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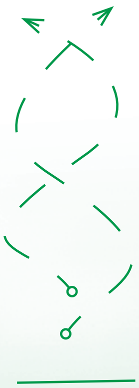

Process Control Instrumentation Technology

Curtis D. Johnson

Eighth Edition

ALWAYS LEARNING

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Pearson New International Edition

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Technology
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Table of Contents

Introduction to Process Control

INSTRUCTIONAL OBJECTIVES

This chapter presents an introduction to process-control concepts and the elements of a process-control system. After you read this chapter and work through the example problems and chapter problems you will be able to:

- Draw a block diagram of a simple process-control loop and identify each element.
- List three typical controlled variables and one controlling variable.
- Describe three criteria to evaluate the performance of a process-control loop.
- Explain the difference between analog and digital control systems.
- Define supervisory control.
- Explain the concept behind process-control networks.
- Define accuracy, hysteresis, and sensitivity.
- List the SI units for length, time, mass, and electric current.
- Recognize the common P&ID symbols.
- Draw a typical first-order time response curve.
- Determine the average and standard deviation of a set of data samples.

1 INTRODUCTION

Human progress from a primitive state to our present complex, technological world has been marked by learning new and improved methods to control the environment. Simply stated, the term *control* means methods to force parameters in the environment to have specific values. This can be as simple as making the temperature in a room stay at 21°C or as complex as manufacturing an integrated circuit or guiding a spacecraft to Jupiter. In general, all the elements necessary to accomplish the control objective are described by the term *control system.*

The purpose of this book is to examine the elements and methods of control system operation used in industry to control industrial processes (hence the term *process control*). This

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chapter will present an overall view of process-control technology and its elements, including important definitions.

2 CONTROL SYSTEMS

The basic strategy by which a control system operates is logical and natural. In fact, the same strategy is employed in living organisms to maintain temperature, fluid flow rate, and a host of other biological functions. This is natural process control.

The technology of artificial control was first developed using a human as an integral part of the control action. When we learned how to use machines, electronics, and computers to replace the human function, the term *automatic control* came into use.

2.1 Process-Control Principles

In process control, the basic objective is to regulate the value of some quantity. To regulate means to maintain that quantity at some desired value regardless of external influences. The desired value is called the *reference value* or *setpoint.*

In this section, a specific system will be used to introduce terms and concepts employed to describe process control.

The Process Figure 1 shows the process to be used for this discussion. Liquid is flowing into a tank at some rate, Q_{in}, and out of the tank at some rate, Q_{out}. The liquid in the tank has some height or level, h. It is known that the output flow rate varies as the square root of the height, $Q_{out} = K\sqrt{h}$, so the higher the level, the faster the liquid flows out. If the output flow rate is not exactly equal to the input flow rate, the level will drop, if $Q_{out} > Q_{in}$, or rise, if $Q_{out} < Q_{in}$.

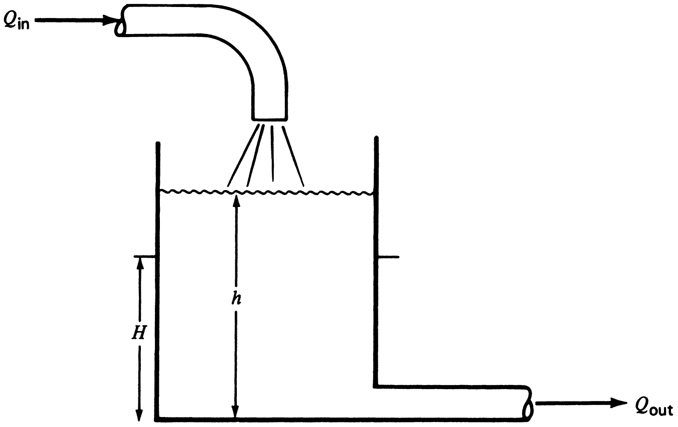


FIGURE 1
The objective is to regulate the level of liquid in the tank, *h,* to the value *H.*

This process has a property called *self-regulation.* This means that for some input flow rate, the liquid height will rise until it reaches a height for which the output flow rate matches the input flow rate. A self-regulating system does not provide regulation of a variable to any particular reference value. In this example, the liquid level will adopt some value for which input and output flow rates are the same, and there it will stay. But if the input flow rate changed, then the level would change also, so it is not regulated to a reference value.

EXAMPLE 1 The tank in Figure 1 has a relationship between flow and level given by $Q_{\text{out}} = K\sqrt{h}$ where h is in feet and $K = 1.156\ (\text{gal/min})/\text{ft}^{1/2}$. Suppose the input flow rate is 2 gal/min. At what value of h will the level stabilize from self-regulation?

Solution

The level will stabilize from self-regulation when $Q_{\text{out}} = Q_{\text{in}}$. Thus, we solve for *h,*

$$h = \left(\frac{Q_{\text{out}}}{K}\right)^2 = \left(\frac{2\ \text{gal/min}}{1.156\ (\text{gal/min})/\text{ft}^{1/2}}\right)^2 = 3\ \text{ft}$$

Suppose we want to maintain the level at some particular value, *H,* in Figure 1, regardless of the input flow rate. Then something more than self-regulation is needed.

Human-Aided Control Figure 2 shows a modification of the tank system to allow artificial regulation of the level by a human. To regulate the level so that it maintains the value *H,* it will be necessary to employ a sensor to measure the level. This has been provided via a "sight tube," *S,* as shown in Figure 2. The actual liquid level or height is called the *controlled variable.* In addition, a valve has been added so that the output flow rate can be changed by the human. The output flow rate is called the *manipulated variable* or *controlling variable.*

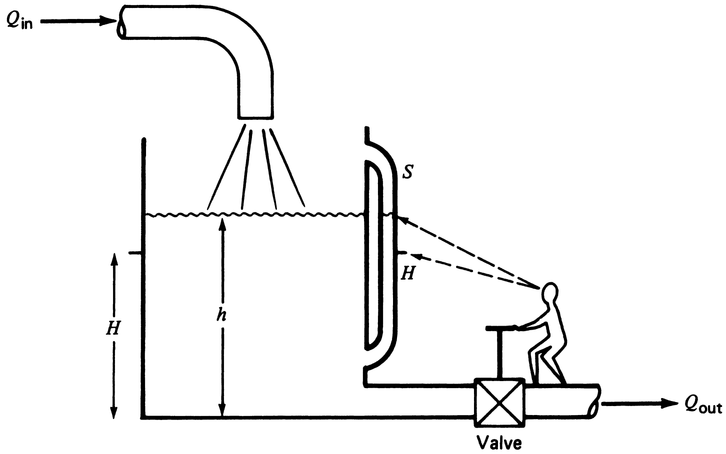


FIGURE 2
A human can regulate the level using a sight tube, *S,* to compare the level, *h,* to the objective, *H,* and adjust a valve to change the level.

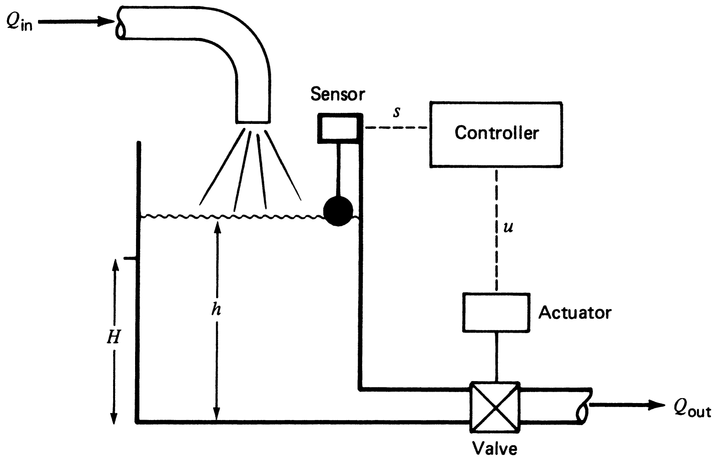


FIGURE 3
An automatic level-control system replaces the human with a controller and uses a sensor to measure the level.

Now the height can be regulated apart from the input flow rate using the following strategy: The human measures the height in the sight tube and compares the value to the setpoint. If the measured value is larger, the human opens the valve a little to let the flow out increase, and thus the level lowers toward the setpoint. If the measured value is smaller than the setpoint, the human closes the valve a little to decrease the flow out and allow the level to rise toward the setpoint.

By a succession of incremental opening and closing of the valve, the human can bring the level to the setpoint value, H, and maintain it there by continuous monitoring of the sight tube and adjustment of the valve. The height is regulated.

Automatic Control To provide automatic control, the system is modified as shown in Figure 3 so that machines, electronics, or computers replace the operations of the human. An instrument called a *sensor* is added that is able to measure the value of the level and convert it into a proportional signal, s. This signal is provided as input to a machine, electronic circuit, or computer called the *controller*. The controller performs the function of the human in evaluating the measurement and providing an output signal, u, to change the valve setting via an *actuator* connected to the valve by a mechanical linkage.

When automatic control is applied to systems like the one in Figure 3, which are designed to regulate the value of some variable to a setpoint, it is called *process control.*

2.2 Servomechanisms

Another commonly used type of control system, which has a slightly different objective from process control, is called a *servomechanism.* In this case, the objective is to force some parameter to vary in a specific manner. This may be called a tracking control system. In-

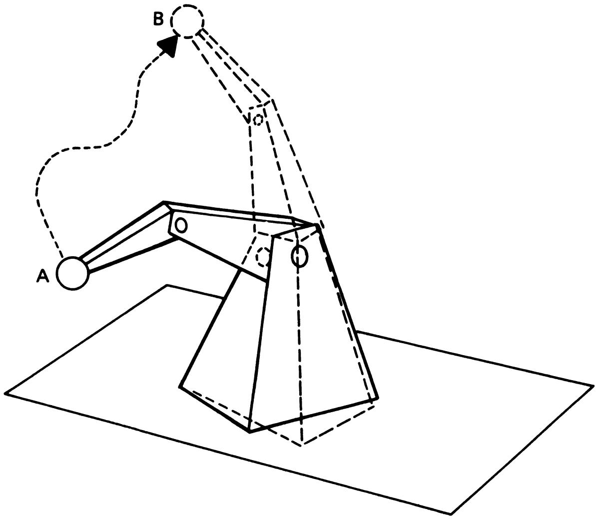


FIGURE 4
Servomechanism-type control systems are used to move a robot arm from point *A* to point *B* in a controlled fashion.

stead of regulating a variable value to a setpoint, the servomechanism forces the controlled variable value to follow variation of the reference value.

For example, in an industrial robot arm like the one shown in Figure 4, servomechanisms force the robot arm to follow a path from point *A* to point *B.* This is done by controlling the speed of motors driving the arm and the angles of the arm parts.

The strategy for servomechanisms is similar to that for process-control systems, but the dynamic differences between regulation and tracking result in differences in design and operation of the control system. This book is directed toward process-control technology.

2.3 Discrete-State Control Systems

This is a type of control system concerned with controlling a *sequence of events* rather than regulation or variation of individual variables. For example, the manufacture of paint might involve the regulation of many variables, such as mixing temperature, flow rate of liquids into mixing tanks, speed of mixing, and so on. Each of these might be expected to be regulated by process-control loops. But there is also a sequence of events that must occur in the overall process of manufacturing the paint. This sequence is described in terms of events that are timed to be started and stopped on a specified schedule. Referring to the paint example, the mixture needs to be heated with a regulated temperature for a certain length of time and then perhaps pumped into a different tank and stirred for another period.

The starting and stopping of events is a discrete-based system because the event is either *true* or *false,* (i.e., started or stopped, open or closed, on or off). This type of control system can also be made automatic and is perfectly suited to computer-based controllers.

These discrete-state control systems are often implemented using specialized computer-based equipment called programmable logic controllers (PLCs).

3 PROCESS-CONTROL BLOCK DIAGRAM

To provide a practical, working description of process control, it is useful to describe the elements and operations involved in more generic terms. Such a description should be independent of a particular application (such as the example presented in the previous section) and thus be applicable to *all* control situations. A model may be constructed using blocks to represent each distinctive element. The characteristics of control operation then may be developed from a consideration of the properties and interfacing of these elements. Numerous models have been employed in the history of process-control description; we will use one that seems most appropriate for a description of modern and developing technology of process control.

3.1 Identification of Elements

The elements of a process-control system are defined in terms of separate functional parts of the system. The following paragraphs define the basic elements of a process-control system and relate them to the example presented in Section 2.

Process In the previous example, the flow of liquid in and out of the tank, the tank itself, and the liquid all constitute a process to be placed under control with respect to the fluid level. In general, a process can consist of a complex assembly of phenomena that relate to some manufacturing sequence. Many variables may be involved in such a process, and it may be desirable to control all these variables at the same time. There are *single-variable* processes, in which only one variable is to be controlled, as well as *multivariable* processes, in which many variables, perhaps interrelated, may require regulation. The process is often also called the *plant.*

Measurement Clearly, to effect control of a variable in a process, we must have information about the variable itself. Such information is found by measuring the variable. In general, a *measurement* refers to the conversion of the variable into some corresponding *analog* of the variable, such as a pneumatic pressure, an electrical voltage or current, or a digitally encoded signal. A sensor is a device that performs the initial measurement and energy conversion of a variable into analogous digital, electrical, or pneumatic information. Further transformation or *signal conditioning* may be required to complete the measurement function. The result of the measurement is a representation of the variable value in some form required by the other elements in the process-control operation.

In the system shown in Figure 3, the controlled variable is the level of liquid in the tank. The measurement is performed by some sensor, which provides a signal, *s,* to the controller. In the case of Figure 2, the sensor is the sight tube showing the level to the human operator as an actual level in the tank.

The sensor is also called a *transducer.* However, the word *sensor* is preferred for the initial measurement device because "transducer" represents a device that converts any signal from one form to another. Thus, for example, a device that converts a voltage into a proportional current would be a transducer. In other words, all sensors are transducers, but not all transducers are sensors.

Error Detector In Figure 2, the human looked at the difference between the actual level, *h,* and the setpoint level, *H,* and deduced an error. This error has both a magnitude and polarity. For the automatic control system in Figure 3, this same kind of error determination must be made before any control action can be taken by the controller. Although the error detector is often a physical part of the controller device, it is important to keep a clear distinction between the two.

Controller The next step in the process-control sequence is to examine the error and determine what action, if any, should be taken. This part of the control system has many names, such as *compensator* or *filter,* but *controller* is the most common. The evaluation may be performed by an operator (as in the previous example), by electronic signal processing, by pneumatic signal processing, or by a computer. In modern control systems, the operations of the controller are typically performed by microprocessor-based computers. The controller requires an input of both a *measured indication* of the controlled variable and a representation of the *reference value* of the variable, expressed in the same terms as the measured value. The reference value of the variable, you will recall, is referred to as the setpoint. Evaluation consists of determining the action required to drive the controlled variable to the setpoint value.

Control Element The final element in the process-control operation is the device that exerts a direct influence on the process; that is, it provides those required changes in the controlled variable to bring it to the setpoint. This element accepts an input from the controller, which is then transformed into some proportional operation performed on the process. In our previous example, the control element is the valve that adjusts the outflow of fluid from the tank. This element is also referred to as the *final control element.*

Often an intermediate operation is required between the controller output and the final control element. This operation is referred to as an *actuator* because it uses the controller signal to actuate the final control element. The actuator translates the small energy signal of the controller into a larger energy action on the process.

3.2 Block Diagram

Figure 5 shows a general block diagram constructed from the elements defined previously. The controlled variable in the process is denoted by c in this diagram, and the measured representation of the controlled variable is labeled b. The controlled variable setpoint is labeled r, for reference. The controller uses the error input to determine an appropriate output signal, p, which is provided as input to the control element. The control element operates on the process by changing the value of the controlling process variable, u.

The error detector is a *subtracting-summing point* that outputs an *error signal,* $e = r - b$, to the controller for comparison and action.

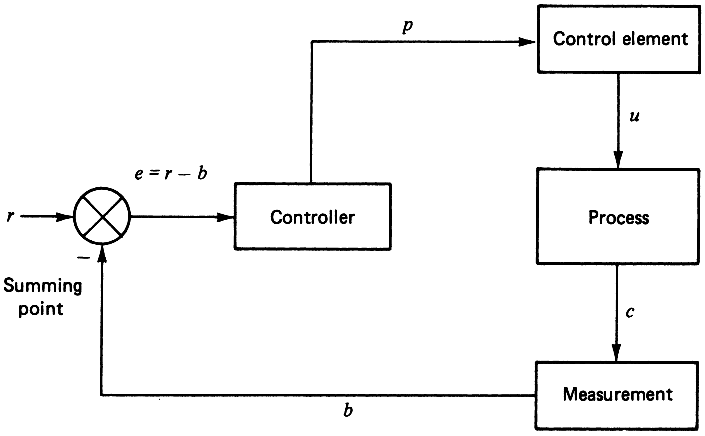


FIGURE 5
This block diagram of a control loop defines all the basic elements and signals involved.

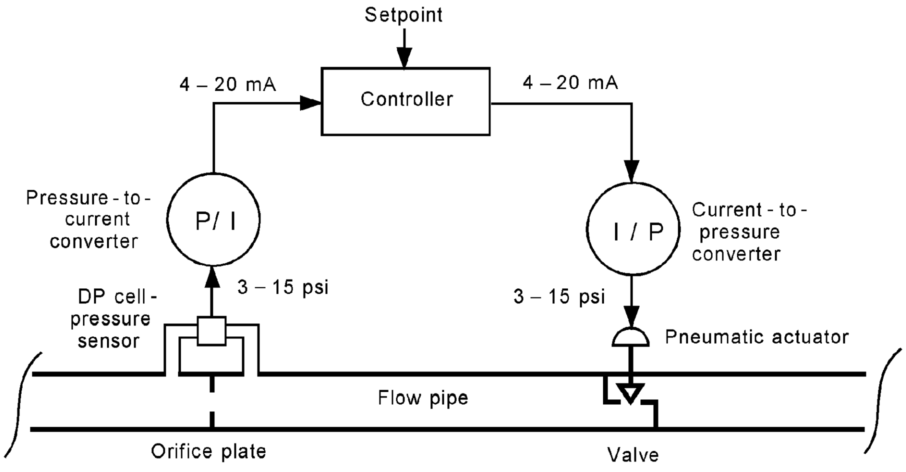
Figure 6 shows how a physical control system is represented as a block diagram. The physical system for control of flow through a pipe is shown in Figure 6. Variation of flow through an obstruction (the orifice plate) produces a pressure-difference variation across the obstruction. This variation is converted to the standard signal range of 3 to 15 psi. The P/I converter changes the pressure to a 4- to 20-mA electric current, which is sent to the controller. The controller outputs a 4- to 20-mA control signal to signify the correct valve setting to provide the correct flow. This current is converted to a 3- to 15-psi pressure signal by the I/P converter and applied to a pneumatic actuator. The actuator then adjusts the valve setting.

Figure 6 shows how all the control system operations are condensed to the standard block diagram operations of measurement, error detection, controller, and final control element.

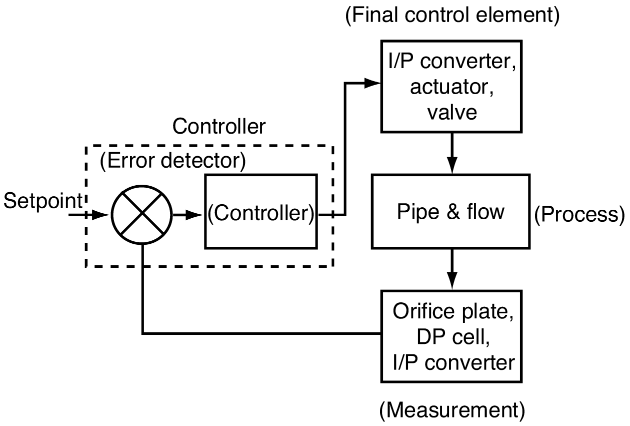
The purpose of a block diagram approach is to allow the process-control system to be analyzed as the interaction of smaller and simpler subsystems. If the characteristics of each element of the system can be determined, then the characteristics of the assembled system can be established by an analytical marriage of these subsystems. The historical development of the system approach in technology was dictated by this practical aspect: first, to specify the characteristics desired of a total system, and then, to delegate the development of subsystems that provide the overall criteria.

It becomes evident that the specification of a process-control system to regulate a variable, *c,* within specified limits and with specified time responses, determines the characteristics the measurement system must possess. This same set of system specifications is reflected in the design of the controller and control element.

From this concept, we conclude that the analysis of a process-control system requires an understanding of the overall system behavior and the reflection of this behavior in the properties of the system elements. Most people find that an understanding of



(a) Physical diagram of a process-control loop



(b) Block diagram of the process-control loop

FIGURE 6
The physical diagram of a control loop and its corresponding block diagram look similar. Note the use of current- and pressure-transmission signals.

the parts leads to a better understanding of the whole. We will proceed with this assumption as a guiding concept.

The Loop Notice in Figure 5 that the signal flow forms a complete circuit from process through measurement, error detector, controller, and final control element. This is called a *loop,* and in general we speak of a process-control loop. In most cases, it is called a *feedback loop,* because we determine an error and feed back a correction to the process.

4 CONTROL SYSTEM EVALUATION

A process-control system is used to regulate the value of some process variable. When such a system is in use, it is natural to ask, How well is it working? This is not an easy question to answer, because it is possible to adjust a control system to provide different kinds of response to errors. This section discusses some methods for evaluating how well the system is working.

The variable used to measure the performance of the control system is the error, $e(t)$, which is the difference between the constant setpoint or reference value, r, and the controlled variable, $c(t)$.

$$e(t) = r - c(t) \tag{1}$$

Since the value of the controlled variable may vary in time, so may the error. (Note that in a servomechanism, the value of r may be forced to vary in time also.)

Control System Objective In principle, the objective of a control system is to make the error in Equation (1) exactly zero, but the control system responds only to errors (i.e., when an error occurs, the control system takes action to drive it to zero). Conversely, if the error were zero and stayed zero, the control system would not be doing anything and would not be needed in the first place. Therefore, this objective can never be perfectly achieved, and there will always be some error. The question of evaluation becomes one of how large the error is and how it varies in time.

A practical statement of control system objective is best represented by three requirements:

1. The system should be stable.
2. The system should provide the best possible steady-state regulation.
3. The system should provide the best possible transient regulation.

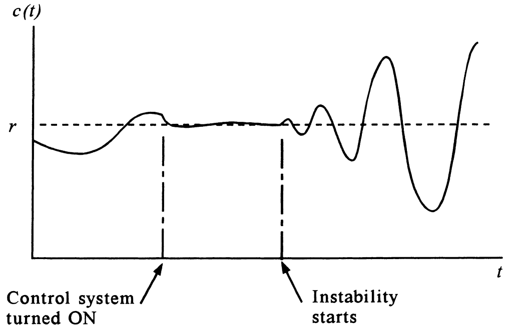
4.1 Stability

The purpose of the control system is to regulate the value of some variable. This requires that action be taken on the process itself in response to a measurement of the variable. If this is not done correctly, the control system can cause the process to become unstable. In fact, the more tightly we try to control the variable, the greater the possibility of instability.

Figure 7 shows that, prior to turning on a control system, the controlled variable drifts in a random fashion and is not regulated. After the control system is turned on, the variable is forced to adopt the setpoint value, and all is well for awhile. Notice that some time later, however, the variable begins to exhibit growing oscillations of value—that is, an instability. This occurs even though the control system is still connected and operational; in fact, it occurs *because* the system is connected and operational.

The first objective, then, simply means that the control system must be designed and adjusted so that the system is stable. Typically, as the control system is adjusted to give better control, the likelihood of instability also increases.

FIGURE 7
A control system can actually cause a system to become unstable.



4.2 Steady-State Regulation

The objective of the best possible steady-state regulation simply means that the steady-state error should be a minimum. Generally, when a control system is specified, there will be some allowable deviation, $\pm\Delta c$, about the setpoint. This means that variations of the variable within this band are expected and acceptable. External influences that tend to cause drifts of the value beyond the allowable deviation are corrected by the control system.

For example, a process-control technologist might be asked to design and implement a control system to regulate temperature at 150°C within ±2°C. This means the setpoint is to be 150°C, but the temperature may be allowed to vary within the range of 148° to 152°C.

4.3 Transient Regulation

What happens to the value of the controlled variable when some sudden transient event occurs that would otherwise cause a large variation? For example, the setpoint could change. Suppose the setpoint in the aforementioned temperature case were suddenly changed to 160°C. Transient regulation specifies how the control system reacts to bring the temperature to this new setpoint.

Another type of transient influence is a sudden change of some other process variable. The controlled variable depends on other process variables. If one of them suddenly changes value, the controlled variable may be driven to change also, so the control system acts to minimize the effect. This is called *transient response.*

4.4 Evaluation Criteria

The question of how well the control system is working is thus answered by (1) ensuring stability, (2) evaluating steady-state response, and (3) evaluating the response to setpoint changes and transient effects. There are many criteria for gauging the response. In general, the term *tuning* is used to indicate how a process-control loop is adjusted to provide the best control.

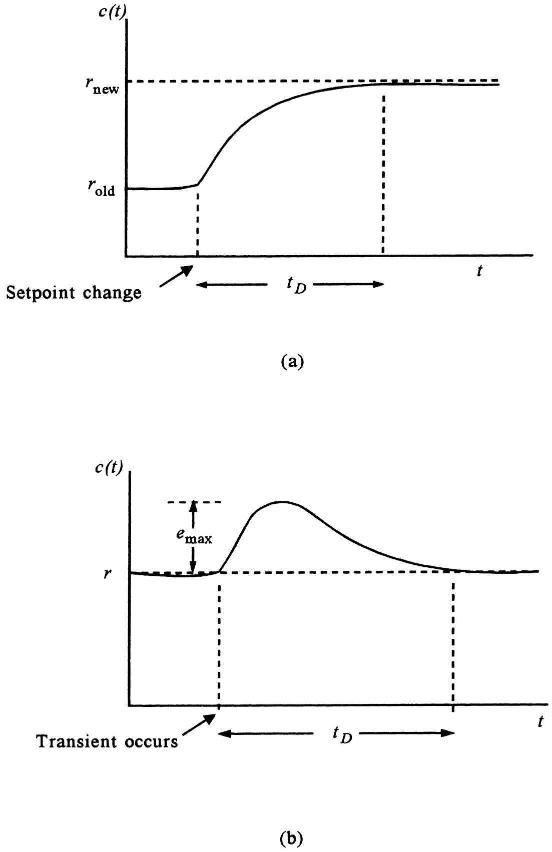


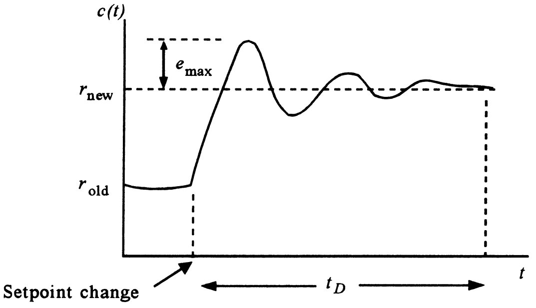
FIGURE 8
One of the measures of control system performance is how the system responds to changes of setpoint or a transient disturbance.

Damped Response One type of criterion requires that the controlled variable exhibit a response such as that shown in Figure 8 for excitations of both setpoint changes and transient effects. Note that the error is of only one polarity (i.e., it never oscillates about the setpoint). For this case, measures of quality are the duration, t_D, of the excursion and, for the transient, the maximum error, $e_{\max}$, for a given input. The duration is usually defined as the time taken for the controlled variable to go from 10% of the change to 90% of the change following a setpoint change. In the case of a transient, the duration is often defined as the time from the start of the disturbance until the controlled variable is again within 4% of the reference.

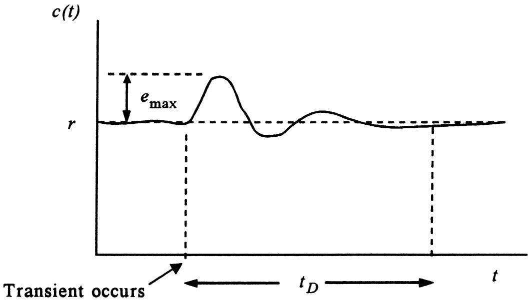
Different tuning will provide different values of $e_{\max}$ and t_D for the same excitation. It is up to the process designers to decide whether the best control is larger duration with smaller peak error, or vice versa, or something in between.

Cyclic Response Another type of criterion applies to those cases in which the response to a setpoint change or transient is as shown in Figure 9. Note that the controlled variable oscillates about the setpoint. In this case, the parameters of interest are the maximum error,

FIGURE 9
In cyclic or underdamped response, the variable will exhibit oscillations about the reference value.



a) Setpoint change oscillations



b) Transient change oscillations

e_{max}, and the duration, t_D, also called the settling time. The duration is measured from the time when the allowable error is first exceeded to the time when it falls within the allowable error and stays.

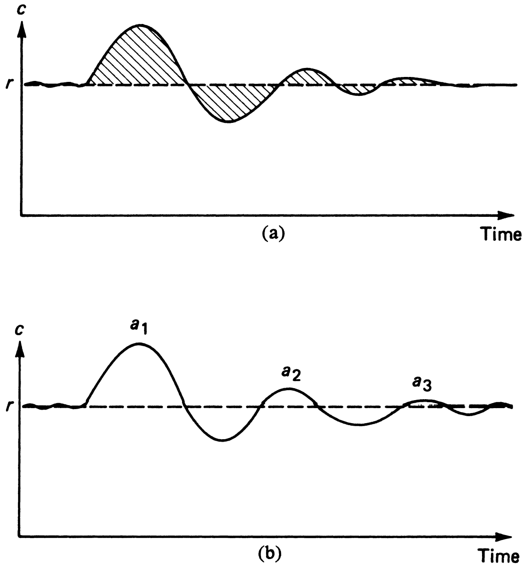
The nature of the response is modified by adjusting the control loop parameters, which is called *tuning*. There may be large maximum error but short duration or long duration with small maximum error, and everything in between.

A number of standard cyclic tuning criteria are used. Two common types are minimum area and quarter amplitude. In *minimum area,* the tuning is adjusted until the net area under the error-time curve is a minimum, for the same degree of excitation (setpoint change or transient). Figure 10 shows the area as a shaded part of the curve. Analytically, this is given by

$$A = \int |e(t)|dt = \text{minimum} \qquad \textbf{(2)}$$

The *quarter-amplitude* criterion, shown in Figure 10, specifies that the ampli tude of each peak of the cyclic response be a quarter of the preceding peak. Thus, $a_2 = a_1/4, a_3 = a_2/4$, and so on.

FIGURE 10
Two criteria for judging the quality of control-system response are the minimum area and quarter amplitude.



5 ANALOG AND DIGITAL PROCESSING

In the past, the functions of the controller in a control system were performed by sophisticated electronic circuits. Data were represented by the magnitude of voltages and currents in such systems. This is referred to as *analog processing.* Most modern control systems now employ digital computers to perform controller operations. In computers, data are represented as binary numbers consisting of a specific number of bits. This is referred to as *digital processing.* The paragraphs that follow contrast the analog and digital approaches to control system operation.

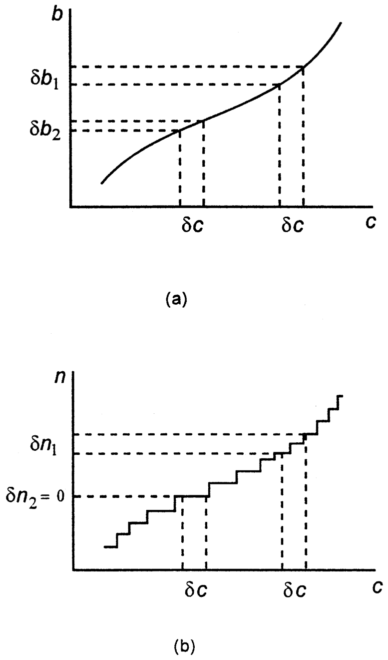
5.1 Data Representation

The representation of data refers to how the magnitude of some physical variable is represented in the control loop. For example, if a sensor outputs a voltage whose magnitude varies with temperature, then the voltage represents the temperature. Analog and digital systems represent data in very different fashions.

Analog Data An analog representation of data means that there is a smooth and continuous variation between a representation of a variable value and the value itself. Figure 11 shows an analog relationship between some variable, *c,* and its representation, *b.* Notice that, for every value of *c* within the range covered, there is a unique value of *b.* If *c* changes by some small amount, δc, then *b* will change by a proportional amount, δb.

The relationship in Figure 11a is called *nonlinear* because the same δc does not result in the same δb, as shown. This is described in more detail later in this chapter.

FIGURE 11
Graph (a) shows how output variable *b* changes as an analog of variable *c*. Graph (b) shows how a digital output variable, *n,* would change with variable *c.*



(a)

(b)

Digital Data Digital data means that numbers are represented in terms of binary digits, also called bits, which take on values of one **(1)** or zero **(0).** When data are represented digitally, some range of analog numbers is encoded by a fixed number of binary digits. The consequence is a loss of information because a fixed number of binary digits has a limited resolution. For example, Table 1 shows how voltage from 0 to 15 volts could be encoded by four binary digits. A change of one volt produces a change of the least significant bit (LSB). You can see that, if the voltage changed by less than one volt, the digital representation would not change. So the representation cannot distinguish between 4.25 V and 4.75 V because both would be represented by $\mathbf{0100_2}$.

Table 1 also shows the hexadecimal (hex) representation of binary numbers in the digital representation. Often we use the hex representation when presenting digital data because it is a more compact and human-friendly way of writing numbers.

The consequence of digital representation of data is that the smooth and continuous relation between the representation and the variable data value is lost. Instead, the digital representation can take on only discrete values. This can be seen in Figure 11, where a variable, *c,* is represented by a digital quantity, *n.* Notice that variations of *c,* such as δc, may not result in any change in *n.* The variable must change by more than some minimum amount, depending on where in the curve the change occurs, before a change in representation is assured.

Data Conversions Special devices are employed to convert analog voltages into a digital representation. These are called *analog-to-digital converters* (ADCs). In a control system, the sensor often produces an analog output such as a voltage. Then an ADC is used

TABLE 1
Decimal-binary-hex encoding

Voltage	Binary	Hex
0	0000	0h
1	0001	1h
2	0010	2h
3	0011	3h
4	0100	4h
5	0101	5h
6	0110	6h
7	0111	7h
8	1000	8h
9	1001	9h
10	1010	Ah
11	1011	Bh
12	1100	Ch
13	1101	Dh
14	1110	Eh
15	1111	Fh

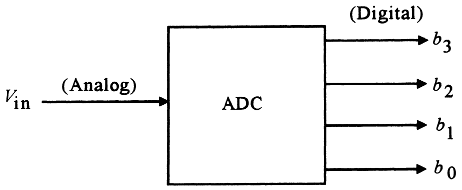


FIGURE 12
An ADC converts analog data, such as voltage, into a digital representation, in this case 4 bits.

to convert that voltage into a digital representation for input to the computer. Figure 12 shows how an ADC might be used to convert voltage into a 4-bit digital signal as illustrated in Table 1.

Digital-to-analog converters (DACs) convert a digital signal into an analog voltage. These devices are used to convert the control output of the computer into a form suitable for the final control element.

5.2 ON/OFF Control

One of the most elementary types of digital processing has been in use for many years, long before the advent of computers, in fact. This is called ON/OFF control because the final control element has only two states, on and off. Thus, the controller output need have only these two states as well. It can be said that the controller output is a digital representation of a single binary digit, **0** or **1.**

Figure 13 shows a diagram of an elementary ON/OFF control system whose objective is to maintain the temperature in a system at some reference value, T_{ref}. A sensor

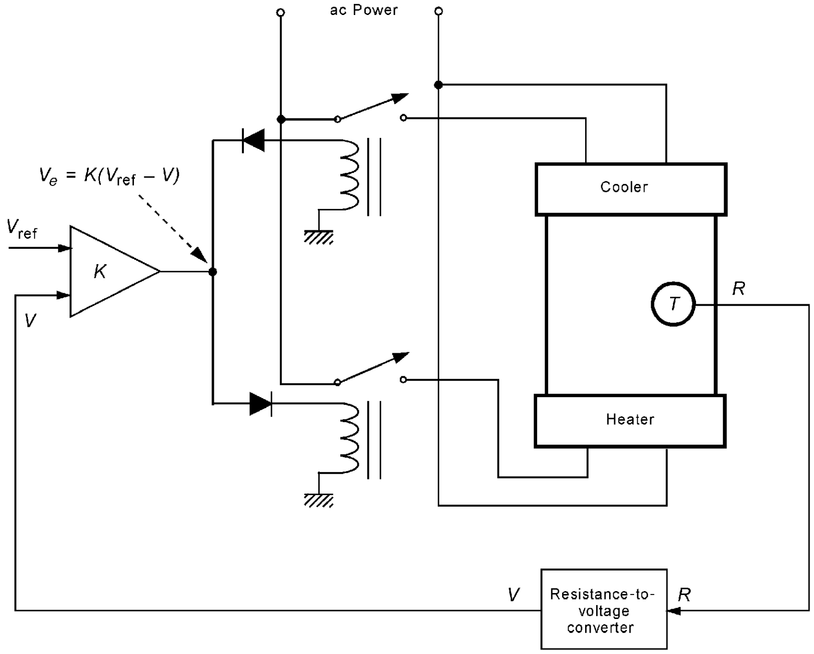


FIGURE 13
This ON/OFF control system can either heat or cool or do neither. No variation of the degree of heating or cooling is possible.

converts temperature values into a resistance in an analog fashion; that is, R varies smoothly and continuously with T. Signal conditioning converts the variable R into an analog voltage, V. Thus, V is an analog of T as well. The differential amplifier multiplies the difference between V and a reference voltage, V_{ref}, by a gain K to produce an error voltage, V_e.

$$V_e = K(V_{ref} - V)$$

V_{ref} is simply defined as the voltage from the converter that would be produced by T_{ref}.

At this point, the system becomes digital, because the relays will either be open or closed so that the heater or cooler will either be on or off. The diodes direct the current to the appropriate relay to produce heating or cooling, based on polarity. This system also exhibits a deadband and a hysteresis, since there is a difference between a relay "pull-in" voltage and the "release" voltage. A *deadband* is a range, of temperature in this case, wherein no action will occur. *Hysteresis* means that the behavior of the system is different at the same value of temperature, depending on whether the temperature is increasing or decreasing.

Our home and auto heaters and air conditioners, home water heaters, and a host of other basic control systems work according to the same ON/OFF mode.

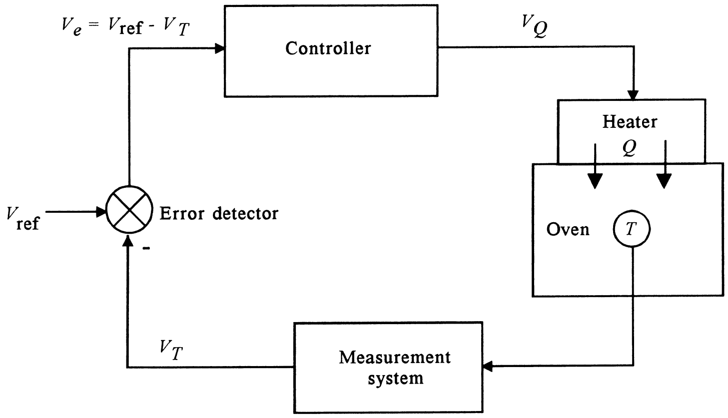


FIGURE 14
An analog control system such as this allows continuous variation of some parameter, such as heat input, as a function of error.

5.3 Analog Control

True analog control exists when all variables in the system are analog representations of another variable. Figure 14 shows a process in which a heater is used to control temperature in an oven. In this case, however, the heater output, Q, is an analog of the excitation voltage, V_Q, and thus heat can be varied continuously. Notice that every signal is an analog: V_T is an analog of T; the error E is an analog of the difference between the reference, V_{ref}, and the temperature voltage, V_T. The reference voltage is simply the voltage that would result from measurement of the specified reference temperature, T_{ref}.

5.4 Digital Control

True digital control involves the use of a computer in modern applications, although in the past, digital logic circuits were also used. There are two approaches to using computers for control.

Supervisory Control When computers were first considered for applications in control systems, they did not have a good reliability; they suffered frequent failures and breakdown. The necessity for continuous operation of control systems precluded the use of computers to perform the actual control operations. Supervisory control emerged as an intermediate step wherein the computer was used to monitor the operation of analog control loops and to determine appropriate setpoints. A single computer could monitor many control loops and use appropriate software to optimize the setpoints for the best overall plant operation. If the computer failed, the analog loops kept the process running using the last setpoints until the computer came back on-line.

Figure 15 shows how a supervisory computer would be connected to the analog heater control system of Figure 14. Notice how the ADC and DAC provide interface between the analog signals and the computer.

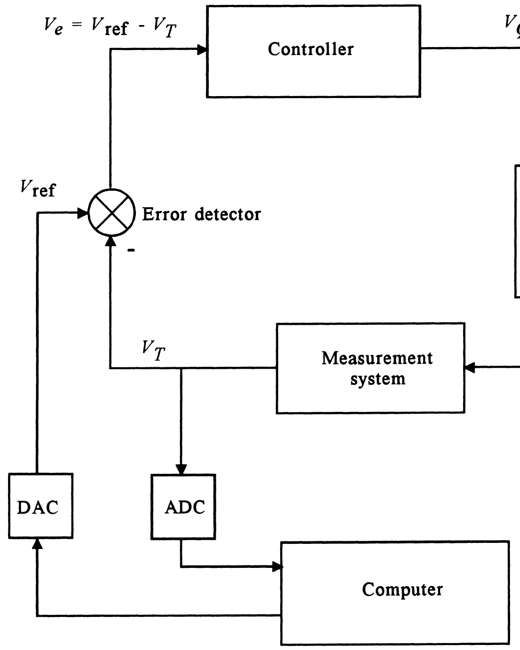


FIGURE 15
In supervisory control, the computer monitors measurements and updates setpoints, but the loops are still analog in nature.

Direct Digital Control (DDC) As computers have become more reliable and miniaturized, they have taken over the controller function. Thus, the analog processing loop is discarded. Figure 16 shows how, in a full computer control system, the operations of the controller have been replaced by software in the computer. The ADC and DAC provide interface with the process measurement and control action. The computer inputs a digital representation of the temperature, N_T, as an analog-to-digital conversion of the voltage, V_T. Error detection and controller action are determined by software. The computer then provides output directly to the heater via digital representation, N_Q, which is converted to the excitation voltage, V_Q, by the DAC.

Smart Sensor Along with the dramatic advances in computer technology has come an equally dramatic advance in the applications of computers to control systems. One of the most remarkable developments has been the integration of a microprocessor-based controller computer directly into the sensor assembly. Using modern integrated circuit technology, the sensor, signal conditioning, ADC, and computer controller are all contained within the sensor housing. In one form, the unit also contains a DAC with a 4- to 20-mA output to be fed to the final control element. The setpoint is programmed by connecting another computer to the unit using a serial interface line.

The most current technology, however, is to interface these smart sensors to a local area network, or field bus, to be described next, which allows the sensor to be connected to other computers and to the final control element over a common digital serial line.

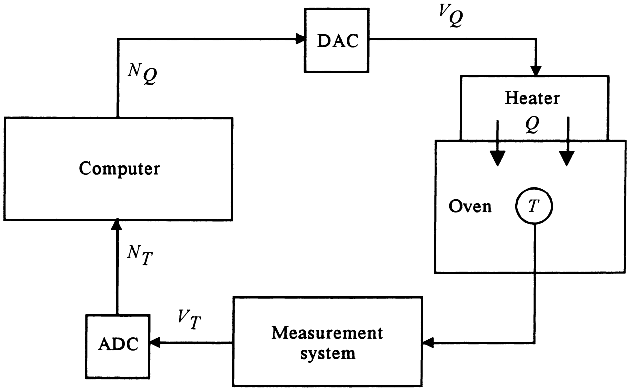


FIGURE 16
This direct digital control system lets the computer perform the error detection and controller functions.

Networked Control Systems When a plant uses DDC, it becomes possible to place the computer-based controller directly at the site of the plant where the control is needed. This is done by using smart sensors or by placing the computer controller in hermetically sealed instrument cases around the plant. In order to have coordinated control of the whole plant, all these DDC units are placed on a local area network (LAN). The LAN commonly provides communication as a serial stream of digital data over a variety of carriers such as wires and fiber optics. The LAN also connects to computers exercising master control of plant operations, fiscal computers for accounting and production control, and engineering computers for monitoring and modifying plant operations as needed. In control systems, these LANs are referred to as a *field bus.*

Figure 17 shows how the LAN or field bus connects the computers in a plant together. Each of the process-control computers operates one or more DDC loops like the one shown in Figure 16. Bus users can monitor the operations of any of the plant process-control loops, and those with authorization can modify control characteristics such as setpoints and gains.

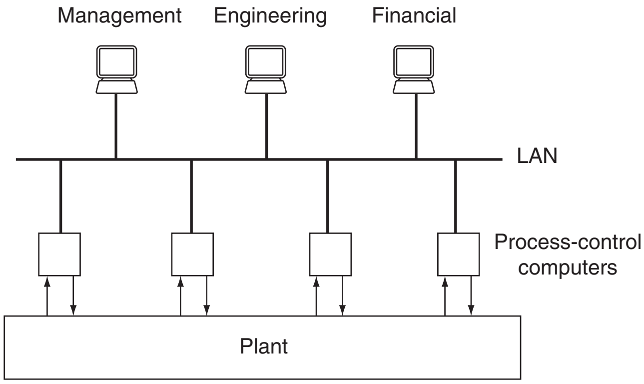


FIGURE 17
Local area networks (LANs) play an important role in modern process-control plants.

Special process-control bus standards have been developed for how data and information are represented and transmitted in these networks. The two most commonly implemented standards are the Foundation Fieldbus and the Profibus (Process Field Bus). The idea behind these standards is to have universal agreement among process equipment manufacturers on how data are represented on the bus line and how data are transmitted and received. This is an extension of the "plug and play" concept used for computer hardware. With standardization, successful interconnection and interfacing of equipment from a variety of manufacturers into a control loop is assured. Fieldbus (primarily in the United States) and Profibus (primarily in Europe) are vying to become the universal standard.

5.5 Programmable Logic Controllers

Many manufacturing operations are ON/OFF in nature; that is, a conveyor or heater is either on or off, a valve is either open or closed, and so on. In the past, these types of discrete control functions were often provided by a system of electrical relays wired according to a complex diagram into what was called a relay logic controller.

In recent years, computers have also taken over the operation of such relay logic controllers, known as *programmable logic controllers* (PLCs). Even though originally designed to control discrete-state (ON/OFF) systems, they are now also used to implement DDC.

Figure 18 shows how the problem of Figure 13 would be implemented using a PLC. Note that thermal-limit switches are used instead of a sensor to indicate when the

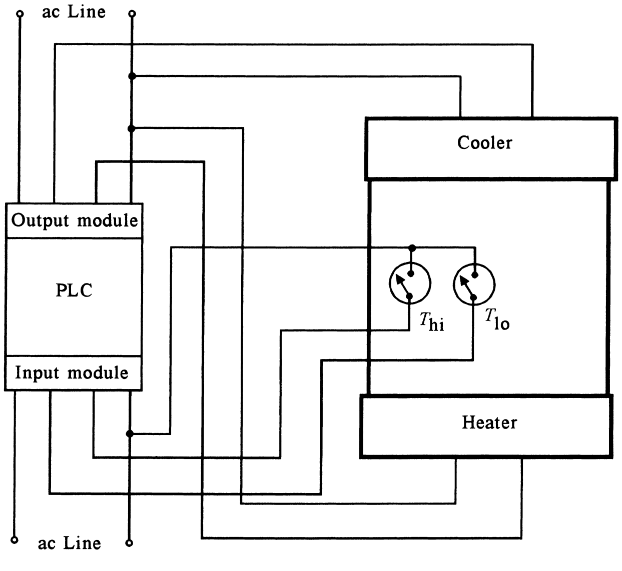


FIGURE 18
A programmable logic controller (PLC) is an outgrowth of ON/OFF-type control environments. In this case the heater and cooler are either ON or OFF.

temperature has risen above or fallen below the limit temperatures. These are simply switches designed to open (or close) when the temperature reaches certain preset limits.

6 UNITS, STANDARDS, AND DEFINITIONS

As in any other technological discipline, the field of process control has many sets of units, standards, and definitions to describe its characteristics. Some of these are a result of historical use, some are for convenience, and some are just confusing. As the discipline grew, there were efforts to standardize terms so that professional workers in process control could effectively communicate among themselves and with specialists in other disciplines. In this section, we summarize the present state of affairs relative to the common units, standards, and definitions.

6.1 Units

To ensure precise technical communication among individuals employed in technological disciplines, it is essential to use a well-defined set of units of measurement. The metric system of units provides such communication and has been adopted by most technical disciplines. In process control, a particular set of metric units is used (which was developed by an international conference) called the International System (SI, Système International D'Unités).

In the United States, the English system of units is still in common use, although use of SI units is gradually occurring more often. It is important for the process-control specialist to know English units and to be able to perform conversions with the SI system.

International System of Units The international system of units is maintained by an international agreement for worldwide standardization. The system is based on seven well-defined base units and two supplementary, dimensionless units. Everything else falls into the category of defined units, which are defined in terms of the seven base and two supplementary units.

Quantity	Unit	Symbol
BASE		
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Amount of substance	Mole	mol
Luminous intensity	Candela	cd
SUPPLEMENTARY		
Plane angle	Radian	rad
Solid angle	Steradian	sr

All other SI units can be derived from these nine units, although in many cases a special name is assigned to the derived quantity. Thus, a force is measured by the newton (N), where $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$; energy is measured by the joule (J) or watt-second $(\text{W}\cdot\text{s})$, given by $1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$; and so on, as shown in Appendix: Units.

Other Units Although the SI system is used in this text, other units remain in common use in some technical areas. The reader, therefore, should be able to identify and translate between the SI system and other systems. The centimeter-gram-second system (CGS) and the English system are also given in Appendix: Units. The following examples illustrate some typical translations of units.

EXAMPLE 2

Express a pressure of $p = 2.1 \times 10^3\ \text{dyne/cm}^2$ in pascals. $1\ \text{Pa} = 1\ \text{N/m}^2$.

Solution

From Appendix: Units, we find $10^2\ \text{cm} = 1\ \text{m}$ and $10^5\ \text{dyne} = 1$ newton; thus,

$$p = (2.1 \times 10^3\ \text{dyne/cm}^2)\left(10^2\,\frac{\text{cm}}{\text{m}}\right)^2\left(\frac{1\ \text{N}}{10^5\ \text{dyne}}\right)$$

$$p = \mathbf{210\ N/m^2 = 210\ pascals\ (Pa)}$$

EXAMPLE 3

Find the number of feet in 5.7 m.

Solution

Reference to the table of conversions in Appendix: Units shows that 1 m = 39.37 in.; therefore,

$$(5.7\ \text{m})\left(39.37\,\frac{\text{in.}}{\text{m}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right) = \mathbf{18.7\ ft}$$

EXAMPLE 4

Express 6.00 ft in meters.

Solution

Using 39.37 in./m and 12 in./ft,

$$(6.00\ \text{ft})\,(12\ \text{in./ft})\left(\frac{1\ \text{m}}{39.37\ \text{in.}}\right) = \mathbf{1.83\ m}$$

EXAMPLE 5

Find the mass in kilograms of a 2-lb object.

Solution

The conversion factor between mass in kilograms and pounds is found from Appendix: Units to be 0.454 kg/lb. Therefore, we have

$$m = (2\ \text{lb})(0.454\ \text{kg/lb})$$
$$m = \mathbf{0.908\ kg}$$

Metric Prefixes With the wide variation of variable magnitudes that occurs in industry, there is a need to abbreviate very large and small numbers. Scientific notation allows the expression of such numbers through powers of 10. A set of standard metric prefixes has been adopted by the SI to express these powers of 10, which are employed to simplify the expression of very large or very small numbers.

EXAMPLE 6 Express 0.0000215 s and 3,781,000,000 W using decimal prefixes.

Solution

We first express the quantities in scientific notation and then find the appropriate decimal prefix from Appendix: Units.

$$0.0000215 \text{ s} = 21.5 \times 10^{-6} \text{s} = \mathbf{21.5\ \mu s}$$

and

$$3,781,000,000 \text{ W} = 3.781 \times 10^{9} \text{ W} = \mathbf{3.781\ GW}$$

6.2 Analog Data Representation

For measurement systems or control systems, part of the specification is the range of the variables involved. Thus, if a system is to measure temperature, there will be a range of temperature specified, for example, 20° to 120°C. Similarly, if the controller is to output a signal to a continuous valve, this signal will be designed to cover the range from fully closed to fully open, with all the various valve settings in between.

Two analog standards are in common use as a means of representing the range of variables in control systems. For electrical systems, we use a range of electric current carried in wires, and for pneumatic systems we use a range of gas pressure carried in pipes. These signals are used primarily to transmit variable information over some distance, such as to and from the control room and the plant. Figure 19 shows a diagram of a process-control

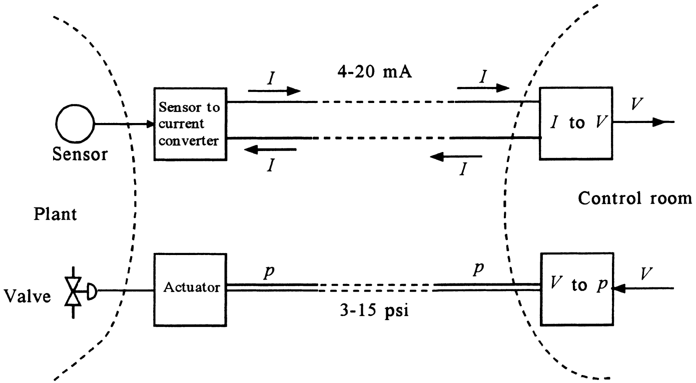


FIGURE 19
Electric current and pneumatic pressures are the most common means of information transmission in the industrial environment.

installation in which current is used to transmit measurement data about the controlled variable to the control room, and gas pressure in pipes is used to transmit a feedback signal to a valve to change flow as the controlling variable.

Current Signal The most common current transmission signal is 4 to 20 mA. Thus, in the following temperature example, 20°C might be represented by 4 mA and 120°C by 20 mA, with all temperatures in between represented by a proportional current.

EXAMPLE 7 Suppose the temperature range 20° to 120°C is linearly converted to the standard current range of 4 to 20 mA. What current will result from 66°C? What temperature does 6.5 mA represent?

Solution

The easiest way to solve this kind of problem is to develop a linear equation between temperature and current. We can write this equation as $I = mT + I_0$, and we know from the given data that $I = 4$ mA when $T = 20^\circ$C and that $I = 20$ mA when $T = 120^\circ$C. Thus, we have two equations in two unknowns:

$$4 \text{ mA} = (20^\circ\text{C})m + I_0$$

$$20 \text{ mA} = (120^\circ\text{C})m + I_0$$

Subtracting the first from the second gives

$$16 \text{ mA} = (100^\circ\text{C})m$$

so that $m = 0.16 \text{ mA}/^\circ\text{C}$. Then we find I_0:

$$I_0 = 4 \text{ mA} - (20^\circ\text{C})(0.16 \text{ mA}/^\circ\text{C})$$

$$I_0 = 0.8 \text{ mA}$$

Thus, the equation relating current and temperature is

$$I = (0.16 \text{ mA}/^\circ\text{C})T + 0.8 \text{ mA}$$

Now answering the questions is easy. For 66°C, we have

$$I = (0.16 \text{ mA}/^\circ\text{C})66^\circ\text{C} + 0.8 \text{ mA} = \mathbf{11.36\ mA}$$

For 6.5 mA, we solve for *T*:

$$6.5 \text{ mA} = (0.16 \text{ mA}/^\circ\text{C})T + 0.8 \text{ mA}$$

for which $T = \mathbf{35.6^\circ C}$.

Current is used instead of voltage because the system is then less dependent on load. The sensor-to-current converter in Figure 19, also called a transmitter, is designed to launch a current into the line regardless of load, to a degree. In Figure 20, a resistor, *R,* has been added to the lines connecting the plant to the control room. In the control room, the incoming current has been converted to a voltage using resistor R_L. Note that if the short around resistor *R* is cut

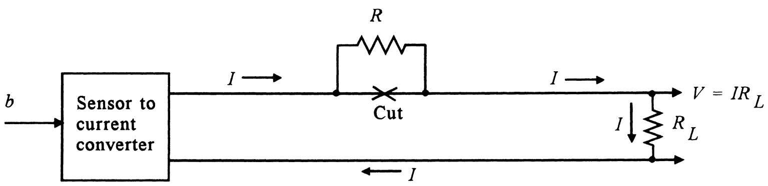


FIGURE 20
One of the advantages of current as a transmission signal is that it is nearly independent of line resistance.

so that *R* is now in the circuit, no change in current will occur. The transmitter is designed to adjust conditions (in this case, output voltage) so that the current is held constant. Practically speaking, most current transmitters can work into any load, from 0 to about 1000 Ω.

Voltage is not used for transmission because of its susceptibility to changes of resistance in the line.

Pneumatic Signals In the United States, the most common standard for pneumatic signal transmission is 3 to 15 psi. In this case, when a sensor measures some variable in a range, it is converted into a proportional pressure of gas in a pipe. The gas is usually dry air. The pipe may be many hundreds of meters long, but as long as there is no leak in the system, the pressure will be propagated down the pipe. This English system standard is still widely used in the United States, despite the move to the SI system of units. The equivalent SI range that will eventually be adopted is 20 to 100 kPa.

6.3 Definitions

This section presents definitions of some of the common terms and expressions used to describe process-control elements.

Error The most important quantity in control systems is the error. When used to describe the results of a measurement, error is the difference between the actual value of a variable and the measured indication of its value. In that case, the *accuracy* of the measurement system places bounds on the possible error.

When used for a controlled variable in a control system, error is the difference between the measured value of the variable and the desired value—that is, the reference or setpoint value.

Block Definitions As noted in Section 3.2, control systems are often described in terms of blocks. One block represents the measurement, another the controller, and so on. In order to work effectively in control systems, one must understand the terms and expressions used to describe the characteristics of a block. Figure 21 shows a block that has an input of some variable, $x(t)$, and an output of another variable, $y(t)$. This model will be used in the following paragraphs to define the characteristics of the block.

Transfer Function The transfer function, $T(x, y, t)$ in Figure 21, describes the relationship between the input and output for the block. The transfer function is often described in two parts, the static part and the dynamic part. The static transfer function describes the

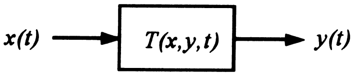


FIGURE 21
A transfer function shows how a system-block output variable varies in response to an input variable, as a function of both static input value and time.

input/output relationship when the input is not changing in time. The dynamic transfer function describes the input/output relationship when there is time variation of the input.

Static transfer functions may be presented in the form of equations, tables, or graphs. For example, a flow meter may relate flow, Q, in gallons per minute, to a differential pressure, Δp, in psi, via an equation such as

$$Q = 119.5\sqrt{\Delta p}$$

However, an RTD temperature sensor is usually represented by a table of resistance versus temperature. Graphs are often used to visually display how input and output vary. Frequently, the transfer is valid only over a certain range of variable values.

The dynamic transfer function is often represented by a differential equation in time. Common examples of simple dynamic transfer functions are presented in Section 7.

Accuracy This term is used to specify the maximum overall error to be expected from a device, such as measurement of a variable. Accuracy is usually expressed as the *inaccuracy* and can appear in several forms:

1. Measured variable; the accuracy is ± 2°C in some temperature measurement. Thus, there would be an uncertainty of ±2°C in any value of temperature measured.
2. Percentage of the instrument full-scale (FS) reading. Thus, an accuracy of ±0.5% FS in a 5-V full-scale range meter would mean the inaccuracy or uncertainty in any measurement is ±0.025 V.
3. Percentage of instrument span—that is, percentage of the range of instrument measurement capability. Thus, for a device measuring ±3% of span for a 20 to 50 psi range of pressure, the accuracy would be $(\pm 0.03)(50 - 20) = \pm 0.9$ psi.
4. Percentage of the actual reading. Thus, for a ±2% of reading voltmeter, we would have an inaccuracy of ±0.04 V for a reading of 2 V.

EXAMPLE 8

A temperature sensor has a span of 20°−250°C. A measurement results in a value of 55°C for the temperature. Specify the error if the accuracy is **(a)** ±0.5% FS, **(b)** ±0.75% of span, and **(c)** ±0.8% of reading. What is the possible temperature in each case?

Solution

Using the given definitions, we find

a. Error $= (\pm 0.005)(250°\mathrm{C}) = \pm \mathbf{1.25°C}$. Thus, the actual temperature is in the range of 53.75° to 56.25°C.
b. Error $= (\pm 0.0075)(250 - 20)°\mathrm{C} = \pm \mathbf{1.725°C}$. Thus, the actual temperature is in the range of 53.275° to 56.725°C.
c. Error $= (\pm 0.008)(55°\mathrm{C}) = \pm \mathbf{0.44°C}$. Thus, the temperature is in the range of 54.56° to 55.44°C.

EXAMPLE 9 A temperature sensor has a transfer function of 5 mV/°C with an accuracy of $\pm 1\%$. Find the possible range of the transfer function.

Solution

The transfer function range will be $(\pm 0.01)(5 \text{ mV/°C}) = \pm 0.05 \text{ mV/°C}$. Thus, the range is 4.95 to 5.05 mV/°C.

EXAMPLE 10 Suppose a reading of 27.5 mV results from the sensor used in Example 9. Find the tem perature that could provide this reading.

Solution

Because the range of transfer function is 4.95 to 5.05 mV/°C, the possible temperature values that could be inferred from a reading of 27.5 mV are

$$(27.5 \text{ mV})\left(\frac{1}{4.95 \text{ mV/°C}}\right) = \mathbf{5.56°C}$$

$$(27.5 \text{ mV})\left(\frac{1}{5.05 \text{ mV/°C}}\right) = \mathbf{5.45°C}$$

Thus, we can be certain only that the temperature is between 5.45°C and 5.56°C.

The application of digital processing has necessitated an accuracy definition compatible with digital signals. In this regard, we are most concerned with the error involved in the digital representation of analog information. Thus, the accuracy is quoted as the percentage deviation of the analog variable per bit of the digital signal. As an example, an A/D converter may be specified as 0.635 volts per bit $\pm$ 1%. This means that a bit will be set for an input voltage change of 0.635 $\pm$ 0.006 V, or 0.629 to 0.641 V.

System Accuracy Often, one must consider the overall accuracy of *many* elements in a process-control loop to represent a process variable. Generally, the best way to do this is to express the accuracy of each element in terms of the transfer functions. For example, suppose we have a process with two transfer functions that act on the dynamic variable to produce an output voltage as shown in Figure 22. We can describe the output as

$$V \pm \Delta V = (K \pm \Delta K)(G \pm \Delta G)C \tag{3}$$

where
V = output voltage
$\pm\Delta V$ = uncertainty in output voltage
K, G = nominal transfer functions
$\Delta K, \Delta G$ = uncertainties in transfer functions
C = dynamic variable

From Equation (3), we can find the output uncertainty to be

$$\Delta V = \pm GC\,\Delta K \pm KC\,\Delta G \pm \Delta K\,\Delta GC \tag{4}$$

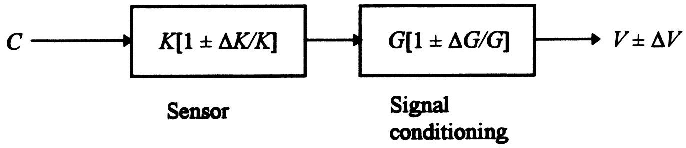


FIGURE 22
Uncertainties in block transfer functions build up as more blocks are involved in the transformation.

Equation (4) can be written in terms of fractional uncertainties by factoring out V. In this case we get,

$$\frac{\Delta V}{V} = \pm\frac{\Delta K}{K} \pm \frac{\Delta G}{G} \pm \left(\frac{\Delta V}{V}\right)\left(\frac{\Delta K}{K}\right)$$

Of course the fractional uncertainties will be small (<< 1) so the last term will be the product of two small numbers and will thus be really small. Therefore, we usually ignore the last term and write the uncertainty as:

$$\frac{\Delta V}{V} = \pm\frac{\Delta K}{K} \pm \frac{\Delta G}{G} \tag{5}$$

Thus Equation (5) shows that the worst-case uncertainty would be the sum of the individual uncertainties.

Statistical analysis teaches us that it is more realistic to use the root-mean-square (rms) representation of system uncertainty. This will give an overall uncertainty somewhat less than worst-case but more likely to reflect the actual value. This is found from the relation

$$\left[\frac{\Delta V}{V}\right]_{\text{rms}} = \pm\sqrt{\left(\frac{\Delta K}{K}\right)^2 + \left(\frac{\Delta G}{G}\right)^2}$$

EXAMPLE 11

Find the system accuracy of a flow process if the transducer transfer function is $10\ \text{mV}/(\text{m}^3/\text{s}) \pm 1.5\%$ and the signal-conditioning system-transfer function is $2\ \text{mA/mV} \pm 0.5\%$.

Solution

Here we have a direct application of

$$\frac{\Delta V}{V} = \pm\left[\frac{\Delta K}{K} + \frac{\Delta G}{G}\right]$$

$$\frac{\Delta V}{V} = \pm[0.015 + 0.005]$$

$$\frac{\Delta V}{V} = \pm 0.02 = \pm 2\%$$

so that the net transfer function is 20 mA/(m^3/s) ± 2%. If we use the more statistically appropriate rms approach, the system accuracy would be

$$\left[\frac{\Delta V}{V}\right]_{\text{rms}} = \pm\sqrt{(0.015)^2 + (0.005)^2}$$
$$= \pm 0.0158$$

So the accuracy is about ±1.6%.

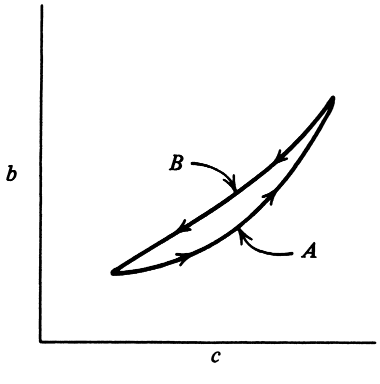
Sensitivity Sensitivity is a measure of the change in output of an instrument for a change in input. Generally speaking, high sensitivity is desirable in an instrument because a large change in output for a small change in input implies that a measurement may be taken easily. Sensitivity must be evaluated together with other parameters, such as *linearity* of output to input, *range,* and *accuracy.* The value of the sensitivity is generally indicated by the transfer function. Thus, when a temperature transducer outputs 5 mV per degree Celsius, the sensitivity is 5 mV/°C.

Hysteresis and Reproducibility Frequently, an instrument will not have the same output value for a given input in repeated trials. Such variation can be due to inherent uncertainties that imply a limit on the *reproducibility* of the device. This variation is random from measurement to measurement and is not predictable.

A similar effect is related to the history of a particular measurement taken with an instrument. In this case, a different reading results for a specific input, depending on whether the input value is approached from higher or lower values. This effect, called *hysteresis,* is shown in Figure 23, where the output of an instrument has been plotted against input. We see that if the input parameter is varied from low to high, curve *A* gives values of the output. If the input parameter is decreasing, curve *B* relates input to output. Hysteresis is usually specified as a percentage of full-scale maximum deviation between the two curves. This effect is predictable if measurement values are always approached from one direction, because hysteresis will not cause measurement errors.

Resolution Inherent in many measurement devices is a minimum measurable value of the input variable. Such a specification is called the *resolution* of the device. This

FIGURE 23
Hysteresis is a predictable error resulting from differences in the transfer function as the input variable increases or decreases.



characteristic of the instrument can be changed only by redesign. A good example is a wire-wound potentiometer in which the slider moves across windings to vary resistance. If one turn of the winding represents a change of ΔR ohms, then the potentiometer cannot provide a resistance change *less* than ΔR. We say that the potentiometer resolution is ΔR. This is often expressed as a percentage of the full-scale range.

EXAMPLE 12 A force sensor measures a range of 0 to 150 N with a resolution of 0.1% FS. Find the smallest change in force that can be measured.

Solution

Because the resolution is 0.1% FS, we have a resolution of $(0.001)(150\text{ N}) = 0.15\text{ N}$, which is the smallest measurable change in force.

In some cases, the resolution of a measurement system is limited by the sensitivity of associated signal conditioning. When this occurs, the resolution can be improved by employing better conditioning.

EXAMPLE 13 A sensor has a transfer function of 5 mV/°C. Find the required voltage resolution of the signal conditioning if a temperature resolution of 0.2°C is required.

Solution

A temperature change of 0.2°C will result in a voltage change of

$$\left(5\frac{\text{mV}}{^\circ\text{C}}\right)(0.2^\circ\text{C}) = \mathbf{1.0\ mV}$$

Thus, the voltage system must be able to resolve 1.0 mV.

In analog systems, the resolution of the system is usually determined by the smallest measurable change in the analog output signal of the measurement system. In digital systems, the resolution is a well-defined quantity that is simply the change in dynamic variable represented by a *1-bit change* in the binary word output. In these cases, resolution can be improved only by a different coding of the analog information or adding more bits to the word.

Linearity In both sensor and signal conditioning, output is represented in some functional relationship to the input. The only stipulation is that this relationship be unique; that is, for each value of the input variable, there exists one unique value of the output variable. For simplicity of design, a linear relationship between input and output is highly desirable. When a linear relationship exists, a straight-line equation can be used to relate the measured variable and measurement output

$$c_m = mc + c_0 \tag{6}$$

where c = variable to be measured
m = slope of straight line
c_0 = offset or intercept of straight line
c_m = output of measurement

No simple relationship such as Equation (6) can usually be found for the nonlinear cases, although in some cases approximations of a linear or quadratic nature are fitted to portions of these curves.

EXAMPLE 14 A sensor resistance changes linearly from 100 to 180 Ω as temperature changes from 20° to 120°C. Find a linear equation relating resistance and temperature.

Solution

Using Equation (6) as a guide, the desired equation would be of the form

$$R = mT + R_0$$

To find the two constants, m and R_0, we form two equations and two unknowns from the facts given:

$$\begin{aligned} 100\ \Omega &= (20^\circ\text{C}\ m + R_0) \\ 180\ \Omega &= (120^\circ\text{C}\ m + R_0) \end{aligned}$$

Subtracting the first equation from the second gives

$$80\ \Omega = (100^\circ\text{C})m \quad \text{or} \quad m = 0.8\ \Omega/^\circ\text{C}$$

Then, from the first equation, we find

$$100\ \Omega = (20^\circ\text{C})(0.8\ \Omega/^\circ\text{C}) + R_0$$

from which

$$R_0 = 84\ \Omega$$

The equation relating resistance and temperature is

$$R = 0.8T + 84$$

One of the specifications of sensor output is the degree to which it is linear with the measured variable and the span over which this occurs. A measure of sensor linearity is to determine the deviation of the sensor output from a best-fit straight line over a particular range. A common specification of linearity is the maximum deviation from a straight line expressed as percent of FS.

Consider a sensor that outputs a voltage as a function of pressure from 0 to 100 psi with a linearity of 5% FS. This means that, at some point on the curve of voltage versus pressure, the deviation between actual pressure and linearly indicated pressure for a given voltage deviates by 5% of 100 psi, or 5 psi.

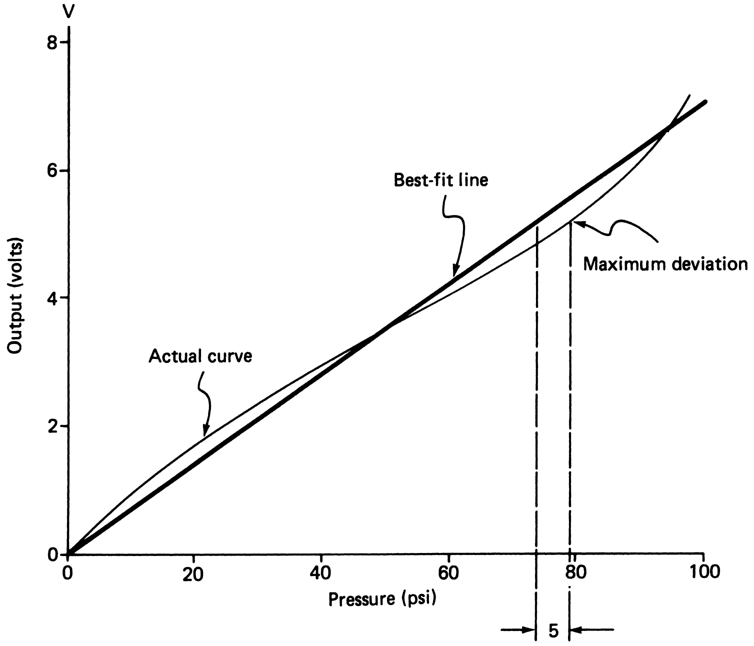


FIGURE 24
Comparison of an actual curve and its best-fit straight line, where the maximum deviation is 5% FS.

Figure 24 indicates this graphically. A straight line has been fitted to the slightly nonlinear sensor curve. One can either specify that, for a given voltage, there is a deviation between actual and linearly predicted pressure or that, for a given pressure, there is a deviation between actual and linearly predicted voltage.

6.4 Process-Control Drawings

An electrical schematic is a drawing that employs a standard set of symbols and definitions so that anyone who knows the standards can understand the operation of the circuit. In just the same way, process control employs a standard set of symbols and definitions to represent a plant and its associated control systems. This standard was developed and approved by a collaboration between the American National Standards Institute (ANSI) and the Instrumentation, Systems, and Automation (ISA) society. The standard is designated ANSI/ISA S5.1-1984 (R1992) Instrumentation Symbols and Identification.

This standard is used for the early design phases of a plant control system to construct simplified process-control diagrams and then to render the final, detailed plant design as a piping and instrumentation diagram (P&ID).

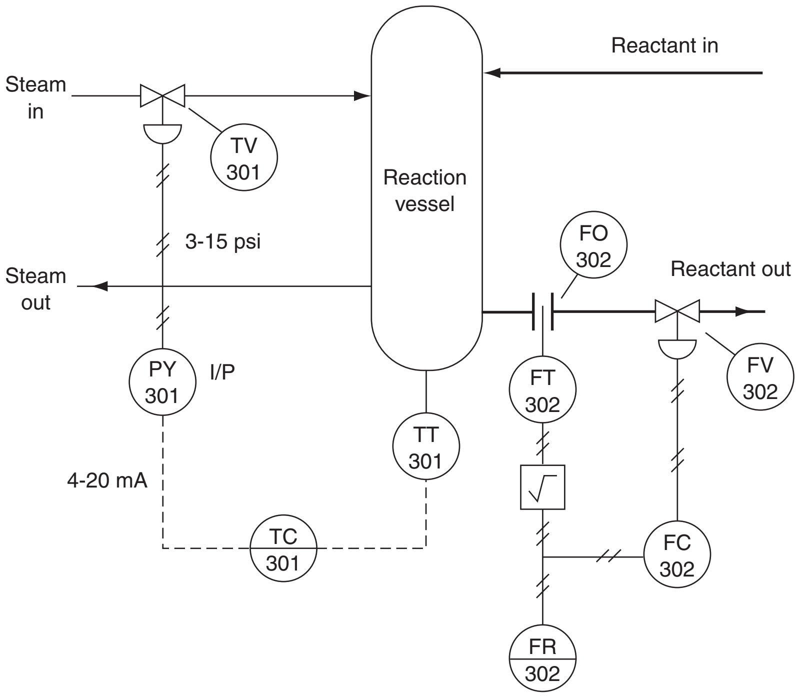


FIGURE 25
A P&ID uses special symbols and lines to show the devices and interconnections in a process-control system.

Essential Elements The P&ID depicts the entire plant and associated control systems. This includes plant operating units, product flow lines, measurement and control signal lines, sensors, controllers, final control elements, computers, and programmable logic controllers (PLCs). Also included are letter and number designations to identify the function of an element and notes to further explain features of the P&ID. Let's use the P&IDs shown in Figures 25 and 26 to summarize some of these features.

Figure 25 shows a section of a plant with a reaction vessel into and out of which a reactant (heavy line) flows. The vessel is heated by steam input. Temperature within the vessel is controlled by controlling the steam input, and a flow control system regulates the reactant flow out of the vessel.

Figure 26 shows a system in which the level in a tank is controlled using a cascade control system. In a cascade control system, the setpoint of one loop is the controller output of another loop. The system in Figure 26 uses computers to provide the controller operations. Again, the primary product flow is shown as a heavy line.

Instrument Lines Symbols Figure 25 shows that the standard 4- to 20-mA current signal is represented as a dashed line in the P&ID, while the pneumatic signal

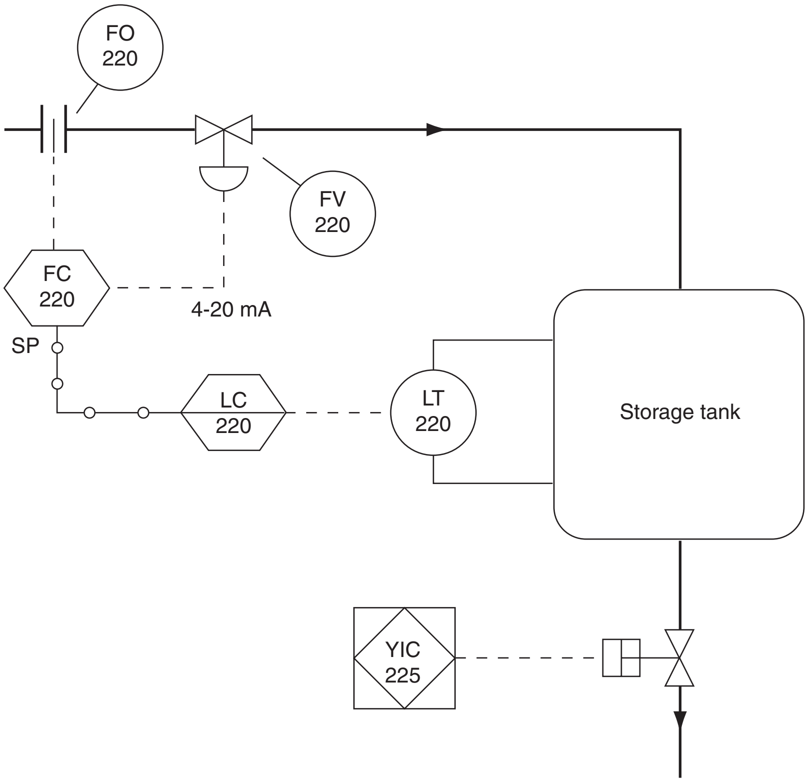


FIGURE 26
Computers and programmable logic controllers are included in the P&ID.

(e.g., 3 to 15 psi) is presented as a line with crosshatches. Figure 26 shows electrical (current) lines and also the digital data feed from a computer as a solid line with small bubbles.

Instrument Symbols The instrumentation associated with control systems varies from sensors and transmitters to controllers, computers, and PLCs. These are drawn as bubbles with or without rectangles. In general, the instrument symbol will be identified by a letter code, which denotes its function, and by a number code assigned by the designers, which may identify the loop or some region of the plant.

Figure 25 shows that the temperature control loop has a temperature sensor and transmitter designated by TT/301, which is connected to a temperature controller, TC/301. The solid line through the controller bubble means it is accessible by an operator, as in a control room panel. When the setpoint is not indicated, that means it is manually set. The controller is connected to an I/P converter designated by PY/301. The flow control loop is all pneumatic and has a flow transmitter (FT/302), a flow controller (FC/302), and a flow recorder (FR/302), which is located in the control room. Note that the sensor actually measures pressure and that the flow is proportional to the square root of pressure. Therefore, a square-root extractor is indicated following the flow transmitter. The flow recorder charts variations of flow rate for an operator.

Figure 26 shows a flow loop under computer control. This flow control computer, FC 220, is located in the field (i.e., at the site of control). This is an all-electrical loop that inputs a 4- to 20-mA signal for flow measurement and outputs a 4- to 20-mA signal to the control valve, FV/220. A level transmitter, LT/220, provides input to a computer, LC/220, which is located in the control room. The output of this computer is the setpoint of the flow loop. Finally, a PLC, shown as a diamond within a box, YIC/225, can open or close the tank drain valve.

Other Symbols Figures 25 and 26 also show the symbols for several other el ements associated with the control systems and plants. For example, the control valves are often identified in terms of what they control. In Figure 25, the steam valve is labeled TV/301 to indicate that it is for temperature control even though it is actually controlling steam flow. Both figures show an orifice plate used to measure flow through a pipe. The valve actuator shown in Figure 26 with the PLC indicates it is spring-loaded and electrically actuated.

7 SENSOR TIME RESPONSE

The static transfer function of a process-control loop element specifies how the output is related to the input if the input is constant. An element also has a time dependence that specifies how the output changes in time when the input is changing in time. It is independent from the static transfer function. This dynamic transfer function, which is independent from the static transfer function, is often simply called the *time response.* It is particularly important for sensors because they are the primary element for providing knowledge of the controlled variable value. This section will discuss the two most common types of sensor time responses.

Figure 27 shows a sensor that produces an output, $b(t)$, as a function of the input, $c(t)$. The static transfer function determines the output when the input is not changing in time. To specify the time response, the nature of the time variation in output, $b(t)$, is given

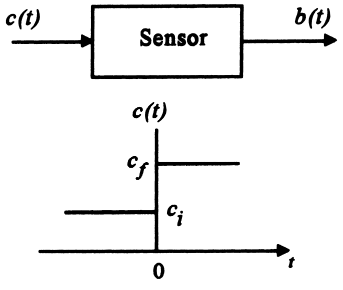
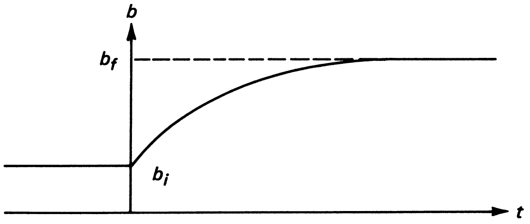


FIGURE 27
The dynamic transfer function specifies how a sensor output varies when the input changes instantaneously in time (i.e., a step change).

FIGURE 28
Characteristic first-order exponential time response of a sensor to a step change of input.



when the input exhibits a step change as shown. Note that at $t = 0$ the input to the sensor is suddenly changed from an initial value, c_i, to a final value, c_f. If the sensor were perfect, its output would be determined by the static transfer function to be b_i before $t = 0$ and b_f after $t = 0$. However, all sensors exhibit some lag between the output and the input and some characteristic variation in time before settling on the final value.

7.1 First-Order Response

The simplest time response is shown in Figure 28 as the output change in time following a step input as in Figure 27. This is called *first order* because, for all sensors of this type, the time response is determined by the solution of a first-order differential equation.

A general equation can be written for this response independent of the sensor, the variable being measured, or the static transfer function. The equation gives the sensor output as a function of time following the step input (i.e., it traces the curve of Figure 28 in time):

$$b(t) = b_i + (b_f - b_i)[1 - e^{-t/\tau}] \tag{7}$$

where b_i = initial sensor output from static transfer function and initial input
b_f = final sensor output from static transfer function and final input
τ = sensor time constant

The sensor output is in error during the transition time of the output from b_i to b_f. The actual variable value was changed instantaneously to a new value at $t = 0$. Equation (7) describes transducer output very well except during the initial time period—that is, at the *start* of the response near $t = 0$. In particular, the actual transducer response generally starts the change with a *zero* slope, and Equation (7) predicts a finite starting slope.

Time Constant The time constant, τ, is part of the specification of the sensor. Its significance can be seen by writing Equation (7) as follows

$$b(t) - b_i = (b_f - b_i)[1 - e^{-t/\tau}] \tag{8}$$

In this equation, the quantity on the left is the *change* in output as a function of time, whereas $(b_f - b_i)$ is the total change that will occur. Thus, the square-bracketed term in Equation (8) is the fraction of total change as a function of time.

Suppose we wish to find the change that has occurred at a time numerically equal to τ. Then we set $t = \tau$ in Equation (8) and find

$$\begin{aligned} b(\tau) - b_i &= (b_f - b_i)[1 - e^{-1}] \\ b(\tau) - b_i &= 0.6321(b_f - b_i) \end{aligned} \tag{9}$$

Thus, we see that one time constant represents the time at which the output value has changed by approximately 63% of the total change.

The time constant τ is sometimes referred to as the 63% time, the *response time,* or the e-folding time. For a step change, the output response has approximately reached its final value after five time constants, since from Equation (8) we find

$$b(5\tau) - b_i = 0.993(b_f - b_i)$$

EXAMPLE 15

A sensor measures temperature linearly with a static transfer function of 33 mV/°C and has a 1.5-s time constant. Find the output 0.75 s after the input changes from 20° to 41°C. Find the error in temperature this represents.

Solution

We first find the initial and final values of the sensor output:

$$\begin{aligned} b_i &= (33\,\text{mV}/^\circ\text{C})(20^\circ\text{C}) \\ b_i &= 660\,\text{mV} \\ b_f &= (33\,\text{mV}/^\circ\text{C})(41^\circ\text{C}) \\ b_f &= 1353\,\text{mV} \end{aligned}$$

Now,

$$\begin{aligned} b(t) &= b_i + (b_f - b_i)[1 - e^{-t/\tau}] \\ b(0.75) &= 660 + (1353 - 660)[1 - e^{-0.75/1.5}] \\ b(0.75) &= \mathbf{932.7\ mV} \end{aligned}$$

This corresponds to a temperature of

$$\begin{aligned} T &= \frac{932.7}{33\,\text{mV}/^\circ\text{C}} \\ T &= \mathbf{28.3^\circ C} \end{aligned}$$

Thus the indicated temperature differs from the actual temperature by -12.7°C because of the lag in sensor output. After a time of about five times constants (~7.5 s) the sensor output would be about 1.353 V, correctly indicating the actual temperature of 41°C.

In many cases, the transducer output may be inversely related to the input. Equation (7) still describes the time response of the element where the final output is less than the initial output.

Note that the time response analysis is *always* applied to the output of the sensor, never the input. In Example 15 the temperature changed suddenly from 20° to 41°C, so it would be wrong to write equations for the temperature in terms of first-order time response. It is only the *output* of the sensor that lagged. Particularly if the sensor output varies nonlinearly with its input, terrible errors will occur if the time response equation is applied to the input.

Real-Time Effects The concept of the exponential time response and associated time constant is based on a sudden discontinuous change of the input value. In the real world, such instantaneous changes occur rarely, if ever, and thus we have presented a worst-case situation in the time response. In general, a sensor should be able to track any changes in the physical dynamic variable in a time less than one time constant.

7.2 Second-Order Response

In some sensors, a step change in the input causes the output to oscillate for a short period of time before settling down to a value that corresponds to the new input. Such oscillation (and the decay of the oscillation itself) is a function of the sensor. This output transient generated by the transducer is an error and must be accounted for in any measurement involving a transducer with this behavior.

This is called a *second-order response* because, for this type of sensor, the time behavior is described by a second-order differential equation. It is not possible to develop a universal solution, as it is for the first-order time response. Instead, we simply describe the general nature of the response.

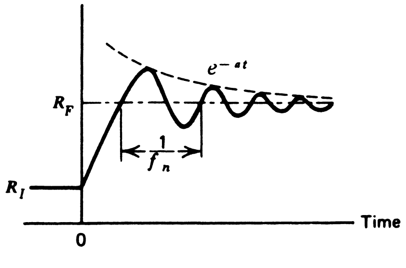
Figure 29 shows a typical output curve that might be expected from a transducer having a second-order response for a discontinuous change in the input. It is impossible to describe this behavior by a simple analytic expression, as it is with the first-order response. However, the general behavior can be described in time as

$$R(t) \propto R_0 e^{-at} \sin(2\pi f_n t) \qquad \textbf{(10)}$$

where $R(t)$ = the transducer output
a = output damping constant
f_n = natural frequency of the oscillation
R_0 = amplitude

This equation shows the basic damped oscillation output of the device. The damping constant, a, and natural frequency, f_n, are characteristics of the transducer itself and must be considered in many applications.

FIGURE 29
Characteristic second-order oscillatory time response of a sensor.



In general, such a transducer can be said to track the input when the input changes in a time that is *greater* than the period represented by the natural frequency. The damping constant defines the time one must wait after a disturbance at $t = 0$ for the transducer output to be a true indication of the transducer input. Thus, we see that in a time of $(1/a)$, the amplitude of the oscillations would be down to e^{-1}, or approximately 37%. More will be said about the effects of natural frequency and damping in the treatment of specific transducers that exhibit this behavior.

8 SIGNIFICANCE AND STATISTICS

Process control is vitally concerned with the value of variables, as the stated objective is to regulate the value of selected variables. It is therefore very important that the true significance of some measured value be understood. We have already seen that inherent errors and accuracy may lend uncertainty to the value indicated by a measurement. In this section we need to consider another feature of measurement that may be misleading about the actual value of a variable, as well as a method to help interpret the significance of measurements.

8.1 Significant Figures

In any measurement, we must be careful not to attach more significance to a variable value than the instrument can support. This is particularly true with the growing use of digital reading instruments and calculators with 8 to 12-digit readouts. Suppose, for example, that a digital instrument measures a resistance as 125 kΩ. Even if we ignore the instrument accuracy, this does not mean that the resistance is 125, 000 Ω. Rather, it means that the resistance is closer to 125,000 than it is to 124,000 or to 126, 000 Ω. We can use the 125 kΩ number in subsequent calculations, but we cannot draw conclusions about results having more than three numbers—that is, three significant figures. The significant figures are the digits (places) actually read or known from a measurement or calculation.

Significance in Measurement When using a measuring instrument, the number of significant figures is indicated either by readability, in the case of analog instruments, or by the number of digits, in a digital instrument. This is not to be confused with accuracy, which supplies an uncertainty to the reading itself. The following example illustrates how significant figures in measurement and accuracy are treated in the same problem.

EXAMPLE 16 A digital multimeter measures the current through a 12.5-kΩ resistor as 2.21 mA, using the 10-mA scale. The instrument accuracy is ±0.2% FS. Find the voltage across the resistor and the uncertainty in the value obtained.

Solution

First, we note that current is given to three significant figures, so no result we find can be significant to more than three digits. Then we see that the given accuracy becomes an uncertainty in the current of ±0.02 mA. From Ohm's law, we find the voltage as

$$V = IR = (2.21\text{ mA})(12.5\text{ k}\Omega) = 27.625\text{ V}$$

But in terms of significant figures, we give this as $V = 27.6$ V. The accuracy means the current could vary from 2.19 to 2.23 mA, which introduces an uncertainty of ±0.25 V. Thus, the complete answer is 27.6 ± 0.3 V, because we must express the uncertainty so that our significance is not changed.

Significance in Calculations In calculations, one must be careful not to obtain a result that has more significance than the numbers employed in the calculation. The answer can have no more significance than the least of the numbers used in the calculation.

EXAMPLE 17

A transducer has a specified transfer function of 22.4 mV/°C for temperature measurement. The measured voltage is 412 mV. What is the temperature?

Solution

Using the values given, we find

$$T = (412\,\text{mV})/(22.4\,\text{mV}/°\text{C}) = 18.392857°\text{C}$$

This was found using an 8-digit calculator, but the two given values are significant to only three places. Thus, our result can be significant to only three places; the answer is **18.4°C**.

Significance in Design The reader should be aware of [the difference in significant figures associated with measurement and conclusions drawn from measurement and significant figures associated with design.] A design is a hypothetical development that makes implicit assumptions about selected values in the design. If the designer specifies a 1.1-kΩ resistor, the assumption is that it is exactly 1100 Ω. If the designer specifies that there are 4.7 V across the resistor, then there are exactly 4.7 V and the current can be calculated as 4.2727272 mA. Now, suppose we measure the resistor when the design is built and find it to be 1.1 kΩ (two significant figures) and measure the voltage and find it to be 4.7 V (two significant figures). In this case, we report the calculated current of 4.3 mA, because we are dealing with two significant figures.

In the examples and problems in this book, we try to maintain the distinction between design values and measurement values. Whenever a problem or example involves design, perfect values are assumed. Thus, if a design specifies a 4.7-kΩ resistor, it is assumed that the value is exactly 4.7 kΩ (i.e., 4700.00 Ω). Whenever measurement is suggested, the figures given are assumed to be the significant figures. If a problem specifies that the measured voltage is 5.0, it is assumed that it is known to only two significant figures.

8.2 Statistics

Often, confidence in the value of a variable can be improved by elementary statistical analysis of measurements. This is particularly true where random errors in measurement cause a distribution of readings of the value of some variable.

Arithmetic Mean If many measurements of a particular variable are taken, the arithmetic mean is calculated to obtain an average value for the variable. There are many instances in process control when such an average value is of interest. For example, one may wish to control the average temperature in a process. The temperature might be measured in 10 locations and averaged to give a controlled variable value for use in the control loop.

Another common application is the calibration of transducers and other process instruments. In such cases, the average gives information about the transfer function. In digital or computer process control, it is often easier to use the average value of process variables. The arithmetic mean of a set of n values, given by $x_1, x_2, x_3, \ldots, x_n$ is defined by the equation

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \tag{11}$$

where $\bar{x}$ = arithmetic mean
n = number of values to be averaged
$x_1, x_2, \ldots, x_n$ = individual values

We often use the symbol Σ to represent a sum of numbers such as that used in Equation (11). Here we would write the equation as

$$\bar{x} = \frac{\Sigma x_i}{n} \tag{12}$$

where Σx_i = symbol for a sum of the values $x_1, x_2, \ldots, x_n$

Standard Deviation Often it is insufficient to know the value of the arithmetic mean of a set of measurements. To interpret the measurements properly, it may be necessary to know something about how the individual values are spread out about the mean. Thus, although the mean of the set (50, 40, 30, 70) is 47.5 and the mean of the set (5, 150, 21, 14) is also 47.5, the second group of numbers is obviously far more spread out. The standard deviation is a measure of this spread. Given a set of n values $x_1, x_2, \ldots, x_n$, we first define a set of deviations by the difference between the individual values and the arithmetic mean of the values, $\bar{x}$. The deviations are

$$d_1 = x_1 - \bar{x}$$
$$d_2 = x_2 - \bar{x}$$

and so on, until

$$d_n = x_n - \bar{x}$$

The set of these n deviations is now used to define the standard deviation according to the equation

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n - 1}} \tag{13}$$

or, using the summation symbol,

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n - 1}} \tag{14}$$

Of course, the larger the standard deviation, the more spread out the numbers from which it is calculated.

EXAMPLE 18 Temperature was measured in eight locations in a room, and the values obtained were 21.2°, 25.0°, 18.5°, 22.1°, 19.7°, 27.1°, 19.0°, and 20.0°C. Find the arithmetic mean of the temperature and the standard deviation.

Solution

Using Equation (11), we have

$$\overline{T} = \frac{21.2 + 25 + 18.5 + 22.1 + 19.7 + 27.1 + 19 + 20}{8}$$

$$\overline{T} = \mathbf{21.6^\circ C}$$

The standard deviation is found from Equation (1.13):

$$\sigma = \sqrt{\frac{(21.2 - 21.6)^2 + (25 - 21.6)^2 + \cdots + (20 - 21.6)^2}{(8 - 1)}}$$

$$\sigma = \mathbf{3.04^\circ C}$$

Interpretation of Standard Deviation Figure 30 shows two curves constructed from many samples of two sensors measuring some variable with a fixed value, *V*. Due to sensor uncertainty the sampled values provided by the sensors exhibit variation about the average; however, both sensors are providing the correct value as the average. Notice that the distribution of readings for sensor *A* is much more narrowly distributed around the average than sensor *B*. This means that any single reading from sensor *A* is more likely to give the actual value of the measured variable. The standard deviation of the readings from sensor *A* would be much smaller than the standard deviation of sensor *B*.

A more quantitative evaluation of spreading can be made if we make certain assumptions about the set of data values used. In particular, we assume that the errors are truly

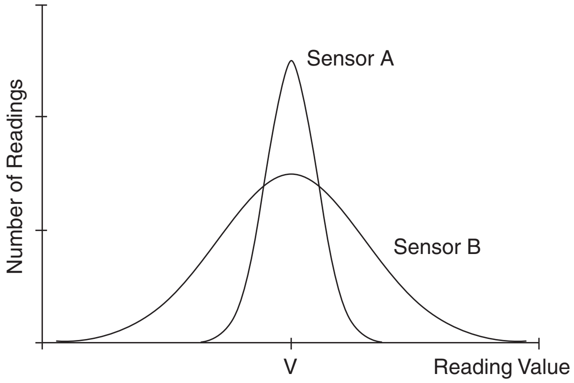


FIGURE 30
Multiple readings are taken of some variable with an actual value, *V*. The distributions show that sensor A has a smaller standard deviation than sensor B.

random and that we have taken a large sample of readings. We then can claim that the standard deviation and data are related to a special curve called the *normal probability curve,* or *bell curve.* If this is true, then

1. 68% of all readings lie within $\pm 1\sigma$ of the mean.
2. 95.5% of all readings lie within $\pm 2\sigma$ of the mean.
3. 99.7% of all readings lie within $\pm 3\sigma$ of the mean.

This gives us the added ability to make quantitative statements about how the data are spread about the mean. Thus, if one set of pressure readings has a mean of 44 psi with a standard deviation of 14 psi and another a mean of 44 psi with a standard deviation of 3 psi, we know the latter is much more peaked about the mean. In fact, 68% of all the readings in the second case lie from 41 to 47 psi, whereas in the first case, 68% of readings lie from 30 to 58 psi.

EXAMPLE 19 A control system was installed to regulate the weight of potato chips dumped into bags in a packaging operation. Given samples of 15 bags drawn from the operation before and after the control system was installed, evaluate the success of the system. Do this by comparing the arithmetic mean and standard deviations before and after. The bags should be 200 g.

Samples before: 201, 205, 197, 185, 202, 207, 215, 220, 179, 201, 197, 221, 202, 200, 195

Samples after: 197, 202, 193, 210, 207, 195, 199, 202, 193, 195, 201, 201, 200, 189, 197

Solution

In the before case, we use Equations (12) and (14) to find the mean and standard devi ation, and get

$$\begin{aligned} \overline{W}_b &= 202\,\text{g} \\ \sigma_b &= 11\,\text{g} \end{aligned}$$

Now the mean and standard deviations are found for the after case:

$$\begin{aligned} \overline{W}_a &= 199\,\text{g} \\ \sigma_a &= 5\,\text{g} \end{aligned}$$

Thus, we see that the control system has brought the average bag weight closer to the ideal of 200 g and that it has cut the spread by a factor of 2. In the before case, 99% of the bags weighed 202 ± 33 g, but with the control system, 99% of the bags weighed in the range of 199 ± 15 g.

SUMMARY

This chapter presents an overview of process control and its elements. Subsequent chapters will examine these topics in more detail and provide a more quantitative understanding.

The following list will help the reader master the key points of the chapter.

1. Process control itself has been described as suitable for application to any situation in which a variable is regulated to some desired value or range of values. Figure 5 shows a block diagram in which the elements of measurement, error detector, controller, and control element are connected to provide the required regulation.
2. Numerous criteria have been discussed that allow the evaluation of process-control loop performance, of which the settling time, peak error, and minimum area are the most indicative of loop characteristics.
3. Both analog and digital processing are used in process-control applications. The current trend is to make analog measurements of the controlled variable, digitize them, and use a digital controller for evaluation. The basic technique of digital encoding allows each bit of a binary word to correspond to a certain quantity of the measured variable. The arrangement of “0” and “1” states in the word then serves as the encoding.
4. The SI system of units forms the basis of computations in this book as well as in the process-control industry in general. However, it is still necessary to understand conversions to other systems, notably the English system (see Appendix: Units).
5. A standard adopted for analog process-control signals is the 4- to 20-mA current range to represent the span of measurements of the dynamic variable.
6. The definitions of accuracy, resolution, and other terms used in process control are necessary and are similar to those in related fields.
7. The concept of transducer time response was introduced. The time constant becomes part of the dynamic properties of a transducer.
8. The use of significant figures is important to properly interpret measurements and conclusions drawn from measurements.
9. Statistics can help interpret the validity of measurements through the use of the arithmetic mean and the standard deviation.
10. P&ID drawings and symbols are the typical representation used to display process-control systems.

PROBLEMS

Section 2

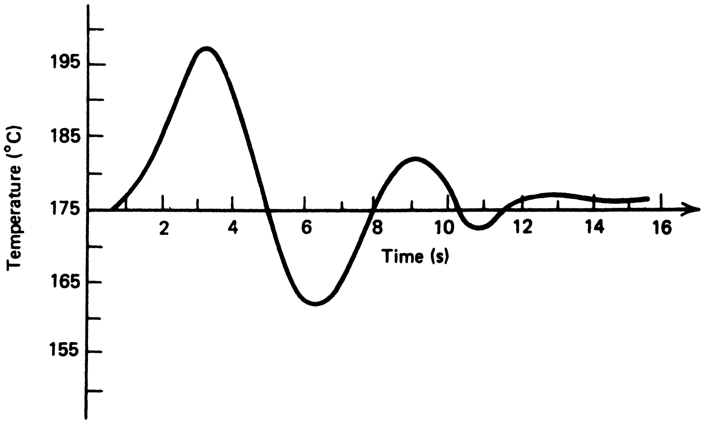
1 Explain how the basic strategy of control is employed in a room air-conditioning system. What is the controlled variable? What is the manipulated variable? Is the system self-regulating?

2 Is driving an automobile best described as a servomechanism or a process-control system? Why?

Section 3

3 Construct a block diagram of a refrigerator control system. Define each block in terms of the refrigerator components. (If you do not know the components, look them up in an encyclopedia or the Internet.)

FIGURE 31
Figure for Problem 4.



Section 4

4 A process-control loop has a setpoint of 175°C and an allowable deviation of ±5°C. A transient causes the response shown in Figure 31. Specify the maximum error and settling time.

5 Two different tunings of a process-control loop result in the transient responses shown in Figure 32. Estimate which would be preferred to satisfy the minimum area criteria.

6 The second cyclic transient error peak of a response test measures 4.4%. For the quarter-amplitude criteria, what error should be the third peak value?

7 Does the response of Figure 31 satisfy the quarter-amplitude criterion?

Section 5

8 An analog sensor converts flow linearly so that flow from 0 to 300 m^3/h becomes a current from 0 to 50 mA. Calculate the current for a flow of 225 m^3/h.

9 What binary word would represent the decimal number 16 if Table 1 were continued?

10 Suppose each bit change in a 4-bit ADC represents a level of 0.15 m.

- **a.** What would the 4 bits be for a level of 1.7 m?
- **b.** Suppose the 4 bits were $\mathbf{1000_2}$. What is the range of possible levels?

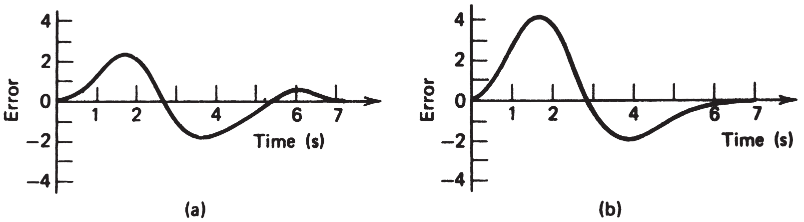


FIGURE 32
Figure for Problem 5.

11 For the process-control system in Figure 13, suppose that the relays close at $|1.5|$ V and open at $|1.1|$ V. This means that as the voltage on the relay reaches ± 1.5 V, it closes, and does not open again until the voltage drops to 1.1 V (i.e., there is a deadband). The amplifier has a gain of 10, the reference is 3 V, and the sensor outputs 150 mV/°C. Calculate the temperatures at which the heater turns on and off and at which the cooler turns on and off.

12 Show how the control system in Figure 6 would be modified to use **(a)** supervisory computer control and **(b)** DDC or computer control.

13 Think about how you adjust the water temperature coming from a single nozzle using the hot and cold hand valves in a kitchen faucet. Construct the block diagram of an automatic system as follows: The desired water temperature is selected by the user, perhaps by a knob and LCD readout. One hand valve turns on the cold water. The hot water valve is automatically set to keep the temperature at the selected value. Describe what elements would be necessary to do this using **(a)** analog control and **(b)** computer control.

Section 6

14 What is your mass in kilograms? What is your height in meters?

15 Atmospheric pressure is about 14.7 lb/in.^{2} (psi). What is this pressure in pascals?

16 An accelerometer is used to measure the constant acceleration of a race car that covers a quarter mile in 7.2 s.

- **a.** Using $x = at^2/2$ to relate distance, *x*, acceleration, *a*, and time, *t*, find the acceleration in ft/s^2.
- **b.** Express this acceleration in m/s^2.
- **c.** Find the car speed, *v*, in m/s at the end of the quarter mile using the relation $v^2 = 2ax$.
- **d.** Find the car energy in joules at the end of the quarter mile if it weighs 2000 lb, where the energy $W = mv^2/2$.

17 Suppose a liquid level ranging from 5.5 to 8.6 m is linearly converted to pneumatic pressure ranging from 3 to 15 psi. What pressure will result from a level of 7.2 m? What level does a pressure of 4.7 psi represent?

18 A controller output is a 4- to 20-mA signal that drives a valve to control flow. The relation between current and flow is $Q = 45[I - 2\text{ mA}]^{1/2}$ gal/min. What is the flow for 12 mA? What current produces a flow of 162 gal/min?

19 An instrument has an accuracy of $\pm 0.5\%$ FS and measures resistance from 0 to 1500 Ω. What is the uncertainty in an indicated measurement of 397 Ω?

20 A sensor has a transfer function of 0.5 mV/°C and an accuracy of $\pm 1\%$. If the temperature is known to be 60°C, what can be said with absolute certainty about the output voltage?

21 The sensor in Problem 20 is used with an amplifier with a gain of 15 ± 0.25 and displayed on a meter with a range of 0 to 2 V at $\pm 1.5\%$ FS. What is the worst-case and rms uncertainty for the total measurement?

22 Using the nominal transfer function values, what is the maximum measurable temperature of the system in Problems 20 and 21?

23 A temperature sensor transfer function is 44.5 mV/°C. The output voltage is measured at 8.86 V on a 3-digit voltmeter. What can you say about the value of the temperature?

24 A level sensor inputs a range from 4.50 to 10.6 ft and outputs a pressure range from 3 to 15 psi. Find an equation such as Equation (6) between level and pressure. What is the pressure for the level of 9.2 ft?

25 Draw Figure 6 in the standard P&ID symbols.

Section 7

26 A temperature sensor has a static transfer function of $0.15 \text{ mV}/°\text{C}$ and a time constant of 3.3 s. If a step change of 22° to 50°C is applied at $t = 0$, find the output voltages at 0.5 s, 2.0 s, 3.3 s, and 9 s. What is the *indicated* temperature at these times?

27 A pressure sensor measures 44 psi just before a sudden change to 70 psi. The sensor measures 52 psi at a time 4.5 s after the change. What is the sensor time constant?

28 A photocell with a 35-ms time constant is used to measure light flashes. How long after a sudden dark-to-light flash before the cell output is 80% of the final value?

29 An alarm light goes ON when a pressure sensor voltage rises above 4.00 V. The pressure sensor outputs 20 mV/kPa and has a time constant of 4.9 s. How long after the pressure rises suddenly from 100 kPa to 400 kPa does the light go ON?

30 A pressure sensor has a resistance that changes with pressure according to $R = (0.15 \text{ k}\Omega/\text{psi})p + 2.5 \text{ k}\Omega$. This resistance is then converted to a voltage with the transfer function

$$V = \frac{10R}{R + 10k} \text{ volts}$$

The sensor time constant is 350 ms. At $t = 0$, the pressure changes suddenly from 40 psi to 150 psi.

a. What is the voltage output at 0.5 s? What is the indicated pressure at this time?
b. At what time does the output reach 5.0 V?

31 At $t = 0$, a temperature sensor was suddenly changed from 25° to 100°C. The sensor outputs voltage given by the expression $V = (0.06 \text{ V}/°\text{C})\,[T - 20°\text{C}]$. The following table gives the voltages measured and the times. Determine the average time constant of the sensor.

t (seconds)	0	0.1	0.2	0.3	0.4	0.5
V (volts)	0.3	1.8	2.8	3.4	3.9	4.2

Section 8

32 A circuit design calls for a 1.5-kΩ resistor to have 4.7 V across its terminals. What would be the expected current? The circuit is built, and the resistance is measured at 1500 Ω and the voltage at 4.7 V. What is the current through the resistor?

33 Flow rate was monitored for a week, and the following values were recorded as gal/min: 10.1, 12.2, 9.7, 8.8, 11.4, 12.9, 10.2, 10.5, 9.8, 11.5, 10.3, 9.3, 7.7, 10.2, 10.0, and 11.3. Find the mean and the standard deviation for these data.

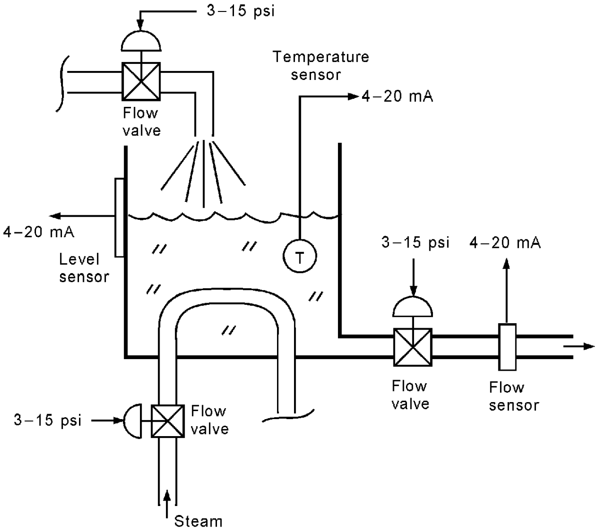


FIGURE 33
Figure for Problem S1.

34 A manufacturer specification sheet lists the transfer function of a pressure sensor as 45 ± 5% mV/kPa with a time constant of 4 ± 10% s. A highly accurate test system applies a step change of pressure from 20 kPa to 100 kPa.

a. What is the range of sensor voltage outputs initially and finally?

b. What range of voltages would be expected to be measured 2 s after the step change is applied?

SUPPLEMENTARY PROBLEMS

S1 Figure 33 shows a manufacturing process diagram. In this process, the following independent control requirements must be satisfied:

a. Control the level at L_{sp}.

b. Control the temperature at T_{sp}.

c. Control the output flow rate at Q_{sp}.

Complete the diagram showing the control loops by using the block diagram error-detector symbols and controller blocks. Include blocks for necessary signal converters.

S2 Prepare a process-control drawing of the system shown in Figure 33 similar to that shown in Figure 25. Be sure to include signal conversions.

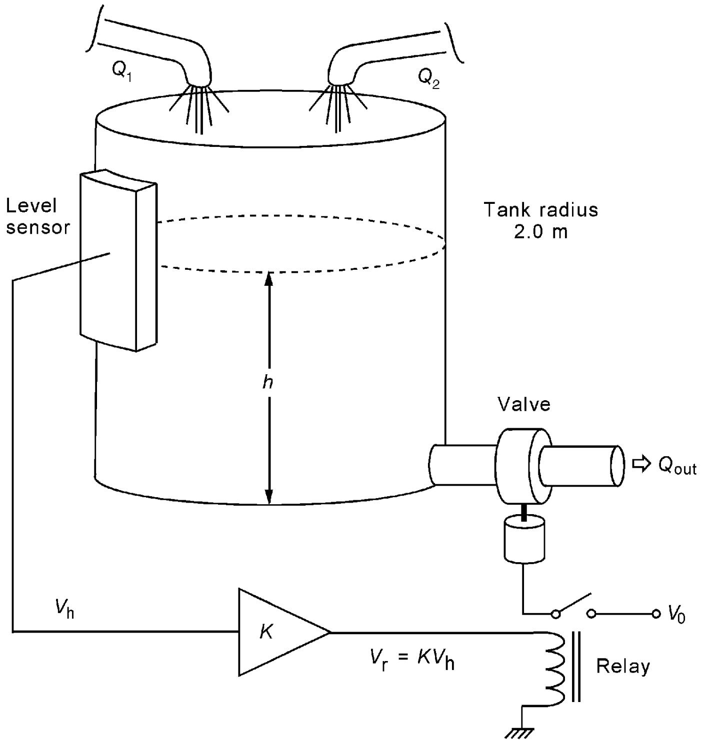


FIGURE 34
Figure for Problem S5.

S3 Explain which of the control loops in Figure 33 are self-regulating; give reasons why and why not.

S4 Assume control systems are in place to provide the requirements of Problem S1. Now, suppose the output-control valve suddenly sticks in the closed state. Explain why the tank will not overflow.

S5 Figure 34 shows a simple level-control system in which a closed relay opens the valve and an open relay closes the valve. Input flow is not controlled. The relay closes at 6.0 V and opens again at 4.8 V. The level sensor has a transfer function of $V_h = 0.8h + 0.4$ V.

a. Find the value of amplifier gain, *K,* required to open the valve when the level reaches 1.5 m.

b. At what level does the valve close?

c. Suppose $Q_1 = 5\ \mathrm{m^3/min}$, $Q_2 = 2\ \mathrm{m^3/min}$, and $Q_{out} = 9\ \mathrm{m^3/min}$ (when open). What is the period of the level oscillation?

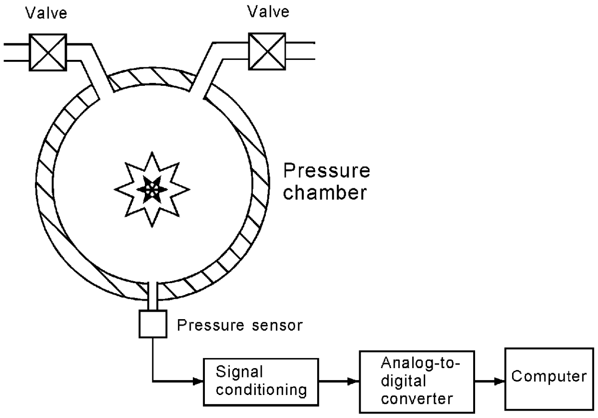


FIGURE 35
Figure for Problem S7.

S6 A pressure-measurement system uses a sensor that converts pressure into voltage according to the transfer functions, $V_p = 0.5\sqrt{p}$. This voltage is then converted into a current. As the pressure varies from 0 to 100 psi, the current varies from 4 to 20 mA.

a. Find the transfer function equation for the conversion of voltage to current.

b. What pressure change, Δp, will cause the current to change by 1 mA from 19 mA to 20 mA?

c. What pressure change, Δp, will cause the current to change by 1 mA from 4 mA to 5 mA? Why is the pressure change not the same as in the previous case, even though the current changed by 1 mA in both cases?

d. Prepare a graph of current versus pressure. Is it linear or nonlinear?

S7 Figure 35 shows a system for measuring the pressure of exploding gases inside a steel chamber. A computer is used to measure the pressure. The pressure sensor has a transfer function of $V_p = 0.05\sqrt{p + 500}$ and a first-order time constant of $\tau = 2.0$ s. When an explosion occurs, the pressure rises virtually instantaneously from 0 to some maximum, $p_{\max}$. At $t = 0$, the explosion occurs, and the computer *must* take a reading at $t = 1$ s and determine the pressure, $p_{\max}$. This is before the sensor signal has stabilized.

a. Explain how $p_{\max}$ can be determined from a measurement taken at $t = 1.0$ s.

b. Suppose the sensor signal at $t = 1.0$ s is 1.45 V. What is the value of $p_{\max}$?

c. Suppose $p_{\max} = 2500$ psi; what value will the sensor voltage have at 1.0 s?

d. What equations will the computer be programmed to use in order to find $p_{\max}$ from the sensor voltage taken at 1.0 s?

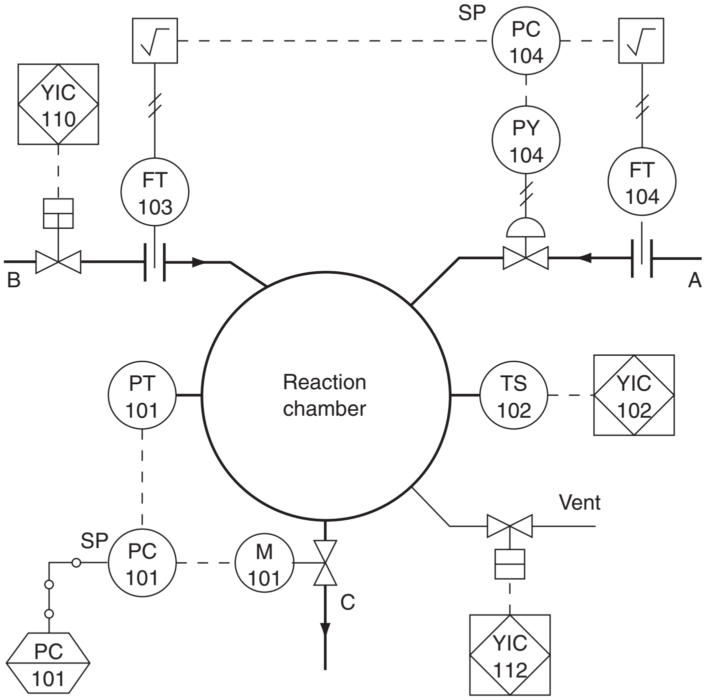


FIGURE 36
Figure for Problem S8.

S8 Figure 36 shows the P&ID for a process wherein materials A and B react in a chamber to create product C. The reaction generates heat and pressure within the chamber.

- **a.** Provide a description of each element in the diagram and the signals that connect the element.
- **b.** Describe the nature of the control loops shown and an overall view of how the process operates.
- **c.** Explain the purpose of the PLC units and the computer.

INTRODUCTION TO PROCESS CONTROL

SOLUTIONS TO ODD-NUMBERED PROBLEMS

5 The area is estimated by rectangular areas; curve *a* is found to give the least area.

7 First peak error is 22.5°C, so the quarter amplitudes should be 5.6 and 1.8. Actual values are 7 and 2, so quarter amplitude is approximately satisfied.

9 1110_2

11 19°C, 19.3°C, 20.7°C, and 21°C

15 101379 Pa

17 $p = 9.6 \text{ psi}, L = 5.9 \text{ m}$

19 $R = 397 \pm 7.5\ \Omega$

21 Worse case $= \pm 4.2\%, RMS = \pm 2.5\%$

23 $T = 199°\text{C}$

25 See Figure S.1.

27 $\tau = 12 \text{ s}$

29 $t = 1.99 \text{ s}$

31 $\tau_{\text{ave}} = 0.25 \text{ s}$

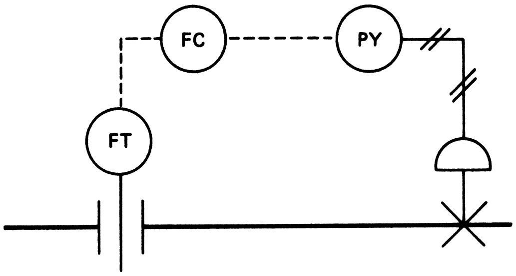
33 $Q_{\text{ave}} = 10.4 \text{ gal/min}, \sigma = 1.24 \text{ gal/min}$

Supplementary Problems

S1 See Figure S.2.

S3 Temperature and flow are self-regulating, level is not.

Figure S.1



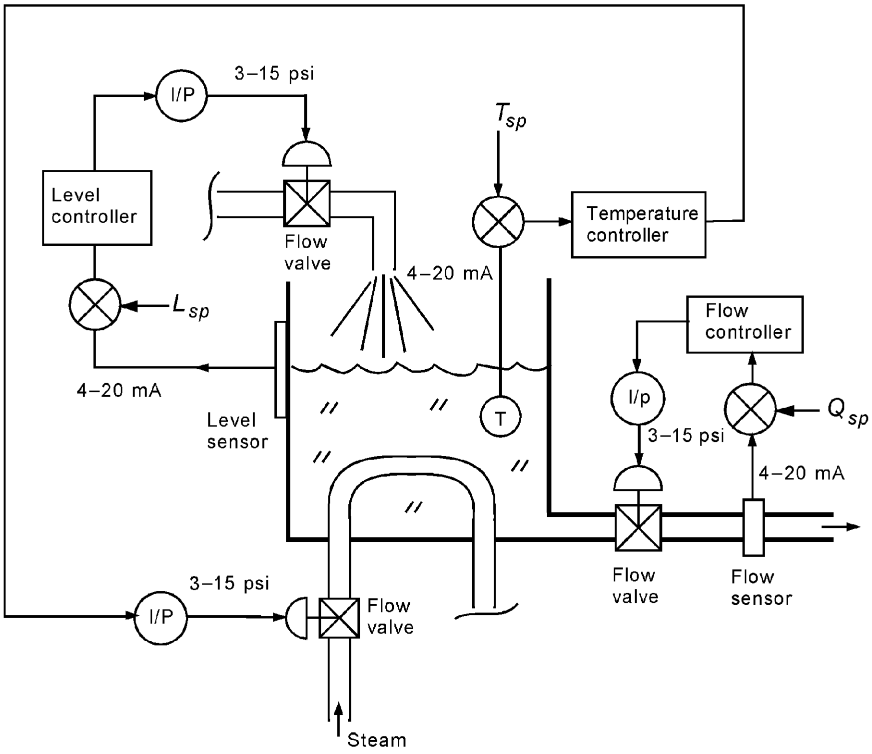


Figure S.2

S5 **a.** $K = 3.75$

b. L (low) = 1.1 m

c. period = 3.23 min

S7 **a.** Use known first-order time-response equation since input is a step.

b. final pressure = 1035 psi

c. $V(1\ \text{s}) = 1.76\ \text{V}$

d. $V_f = 1.12 + \dfrac{V(1) - 1.12}{1 - e^{-0.5}}$; then $p = \left(\dfrac{Vf}{0.05}\right)^2 - 500$

Analog Signal Conditioning

INSTRUCTIONAL OBJECTIVES

The purpose of this chapter is to introduce the reader to a variety of analog signal-conditioning methods used in process-control systems. Both passive methods and active methods, based upon the use of op amps, are defined. After reading this chapter and working through the examples and problems you will be able to:

- Explain the purpose of analog signal conditioning.
- Design a Wheatstone bridge circuit to convert resistance change to voltage change.
- Design RC low-pass and high-pass filter circuits to eliminate unwanted signals.
- Draw the schematics of four common op amp circuits and provide the transfer functions.
- Explain the operation of an instrumentation amplifier and draw its schematic.
- Design an analog signal-conditioning system to convert an input range of voltages to some desired output range of voltage.
- Design analog signal conditioning so that some range of resistance variation is converted into a desired range of voltage variation.

1 INTRODUCTION

Signal conditioning refers to operations performed on signals to convert them to a form suitable for interfacing with other elements in the process-control loop. In this chapter, we are concerned only with analog conversions, where the conditioned output is still an analog representation of the variable. Even in applications involving digital processing, some type of analog conditioning is usually required before analog-to-digital conversion is made.

From Chapter 2 of *Process Control Instrumentation Technology*, Eighth Edition. Curtis D. Johnson.

2 PRINCIPLES OF ANALOG SIGNAL CONDITIONING

A sensor measures a variable by converting information about that variable into a dependent signal of either electrical or pneumatic nature. To develop such transducers, we take advantage of fortuitous circumstances in nature where a dynamic variable influences some characteristic of a material. Consequently, there is little choice of the type or extent of such proportionality. For example, once we have researched nature and found that cadmium sulfide resistance varies inversely and nonlinearly with light intensity, we must then learn to employ this device for light measurement within the confines of that dependence. Analog signal conditioning provides the operations necessary to transform a sensor output into a form necessary to interface with other elements of the process-control loop. We will confine our attention to electrical transformations.

We often describe the effect of the signal conditioning by the term *transfer function.* By this term we mean the effect of the signal conditioning on the input signal. Thus, a simple voltage amplifier has a transfer function of some constant that, when multiplied by the input voltage, gives the output voltage.

It is possible to categorize signal conditioning into several general types.

2.1 Signal-Level and Bias Changes

One of the most common types of signal conditioning involves adjusting the level (magnitude) and bias (zero value) of some voltage representing a process variable. For example, some sensor output voltage may vary from 0.2 to 0.6 V as a process variable changes over a measurement range. However, equipment to which this sensor output must be connected perhaps requires a voltage that varies from 0 to 5 V for the *same* variation of the process variable.

We perform the required signal conditioning by first changing the zero to occur when the sensor output is 0.2 V. This can be done by simply subtracting 0.2 from the sensor output, which is called a *zero shift,* or a bias adjustment.

Now we have a voltage that varies from 0 to 0.4 V, so we need to make the voltage larger. If we multiply the voltage by 12.5, the new output will vary from 0 to 5 V as required. This is called *amplification,* and 12.5 is called the *gain.* In some cases, we need to make a sensor output smaller, which is called *attenuation.* You should note that the circuit that does either chore is called an *amplifier.* We distinguish between amplification and attenuation by noting whether the gain of the amplifier is greater than or less than unity.

In designing bias and amplifier circuits, we must be concerned with issues such as the frequency response, output impedance, and input impedance.

2.2 Linearization

As pointed out at the beginning of this section, the process-control designer has little choice of the characteristics of a sensor output versus a process variable. Often, the dependence that exists between input and output is nonlinear. Even those devices that are approximately linear may present problems when precise measurements of the variable are required.

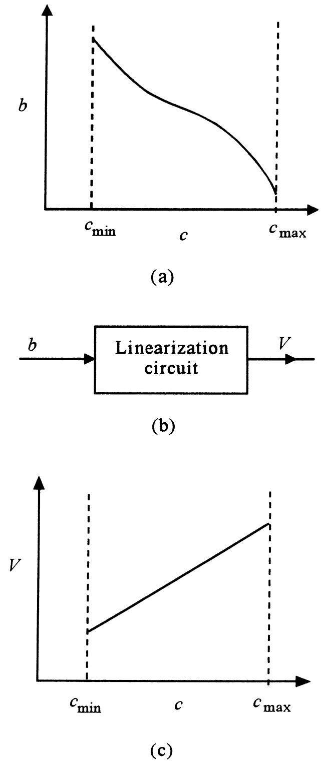


FIGURE 1
The purpose of linearization is to provide an output that varies linearly with some variable even if the sensor output does not.

Historically, specialized analog circuits were devised to linearize signals. For example, suppose a sensor output varied nonlinearly with a process variable, as shown in Figure 1a. A linearization circuit, indicated symbolically in Figure 1b, would ideally be one that conditioned the sensor output so that a voltage was produced which was linear with the process variable, as shown in Figure 1c. Such circuits are difficult to design and usually operate only within narrow limits.

The modern approach to this problem is to provide the nonlinear signal as input to a computer and perform the linearization using software. Virtually any nonlinearity can be handled in this manner and, with the speed of modern computers, in nearly real time.

2.3 Conversions

Often, signal conditioning is used to convert one type of electrical variation into another. Thus, a large class of sensors exhibit changes of resistance with changes in a dynamic variable. In these cases, it is necessary to provide a circuit to convert this resistance change either to a voltage or a current signal. This is generally accomplished by bridges when the fractional resistance change is small and/or by amplifiers whose gain varies with resistance.

Signal Transmission An important type of conversion is associated with the process-control standard of transmitting signals as 4- to 20-mA current levels in wire. This gives rise to the need for converting resistance and voltage levels to an appropriate current level at the transmitting end and for converting the current back to voltage at the receiving end. Of course, current transmission is used because such a signal is independent of load variations other than accidental shunt conditions that may draw off some current. Thus, voltage-to-current and current-to-voltage converters are often required.

Digital Interface The use of computers in process control requires conversion of analog data into a digital format by integrated circuit devices called analog-to-digital converters (ADCs). Analog signal conversion is usually required to adjust the analog measurement signal to match the input requirements of the ADC. For example, the ADC may need a voltage that varies between 0 and 5 V, but the sensor provides a signal that varies from 30 to 80 mV. Signal conversion circuits can be developed to interface the output to the required ADC input.

2.4 Filtering and Impedance Matching

Two other common signal-conditioning requirements are filtering and matching impedance.

Often, spurious signals of considerable strength are present in the industrial environment, such as the 60-Hz line frequency signals. Motor start transients may also cause pulses and other unwanted signals in the process-control loop. In many cases, it is necessary to use high-pass, low-pass, or notch *filters* to eliminate unwanted signals from the loop. Such filtering can be accomplished by *passive* filters, using only resistors, capacitors, and inductors, or *active* filters, using gain and feedback.

Impedance matching is an important element of signal conditioning when transducer internal impedance or line impedance can cause errors in measurement of a dynamic variable. Both active and passive networks are employed to provide such matching.

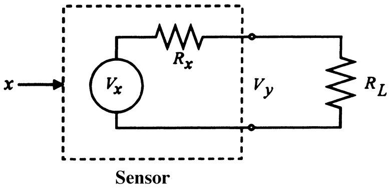
2.5 Concept of Loading

One of the most important concerns in analog signal conditioning is the loading of one circuit by another. This introduces uncertainty in the amplitude of a voltage as it is passed through the measurement process. If this voltage represents some process variable, then we have uncertainty in the value of the variable.

Qualitatively, loading can be described as follows. Suppose the open-circuit output of some element is a voltage, say V_x, when the element input is some variable of value x. This element could be a sensor or some other part of the signal-conditioning circuit, such as a bridge circuit or amplifier. *Open circuit* means that nothing is connected to the output. Loading occurs when we do connect something, a load, across the output, and the output voltage of the element drops to some value, $V_y < V_x$. Different loads result in different drops.

Quantitatively, we can evaluate loading as follows. Thévenin's theorem tells us that the output terminals of any two terminal elements can be defined as a voltage source in se-

FIGURE 2
The Thévenin equivalent circuit of a sensor allows easy visualization of how loading occurs.



ries with an output impedance. Let's assume this is a resistance (the output resistance) to make the description easier to follow. This is called the Thévenin equivalent circuit model of the element.

Figure 2 shows such an element modeled as a voltage V_x and a resistance R_x. Now suppose a load, R_L, is connected across the output of the element as shown in Figure 2. This could be the input resistance of an amplifier, for example. A current will flow, and voltage will be dropped across R_x. It is easy to calculate that the loaded output voltage will thus be given by

$$V_y = V_x\left(1 - \frac{R_x}{R_L + R_x}\right) \tag{1}$$

The voltage that appears across the load is reduced by the voltage dropped across the internal resistance.

This equation shows how the effects of loading can be reduced. Clearly, the objective will be to make R_L much larger than R_x—that is, $R_L \gg R_x$. The following example shows how the effects of loading can compromise our measurements.

EXAMPLE 1

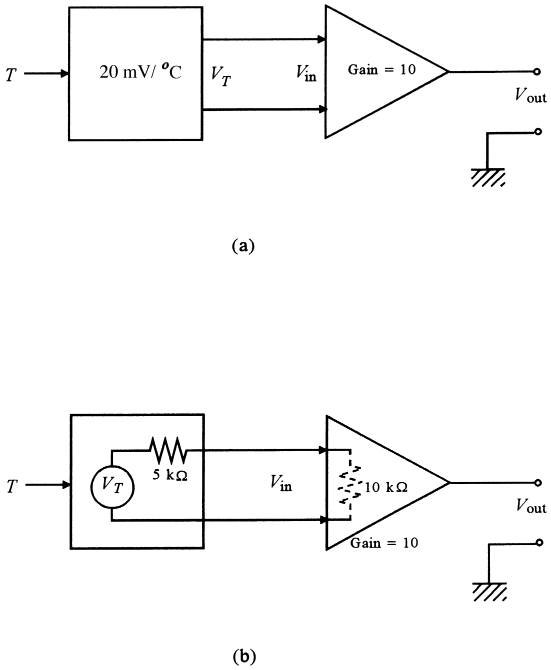
An amplifier outputs a voltage that is 10 times the voltage on its input terminals. It has an input resistance of 10 kΩ. A sensor outputs a voltage proportional to temperature with a transfer function of 20 mV/°C. The sensor has an output resistance of 5.0 kΩ. If the temperature is 50°C, find the amplifier output.

Solution

The naive solution is represented by Figure 3a. The unloaded output of the sensor is simply $V_T = (20\ \text{mV}/^\circ\text{C})50^\circ\text{C} = 1.0\ \text{V}$. Since the amplifier has a gain of 10, the output of the amplifier appears to be $V_{\text{out}} = 10\ V_{\text{in}} = (10)1.0\ \text{V} = 10\ \text{V}$. But this is wrong, because of loading!

Figure 3b shows the correct analysis. Here we see that there will be a voltage dropped across the output resistance of the sensor. The actual amplifier input voltage will be given by Equation (1),

$$V_{\text{in}} = V_T\left(1 - \frac{5.0\ \text{k}\Omega}{5.0\ \text{k}\Omega + 10\ \text{k}\Omega}\right)$$



(a)

(b)

FIGURE 3
If loading is ignored, serious errors can occur in expected outputs of circuits and gains of amplifiers.

where $V_T = 1.0$ V, so that $V_{in} = 0.67$ V. Thus, the output of the amplifier is actually $V_{out} = 10(0.67\text{ V}) = 6.7$ V.

This concept plays an important role in analog signal conditioning.

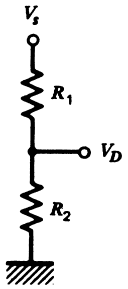
If the electrical quantity of interest is frequency or a digital signal, then loading is not such a problem. That is, if there is enough signal left after loading to measure the frequency or to distinguish ones from zeros, there will be no error. Loading is important mostly when signal amplitudes are important.

3 PASSIVE CIRCUITS

Bridge and divider circuits are two passive techniques that have been extensively used for signal conditioning for many years. Although modern active circuits often replace these techniques, there are still many applications where their particular advantages make them useful.

Bridge circuits are used primarily as an accurate means of measuring changes in impedance. Such circuits are particularly useful when the fractional changes in impedance are very small.

FIGURE 4
The simple voltage divider can often be used to convert resistance variation into voltage variation.



Another common type of passive circuit involved in signal conditioning is for filtering unwanted frequencies from the measurement signal. It is quite common in the industrial environment to find signals that possess high- and/or low-frequency noise as well as the desired measurement data. For example, a transducer may convert temperature information into a dc voltage, proportional to temperature. Because of the ever-present ac power lines, however, there may be a 60-Hz noise voltage impressed on the output that makes determination of the temperature difficult. A passive circuit consisting of a resistor and capacitor often can be used to eliminate both high- and low-frequency noise without changing the desired signal information.

3.1 Divider Circuits

The elementary voltage divider shown in Figure 4 often can be used to provide conver sion of resistance variation into a voltage variation. The voltage of such a divider is given by the well-known relationship

$$V_D = \frac{R_2 V_s}{R_1 + R_2} \tag{2}$$

where V_s = supply voltage
R_1, R_2 = divider resistors

Either R_1 or R_2 can be the sensor whose resistance varies with some measured variable.

It is important to consider the following issues when using a divider for conversion of resistance to voltage variation:

1. The variation of V_D with either R_1 or R_2 is nonlinear; that is, even if the resistance varies linearly with the measured variable, the divider voltage will not vary linearly.
2. The effective output impedance of the divider is the parallel combination of R_1 and R_2. This may not necessarily be high, so loading effects must be considered.
3. In a divider circuit, current flows through both resistors; that is, power will be dissipated by both, including the sensor. The power rating of both the resistor and sensor must be considered.

EXAMPLE 2 The divider of Figure 4 has $R_1 = 10.0\ \text{k}\Omega$ and $V_s = 5.00\ \text{V}$. Suppose R_2 is a sensor whose resistance varies from 4.00 to 12.0 kΩ as some dynamic variable varies over a range. Then find **(a)** the minimum and maximum of V_D, **(b)** the range of output impedance, and **(c)** the range of power dissipated by R_2.

Solution

a. The solution is given by Equation (2). For $R_2 = 4\ \text{k}\Omega$, we have

$$V_D = \frac{(5\ \text{V})(4\ \text{k}\Omega)}{10\ \text{k}\Omega + 4\ \text{k}\Omega} = 1.43\ \text{V}$$

For $R_2 = 12\ \text{k}\Omega$, the voltage is

$$V_D = \frac{(5\ \text{V})(12\ \text{k}\Omega)}{10\ \text{k}\Omega + 12\ \text{k}\Omega} = 2.73\ \text{V}$$

b. Thus, the voltage varies from 1.43 to 2.73 V.

c. The range of output impedance is found from the parallel combination of R_1 and R_2 for the minimum and maximum of R_2. Simple parallel resistance computation shows that this will be from 2.86 to 5.45 kΩ.

d. The power dissipated by the sensor can be determined most easily from V^2/R_2, as the voltage across R_2 has been calculated. The power dissipated varies from 0.51 to 0.62 mW.

3.2 Bridge Circuits

Bridge circuits are used to convert impedance variations into voltage variations. One of the advantages of the bridge for this task is that it can be designed so the voltage produced varies around zero. This means that amplification can be used to increase the voltage level for increased sensitivity to variation of impedance.

Another application of bridge circuits is in the precise static measurement of an impedance.

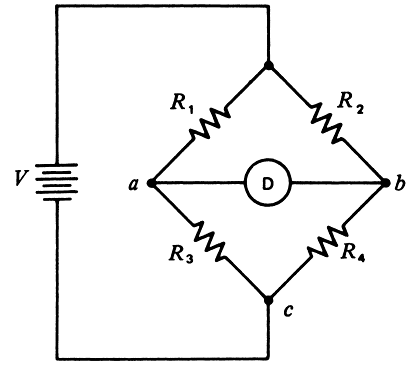
Wheatstone Bridge The simplest and most common bridge circuit is the dc Wheatstone bridge, as shown in Figure 5. This network is used in signal-conditioning applications where a sensor changes resistance with process variable changes. Many modifications of this basic bridge are employed for other specific applications. In Figure 5, the object labeled *D* is a *voltage detector* used to compare the potentials of points *a* and *b* of the network. In most modern applications, the detector is a very high-input impedance differential amplifier. In some cases, a highly sensitive galvanometer with a relatively low impedance may be used, especially for calibration purposes and spot measurement instruments.

For our initial analysis, assume the detector impedance is infinite—that is, an open circuit.

In this case, the potential difference, ΔV, between points *a* and *b* is simply

$$\Delta V = V_a - V_b \tag{3}$$

FIGURE 5
The basic dc Wheatstone bridge.



where V_a = potential of point *a* with respect to *c*

V_b = potential of point *b* with respect to *c*

The values of V_a and V_b now can be found by noting that V_a is just the supply voltage, *V*, divided between R_1 and R_3.

$$V_a = \frac{VR_3}{R_1 + R_3} \tag{4}$$

In a similar fashion, V_b is a divided voltage given by

$$V_b = \frac{VR_4}{R_2 + R_4} \tag{5}$$

where V = bridge supply voltage

If we now combine Equations (3), (4), and (5), the voltage difference or voltage offset can be written

$$\Delta V = \frac{VR_3}{R_1 + R_3} - \frac{VR_4}{R_2 + R_4} \tag{6}$$

Using algebra, the reader can show that this equation reduces to

$$\Delta V = V\frac{R_3R_2 - R_1R_4}{(R_1 + R_3)\cdot(R_2 + R_4)} \tag{7}$$

Equation (7) shows how the difference in potential across the detector is a function of the supply voltage and the values of the resistors. Because a difference appears in the numerator of Equation (7), it is clear that a particular combination of resistors can be found that will result in zero difference and zero voltage across the detector—that is, a *null.* Obviously, this combination, from examination of Equation (7), is

$$R_3R_2 = R_1R_4 \tag{8}$$

Equation (8) indicates that whenever a Wheatstone bridge is assembled and resistors are adjusted for a detector null, the resistor values must satisfy the indicated equality. It does not matter if the supply voltage drifts or changes; the null is maintained. Equations (7) and (8) underlie the application of Wheatstone bridges to process-control applications using high-input impedance detectors.

EXAMPLE 3 If a Wheatstone bridge, as shown in Figure 5, nulls with $R_1 = 1000\ \Omega$, $R_2 = 842\ \Omega$, and $R_3 = 500\ \Omega$, find the value of R_4.

Solution

Because the bridge is nulled, find R_4 using Equation (8):

$$R_1 R_4 = R_3 R_2$$

$$R_4 = \frac{R_3 R_2}{R_1} = \frac{(500\ \Omega)\,(842\ \Omega)}{1000\ \Omega}$$

$$R_4 = \mathbf{421}\ \Omega$$

EXAMPLE 4 The resistors in a bridge are given by $R_1 = R_2 = R_3 = 120\ \Omega$ and $R_4 = 121\ \Omega$. If the supply is 10.0 V, find the voltage offset.

Solution

Assuming the detector impedance to be very high, we find the offset from

$$\Delta V = V \frac{R_3 R_2 - R_1 R_4}{(R_1 + R_3)\cdot(R_2 + R_4)}$$

$$\Delta V = 10\ \text{V}\ \frac{(120\ \Omega)(120\ \Omega) - (120\ \Omega)(121\ \Omega)}{(120\ \Omega + 120\ \Omega)\cdot(120\ \Omega + 121\ \Omega)}$$

$$\Delta V = \mathbf{-20.7\ mV}$$

Notice that the result in Example 4 is a negative voltage. Remember that ΔV is simply the difference between V_a and V_b in Figure 5, as given by Equation (3). So the fact that ΔV is negative in this example simply means that V_b is larger than V_a.

Galvanometer Detector The use of a galvanometer as a null detector in the bridge circuit introduces some differences in our calculations because the detector resistance may be low and because we must determine the bridge offset as current offset. When the bridge is nulled, Equation (8) still defines the relationship between the resistors in the bridge arms. Equation (7) must be modified to allow determination of current drawn by the galvanometer when a null condition is not present. Perhaps the easiest way to determine this offset current is first to find the Thévenin equivalent circuit between points *a* and *b* of the bridge (as drawn in Figure 5 with the detector removed). The Thévenin voltage is simply the open-circuit voltage difference between points *a* and *b* of the circuit. But wait! Equation (7) is the open-circuit voltage, so

$$V_{\text{Th}} = V \frac{R_3 R_2 - R_1 R_4}{(R_1 + R_3)(R_2 + R_4)} \tag{9}$$