

Pearson New International Edition

Statistics for Psychology

Arthur Aron Elaine Aron Elliot Coups
Sixth Edition

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PEARSON®

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Displaying the Order in a Group of Numbers Using Tables and Graphs

Chapter Outline

- ❖ The Two Branches of Statistical Methods
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- ❖ Controversy: Misleading Graphs
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Welcome to *Statistics for Psychology*. We imagine you to be like other students we have known who have taken this course. You have chosen to major in psychology or a related field because you are fascinated by people: fascinated by their visible behaviors and inner lives, as well as perhaps by your own psychology. Some of you are highly scientific sorts; others are more intuitive. Some of you are fond of math; others are less so. Whatever your style, we welcome you.

More importantly, we want to assure you that whatever your reason for studying psychology or a related field, this course is not a waste of time. Learning about statistics helps you to read the work of other psychologists, to do your own research if you so choose, and to hone both your reasoning and intuition. It also helps you to evaluate reports of scientific research in the media. Statistics really can make an important contribution to the next phase of your life.

From Chapter 1 of *Statistics for Psychology*, Sixth Edition. Arthur Aron, Elliot J. Coups, and Elaine N. Aron. Copyright © 2013 by Pearson Education, Inc. All rights reserved.

Displaying the Order in a Group of Numbers Using Tables and Graphs

Formally, **statistics** is a branch of mathematics that focuses on the organization, analysis, and interpretation of a group of numbers. But really what is statistics? Think of statistics as a tool that has evolved from a basic thinking process employed by every human: you observe a thing; you wonder what it means or what caused it; you have an insight or make an intuitive guess; you observe again, but now in detail, or you try making little changes in the process to test your intuition. Then you face the eternal problem: was your hunch confirmed or not? What are the chances that what you observed this second time will happen again and again, so that you can announce your insight to the world as something probably true?

In other words, statistics is a method of pursuing truth. At a minimum, statistics can tell you the likelihood that your hunch is true in this time and place and with these sorts of people. This type of pursuit of truth, especially in the form of an event's future likelihood, is the essence of psychology, of science, and of human evolution. Think of the first research questions: What will the mammoths do next spring? What will happen if I eat this root? It is easy to see how the early accurate "researchers" survived. You are here today because your ancestors exercised brains as well as brawn. Do those who come after you the same favor: think carefully about outcomes. Statistics is one good way to do that.

TIP FOR SUCCESS

The issue of how to design good research is a topic in itself, summarized in our Web Chapter W1 (*Overview of the Logic and Language of Psychology Research*) available at www.pearsonhighered.com/aron (and also at www.mystatlab.com for MyStatLab users). But in this text we mainly focus on the statistical methods for making sense of the data collected through research.

Psychologists use statistical methods to help them make sense of the numbers they collect when conducting research. Psychologists usually use statistical software to carry out statistical procedure. However, the best way to develop a solid understanding of statistics is actually to do the procedures by hand for a while (with the help of a calculator—it's not the multiplying and adding that you learn from, but the going through all the steps). To minimize the amount of mindless figuring you have to do, we use relatively small groups of simple numbers in each chapter's examples and practice problems. All of this allows you to focus on the *underlying principles and logic* of the statistical procedures as you manipulate the numbers. Having said that, we also recognize the importance for many of you of learning how to do statistical procedures on a computer so that, for example, you can some day conduct your own research, maybe starting with a senior project. Therefore, at the end of the chapter there is a section called Using SPSS. SPSS statistical software is commonly used by psychologists and other behavioral and social scientists to carry out statistical analyses. Check with your instructor to see if you have access to SPSS at your institution. (There are also a number of other statistical programs used by researchers, and many basic statistical procedures can be carried out using standard spreadsheet programs such as Excel; for those of you not using SPSS, the SPSS sections will still be helpful in giving you the general idea of how one does such problems on a computer.)

statistics branch of mathematics that focuses on the organization, analysis, and interpretation of a group of numbers.

descriptive statistics procedures for summarizing a group of scores or otherwise making them more understandable.

inferential statistics procedures for drawing conclusions based on the scores collected in a research study but going beyond them.

The Two Branches of Statistical Methods

There are two main branches of statistical methods.

1. **Descriptive statistics:** Psychologists use descriptive statistics to summarize and describe a group of numbers from a research study.
2. **Inferential statistics:** Psychologists use inferential statistics to draw conclusions and to make inferences that are based on the numbers from a research study but that go beyond the numbers. For example, inferential statistics

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allow researchers to make inferences about a large group of individuals based on a research study in which a much smaller number of individuals took part.

In this chapter, we focus on descriptive statistics. This topic is important in its own right, but it also prepares you to understand inferential statistics.

In this chapter we introduce you to some basic concepts, and then you will learn to use tables and graphs to describe a group of numbers. The purpose of descriptive statistics is to make a group of numbers easy to understand. As you will see, tables and graphs help a great deal.

Some Basic Concepts

Variables, Values, and Scores

As part of a larger study (Aron, Paris, & Aron, 1995), researchers gave a questionnaire to students in an introductory statistics class during the first week of the course. One question asked was, “How stressed have you been in the last 2½ weeks, on a scale of 0 to 10, with 0 being *not at all stressed* and 10 being *as stressed as possible?*” (How would *you* answer?) In this study, the researchers used a survey to examine students’ level of stress. Other methods that researchers use to study stress include creating stress with laboratory tasks (such as having to be videotaped giving a talk for humans or swimming in water for rats) and measuring stress-related hormones or brain changes.

In this example, level of stress is a **variable**, which can have **values** from 0 to 10, and the value of any particular person’s answer is the person’s **score**. If you answered 6, your score is 6; your score has a value of 6 on the variable called “level of stress.”

More formally, a variable is a condition or characteristic that can have different values. In short, it can *vary*. In our example, the variable was level of stress, which can have the values of 0 through 10. Height is a variable, social class is a variable, score on a creativity test is a variable, type of psychotherapy received by patients is a variable, speed on a reaction time test is a variable, number of people absent from work on a given day is a variable, and so forth.

A value is just a number, such as 4, –81, or 367.12. A value can also be a category, such as male or female, or a psychiatric diagnosis—major depression, post-traumatic stress disorder—and so forth.

Finally, on any variable, each person studied has a particular number or *score* that is his or her value on the variable. As we’ve said, your score on the stress variable might have a value of 6. Another student’s score might have a value of 8.

Psychology research is about variables, values, and scores (see Table 1). The formal definitions are a bit abstract, but in practice, the meaning is usually clear.

variable characteristic that can have different values.

values possible number or category that a score can have.

score particular person’s value on a variable.

Table 1 Some Basic Terminology

Term	Definition	Examples
Variable	Condition or characteristic that can have different values	Stress level; age; gender; religion
Value	Number or category	0, 1, 2, 3, 4; 25, 85; female; Catholic
Score	A particular person’s value on a variable	0, 1, 2, 3, 4; 25, 85; female; Catholic

Levels of Measurement (Kinds of Variables)

Most of the variables psychologists use are like those in the stress ratings example: the scores are numbers that tell you how much there is of what is being measured. In the stress ratings example, the higher the number is, the more stress there is. This is an example of a **numeric variable**. Numeric variables are also called *quantitative variables*.

There are several kinds of numeric variables. In psychology research the most important distinction is between two types: equal-interval variables and rank-order variables. An **equal-interval variable** is a variable in which the numbers stand for approximately equal amounts of what is being measured. For example, grade point average (GPA) is a roughly equal-interval variable, since the difference between a GPA of 2.5 and 2.8 means about as much as the difference between a GPA of 3.0 and 3.3 (each is a difference of 0.3 of a GPA). Most psychologists also consider scales like the 0-to-10 stress ratings as roughly equal interval. So, for example, a difference between stress ratings of 4 and 6 means about as much as the difference between 7 and 9.

Some equal-interval variables are measured on what is called a **ratio scale**. An equal-interval variable is measured on a ratio scale if it has an *absolute zero point*. An absolute zero point means that the value of zero on the variable indicates a complete absence of the variable. Most counts or accumulations of things use a ratio scale. For example, the number of siblings a person has is measured on a ratio scale, because a zero value means having no siblings. With variables that are measured on a ratio scale, you can make statements about the difference in magnitude between values. So, we can say that a person with four siblings has twice as many siblings as a person with two siblings. Other examples of variables that are measured on a ratio scale include distance, time, and weight. However, most of the variables in psychology are not on a ratio scale.

The other main type of numeric variable, a **rank-order variable**, is a variable in which the numbers stand only for relative ranking. (Rank-order variables are also called *ordinal variables*.) A student's standing in his or her graduating class is an example. The amount of difference in underlying GPA between being second and third in class standing could be very unlike the amount of difference between being eighth and ninth.

A rank-order variable provides less information than an equal-interval variable. That is, the difference from one rank to the next doesn't tell you the exact difference in amount of what is being measured. However, psychologists often use rank-order variables because they are the only information available. Also, when people are being asked to rate something, it is sometimes easier and less arbitrary for them to make rank-order ratings. For example, when rating how much you like each of your friends, it may be easier to rank them by how much you like them than to rate your liking for each of them on a scale. Yet another reason researchers often use rank-order variables is that asking people to do rankings forces them to make distinctions. For example, if asked to rate how much you like each of your friends on a 1-to-10 scale, you might rate several of them at exactly the same level, but ranking would avoid such ties.

Another major type of variable used in psychology research, which is not a numeric variable at all, is a **nominal variable** in which the values are names or categories. The term *nominal* comes from the idea that its values are names. (Nominal variables are also called *categorical variables* because their values are categories.) For example, for the nominal variable gender, the values are female and male. A person's "score" on the variable gender is one of these two values. Another example is psychiatric diagnosis, which has values such as major depression, post-traumatic stress disorder, schizophrenia, and obsessive-compulsive disorder.

These different kinds of variables represent different **levels of measurement** (see Table 2). Researchers sometimes have to decide how they will measure a particular

numeric variable variable whose values are numbers (as opposed to a nominal variable). Also called *quantitative variable*.

equal-interval variable variable in which the numbers stand for approximately equal amounts of what is being measured.

ratio scale an equal-interval variable is measured on a ratio scale if it has an *absolute zero point*, meaning that the value of zero on the variable indicates a complete absence of the variable.

rank-order variable numeric variable in which the values are ranks, such as class standing or place finished in a race. Also called *ordinal variable*.

nominal variable variable with values that are categories (that is, they are names rather than numbers). Also called *categorical variable*.

levels of measurement types of underlying numerical information provided by a measure, such as equal-interval, rank-order, and nominal (categorical).

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Level	Definition	Example
Equal-interval	Numeric variable in which differences between values correspond to differences in the underlying thing being measured	Stress level; age
Rank-order	Numeric variable in which values correspond to the relative position of things measured	Class standing; position finished in a race
Nominal	Variable in which the values are categories	Gender; religion

variable. For example, they might use an equal-interval scale, a rank-order scale, or a nominal scale. The level of measurement selected affects the type of statistics that can be used with a variable. Suppose a researcher is studying the effects of a particular type of brain injury on being able to recognize objects. One approach the researcher might take would be to measure the number of different objects an injured person can observe at once. This is an example of an equal-interval level of measurement. Alternately, the researcher might rate people as able to observe no objects (rated 0), only one object at a time (rated 1), one object with a vague sense of other objects (rated 2), or ordinary vision (rated 3). This would be a rank-order approach. Finally, the researcher might divide people into those who can identify the location of an object but not what the object is (rated L), those who can identify what the object is but not locate it in space (rated I), those who can both locate and identify an object but have other abnormalities of object perception (rated O), those who are completely blind (rated B), and those with normal visual perception (rated N). This is a nominal level of measurement.

In this chapter, as in most psychology research, we focus mainly on numeric, equal-interval variables (or variables that roughly approximate equal-interval variables).

Another distinction that researchers sometimes make is between discrete variables and continuous variables. A **discrete variable** is one that has specific values and cannot have values between the specific values. For example, the number of times you went to the dentist in the last 12 months is a discrete variable. You may have gone 0, 1, 2, 3, or more times, but you can't have gone 1.72 times or 2.34 times. Nominal variables, such as gender, religious affiliation, and college major can also be considered to be discrete variables. With a **continuous variable**, there are in theory an infinite number of values between any two values. So, even though we usually answer the question "How old are you?" with a specific age, such as 19 or 20, you could also answer it by saying that you are 19.26 years old. Height, weight, and time are examples of other continuous variables.

discrete variable variable that has specific values and that cannot have values between these specific values.

continuous variable variable for which, in theory, there are an infinite number of values between any two values.

How are you doing?

1. A father rates his daughter as a 2 on a 7-point scale (from 1 to 7) of crankiness. In this example, (a) what is the variable, (b) what is the score, and (c) what is the range of values?
2. What is the difference between a numeric and a nominal variable?
3. Give the level of measurement of each of the following variables: (a) a person's nationality (Mexican, Chinese, Ethiopian, Australian, etc.), (b) a person's score on a standard IQ test, (c) a person's place on a waiting list (first in line, second in line, etc.).
4. What is the difference between a discrete and a continuous variable?

Answers

1. (a) crankiness, (b) 2, (c) 1 to 7.
2. A numeric variable has values that are numbers that tell you the degree or extent of what the variable measures; a nominal variable has values that are different categories and have no particular numerical order.
3. (a) nominal, (b) equal-interval, (c) rank-order.
4. A discrete variable has specific values and has no values between the specific values. A continuous variable has, in theory, an infinite number of values between any two values.

BOX 1 Important Trivia for Poetic Statistics Students

The word *statistics* comes from the Italian word *statista*, a person dealing with affairs of state (from *stato*, “state”). It was originally called “state arithmetic,” involving the tabulation of information about nations, especially for the purpose of taxation and planning the feasibility of wars.

Statistics were needed in ancient times to figure the odds of shipwrecks and piracy for marine insurance that would encourage voyages of commerce and exploration to far-flung places. The modern study of mortality rates and life insurance descended from the 17th-century plague pits—counting the bodies of persons cut down in the bloom of youth. The theory of errors began in astronomy, that is, with stargazing; the theory of correlation has its roots in biology, from the observation of parent and child differences. Probability theory arose in the tense environs of the gambling table. The theory of analysis of experiments began in breweries and out among waving fields of wheat, where correct guesses determined not only the survival of a tasty beer but of thousands of marginal farmers. Theories of measurement and factor analysis derived from personality psychology, where the depths of human character were first explored with numbers. And chi-square came to us from sociology, where it was often a question of class.

In the early days of statistics, it was popular to use the new methods to prove the existence of God. For example, John Arbuthnot discovered that more male than female babies were born in London between 1629 and 1710. In what is considered the first use of a statistical

test, he proved that the male birthrate was higher than could be expected by chance (assuming that 50:50 was chance) and concluded that there was a plan operating, since males face more danger to obtain food for their families, and only God, he said, could do such planning.

In 1767, John Michell also used probability theory to prove the existence of God when he argued that the odds were 500,000 to 1 against six stars being placed as close together as those in the constellation Pleiades; so their placement had to have been a deliberate act of the Creator.

Statistics in the “state arithmetic” sense are legally endorsed by most governments today. For example, the first article of the U.S. Constitution requires a census. And statistics helped the United States win the Revolutionary War. John Adams obtained critical aid from Holland by pointing out certain vital statistics, carefully gathered by the clergy in local parishes, demonstrating that the colonies had doubled their population every 18 years, adding 20,000 fighting men per annum. “Is this the case of our enemy, Great Britain?” Adams wrote. “Which then can maintain the war the longest?”

Similar statistics were observed by U.S. President Thomas Jefferson in 1786. He wrote that his people “become uneasy” when there are more of them than 10 per square mile and that given the population growth of the new country, within 40 years these restless souls would fill up all of their country’s “vacant land.” Some 17 years later, Jefferson doubled the size of the United States’ “vacant” land through the Louisiana Purchase.

Frequency Tables

An Example

Let's return to the stress ratings example. Recall that in this study, students in an introductory statistics class during the first week of the course answered the question, "How stressed have you been in the last 2½ weeks, on a scale of 0 to 10, with 0 being *not at all stressed* and 10 being *as stressed as possible*?" The actual study included scores from 151 students. To ease the learning for this example, we are going to use a representative subset of scores from 30 of the 151 students (this also saves you time if you want to try it for yourself). The 30 students' scores (their ratings on the scale) are:

8, 7, 4, 10, 8, 6, 8, 9, 9, 7, 3, 7, 6, 5, 0, 9, 10, 7, 7, 3, 6, 7, 5, 2, 1, 6, 7, 10, 8, 8.

Looking through all these scores gives some sense of the overall tendencies, but this is hardly an accurate method. One solution is to make a table showing how many students used each of the 11 values that the ratings can have (0, 1, 2, and so on, through 10). That is, the number of students who used each particular rating is the *frequency* of that value. We have done this in Table 3. We also figured the percentage that each value's frequency is of the total number of scores. Tables like this sometimes give only the raw-number frequencies, not the percentages, or only the percentages and not the raw-number frequencies.

In addition, some frequency tables include, for each value, the total number of scores with that value and all values preceding it. These are called *cumulative frequencies* because they tell how many scores are accumulated up to this point on the table. If percentages are used, cumulative percentages also may be included (for an example, see Figure 19 in the Using SPSS section). Cumulative percentages give, for each value, the percentage of scores up to and including that value. The cumulative percentage for any given value (or for a score having that value) is also called a *percentile*. Cumulative frequencies and cumulative percentages allow you to see where a particular score falls in the overall group of scores.

Table 3 is called a **frequency table** because it shows how frequently (how many times) each score was used. A frequency table makes the pattern of numbers easy to see. In this example, you can see that most of the students rated their stress level around 7 or 8, with few rating it very low.

Table 3 Frequency Table of Number of Students Rating Each Value of the Stress Scale

Stress Rating	Frequency	Percent
0	1	3.3
1	1	3.3
2	1	3.3
3	2	6.7
4	1	3.3
5	2	6.7
6	4	13.3
7	7	23.3
8	5	16.7
9	3	10.0
10	3	10.0

Source: Data based on Aron et al. (1995).

How to Make a Frequency Table

There are four steps in making a frequency table.

- ❶ **Make a list down the page of each possible value, from lowest to highest.** In the stress ratings results, the list goes from 0, the lowest possible rating, up to 10, the highest possible rating.¹ Note that even if one of the ratings between 0 and 10 is not used, you still include that value in the listing, showing it as having a frequency of 0. For example, if no one gave a stress rating of 2, you still include 2 as one of the values on the frequency table.
- ❷ **Go one by one through the scores, making a mark for each next to its value on your list.** This is shown in Figure 1.
- ❸ **Make a table showing how many times each value on your list is used.** That is, add up the number of marks beside each value.
- ❹ **Figure the percentage of scores for each value.** To do this, take the frequency for that value, divide it by the total number of scores, and multiply by 100. You may need to round off the percentage. We recommend that you

frequency table ordered listing of number of individuals having each of the different values for a particular variable.

Displaying the Order in a Group of Numbers Using Tables and Graphs

TIP FOR SUCCESS

When doing Step 2, cross off each score as you mark it on the list. This should help you avoid mistakes, which are common in this step.

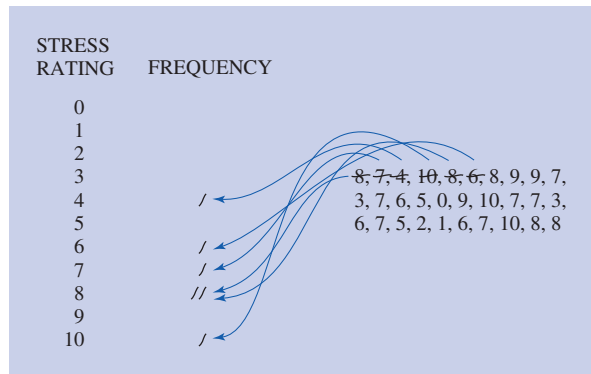


Figure 1 Making a frequency table for the stress ratings scores. (Data based on Aron, Paris, and Aron, 1995.)

round percentages to one decimal place. Note that because of the rounding, your percentages do not usually add up to exactly 100% (but the total should be very close to 100%).

Table 4 Frequency Table for a Nominal Variable: Closest Person in Life for 208 Students

Closest Person	Frequency	Percent
Family member	33	15.9
Nonromantic friend	76	36.5
Romantic partner	92	44.2
Other	7	3.4

Source: Data from Aron et al. (1992).

Frequency Tables for Nominal Variables

The preceding steps assume you are using numeric variables, the most common situation. However, you can also use a frequency table to show the number of scores for each value (that is, for each category) of a nominal variable. For example, researchers (Aron, Aron, & Smollan, 1992) asked 208 students to name the closest person in their life. As shown in Table 4, 33 students selected a family member, 76 a nonromantic friend, 92 a romantic partner, and 7 selected some other person. Also in Table 4, the values listed on the left hand side of the frequency table are the values (the categories) of the variable.

Another Example

Tracy McLaughlin-Volpe and her colleagues (2001) had 94 introductory psychology students keep a diary of their social interactions for a week during the regular semester. Each time a participant had a social interaction lasting 10 minutes or longer, he or she would fill out a card. The card had questions about various aspects of the conversation and the conversation partner. Excluding family and work situations, the number of social interactions 10 minutes or longer over a week for these students were as follows:

48, 15, 33, 3, 21, 19, 17, 16, 44, 25, 30, 3, 5, 9, 35, 32, 26, 13, 14, 14, 47, 47, 18, 11, 5, 19, 24, 17, 6, 25, 8, 18, 29, 1, 18, 22, 3, 22, 29, 2, 6, 10, 29, 10, 29, 21, 38, 41, 16, 17, 8, 40, 8, 10, 18, 7, 4, 4, 8, 11, 3, 23, 10, 19, 21, 13, 12, 10, 4, 17, 11, 21, 9, 8, 7, 5, 3, 22, 14, 25, 4, 11, 10, 18, 1, 28, 27, 19, 24, 35, 9, 30, 8, 26.

Now, let's follow our four steps for making a frequency table.

- 1 Make a list down the page of each possible value, from lowest to highest.** The lowest possible number of interactions is 0. In this study, the highest number of interactions could be any number. However, the highest actual number in this group is 48; so we can use 48 as the highest value. Thus, the first step

Displaying the Order in a Group of Numbers Using Tables and Graphs

is to list these values down a page. (It might be good to use several columns so that you can have all the scores on a single page.)

- 2 **Go one by one through the scores, making a mark for each next to its value on your list.** Figure 2 shows the results of this step.
- 3 **Make a table showing how many times each value on your list is used.** Table 5 is the result.
- 4 **Figure the percentage of scores for each value.** We have *not* done so in this example because it would not help much for seeing the pattern of scores. However, if you want to check your understanding of this step, the first five percentages would be 0.0%, 2.1%, 1.1%, 5.3%, and 4.3%. (These are the percentages for frequencies of 0, 2, 1, 5, and 4, rounded to one decimal place.)

0 -	17 - ////	34 -
1 - //	18 - ////	35 - //
2 - /	19 - ////	36 -
3 - ////	20 -	37 -
4 - ////	21 - ////	38 - /
5 - ///	22 - ///	39 -
6 - //	23 - /	40 - /
7 - //	24 - //	41 - /
8 - //// /	25 - ///	42 -
9 - ///	26 - //	43 -
10 - //// /	27 - /	44 - /
11 - ////	28 - /	45 -
12 - /	29 - ////	46 -
13 - //	30 - //	47 - //
14 - ///	31 -	48 - /
15 - /	32 - /	
16 - //	33 - /	

Figure 2 Making a frequency table of students' social interactions over a week. (Data from McLaughlin-Volpe et al., 2001.)

Grouped Frequency Tables

Sometimes there are so many possible values that an ordinary frequency table is too awkward to give a simple picture of the scores. The last example was a bit like that, wasn't it? The solution is to make groupings of values that include all values in a certain range. Consider the stress ratings example. Instead of having a separate frequency figure for the group of students who rated their stress as 8 and another for those who rated it as 9, you could have a combined category of 8 and 9. This combined category is a range of values that includes these two values. A combined category like this is called an **interval**. This particular interval of 8 and 9 has a frequency of 8 (the 5 scores with a value of 8 plus the 3 scores with a value of 9).

A frequency table that uses intervals is called a **grouped frequency table**. Table 6 is a grouped frequency table for the stress ratings example. (Note that in this example the full frequency table has only 11 different values. Thus, a grouped

TIP FOR SUCCESS

Be sure to check your work by adding the frequencies for all of the scores. This sum should equal the total number of scores you started with.

Score	Frequency	Score	Frequency	Score	Frequency
0	0	17	4	34	0
1	2	18	5	35	2
2	1	19	4	36	0
3	5	20	0	37	0
4	4	21	4	38	1
5	3	22	3	39	0
6	2	23	1	40	1
7	2	24	2	41	1
8	6	25	3	42	0
9	3	26	2	43	0
10	6	27	1	44	1
11	4	28	1	45	0
12	1	29	4	46	0
13	2	30	2	47	2
14	3	31	0	48	1
15	1	32	1		
16	2	33	1		

Source: Data from McLaughlin-Volpe et al. (2001).

Stress Rating Interval	Frequency	Percent
0-1	2	6.7
2-3	3	10.0
4-5	3	10.0
6-7	11	36.7
8-9	8	26.7
10-11	3	10.0

Source: Data based on Aron et al. (1995).

interval range of values in a grouped frequency table that are grouped together. (For example, if the interval size is 10, one of the intervals might be from 10 to 19.)

grouped frequency table frequency table in which the number of individuals (frequency) is given for each interval of values.

Displaying the Order in a Group of Numbers Using Tables and Graphs

Table 7 Grouped Frequency Table for Numbers of Social Interactions During a Week for 94 College Students

Interval	Frequency	Percent
0–4	12	12.8
5–9	16	17.0
10–14	16	17.0
15–19	16	17.0
20–24	10	10.6
25–29	11	11.7
30–34	4	4.3
35–39	3	3.2
40–44	3	3.2
45–49	3	3.2

Source: Data from McLaughlin-Volpe et al. (2001).

frequency table is not really necessary.) Table 7 is a grouped frequency table for the 94 students' number of social interactions over a week.

A grouped frequency table can make information even more directly understandable than an ordinary frequency table can. Of course, the greater understandability of a grouped frequency table is at a cost. You lose some information: the details of the breakdown of frequencies in each interval.

When you are setting up a grouped frequency table, it makes a big difference how many intervals you use. There are guidelines to help researchers with this, but in practice it is done automatically by the researcher's computer (see this chapter's Using SPSS section for instructions on how to create frequency tables using statistical software). However, should you have to make a grouped frequency table on your own, the key is to experiment with the interval size until you come up with one that is a round number (such as 2, 3, 5, or 10) and that creates about 5 to 15 intervals. Then, when actually setting up the table, be sure you set the start of each interval to a multiple of the interval size and the top end of each interval to the number just below the start of the next interval. For example, Table 6 uses six intervals with an interval size of 2. The intervals are 0–1, 2–3, 4–5, 6–7, 8–9, and 10–11. Note that each interval starts with a multiple of 2 (0, 2, 4, 6, 8, 10) and the top end of each interval (1, 3, 5, 7, 9) is the number just below the start of the next interval (2, 4, 6, 8, 10). Table 7 uses 10 intervals with an interval size of 5. The intervals are 0–4, 5–9, 10–14, 15–19, and so on, with a final interval of 45–49. Note that each interval starts with a multiple of 5 (0, 5, 10, 15, and so on) and that the top end of each interval (4, 9, 14, 19, and so on) is the number just below the start of the next interval (5, 10, 15, 20, and so on).

How are you doing?

1. What is a frequency table?
2. Why would a researcher want to make a frequency table?
3. Make a frequency table for the following scores: 5, 7, 4, 5, 6, 5, 4.
4. What does a grouped frequency table group?

Answers

1. A frequency table is a systematic listing of the number of scores (the frequency) of each value in the group studied.
2. A frequency table makes it easy to see the pattern in a large group of scores.

3. Value	Frequency	Percent
4	2	28.6
5	3	42.9
6	1	14.3
7	1	14.3

4. A grouped frequency table groups the frequencies of adjacent values into intervals.

Histograms

A graph is another good way to make a large group of scores easy to understand. A picture may be worth a thousand words, but it is also sometimes worth a thousand numbers. A straightforward approach is to make a graph of the frequency table. One kind of graph of the information in a frequency table is a kind of bar chart called a **histogram**. In a histogram, the height of each bar is the frequency of each value in the frequency table. Ordinarily, in a histogram, all the bars are put next to each other with no space in between. The result is that a histogram looks a bit like

histogram barlike graph of a frequency distribution in which the values are plotted along the horizontal axis and the height of each bar is the frequency of that value; the bars are usually placed next to each other without spaces, giving the appearance of a city skyline.

Displaying the Order in a Group of Numbers Using Tables and Graphs

a city skyline. Figure 3 shows two histograms based on the stress ratings example (one based on the ordinary frequency table and one based on the grouped frequency table). Figure 4 shows a histogram based on the grouped frequency table for the example of the numbers of students' social interactions in a week.

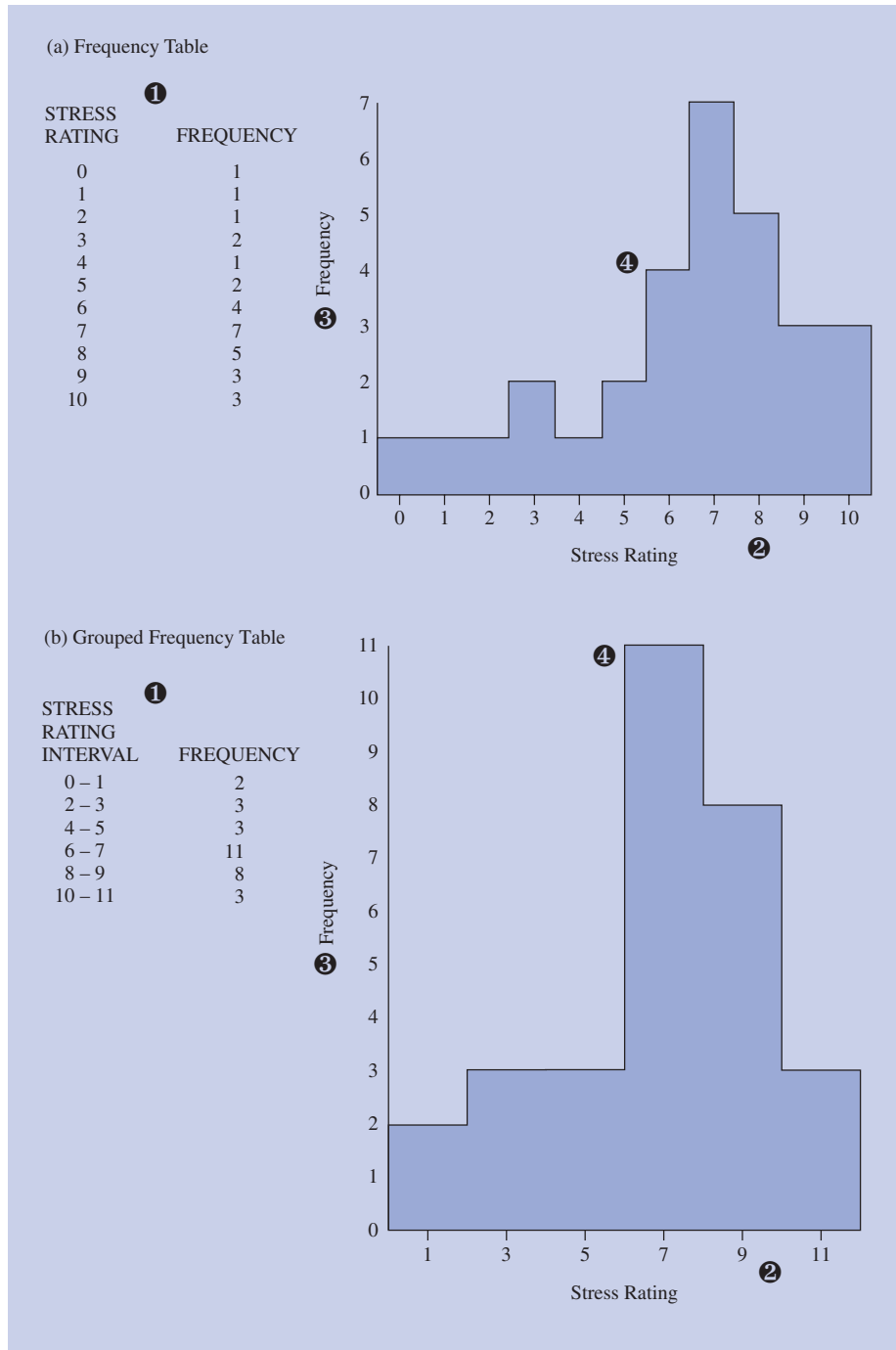


Figure 3 Histograms based on (a) frequency table and (b) a grouped frequency for the stress ratings example. (Data based on Aron et al., 1995.)

Displaying the Order in a Group of Numbers Using Tables and Graphs

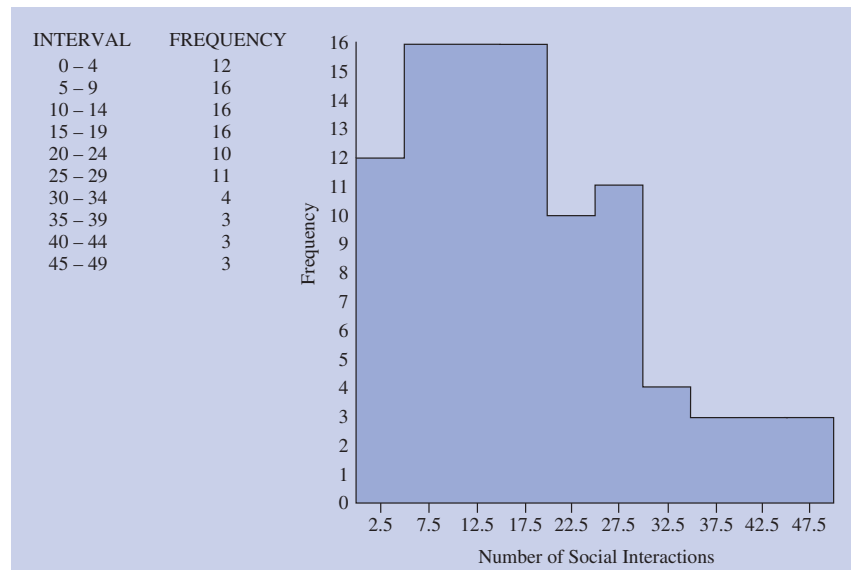


Figure 4 Histogram for number of social interactions during a week for 94 college students, based on grouped frequencies. (Data from McLaughlin-Volpe et al., 2001.)

BOX 2 Math Anxiety, Statistics Anxiety, and You: A Message for Those of You Who Are Truly Worried About This Course

Many of you are probably not looking forward to this course, and some of you may even have a certain degree of “statistics anxiety” (Onwuegbuzie, 2000; Zeidner, 1991). You are not alone. The study of statistics anxiety is a growing field. There are even questionnaire measures of statistics anxiety (e.g., Statistical Anxiety Rating Scale; Cruise, Cash, & Bolton, 1985) and the *Statistical Education Research Journal*, begun in 2002, devotes many articles to the problem and how instructors can reduce it. What can you do?

First, take hope. In fact, several studies of statistics anxiety actually bring home the importance of hope (e.g., Kauer, 2010; Onwuegbuzie & Snyder, 2000). There’s reason for hope because this course is a chance for a fresh start with digits. Your past performance in (or avoidance of) geometry, trigonometry, calculus, or similar courses need not influence in any way how well you comprehend statistics. This is largely a different subject.

Second, a little anxiety is not a bad thing. A study of statistics anxiety at Auburn University (Keeley, Zayac, & Correia, 2008) confirmed one of the most solid findings in psychology, the Yerkes-Dodson (1908) law. That law says that performance and comfort are highest when you are at your optimal level of arousal, neither under aroused and bored or over aroused and anxious, but just

right. (Maybe it should be the Goldilocks’ Law.) In the Auburn study, students with some anxiety out performed those with too much anxiety and also those with too little. So go ahead, worry a little.

If, however, your anxiety is interfering with studying or taking tests, then you need to determine where it is coming from. Is it statistics anxiety, anxiety about math in general, test anxiety, general anxiety, low self-confidence? Each can play its role (Cooper & Robinson, 1989; Dwinell & Higbee, 1991; Onwuegbuzie, 2000).

Is your problem statistics anxiety or general math anxiety? If you only fear statistics, we promise it will be no harder than any other math course (actually, much *less* purely mathematical than most). For general math anxiety, go to the Internet. A search will yield hundreds of wonderful books and Web sites to help you. We highly recommend Sheila Tobias’s classics, *Overcoming Math Anxiety* (1995) and *Succeed with Math: Every Student’s Guide to Conquering Math Anxiety* (1987). Tobias, a former math avoider herself, suggests that your goal should be “math mental health,” which she defines as “the willingness to learn the math you need when you need it” (1995, p. 12). Another fine book on the subject is *Conquering Math Anxiety* by Carol Arem (2009). Both authors emphasize the importance of looking back over your history with math to

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when it became a problem. They also teach ways to deal with the emotions as they rise up while you are studying.

Is your problem test anxiety? Test taking can easily push you out of your optimal level of arousal, especially when the test is timed. Your mind starts to race, creating more anxiety, more arousal, and so on. Maybe the thought comes that you are “no good and never will be.” So think through that thought right now so that you can forget it. You are in college. You are definitely “good.” If you have had some low grades, that may reflect your problems with tests more than your abilities.

What more can you do about test anxiety? There are many ways to reduce anxiety and arousal in general, such as learning to breathe properly and to take a brief break to relax deeply. Your counseling center should be able to help you or direct you to some good books on the subject. Again, many Web sites deal with reducing anxiety.

We think that test anxiety is best solved by over preparing for a few tests, so that you go in with the certainty that you cannot possibly fail, no matter how aroused you become. The ideal time to begin applying this tactic is the first test of this course. There will be no old material to review, success will not depend on having understood previous material, and initial success will help you do well throughout the course.

You also might enlist the sympathy of your instructor or teaching assistant about your test anxiety. Bring in a list of what you have studied, explain why you are being so exacting, and ask if you have missed anything. Your preparation must be ridiculously thorough, but only for a few exams. After these successes, your test anxiety should decline. Also, reduce your arousal by making the testing situation more familiar. Create a practice test situation as similar to a real test as possible, making a special effort to duplicate the aspects that bother you most. If feeling rushed is the troubling part, once you think you are well prepared, set yourself a time limit for solving some homework problems. Make yourself write out answers fully and legibly—this may be part of what makes you feel slow during a test. If the presence of others bothers you—the sound of their scurrying pencils while yours is frozen in midair—do your practice test with others in your course. If the sense of competition is behind your fear, add the twist of making it an explicit contest—then laugh about it afterwards.

Is your problem a general lack of confidence? Maybe something else in your life is causing you to worry or feel bad about yourself. In that case, it is time you tried your friendly college counseling center.

Lastly, could you be highly sensitive? A final word about anxiety and arousal. About 20% of us seem to be born “highly sensitive” (Aron, Aron, & Jagiellowicz, 2012). Traditionally seen as shyness, hesitancy, or introversion (Eysenck, 1981; Kagan, 1994), this temperament trait actually seems due to a preference to observe, notice subtle stimulation, and process this information deeply, along with having a heightened emotionality that serves to motivate that processing (Aron, 1996; Aron & Aron, 1997; Jagiellowicz et al., 2011). This trait is normal and found in over one hundred other species (Wolf, van Doorn, & Weissing, 2008). Highly sensitive persons (HSPs) tend to be very intuitive or even gifted. But they are more easily over aroused by high levels of stimulation, like tests.

You might want to find out if you are an HSP (at www.hsperson.com). If you are, appreciate the trait’s assets and make some allowances for its one disadvantage, this tendency to become easily over aroused, which can affect test performance. What matters is what you actually know, which is probably quite a bit. This simple act of self-acceptance—that you are *not* less smart but *are* more sensitive—may in itself help ease your arousal when trying to express your statistical knowledge.

More generally. Is there still a lingering feeling that you are “no good” at math? That some people are just naturally good at math and you are not one of them? A study cited by Tobias (1995) compared students in Asia and the United States on an international mathematics test and found the U.S. students were thoroughly outperformed. Why? Interviews revealed that Asian students saw math as an ability fairly equally distributed among people and thought that differences in performance were due to hard work. In contrast, U.S. students thought some people are just born better at math; so hard work matters little.

Learn from those good at math. Do not feel nervous when you are stuck. Stay calm and go back to the last place where you understood things, or ask for help from someone who will not make you nervous. One of the worst feelings is having someone explain something, and you still don’t get it. But if that happens, remember that it is just that you are getting anxious and over aroused, not that you are “bad at math.” Keep looking for the right kind of help. Some people love to teach and know the importance of praising you when you get it right. Find them. Then, once you understand something, try to explain it to someone else right away. Teaching is a great way to really grasp an idea.

And breathe deeply. Stay calm. Maybe you can actually enjoy the process of becoming someone who truly understands this subject.

How to Make a Histogram

There are four steps in making a histogram.

TIP FOR SUCCESS

Now try this yourself! Work out the interval midpoints for the grouped frequency table for the stress ratings example shown in Table 6. Remember, you are not finding the middle point of the interval, but rather the middle point between where one interval starts and the next one begins. Your answers should be the same as the values shown along the bottom of Figure 3b.

- ❶ **Make a frequency table (or grouped frequency table).**
- ❷ **Put the values along the bottom of the page, from left to right, from lowest to highest.** If you are making a histogram from a grouped frequency table, the values you put along the bottom of the page are the interval midpoints. The midpoint of an interval is halfway between the start of that interval and the start of the next highest interval. So, in Figure 4, the midpoint for the 0–4 interval is 2.5, because 2.5 is halfway between 0 (the start of the interval) and 5 (the start of the next highest interval). For the 5–9 interval, the midpoint is 7.5 because 7.5 is halfway between 5 (the start of the interval) and 10 (the start of the next highest interval). Do this for each interval. When you get to the last interval, find the midpoint between the start of the interval and the start of what would be the next highest interval. So, in Figure 4, the midpoint for the 45–49 interval is halfway between 45 (the start of the interval) and 50 (the start of what would be the next interval), which is 47.5.
- ❸ **Make a scale of frequencies along the left edge of the page that goes from 0 at the bottom to the highest frequency for any value.**
- ❹ **Make a bar above each value with a height for the frequency of that value.** For each bar, make sure that the middle of the bar is above its value.

When you have a nominal variable, the histogram is called a *bar graph*. Since the values of a nominal variable are not in any particular order, you leave a space between the bars. Figure 5 shows a bar graph based on the frequency table in Table 4.

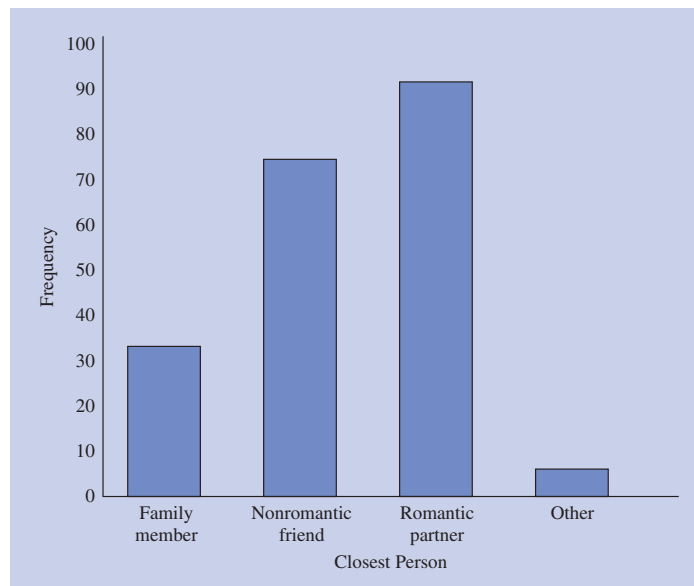


Figure 5 Bar graph for the closest person in life for 208 students (see Table 4). (Data from Aron et al., 1995.)

How are you doing?

1. Why do researchers make histograms?
2. When making a histogram from a frequency table, (a) what goes along the bottom, (b) what goes along the left edge, and (c) what goes above each value?
3. Make a histogram based on the following frequency table:

Value	Frequency
1	3
2	4
3	8
4	5
5	2

4. How is a histogram based on a nominal variable different from one based on a numeric equal-interval variable?

Answers

1. Researchers make histograms to show the pattern visually in a frequency table.
2. (a) The values, from lowest to highest go along the bottom; (b) the frequencies from 0 at the bottom to the highest frequency of any value at the top go along the left edge; (c) above each value is a bar with a height of the frequency for that value.
3. See Figure 6.

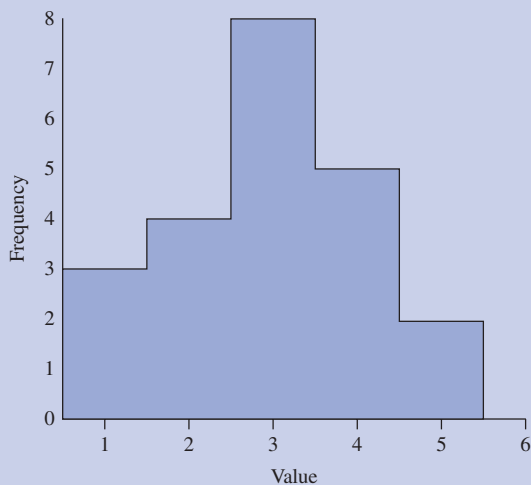


Figure 6
Histogram for “How are you doing?” question 3.

4. A histogram based on a nominal variable has gaps between the bars and is called a bar graph.

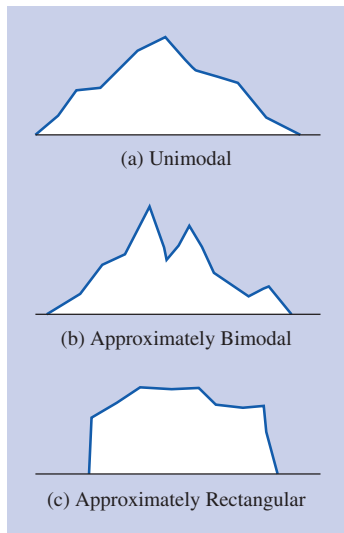


Figure 7 Examples of (a) unimodal, (b) approximately bimodal, and (c) approximately rectangular frequency polygons.

frequency distribution pattern of frequencies over the various values; what a frequency table, histogram, or frequency polygon describes.

unimodal distribution frequency distribution with one value clearly having a larger frequency than any other.

bimodal distribution frequency distribution with two approximately equal frequencies, each clearly larger than any of the others.

multimodal distribution frequency distribution with two or more high frequencies separated by a lower frequency; a bimodal distribution is the special case of two high frequencies.

rectangular distribution frequency distribution in which all values have approximately the same frequency.

symmetrical distribution distribution in which the pattern of frequencies on the left and right side are mirror images of each other.

skewed distribution distribution in which the scores pile up on one side of the middle and are spread out on the other side; distribution that is not symmetrical.

Shapes of Frequency Distributions

A **frequency distribution** shows the pattern of frequencies over the various values. A frequency table or histogram describes a frequency distribution because each shows the pattern or shape of how the frequencies are spread out, or “distributed.” Psychologists also describe this shape in words. Describing the shape of a distribution is important.

Unimodal and Bimodal Frequency Distributions

One question is whether a distribution’s shape has only one main high point: one high “tower” in the histogram. For example, in the stress ratings study, the most frequent value is 7, giving a graph only one very high area. This is a **unimodal distribution**. If a distribution has two fairly equal high points, it is a **bimodal distribution**. Any distribution with two or more high points is called a **multimodal distribution**. (Strictly speaking, a distribution is bimodal or multimodal only if the peaks are exactly equal. However, psychologists use these terms more informally to describe the general shape.) Finally, a distribution with values of all about the same frequency is a **rectangular distribution**. Figure 7 shows examples of these frequency distribution shapes. As you will see, the graphs in Figure 7 are not histograms, but special line graphs called *frequency polygons*, which are another way to graph a frequency table. In a frequency polygon, the line moves from point to point. The height of each point shows the number of scores with that value. This creates a mountain-peak skyline.

The scores from most psychology studies are usually an approximately unimodal distribution. Bimodal and other multimodal distributions occasionally turn up. A bimodal example is the distribution of the ages of people in a toddlers’ play area in a park, who are mostly either toddlers of ages around 2 to 4 years or caretakers with ages of 20 to 40 years or so (with few infants, a few siblings aged 5 to 19 years, and a few grandparents above 40 years). Thus, if you make a frequency distribution of these ages, the large frequencies are at the values for toddler ages (2 to 4) and for higher ages (20 to 40 or so). An example of a rectangular distribution is the number of children at each grade level at an elementary school; there is about the same number in first grade, second grade, and so on. Figure 8 shows these examples.

Symmetrical and Skewed Distributions

Look again at the histograms of the stress ratings example (Figure 3). The distribution is lopsided, with more scores near the high end. This is somewhat unusual. Most things we measure in psychology have about equal numbers on both sides of the middle. That is, most of the time in psychology, the scores follow an approximately **symmetrical distribution** (if you fold the graph of a symmetrical distribution in half, the two halves look the same).

A distribution that clearly is not symmetrical is called a **skewed distribution**. The stress ratings distribution is an example. A skewed distribution has one side that is long and spread out, somewhat like a tail. The side with the *fewer* scores (the side that looks like a tail) is considered the direction of the skew. Thus, the stress study example, which has too few scores at the low end, is skewed to the left. However, the social interactions example, which has too few scores at the high end, is skewed to the right (see Figure 4). Figure 9 shows examples of approximately symmetrical and skewed distributions.

Displaying the Order in a Group of Numbers Using Tables and Graphs

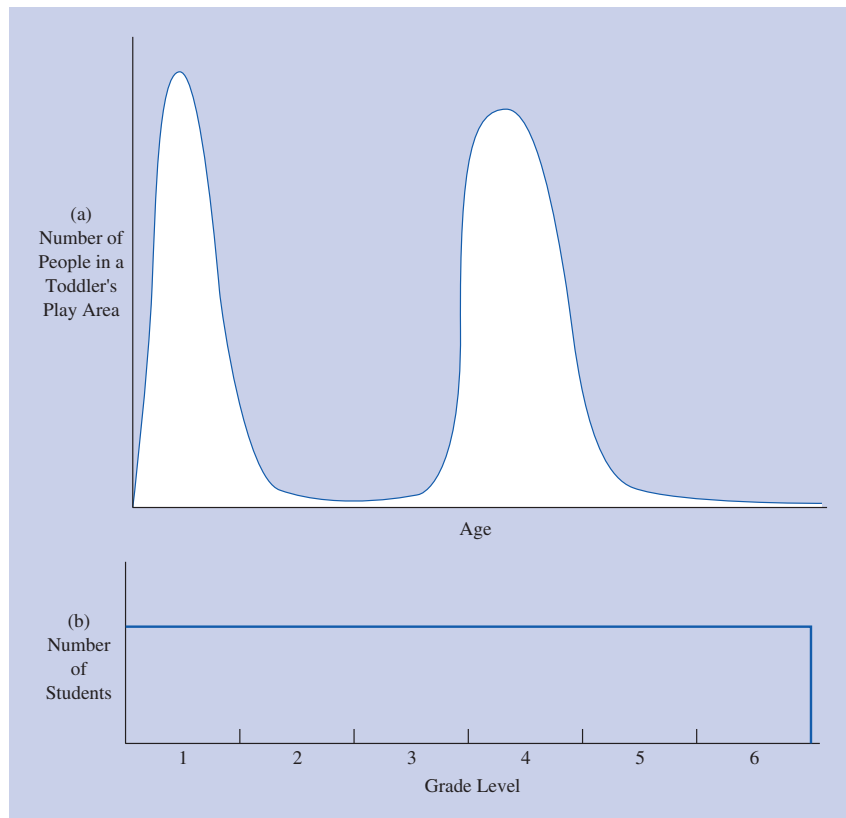


Figure 8 Fictional examples of distributions that are not unimodal: (a) A bimodal distribution showing the possible frequencies for people of different ages in a toddler's play area. (b) A regular distribution showing the possible frequencies of students at different grade levels in an elementary school.

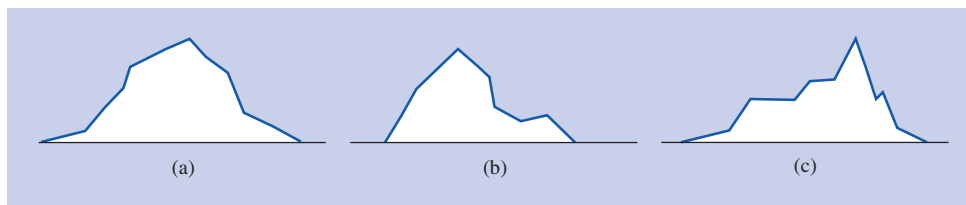


Figure 9 Examples of frequency polygons of distributions that are (a) approximately symmetrical, (b) skewed to the right (positively skewed), and (c) skewed to the left (negatively skewed).

TIP FOR SUCCESS

It helps you remember the direction of the skew to know that the word *skew* comes from the French *queue*, which means line or tail. Thus, the direction of the skew is the side that has the long line, or tail.

A distribution that is skewed to the right is also called *positively skewed*. A distribution skewed to the left is also called *negatively skewed*.

Strongly skewed distributions come up in psychology research mainly when what is being measured has some upper or lower limit. For example, a family cannot have fewer than zero children. When many scores pile up at the low end because it is impossible to have a lower score, the result is called a **floor effect**. A skewed distribution caused by a lower limit is shown in Figure 10a.

floor effect situation in which many scores pile up at the low end of a distribution (creating skewness to the right) because it is not possible to have any lower score.

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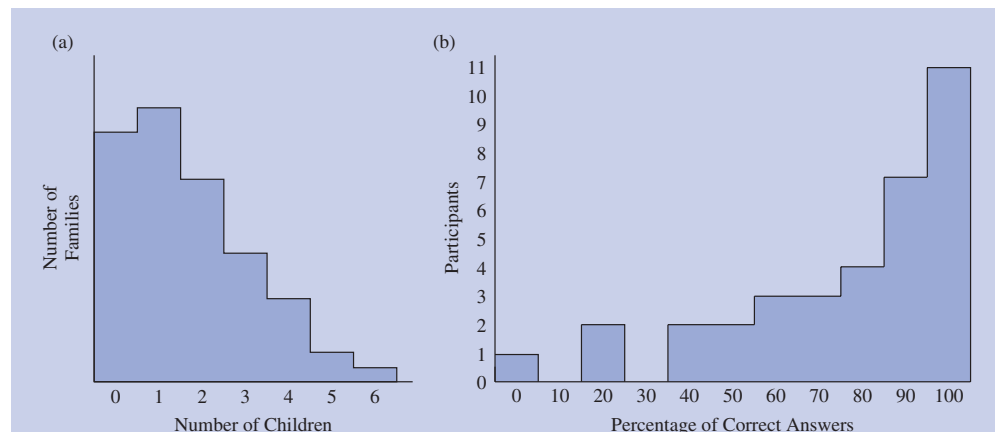


Figure 10 (a) A distribution skewed to the right due to a floor effect: fictional distribution of the number of children in families. (b) A distribution skewed to the left due to a ceiling effect: fictional distribution of adults' scores on a multiplication table test.

A skewed distribution caused by an upper limit is shown in Figure 10b. This is a distribution of adults' scores on a multiplication table test. This distribution is strongly skewed to the left. Most of the scores pile up at the right, the high end (a perfect score). This shows a **ceiling effect**. The stress ratings example also shows a mild ceiling effect because many students had high levels of stress, the maximum rating was 10, and people often do not like to use ratings right at the maximum.

Normal and Kurtotic Distributions

Psychologists also describe a distribution in terms of whether the middle of the distribution is particularly peaked or flat. The standard of comparison is a bell-shaped curve. In psychology research and in nature generally, distributions often are similar to this bell-shaped standard, called the **normal curve**. However, the important thing is that the normal curve is a unimodal, symmetrical curve with an average peak—the sort of bell shape shown in Figure 11a. Both the stress ratings and the social interactions examples are somewhat skewed. In our experience, most distributions that result from psychology research are closer to the normal curve than are these two examples.

Kurtosis is how much the shape of a distribution differs from a normal curve in terms of whether its curve in the middle is more peaked or flat than the normal curve (DeCarlo, 1997). Kurtosis comes from the Greek word *kyrtos*, “curve.” Figure 11b shows a kurtotic distribution with a more extreme peak than the normal curve. Figure 11c shows an extreme example of a kurtotic distribution, one with a very flat distribution. (A rectangular distribution would be even more extreme.)

Distributions that are more peaked or flat than a normal curve also tend to have a different shape in the tails. Those with a very peaked curve usually have more scores in the tails of the distribution than the normal curve (see Figure 11b). It is as if the normal curve got pinched in the middle and some of it went up into a sharp peak and the rest spread out into thick tails. Distributions with a flatter curve usually have fewer scores in the tails of the distribution than the normal curve (see Figure 11c). It is as if the tails and the top of the curve both got sucked in toward the middle on both sides. Although it is often easiest to identify kurtosis in terms of how peaked or flat the distribution is, the number of scores in the tails is what matters.

ceiling effect situation in which many scores pile up at the high end of a distribution (creating skewness to the left) because it is not possible to have a higher score.

normal curve specific, mathematically defined, bell-shaped frequency distribution that is symmetrical and unimodal; distributions observed in nature and in research commonly approximate it.

kurtosis extent to which a frequency distribution deviates from a normal curve in terms of whether its curve in the middle is more peaked or flat than the normal curve.

Displaying the Order in a Group of Numbers Using Tables and Graphs

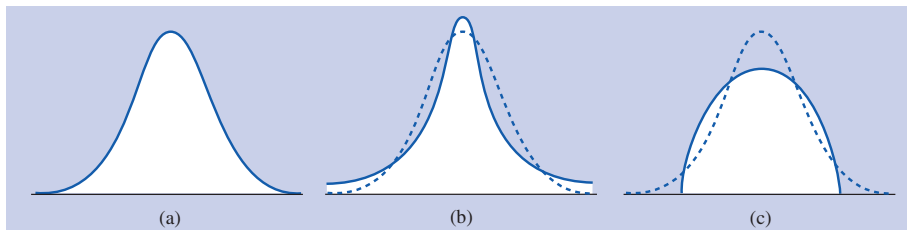


Figure 11 Examples of (a) normal, (b) heavy-tailed, and (c) light-tailed distributions. The normal distribution is shown as a dashed line in (b) and (c).

Source: Adapted from DeCarlo, T. (1997). On the meaning and use of kurtosis. *Psychological Methods*, 3, 292–307. Copyright © 1997 by the American Psychological Association. Adapted with permission. The use of APA information does not imply endorsement by APA.

How are you doing?

1. Describe the difference between a unimodal and multimodal distribution in terms of (a) a frequency graph and (b) a frequency table.
2. What does it mean to say that a distribution is skewed to the left?
3. What kind of skew is created by (a) a floor effect and (b) a ceiling effect?
4. When a distribution is described as being peaked or flat, what is it being compared to?

Answers

1. (a) A unimodal distribution has one main high point; a multimodal distribution has more than one main high point. (b) A unimodal distribution has one value with a higher frequency than all the other frequencies; a multimodal distribution has more than one value with large frequencies compared to the values around it.
2. When a distribution is skewed to the left, fewer scores have low values than have high values.
3. (a) A floor effect creates a distribution that is skewed to the right; (b) a ceiling effect creates a distribution that is skewed to the left.
4. The distribution is being compared to a normal curve.

Controversy: Misleading Graphs

The most serious controversy about frequency tables and histograms is not among psychologists, but among the general public. The misuse of these procedures by some public figures, advertisers, and the media seems to have created skepticism about the trustworthiness of statistics in general and of statistical tables and charts in particular. Everyone has heard that “statistics lie.”

Of course, people can and do lie with statistics. It is just as easy to lie with words, but you may be less sure of your ability to recognize lies with numbers. In this section, we note two ways in which frequency tables and graphs can be misused and tell how to recognize such misuses. Much of this material is based on the classic discussion of these issues in Tufte (1983). There is also a substantial research literature on the misuse of graphs in corporate and government technical reports on an array of important scientific and social topics (e.g., Jones, 2011).

Failure to Use Equal Interval Sizes

A key requirement of a grouped frequency table or graph is that the size of the intervals be equal. If they are not equal, the table or graph can be very misleading. Tufte (1983) gives an example, shown in Figure 12, from the respectable (and usually very accurate) *New York Times*. This chart gives the impression that commissions paid to travel agents dropped dramatically in 1978. However, a close reading of the graph shows that the third bar for each airline is for only the first half of 1978. Thus, only half a year is being compared to each of the preceding full years. Assuming that the second half of 1978 was like the first half, the information in this graph actually tells us that 1978 shows an increase rather than a decrease. For example, Delta Airlines estimated a full-year 1978 figure of \$72 million, much higher than 1977's \$57 million.

Exaggeration of Proportions

The height of a histogram or bar graph (or frequency polygon) usually begins at 0 or the lowest value of the scale and continues to the highest value of the scale. Figure 13a shows a bar graph that does not follow this standard. The bar graph shows the mean housing price in a particular region over a 4-year period (from 2008 to 2011). By starting the vertical axis at \$150,000 (instead of 0, as is customary), the graph appears to exaggerate the changes in housing price over time. Figure 13b shows the same results with the vertical axis starting at \$0. You can still see the changes in housing price from year to year in Figure 13b but the figure does a better job of showing the size of those changes.

The overall proportion of a histogram or bar graph should be about 1 to 1.5 times as wide as it is tall, as in Figure 14a for the stress ratings example. But look what

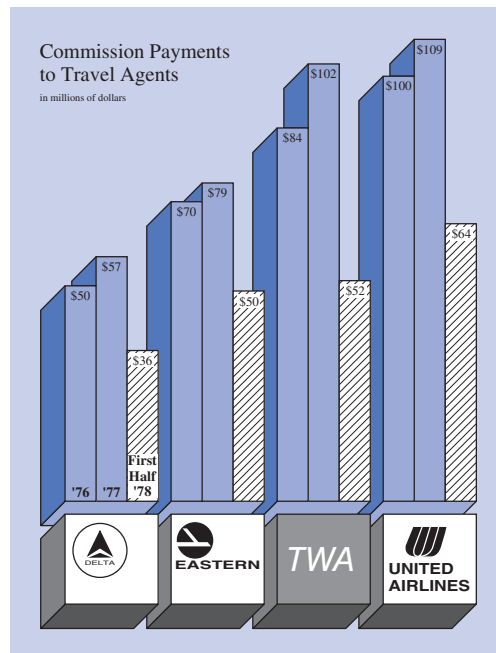


Figure 12 Misleading illustration of a frequency distribution due to unequal interval sizes.

Source: "Commission Payments to Travel Agents," From *The New York Times*, August 8, 1978. © 1978 The New York Times. Used by permission and protected by the Copyright Laws of the United States. The printing, copying, redistribution, or retransmission of the Material without express written permission is prohibited. www.nytimes.com

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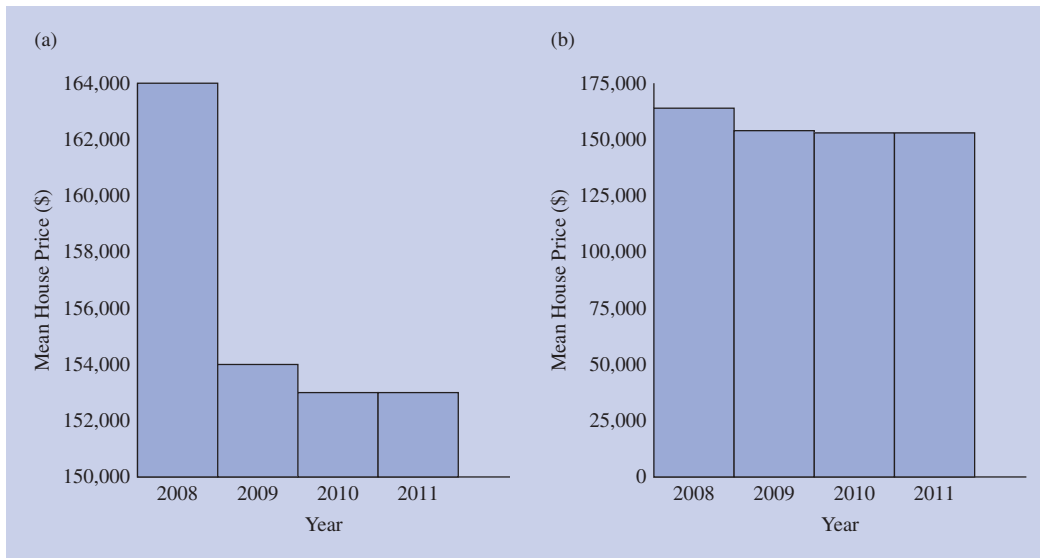


Figure 13 Misleading bar graph due to not starting at zero. The vertical axis starts at \$150,000 for figure (a) compared to \$0 for figure (b).

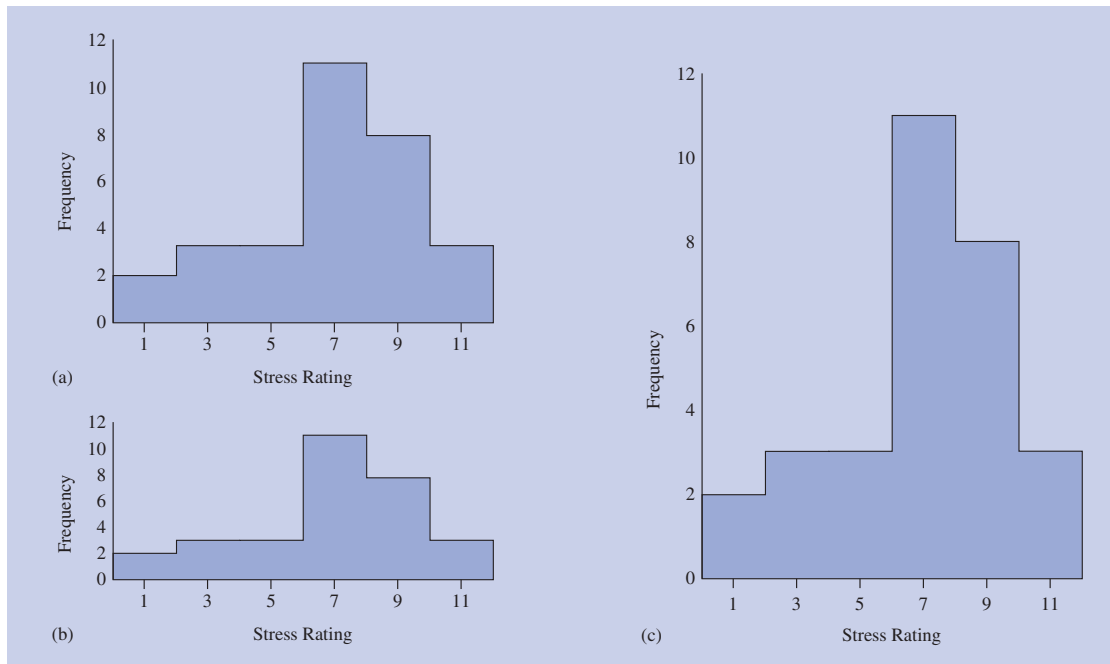


Figure 14 Histogram of students' stress ratings distorted from the standard of width 1 to 1.5 times height. (Data based on Aron et al., 1995.)

happens if we make the graph much shorter or taller, as shown in Figures 14b and 14c. The effect is like using computer software to alter a photograph of a person: the true picture is distorted. Any particular shape of the histogram is in a sense accurate. But the 1-to-1.5 proportion has been adopted to give people a standard for comparison. Changing this proportion misleads the eye.

Frequency Tables and Histograms in Research Articles

Psychology researchers mainly use frequency tables and histograms as an important first step before conducting more elaborate statistical analyses. Knowing the distribution of the variables in your research study can influence the type of statistical analysis that you conduct. The best way to identify the distribution of a variable is to look at it visually, using a frequency table or histogram. However, frequency tables and histograms are usually not included in research articles, and when they are, just because they are so rare, they are often not standard in some way. Hether and Murphy (2010) conducted an interesting study, in which they examined the health-related plots from three seasons of 10 prime-time U.S. television programs. They were particularly interested in whether male and female characters on the programs experienced different types of health issues. The researchers devised a detailed coding scheme that graduate students then used to code the health issues experienced by male and female characters in the television programs. Table 8 is a frequency table showing the 10 most common health issues

Rank	Health Issue	Freq	% Within Sex
Males			
1	Homicide	245	33
2	Unusual illness/disease	107	15
3	Unintentional injury	43	6
4	Motor vehicle related	37	5
5	Illegal substance abuse	30	4
6	Heart disease	28	4
7	Mental health	23	3
8	Toxic substance exposure	22	3
9	Unintentional falls	20	3
10	Prescription medication abuse	19	3
<i>N</i>		732	
Females			
1	Homicide	151	27
2	Unusual illness/disease	89	16
3	Pregnancy related	47	9
4	Rape/sexual assault	29	5
5	Unintentional injury	23	4
6	Mental health	22	4
7	Cancer	21	4
8	Motor vehicle related	21	4
9	Heart disease	16	3
10	Alcohol abuse	14	3
		554	

Source: Hether, H. J., & Murphy, S. T. (2010). Sex roles in health storylines on prime time television: A content analysis. *Sex Roles, 62*, 810–821. Reprinted by permission of Springer publishers.

Displaying the Order in a Group of Numbers Using Tables and Graphs

faced by the male and female characters. The table shows, for example, that the most common health issue was homicide, which represented 33% and 27% of the health issues experienced by male and female characters, respectively.

Histograms are even more rare in research articles (except in articles *about* statistics), but they do appear occasionally. Maggi and colleagues (2007) conducted a study of age-related changes in cigarette smoking behaviors in Canadian adolescents. As shown in Figure 15, they created a histogram—from a grouped frequency table—to display their results. Their histogram shows the results from the two samples they studied (one shown in the light colored bars and the other in the dark colored bars). As you can see in the figure, less than 10% of the 10- and 11-year-olds reported that they had tried smoking, but more than half of the 16- and 17-year-olds said they had tried smoking. As already mentioned, such figures are often not standard in some way. In this example, the researchers drew the histogram with gaps between the bars, whereas it is standard not to use gaps (unless you are drawing a bar graph for a nominal variable). However, the histogram still does a good job of showing the distribution. Also, the researchers, to allow for a fair comparison of how the rate of smoking differed among adolescents of varying ages, plotted the percentage of adolescents on the vertical axis instead of the actual number of adolescents. (This was a very appropriate change from the standard in this case. Plotting the actual number of adolescents who reported smoking would have been misleading, because there were not the same number of individuals in each of the age groups.)

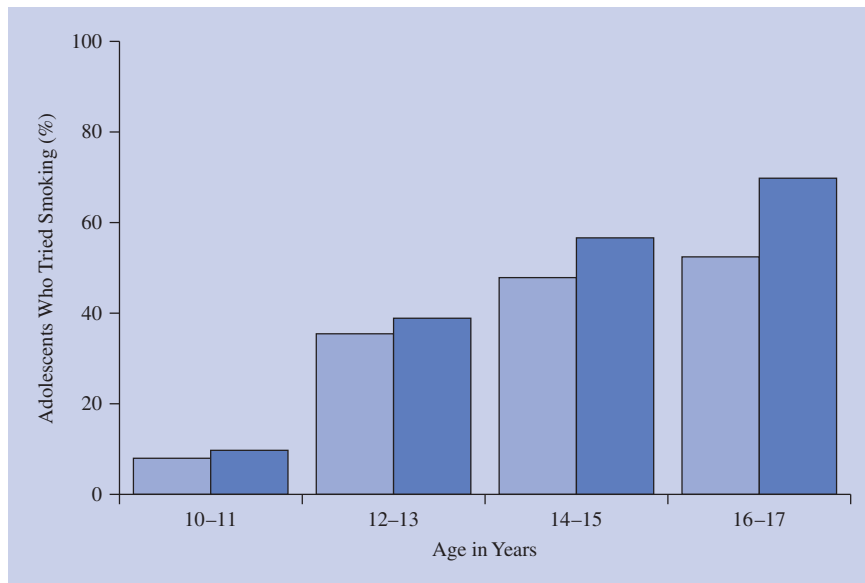


Figure 15 Change in the percentage of adolescents surveyed in the Canadian National Longitudinal Survey of Children and Youth longitudinal sample ($N = 9,667$; light blue) and cross-sectional sample ($N = 7,965$; dark blue), who had tried smoking between the ages of 10 and 17 years.

Source: Maggi, S., Hertzman, C., & Vaillancourt, T. (2007). Changes in smoking behaviors from late childhood to adolescence: Insights from the Canadian National Longitudinal Survey of Children and Youth. *Health Psychology, 26*, 232–240. Published by the American Psychological Association. Reprinted with permission.

Learning Aids

Summary

1. Psychologists use descriptive statistics to describe and summarize a group of numbers from a research study.
2. A value is a number or category; a variable is a characteristic that can have different values; a score is a particular person's value on the variable.
3. Most variables in psychology research are numeric with approximately equal intervals. However, some numeric variables are rank-ordered (the values are ranks), and some variables are not numeric at all (the values are categories).
4. A frequency table organizes the scores into a table of each of the possible values with the frequency and percentage of scores with that value.
5. When there are many different values, a grouped frequency table is useful. It is like an ordinary frequency table except that the frequencies are given for intervals that include a range of values.
6. The pattern of frequencies in a distribution can be shown visually with a histogram (or bar graph for a nominal variable), in which the height of each bar is the frequency for a particular value.
7. The general shape of a histogram can be unimodal (having a single peak), bimodal (having two peaks), multimodal (including bimodal), or rectangular (having no peak); it can be symmetrical or skewed (having a long tail) to the right or the left; and, compared to the bell-shaped normal curve, it can be kurtotic (having a peaked or flat distribution).
8. Statistical graphs for the general public are sometimes distorted in ways that mislead the eye, such as failing to use equal intervals or exaggerating proportions.
9. Frequency tables and histograms are very important for researchers in the process of interpreting their results, but are rarely shown in published research articles. When they are, they often follow nonstandard formats or involve frequencies (or percentages) for a nominal variable. The shapes of distributions (normal, skewed, and so on) are more often described.

Key Terms

statistics
descriptive statistics
inferential statistics
variable
values
score
numeric variable
equal-interval variable
ratio scale
rank-order variable

nominal variable
levels of measurement
discrete variable
continuous variable
frequency table
interval
grouped frequency table
histogram
frequency distribution
unimodal distribution

bimodal distribution
multimodal distribution
rectangular distribution
symmetrical distribution
skewed distribution
floor effect
ceiling effect
normal curve
kurtosis

Example Worked-Out Problems

Ten first-year university students rated their interest in graduate school on a scale from 1 = *no interest at all* to 6 = *high interest*. Their scores were as follows: 2, 4, 5, 5, 1, 3, 6, 3, 6, 6.

Making a Frequency Table

See Figure 16.

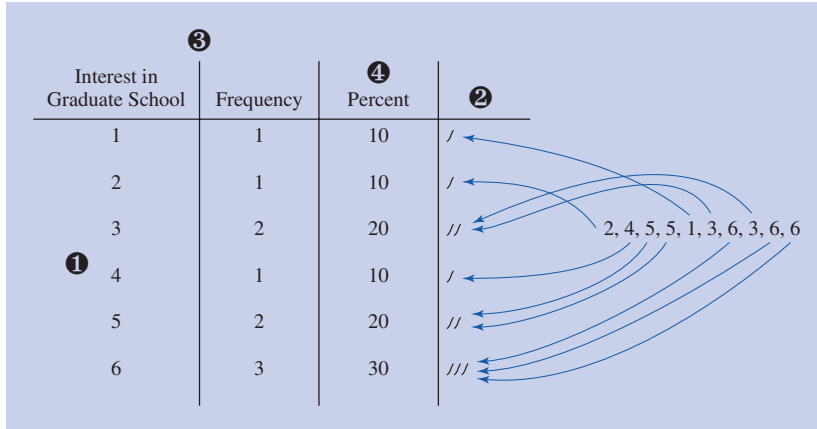


Figure 16 Answer to Example Worked-Out Problem for making a frequency table. ① Make a list down the page of each possible value, from lowest to highest. ② Go one by one through the scores, making a mark for each next to its value on your list. ③ Make a table showing how many times each value on your list is used. ④ Figure the percentage of scores for each value.

Making a Histogram

See Figure 17.

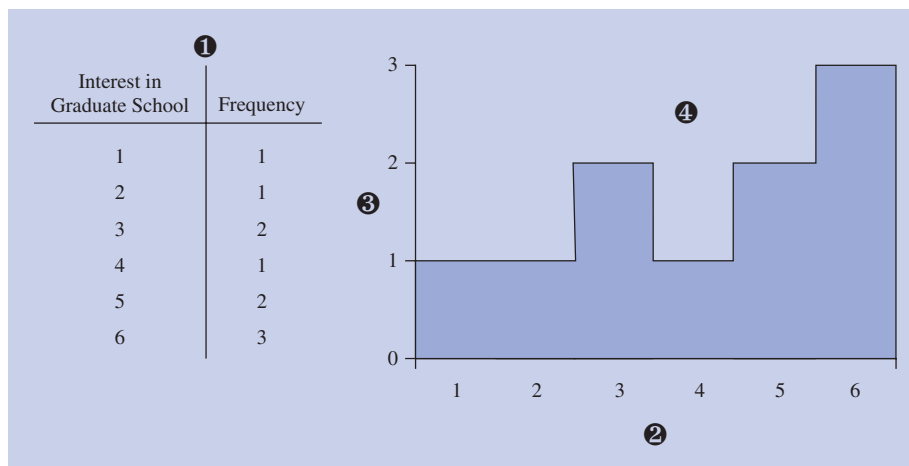


Figure 17 Answer to Example Worked-Out Problem for making a histogram. ① Make a frequency table. ② Put the values along the bottom of the page (from left to right, from lowest to highest). ③ Make a scale of frequencies along the left edge of the page (going from 0 at the bottom to the highest frequency for any value). ④ Make a bar for each value (with a height for the frequency of that value).

Practice Problems

These problems involve tabulation and making graphs. Most real-life statistics problems are done with special statistical software. Even if you have such software, do these problems by hand to ingrain the method in your mind. To learn how to use a computer to solve statistics problems like those in this chapter, refer to the Using SPSS section at the end of this chapter.

All data are fictional unless an actual citation is given.

Set I

MyStatLab

1. A client rates her satisfaction with her vocational counselor as a 3 on a 4-point scale from 1 = *not at all satisfied* to 4 = *very satisfied*. What is the (a) variable, (b) possible values, and (c) this client's score?
2. Give the level of measurement for each of the following variables: (a) ethnic group to which a person belongs, (b) number of times a mouse makes a wrong turn in a laboratory maze, and (c) position a runner finishes in a race.

MyStatLab

3. A particular block in a suburban neighborhood has 20 households. The number of children in these households is as follows:

2, 4, 2, 1, 0, 3, 6, 0, 1, 1, 2, 3, 2, 0, 1, 2, 1, 0, 2, 2

Make (a) a frequency table and (b) a histogram. Then (c) describe the general shape of the distribution.

MyStatLab

4. Fifty students were asked how many hours they studied this weekend. Here are their answers:

11, 2, 0, 13, 5, 7, 1, 8, 12, 11, 7, 8, 9, 10, 7, 4, 6, 10, 4, 7, 8, 6, 7, 10, 7, 3,
11, 18, 2, 9, 7, 3, 8, 7, 3, 13, 9, 8, 7, 7, 10, 4, 15, 3, 5, 6, 9, 7, 10, 6

Make (a) a frequency table and (b) a histogram. Then (c) describe the general shape of the distribution.

MyStatLab

5. These are the scores on a test of sensitivity to smell taken by 25 chefs attending a national conference:

96, 83, 59, 64, 73, 74, 80, 68, 87, 67, 64, 92, 76, 71, 68, 50, 85,
75, 81, 70, 76, 91, 69, 83, 75

Make (a) a frequency table and (b) histogram. (c) Make a grouped frequency table using intervals of 50–59, 60–69, 70–79, 80–89, and 90–99. Based on the grouped frequency table, (d) make a histogram and (e) describe the general shape of the distribution.

MyStatLab

6. The following data are the number of minutes it took each of a group of 34 10-year-olds to do a series of abstract puzzles:

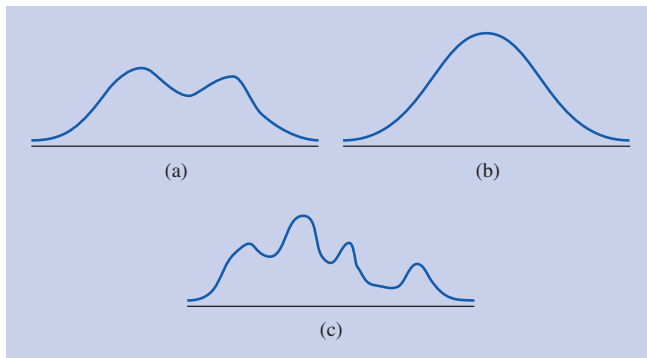
24, 83, 36, 22, 81, 39, 60, 62, 38, 66, 38, 36, 45, 20, 20, 67, 41, 87,
41, 82, 35, 82, 28, 80, 80, 68, 40, 27, 43, 80, 31, 89, 83, 24

Make (a) a frequency table and (b) a grouped frequency table using intervals of 20–29, 30–39, 40–49, 50–59, 60–69, 70–79, and 80–89. Based on the grouped frequency table, (c) make a histogram and (d) describe the general shape of the distribution.

Displaying the Order in a Group of Numbers Using Tables and Graphs

7. Describe the shapes of the three distributions illustrated.

MyStatLab



8. Make up and draw an example of each of the following distributions: (a) symmetrical, (b) rectangular, and (c) skewed to the right.
9. Explain to a person who has never had a course in statistics what is meant by (a) a symmetrical unimodal distribution and (b) a negatively skewed unimodal distribution. (Be sure to include in your first answer an explanation of what *distribution* means.)
10. McKee and Ptacek (2001) asked 90 college students about a time they had delivered bad news to someone. Table 9 shows the results for the type of bad news given. (a) Using this table as an example, explain the idea of a frequency table to a person who has never had a course in statistics. (b) Explain the general meaning of the pattern of results.

MyStatLab

MyStatLab

Set II

11. A participant in a cognitive psychology study is given 50 words to remember and later asked to recall as many of the words as she can. She recalls 17 words. What is the (a) variable, (b) possible values, and (c) score?

MyStatLab

Table 9 Descriptive Statistics for the Type of News Given

Category	Frequency	Percentage
1. Relationship with family	19	21.1
2. School	1	1.1
3. Job/work	6	6.7
4. Relationship with actual/potential girlfriend/boyfriend	17	18.9
5. Personal health	1	1.1
6. Finance	1	1.1
7. Relationship with friends	21	23.3
8. Health of family member/friend	23	25.6
9. Other	1	1.1

Source: McKee, T. L. E., & Ptacek, J. T. (2001). I'm afraid I have something bad to tell you: Breaking bad news from the perspective of the giver. *Journal of Applied Social Psychology, 31*, 246–273. Copyright © 2001. Reprinted by permission of Blackwell Publishers Journals.

Displaying the Order in a Group of Numbers Using Tables and Graphs

MyStatLab

12. Explain and give an example for each of the following types of variables: (a) equal-interval, (b) rank-order, (c) nominal, (d) ratio scale, (e) continuous; (f) discrete.

13. An organizational psychologist asks 20 employees in a company to rate their job satisfaction on a 5-point scale from 1 = *very unsatisfied* to 5 = *very satisfied*. The ratings are as follows:

3, 2, 3, 4, 1, 3, 3, 4, 5, 2, 3, 5, 2, 3, 3, 4, 1, 3, 2, 4

Make (a) a frequency table and (b) a histogram. Then (c) describe the general shape of the distribution.

14. A social psychologist asked 15 college students how many times they “fell in love” before they were 11 years old. The numbers of times were as follows:

2, 0, 6, 0, 3, 1, 0, 4, 9, 0, 5, 6, 1, 0, 2

Make (a) a frequency table and (b) a histogram. Then (c) describe the general shape of the distribution.

15. Following are the speeds of 40 cars clocked by radar on a particular road in a 35-mph zone on a particular afternoon:

30, 36, 42, 36, 30, 52, 36, 34, 36, 33, 30, 32, 35, 32, 37, 34, 36, 31, 35, 20, 24, 46, 23, 31, 32, 45, 34, 37, 28, 40, 34, 38, 40, 52, 31, 33, 15, 27, 36, 40

Make (a) a grouped frequency table and (b) a histogram based on the grouped frequency table. Then (c) describe the general shape of the distribution.

16. Here are the number of holiday gifts purchased by 25 families randomly interviewed at a local mall at the end of the holiday season:

22, 18, 22, 26, 19, 14, 23, 27, 2, 18, 28, 28, 11, 16, 34, 28, 13, 21, 32, 17, 6, 29, 23, 22, 19

Make (a) a frequency table and (b) a grouped frequency table using intervals of 0–4, 5–9, 10–14, 15–19, 20–24, 25–29, and 30–34. Based on the grouped frequency table, (c) make a histogram and (d) describe the general shape of the distribution.

MyStatLab

17. Pick a book and a page number of your choice. (Select a page with at least 30 lines; *do not pick a textbook or any book with tables or illustrations.*) Make a list of the number of words on each line; use that list as your group of scores. Make (a) a frequency table and (b) a histogram. Then (c) describe the general shape of the distribution. (Be sure to give the name, author, publisher, and year of the book you used, along with the page number, with your answer.)

MyStatLab

18. Explain to a person who has never taken a course in statistics the meaning of a grouped frequency table.

19. Make up and draw an example of each of the following distributions: (a) bimodal, (b) approximately rectangular, and (c) skewed to the right. Do not use an example given in class.

20. Find an example in a newspaper, magazine, or news website of a graph that misleads by failing to use equal interval sizes or by exaggerating proportions.

MyStatLab

21. Raskauskas and Stoltz (2007) asked a group of 84 adolescents about their involvement in traditional and electronic bullying. The researchers defined electronic bullying as “... a means of bullying in which peers use electronics [such as text messages, emails, and defaming Web sites] to taunt, threaten,

Displaying the Order in a Group of Numbers Using Tables and Graphs

Table 10 Incidence of Traditional and Electronic Bullying and Victimization ($N = 84$)

Form of Bullying	<i>N</i>	%
Electronic victims	41	48.8
Text-message victim	27	32.1
Internet victim (Web sites, chatrooms)	13	15.5
Picture-phone victim	8	9.5
Traditional victims	60	71.4
Physical victim	38	45.2
Teasing victim	50	59.5
Rumors victim	32	38.6
Exclusion victim	30	50.0
Electronic bullies	18	21.4
Text-message bully	18	21.4
Internet bully	11	13.1
Traditional bullies	54	64.3
Physical bully	29	34.5
Teasing bully	38	45.2
Rumor bully	22	26.2
Exclusion bully	35	41.7

Source: Raskauskas, J., & Stoltz, A. D. (2007). Involvement in traditional and electronic bullying among adolescents. *Developmental Psychology*, 43, 564–575. Published by the American Psychological Association. Reprinted with permission.

harass, and/or intimidate a peer” (p. 565). Table 10 is a frequency table showing the adolescents’ reported incidence of being victims or perpetrators of traditional and electronic bullying. (a) Using this table as an example, explain the idea of a frequency table to a person who has never had a course in statistics. (b) Explain the general meaning of the pattern of results.

22. Mouradian (2001) surveyed college students selected from a screening session to include two groups: (a) “Perpetrators”—students who reported at least one violent act (hitting, shoving, etc.) against their partner in their current or most recent relationship—and (b) “Comparisons”—students who did not report any such uses of violence in any of their last three relationships. At the actual testing session, the students first read a description of an aggressive behavior such as, “Throw something at his or her partner” or “Say something to upset his or her partner.” They then were asked to write “as many examples of circumstances of situations as [they could] in which a person might engage in behaviors or acts of this sort with or towards their significant other.” Table 11 shows the “Dominant Category of Explanation” (the category a participant used most) for females and males, broken down by comparisons and perpetrators. (a) Using this table as an example, explain the idea of a frequency table to a person who has never had a course in statistics. (b) Explain the general meaning of the pattern of results.


MyStatLab

Table 11 Dominant Category of Explanation for Intimate Aggression by Gender and Perpetrator Status







Category	Group							
	Female				Male			
	Comparisons (n = 36)		Perpetrators (n = 33)		Comparisons (n = 32)		Perpetrators (n = 25)	
	f	%	f	%	f	%	f	%
Self-defense	2	6	3	9	3	9	1	4
Control motives	8	22	9	27	9	28	3	12
Expressive aggression	4	11	3	9	3	9	8	32
Face/self-esteem preservation	1	3	2	6	2	6	3	12
Exculpatory explanations	5	14	3	9	3	9	3	12
Rejection of perpetrator or act	12	33	6	18	10	31	7	28
Prosocial/acceptable explanations	0	0	0	0	0	0	0	0
Tied categories	4	11	7	21	2	6	0	0


Note: f = frequency. % = percentage of respondents in a given group who provided a particular category of explanation.
Source: Mouradian, V. E. (2001). Applying schema theory to intimate aggression: Individual and gender differences in representation of contexts and goals. *Journal of Applied Social Psychology, 31*, 376–408. Copyright © 2001 by Blackwell Publishing. Reprinted by permission of Blackwell Publishers Journals.

Using SPSS

The  in the following steps indicates a mouse click. (We used SPSS version 19.0 to carry out these analyses. The steps and output may be slightly different for other versions of SPSS.)

Creating a Frequency Table

- 1 Enter the scores from your distribution in one column of the data window.
- 2  Analyze.
- 3  Descriptive statistics.
- 4  Frequencies.
- 5  the variable you want to make a frequency table of and then  the arrow.
- 6  OK.

Practice the preceding steps by creating a frequency table for the social interactions example in this chapter (the scores are also listed). After Step 5, your screen should look like Figure 18. Your output window (which appears after you  OK in Step 6) should look like Figure 19. As you will see, SPSS automatically produces a column with the cumulative percentage (or percentile) for each value. (Note that it is possible to create grouped frequency tables in SPSS, but since it is not a straightforward process, we do not cover it here.)

Displaying the Order in a Group of Numbers Using Tables and Graphs

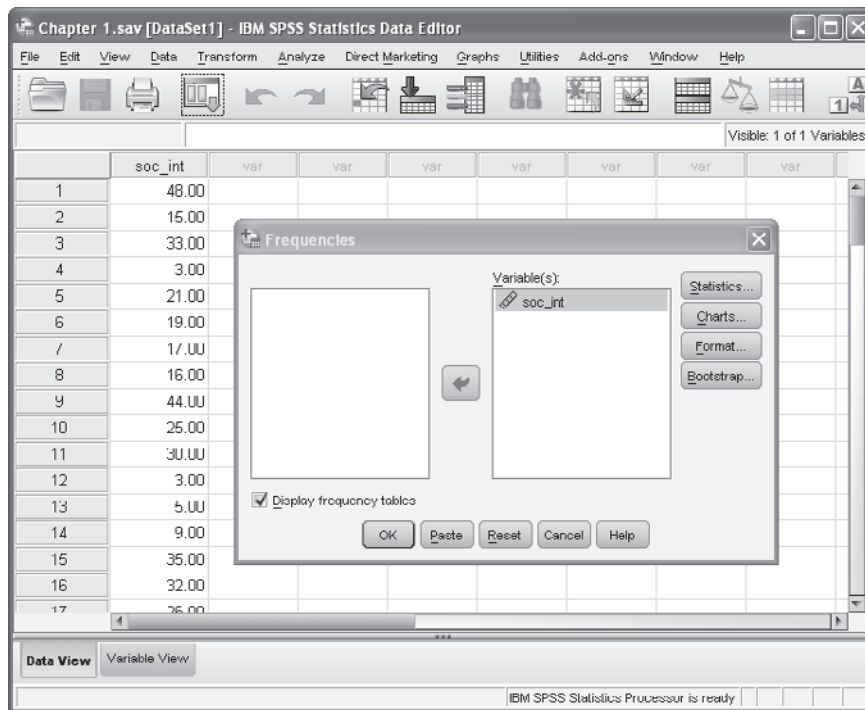


Figure 18 SPSS data window and frequencies window for the social interactions example. (Data from McLaughlin-Volpe et al., 2001.)

Creating a Histogram

- 1 Enter the scores from your distribution in one column of the data window.
- 2 Analyze.
- 3 Descriptive statistics.
- 4 Frequencies.
- 5 the variable you want to make a histogram of and then the arrow.
- 6 Charts, Histograms, Continue.
- 7 Optional: To instruct SPSS *not* to produce a frequency table, the box labeled *Display frequency tables* (this *unchecks* the box).
- 8 OK.

Practice these steps by creating a histogram for the social interactions example in this chapter (the scores are also listed). Your output window should look like Figure 20. Notice that SPSS automatically creates a histogram based on a grouped frequency table. (Should you wish, you can change the number of intervals or the

Displaying the Order in a Group of Numbers Using Tables and Graphs

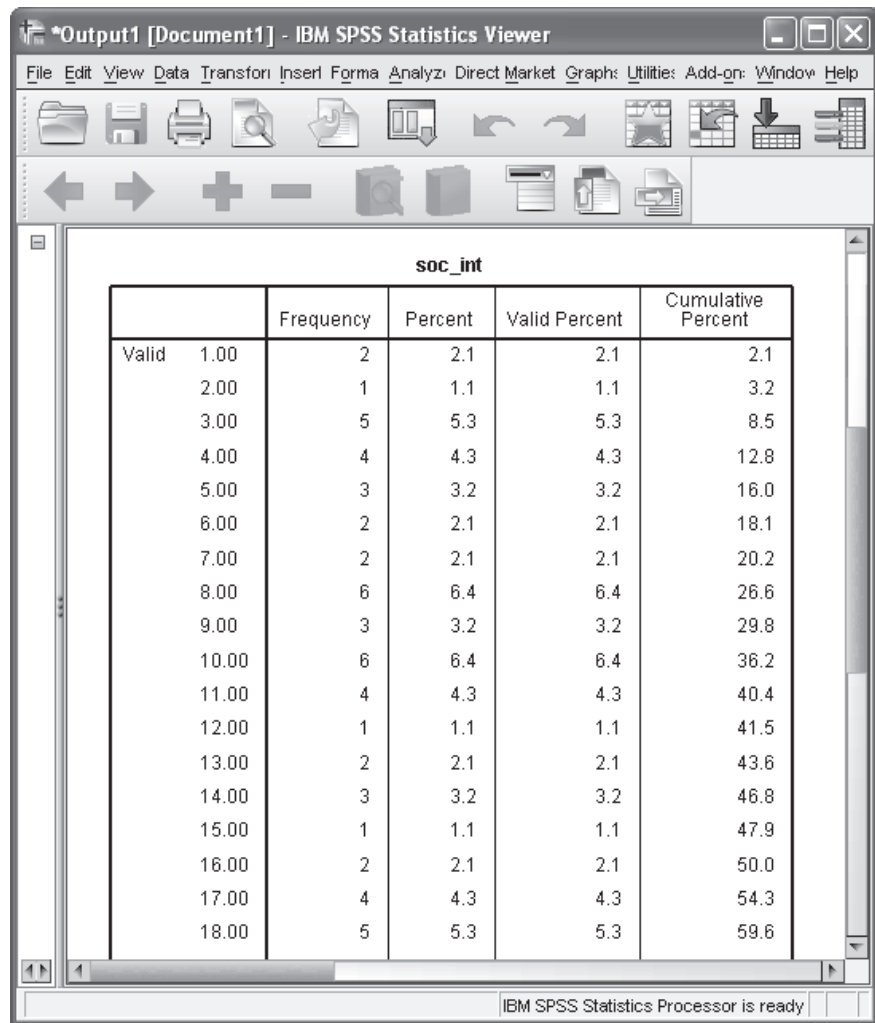


Figure 19 SPSS frequency table for the social interactions example. (Data from McLaughlin-Volpe et al., 2001.)

interval size for the histogram by doing the following: Place your mouse cursor on the histogram and double-click to bring up a Chart Editor window; place your mouse cursor over one of the bars in the histogram and double-click to bring up a Properties window; *Custom*; *Number of intervals* or *Interval Width*; then enter the number of intervals or the interval size you want; *Apply*. If you want a nongrouped histogram, type in “1” for the interval width.)

Displaying the Order in a Group of Numbers Using Tables and Graphs

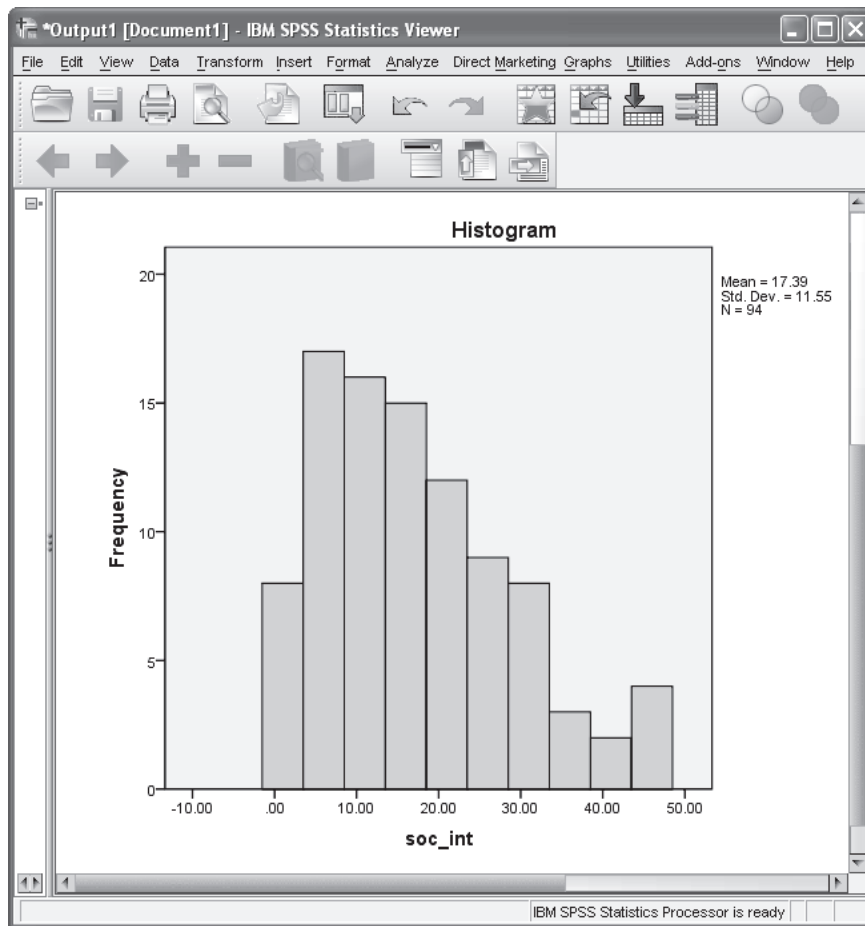


Figure 20 SPSS histogram for the social interactions example. (Data from McLaughlin-Volpe et al., 2001.)

Chapter Note

1. Most research articles follow the procedure we recommend here: going from lowest at the top to highest at the bottom. However, some statistics authorities recommend going from highest at the top to lowest at the bottom.

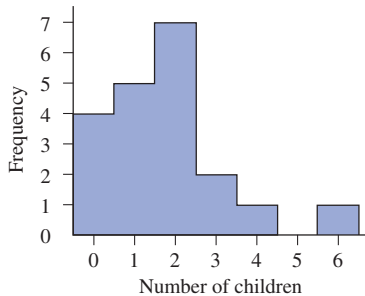
Answers to Set I Practice Problems

Chapter 1

- (a) Satisfaction with the vocational counselor; (b) 1, 2, 3, or 4; (c) 3.
- (a) Nominal (or categorical); (b) numeric (or quantitative)—more precisely, equal interval; (c) numeric (or quantitative)—more precisely, rank order (or ordinal).
- (a) Frequency table:

Number of Children	Frequency	Percent
0	4	20
1	5	25
2	7	35
3	2	10
4	1	5
5	0	0
6	1	5

(b) Histogram:



(c) Unimodal, skewed to the right.

4. (a) Frequency table:

Hours	Frequency	Percent	Hours	Frequency	Percent
0	1	2	10	5	10
1	1	2	11	3	6
2	2	4	12	1	2
3	4	8	13	2	4
4	3	6	14	0	0
5	2	4	15	1	2
6	4	8	16	0	0
7	11	22	17	0	0
8	5	10	18	1	2
9	4	8			

(b) Histogram is based on the preceding frequency table. See answer to question 3b for an example.

(c) Approximately unimodal, slightly skewed to the right.

5. (a) Frequency table:

Score	Frequency	Percent	Score	Frequency	Percent
50	1	4	74	1	4
51	0	0	75	2	8
52	0	0	76	2	8
53	0	0	77	0	0
54	0	0	78	0	0
55	0	0	79	0	0
56	0	0	80	1	4
57	0	0	81	1	4
58	0	0	82	0	0
59	1	4	83	2	8
60	0	0	84	0	0
61	0	0	85	1	4
62	0	0	86	0	0
63	0	0	87	1	4
64	2	8	88	0	0
65	0	0	89	0	0
66	0	0	90	0	0
67	1	4	91	1	4
68	2	8	92	1	4
69	1	4	93	0	0
70	1	4	94	0	0
71	1	4	95	0	0
72	0	0	96	1	4
73	1	4			

(b) Based on the preceding frequency table. See answer to question 3b for an example.

(c) Grouped frequency table:

Interval	Frequency	Percent
50–59	2	8
60–69	6	24
70–79	8	32
80–89	6	24
90–99	3	12

Displaying the Order in a Group of Numbers Using Tables and Graphs

(d) Based on frequency table in (c) above. See answer to question 3b for an example.

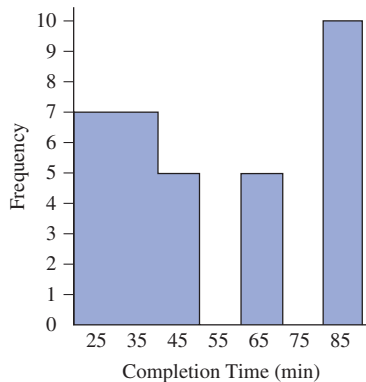
(e) Unimodal, approximately symmetrical (slightly skewed to the left).

6. (a) Similar to question 5a.

(b) Grouped frequency table:

Interval	Frequency	Percent
20–29	7	20.6
30–39	7	20.6
40–49	5	14.7
50–59	0	0.0
60–69	5	14.7
70–79	0	0.0
80–89	10	29.4

(c) Histogram:



(d) Roughly rectangular.

7. (a) Bimodal; (b) approximately normal (or unimodal or symmetrical); (c) multimodal.

8. (a) Any distribution that is symmetrical (that is, the pattern of frequencies on the left and right side of the distribution are mirror images of each other); (b) any distribution in which all values have approximately the same frequency; (c) any distribution in which the scores are piled up on the left side of the distribution and are spread out on the right side of the distribution.

9. (a) A distribution is the way a group of numbers is spread out over the possible values the numbers can have. You can describe such a distribution with a graph, called a histogram—a kind of bar graph with one bar for each possible value with one unit of height for each time its particular value occurs. In a histogram, a symmetrical distribution has a symmetrical shape (the right and left halves are mirror images). A unimodal distribution is one in which this graph has a single high point, with the other values gradually decreasing around it.

(b) A negatively skewed unimodal distribution has a single high point, is not symmetrical, and its tail—the long, low side—extends to the left (where the negative scores go on the graph).

10. (a) This is called a frequency table because it lays out how frequently (how many times) each category occurs for nine different categories. A frequency table makes the pattern of numbers easy to see. For example, of the 90 college students in the study, 19 gave bad news about Relationship with family (the first category). The table also gives the percentages. For example, 19 students is 19/90 of the total, or 21.1 percent.

(b) The most bad news is given in four of the nine categories: Relationship with family, Relationship with actual/potential girlfriend/boyfriend, Relationship with friends, and Health of family member/friend. All of these categories had to do with family members or friends and most with relationships, and there were few cases in the other categories (which had little directly to do with family or friends).

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Central Tendency and Variability

From Chapter 2 of *Statistics for Psychology*, Sixth Edition. Arthur Aron, Elliot J. Coups, and Elaine N. Aron. Copyright © 2013 by Pearson Education, Inc. All rights reserved.

Central Tendency and Variability

Chapter Outline

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TIP FOR SUCCESS

Before beginning this chapter, you should be sure that you are comfortable with the key terms *variable*, *score*, and *value*.

Several recent studies have asked students to record their dreams each morning for a week or two (e.g., Mikulincer, Shaver, & Avihou-Kanza, 2011). In a typical study, the total number of dreams per week for 10 students might be as follows: 7, 8, 8, 7, 3, 1, 6, 9, 3, 8. How might you make this group of scores more understandable?

One way to gain that understanding is to use the tables and graphs. Another way, which you will learn in this chapter, is to use numbers to describe and summarize a group of scores such as this.

First, you can describe a group of scores in terms of a *representative* (or *typical*) *value*, such as an average. A representative value gives the *central tendency* of a group of scores. A representative value is a simple way, with a single number, to describe a group of scores (and there may be hundreds—or even thousands—of scores). The main representative value we consider is the *mean*. Next, we focus on ways of describing how the numbers are spread out in a group of scores. In other words, we consider the amount of variation, or *variability*, among the scores. The two measures of variability you will learn about are called the *variance* and *standard deviation*.

In this chapter, you will use statistical formulas. Such formulas are not here to confuse you. Hopefully, you will come to see that they actually simplify things and provide a very straightforward, concise way of describing statistical procedures. To help you grasp what such formulas mean in words, whenever we present formulas, we always also give the “translation” in ordinary English.

Central Tendency

The **central tendency** of a group of scores (a distribution) refers to the middle of the group of scores. You will learn about three measures of central tendency: *mean*, *mode*, and *median*. Each measure of central tendency uses its own method to come up with a single number describing the middle of a group of scores. We start with the mean, the most commonly used measure of central tendency. Understanding the mean is also an important foundation for much of what you learn later.

The Mean

Usually the best measure of central tendency is the ordinary average, the sum of all the scores divided by the number of scores. In statistics, this is called the **mean**. The average, or mean, of a group of scores is a representative value. Consider again the example from the start of the chapter, in which 10 students recorded their total number of dreams during the last weeks. The mean of the 10 scores (7, 8, 8, 7, 3, 1, 6, 9, 3, 8) is 6 (the sum of 60 dreams divided by 10 students). That is, on the average, each student had 6 dreams in the past week. The information for the 10 students is thus summarized by the single number 6.

You can think of the mean as a kind of balancing point for the distribution of scores. Try it by visualizing a board balanced over a log, like a rudimentary teeter-totter. Imagine piles of blocks set along the board according to their values, one for each score in the distribution (like a histogram made of blocks). The mean is the point on the board where the weight of the blocks on one side balances exactly with the weight on the other side. Figure 1 shows this for the number of dreams for the 10 students.

Mathematically, you can think of the mean as the point at which the total distance to all the scores above that point equals the total distance to all the scores below that point. Let's first figure the total distance from the mean to all the scores above the mean for the dreams example shown in Figure 1. There are two scores of 7, each of which is 1 unit above 6 (the mean). There are three scores of 8, each of which is 2 units above 6. And, there is one score of 9, which is 3 units above 6. This gives a total distance of 11 units ($1 + 1 + 2 + 2 + 2 + 3$) from the mean to all the scores above the mean. Now, let's look at the scores below the mean. There are two scores of 3, each of which is 3 units below 6 (the mean). And there is one score of 1, which is 5 units below 6. This gives a total distance of 11 units ($3 + 3 + 5$) from the mean to all of the scores below the mean. Thus, you can see that the total distance from the mean to the scores above the mean is the same as the total distance from the mean to the scores below the mean. The scores above the mean balance out the scores below the mean (and vice-versa).

central tendency typical or most representative value of a group of scores.

mean arithmetic average of a group of scores; sum of the scores divided by the number of scores.

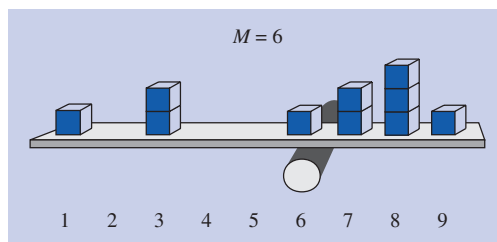


Figure 1 Mean of the distribution of the number of dreams during a week for 10 students, illustrated using blocks on a board balanced on a log.

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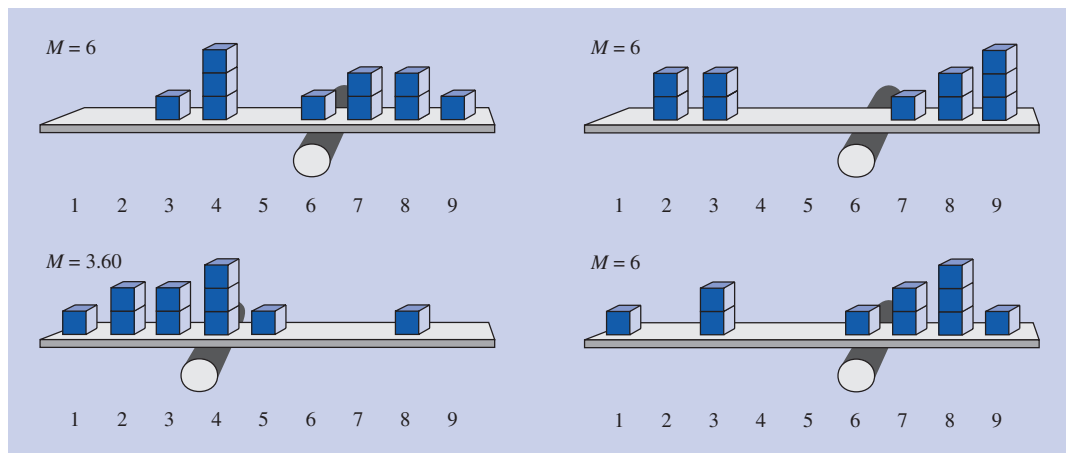


Figure 2 Means of various distributions illustrated with blocks on a board balanced on a log.

Some other examples are shown in Figure 2. Notice that there doesn't have to be a block right at the balance point. That is, the mean doesn't have to be a score actually in the distribution. The mean is the average of the scores, the balance point. The mean can be a decimal number, even if all the scores in the distribution have to be whole numbers (a mean of 2.30 children, for example). For each distribution in Figure 2, the total distance from the mean to the scores above the mean is the same as the total distance from the mean to the scores below the mean. (By the way, this analogy to blocks on a board, in reality, works out precisely only if the board has no weight of its own.)

Formula for the Mean and Statistical Symbols

The rule for figuring the mean is to add up all the scores and divide by the number of scores. Here is how this rule is written as a formula:

The mean is the sum of the scores divided by the number of scores.

$$M = \frac{\sum X}{N} \quad (1)$$

M is a symbol for the mean. An alternative symbol, \bar{X} ("X-bar"), is sometimes used. However, M is almost always used in published research articles, as recommended by the style guidelines of the American Psychological Association (2009). You will see \bar{X} used mostly in advanced statistics books and in articles *about* statistics. In fact, there is not a standard agreement for many of the symbols used in statistics. (In this text we generally use the symbols most widely found in psychology research articles.)

\sum , the capital Greek letter sigma, is the symbol for "sum of." It means "add up all the numbers for whatever follows." It is the most common special arithmetic symbol used in statistics.

X stands for the scores in the distribution of the variable X . We could have picked any letter. However, if there is only one variable, it is usually called X .

$\sum X$ is "the sum of X ." This tells you to add up all the scores in the distribution of the variable X . Suppose X is the number of dreams of our 10 students: $\sum X$ is $7 + 8 + 8 + 7 + 3 + 1 + 6 + 9 + 3 + 8$, which is 60.

TIP FOR SUCCESS

Think of each formula as a statistical recipe, with statistical symbols as ingredients. Before you use each formula, be sure you know what each symbol stands for. Then carefully follow the formula to come up with the end result.

M mean.

\sum sum of; add up all the scores following this symbol.

X scores in the distribution of the variable X .

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N stands for number—the number of scores in a distribution. In our example, there are 10 scores. Thus, N equals 10.¹

Overall, the formula says to divide the sum of all the scores in the distribution of the variable X by the total number of scores, N . In the dreams example, this means you divide 60 by 10. Put in terms of the formula,

$$M = \frac{\sum X}{N} = \frac{60}{10} = 6$$

Additional Examples of Figuring the Mean

Consider the following example: The stress ratings of the 30 students in the first week of their statistics class (based on Aron et al., 1995) were:

8, 7, 4, 10, 8, 6, 8, 9, 9, 7, 3, 7, 6, 5, 0, 9, 10, 7, 7, 3, 6, 7, 5, 2, 1, 6, 7, 10, 8, 8

You can now summarize all this information as a single number by figuring the mean. Figure the mean by adding up all the stress ratings and dividing by the number of stress ratings. That is, you add up the 30 stress ratings: $8 + 7 + 4 + 10 + 8 + 6 + 8 + 9 + 9 + 7 + 3 + 7 + 6 + 5 + 0 + 9 + 10 + 7 + 7 + 3 + 6 + 7 + 5 + 2 + 1 + 6 + 7 + 10 + 8 + 8$, for a total of 193. Then you divide this total by the number of scores, 30. In terms of the formula,

$$M = \frac{\sum X}{N} = \frac{193}{30} = 6.43$$

This tells you that the average stress rating was 6.43 (after rounding off). This is clearly higher than the middle of the 0–10 scale. You can also see this on a graph. Think again of the histogram as a pile of blocks on a board and the mean of 6.43 as the point where the board balances on the fulcrum (see Figure 3). This single representative value simplifies the information in the 30 stress scores.

TIP FOR SUCCESS

When an answer is not a whole number, we suggest that you use two more decimal places in the answer than for the original numbers. In this example, the original numbers did not use decimals, so we rounded the answer to two decimal places.

N number of scores in a distribution.

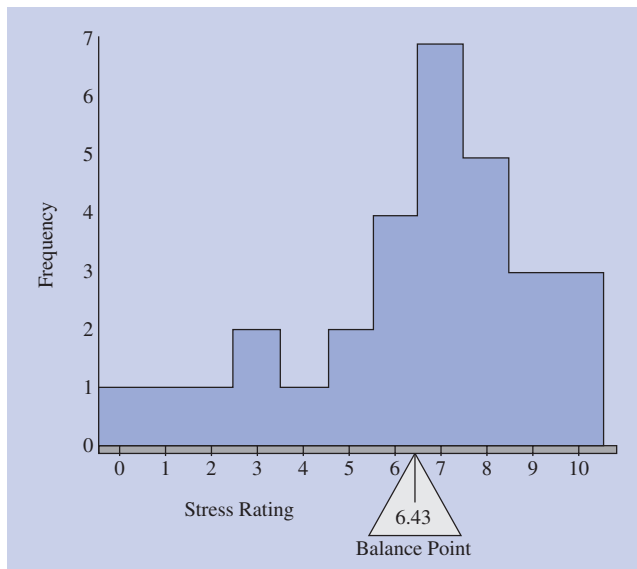


Figure 3 Analogy of blocks on a board balanced on a fulcrum showing the means for 30 statistics students' ratings of their stress level. (Data based on Aron et al., 1995.)

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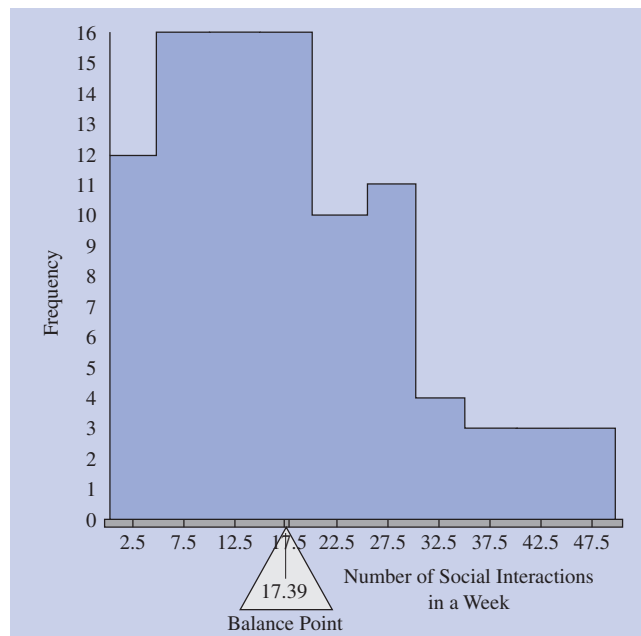


Figure 4 Analogy of blocks on a board balanced on a fulcrum illustrating the mean for number of social interactions during a week for 94 college students. (Data from McLaughlin-Volpe et al., 2001.)

Similarly, consider example of students' social interactions (McLaughlin-Volpe et al., 2001). The actual number of interactions over a week for the 94 students are as follows: 48, 15, 33, 3, 21, 19, 17, 16, 44, 25, 30, 3, 5, 9, 35, 32, 26, 13, 14, 14, 47, 47, 18, 11, 5, 19, 24, 17, 6, 25, 8, 18, 29, 1, 18, 22, 3, 22, 29, 2, 6, 10, 29, 10, 29, 21, 38, 41, 16, 17, 8, 40, 8, 10, 18, 7, 4, 4, 8, 11, 3, 23, 10, 19, 21, 13, 12, 10, 4, 17, 11, 21, 9, 8, 7, 5, 3, 22, 14, 25, 4, 11, 10, 18, 1, 28, 27, 19, 24, 35, 9, 30, 8, 26. We can now take those same 94 scores, add them up, and divide by 94 to figure the mean:

$$M = \frac{\sum X}{N} = \frac{1,635}{94} = 17.39$$

This tells us that during this week these students had an average of 17.39 social interactions. Figure 4 shows the mean of 17.39 as the balance point for the 94 social interaction scores.

Steps for Figuring the Mean

Figure the mean in two steps.

- ➊ **Add up all the scores.** That is, figure $\sum X$.
- ➋ **Divide this sum by the number of scores.** That is, divide $\sum X$ by N .

The Mode

The **mode** is another measure of central tendency. The mode is the most common single value in a distribution. In our dreams example, the mode is 8. This is because there are three students with 8 dreams and no other particular number of dreams

mode value with the greatest frequency in a distribution.

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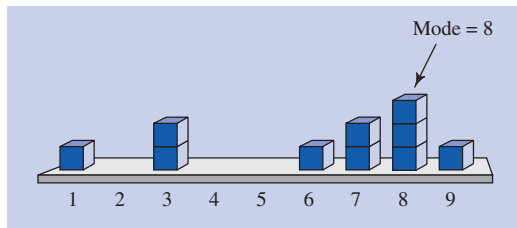


Figure 5 Mode as the high point in a distribution's histogram, using the example of the number of dreams during a week for 10 students.

with as many students. Another way to think of the mode is that it is the particular value with the largest frequency in a frequency table, the high point or peak of a distribution's histogram (as shown in Figure 5).

In a perfectly symmetrical unimodal distribution, the mode is the same as the mean. However, what happens when the mean and the mode are not the same? In that situation, the mode is usually not a very good way of describing the central tendency of the scores in the distribution. In fact, sometimes researchers compare the mode to the mean to show that the distribution is *not* perfectly symmetrical. Also, the mode can be a particularly poor representative value because it does not reflect many aspects of the distribution. For example, you can change some of the scores in a distribution without affecting the mode—but this is not true of the mean, which is affected by any changes in the scores in the distribution (see Figure 6).

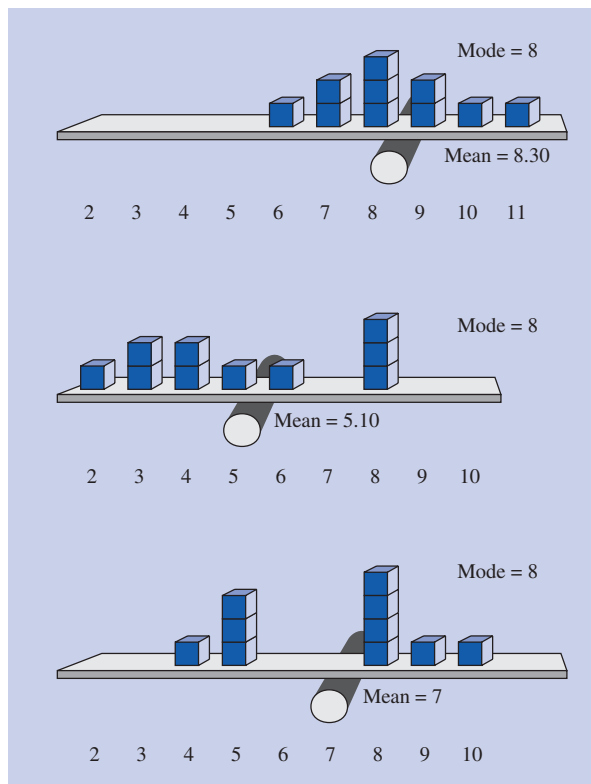


Figure 6 Effect on the mean and on the mode of changing some scores, using the example of the number of dreams during a week for 10 students.

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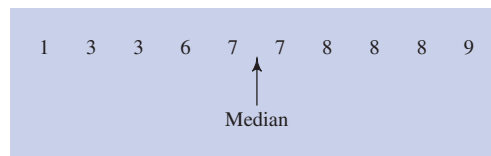


Figure 7 The median is the middle score when scores are lined up from lowest to highest, using the example of the number of dreams during a week for 10 students.

On the other hand, the mode *is* the usual way of describing the central tendency for a nominal variable. For example, if you know the religions of a particular group of people, the mode tells you for this group which religion has the most people in it. However, when it comes to figuring central tendency for numerical variables in psychology research, the mode is rarely used.

The Median

TIP FOR SUCCESS

When figuring the median, remember that the first step is to line up the scores from lowest to highest. Forgetting to do this is the most common mistake students make when figuring the median.

Another alternative to the mean is the **median**. If you line up all the scores from lowest to highest, the middle score is the median. Figure 7 shows the scores for the number of dreams lined up from lowest to highest. In this example, the fifth and sixth scores (the two middle ones) are both 7s. Either way, the median is 7.

When you have an even number of scores, the median can be between two different scores. In that situation, the median is the average (the mean) of those two scores.

Steps for Finding the Median

Finding the median takes three steps.

- 1 **Line up all the scores from lowest to highest.**
- 2 **Figure how many scores there are to the middle score by adding 1 to the number of scores and dividing by 2.** For example, with 29 scores, adding 1 and dividing by 2 gives you 15. The 15th score is the middle score. If there are 50 scores, adding 1 and dividing by 2 gives you 25.5. Because there are no half scores, the 25th and 26th scores (the scores on either side of 25.5) are the middle scores.
- 3 **Count up to the middle score or scores.** If you have one middle score, this is the median. If you have two middle scores, the median is the average (the mean) of these two scores.

Comparing the Mean, Mode, and Median

Sometimes, the median is better than the mean (or mode) as a representative value for a group of scores. This happens when a few extreme scores would strongly affect the mean but would not affect the median. Reaction time scores are a common example in psychology research. Suppose you are asked to press a key as quickly as possible when a picture of a particular political candidate is shown on the computer screen. On five showings of the candidate, your times (in seconds) to respond are .74, .86, 2.32, .79, and .81. The mean of these five scores is 1.1040: that is, $(\sum X)/N = 5.52/5 = 1.1040$. However, this mean is very much influenced by the one very long time (2.32 seconds). (Perhaps you were distracted just when the candidate's picture was shown.) The median is much less affected by the extreme score. The median of these five scores is .81—a value that is much more representative of most of the scores. Thus, using the median deemphasizes the one extreme time, which is probably appropriate. An extreme score like this is called an **outlier**. In this example, the outlier was much higher than the other scores, but in other cases an outlier may be much lower than the other scores in the distribution.

median middle score when all the scores in a distribution are arranged from lowest to highest.

outlier score with an extreme value (very high or very low) in relation to the other scores in the distribution.

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The importance of whether you use the mean, mode, or median can be seen in a controversy among psychologists studying the evolutionary basis of human mate choice. One set of theorists (e.g., Buss & Schmitt, 1993) argued that over their lives, men should prefer to have many partners, but women should prefer to have just one reliable partner. This is because a woman can have only a small number of children in a lifetime and her genes are most likely to survive if those few children are well taken care of. Men, however, can have a great many children in a lifetime. Therefore, according to the theory, a shotgun approach is best for men, because their genes are most likely to survive if they have a great many partners. Consistent with this assumption, evolutionary psychologists have found that men report wanting far more partners than do women.

Other theorists (e.g., Miller & Fishkin, 1997), however, have questioned this view. They argue that women and men should prefer about the same number of partners. This is because individuals with a basic predisposition to seek a strong intimate bond are most likely to survive infancy. This desire for strong bonds, they argue, remains in adulthood. These theorists also asked women and men how many partners they wanted. They found the same result as the previous researchers when using the mean: men wanted an average of 64.3 partners, women an average of 2.8 partners. However, the picture looks drastically different if you look at the median or mode (see Table 1). Figure 8, taken directly from their article, shows why. Most women and most men want

Table 1 Responses of 106 Men and 160 Women to the Question, "How many partners would you ideally desire in the next 30 years?"

	Mean	Median	Mode
Women	2.8	1	1
Men	64.3	1	1

Source: Data from Miller & Fishkin (1997).

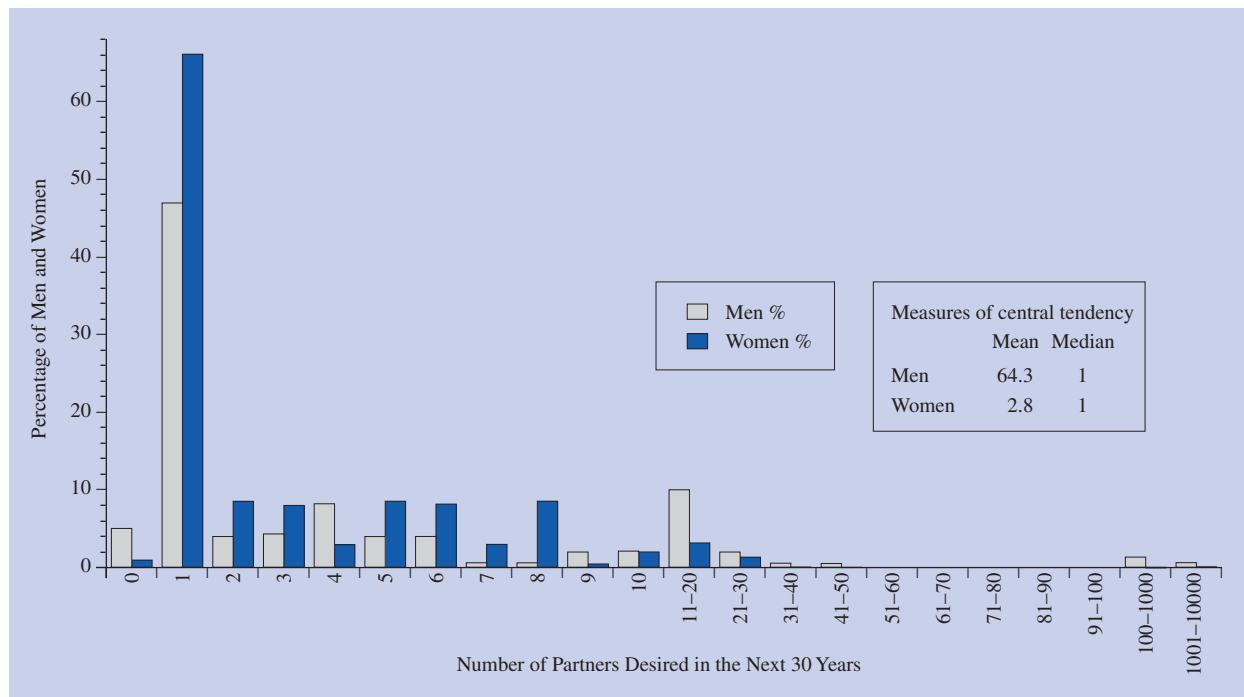


Figure 8 Distributions for men and women for the ideal number of partners desired over 30 years. *Note:* To include all the data, we collapsed across categories farther out on the tail of these distributions. If every category represented a single number, it would be more apparent that the tail is very flat and that distributions are even more skewed than is apparent here.

Source: Miller, L. C., & Fishkin, S. A. (1997). On the dynamics of human bonding and reproductive success: Seeking windows on the adapted-for-human-environmental interface. In J. Simpson & D. T. Kenrick (Eds.), *Evolutionary social psychology* (pp. 197–235). Mahwah, NJ: Erlbaum.

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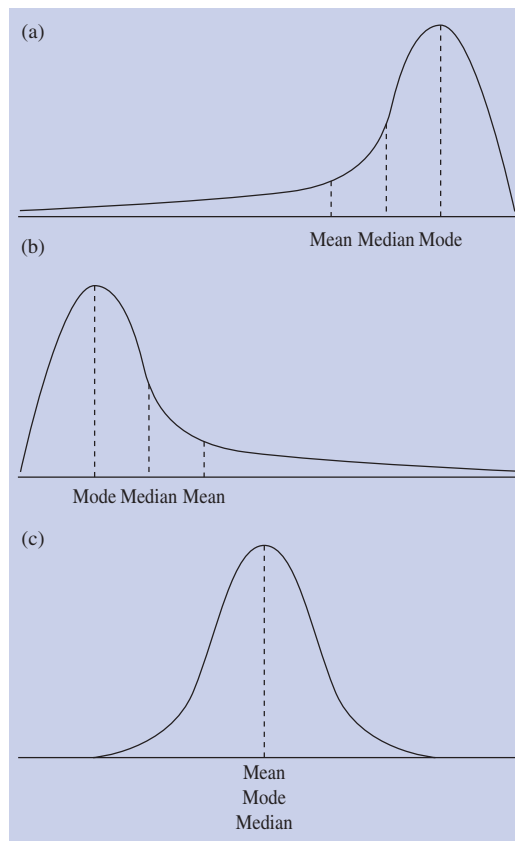


Figure 9 Locations of the mean, mode, and median on (a) a distribution skewed to the left, (b) a distribution skewed to the right, and (c) a normal curve.

just one partner. A few want more, some many more. The big difference is that there are a lot more men in the small group that want many more than one partner. These results have also been replicated in subsequent studies (Pedersen et al., 2002).

So, which theory is right? You could argue either way from these results. (For a more recent discussion of this controversy regarding the theoretical and statistical issues, see Smith & Konik, 2011.) The point is that focusing just on the mean can clearly misrepresent the reality of the distribution. As this example shows, the median is most likely to be used when a few extreme scores would make the mean unrepresentative of the main body of scores. Figure 9 illustrates this point, by showing the relative location of the mean, mode, and median for three types of distribution. The distribution in Figure 9a is skewed to the left (negatively skewed); the long tail of the distribution points to the left. The mode in this distribution is the highest point of the distribution, which is on the far right hand side of the distribution. The median is the point at which half of the scores are above that point and half are below. As you can see, for that to happen, the median must be a lower value than the mode. Finally, the mean is strongly influenced by the very low scores in the long tail of the distribution and is thus a lower value than the median. Figure 9b shows the location of the mean, mode, and median for a distribution that is skewed to the right (positively skewed). In this case, the mean is a higher value than either the mode or median because the mean is strongly influenced by the very *high* scores in the long tail of the distribution. Again, the mode is the highest point of the distribution, and

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Table 2 Summary of Measures of Central Tendency

Measure	Definition	When Used
Mean	Sum of the scores divided by the number of scores	<ul style="list-style-type: none">• With equal-interval variables• Very commonly used in psychology research
Mode	Value with the greatest frequency in a distribution	<ul style="list-style-type: none">• With nominal variables• Rarely used in psychology research
Median	Middle score when all the scores in a distribution are arranged from lowest to highest	<ul style="list-style-type: none">• With rank-ordered variables• When a distribution has one or more outliers• Rarely used in psychology research

the median is between the mode and the mean. In Figures 9a and 9b, the mean is not a good representative value of the scores, because it is overly influenced by the extreme scores.

Figure 9c shows a normal curve. As for any distribution, the mode is the highest point in the distribution. For a normal curve, the highest point falls exactly at the midpoint of the distribution. This midpoint is the median value, since half of the scores in the distribution are below that point and half are above it. The mean also falls at the same point because the normal curve is symmetrical about the midpoint, and every score in the left hand side of the curve has a matching score on the right hand side. So, for a perfect normal curve, the mean, mode, and median are always the same value.

In some situations, psychologists use the median as part of more complex statistical methods. Also, the median is the usual way of describing the central tendency for a rank-order variable. Otherwise, unless there are extreme scores, psychologists almost always use the mean as the representative value of a group of scores. In fact, as you will learn, the mean is a fundamental building block for most other statistical techniques.

A summary of the mean, mode, and median as measures of central tendency is shown in Table 2.

How are you doing?

1. Name and define three measures of central tendency.
2. Write the formula for the mean and define each of the symbols.
3. Figure the mean of the following scores: 2, 3, 3, 6, and 6.
4. For the following scores, find (a) the mean, (b) the mode, and (c) the median: 5, 3, 2, 13, 2. (d) Why is the mean different from the median?

Answers

1. The mean is the ordinary average, the sum of the scores divided by the number of scores. The mode is the most frequent score in a distribution. The median is the middle score; that is, if you line the scores up from lowest to highest, it is the halfway score.
2. The formula for the mean is $M = (\sum X)/N$. M is the mean; \sum is the symbol for "sum of"—add up all the scores that follow; X is the variable whose scores you are adding up; N is the number of scores.
3. $M = (\sum X)/N = (2 + 3 + 3 + 6 + 6)/5 = 4$.
4. (a) The mean is 5; (b) the mode is 2; (c) the median is 3; (d) The mean is different from the median because the extreme score (13) makes the mean higher than the median.

Variability

Researchers also want to know how spread out the scores are in a distribution. This shows the amount of variability in the distribution. For example, suppose you were asked, “How old are the students in your statistics class?” At a city-based university with many returning and part-time students, the mean age might be 29 years. You could answer, “The average age of the students in my class is 29.” However, this would not tell the whole story. You could have a mean of 29 because every student in the class was exactly 29 years old. If this is the case, the scores in the distribution are not spread out at all. In other words, there is no variation, or *variability*, among the scores. You could also have a mean of 29 because exactly half the class members were 19 and the other half 39. In this situation, the distribution is much more spread out; there is considerable variability among the scores in the distribution.

You can think of the variability of a distribution as the amount of spread of the scores around the mean. In other words, how close or far from the mean are the scores in a distribution? If the scores are mostly quite close to the mean, then the distribution has less variability than if the scores are further from the mean. Distributions with the same mean can have very different amounts of spread around the mean; Figure 10a shows histograms for three different frequency distributions with the same mean but different amounts of spread around the mean. A real-life example of this is shown in Figure 11, which shows the distributions of the housing prices in two

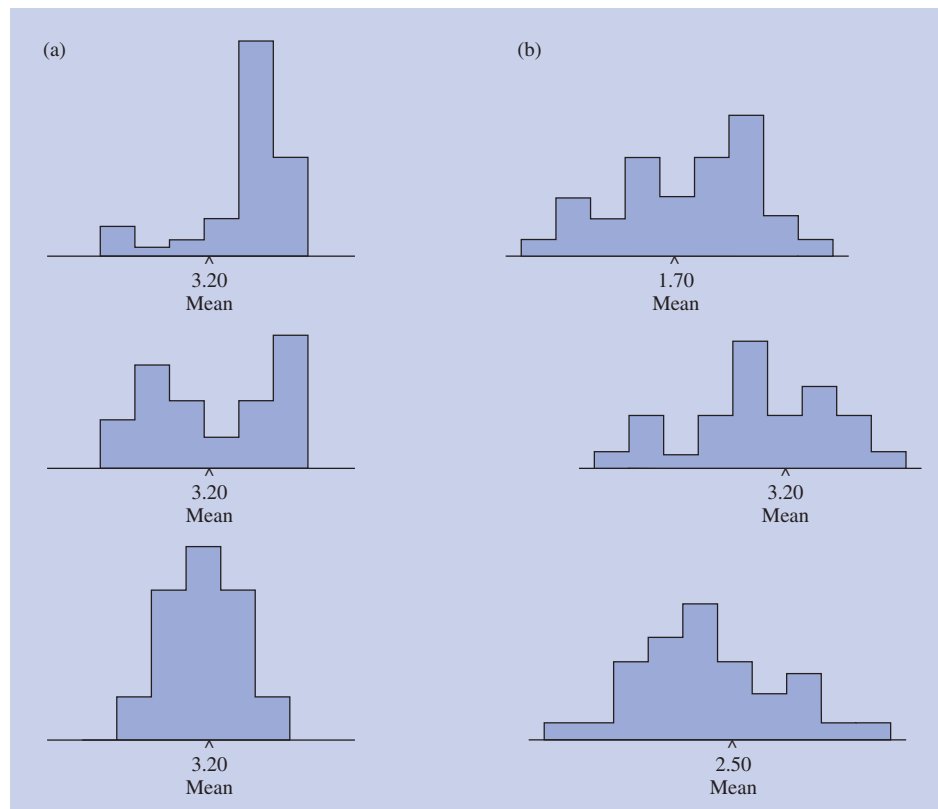


Figure 10 Examples of distributions with (a) the same mean but different amounts of spread, and (b) different means but the same amount of spread.

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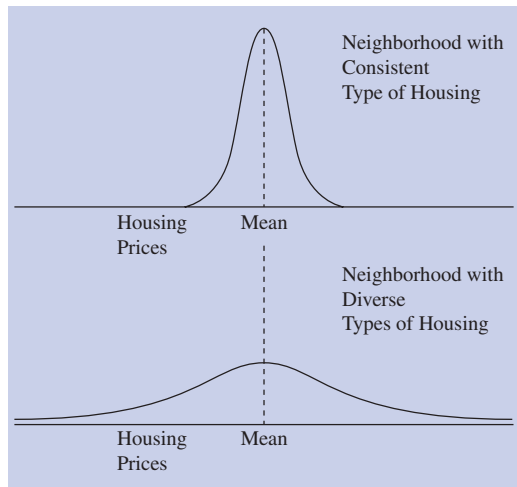


Figure 11 Example of two distributions with the same mean but different amounts of spread: housing prices for a neighborhood with diverse types of housing and for a neighborhood with a consistent type of housing.

neighborhoods: one with diverse housing types and the other with a consistent type of housing. As with Figure 10a, the mean housing price is the same in each neighborhood. However, the distribution for the neighborhood with diverse housing types is much more spread out around the mean than the distribution for the neighborhood that has a consistent type of housing. This tells you that there is much greater variability in the prices of housing in the neighborhood with diverse types of housing than in the neighborhood with a consistent housing type. Also, distributions with different means can have the same amount of spread around the mean. Figure 10b shows three different distributions with different means but the same amount of spread. So, although the mean provides a representative value of a group of scores, it doesn't tell you about the variability of the scores. You will now learn about two measures of the variability of a group of scores: the *variance* and *standard deviation*.²

The Variance

The **variance** of a group of scores is one kind of number that tells you how spread out the scores are around the mean. To be precise, the variance is the average of each score's squared difference from the mean.

Here are the four steps to figure the variance:

- 1 **Subtract the mean from each score.** This gives each score's **deviation score**, which is how far away the score is from the mean.
- 2 **Square each of these deviation scores** (multiply each by itself). This gives each score's **squared deviation score**.
- 3 **Add up the squared deviation scores.** This total is called the **sum of squared deviations**.
- 4 **Divide the sum of squared deviations by the number of scores.** This gives the average (the mean) of the squared deviations, called the variance.

Suppose one distribution is more spread out than another. The more spread-out distribution has a larger variance because being spread out makes the deviation scores bigger. If the deviation scores are bigger, the squared deviation scores and the average of the squared deviation scores (the variance) are also bigger.

variance measure of how spread out a set of scores are; average of the squared deviations from the mean.

deviation score score minus the mean.

squared deviation score square of the difference between a score and the mean.

sum of squared deviations total of each score's squared difference from the mean.

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In the example of the class in which everyone was exactly 29 years old, the variance would be exactly 0. That is, there would be no variance (which makes sense, because there is no variability among the ages). (In terms of the numbers, each person's deviation score would be $29 - 29 = 0$; 0 squared is 0. The average of a bunch of zeros is 0.) By contrast, the class of half 19-year-olds and half 39-year-olds would have a rather large variance of 100. (The 19-year-olds would each have deviation scores of $19 - 29 = -10$. The 39-year-olds would have deviation scores of $39 - 29 = 10$. All the squared deviation scores, which are either -10 squared or 10 squared, come out to 100. The average of all 100s is 100.)

The variance is extremely important in many statistical procedures you will learn about later. However, the variance is rarely used as a *descriptive statistic*. This is because the variance is based on *squared* deviation scores, which do not give a very easy-to-understand sense of how spread out the actual, nonsquared scores are. For example, a class with a variance of 100 clearly has a more spread-out distribution than one whose variance is 10. However, the number 100 does not give an obvious insight into the actual variation among the ages, none of which is anywhere near 100 years away from the others.³

The Standard Deviation

The most widely used number to *describe* the spread of a group of scores is the **standard deviation**. The standard deviation is simply the square root of the variance. There are two steps in figuring the standard deviation.

- 1 **Figure the variance.**
- 2 **Take the square root.** The standard deviation is the *positive* square root of the variance. (Any number has both a positive and a negative square root. For example, the square root of 9 is both $+3$ and -3 . But for the standard deviation we always use the positive square root.)

If the variance of a group of scores is 100, the standard deviation is 10. If the variance is 9, the standard deviation is 3.

The variance is about squared deviations from the mean. Therefore, its square root, the standard deviation, is about direct, ordinary, not-squared deviations from the mean. *Roughly speaking, the standard deviation is the average amount that scores differ from the mean.* For example, consider a class where the ages have a standard deviation of 10 years. This tells you that the ages are spread out, on the average, about 10 years in each direction from the mean. Knowing the standard deviation gives you a general sense of the degree of spread.⁴

The standard deviation does not, however, perfectly describe the shape of the distribution. For example, suppose the distribution of the number of children in families in a particular country has a mean of 4 and standard deviation of 1. Figure 12 shows several possibilities of the distribution of number of children, all with a mean of 4 and a standard deviation of 1.

standard deviation square root of the average of the squared deviations from the mean; the most common descriptive statistic for variation; approximately the average amount that scores in a distribution vary from the mean.

The variance is the sum of the squared deviations of the scores from the mean, divided by the number of scores.

Formulas for the Variance and the Standard Deviation

We have seen that the variance is the average squared deviation from the mean. Here is the formula for the variance.

$$SD^2 = \frac{\sum(X - M)^2}{N} \quad (2)$$

Central Tendency and Variability

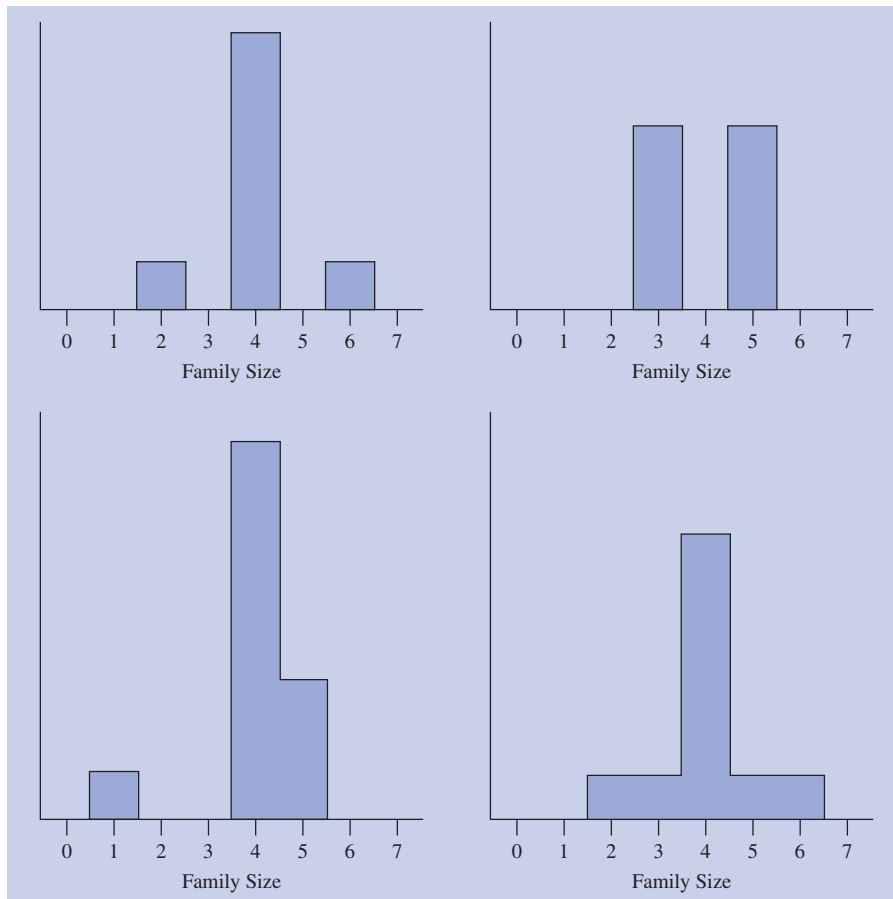


Figure 12 Some possible distributions for family size in a country where the mean is 4 and the standard deviation is 1.

SD^2 is the symbol for the *variance*. This may seem surprising. SD is short for *standard deviation*. The symbol SD^2 emphasizes that the variance is the standard deviation squared. (Later, you will learn other symbols for the variance, S^2 and σ^2 —the lowercase Greek letter sigma squared. The different symbols are for different situations in which the variance is used. In some cases, it is figured slightly differently.)

The top part of the formula is the *sum of squared deviations*. X is for each score and M is the mean. Thus, $X - M$ is the score minus the mean, the deviation score. The superscript number (2) tells you to square each deviation score. Finally, the sum sign (Σ) tells you to add up all these squared-deviation scores.

The sum of squared deviations of the scores from the mean, which is called the **sum of squares** for short, has its own symbol, SS . Thus, the variance formula can be written using SS instead of $\Sigma(X - M)^2$:

$$SD^2 = \frac{SS}{N}$$

Whether you use the simplified symbol SS or the full description of the sum of squared deviations, the bottom part of the formula is just N , the number of scores. That is, the formula says to divide the sum of the squared deviation scores by the number of scores in the distribution.

SD^2 variance.

SD standard deviation.

sum of squares (SS) sum of squared deviations.

TIP FOR SUCCESS

The sum of squared deviations is an important part of many of the procedures you will learn later; so be sure that you fully understand it, as well as how it is figured.

(3) ←

The variance is the sum of squares divided by the number of scores.

Central Tendency and Variability

The standard deviation is the square root of the variance. So, if you already know the variance, the formula is

The standard deviation is the square root of the variance.

$$SD = \sqrt{SD^2} \quad (4)$$

The formula for the standard deviation, starting from scratch, is the square root of what you figure for the variance:

The standard deviation is the square root of the result of taking the sum of the squared deviations of the scores from the mean divided by the number of scores.

$$SD = \sqrt{\frac{\sum(X - M)^2}{N}} \quad (5)$$

or

The standard deviation is the square root of the result of taking the sum of squares divided by the number of scores.

$$SD = \sqrt{\frac{SS}{N}} \quad (6)$$

Examples of Figuring the Variance and Standard Deviation

Table 3 shows the figuring for the variance and standard deviation for the number of dreams example. (The table assumes you have already figured the mean to be 6 dreams.) Usually, it is easiest to do your figuring using a calculator, especially one with a square-root key. The standard deviation of 2.57 tells you that roughly speaking, on the average, the number of dreams vary by about 2½ from the mean of 6.

TIP FOR SUCCESS

Always check that your answers make *intuitive sense*. For example, looking at the scores for the dreams example, a standard deviation—which, roughly speaking, represents the average amount that the scores vary from the mean—of 2.57 makes sense. If your answer had been 31.23, however, it would mean that, on average, the number of dreams varied by more than 30 from the mean of 6. Looking at the group of scores, that just couldn't be true.

Table 4 shows the figuring for the variance and standard deviation for the example of students' number of social interactions during a week (McLaughlin-Volpe et al., 2001). (To save space, the table shows only the first few and last few scores.) Roughly speaking, this result tells you that a student's number of social interactions in a week varies from the mean (of 17.39) by an average of 11.49 social interactions. This can also be shown on a histogram (see Figure 13).

Measures of variability, such as the variance and standard deviation, are heavily influenced by the presence of one or more outliers (extreme values) in a distribution.

TIP FOR SUCCESS

Notice that in Table 3 that the deviation scores (shown in the third column) add up to 0. The sum of the deviation scores is *always* 0 (or very close to 0, allowing for rounding error). So, to check your figuring, always sum the deviation scores. If they do not add up to 0, do your figuring again!

Table 3 Figuring the Variance and Standard Deviation in the Number of Dreams Example

Score (Number of Dreams)	–	Mean Score (Mean Number of Dreams)	=	Deviation Score	Squared Deviation Score
7		6		1	1
8		6		2	4
8		6		2	4
7		6		1	1
3		6		–3	9
1		6		–5	25
6		6		0	0
9		6		3	9
3		6		–3	9
8		6		2	4
				∑: 0	66

Variance = $SD^2 = \frac{\sum(X - M)^2}{N} = \frac{SS}{N} = \frac{66}{10} = 6.60$
 Standard deviation = $SD = \sqrt{SD^2} = \sqrt{6.60} = 2.57$

Central Tendency and Variability

Table 4 Figuring the Variance and Standard Deviation for Number of Social Interactions During a Week for 94 College Students

Number of Interactions	–	Mean Number of Interactions	=	Deviation Score	Squared Deviation Score
48		17.39		30.61	936.97
15		17.39		–2.39	5.71
33		17.39		15.61	243.67
3		17.39		–14.39	207.07
21		17.39		3.61	13.03
-		-		-	-
-		-		-	-
-		-		-	-
35		17.39		17.61	310.11
9		17.39		–8.39	70.39
30		17.39		12.61	159.01
8		17.39		–9.39	88.17
26		17.39		8.61	74.13
				Σ: 0.00	12,406.44

Variance = $SD^2 = \frac{\sum(X - M)^2}{N} = \frac{12,406.44}{94} = 131.98$
 Standard deviation = $\sqrt{SD^2} = \sqrt{131.98} = 11.49$

TIP FOR SUCCESS

When figuring the variance and standard deviation, lay your work out as in Tables 3 and 4. This helps you follow all the steps and end up with the correct answers.

Source: Data from McLaughlin-Volpe et al. (2001).

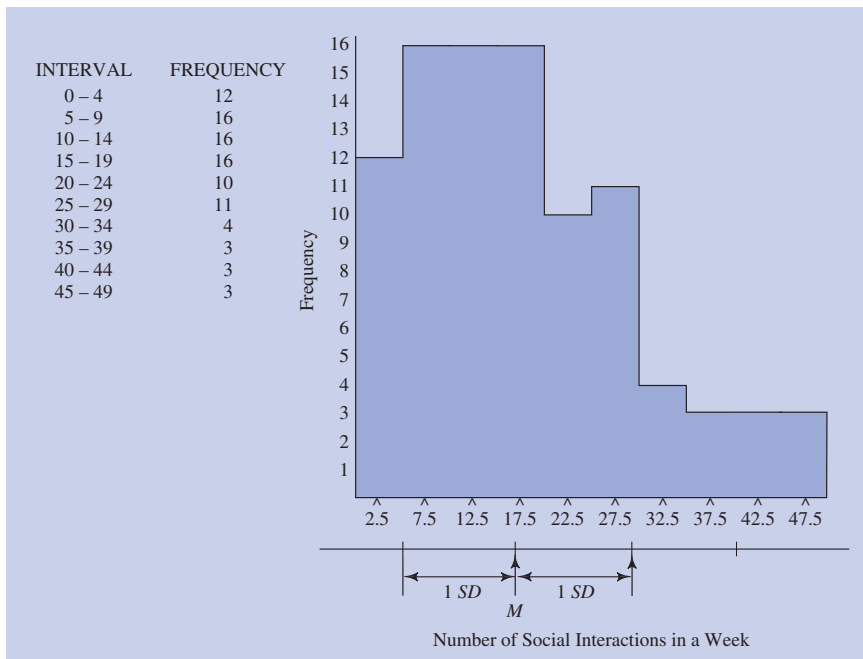


Figure 13 The standard deviation as the distance along the base of a histogram, using the example of number of social interactions in a week. (Data from McLaughlin-Volpe et al., 2001.)

TIP FOR SUCCESS

A common mistake when figuring the standard deviation is to jump straight from the sum of squared deviations to the standard deviation (by taking the square root of the sum of squared deviations). Remember, before finding the standard deviation, first figure the variance (by dividing the sum of squared deviations by the number of scores, N). Then take the square root of the variance to find the standard deviation.

The variance is the sum of the squared scores minus the result of taking the sum of all the scores, squaring this sum and dividing by the number of scores, and then taking this whole difference and dividing it by the number of scores.

The scores in the number of dreams example were 7, 8, 8, 7, 3, 1, 6, 9, 3, 8, and we figured the standard deviation of the scores to be 2.57. Now imagine that one additional person is added to the study and that the person reports having 21 dreams in the past week. The standard deviation of the scores would now be 4.96; adding this single score almost doubled the size of the standard deviation.

Computational and Definitional Formulas

In actual research situations, psychologists must often figure the variance and the standard deviation for distributions with many scores, often involving decimals or large numbers. In the days before computers, this could make the whole process quite time-consuming, even with a calculator. To deal with this problem, in the old days researchers developed various shortcuts to simplify the figuring. A shortcut formula of this type is called a **computational formula**.

The traditional computational formula for the variance (of the kind we are discussing in this chapter) is as follows:

$$\rightarrow SD^2 = \frac{\sum X^2 - [(\sum X)^2/N]}{N} \tag{7}$$

$\sum X^2$ means that you square each score and then take the sum of the squared scores. However, $(\sum X)^2$ means that you first add up all the scores and then take the square of this sum. Although this sounds complicated, this formula was actually easier to use than the one you learned before if a researcher was figuring the variance for a lot of numbers by hand or even with an old-fashioned handheld calculator, because the researcher did not have to first find the deviation score for each score.

However, these days computational formulas are mainly of historical interest. In fact, today, even many handheld calculators are set up so that you need only enter the scores and press a button or two to get the variance and the standard deviation. (Also, spreadsheet programs such as Excel commonly include procedures for most of the statistical procedure.)

In this text, we give a few computational formulas, because you may run into them and be confused should you look at an old book on statistical methods. However, we very definitely recommend *not* using the computational formulas when you are learning statistics, even if they might save you a few minutes of figuring a practice problem. The computational formulas usually make it much harder to understand the *meaning* of what you are figuring. The only reason for figuring problems at all by hand when you are learning statistics is to reinforce the underlying principles. Thus, you would be undermining the whole point of the practice problems if you use a formula that had a complex relation to the basic logic. The formulas we give you for the practice problems and for all the examples in this text are designed to help strengthen your understanding of what the figuring *means*. Thus, the usual formula we give for each procedure is what statisticians call a **definitional formula**.

computational formula equation mathematically equivalent to the definitional formula. Easier to use for figuring by hand, it does not directly show the meaning of the procedure.

definitional formula equation for a statistical procedure directly showing the meaning of the procedure.

The Importance of Variability in Psychology Research

Variability is an important topic in psychology research because much of the research focuses on explaining variability. We will use a couple of examples to show what we mean by “explaining variability.” As you might imagine, different students experience different levels of stress with regard to learning statistics: Some experience little stress; for other students, learning statistics can be a source of great stress. So, in this example, explaining variability means identifying the

factors that explain why students differ in the amount of stress they experience. Perhaps how much experience students have had with math explains some of the variability. That is, according to this explanation, the differences (the variability) among students in amount of stress are partially due to the differences (the variability) among students in the amount of experience they have had with math. Thus, the variation in math experience partially explains, or accounts for, the variation in stress. What factors might explain the variation in students' number of weekly social interactions? Perhaps a factor is variation in the extraversion of students, with more extraverted students tending to have more interactions. Or perhaps it is variation in gender, with one gender having consistently more interactions than the other.

The Variance as the Sum of Squared Deviations Divided by $N - 1$

Researchers often use a slightly different kind of variance. We have defined the variance as the average of the squared deviation scores. Using that definition, you divide the sum of the squared deviation scores by the number of scores (that is, the variance is SS/N). For many purposes it is better to define the variance as the sum of squared deviation scores divided by *1 less than the number of scores*. In other words, for those purposes the variance is the sum of squared deviations divided by $N - 1$ (that is, variance is $SS/[N - 1]$). (You use this dividing by $N - 1$ approach when you have scores from a particular group of people and you want to estimate what the variance would be for the larger group of people whom these individuals represent.)

The variances and standard deviations given in research articles are usually figured using $SS/(N - 1)$. Also, when calculators or computers give the variance or the standard deviation automatically, they are usually figured in this way (for example, see the Using SPSS section at the end of this chapter). But don't worry. The approach you are learning in this chapter of dividing by N (that is, figuring variance as SS/N) is entirely correct for our purpose here, which is to use descriptive statistics to describe the variation in a particular group of scores. We mention this other approach (variance as $SS/[N - 1]$) now only so that you will not be confused if you read about variance or standard deviation in other places or if your calculator or a computer program gives a surprising result.

How are you doing?

1. (a) Define the variance and (b) indicate what it tells you about a distribution and how this is different from what the mean tells you.
2. (a) Define the standard deviation; (b) describe its relation to the variance; and (c) explain what it tells you approximately about a group of scores.
3. Give the full formula for the variance and indicate what each of the symbols means.
4. Figure the (a) variance and (b) standard deviation for the following scores: 2, 4, 3, and 7 ($M = 4$).
5. Explain the difference between a definitional formula and a computational formula.

6. What is the difference between the formula for the variance you learned in this chapter and the formula that is typically used to figure the variance in research articles?

Answers

1. (a) The variance is the average of the squared deviation of each score from the mean. (b) The variance tells you about how spread out the scores are (that is, their variability), while the mean tells you the central tendency of the distribution.
2. (a) The standard deviation is the square root of the average of the squared deviations from the mean. (b) The standard deviation is the square root of the variance. (c) The standard deviation tells you approximately the average amount that scores differ from the mean.
3. $SD^2 = [\sum(X - M)^2]/N$. SD^2 is the variance. \sum tells you to take the sum of what follows. X is for the scores for the variable being studied. M is the mean of the scores. N is the number of scores.
4. (a) Variance: $SD^2 = [\sum(X - M)^2]/N = [(2 - 4)^2 + (4 - 4)^2 + (3 - 4)^2 + (7 - 4)^2]/4 = 14/4 = 3.50$.
(b) Standard deviation: $SD = \sqrt{SD^2} = \sqrt{3.50} = 1.87$.
5. A definitional formula is the standard formula in the straightforward form that shows the meaning of what the formula is figuring. A computational formula is a mathematically equivalent variation of the definitional formula. However, the computational formula tends not to show the underlying meaning. Computational formulas were often used before computers were available and researchers had to do their figuring by hand with a lot of scores.
6. The formula for the variance in this chapter divides the sum of squares by the number of scores (that is, SS/N). The variance in research articles is usually figured by dividing the sum of squares by one less than the number of scores (that is, $SS/[N - 1]$).

BOX 1 The Sheer Joy (Yes, Joy) of Statistical Analysis

You are learning statistics for the fun of it, right? No? Or maybe so, after all. If you become a psychologist, at some time or other you will form a hypothesis, gather data, and analyze them. (Even if you plan a career as a psychotherapist or other mental health practitioner, you will probably eventually wish to test an idea about the nature of your patients and their difficulties.) That hypothesis—your own original idea—and the data you gather to test it are going to be very important to you. Your heart may well be pounding with excitement as you analyze the data.

Consider some of the comments of social psychologists we interviewed some years ago for our book *The Heart of Social Psychology* (Aron & Aron, 1989). Deborah Richardson, who studies interpersonal aggression, confided that her favorite part of being a social psychologist is looking at the statistical output of the computer analyses:

It's like putting together a puzzle. . . . It's a highly arousing, positive experience for me. I often go through periods of euphoria. Even when the data don't do what I want them to do . . . [there's a] physiological response. . . . It's exciting to see the numbers come off—Is it actually the way I thought it would be?—then thinking about the alternatives.

Harry Reis, recent President of the Society for Personality and Social Psychology, sees his profession the same way:

By far the most rewarding part is when you get a new data set and start analyzing it and things pop out, partly a confirmation of what led you into the study in the first place, but then also other things "Why is that?" Trying to make sense of it. The kind of ideas that come from data. . . . I love analyzing data.

Bibb Latane, winner (twice!) of the Behavioral Science Award from the American Association for the Advancement of Science, is known for, among other things, his work on why people don't always intervene to help others who are in trouble. He reports eagerly awaiting

. . . the first glimmerings of what came out . . . [and] using them to shape what the next question should be . . . You need to use everything you've got, . . . every bit of your experience and intuition. It's where you have the biggest effect, it's the least routine. You're in the room

with the tiger, face to face with the core of what you are doing, at the moment of truth.

Bill Graziano, at Purdue University, whose work integrates developmental, personality, and social psychology, calls the analysis of his data "great fun, just great fun." And in the same vein, Margaret Clark, at Yale University, who studies emotion and close relationships, declares that "the most fun of all is getting the data and looking at them."

So, you see? Statistics in the service of your own creative ideas can be a pleasure indeed.

Controversy: The Tyranny of the Mean

Looking in the behavioral and social science research journals, you would think that statistical methods are their sole tool and language, but there have also been rebellions against the reign of statistics. We are most familiar with this issue in psychology, where the most unexpected opposition came from the leader of behaviorism, the school in the history of psychology most dedicated to keeping the field strictly scientific.

Behaviorism opposed the study of inner states because inner events are impossible to observe objectively. (Today most research psychologists claim to measure inner events indirectly but objectively.) Behaviorism's most famous advocate, B. F. Skinner, was quite opposed to statistics. Skinner even said, "I would much rather see a graduate student in psychology taking a course in physical chemistry than in statistics. And I would include [before statistics] other sciences, even poetry, music, and art" (Evans, 1976, p. 93).

Skinner was constantly pointing to the information lost by averaging the results of a number of cases. For instance, Skinner (1956) cited the example of three overeating mice—one naturally obese, one poisoned with gold, and one whose hypothalamus had been altered. Each had a different curve for learning to press a bar for food. If these learning curves had been merged statistically, the result would have represented no actual eating habits of any real mouse. As Skinner said, "These three individual curves contain more information than could probably ever be generated with measures requiring statistical treatment, yet they will be viewed with suspicion by many psychologists because they are single cases" (p. 232).

In clinical psychology and the study of personality, voices have always been raised in favor of the in-depth study of one person instead of or as well as the averaging of persons. The philosophical underpinnings of the in-depth study of individuals can be found in phenomenology, which began in Europe after World War I (Husserl, 1970). This viewpoint has been important throughout the social sciences, not just in psychology.

Today, the rebellion in psychology is led by qualitative research methodologists (e.g., Willig & Stainton-Rogers, 2008), an approach that is much more prominent in other behavioral and social sciences, such as communication. The qualitative research methods, developed mainly in cultural anthropology, can involve long interviews or observations of a few individuals. The highly skilled researcher decides, as the event is taking place, what is important to remember, record, and pursue through more questions or observations. The mind of the researcher is the main tool because, according to this approach, only that mind can find the important relationships among the many categories of events arising in the respondent's speech.

Central Tendency and Variability

Many who favor qualitative methods argue for a blend: First, discover the important categories through a qualitative approach. Then, determine their incidence in the larger population through quantitative methods. Too often, these advocates argue, quantitative researchers jump to conclusions about a phenomenon without first exploring the human experience of it through free-response interviews or observations.

Finally, Carl Jung, founder of Jungian psychology, sometimes spoke of the “statistical mood” and its effect on a person’s feeling of uniqueness. Jung had no problem with statistics—he used them in his own research. He was concerned about the cultural impact of this “statistical mood”—much like the impact of being on a jammed subway and observing the hundreds of blank faces and feeling diminished, “one of a crowd.” He held that the important contributions to culture tend to come from people who feel unique and not ordinary. As we increasingly describe ourselves statistically—“90% of men under thirty think . . .”—we tend to do just that, think like 90% of men under thirty. To counteract this mood, Jungian analyst Marie Louise von Franz (1979) wrote, “An act of loyalty is required towards one’s own feelings” (pp. IV-18). Feeling “makes your life and your relationships and deeds feel unique and gives them a definite value” (pp. IV-18–IV-19). Your beloved is like no one else. Your own death is a face behind a door. And the meaning of ‘civilian deaths this month due to the war were 20,964’ is unfathomable horror—not a number.

In short, there have been many who have questioned an exclusively statistical view of our subject matter, and their voices should be considered too as you proceed with your study of what has become the predominant, but not exclusive, means of doing psychology research.

BOX 2 Gender, Ethnicity, and Math Performance

From time to time someone tries to argue that because some groups of people score better on standardized math tests and make careers out of mathematics, these groups have a genetic advantage in math (or statistics), implying others are innately inferior at it. The issue comes up about gender, about racial and ethnic groups, and of course in arguments about overall intelligence as well as math. There’s little evidence for such genetic differences (a must-see classic article is Block, 1995), but the stereotypes persist.

The impact of these stereotypes was well established in research by Steele and his colleagues (e.g., 1997), who have done numerous studies on what they call “stereotype threat,” the risk of confirming, as a self-characteristic, a negative stereotype about a group you happen to belong to. Steele argues that it is not so much that you have internalized a negative attitude about yourself, but that certain situations create the threat. A typical experiment creating stereotype threat (Spencer et al., 1999) involved women taking a difficult math test. Half were told that men generally do better on the test, and the other half that women generally do equally well. Those who were told that women do worse did indeed score substantially lower than the other group. In the

other condition, there was no difference. (In fact, in two separate studies, men performed a little worse when they were told there was no gender difference, as if they had lost some of their confidence.)

In another typical experiment creating stereotype threat, fifth- and sixth-grade students were randomly assigned to fill out a form asking for their race before taking their end-of-year mathematics test (Alter et al., 2010). African American students who filled out this form only answered about half as many math questions correctly as African American students who did not.

Stereotype threat has also been found to occur in the United States for Latinos (Gonzales et al., 2002) and the poor (Croizet & Claire, 1998), but they can occur in any culture, to any subgroup that is the target of negative stereotypes. Stereotype threat plays the main role in lower test scores among minorities (Suzuki & Aronson, 2005). For example, the same difference of 15 IQ points between a dominant and minority group has been found all over the world, even when there is no genetic difference between the groups. And in cases where opportunities for a group have changed, such as when they emigrate, differences have rapidly disappeared (Block, 1995). Hyde and

Mertz (2009) found that the more gender equality there is in a society, the more that women are equal to men in their math performance, and the greater likelihood that women are found to have “profound mathematical talent.”

What Can You Do for Yourself?

Thanks to a better understanding of the brain, psychologists (e.g., Forbes & Schmader, 2010) are learning more about how stereotype threat works to hurt your performance by looking separately at the effects of attitudes (occurring in parts of the brain associated with emotions) and stereotypes (affecting parts of the brain that interpret experience, a constant activity). Sometimes it is said that minorities fail because they give up or have a “negative attitude,” but this research sorts that out. In a series of experiments involving “implicit association,” women were trained without being aware of it to have a more positive attitude towards math or to have a more positive stereotype of women as good at math. The goal was to sort out attitude and stereotype. The study found that positive attitude did not help with math performance. In fact, when under stereotype threat (a man tells them they are going to take a math test and asks them their gender), these women were even more interested in math and more motivated to do it than when not threatened.

Additional experiments revealed that scores improved only when the women were taught to believe (through implicit retraining) that, as a woman, they could actually do math equally as well as men.

Stereotypes, not attitudes, were the problem because countering stereotypes, unlike maintaining a positive attitude, requires constant mental effort. Stereotypes operate in the part of the brain where you are continually interpreting experience, so you have to keep telling yourself, “I can do this.” That takes up some of your working memory, and the less working memory, the more difficult it is to solve math problems.

Change Those Beliefs

This research says that you *must* work on your wrong belief that because of being part of some minority you cannot do math as well as others. This is the only protection against those conditions that remind you of the negative stereotypes about your group and math. To begin that project, let’s look at some facts. There is no indication that, on average, men and women differ in their math abilities. In a recent comprehensive review, Lindberg and colleagues (2010) combined the results of 242 studies that together included data from more

than 1 million men and women. Overall, they found no evidence of gender differences in mathematics performance. However, it is true that there are currently more men than women employed in math-intensive academic careers, such as engineering, physics, chemistry, and mathematics. Why might that be? According to a recent analysis by Ceci and Williams (2010), it does not appear to be due to gender differences in math ability or current gender discrimination (although discrimination in the past likely helps to explain historical trends). Instead, men and women’s different preferences and choices seem to be the primary reason why fewer women pursue a career in math-intensive fields. As adolescents, fewer girls than boys are interested in math-intensive careers. And among individuals who pursue graduate level education in math-intensive fields, women are more likely to switch fields or drop out than men.

As for African-Americans in the United States, Steele (1997) found that differences in performance from the majority all had to do with the situation. For example, when African American students were enrolled in a transition-to-college program emphasizing that they were the cream of the crop and much was expected of them, their grades rose substantially. African American students at the same school who were enrolled in a remedial program for minorities received considerable attention, but their grades improved very little, and many more of them dropped out of school than in the other group. Steele argues that the very idea of a remedial program exposed those students to a subtle stereotype threat.

Treat Stereotype Threat as Another Form of Anxiety

While you are working on changing your stereotypes, you can try to be alert to stereotype threat. But research on stereotypes shows that they can be activated without our awareness (Fiske, 1998), so another approach is to try to counter its effects on working memory, which are the same as those caused by any form of test anxiety. Cognitive research on stereotype threat has demonstrated that it most affects math problems relying on long-term memory, and it spills over into subsequent tasks not normally affected by stereotype threat (Beilock et al., 2007). That means that what you have studied can be easily forgotten unless you over prepare, and that in turn can make you doubt your ability even to add $2 + 2$. So do all that you can to keep yourself in an optimal level of arousal, such as making the testing situation familiar, associated with success, and even comfortable. You know you can do it.

Be Empowered

Finally, although increasing your motivation alone does not help very much, it can't hurt. Fight for your right to know this subject. Consider these words from the former president of the Mathematics Association of America:

The paradox of our times is that as mathematics becomes increasingly powerful, only the powerful seem to benefit from it [italics added]. The ability to think mathematically—broadly interpreted—is absolutely

crucial to advancement in virtually every career. Confidence in dealing with data, skepticism in analyzing arguments, persistence in penetrating complex problems, and literacy in communicating about technical matters are the enabling arts offered by the new mathematical sciences. (Steen, 1987, p. xviii)

Do not let anyone take away your power to change the world, which most emphatically requires you to know how to use and interpret statistics.

Central Tendency and Variability in Research Articles

The mean and the standard deviation are very commonly reported in research articles. However, the mode, median, and variance are only occasionally reported. Sometimes, the mean and standard deviation are included in the text of an article. For our dreams example, the researcher might write, “The mean number of dreams in the last week for the 10 students was 6.00 ($SD = 2.57$).” Means and standard deviations are also often listed in tables, especially if a study includes several groups or several different variables. For example, Kujath (2011) conducted a study with 183 college students who reported using Facebook or MySpace. As shown in Table 5 (reproduced from Kujath’s article), the students reported on their use of each social networking site. Notice that the table is one of those relatively rare examples that includes the median as well as the mean and standard deviation (usually just the mean and standard deviation are shown). As the table shows, students reported using MySpace for a greater amount of time per day (mean = 67.4 minutes) than Facebook (mean = 39.1 minutes). Also notice that there was greater variability in the total time per day students spent on MySpace ($SD = 112.7$ minutes) compared to Facebook ($SD = 77.0$). How does your usage of social networking sites compare to the usage reported in this study?

Another interesting example is shown in Table 6 (reproduced from Norcross et al., 2005). The table shows the application and enrollment statistics for psychology doctoral programs in the United States, broken down by area of psychology and by year (1973, 1979, 1992, and 2003). The table does not give standard deviations, but it does give both means and medians. For example, in 2003 the mean number of applicants to doctoral counseling psychology programs was 71.0, but the median was only 59. This suggests that some programs had very high numbers of applicants that skewed the distribution. In fact, you can see from the table that for almost every

Table 5 Facebook and MySpace Utilization

	Facebook			MySpace		
	Mn	Mdn	SD	Mn	Mdn	SD
Membership months	21.9	24.0	13.4	28.2	24.0	15.5
Times used per day	1.2	1.0	1.3	1.6	1.0	1.5
Minutes spent per use	31.5	17.5	58.8	38.8	30.0	57.1
Total minutes per day	39.1	15.0	77.0	67.4	30.0	112.7

Source: Kujath, C. L. (2011). Facebook and MySpace: Complement or substitute for face-to-face interaction? *Cyberpsychology, Behavior, and Social Networking*, 14, 75–78. Reprinted by permission of Mary Ann Liebert, Inc. publishers.

Table 6 Application and Enrollment Statistics by Area and Year: Doctoral Programs

Program	N of programs				Applications								Enrollments			
					M				Mdn				M		Mdn	
	1973	1979	1992	2003	1973	1979	1992	2003	1973	1979	1992	2003	1992	2003	1992	2003
Clinical	105	130	225	216	314.4	252.6	191.1	142.0	290	234	168	126	12.0	15.4	8	8
Clinical neuro				20				72.3				37		10.7		6
Community	4	2	5	13	90.5		24.4	23.5	60		23	21	3.2	3.3	2	3
Counseling	29	43	62	66	133.4	90.9	120.2	71.0	120	84	110	59	7.3	6.8	6	7
Health			7	13			40.7	71.2			30	56	4.4	6.7	5	4
School	30	39	56	57	78.5	54.0	31.3	38.7	53	34	32	31	5.4	6.9	5	5
Other health service provider subfield				52				83.5				48		9.2		7
Cognitive			47	104			24.6	30.1			22	22	2.6	3.4	2	3
Developmental	56	72	97	111	54.1	38.9	27.6	25.5	41	30	24	22	2.8	3.4	2	3
Educational	23	28	30	35	67.8	39.7	20.0	19.7	34	26	12	13	6.0	4.9	4	4
Experimental	118	127	78	40	56.2	33.2	31.3	26.7	42	25	26	17	4.4	4.1	3	3
I/O	20	25	49	60	39.9	54.7	66.2	46.9	37	48	70	41	4.9	4.7	4	4
Neuroscience				53				22.0				16		2.8		2
Personality	23	15	10	18	42.5	24.7	12.3	47.8	33	17	6	31	1.0	2.8	1	2
Psychobiological/physiological				18				21.1				17		2.4		2
Quantitative	40	43	76	17	33.2	29.3	20.0	11.2	29	24	20	11	3.9	1.9	2	1
Social	58	72	59	85	48.7	30.9	47.1	43.1	40	24	37	35	3.3	3.2	3	3
Other fields	60	47	288	101	61.6	74.1	26.6	26.0	27	25	15	17	3.3	3.8	2	3
Total	566	645	1,089	1,079	106.1	85.2	69.4	59.6			31	33	5.6	6.7	4	4

Note: The academic years correspond to the 1975–1976, 1981–1982, 1994 and 2005 editions of *Graduate Study in Psychology*, respectively. Clinical neuro = clinical neuropsychology; I/O = industrial-organizational.

Source: Norcross, J. C., Kohout, J. L., & Wicherski, M. (2005). Graduate study in psychology: 1971 to 2004. *American Psychologist*, 60, 959–975. Published by the American Psychological Association. Reprinted with permission.

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kind of program and for both applications and enrollments, the means are typically higher than the medians. You may also be struck by just how competitive it is to get into doctoral programs in many areas of psychology. It is our experience that one of the factors that makes a lot of difference is doing well in statistics courses!

Learning Aids

Summary

1. The mean is the most commonly used measure of central tendency of a distribution of scores. The mean is the ordinary average—the sum of the scores divided by the number of scores. In symbols, $M = (\sum X)/N$.
2. Other, less commonly used ways of describing the central tendency of a distribution of scores are the mode (the most common single value) and the median (the value of the middle score when all the scores are lined up from lowest to highest).
3. The variability of a group of scores can be described by the variance and the standard deviation.
4. The variance is the average of the squared deviation of each score from the mean. In symbols, $SD^2 = [\sum(X - M)^2]/N$. The sum of squared deviations, $\sum(X - M)^2$, is also symbolized as SS . Thus $SD^2 = SS/N$.
5. The standard deviation is the square root of the variance. In symbols, $SD = \sqrt{SD^2}$. It is approximately the average amount that scores differ from the mean.
6. There have always been a few psychologists who have warned against statistical methodology because in the process of creating averages, knowledge about the individual case is lost.
7. Means and standard deviations are often given in research articles in the text or in tables.

Key Terms

central tendency
mean (M)
 \sum (sum of)
 X
 N (number of scores)
mode

median
outlier
variance
deviation score
squared deviation score
sum of squared deviations

standard deviation (SD)
 SD^2
 SD
sum of squares (SS)
computational formula
definitional formula

Example Worked-Out Problems

Figuring the Mean

Find the mean for the following scores: 8, 6, 6, 9, 6, 5, 6, 2.

Answer

You can figure the mean using the formula or the steps.

Using the formula: $M = (\sum X)/N = 48/8 = 6$.

Using the steps:

- ① Add up all the scores. $8 + 6 + 6 + 9 + 6 + 5 + 6 + 2 = 48$.
- ② Divide this sum by the number of scores. $48/8 = 6$.

Finding the Median

Find the median for the following scores: 1, 7, 4, 2, 3, 6, 2, 9, 7.

Answer

- ① Line up all the scores from lowest to highest. 1, 2, 2, 3, 4, 6, 7, 7, 9.
- ② Figure how many scores there are to the middle score by adding 1 to the number of scores and dividing by 2. There are 9 scores; so the middle score is the result of adding 1 to 9 and then dividing by 2, which is 5. The middle score is the fifth score.
- ③ Count up to the middle score or scores. The fifth score from the bottom is 4; so the median is 4.

Figuring the Sum of Squares and the Variance

Find the sum of squares and the variance for the following scores: 8, 6, 6, 9, 6, 5, 6, 2. (These are the same scores used in the previous example for the mean: $M = 6$.)

Answer

You can figure the sum of squares and the variance using the formulas or the steps. Using the formulas:

$$\begin{aligned}
 SS &= \sum(X - M)^2 = (8 - 6)^2 + (6 - 6)^2 + (6 - 6)^2 \\
 &\quad + (9 - 6)^2 + (6 - 6)^2 + (5 - 6)^2 + (6 - 6)^2 + (2 - 6)^2 \\
 &= 2^2 + 0^2 + 0^2 + 3^2 + 0^2 + (-1)^2 + 0^2 + (-4)^2 \\
 &= 4 + 0 + 0 + 9 + 0 + 1 + 0 + 16 \\
 &= 30 \\
 SD^2 &= SS/N = 30/8 = 3.75.
 \end{aligned}$$

Table 7 shows the figuring, using the following steps:

- ① Subtract the mean from each score.
- ② Square each of these deviation scores.

Score	Mean	① Deviation	② Squared Deviation
8	6	2	4
6	6	0	0
6	6	0	0
9	6	3	9
6	6	0	0
5	6	-1	1
6	6	0	0
2	6	-4	16
			$\Sigma = SS = 30$ ③
④ Variance = $30/8 = 3.75$			

Central Tendency and Variability

- ③ **Add up the squared deviation scores.** This gives the sum of squares (SS).
- ④ **Divide the sum of squared deviations by the number of scores.** This gives the variance (SD^2).

Figuring the Standard Deviation

Find the standard deviation for the following scores: 8, 6, 6, 9, 6, 5, 6, 2. (These are the same scores used above for the mean, sum of squares, and variance. $SD^2 = 3.75$.)

Answer

You can figure the standard deviation using the formula or the steps.

Using the formula: $SD = \sqrt{SD^2} = \sqrt{3.75} = 1.94$.

Using the steps:

- ① **Figure the variance.** The variance (from above) is 3.75.
- ② **Take the square root.** The square root of 3.75 is 1.94.

Outline for Writing Essays on Finding the Mean, Variance, and Standard Deviation

1. Explain that the mean is a measure of the central tendency of a group of scores. Mention that the mean is the ordinary average, the sum of the scores divided by the number of scores.
2. Explain that the variance and standard deviation both measure the amount of variability (or spread) among a group of scores.
3. The variance is the average of each score's squared difference from the mean. Describe the steps for figuring the variance.
4. Roughly speaking, the standard deviation is the average amount that scores differ from the mean. Explain that the standard deviation is directly related to the variance and is figured by taking the square root of the variance.

Practice Problems

These problems involve figuring. Most real-life statistics problems are done with special statistical software. Even if you have such software, do these problems by hand to ingrain the method in your mind. To learn how to use a computer to solve statistics problems like those in this chapter, refer to the Using SPSS section at the end of this chapter.

All data are fictional unless an actual citation is given.

Set I

MyStatLab

1. For the following scores, find the (a) mean, (b) median, (c) sum of squared deviations, (d) variance, and (e) standard deviation:

32, 28, 24, 28, 28, 31, 35, 29, 26

MyStatLab

2. For the following scores, find the (a) mean, (b) median, (c) sum of squared deviations, (d) variance, and (e) standard deviation:

6, 1, 4, 2, 3, 4, 6, 6

MyStatLab

3. For the following scores, find the (a) mean, (b) median, (c) sum of squared deviations, (d) variance, and (e) standard deviation:

2.13, 6.01, 3.33, 5.78

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4. Here are the noon temperatures (in degrees Celsius) in a particular Canadian city on Thanksgiving Day for the 10 years from 2002 through 2011: 0, 3, 6, 8, 2, 9, 7, 6, 4, 5. Describe the typical temperature and the amount of variation to a person who has never had a course in statistics. Give three ways of describing the representative temperature and two ways of describing its variation, explaining the differences and how you figured each.
5. A researcher is studying the amygdala (a part of the brain involved in emotion). Six participants in a particular fMRI (brain scan) study are measured for the increase in activation of their amygdala while they are viewing pictures of violent scenes. The activation increases are .43, .32, .64, .21, .29, and .51. Figure the (a) mean and (b) standard deviation for these six activation increases. (c) Explain what you have done and what the results mean to a person who has never had a course in statistics.
6. Describe and explain the location of the mean, mode, and median for a normal curve.
7. A researcher studied the number of anxiety attacks recounted over a two-week period by 30 people in psychotherapy for an anxiety disorder. In an article describing the results of the study, the researcher reports: "The mean number of anxiety attacks was 6.84 ($SD = 3.18$)." Explain these results to a person who has never had a course in statistics.
8. In a study by Gonzaga et al. (2001), romantic couples answered questions about how much they loved their partner and also were videotaped while revealing something about themselves to their partner. The videotapes were later rated by trained judges for various signs of affiliation. Table 8 shows some of the results. Explain to a person who has never had a course in statistics the results for self-reported love for the partner and for the number of seconds "leaning toward the partner."

MyStatLab

MyStatLab

Set II

9. (a) Describe and explain the difference between the mean, median, and mode. (b) Make up an example (not in your lectures) in which the median would be the preferred measure of central tendency.

MyStatLab

Table 8 Mean Levels of Emotions and Cue Display in Study 1

Indicator	Women ($n = 60$)		Men ($n = 60$)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Emotion reports				
Self-reported love	5.02	2.16	5.11	2.08
Partner-estimated love	4.85	2.13	4.58	2.20
Affiliation-cue display				
Affirmative head nods	1.28	2.89	1.21	1.91
Duchenne smiles	4.45	5.24	5.78	5.59
Leaning toward partner	32.27	20.36	31.36	21.08
Gesticulation	0.13	0.40	0.25	0.77

Note: Emotions are rated on a scale of 0 (*none*) to 8 (*extreme*). Cue displays are shown as mean seconds displayed per 60 s.

Source: Gonzaga, G. C., Keltner, D., Londahl, E. A., & Smith, M. D. (2001). Love and the commitment problem in romantic relationships and friendship. *Journal of Personality and Social Psychology, 81*, 247–262. Published by the American Psychological Association. Reprinted with permission.

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MyStatLab

10. (a) Describe the variance and standard deviation. (b) Explain why the standard deviation is more often used as a descriptive statistic than the variance.

MyStatLab

11. For the following scores, find the (a) mean, (b) median, (c) sum of squared deviations, (d) variance, and (e) standard deviation:

2, 2, 0, 5, 1, 4, 1, 3, 0, 0, 1, 4, 4, 0, 1, 4, 3, 4, 2, 1, 0

MyStatLab

12. For the following scores, find the (a) mean, (b) median, (c) sum of squared deviations, (d) variance, and (e) standard deviation:

1,112; 1,245; 1,361; 1,372; 1,472

MyStatLab

13. For the following scores, find the (a) mean, (b) median, (c) sum of squared deviations, (d) variance, and (e) standard deviation:

3.0, 3.4, 2.6, 3.3, 3.5, 3.2

MyStatLab

14. For the following scores, find the (a) mean, (b) median, (c) sum of squared deviations, (d) variance, and (e) standard deviation:

8, -5, 7, -10, 5

MyStatLab

15. Make up three sets of scores: (a) one with the mean greater than the median, (b) one with the median and the mean the same, and (c) one with the mode greater than the median. (Each made-up set of scores should include at least five scores.)

16. A psychologist interested in political behavior measured the square footage of the desks in the official office of four U.S. governors and of four chief executive officers (CEOs) of major U.S. corporations. The figures for the governors were 44, 36, 52, and 40 square feet. The figures for the CEOs were 32, 60, 48, and 36 square feet. (a) Figure the means and standard deviations for the governors and for the CEOs. (b) Explain, to a person who has never had a course in statistics, what you have done. (c) Note the ways in which the means and standard deviations differ, and speculate on the possible meaning of these differences, presuming that they are representative of U.S. governors and large corporations' CEOs in general.

MyStatLab

17. A developmental psychologist studies the number of words that seven infants have learned at a particular age. The numbers are 10, 12, 8, 0, 3, 40, and 18. Figure the (a) mean, (b) median, and (c) standard deviation for the number of words learned by these seven infants. (d) Explain what you have done and what the results mean to a person who has never had a course in statistics.

18. Describe and explain the location of the mean, mode, and median of a distribution of scores that is strongly skewed to the left.

19. You figure the variance of a distribution of scores to be -4.26 . Explain why your answer cannot be correct.

MyStatLab

20. A study involves measuring the number of days absent from work for 216 employees of a large company during the preceding year. As part of the results, the researcher reports, "The number of days absent during the preceding year ($M = 9.21$; $SD = 7.34$) was . . ." Explain what is written in parentheses to a person who has never had a course in statistics.

21. Selwyn (2007) conducted a study of gender-related perceptions of information and communication technologies (such as video game systems, DVD players, and cell phones). The researcher asked 406 college students in Wales to rate 8 technologies in terms of their level of masculinity or femininity. The students rated each technology using a 7-point response scale, from -3 for very feminine to

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+3 for very masculine, with a midpoint of 0 for neither masculine or feminine. Table 9 (reproduced from Selwyn's article) shows the mean, standard deviation, and variance of the students' ratings of each technology. Explain the results to a person who has never had a course in statistics. (Be sure to explain some specific numbers, as well as the general principle of the mean, standard deviation, and variance.)


Table 9 Mean Scores for Each Technology

	N	Mean	S.D.	Variance
Games machine (e.g., Playstation)	403	1.92	1.00	.98
DVD Player	406	.44	.85	.73
Personal Computer (PC)	400	.36	.82	.68
Digital radio (DAB)	399	.34	.99	.98
Television set	406	.26	.78	.62
Radio	404	-.01	.81	.65
Mobile phone	399	-.19	.88	.77
Landline telephone	404	-.77	1.03	1.07













Note: Mean scores range from -3 (*very feminine*) to +3 (*very masculine*). The midpoint score of .0 denotes "neither masculine nor feminine."

Source: Selwyn, N. (2007). Hi-tech = guy-tech? An exploration of undergraduate students' gendered perceptions of information and communication technologies. *Sex Roles, 56*, 525-536. Copyright © 2007. Reprinted by permission of Springer Science and Business Media.

Using SPSS

The  in the following steps indicates a mouse click. (We used SPSS version 19.0 to carry out these analyses. The steps and output may be slightly different for other versions of SPSS.)

Finding the Mean, Mode, and Median

- 1 Enter the scores from your distribution in one column of the data window.
- 2  *Analyze*.
- 3  *Descriptive statistics*.
- 4  *Frequencies*.
- 5  the variable for which you want to find the mean, mode, and median, and then  the arrow.
- 6  *Statistics*.
- 7  *Mean*,  *Median*,  *Mode*,  *Continue*.
- 8 Optional: To instruct SPSS *not* to produce a frequency table,  the box labeled *Display frequency tables* (this unchecks the box).
- 9  *OK*.

Practice the preceding steps by finding the mean, mode, and median for the number of dreams example at the start of the chapter (the scores are 7, 8, 8, 7, 3, 1, 6, 9, 3, 8). Your output window should look like Figure 14. (If you instructed SPSS not to show the frequency table, your output will show only the mean, median, and mode.)

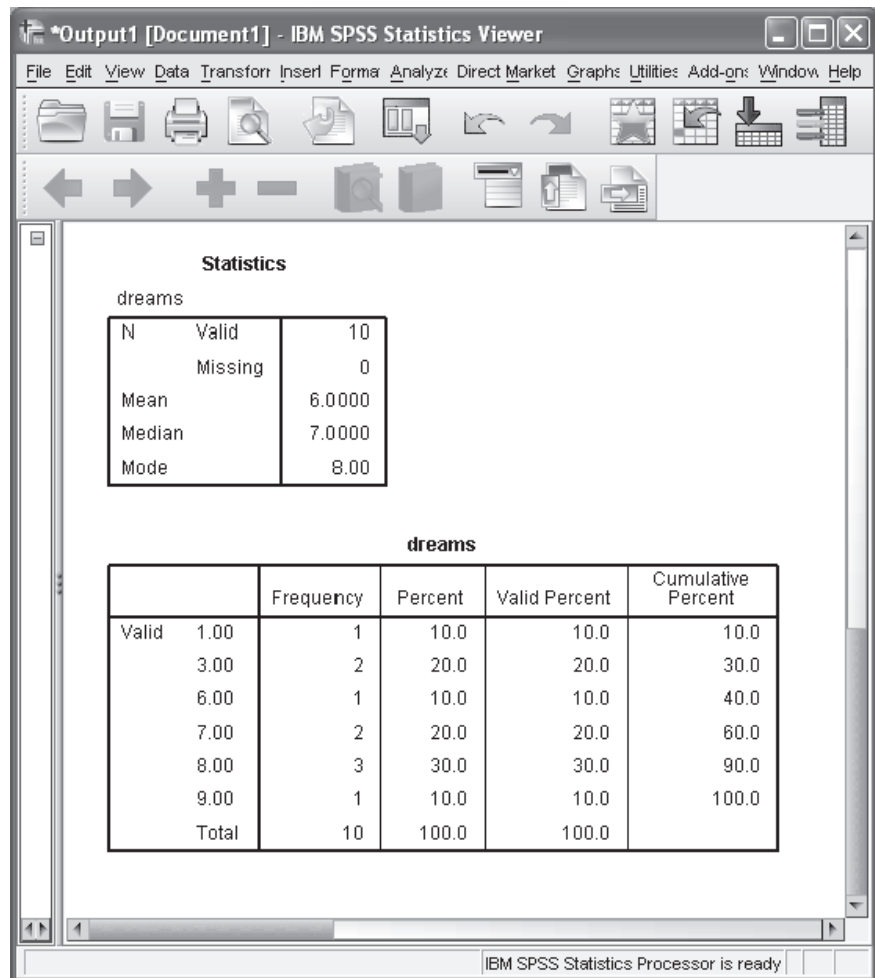


Figure 14 Using SPSS to find the mean, median, and mode for the number of dreams example.

Finding the Variance and Standard Deviation

As mentioned earlier in the chapter, most calculators and computer software—including SPSS—calculate the variance and standard deviation using a formula that involves dividing by $N - 1$ instead of N . So, if you request the variance and standard deviation directly from SPSS (for example, by clicking *Variance* and *Std. deviation* in Step 7), the answers provided by SPSS will be slightly different from the answers in this chapter.⁵ The following steps show you how to use SPSS to figure the variance and standard deviation using the dividing-by- N method you learned in this chapter. It is easier to learn these steps using actual numbers; so we will use the number of dreams example again.

- 1 Enter the scores for which you want to compute the variance and standard deviation in one column of the data window (the scores are 7, 8, 8, 7, 3, 1, 6, 9, 3, 8). We will call this variable “dreams.” (This is the same first step for finding the mean of a distribution of scores.)