may remark on the Solution, may give an alternate approach, or mention an application.

Step-by-Step Examples: After many Problem Solving Strategies (more than 20 in the book), the next Example is done step-by-step following precisely the steps just seen.

Exercises within the text, after an Example or derivation, give students a chance to see if they have understood enough to answer a simple question or do a simple calculation. Many are multiple choice.

Greater clarity: No topic, no paragraph in this book was overlooked in the search to improve the clarity and conciseness of the presentation. Phrases and sentences that may slow down the principal argument have been eliminated: keep to the essentials at first, give the elaborations later.

$\vec{\mathbf{F}}, \vec{\mathbf{v}}, \vec{\mathbf{B}}$

Vector notation, arrows: The symbols for vector quantities in the text and Figures now have a tiny arrow over them, so they are similar to what we write by hand.

Cosmological Revolution: With generous help from top experts in the field, readers have the latest results.

Page layout: more than in the previous edition, serious attention has been paid to how each page is formatted. Examples and all important derivations and arguments are on facing pages. Students then don't have to turn back and forth. Throughout, readers see, on two facing pages, an important slice of physics.

New Applications: LCDs, digital cameras and electronic sensors (CCD, CMOS), electric hazards, GFCIs, photocopiers, inkjet and laser printers, metal detectors, underwater vision, curve balls, airplane wings, DNA, how we actually *see* images. (Turn back a page to see a longer list.)

Examples modified: more math steps are spelled out, and many new Examples added. About 10% of all Examples are Estimation Examples.

This Book is Shorter than other complete full-service books at this level. Shorter explanations are easier to understand and more likely to be read.

Content and Organizational Changes

- **Rotational Motion**: Chapters 10 and 11 have been reorganized. All of angular momentum is now in Chapter 11.
- **First law of thermodynamics**, in Chapter 19, has been rewritten and extended. The full form is given: $\Delta K + \Delta U + \Delta E_{\text{int}} = Q - W$, where internal energy is E_{int}, and U is potential energy; the form $Q - W$ is kept so that $dW = P\,dV$.
- Kinematics and Dynamics of Circular Motion are now treated together in Chapter 5.
- Work and Energy, Chapters 7 and 8, have been carefully revised.
- Work done by friction is discussed now with energy conservation (energy terms due to friction).
- Chapters on Inductance and AC Circuits have been combined into one: Chapter 30.
- Graphical Analysis and Numerical Integration is a new optional Section 2–9. Problems requiring a computer or graphing calculator are found at the end of most Chapters.
- Length of an object is a script ℓ rather than normal *l*, which looks like 1 or I (moment of inertia, current), as in $F = I\ell B$. Capital L is for angular momentum, latent heat, inductance, dimensions of length $[L]$.
- Newton's law of gravitation remains in Chapter 6. Why? Because the $1/r^2$ law is too important to relegate to a late chapter that might not be covered at all late in the semester; furthermore, it is one of the basic forces in nature. In Chapter 8 we can treat real gravitational potential energy and have a fine instance of using $U = -\int \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}}$.
- New Appendices include the differential form of Maxwell's equations and more on dimensional analysis.
- Problem Solving Strategies are found on pages 30, 58, 64, 96, 102, 125, 166, 198, 229, 261, 314, 504, 551, 571, 600, 685, 716, 740, 763, 849, 871, and 913.

Organization

Some instructors may find that this book contains more material than can be covered in their courses. The text offers great flexibility. Sections marked with a star * are considered optional. These contain slightly more advanced physics material, or material not usually covered in typical courses and/or interesting applications; they contain no material needed in later Chapters (except perhaps in later optional Sections). For a brief course, all optional material could be dropped as well as major parts of Chapters 1, 13, 16, 26, 30, and 35, and selected parts of Chapters 9, 12, 19, 20, 33, and the modern physics Chapters. Topics not covered in class can be a valuable resource for later study by students. Indeed, this text can serve as a useful reference for years because of its wide range of coverage.

Versions of this Book

Complete version: 44 Chapters including 9 Chapters of modern physics.

Classic version: 37 Chapters including one each on relativity and quantum theory.

3 Volume version: Available separately or packaged together (Vols. 1 & 2 or all 3 Volumes):

Volume 1: Chapters 1–20 on mechanics, including fluids, oscillations, waves, plus heat and thermodynamics.

Volume 2: Chapters 21–35 on electricity and magnetism, plus light and optics.

Volume 3: Chapters 36–44 on modern physics: relativity, quantum theory, atomic physics, condensed matter, nuclear physics, elementary particles, cosmology and astrophysics.

Thanks

Many physics professors provided input or direct feedback on every aspect of this textbook. They are listed below, and I owe each a debt of gratitude.

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The final responsibility for all errors lies with me. I welcome comments, corrections, and suggestions as soon as possible to benefit students for the next reprint.

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Douglas C. Giancoli obtained his BA in physics (summa cum laude) from the University of California, Berkeley, his MS in physics at the Massachusetts Institute of Technology, and his PhD in elementary particle physics at the University of California, Berkeley. He spent 2 years as a post-doctoral fellow at UC Berkeley's Virus lab developing skills in molecular biology and biophysics. His mentors include Nobel winners Emilio Segrè and Donald Glaser.

He has taught a wide range of undergraduate courses, traditional as well as innovative ones, and continues to update his texbooks meticulously, seeking ways to better provide an understanding of physics for students.

Doug's favorite spare-time activity is the outdoors, especially climbing peaks (here on a dolomite summit, Italy). He says climbing peaks is like learning physics: it takes effort and the rewards are great.

Online Supplements (partial list)

MasteringPhysics™ (www.masteringphysics.com)
is a sophisticated online tutoring and homework system developed specially for courses using calculus-based physics. Originally developed by David Pritchard and collaborators at MIT, MasteringPhysics provides **students** with individualized online tutoring by responding to their wrong answers and providing hints for solving multi-step problems when they get stuck. It gives them immediate and up-to-date assessment of their progress, and shows where they need to practice more. MasteringPhysics provides **instructors** with a fast and effective way to assign tried-and-tested online homework assignments that comprise a range of problem types. The powerful post-assignment diagnostics allow instructors to assess the progress of their class as a whole as well as individual students, and quickly identify areas of difficulty.

WebAssign (www.webassign.com)

CAPA and LON-CAPA (www.lon-capa.org)

Student Supplements (partial list)

Student Study Guide & Selected Solutions Manual (Volume I: 0-13-227324-1, Volumes II & III: 0-13-227325-X) by Frank Wolfs

Student Pocket Companion (0-13-227326-8) by Biman Das

Tutorials in Introductory Physics (0-13-097069-7)
by Lillian C. McDermott, Peter S. Schaffer, and the Physics Education Group at the University of Washington

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Mathematics for Physics with Calculus (0-13-191336-0)
by Biman Das

To Students

HOW TO STUDY

1. Read the Chapter. Learn new vocabulary and notation. Try to respond to questions and exercises as they occur.
2. Attend all class meetings. Listen. Take notes, especially about aspects you do not remember seeing in the book. Ask questions (everyone else wants to, but maybe you will have the courage). You will get more out of class if you read the Chapter first.
3. Read the Chapter again, paying attention to details. Follow derivations and worked-out Examples. Absorb their logic. Answer Exercises and as many of the end of Chapter Questions as you can.
4. Solve 10 to 20 end of Chapter Problems (or more), especially those assigned. In doing Problems you find out what you learned and what you didn't. Discuss them with other students. Problem solving is one of the great learning tools. Don't just look for a formula—it won't cut it.

NOTES ON THE FORMAT AND PROBLEM SOLVING

1. Sections marked with a star (*) are considered **optional**. They can be omitted without interrupting the main flow of topics. No later material depends on them except possibly later starred Sections. They may be fun to read, though.
2. The customary **conventions** are used: symbols for quantities (such as m for mass) are italicized, whereas units (such as m for meter) are not italicized. Symbols for vectors are shown in boldface with a small arrow above: $\vec{\mathbf{F}}$.
3. Few equations are valid in all situations. Where practical, the **limitations** of important equations are stated in square brackets next to the equation. The equations that represent the great laws of physics are displayed with a tan background, as are a few other indispensable equations.
4. At the end of each Chapter is a set of **Problems** which are ranked as Level I, II, or III, according to estimated difficulty. Level I Problems are easiest, Level II are standard Problems, and Level III are "challenge problems." These ranked Problems are arranged by Section, but Problems for a given Section may depend on earlier material too. There follows a group of General Problems, which are not arranged by Section nor ranked as to difficulty. Problems that relate to optional Sections are starred (*). Most Chapters have 1 or 2 Computer/Numerical Problems at the end, requiring a computer or graphing calculator. Answers to odd-numbered Problems are given at the end of the book.
5. Being able to solve **Problems** is a crucial part of learning physics, and provides a powerful means for understanding the concepts and principles. This book contains many aids to problem solving: (a) worked-out **Examples** and their solutions in the text, which should be studied as an integral part of the text; (b) some of the worked-out Examples are **Estimation Examples**, which show how rough or approximate results can be obtained even if the given data are sparse (see Section 1–6); (c) special **Problem Solving Strategies** placed throughout the text to suggest a step-by-step approach to problem solving for a particular topic—but remember that the basics remain the same; most of these "Strategies" are followed by an Example that is solved by explicitly following the suggested steps; (d) special problem-solving Sections; (e) "Problem Solving" marginal notes which refer to hints within the text for solving Problems; (f) **Exercises** within the text that you should work out immediately, and then check your response against the answer given at the bottom of the last page of that Chapter; (g) the Problems themselves at the end of each Chapter (point 4 above).
6. **Conceptual Examples** pose a question which hopefully starts you to think and come up with a response. Give yourself a little time to come up with your own response before reading the Response given.
7. **Math** review, plus some additional topics, are found in Appendices. Useful data, conversion factors, and math formulas are found inside the front and back covers.

An early science fantasy book (1940), called *Mr Tompkins in Wonderland* by physicist George Gamow, imagined a world in which the speed of light was only 10 m/s (20 mi/h). Mr Tompkins had studied relativity and when he began "speeding" on a bicycle, he "expected that he would be immediately shortened, and was very happy about it as his increasing figure had lately caused him some anxiety. To his great surprise, however, nothing happened to him or to his cycle. On the other hand, the picture around him completely changed. The streets grew shorter, the windows of the shops began to look like narrow slits, and the policeman on the corner became the thinnest man he had ever seen. 'By Jove!' exclaimed Mr Tompkins excitedly, 'I see the trick now. This is where the word *relativity* comes in.' "

Relativity does indeed predict that objects moving relative to us at high speed, close to the speed of light c, are shortened in length. We don't notice it as Mr Tompkins did, because $c = 3 \times 10^8$ m/s is incredibly fast. We will study length contraction, time dilation, simultaneity non-agreement, and how energy and mass are equivalent $(E = mc^2)$.

Cambridge University Press; "The City Blocks Became Still Shorter" photo from page 4 of the book "Mr Tompkins in Paperback" by George Gamow. Reprinted with the permission of Cambridge University Press

The Special Theory of Relativity

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

A rocket is headed away from Earth at a speed of 0.80c. The rocket fires a missile at a speed of 0.70c (the missile is aimed away from Earth and leaves the rocket at 0.70c relative to the rocket). How fast is the missile moving relative to Earth?

(a) 1.50c;
(b) a little less than 1.50c;
(c) a little over c;
(d) a little under c;
(e) 0.75c.

Physics at the end of the nineteenth century looked back on a period of great progress. The theories developed over the preceding three centuries had been very successful in explaining a wide range of natural phenomena. Newtonian mechanics beautifully explained the motion of objects on Earth and in the heavens. Furthermore, it formed the basis for successful treatments of fluids, wave motion, and sound. Kinetic theory explained the behavior of gases and other materials. Maxwell's theory of electromagnetism not only brought together and explained electric and magnetic phenomena, but it predicted the existence of electromagnetic waves that would behave in every way just like light—so light came to be thought of as an electromagnetic wave. Indeed, it seemed that the natural world, as seen through the eyes of physicists, was very well explained. A few puzzles remained, but it was felt that these would soon be explained using already known principles.

CONTENTS

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

From Chapter 36 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

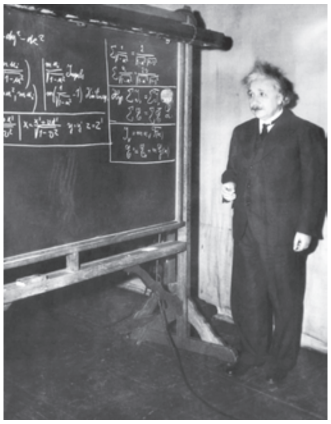

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FIGURE 1 Albert Einstein (1879–1955), one of the great minds of the twentieth century, was the creator of the special and general theories of relativity.

It did not turn out so simply. Instead, these puzzles were to be solved only by the introduction, in the early part of the twentieth century, of two revolutionary new theories that changed our whole conception of nature: the *theory of relativity* and *quantum theory.*

Physics as it was known at the end of the nineteenth century (what we've covered up to now in this book) is referred to as **classical physics**. The new physics that grew out of the great revolution at the turn of the twentieth century is now called **modern physics**. In this Chapter, we present the special theory of relativity, which was first proposed by Albert Einstein (1879–1955; Fig. 1) in 1905.

1 Galilean–Newtonian Relativity

Einstein's special theory of relativity deals with how we observe events, particularly how objects and events are observed from different frames of reference. This subject had, of course, already been explored by Galileo and Newton.

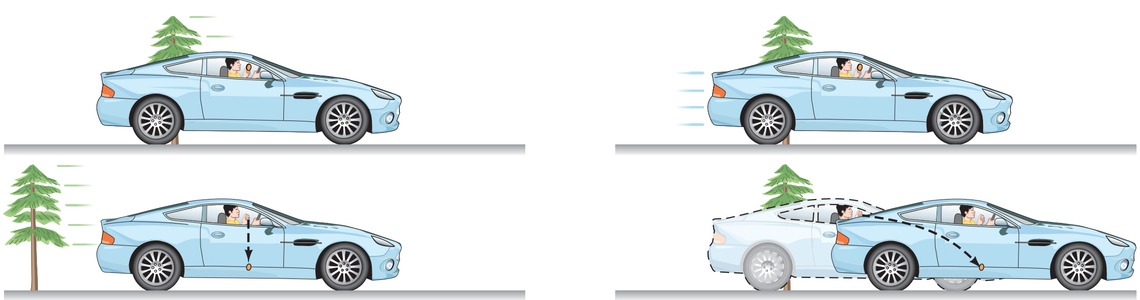
The special theory of relativity deals with events that are observed and measured from so-called **inertial reference frames,** which are reference frames in which Newton's first law is valid: if an object experiences no net force, the object either remains at rest or continues in motion with constant speed in a straight line. It is usually easiest to analyze events when they are observed and measured by observers at rest in an inertial frame. The Earth, though not quite an inertial frame (it rotates), is close enough that for most purposes we can consider it an inertial frame. Rotating or otherwise accelerating frames of reference are noninertial frames,† and won't concern us in this Chapter (they are dealt with in Einstein's general theory of relativity).

A reference frame that moves with constant velocity with respect to an inertial frame is itself also an inertial frame, since Newton's laws hold in it as well. When we say that we observe or make measurements from a certain reference frame, it means that we are at rest in that reference frame.

Both Galileo and Newton were aware of what we now call the **relativity principle** applied to mechanics: that *the basic laws of physics are the same in all inertial reference frames*. You may have recognized its validity in everyday life. For example, objects move in the same way in a smoothly moving (constant-velocity) train or airplane as they do on Earth. (This assumes no vibrations or rocking which would make the reference frame noninertial.) When you walk, drink a cup of soup, play pool, or drop a pencil on the floor while traveling in a train, airplane, or ship moving at constant velocity, the objects move just as they do when you are at rest on Earth. Suppose you are in a car traveling rapidly at constant velocity. If you drop a coin from above your head inside the car, how will it fall? It falls straight downward with respect to the car, and hits the floor directly below the point of release, Fig. 2a.

†On a rotating platform (say a merry-go-round), for example, an object at rest starts moving outward even though no object exerts a force on it. This is therefore not an inertial frame.

FIGURE 2 A coin is dropped by a person in a moving car. The upper views show the moment of the coin's release, the lower views are a short time later. (a) In the reference frame of the car, the coin falls straight down (and the tree moves to the left). (b) In a reference frame fixed on the Earth, the coin has an initial velocity (= to car's) and follows a curved (parabolic) path.



(a) Reference frame = car

(b) Reference frame = Earth

This is just how objects fall on the Earth—straight down—and thus our experiment in the moving car is in accord with the relativity principle. (If you drop the coin out the car's window, this won't happen because the moving air drags the coin backward relative to the car.)

Note in this example, however, that to an observer on the Earth, the coin follows a curved path, Fig. 2b. The actual path followed by the coin is different as viewed from different frames of reference. This does not violate the relativity principle because this principle states that the *laws* of physics are the same in all inertial frames. The same law of gravity, and the same laws of motion, apply in both reference frames. The acceleration of the coin is the same in both reference frames. The difference in Figs. 2a and b is that in the Earth's frame of reference, the coin has an initial velocity (equal to that of the car). The laws of physics therefore predict it will follow a parabolic path like any projectile. In the car's reference frame, there is no initial velocity, and the laws of physics predict that the coin will fall straight down. The laws are the same in both reference frames, although the specific paths are different.

CAUTION

Laws are the same, but paths may be different in different reference frames

Galilean–Newtonian relativity involves certain unprovable assumptions that make sense from everyday experience. It is assumed that the lengths of objects are the same in one reference frame as in another, and that time passes at the same rate in different reference frames. In classical mechanics, then, space and time intervals are considered to be **absolute**: their measurement does not change from one reference frame to another. The mass of an object, as well as all forces, are assumed to be unchanged by a change in inertial reference frame.

The position of an object, however, is different when specified in different reference frames, and so is velocity. For example, a person may walk inside a bus toward the front with a speed of 2 m/s. But if the bus moves 10 m/s with respect to the Earth, the person is then moving with a speed of 12 m/s with respect to the Earth. The acceleration of an object, however, is the same in any inertial reference frame according to classical mechanics. This is because the change in velocity, and the time interval, will be the same. For example, the person in the bus may accelerate from 0 to 2 m/s in 1.0 seconds, so $a = 2\,\mathrm{m/s^2}$ in the reference frame of the bus. With respect to the Earth, the acceleration is $(12\,\mathrm{m/s} - 10\,\mathrm{m/s})/(1.0\,\mathrm{s}) = 2\,\mathrm{m/s^2}$, which is the same.

CAUTION

Position and velocity are different in different reference frames, but length is the same (classical)

Since neither F, m, nor a changes from one inertial frame to another, then Newton's second law, $F = ma$, does not change. Thus Newton's second law satisfies the relativity principle. It is easily shown that the other laws of mechanics also satisfy the relativity principle.

That the laws of mechanics are the same in all inertial reference frames implies that no one inertial frame is special in any sense. We express this important conclusion by saying that **all inertial reference frames are equivalent** for the description of mechanical phenomena. No one inertial reference frame is any better than another. A reference frame fixed to a car or an aircraft traveling at constant velocity is as good as one fixed on the Earth. When you travel smoothly at constant velocity in a car or airplane, it is just as valid to say you are at rest and the Earth is moving as it is to say the reverse.[†] There is no experiment you can do to tell which frame is "really" at rest and which is moving. Thus, there is no way to single out one particular reference frame as being at absolute rest.

A complication arose, however, in the last half of the nineteenth century. Maxwell's comprehensive and successful theory of electromagnetism predicted that light is an electromagnetic wave. Maxwell's equations gave the velocity of light c as $3.00 \times 10^8\,\mathrm{m/s}$; and this is just what is measured. The question then arose: in what reference frame does light have precisely the value predicted by Maxwell's theory? It was assumed that light would have a different speed in different frames of reference. For example, if observers were traveling on a rocket ship at a speed of $1.0 \times 10^8\,\mathrm{m/s}$ away from a source of light, we might expect them to measure the speed of the light reaching them to be $(3.0 \times 10^8\,\mathrm{m/s}) - (1.0 \times 10^8\,\mathrm{m/s}) = 2.0 \times 10^8\,\mathrm{m/s}$. But Maxwell's equations have no provision for relative velocity. They predicted the speed of light to be $c = 3.0 \times 10^8\,\mathrm{m/s}$, which seemed to imply that there must be some preferred reference frame where c would have this value.

[†]We are ignoring the rotation and curvature of the Earth.

Waves can travel on water and along ropes or strings, and sound waves travel in air and other materials. Nineteenth-century physicists viewed the material world in terms of the laws of mechanics, so it was natural for them to assume that light too must travel in some *medium*. They called this transparent medium the **ether** and assumed it permeated all space.† It was therefore assumed that the velocity of light given by Maxwell's equations must be with respect to the ether.

At first it appeared that Maxwell's equations did *not* satisfy the relativity principle. They were simplest in the frame where $c = 3.00 \times 10^8\,\text{m/s}$; that is, in a reference frame at rest in the ether. In any other reference frame, extra terms would have to be added to take into account the relative velocity. Thus, although most of the laws of physics obeyed the relativity principle, the laws of electricity and magnetism apparently did not. Einstein's second postulate (Section 3) resolved this problem: Maxwell's equations do satisfy relativity.

Scientists soon set out to determine the speed of the Earth relative to this absolute frame, whatever it might be. A number of clever experiments were designed. The most direct were performed by A. A. Michelson and E. W. Morley in the 1880s. They measured the difference in the speed of light in different directions using Michelson's interferometer. They expected to find a difference depending on the orientation of their apparatus with respect to the ether. For just as a boat has different speeds relative to the land when it moves upstream, downstream, or across the stream, so too light would be expected to have different speeds depending on the velocity of the ether past the Earth.

Strange as it may seem, they detected no difference at all. This was a great puzzle. A number of explanations were put forth over a period of years, but they led to contradictions or were otherwise not generally accepted. This **null result** was one of the great puzzles at the end of the nineteenth century.

Then in 1905, Albert Einstein proposed a radical new theory that reconciled these many problems in a simple way. But at the same time, as we shall see, it completely changed our ideas of space and time.

*2 The Michelson–Morley Experiment

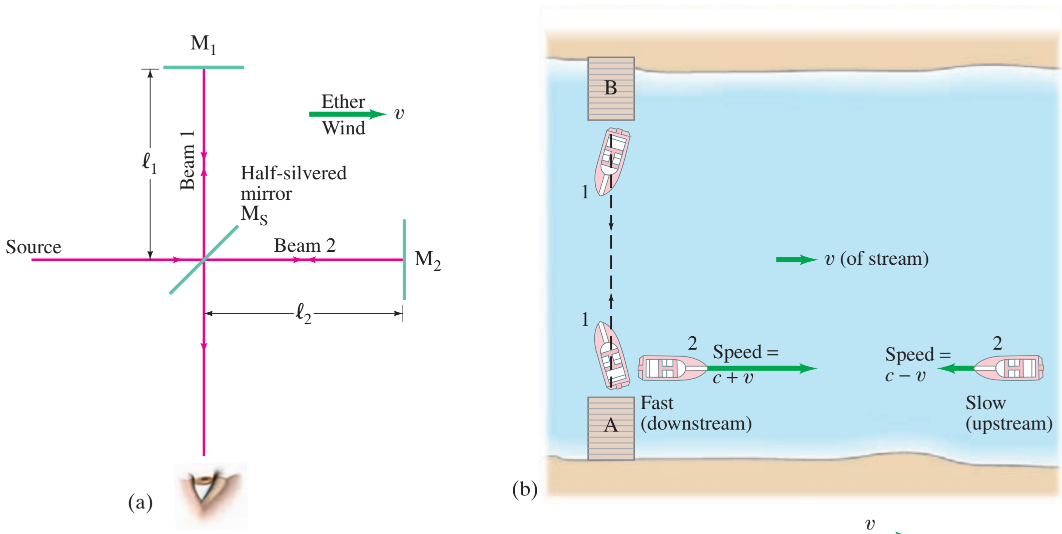
The Michelson–Morley experiment was designed to measure the speed of the *ether*—the medium in which light was assumed to travel—with respect to the Earth. The experimenters thus hoped to find an absolute reference frame, one that could be considered to be at rest.

One of the possibilities nineteenth-century scientists considered was that the ether is fixed relative to the Sun, for even Newton had taken the Sun as the center of the universe. If this were the case (there was no guarantee, of course), the Earth's speed of about $3 \times 10^4\,\text{m/s}$ in its orbit around the Sun could produce a change of 1 part in 10^4 in the speed of light $(3.0 \times 10^8\,\text{m/s})$. Direct measurement of the speed of light to this precision was not possible. But A. A. Michelson, later with the help of E. W. Morley, was able to use his interferometer to measure the difference in the speed of light in different directions to this precision.

This famous experiment is based on the principle shown in Fig. 3. Part (a) is a diagram of the Michelson interferometer, and it is assumed that the "ether wind" is moving with speed v to the right. (Alternatively, the Earth is assumed to move to the left with respect to the ether at speed v.) The light from a source is split into two beams by a half-silvered mirror M_S. One beam travels to mirror M_1 and the other to mirror M_2. The beams are reflected by M_1 and M_2 and are joined again after passing through M_S. The now superposed beams interfere with each other and the resultant is viewed by the observer's eye as an interference pattern.

Whether constructive or destructive interference occurs at the center of the interference pattern depends on the relative phases of the two beams after they have traveled their separate paths. Let us consider an analogy of a boat traveling up and

†The medium for light waves could not be air, since light travels from the Sun to Earth through nearly empty space. Therefore, another medium was postulated, the ether. The ether was not only transparent but, because of difficulty in detecting it, was assumed to have zero density.



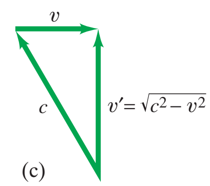


FIGURE 3 The Michelson–Morley experiment. (a) Michelson interferometer. (b) Boat analogy: boat 1 goes across the stream and back; boat 2 goes downstream and back upstream (boat has speed c relative to the water). (c) Calculation of the velocity of boat (or light beam) traveling perpendicular to the current (or ether wind).

down, and across, a river whose current moves with speed v, as shown in Fig. 3b. In still water, the boat can travel with speed c (not the speed of light in this case).

First we consider beam 2 in Fig. 3a, which travels parallel to the "ether wind." In its journey from M_S to M_2, the light would travel with speed $c + v$, according to classical physics, just as for a boat traveling downstream (see Fig. 3b) we add the speed of the river water to the boat's own speed (relative to the water) to get the boat's speed relative to the shore. Since the beam travels a distance ℓ_2, the time it takes to go from M_S to M_2 would be $t = \ell_2/(c + v)$. To make the return trip from M_2 to M_S, the light moves against the ether wind (like the boat going upstream), so its relative speed is expected to be $c - v$. The time for the return trip would be $\ell_2/(c - v)$. The total time for beam 2 to travel from M_S to M_2 and back to M_S is

$$t_2 = \frac{\ell_2}{c + v} + \frac{\ell_2}{c - v} = \frac{2\ell_2}{c\left(1 - v^2/c^2\right)}.$$

Now let us consider beam 1, which travels crosswise to the ether wind. Here the boat analogy (Fig. 3b) is especially helpful. The boat is to go from wharf A to wharf B directly across the stream. If it heads directly across, the stream's current will drag it downstream. To reach wharf B, the boat must head at an angle upstream. The precise angle depends on the magnitudes of c and v, but is of no interest to us in itself. Part (c) of Fig. 3 shows how to calculate the velocity v' of the boat relative to Earth as it crosses the stream. Since c, v, and v' form a right triangle, we have that $v' = \sqrt{c^2 - v^2}$. The boat has the same speed when it returns. If we now apply these principles to light beam 1 in Fig. 3a, we expect the beam to travel with speed $\sqrt{c^2 - v^2}$ in going from M_S to M_1 and back again. The total distance traveled is $2\ell_1$, so the time required for beam 1 to make the round trip would be $2\ell_1/\sqrt{c^2 - v^2}$, or

$$t_1 = \frac{2\ell_1}{c\sqrt{1 - v^2/c^2}}.$$

Notice that the denominator in this equation for t_1 involves a square root, whereas that for t_2 does not.

If $\ell_1 = \ell_2 = \ell$, we see that beam 2 will lag behind beam 1 by an amount

$$\Delta t = t_2 - t_1 = \frac{2\ell}{c}\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right).$$

If $v = 0$, then $\Delta t = 0$, and the two beams will return in phase since they were initially in phase. But if $v \neq 0$, then $\Delta t \neq 0$, and the two beams will return out of phase. If this change of phase from the condition $v = 0$ to that for $v \neq 0$ could be measured, then v could be determined. But the Earth cannot be stopped. Furthermore, we should not be too quick to assume that lengths are not affected by motion and therefore to assume $\ell_1 = \ell_2$.

Michelson and Morley realized that they could detect the difference in phase (assuming that $v \neq 0$) if they rotated their apparatus by 90°, for then the interference pattern between the two beams should change. In the rotated position, beam 1 would now move parallel to the ether and beam 2 perpendicular to it. Thus the roles could be reversed, and in the rotated position the times (designated by primes) would be

$$t_1' = \frac{2\ell_1}{c\left(1 - v^2/c^2\right)} \quad \text{and} \quad t_2' = \frac{2\ell_2}{c\sqrt{1 - v^2/c^2}}.$$

The time lag between the two beams in the nonrotated position (unprimed) would be

$$\Delta t = t_2 - t_1 = \frac{2\ell_2}{c\left(1 - v^2/c^2\right)} - \frac{2\ell_1}{c\sqrt{1 - v^2/c^2}}.$$

In the rotated position, the time difference would be

$$\Delta t' = t_2' - t_1' = \frac{2\ell_2}{c\sqrt{1 - v^2/c^2}} - \frac{2\ell_1}{c\left(1 - v^2/c^2\right)}.$$

When the rotation is made, the fringes of the interference pattern will shift an amount determined by the difference:

$$\Delta t - \Delta t' = \frac{2}{c}\left(\ell_1 + \ell_2\right)\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right).$$

This expression can be considerably simplified if we assume that $v/c \ll 1$. In this case we can use the binomial expansion, so

$$\frac{1}{1 - v^2/c^2} \approx 1 + \frac{v^2}{c^2} \quad \text{and} \quad \frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}.$$

Then

$$\begin{aligned}\Delta t - \Delta t' &\approx \frac{2}{c}\left(\ell_1 + \ell_2\right)\left(1 + \frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2}\right)\\ &\approx \left(\ell_1 + \ell_2\right)\frac{v^2}{c^3}.\end{aligned}$$

Now we assume $v = 3.0 \times 10^4\,\text{m/s}$, the speed of the Earth in its orbit around the Sun. In Michelson and Morley's experiments, the arms ℓ_1 and ℓ_2 were about 11 m long. The time difference would then be about

$$\frac{(22\,\text{m})\left(3.0 \times 10^4\,\text{m/s}\right)^2}{\left(3.0 \times 10^8\,\text{m/s}\right)^3} \approx 7.3 \times 10^{-16}\,\text{s}.$$

For visible light of wavelength $\lambda = 5.5 \times 10^{-7}\,\text{m}$, say, the frequency would be $f = c/\lambda = \left(3.0 \times 10^8\,\text{m/s}\right)/\left(5.5 \times 10^{-7}\,\text{m}\right) = 5.5 \times 10^{14}\,\text{Hz}$, which means that wave crests pass by a point every $1/\left(5.5 \times 10^{14}\,\text{Hz}\right) = 1.8 \times 10^{-15}\,\text{s}$. Thus, with a time difference of $7.3 \times 10^{-16}\,\text{s}$, Michelson and Morley should have noted a movement in the interference pattern of $\left(7.3 \times 10^{-16}\,\text{s}\right)/\left(1.8 \times 10^{-15}\,\text{s}\right) = 0.4$ fringe. They could easily have detected this, since their apparatus was capable of observing a fringe shift as small as 0.01 fringe.

But they found *no significant fringe shift whatever*! They set their apparatus at various orientations. They made observations day and night so that they would be at various orientations with respect to the Sun (due to the Earth's rotation).

They tried at different seasons of the year (the Earth at different locations due to its orbit around the Sun). Never did they observe a significant fringe shift.

This **null result** was one of the great puzzles of physics at the end of the nineteenth century. To explain it was a difficult challenge. One possibility to explain the null result was put forth independently by G. F. Fitzgerald and H. A. Lorentz (in the 1890s) in which they proposed that any length (including the arm of an interferometer) contracts by a factor $\sqrt{1 - v^2/c^2}$ in the direction of motion through the ether. According to Lorentz, this could be due to the ether affecting the forces between the molecules of a substance, which were assumed to be electrical in nature. This theory was eventually replaced by the far more comprehensive theory proposed by Albert Einstein in 1905—the special theory of relativity.

3 Postulates of the Special Theory of Relativity

The problems that existed at the start of the twentieth century with regard to electromagnetic theory and Newtonian mechanics were beautifully resolved by Einstein's introduction of the theory of relativity in 1905. Unaware of the Michelson–Morley null result, Einstein was motivated by certain questions regarding electromagnetic theory and light waves. For example, he asked himself: "What would I see if I rode a light beam?" The answer was that instead of a traveling electromagnetic wave, he would see alternating electric and magnetic fields at rest whose magnitude changed in space, but did not change in time. Such fields, he realized, had never been detected and indeed were not consistent with Maxwell's electromagnetic theory. He argued, therefore, that it was unreasonable to think that the speed of light relative to any observer could be reduced to zero, or in fact reduced at all. This idea became the second postulate of his theory of relativity.

In his famous 1905 paper, Einstein proposed doing away completely with the idea of the ether and the accompanying assumption of a preferred or absolute reference frame at rest. This proposal was embodied in two postulates. The first postulate was an extension of the Galilean–Newtonian relativity principle to include not only the laws of mechanics but also those of the rest of physics, including electricity and magnetism:

> ***First postulate (the relativity principle)*: The laws of physics have the same form in all inertial reference frames.**

The first postulate can also be stated as: *There is no experiment you can do in an inertial reference frame to tell if you are at rest or moving uniformly at constant velocity*.

The second postulate is consistent with the first:

> ***Second postulate (constancy of the speed of light)*: Light propagates through empty space with a definite speed *c* independent of the speed of the source or observer.**

These two postulates form the foundation of Einstein's **special theory of relativity**. It is called "special" to distinguish it from his later "general theory of relativity," which deals with noninertial (accelerating) reference frames. The special theory, which is what we discuss here, deals only with inertial frames.

The second postulate may seem hard to accept, for it seems to violate common sense. First of all, we have to think of light traveling through empty space. Giving up the ether is not too hard, however, since it had never been detected. But the second postulate also tells us that the speed of light in vacuum is always the same, 3.00×10^8 m/s, no matter what the speed of the observer or the source. Thus, a person traveling toward or away from a source of light will measure the same speed for that light as someone at rest with respect to the source. This conflicts with our everyday experience: we would expect to have to add in the velocity of the observer. On the other hand, perhaps we can't expect our everyday experience to be helpful when dealing with the high velocity of light. Furthermore, the null result of the Michelson–Morley experiment is fully consistent with the second postulate.[†]

[†]The Michelson–Morley experiment can also be considered as evidence for the first postulate, since it was intended to measure the motion of the Earth relative to an absolute reference frame. Its failure to do so implies the absence of any such preferred frame.

Einstein's proposal has a certain beauty. By doing away with the idea of an absolute reference frame, it was possible to reconcile classical mechanics with Maxwell's electromagnetic theory. The speed of light predicted by Maxwell's equations *is* the speed of light in vacuum in *any* reference frame.

Einstein's theory required us to give up common sense notions of space and time, and in the following Sections we will examine some strange but interesting consequences of special relativity. Our arguments for the most part will be simple ones. We will use a technique that Einstein himself did: we will imagine very simple experimental situations in which little mathematics is needed. In this way, we can see many of the consequences of relativity theory without getting involved in detailed calculations. Einstein called these "thought" experiments.

4 Simultaneity

An important consequence of the theory of relativity is that we can no longer regard time as an absolute quantity. No one doubts that time flows onward and never turns back. But the time interval between two events, and even whether or not two events are simultaneous, depends on the observer's reference frame. By an **event**, which we use a lot here, we mean something that happens at a particular place and at a particular time.

Two events are said to occur simultaneously if they occur at exactly the same time. But how do we know if two events occur precisely at the same time? If they occur at the same point in space—such as two apples falling on your head at the same time—it is easy. But if the two events occur at widely separated places, it is more difficult to know whether the events are simultaneous since we have to take into account the time it takes for the light from them to reach us. Because light travels at finite speed, a person who sees two events must calculate back to find out when they actually occurred. For example, if two events are *observed* to occur at the same time, but one actually took place farther from the observer than the other, then the more distant one must have occurred earlier, and the two events were not simultaneous.

We now imagine a simple thought experiment. Assume an observer, called O, is located exactly halfway between points A and B where two events occur, Fig. 4. Suppose the two events are lightning that strikes the points A and B, as shown. For brief events like lightning, only short pulses of light (blue in Fig. 4) will travel outward from A and B and reach O. Observer O "sees" the events when the pulses of light reach point O. If the two pulses reach O at the same time, then the two events had to be simultaneous. This is because the two light pulses travel at the same speed (postulate 2), and since the distance OA equals OB, the time for the light to travel from A to O and B to O must be the same. Observer O can then definitely state that the two events occurred simultaneously. On the other hand, if O sees the light from one event before that from the other, then the former event occurred first.

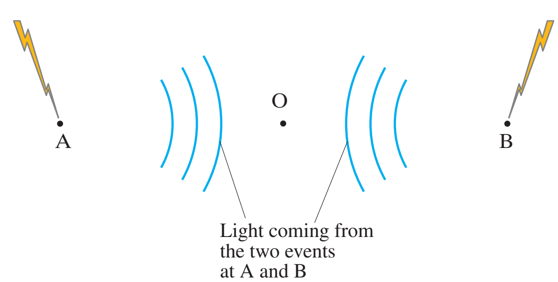


FIGURE 4 A moment after lightning strikes at points A and B, the pulses of light (shown as blue waves) are traveling toward the observer O, but O "sees" the lightning only when the light reaches O.

The question we really want to examine is this: if two events are simultaneous to an observer in one reference frame, are they also simultaneous to another observer moving with respect to the first? Let us call the observers O_1 and O_2 and assume they are fixed in reference frames 1 and 2 that move with speed v relative to one another. These two reference frames can be thought of as two rockets or two trains (Fig. 5). O_2 says that O_1 is moving to the right with speed v, as in Fig. 5a; and O_1 says O_2 is moving to the left with speed v, as in Fig. 5b. Both viewpoints are legitimate according to the relativity principle. [There is no third point of view which will tell us which one is "really" moving.]

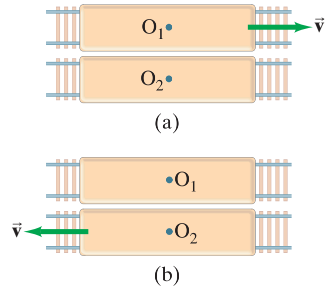


FIGURE 5 Observers O_1 and O_2, on two different trains (two different reference frames), are moving with relative speed v. O_2 says that O_1 is moving to the right (a); O_1 says t hat O_2 is moving to the left (b). Both viewpoints are legitimate: it all depends on your reference frame.

Now suppose that observers O_1 and O_2 observe and measure two lightning strikes. The lightning bolts mark both trains where they strike: at A_1 and B_1 on O_1's train, and at A_2 and B_2 on O_2's train, Fig. 6a. For simplicity, we assume that O_1 is exactly halfway between A_1 and B_1, and that O_2 is halfway between A_2 and B_2. Let us first put ourselves in O_2's reference frame, so we observe O_1 moving to the right with speed v. Let us also assume that the two events occur *simultaneously* in O_2's frame, and just at the instant when O_1 and O_2 are opposite each other, Fig. 6a. A short time later, Fig. 6b, the light from A_2 and B_2 reaches O_2 at the same time (we assumed this). Since O_2 knows (or measures) the distances O_2A_2 and O_2B_2 as equal, O_2 knows the two events are simultaneous in the O_2 reference frame.

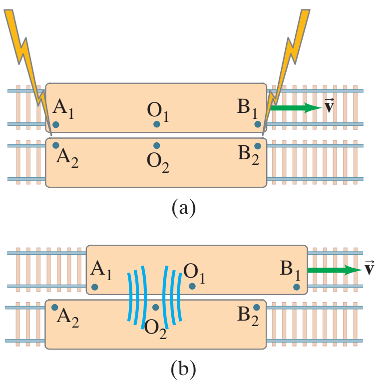


FIGURE 6 Thought experiment on simultaneity. In both (a) and (b) we are in the reference frame of observer O_2, who sees the reference frame of O_1 moving to the right. In (a), one lightning bolt strikes the two reference frames at A_1 and A_2, and a second lightning bolt strikes at B_1 and B_2. (b) A moment later, the light (shown in blue) from the two events reaches O_2 at the same time. So according to observer O_2, the two bolts of lightning struck simultaneously. But in O_1's reference frame, the light from B_1 has already reached O_1, whereas the light from A_1 has not yet reached O_1. So in O_1's reference frame, the event at B_1 must have preceded the event at A_1. Simultaneity in time is not absolute.

But what does observer O_1 observe and measure? From our (O_2) reference frame, we can predict what O_1 will observe. We see that O_1 moves to the right during the time the light is traveling to O_1 from A_1 and B_1. As shown in Fig. 6b, we can see from our O_2 reference frame that the light from B_1 has already passed O_1, whereas the light from A_1 has not yet reached O_1. That is, O_1 observes the light coming from B_1 before observing the light coming from A_1. Given (1) that light travels at the same speed c in any direction and in any reference frame, and (2) that the distance O_1A_1 equals O_1B_1, then observer O_1 can only conclude that the event at B_1 occurred before the event at A_1. The two events are *not* simultaneous for O_1, even though they are for O_2.

We thus find that two events which take place at different locations and are simultaneous to one observer, are actually not simultaneous to a second observer who moves relative to the first.

It may be tempting to ask: "Which observer is right, O_1 or O_2?" The answer, according to relativity, is that they are *both* right. There is no "best" reference frame we can choose to determine which observer is right. Both frames are equally good. We can only conclude that *simultaneity is not an absolute concept*, but is relative. We are not aware of this lack of agreement on simultaneity in everyday life because the effect is noticeable only when the relative speed of the two reference frames is very large (near c), or the distances involved are very large.

EXERCISE A Examine the experiment of Fig. 6 from O_1's reference frame. In this case, O_1 will be at rest and will see event B_1 occur before A_1. Will O_1 recognize that O_2, who is moving with speed v to the left, will see the two events as simultaneous? [*Hint*: Draw a diagram equivalent to Fig. 6.]

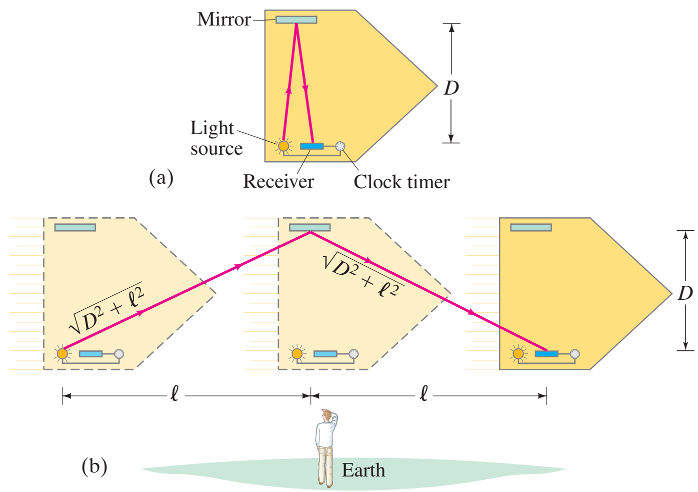


FIGURE 7 Time dilation can be shown by a thought experiment: the time it takes for light to travel across a spaceship and back is longer for the observer on Earth (b) than for the observer on the spaceship (a).

5 Time Dilation and the Twin Paradox

The fact that two events simultaneous to one observer may not be simultaneous to a second observer suggests that time itself is not absolute. Could it be that time passes differently in one reference frame than in another? This is, indeed, just what Einstein's theory of relativity predicts, as the following thought experiment shows.

Figure 7 shows a spaceship traveling past Earth at high speed. The point of view of an observer on the spaceship is shown in part (a), and that of an observer on Earth in part (b). Both observers have accurate clocks. The person on the spaceship (Fig. 7a) flashes a light and measures the time it takes the light to travel directly across the spaceship and return after reflecting from a mirror (the rays are drawn at a slight angle for clarity). In the reference frame of the spaceship, the light travels a distance $2D$ at speed c; so the time required to go across and back, which we call Δt_0, is

$$\Delta t_0 = 2D/c.$$

The observer on Earth, Fig. 7b, observes the same process. But to this observer, the spaceship is moving. So the light travels the diagonal path shown going across the spaceship, reflecting off the mirror, and returning to the sender. Although the light travels at the same speed to this observer (the second postulate), it travels a greater distance. Hence the time required, as measured by the observer on Earth, will be *greater* than that measured by the observer on the spaceship.

Let us determine the time interval Δt measured by the observer on Earth between sending and receiving the light. In time Δt, the spaceship travels a distance $2\ell = v\,\Delta t$ where v is the speed of the spaceship (Fig. 7b). The light travels a total distance on its diagonal path (Pythagorean theorem) of $2\sqrt{D^2+\ell^2}$, where $\ell = v\,\Delta t/2$. Therefore

$$c = \frac{2\sqrt{D^2+\ell^2}}{\Delta t} = \frac{2\sqrt{D^2+v^2(\Delta t)^2/4}}{\Delta t}.$$

We square both sides,

$$c^2 = \frac{4D^2}{(\Delta t)^2} + v^2,$$

and solve for Δt, to find

$$\Delta t = \frac{2D}{c\sqrt{1-v^2/c^2}}.$$

We combine this equation for Δt with the formula above, $\Delta t_0 = 2D/c$, and find

TIME DILATION

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}}. \qquad \textbf{(1a)}$$

Since $\sqrt{1-v^2/c^2}$ is always less than 1, we see that $\Delta t > \Delta t_0$. That is, the time interval between the two events (the sending of the light, and its reception on the spaceship) is *greater* for the observer on Earth than for the observer on the spaceship. This is a general result of the theory of relativity, and is known as **time dilation**. Stated simply, the time dilation effect says that

> **clocks moving relative to an observer are measured to run more slowly (as compared to clocks at rest relative to that observer).**

However, we should not think that the clocks are somehow at fault. Time is actually measured to pass more slowly in any moving reference frame as compared to your own. This remarkable result is an inevitable outcome of the two postulates of the theory of relativity.

The factor $1/\sqrt{1 - v^2/c^2}$ occurs so often in relativity that we often give it the shorthand symbol γ (the Greek letter "gamma"), and write Eq. 1a as

$$\Delta t = \gamma\, \Delta t_0 \tag{1b}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{2}$$

Note that γ is never less than one, and has no units. At normal speeds, $\gamma = 1$ to a few decimal places; in general, $\gamma \geq 1$.

The concept of time dilation may be hard to accept, for it contradicts our experience. We can see from Eqs. 1 that the time dilation effect is indeed negligible unless v is reasonably close to c. If v is much less than c, then the term v^2/c^2 is much smaller than the 1 in the denominator of Eq. 1a, and then $\Delta t \approx \Delta t_0$ (see Example 2). The speeds we experience in everyday life are much smaller than c, so it is little wonder we don't ordinarily notice time dilation. Experiments have tested the time dilation effect, and have confirmed Einstein's predictions. In 1971, for example, extremely precise atomic clocks were flown around the Earth in jet planes. The speed of the planes $(10^3\ \text{km/h})$ was much less than c, so the clocks had to be accurate to nanoseconds $(10^{-9}\ \text{s})$ in order to detect any time dilation. They were this accurate, and they confirmed Eqs. 1 to within experimental error. Time dilation had been confirmed decades earlier, however, by observations on "elementary particles" which have very small masses (typically 10^{-30} to 10^{-27} kg) and so require little energy to be accelerated to speeds close to the speed of light, c. Many of these elementary particles are not stable and decay after a time into lighter particles. One example is the muon, whose mean lifetime is $2.2\ \mu\text{s}$ when at rest. Careful experiments showed that when a muon is traveling at high speeds, its lifetime is measured to be longer than when it is at rest, just as predicted by the time dilation formula.

EXAMPLE 1 **Lifetime of a moving muon.** (*a*) What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c = 1.80 \times 10^8\ \text{m/s}$ with respect to the laboratory? Its mean lifetime at rest is $2.20\ \mu\text{s} = 2.20 \times 10^{-6}\ \text{s}$. (*b*) How far does a muon travel in the laboratory, on average, before decaying?

APPROACH If an observer were to move along with the muon (the muon would be at rest to this observer), the muon would have a mean life of 2.20×10^{-6} s. To an observer in the lab, the muon lives longer because of time dilation. We find the mean lifetime using Eq. 1a and the average distance using $d = v\,\Delta t$.

SOLUTION (*a*) From Eq. 1a with $v = 0.60c$, we have

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{2.20 \times 10^{-6}\ \text{s}}{\sqrt{1 - 0.36c^2/c^2}} = \frac{2.20 \times 10^{-6}\ \text{s}}{\sqrt{0.64}} = 2.8 \times 10^{-6}\ \text{s}.$$

(*b*) Relativity predicts that a muon with speed 1.80×10^8 m/s would travel an average distance $d = v\,\Delta t = (1.80 \times 10^8\ \text{m/s})(2.8 \times 10^{-6}\ \text{s}) = 500\ \text{m}$, and this is the distance that is measured experimentally in the laboratory.

NOTE At a speed of $1.8 \times 10^8\ \text{m/s}$, classical physics would tell us that with a mean life of $2.2\ \mu\text{s}$, an average muon would travel $d = vt = (1.8 \times 10^8\ \text{m/s})(2.2 \times 10^{-6}\ \text{s}) = 400\ \text{m}$. This is shorter than the distance measured.

EXERCISE B What is the muon's mean lifetime (Example 1) if it is traveling at $v = 0.90c$? (*a*) $0.42\ \mu\text{s}$; (*b*) $2.3\ \mu\text{s}$; (*c*) $5.0\ \mu\text{s}$; (*d*) $5.3\ \mu\text{s}$; (*e*) $12.0\ \mu\text{s}$.

⚠ **CAUTION**
Proper time Δt_0 is for 2 events at the same point in space.

We need to clarify how to use Eqs. 1, and the meaning of Δt and Δt_0. The equation is true only when Δt_0 represents the time interval between the two events in a reference frame where the two events occur at *the same point in space* (as in Fig. 7a where the two events are the light flash being sent and being received). This time interval, Δt_0, is called the **proper time**. Then Δt in Eqs. 1 represents the time interval between the two events as measured in a reference frame moving with speed v with respect to the first. In Example 1 above, Δt_0 (and not Δt) was set equal to 2.2×10^{-6} s because it is only in the rest frame of the muon that the two events ("birth" and "decay") occur at the same point in space. The proper time Δt_0 is the shortest time between the events any observer can measure. In any other moving reference frame, the time Δt is greater.

EXAMPLE 2 **Time dilation at 100 km/h.** Let us check time dilation for everyday speeds. A car traveling 100 km/h covers a certain distance in 10.00 s according to the driver's watch. What does an observer at rest on Earth measure for the time interval?

APPROACH The car's speed relative to Earth is $100\text{ km/h} = (1.00 \times 10^5\text{ m})/(3600\text{ s}) = 27.8\text{ m/s}$. The driver is at rest in the reference frame of the car, so we set $\Delta t_0 = 10.00\text{ s}$ in the time dilation formula.

SOLUTION We use Eq. 1a:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10.00\text{ s}}{\sqrt{1 - \left(\frac{27.8\text{ m/s}}{3.00 \times 10^8\text{ m/s}}\right)^2}} = \frac{10.00\text{ s}}{\sqrt{1 - (8.59 \times 10^{-15})}}.$$

If you put these numbers into a calculator, you will obtain $\Delta t = 10.00\text{ s}$, since the denominator differs from 1 by such a tiny amount. Indeed, the time measured by an observer on Earth would show no difference from that measured by the driver, even with the best instruments. A computer that could calculate to a large number of decimal places would reveal a difference between Δt and Δt_0. We can estimate the difference using the binomial expansion,

PROBLEM SOLVING
Use of the binomial expansion

$$(1 \pm x)^n \approx 1 \pm nx. \qquad [\text{for } x \ll 1]$$

In our time dilation formula, we have the factor $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$. Thus

$$\Delta t = \gamma\,\Delta t_0 = \Delta t_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx \Delta t_0\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$\approx 10.00\text{ s}\left[1 + \frac{1}{2}\left(\frac{27.8\text{ m/s}}{3.00 \times 10^8\text{ m/s}}\right)^2\right] \approx 10.00\text{ s} + 4 \times 10^{-14}\text{ s}.$$

So the difference between Δt and Δt_0 is predicted to be 4×10^{-14} s, an extremely small amount.

EXERCISE C A certain atomic clock keeps perfect time on Earth. If the clock is taken on a spaceship traveling at a speed $v = 0.60c$, does this clock now run slow according to the people (*a*) on the spaceship, (*b*) on Earth?

EXAMPLE 3 **Reading a magazine on a spaceship.** A passenger on a high-speed spaceship traveling between Earth and Jupiter at a steady speed of $0.75c$ reads a magazine which takes 10.0 min according to her watch. (*a*) How long does this take as measured by Earth-based clocks? (*b*) How much farther is the spaceship from Earth at the end of reading the article than it was at the beginning?

APPROACH (*a*) The time interval in one reference frame is related to the time interval in the other by Eq. 1a or b. (*b*) At constant speed, distance is speed × time. Since there are two times (a Δt and a Δt_0) we will get two distances, one for each reference frame. [This surprising result is explored in the next Section (6).]

SOLUTION (*a*) The given 10.0-min time interval is the proper time—starting and finishing the magazine happen at the same place on the spaceship. Earth clocks measure

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10.00\text{ min}}{\sqrt{1 - (0.75)^2}} = 15.1\text{ min}.$$

(*b*) In the Earth frame, the rocket travels a distance $D = v\,\Delta t = (0.75c)(15.1\text{ min}) = (0.75)(3.0 \times 10^8\text{ m/s})(15.1\text{ min} \times 60\text{ s/min}) = 2.04 \times 10^{11}\text{ m}$. In the spaceship's frame, the Earth is moving away from the spaceship at $0.75c$, but the time is only 10.0 min, so the distance is measured to be $D_0 = v\,\Delta t_0 = (2.25 \times 10^8\text{ m/s})(600\text{s}) = 1.35 \times 10^{11}\text{ m}$.

Values for $\gamma = 1/\sqrt{1 - v^2/c^2}$ at a few speeds v are given in Table 1.

TABLE 1 Values of γ

v	γ
0	1.000
$0.01c$	1.000
$0.10c$	1.005
$0.50c$	1.15
$0.90c$	2.3
$0.99c$	7.1

Space Travel?

Time dilation has aroused interesting speculation about space travel. According to classical (Newtonian) physics, to reach a star 100 light-years away would not be possible for ordinary mortals (1 light-year is the distance light can travel in 1 year $= 3.0 \times 10^8\text{ m/s} \times 3.16 \times 10^7\text{ s} = 9.5 \times 10^{15}\text{ m}$). Even if a spaceship could travel at close to the speed of light, it would take over 100 years to reach such a star. But time dilation tells us that the time involved could be less. In a spaceship traveling at $v = 0.999c$, the time for such a trip would be only about $\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = (100\text{ yr})\sqrt{1 - (0.999)^2} = 4.5\text{ yr}$. Thus time dilation allows such a trip, but the enormous practical problems of achieving such speeds may not be possible to overcome, certainly not in the near future.

In this example, 100 years would pass on Earth, whereas only 4.5 years would pass for the astronaut on the trip. Is it just the clocks that would slow down for the astronaut? No. All processes, including aging and other life processes, run more slowly for the astronaut according to the Earth observer. But to the astronaut, time would pass in a normal way. The astronaut would experience 4.5 years of normal sleeping, eating, reading, and so on. And people on Earth would experience 100 years of ordinary activity.

Twin Paradox

Not long after Einstein proposed the special theory of relativity, an apparent paradox was pointed out. According to this **twin paradox**, suppose one of a pair of 20-year-old twins takes off in a spaceship traveling at very high speed to a distant star and back again, while the other twin remains on Earth. According to the Earth twin, the astronaut twin will age less. Whereas 20 years might pass for the Earth twin, perhaps only 1 year (depending on the spacecraft's speed) would pass for the traveler. Thus, when the traveler returns, the earthbound twin could expect to be 40 years old whereas the traveling twin would be only 21.

This is the viewpoint of the twin on the Earth. But what about the traveling twin? If all inertial reference frames are equally good, won't the traveling twin make all the claims the Earth twin does, only in reverse? Can't the astronaut twin claim that since the Earth is moving away at high speed, time passes more slowly on Earth and the twin on Earth will age less? This is the opposite of what the Earth twin predicts. They cannot both be right, for after all the spacecraft returns to Earth and a direct comparison of ages and clocks can be made.

There is, however, no contradiction here. One of the viewpoints is indeed incorrect. The consequences of the special theory of relativity—in this case, time dilation—can be applied only by observers in an inertial reference frame. The Earth is such a frame (or nearly so), whereas the spacecraft is not. The spacecraft accelerates at the start and end of its trip and when it turns around at the far point of its journey. During the acceleration, the twin on the spacecraft is not in an inertial frame. In between, the astronaut twin may be in an inertial frame (and is justified in saying the Earth twin's clocks run slow), but it is not always the same frame. So she cannot use special relativity to predict their relative ages when she returns to Earth. The Earth twin stays in the same inertial frame, and we can thus trust her predictions based on special relativity. Thus, there is no paradox. The prediction of the Earth twin that the traveling twin ages less is the proper one.

*Global Positioning System (GPS)

PHYSICS APPLIED
Global positioning system (GPS)

Airplanes, cars, boats, and hikers use **global positioning system (GPS)** receivers to tell them quite accurately where they are, at a given moment. The 24 global positioning system satellites send out precise time signals using atomic clocks. Your receiver compares the times received from at least four satellites, all of whose times are carefully synchronized to within 1 part in 10^{13}. By comparing the time differences with the known satellite positions and the fixed speed of light, the receiver can determine how far it is from each satellite and thus where it is on the Earth. It can do this to a typical accuracy of 15 m, if it has been constructed to make corrections such as the one below due to special relativity.

CONCEPTUAL EXAMPLE 4 **A relativity correction to GPS.** GPS satellites move at about $4\ \text{km/s} = 4000\ \text{m/s}$. Show that a good GPS receiver needs to correct for time dilation if it is to produce results consistent with atomic clocks accurate to 1 part in 10^{13}.

RESPONSE Let us calculate the magnitude of the time dilation effect by inserting $v = 4000\ \text{m/s}$ into Eq. 1a:

$$\Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\,\Delta t_0 = \frac{1}{\sqrt{1 - \left(\frac{4 \times 10^3\ \text{m/s}}{3 \times 10^8\ \text{m/s}}\right)^2}}\,\Delta t_0 = \frac{1}{\sqrt{1 - 1.8 \times 10^{-10}}}\,\Delta t_0 .$$

We use the binomial expansion: $(1 \pm x)^n \approx 1 \pm nx$ for $x \ll 1$ which here is $(1 - x)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}x$. That is

$$\Delta t = \left(1 + \tfrac{1}{2}(1.8 \times 10^{-10})\right)\Delta t_0 = \left(1 + 9 \times 10^{-11}\right)\Delta t_0 .$$

The time "error" divided by the time interval is

$$\frac{(\Delta t - \Delta t_0)}{\Delta t_0} = 1 + 9 \times 10^{-11} - 1 = 9 \times 10^{-11} \approx 1 \times 10^{-10}.$$

Time dilation, if not accounted for, would introduce an error of about 1 part in 10^{10}, which is 1000 times greater than the precision of the atomic clocks. Not correcting for time dilation means a receiver could give much poorer position accuracy.

NOTE GPS devices must make other corrections as well, including effects associated with general relativity.

6 Length Contraction

Time intervals are not the only things different in different reference frames. Space intervals—lengths and distances—are different as well, according to the special theory of relativity, and we illustrate this with a thought experiment.

Observers on Earth watch a spacecraft traveling at speed v from Earth to, say, Neptune, Fig. 8a. The distance between the planets, as measured by the Earth observers, is ℓ_0. The time required for the trip, measured from Earth, is

$$\Delta t = \frac{\ell_0}{v}. \qquad \text{[Earth observer]}$$

In Fig. 8b we see the point of view of observers on the spacecraft. In this frame of reference, the spaceship is at rest; Earth and Neptune move† with speed v. The time between departure of Earth and arrival of Neptune (observed from the spacecraft) is the "proper time," since the two events occur at the same point in space (i.e., on the spacecraft). Therefore the time interval is less for the spacecraft

†We assume v is much greater than the relative speed of Neptune and Earth, so the latter can be ignored.

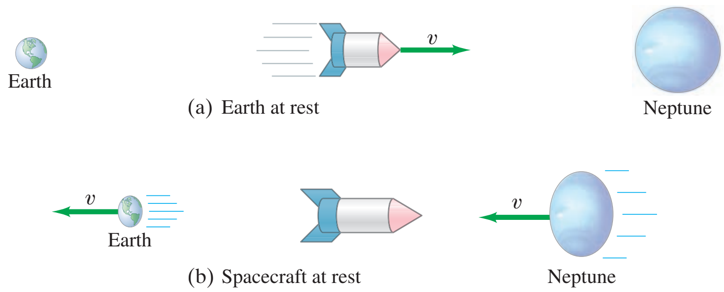


FIGURE 8 (a) A spaceship traveling at very high speed from Earth to the planet Neptune, as seen from Earth's frame of reference. (b) According to an observer on the spaceship, Earth and Neptune are moving at the very high speed v: Earth leaves the spaceship, and a time Δt_0 later Neptune arrives at the spaceship.

observers than for the Earth observers. That is, because of time dilation (Eq. 1a), the time for the trip as viewed by the spacecraft is

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = \Delta t/\gamma. \qquad \text{[spacecraft observer]}$$

Because the spacecraft observers measure the same speed but less time between these two events, they also measure the distance as less. If we let ℓ be the distance between the planets as viewed by the spacecraft observers, then $\ell = v\,\Delta t_0$, which we can rewrite as $\ell = v\,\Delta t_0 = v\,\Delta t\sqrt{1 - v^2/c^2} = \ell_0\sqrt{1 - v^2/c^2}$. Thus we have the important result that

$$\ell = \ell_0\sqrt{1 - v^2/c^2} \qquad \textbf{(3a)}$$

LENGTH CONTRACTION

or, using γ (Eq. 2),

$$\ell = \frac{\ell_0}{\gamma}. \qquad \textbf{(3b)}$$

This is a general result of the special theory of relativity and applies to lengths of objects as well as to distance between objects. The result can be stated most simply in words as:

> **the length of an object moving relative to an observer is measured to be shorter along its direction of motion than when it is at rest.**

This is called **length contraction**. The length ℓ_0 in Eqs. 3 is called the **proper length**. It is the length of the object (or distance between two points whose positions are measured at the same time) as determined by *observers at rest* with respect to the object. Equations 3 give the length ℓ that will be measured by observers when the object travels past them at speed v.

CAUTION
Proper length is measured in reference frame where the two positions are at rest

It is important to note that length contraction occurs *only along the direction of motion*. For example, the moving spaceship in Fig. 8a is shortened in length, but its height is the same as when it is at rest.

Length contraction, like time dilation, is not noticeable in everyday life because the factor $\sqrt{1 - v^2/c^2}$ in Eq. 3a differs from 1.00 significantly only when v is very large.

EXAMPLE 5 **Painting's contraction.** A rectangular painting measures 1.00 m tall and 1.50 m wide. It is hung on the side wall of a spaceship which is moving past the Earth at a speed of 0.90c. See Fig. 9a. (*a*) What are the dimensions of the picture according to the captain of the spaceship? (*b*) What are the dimensions as seen by an observer on the Earth?

APPROACH We apply the length contraction formula, Eq. 3a, to the dimension parallel to the motion; v is the speed of the painting relative to the observer.

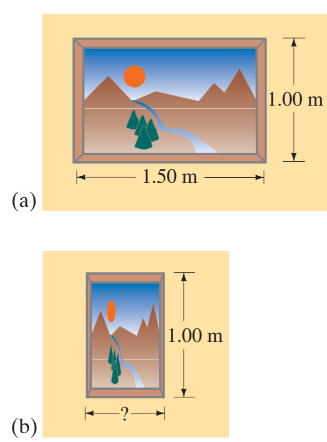
SOLUTION (*a*) The painting is at rest $(v = 0)$ on the spaceship so it (as well as everything else in the spaceship) looks perfectly normal to everyone on the spaceship. The captain sees a 1.00-m by 1.50-m painting.

(*b*) Only the dimension in the direction of motion is shortened, so the height is unchanged at 1.00 m, Fig. 9b. The length, however, is contracted to

$$\begin{aligned}\ell &= \ell_0\sqrt{1 - \frac{v^2}{c^2}}\\ &= (1.50\,\text{m})\sqrt{1 - (0.90)^2} = 0.65\,\text{m}.\end{aligned}$$

So the picture has dimensions 1.00 m × 0.65 m.

FIGURE 9 Example 5.



EXAMPLE 6 **A fantasy supertrain.** A very fast train with a proper length of 500 m is passing through a 200-m-long tunnel. Let us imagine the train's speed to be so great that the train fits completely within the tunnel as seen by an observer at rest on the Earth. That is, the engine is just about to emerge from one end of the tunnel at the time the last car disappears into the other end. What is the train's speed?

APPROACH Since the train just fits inside the tunnel, its length measured by the person on the ground is 200 m. The length contraction formula, Eq. 3a or b, can thus be used to solve for v.

SOLUTION Substituting $\ell = 200\text{ m}$ and $\ell_0 = 500\text{ m}$ into Eq. 3a gives

$$200\text{ m} = 500\text{ m}\sqrt{1 - \frac{v^2}{c^2}};$$

dividing both sides by 500 m and squaring, we get

$$(0.40)^2 = 1 - \frac{v^2}{c^2}$$

or

$$\frac{v}{c} = \sqrt{1 - (0.40)^2}$$

and

$$v = 0.92c.$$

NOTE No real train could go this fast. But it is fun to think about.

NOTE An observer on the *train* would *not* see the two ends of the train inside the tunnel at the same time. Recall that observers moving relative to each other do not agree about simultaneity.

EXERCISE D What is the length of the tunnel as measured by observers on the train in Example 6?

CONCEPTUAL EXAMPLE 7 **Resolving the train and tunnel length.** Observers at rest on the Earth see a very fast 200-m-long train pass through a 200-m-long tunnel (as in Example 6) so that the train momentarily disappears from view inside the tunnel. Observers on the train measure the train's length to be 500 m and the tunnel's length to be only 80 m (Exercise D, using Eq. 3a). Clearly a 500-m-long train cannot fit inside an 80-m-long tunnel. How is this apparent inconsistency explained?

RESPONSE Events simultaneous in one reference frame may not be simultaneous in another. Let the engine emerging from one end of the tunnel be "event A," and the last car disappearing into the other end of the tunnel "event B." To observers in the Earth frame, events A and B are simultaneous. To observers on the train, however, the events are not simultaneous. In the train's frame, event A occurs before event B. As the engine emerges from the tunnel, observers on the train observe the last car as still $500\text{ m} - 80\text{ m} = 420\text{ m}$ from the entrance to the tunnel.

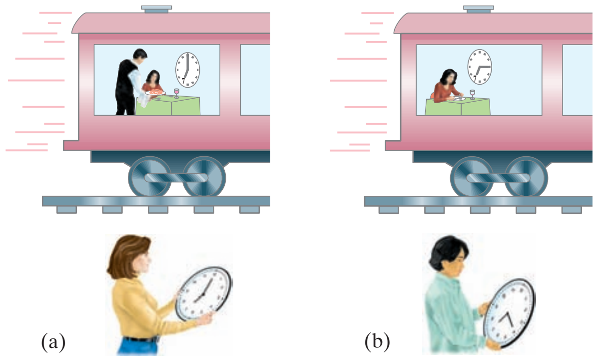


FIGURE 10 According to an accurate clock on a fast-moving train, a person (a) begins dinner at 7:00 and (b) finishes at 7:15. At the beginning of the meal, two observers on Earth set their watches to correspond with the clock on the train. These observers measure the eating time as 20 minutes.

7 Four-Dimensional Space–Time

Let us imagine a person is on a train moving at a very high speed, say $0.65c$, Fig. 10. This person begins a meal at 7:00 and finishes at 7:15, according to a clock on the train. The two events, beginning and ending the meal, take place at the same point on the train. So the proper time between these two events is 15 min. To observers on Earth, the meal will take longer—20 min according to Eqs. 1. Let us assume that the meal was served on a 20-cm-diameter plate. To observers on the Earth, the plate is only 15 cm wide (length contraction). Thus, to observers on the Earth, the meal looks smaller but lasts longer.

In a sense the two effects, time dilation and length contraction, balance each other. When viewed from the Earth, what an object seems to lose in size it gains in length of time it lasts. Space, or length, is exchanged for time.

Considerations like this led to the idea of **four-dimensional space–time**: space takes up three dimensions and time is a fourth dimension. Space and time are intimately connected. Just as when we squeeze a balloon we make one dimension larger and another smaller, so when we examine objects and events from different reference frames, a certain amount of space is exchanged for time, or vice versa.

Although the idea of four dimensions may seem strange, it refers to the idea that any object or event is specified by four quantities—three to describe where in space, and one to describe when in time. The really unusual aspect of four-dimensional space–time is that space and time can intermix: a little of one can be exchanged for a little of the other when the reference frame is changed.

It is difficult for most of us to understand the idea of four-dimensional space–time. Somehow we feel, just as physicists did before the advent of relativity, that space and time are completely separate entities. Yet we have found in our thought experiments that they are not completely separate. And think about Galileo and Newton. Before Galileo, the vertical direction, that in which objects fall, was considered to be distinctly different from the two horizontal dimensions. Galileo showed that the vertical dimension differs only in that it happens to be the direction in which gravity acts. Otherwise, all three dimensions are equivalent, a viewpoint we all accept today. Now we are asked to accept one more dimension, time, which we had previously thought of as being somehow different. This is not to say that there is no distinction between space and time. What relativity has shown is that space and time determinations are not independent of one another.

In Galilean–Newtonian relativity, the time interval between two events, Δt, and the distance between two events or points, Δx, are invariant quantities no matter what inertial reference frame they are viewed from. Neither of these quantities is invariant according to Einstein's relativity. But there is an invariant quantity in four-dimensional space–time, called the **space–time interval**, which is $(\Delta s)^2 = (c\,\Delta t)^2 - (\Delta x)^2$. We leave it as a Problem (97) to show that this quantity is indeed invariant under a Lorentz transformation (Section 8).

8 Galilean and Lorentz Transformations

We now examine in detail the mathematics of relating quantities in one inertial reference frame to the equivalent quantities in another. In particular, we will see how positions and velocities *transform* (that is, change) from one frame to the other.

We begin with the classical or Galilean viewpoint. Consider two inertial reference frames S and S′ which are each characterized by a set of coordinate axes, Fig. 11. The axes x and y (z is not shown) refer to S and x' and y' to S′. The x' and x axes overlap one another, and we assume that frame S′ moves to the right in the x direction at constant speed v with respect to S. For simplicity let us assume the origins 0 and 0′ of the two reference frames are superimposed at time $t = 0$.

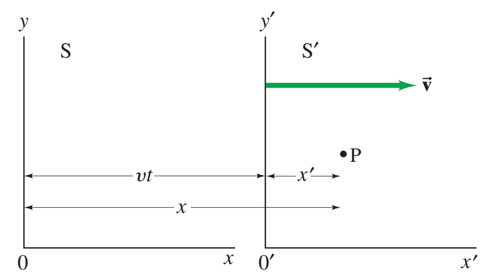


FIGURE 11 Inertial reference frame S′ moves to the right at constant speed v with respect to frame S.

Now consider an event that occurs at some point P (Fig. 11) represented by the coordinates x', y', z' in reference frame S′ at the time t'. What will be the coordinates of P in S? Since S and S′ initially overlap precisely, after a time t', S′ will have moved a distance vt'. Therefore, at time t', $x = x' + vt'$. The y and z coordinates, on the other hand, are not altered by motion along the x axis; thus $y = y'$ and $z = z'$. Finally, since time is assumed to be absolute in Galilean–Newtonian physics, clocks in the two frames will agree with each other; so $t = t'$. We summarize these in the following **Galilean transformation equations**:

$$\begin{aligned} x &= x' + vt' \\ y &= y' \\ z &= z' \\ t &= t'. \end{aligned} \qquad \text{[Galilean]} \quad \mathbf{(4)}$$

These equations give the coordinates of an event in the S frame when those in the S′ frame are known. If those in the S frame are known, then the S′ coordinates are obtained from

$$x' = x - vt, \qquad y' = y, \qquad z' = z, \qquad t' = t. \quad \text{[Galilean]}$$

These four equations are the "inverse" transformation and are very easily obtained from Eqs. 4. Notice that the effect is merely to exchange primed and unprimed quantities and replace v by $-v$. This makes sense because from the S′ frame, S moves to the left (negative x direction) with speed v.

Now suppose the point P in Fig. 11 represents a particle that is moving. Let the components of its velocity vector in S′ be u'_x, u'_y, u'_z. (We use u to distinguish it from the relative velocity of the two frames, v.) Now $u'_x = dx'/dt'$, $u'_y = dy'/dt'$ and $u'_z = dz'/dt'$. The velocity of P as seen from S will have components u_x, u_y, and u_z. We can show how these are related to the velocity components in S′ by differentiating Eqs. 4. For u_x we get

$$u_x = \frac{dx}{dt} = \frac{d(x' + vt')}{dt'} = u'_x + v$$

since v is assumed constant. For the other components, $u'_y = u_y$ and $u'_z = u_z$, so we have

$$\begin{aligned} u_x &= u'_x + v \\ u_y &= u'_y \\ u_z &= u'_z. \end{aligned} \qquad \text{[Galilean]} \quad \textbf{(5)}$$

These are known as the **Galilean velocity transformation equations**. We see that the y and z components of velocity are unchanged, but the x components differ by v: $u_x = u'_x + v$. This is just what you may have seen when dealing with relative velocity.

The Galilean transformations, Eqs. 4 and 5, are valid only when the velocities involved are much less than c. We can see, for example, that the first of Eqs. 5 will not work for the speed of light: light traveling in S′ with speed $u'_x = c$ would have speed $c + v$ in S, whereas the theory of relativity insists it must be c in S. Clearly, then, a new set of transformation equations is needed to deal with relativistic velocities.

We derive the required equation, looking again at Fig. 11. We will try the simple assumption that the transformation is linear and of the form

$$x = \gamma(x' + vt'), \qquad y = y', \qquad z = z'. \qquad \textbf{(i)}$$

That is, we modify the first of Eqs. 4 by multiplying by a constant γ which is yet to be determined† ($\gamma = 1$ non-relativistically). But we assume the y and z equations are unchanged since there is no length contraction in these directions. We will not assume a form for t, but will derive it. The inverse equations must have the same form with v replaced by $-v$. (The principle of relativity demands it, since S′ moving to the right with respect to S is equivalent to S moving to the left with respect to S′.) Therefore

$$x' = \gamma(x - vt). \qquad \textbf{(ii)}$$

Now if a light pulse leaves the common origin of S and S′ at time $t = t' = 0$, after a time t it will have traveled a distance $x = ct$ or $x' = ct'$ along the x axis. Therefore, from Eqs. (i) and (ii) above,

$$ct = \gamma(ct' + vt') = \gamma(c + v)\,t', \qquad \textbf{(iii)}$$

$$ct' = \gamma(ct - vt) = \gamma(c - v)t. \qquad \textbf{(iv)}$$

We substitute t' from Eq. (iv) into Eq. (iii) and find $ct = \gamma(c + v)\gamma(c - v)(t/c) = \gamma^2(c^2 - v^2)t/c$. We cancel out the t on each side and solve for γ to find

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

The constant γ here has the same value as the γ we used before, Eq. 2. Now that we have found γ, we need only find the relation between t and t'. To do so, we combine $x' = \gamma(x - vt)$ with $x = \gamma(x' + vt')$:

$$x' = \gamma(x - vt) = \gamma(\gamma[x' + vt'] - vt).$$

We solve for t and find $t = \gamma(t' + vx'/c^2)$. In summary,

$$\begin{aligned} x &= \gamma(x' + vt') \\ y &= y' \\ z &= z' \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned} \qquad \textbf{(6)}$$

LORENTZ TRANSFORMATIONS

These are called the **Lorentz transformation equations**. They were first proposed, in a slightly different form, by Lorentz in 1904 to explain the null result of the Michelson–Morley experiment and to make Maxwell's equations take the same form in all inertial reference frames. A year later Einstein derived them independently based on his theory of relativity. Notice that not only is the x equation modified as compared to the Galilean transformation, but so is the t equation; indeed, we see directly in this last equation how the space and time coordinates mix.

†γ here is not assumed to be given by Eq. 2.