


Pearson New International Edition

Physics for Scientists & Engineers with Modern Physics Volume 1

Douglas C. Giancoli

Fourth Edition


ALWAYS LEARNING

PEARSON

Pearson New International Edition

Physics for Scientists & Engineers
with Modern Physics Volume 1
Douglas C. Giancoli
Fourth Edition

PEARSON

Pearson Education Limited
Edinburgh Gate
Harlow
Essex CM20 2JE
England and Associated Companies throughout the world

Visit us on the World Wide Web at: www.pearsoned.co.uk

PEARSON

ISBN 10: 1-292-02443-7
ISBN 13: 978-1-292-02443-1

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

Printed in the United States of America

Table of Contents

This page intentionally left blank

Preface

Preface

I was motivated from the beginning to write a textbook different from others that present physics as a sequence of facts, like a Sears catalog: "here are the facts and you better learn them." Instead of that approach in which topics are begun formally and dogmatically, I have sought to begin each topic with concrete observations and experiences students can relate to: start with specifics and only then go to the great generalizations and the more formal aspects of a topic, showing *why* we believe what we believe. This approach reflects how science is actually practiced.

Why a Fourth Edition?

Two recent trends in physics texbooks are disturbing: (1) their revision cycles have become short—they are being revised every 3 or 4 years; (2) the books are getting larger, some over 1500 pages. I don't see how either trend can be of benefit to students. My response: (1) It has been 8 years since the previous edition of this book. (2) This book makes use of physics education research, although it avoids the detail a Professor may need to say in class but in a book shuts down the reader. And this book still remains among the shortest.

This new edition introduces some important new pedagogic tools. It contains new physics (such as in cosmology) and many new appealing applications (list on previous page). Pages and page breaks have been carefully formatted to make the physics easier to follow: no turning a page in the middle of a derivation or Example. Great efforts were made to make the book attractive so students will want to *read* it.

Some of the new features are listed below.

What's New

Chapter-Opening Questions: Each Chapter begins with a multiple-choice question, whose responses include common misconceptions. Students are asked to answer before starting the Chapter, to get them involved in the material and to get any preconceived notions out on the table. The issues reappear later in the Chapter, usually as Exercises, after the material has been covered. The Chapter-Opening Questions also show students the power and usefulness of Physics.

APPROACH paragraph in worked-out numerical Examples: A short introductory paragraph before the Solution, outlining an approach and the steps we can take to get started. Brief NOTES after the Solution may remark on the Solution, may give an alternate approach, or mention an application.

Step-by-Step Examples: After many Problem Solving Strategies (more than 20 in the book), the next Example is done step-by-step following precisely the steps just seen.

Exercises within the text, after an Example or derivation, give students a chance to see if they have understood enough to answer a simple question or do a simple calculation. Many are multiple choice.

Greater clarity: No topic, no paragraph in this book was overlooked in the search to improve the clarity and conciseness of the presentation. Phrases and sentences that may slow down the principal argument have been eliminated: keep to the essentials at first, give the elaborations later.

$\vec{\mathbf{F}}, \vec{\mathbf{v}}, \vec{\mathbf{B}}$ ***Vector notation, arrows***: The symbols for vector quantities in the text and Figures now have a tiny arrow over them, so they are similar to what we write by hand.

Cosmological Revolution: With generous help from top experts in the field, readers have the latest results.

Page layout: more than in the previous edition, serious attention has been paid to how each page is formatted. Examples and all important derivations and arguments are on facing pages. Students then don't have to turn back and forth. Throughout, readers see, on two facing pages, an important slice of physics.

New Applications: LCDs, digital cameras and electronic sensors (CCD, CMOS), electric hazards, GFCIs, photocopiers, inkjet and laser printers, metal detectors, underwater vision, curve balls, airplane wings, DNA, how we actually *see* images. (Turn back a page to see a longer list.)

Examples modified: more math steps are spelled out, and many new Examples added. About 10% of all Examples are Estimation Examples.

This Book is Shorter than other complete full-service books at this level. Shorter explanations are easier to understand and more likely to be read.

Content and Organizational Changes

- **Rotational Motion**: Chapters 10 and 11 have been reorganized. All of angular momentum is now in Chapter 11.
- **First law of thermodynamics**, in Chapter 19, has been rewritten and extended. The full form is given: $\Delta K + \Delta U + \Delta E_{\text{int}} = Q - W$, where internal energy is E_{int}, and U is potential energy; the form $Q - W$ is kept so that $dW = P\,dV$.
- Kinematics and Dynamics of Circular Motion are now treated together in Chapter 5.
- Work and Energy, Chapters 7 and 8, have been carefully revised.
- Work done by friction is discussed now with energy conservation (energy terms due to friction).
- Chapters on Inductance and AC Circuits have been combined into one: Chapter 30.
- Graphical Analysis and Numerical Integration is a new optional Section 2–9. Problems requiring a computer or graphing calculator are found at the end of most Chapters.
- Length of an object is a script ℓ rather than normal l, which looks like 1 or I (moment of inertia, current), as in $F = I\ell B$. Capital L is for angular momentum, latent heat, inductance, dimensions of length $[L]$.
- Newton's law of gravitation remains in Chapter 6. Why? Because the $1/r^2$ law is too important to relegate to a late chapter that might not be covered at all late in the semester; furthermore, it is one of the basic forces in nature. In Chapter 8 we can treat real gravitational potential energy and have a fine instance of using $U = -\int \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}}$.
- New Appendices include the differential form of Maxwell's equations and more on dimensional analysis.
- Problem Solving Strategies are found on pages 30, 58, 64, 96, 102, 125, 166, 198, 229, 261, 314, 504, 551, 571, 600, 685, 716, 740, 763, 849, 871, and 913.

Organization

Some instructors may find that this book contains more material than can be covered in their courses. The text offers great flexibility. Sections marked with a star * are considered optional. These contain slightly more advanced physics material, or material not usually covered in typical courses and/or interesting applications; they contain no material needed in later Chapters (except perhaps in later optional Sections). For a brief course, all optional material could be dropped as well as major parts of Chapters 1, 13, 16, 26, 30, and 35, and selected parts of Chapters 9, 12, 19, 20, 33, and the modern physics Chapters. Topics not covered in class can be a valuable resource for later study by students. Indeed, this text can serve as a useful reference for years because of its wide range of coverage.

Versions of this Book

Complete version: 44 Chapters including 9 Chapters of modern physics.

Classic version: 37 Chapters including one each on relativity and quantum theory.

3 Volume version: Available separately or packaged together (Vols. 1 & 2 or all 3 Volumes):

Volume 1: Chapters 1–20 on mechanics, including fluids, oscillations, waves, plus heat and thermodynamics.

Volume 2: Chapters 21–35 on electricity and magnetism, plus light and optics.

Volume 3: Chapters 36–44 on modern physics: relativity, quantum theory, atomic physics, condensed matter, nuclear physics, elementary particles, cosmology and astrophysics.

Thanks

Many physics professors provided input or direct feedback on every aspect of this textbook. They are listed below, and I owe each a debt of gratitude.

Mario Affatigato, Coe College
Lorraine Allen, United States Coast Guard Academy
Zaven Altounian, McGill University
Bruce Barnett, Johns Hopkins University
Michael Barnett, Lawrence Berkeley Lab
Anand Batra, Howard University
Cornelius Bennhold, George Washington University
Bruce Birkett, University of California Berkeley
Dr. Robert Boivin, Auburn University
Subir Bose, University of Central Florida
David Branning, Trinity College
Meade Brooks, Collin County Community College
Bruce Bunker, University of Notre Dame
Grant Bunker, Illinois Institute of Technology
Wayne Carr, Stevens Institute of Technology
Charles Chiu, University of Texas Austin
Robert Coakley, University of Southern Maine
David Curott, University of North Alabama
Biman Das, SUNY Potsdam
Bob Davis, Taylor University
Kaushik De, University of Texas Arlington
Michael Dennin, University of California Irvine
Kathy Dimiduk, University of New Mexico
John DiNardo, Drexel University
Scott Dudley, United States Air Force Academy
John Essick, Reed College
Cassandra Fesen, Dartmouth College
Alex Filippenko, University of California Berkeley
Richard Firestone, Lawrence Berkeley Lab
Mike Fortner, Northern Illinois University
Tom Furtak, Colorado School of Mines
Edward Gibson, California State University Sacramento
John Hardy, Texas A&M
J. Erik Hendrickson, University of Wisconsin Eau Claire
Laurent Hodges, Iowa State University
David Hogg, New York University
Mark Hollabaugh, Normandale Community College
Andy Hollerman, University of Louisiana at Lafayette
William Holzapfel, University of California Berkeley
Bob Jacobsen, University of California Berkeley
Teruki Kamon, Texas A&M
Daryao Khatri, University of the District of Columbia
Jay Kunze, Idaho State University
Jim LaBelle, Dartmouth College
M.A.K. Lodhi, Texas Tech
Bruce Mason, University of Oklahoma
Dan Mazilu, Virginia Tech
Linda McDonald, North Park College
Bill McNairy, Duke University
Raj Mohanty, Boston University
Giuseppe Molesini, Istituto Nazionale di Ottica Florence
Lisa K. Morris, Washington State University
Blaine Norum, University of Virginia
Alexandria Oakes, Eastern Michigan University
Michael Ottinger, Missouri Western State University
Lyman Page, Princeton and WMAP
Bruce Partridge, Haverford College
R. Daryl Pedigo, University of Washington
Robert Pelcovitz, Brown University
Vahe Peroomian, UCLA
James Rabchuk, Western Illinois University
Michele Rallis, Ohio State University
Paul Richards, University of California Berkeley
Peter Riley, University of Texas Austin
Larry Rowan, University of North Carolina Chapel Hill
Cindy Schwarz, Vassar College
Peter Sheldon, Randolph-Macon Woman's College
Natalia A. Sidorovskaia, University of Louisiana at Lafayette
James Siegrist, UC Berkeley, Director Physics Division LBNL
George Smoot, University of California Berkeley
Mark Sprague, East Carolina University
Michael Strauss, University of Oklahoma
Laszlo Takac, University of Maryland Baltimore Co.
Franklin D. Trumpy, Des Moines Area Community College
Ray Turner, Clemson University
Som Tyagi, Drexel University
John Vasut, Baylor University
Robert Webb, Texas A&M
Robert Weidman, Michigan Technological University
Edward A. Whittaker, Stevens Institute of Technology
John Wolbeck, Orange County Community College
Stanley George Wojcicki, Stanford University
Edward Wright, UCLA
Todd Young, Wayne State College
William Younger, College of the Albemarle
Hsiao-Ling Zhou, Georgia State University

I owe special thanks to Prof. Bob Davis for much valuable input, and especially for working out all the Problems and producing the Solutions Manual for all Problems, as well as for providing the answers to odd-numbered Problems at the end of this book. Many thanks also to J. Erik Hendrickson who collaborated with Bob Davis on the solutions, and to the team they managed (Profs. Anand Batra, Meade Brooks, David Currott, Blaine Norum, Michael Ottinger, Larry Rowan, Ray Turner, John Vasut, William Younger). I am grateful to Profs. John Essick, Bruce Barnett, Robert Coakley, Biman Das, Michael Dennin, Kathy Dimiduk, John DiNardo, Scott Dudley, David Hogg, Cindy Schwarz, Ray Turner, and Som Tyagi, who inspired many of the Examples, Questions, Problems, and significant clarifications.

Crucial for rooting out errors, as well as providing excellent suggestions, were Profs. Kathy Dimiduk, Ray Turner, and Lorraine Allen. A huge thank you to them and to Prof. Giuseppe Molesini for his suggestions and his exceptional photographs for optics.

For Chapters 43 and 44 on Particle Physics and Cosmology and Astrophysics, I was fortunate to receive generous input from some of the top experts in the field, to whom I owe a debt of gratitude: George Smoot, Paul Richards, Alex Filippenko, James Siegrist, and William Holzapfel (UC Berkeley), Lyman Page (Princeton and WMAP), Edward Wright (UCLA and WMAP), and Michael Strauss (University of Oklahoma).

I especially wish to thank Profs. Howard Shugart, Chair Frances Hellman, and many others at the University of California, Berkeley, Physics Department for helpful discussions, and for hospitality. Thanks also to Prof. Tito Arecchi and others at the Istituto Nazionale di Ottica, Florence, Italy.

Finally, I am grateful to the many people at Prentice Hall with whom I worked on this project, especially Paul Corey, Karen Karlin, Christian Botting, John Christiana, and Sean Hogan.

The final responsibility for all errors lies with me. I welcome comments, corrections, and suggestions as soon as possible to benefit students for the next reprint.

D.C.G.

email: Paul.Corey@Pearson.com

Post: Paul Corey
One Lake Street
Upper Saddle River, NJ 07458

About the Author

Douglas C. Giancoli obtained his BA in physics (summa cum laude) from the University of California, Berkeley, his MS in physics at the Massachusetts Institute of Technology, and his PhD in elementary particle physics at the University of California, Berkeley. He spent 2 years as a post-doctoral fellow at UC Berkeley's Virus lab developing skills in molecular biology and biophysics. His mentors include Nobel winners Emilio Segrè and Donald Glaser.

He has taught a wide range of undergraduate courses, traditional as well as innovative ones, and continues to update his textbooks meticulously, seeking ways to better provide an understanding of physics for students.

Doug's favorite spare-time activity is the outdoors, especially climbing peaks (here on a dolomite summit, Italy). He says climbing peaks is like learning physics: it takes effort and the rewards are great.

Online Supplements (partial list)

MasteringPhysics™ (www.masteringphysics.com)
is a sophisticated online tutoring and homework system developed specially for courses using calculus-based physics. Originally developed by David Pritchard and collaborators at MIT, MasteringPhysics provides **students** with individualized online tutoring by responding to their wrong answers and providing hints for solving multi-step problems when they get stuck. It gives them immediate and up-to-date assessment of their progress, and shows where they need to practice more. MasteringPhysics provides **instructors** with a fast and effective way to assign tried-and-tested online homework assignments that comprise a range of problem types. The powerful post-assignment diagnostics allow instructors to assess the progress of their class as a whole as well as individual students, and quickly identify areas of difficulty.

WebAssign (www.webassign.com)

CAPA and LON-CAPA (www.lon-capa.org)

Student Supplements (partial list)

Student Study Guide & Selected Solutions Manual (Volume I: 0-13-227324-1, Volumes II & III: 0-13-227325-X) by Frank Wolfs

Student Pocket Companion (0-13-227326-8) by Biman Das

Tutorials in Introductory Physics (0-13-097069-7)
by Lillian C. McDermott, Peter S. Schaffer, and the Physics Education Group at the University of Washington

Physlet® Physics (0-13-101969-4)
by Wolfgang Christian and Mario Belloni

Ranking Task Exercises in Physics, Student Edition (0-13-144851-X)
by Thomas L. O'Kuma, David P. Maloney, and Curtis J. Hieggelke

E&M TIPERs: Electricity & Magnetism Tasks Inspired by Physics Education Research (0-13-185499-2) by Curtis J. Hieggelke, David P. Maloney, Stephen E. Kanim, and Thomas L. O'Kuma

Mathematics for Physics with Calculus (0-13-191336-0)
by Biman Das

To Students

HOW TO STUDY

1. **1.** Read the Chapter. Learn new vocabulary and notation. Try to respond to questions and exercises as they occur.
2. **2.** Attend all class meetings. Listen. Take notes, especially about aspects you do not remember seeing in the book. Ask questions (everyone else wants to, but maybe you will have the courage). You will get more out of class if you read the Chapter first.
3. **3.** Read the Chapter again, paying attention to details. Follow derivations and worked-out Examples. Absorb their logic. Answer Exercises and as many of the end of Chapter Questions as you can.
4. **4.** Solve 10 to 20 end of Chapter Problems (or more), especially those assigned. In doing Problems you find out what you learned and what you didn't. Discuss them with other students. Problem solving is one of the great learning tools. Don't just look for a formula—it won't cut it.

NOTES ON THE FORMAT AND PROBLEM SOLVING

1. Sections marked with a star (*) are considered **optional**. They can be omitted without interrupting the main flow of topics. No later material depends on them except possibly later starred Sections. They may be fun to read, though.

2. The customary **conventions** are used: symbols for quantities (such as m for mass) are italicized, whereas units (such as m for meter) are not italicized. Symbols for vectors are shown in boldface with a small arrow above: $\vec{\mathbf{F}}$.

3. Few equations are valid in all situations. Where practical, the **limitations** of important equations are stated in square brackets next to the equation. The equations that represent the great laws of physics are displayed with a tan background, as are a few other indispensable equations.

4. At the end of each Chapter is a set of **Problems** which are ranked as Level I, II, or III, according to estimated difficulty. Level I Problems are easiest, Level II are standard Problems, and Level III are "challenge problems." These ranked Problems are arranged by Section, but Problems for a given Section may depend on earlier material too. There follows a group of General Problems, which are not arranged by Section nor ranked as to difficulty. Problems that relate to optional Sections are starred (*). Most Chapters have 1 or 2 Computer/Numerical Problems at the end, requiring a computer or graphing calculator. Answers to odd-numbered Problems are given at the end of the book.

5. Being able to solve **Problems** is a crucial part of learning physics, and provides a powerful means for understanding the concepts and principles. This book contains many aids to problem solving: (a) worked-out **Examples** and their solutions in the text, which should be studied as an integral part of the text; (b) some of the worked-out Examples are **Estimation Examples**, which show how rough or approximate results can be obtained even if the given data are sparse (see Section 1–6); (c) special **Problem Solving Strategies** placed throughout the text to suggest a step-by-step approach to problem solving for a particular topic—but remember that the basics remain the same; most of these "Strategies" are followed by an Example that is solved by explicitly following the suggested steps; (d) special problem-solving Sections; (e) "Problem Solving" marginal notes which refer to hints within the text for solving Problems; (f) **Exercises** within the text that you should work out immediately, and then check your response against the answer given at the bottom of the last page of that Chapter; (g) the Problems themselves at the end of each Chapter (point 4 above).

6. **Conceptual Examples** pose a question which hopefully starts you to think and come up with a response. Give yourself a little time to come up with your own response before reading the Response given.

7. **Math** review, plus some additional topics, are found in Appendices. Useful data, conversion factors, and math formulas are found inside the front and back covers.

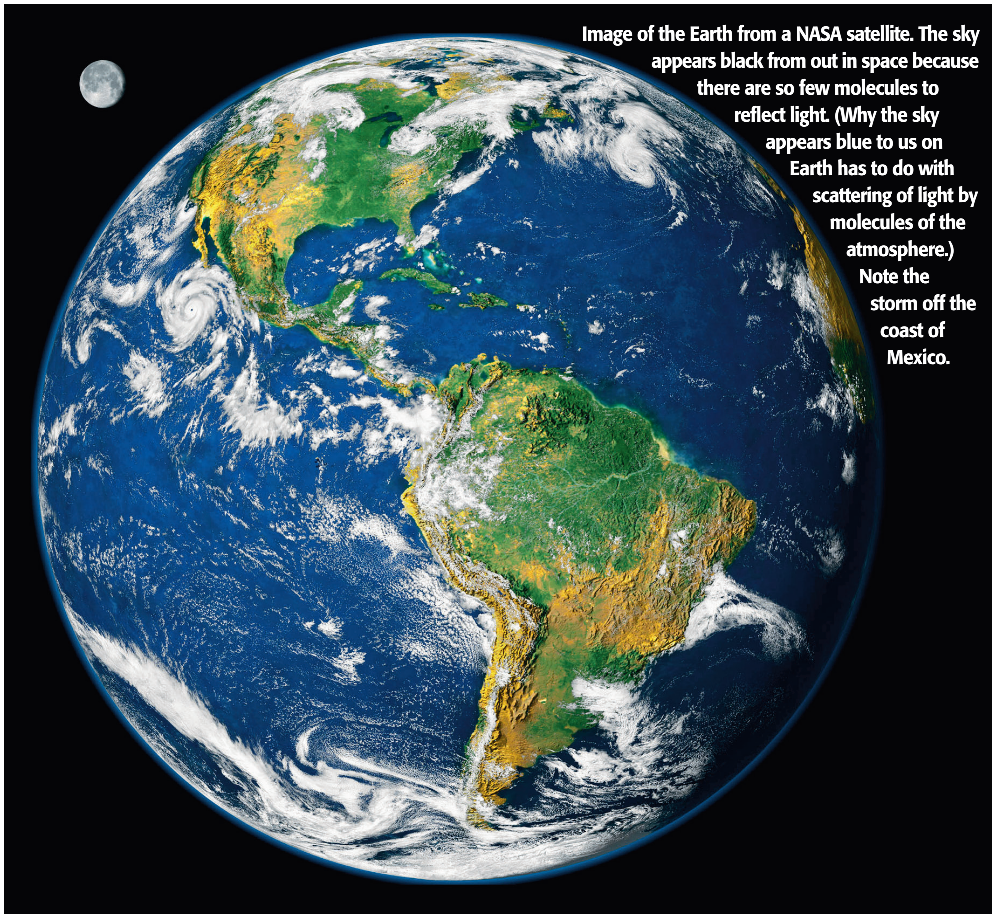


Image of the Earth from a NASA satellite. The sky appears black from out in space because there are so few molecules to reflect light. (Why the sky appears blue to us on Earth has to do with scattering of light by molecules of the atmosphere.) Note the storm off the coast of Mexico.

© Reuters/Corbis

Introduction, Measurement, Estimating

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Suppose you wanted to actually measure the radius of the Earth, at least roughly, rather than taking other people's word for what it is. Which response below describes the best approach?

(a) Give up; it is impossible using ordinary means.
(b) Use an extremely long measuring tape.
(c) It is only possible by flying high enough to see the actual curvature of the Earth.
(d) Use a standard measuring tape, a step ladder, and a large smooth lake.
(e) Use a laser and a mirror on the Moon or on a satellite.

CONTENTS

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

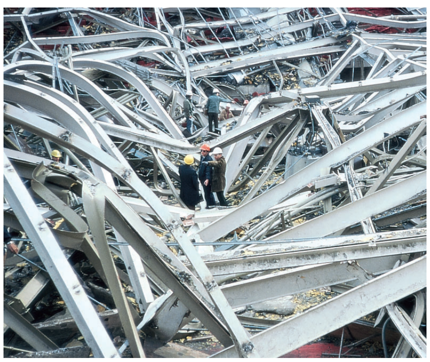
Physics is the most basic of the sciences. It deals with the behavior and structure of matter. The field of physics is usually divided into *classical physics* which includes motion, fluids, heat, sound, light, electricity and magnetism; and *modern physics* which includes the topics of relativity, atomic structure, condensed matter, nuclear physics, elementary particles, and cosmology and astrophysics.

An understanding of physics is crucial for anyone making a career in science or technology. Engineers, for example, must know how to calculate the forces within a structure to design it so that it remains standing (Fig. 1a). Indeed, a simple physics calculation—or even intuition based on understanding the physics of forces—can save hundreds of lives (Fig. 1b). Physics is useful in many fields, and in everyday life.

Philip H. Coblentz/World Travel Images, Inc.

(a)

Antranig M. Ouzoonian, P.E./Weidlinger Associates, Inc.



(b)

FIGURE 1 (a) This Roman aqueduct was built 2000 years ago and still stands. (b) The Hartford Civic Center collapsed in 1978, just two years after it was built.

1 The Nature of Science

The principal aim of all sciences, including physics, is generally considered to be the search for order in our observations of the world around us. Many people think that science is a mechanical process of collecting facts and devising theories. But it is not so simple. Science is a creative activity that in many respects resembles other creative activities of the human mind.

One important aspect of science is **observation** of events, which includes the design and carrying out of experiments. But observation and experiment require imagination, for scientists can never include everything in a description of what they observe. Hence, scientists must make judgments about what is relevant in their observations and experiments.

Consider, for example, how two great minds, Aristotle (384–322 B.C.) and Galileo (1564–1642), interpreted motion along a horizontal surface. Aristotle noted that objects given an initial push along the ground (or on a tabletop) always slow down and stop. Consequently, Aristotle argued that the natural state of an object is to be at rest. Galileo, in his reexamination of horizontal motion in the 1600s, imagined that if friction could be eliminated, an object given an initial push along a horizontal surface would continue to move indefinitely without stopping. He concluded that for an object to be in motion was just as natural as for it to be at rest. By inventing a new approach, Galileo founded our modern view of motion, and he did so with a leap of the imagination. Galileo made this leap conceptually, without actually eliminating friction.

Observation, with careful experimentation and measurement, is one side of the scientific process. The other side is the invention or creation of theories to explain and order the observations. Theories are never derived directly from observations. Observations may help inspire a theory, and theories are accepted or rejected based on the results of observation and experiment.

The great theories of science may be compared, as creative achievements, with great works of art or literature. But how does science differ from these other creative activities? One important difference is that science requires **testing** of its ideas or theories to see if their predictions are borne out by experiment.

Although the testing of theories distinguishes science from other creative fields, it should not be assumed that a theory is "proved" by testing. First of all, no measuring instrument is perfect, so exact confirmation is not possible. Furthermore, it is not possible to test a theory in every single possible circumstance. Hence a theory cannot be absolutely verified. Indeed, the history of science tells us that long-held theories can be replaced by new ones.

2 Models, Theories, and Laws

When scientists are trying to understand a particular set of phenomena, they often make use of a **model**. A model, in the scientist's sense, is a kind of analogy or mental image of the phenomena in terms of something we are familiar with. One example is the wave model of light. We cannot see waves of light as we can water waves. But it is valuable to think of light as made up of waves because experiments indicate that light behaves in many respects as water waves do.

The purpose of a model is to give us an approximate mental or visual picture—something to hold on to—when we cannot see what actually is happening. Models often give us a deeper understanding: the analogy to a known system (for instance, water waves in the above example) can suggest new experiments to perform and can provide ideas about what other related phenomena might occur.

You may wonder what the difference is between a theory and a model. Usually a model is relatively simple and provides a structural similarity to the phenomena being studied. A **theory** is broader, more detailed, and can give quantitatively testable predictions, often with great precision.

It is important, however, not to confuse a model or a theory with the real system or the phenomena themselves.

Scientists give the title **law** to certain concise but general statements about how nature behaves (that energy is conserved, for example). Sometimes the statement takes the form of a relationship or equation between quantities (such as Newton's second law, $F = ma$).

To be called a law, a statement must be found experimentally valid over a wide range of observed phenomena. For less general statements, the term **principle** is often used (such as Archimedes' principle).

Scientific laws are different from political laws in that the latter are *prescriptive*: they tell us how we ought to behave. Scientific laws are *descriptive*: they do not say how nature *should* behave, but rather are meant to describe how nature *does* behave. As with theories, laws cannot be tested in the infinite variety of cases possible. So we cannot be sure that any law is absolutely true. We use the term "law" when its validity has been tested over a wide range of cases, and when any limitations and the range of validity are clearly understood.

Scientists normally do their research as if the accepted laws and theories were true. But they are obliged to keep an open mind in case new information should alter the validity of any given law or theory.

3 Measurement and Uncertainty; Significant Figures

In the quest to understand the world around us, scientists seek to find relationships among physical quantities that can be measured.

Uncertainty

Reliable measurements are an important part of physics. But no measurement is absolutely precise. There is an uncertainty associated with every measurement. Among the most important sources of uncertainty, other than blunders, are the limited accuracy of every measuring instrument and the inability to read an instrument beyond some fraction of the smallest division shown. For example, if you were to use a centimeter ruler to measure the width of a board (Fig. 2), the result could be claimed to be precise to about 0.1 cm (1 mm), the smallest division on the ruler, although half of this value might be a valid claim as well. The reason is that it is difficult for the observer to estimate (or interpolate) between the smallest divisions. Furthermore, the ruler itself may not have been manufactured to an accuracy very much better than this.

FIGURE 2 Measuring the width of a board with a centimeter ruler. The uncertainty is about ± 1 mm.

Mary Teresa Giancoli

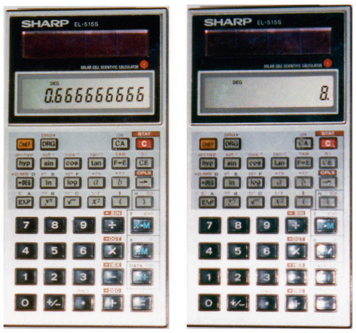
When giving the result of a measurement, it is important to state the **estimated uncertainty** in the measurement. For example, the width of a board might be written as 8.8 ± 0.1 cm. The ± 0.1 cm ("plus or minus 0.1 cm") represents the estimated uncertainty in the measurement, so that the actual width most likely lies between 8.7 and 8.9 cm. The **percent uncertainty** is the ratio of the uncertainty to the measured value, multiplied by 100. For example, if the measurement is 8.8 and the uncertainty about 0.1 cm, the percent uncertainty is

$$\frac{0.1}{8.8} \times 100\% \approx 1\%,$$

where $\approx$ means "is approximately equal to."

Often the uncertainty in a measured value is not specified explicitly. In such cases, the uncertainty is generally assumed to be one or a few units in the last digit specified. For example, if a length is given as 8.8 cm, the uncertainty is assumed to be about 0.1 cm or 0.2 cm. It is important in this case that you do not write 8.80 cm, for this implies an uncertainty on the order of 0.01 cm; it assumes that the length is probably between 8.79 cm and 8.81 cm, when actually you believe it is between 8.7 and 8.9 cm.

Douglas C. Giancoli



(a) (b)

FIGURE 3 These two calculators show the wrong number of significant figures. In (a), 2.0 was divided by 3.0. The correct final result would be 0.67. In (b), 2.5 was multiplied by 3.2. The correct result is 8.0.

Significant Figures

The number of reliably known digits in a number is called the number of **significant figures**. Thus there are four significant figures in the number 23.21 cm and two in the number 0.062 cm (the zeros in the latter are merely place holders that show where the decimal point goes). The number of significant figures may not always be clear. Take, for example, the number 80. Are there one or two significant figures? We need words here: If we say it is *roughly* 80 km between two cities, there is only one significant figure (the 8) since the zero is merely a place holder. If there is no suggestion that the 80 is a rough approximation, then we can often assume that it is 80 km within an accuracy of about 1 or 2 km, and then the 80 has two significant figures. If it is precisely 80 km, to within ± 0.1 km, then we write 80.0 km (three significant figures).

When making measurements, or when doing calculations, you should avoid the temptation to keep more digits in the final answer than is justified. For example, to calculate the area of a rectangle 11.3 cm by 6.8 cm, the result of multiplication would be 76.84 cm^2. But this answer is clearly not accurate to 0.01 cm^2, since (using the outer limits of the assumed uncertainty for each measurement) the result could be between $11.2\,\text{cm} \times 6.7\,\text{cm} = 75.04\,\text{cm}^2$ and $11.4\,\text{cm} \times 6.9\,\text{cm} = 78.66\,\text{cm}^2$. At best, we can quote the answer as 77 cm^2, which implies an uncertainty of about 1 or 2 cm^2. The other two digits (in the number 76.84 cm^2) must be dropped because they are not significant. As a rough general rule (i.e., in the absence of a detailed consideration of uncertainties), we can say that *the final result of a multiplication or division should have only as many digits as the number with the least number of significant figures used in the calculation*. In our example, 6.8 cm has the least number of significant figures, namely two. Thus the result 76.84 cm^2 needs to be rounded off to 77 cm^2.

PROBLEM SOLVING
Significant figure rule: Number of significant figures in final result should be same as the least significant input value

EXERCISE A The area of a rectangle 4.5 cm by 3.25 cm is correctly given by (*a*) 14.625 cm^2; (*b*) 14.63 cm^2; (*c*) 14.6 cm^2; (*d*) 15 cm^2.

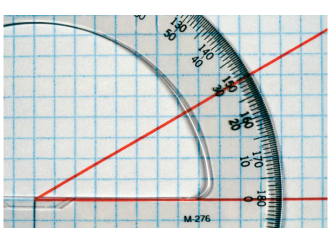
When adding or subtracting numbers, the final result is no more precise than the least precise number used. For example, the result of subtracting 0.57 from 3.6 is 3.0 (and not 3.03).

CAUTION
Calculators err with significant figures

PROBLEM SOLVING
Report only the proper number of significant figures in the final result. Keep extra digits during the calculation

Keep in mind when you use a calculator that all the digits it produces may not be significant. When you divide 2.0 by 3.0, the proper answer is 0.67, and not some such thing as 0.666666666. Digits should not be quoted in a result, unless they are truly significant figures. However, to obtain the most accurate result, you should normally *keep one or more extra significant figures throughout a calculation, and round off only in the final result*. (With a calculator, you can keep all its digits in intermediate results.) Note also that calculators sometimes give too few significant figures. For example, when you multiply 2.5×3.2, a calculator may give the answer as simply 8. But the answer is accurate to two significant figures, so the proper answer is 8.0. See Fig. 3.

FIGURE 4 Example 1. A protractor used to measure an angle.



Paul Silverman/Fundamental Photographs, NYC

CONCEPTUAL EXAMPLE 1 **Significant figures.** Using a protractor (Fig. 4), you measure an angle to be 30°. (*a*) How many significant figures should you quote in this measurement? (*b*) Use a calculator to find the cosine of the angle you measured.

RESPONSE (*a*) If you look at a protractor, you will see that the precision with which you can measure an angle is about one degree (certainly not 0.1°). So you can quote two significant figures, namely, 30° (not 30.0°). (*b*) If you enter cos 30° in your calculator, you will get a number like 0.866025403. However, the angle you entered is known only to two significant figures, so its cosine is correctly given by 0.87; you must round your answer to two significant figures.

EXERCISE B Do 0.00324 and 0.00056 have the same number of significant figures?

Be careful not to confuse significant figures with the number of decimal places.

EXERCISE C For each of the following numbers, state the number of significant figures and the number of decimal places: (*a*) 1.23; (*b*) 0.123; (*c*) 0.0123.

Scientific Notation

We commonly write numbers in "powers of ten," or "scientific" notation—for instance 36,900 as 3.69×10^4, or 0.0021 as 2.1×10^{-3}. One advantage of scientific notation is that it allows the number of significant figures to be clearly expressed. For example, it is not clear whether 36,900 has three, four, or five significant figures. With powers of ten notation the ambiguity can be avoided: if the number is known to three significant figures, we write 3.69×10^4, but if it is known to four, we write 3.690×10^4.

EXERCISE D Write each of the following in scientific notation and state the number of significant figures for each: (*a*) 0.0258, (*b*) 42,300, (*c*) 344.50.

Percent Uncertainty versus Significant Figures

The significant figures rule is only approximate, and in some cases may underestimate the accuracy (or uncertainty) of the answer. Suppose for example we divide 97 by 92:

$$\frac{97}{92} = 1.05 \approx 1.1.$$

Both 97 and 92 have two significant figures, so the rule says to give the answer as 1.1. Yet the numbers 97 and 92 both imply an uncertainty of ± 1 if no other uncertainty is stated. Now 92 ± 1 and 97 ± 1 both imply an uncertainty of about 1% ($1/92 \approx 0.01 = 1\%$). But the final result to two significant figures is 1.1, with an implied uncertainty of ± 0.1, which is an uncertainty of $0.1/1.1 \approx 0.1 \approx 10\%$. In this case it is better to give the answer as 1.05 (which is three significant figures). Why? Because 1.05 implies an uncertainty of ± 0.01 which is $0.01/1.05 \approx 0.01 \approx 1\%$, just like the uncertainty in the original numbers 92 and 97.

SUGGESTION: Use the significant figures rule, but consider the % uncertainty too, and add an extra digit if it gives a more realistic estimate of uncertainty.

Approximations

Much of physics involves approximations, often because we do not have the means to solve a problem precisely. For example, we may choose to ignore air resistance or friction in doing a Problem even though they are present in the real world, and then our calculation is only an approximation. In doing Problems, we should be aware of what approximations we are making, and be aware that the precision of our answer may not be nearly as good as the number of significant figures given in the result.

Accuracy versus Precision

There is a technical difference between "precision" and "accuracy." **Precision** in a strict sense refers to the repeatability of the measurement using a given instrument. For example, if you measure the width of a board many times, getting results like 8.81 cm, 8.85 cm, 8.78 cm, 8.82 cm (interpolating between the 0.1 cm marks as best as possible each time), you could say the measurements give a *precision* a bit better than 0.1 cm. **Accuracy** refers to how close a measurement is to the true value. For example, if the ruler shown in Fig. 2 was manufactured with a 2% error, the accuracy of its measurement of the board's width (about 8.8 cm) would be about 2% of 8.8 cm or about ± 0.2 cm. Estimated uncertainty is meant to take both accuracy and precision into account.

4 Units, Standards, and the SI System

The measurement of any quantity is made relative to a particular standard or **unit**, and this unit must be specified along with the numerical value of the quantity. For example, we can measure length in British units such as inches, feet, or miles, or in the metric system in centimeters, meters, or kilometers. To specify that the length of a particular object is 18.6 is meaningless. The unit *must* be given; for clearly, 18.6 meters is very different from 18.6 inches or 18.6 millimeters.

For any unit we use, such as the meter for distance or the second for time, we need to define a **standard** which defines exactly how long one meter or one second is. It is important that standards be chosen that are readily reproducible so that anyone needing to make a very accurate measurement can refer to the standard in the laboratory.

TABLE 1 Some Typical Lengths or Distances (order of magnitude)

Length (or Distance)	Meters (approximate)
Neutron or proton (diameter)	10^{-15} m
Atom (diameter)	10^{-10} m
Virus [see Fig. 5a]	10^{-7} m
Sheet of paper (thickness)	10^{-4} m
Finger width	10^{-2} m
Football field length	10^{2} m
Height of Mt. Everest [see Fig. 5b]	10^{4} m
Earth diameter	10^{7} m
Earth to Sun	10^{11} m
Earth to nearest star	10^{16} m
Earth to nearest galaxy	10^{22} m
Earth to farthest galaxy visible	10^{26} m

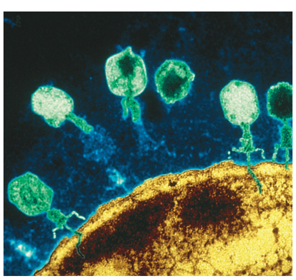
Length

The first truly international standard was the **meter** (abbreviated m) established as the standard of **length** by the French Academy of Sciences in the 1790s. The standard meter was originally chosen to be one ten-millionth of the distance from the Earth's equator to either pole,† and a platinum rod to represent this length was made. (One meter is, very roughly, the distance from the tip of your nose to the tip of your finger, with arm and hand stretched out to the side.) In 1889, the meter was defined more precisely as the distance between two finely engraved marks on a particular bar of platinum–iridium alloy. In 1960, to provide greater precision and reproducibility, the meter was redefined as 1,650,763.73 wavelengths of a particular orange light emitted by the gas krypton-86. In 1983 the meter was again redefined, this time in terms of the speed of light (whose best measured value in terms of the older definition of the meter was 299,792,458 m/s, with an uncertainty of 1 m/s). The new definition reads: "The meter is the length of path traveled by light in vacuum during a time interval of 1/299,792,458 of a second."‡

British units of length (inch, foot, mile) are now defined in terms of the meter. The inch (in.) is defined as precisely 2.54 centimeters (cm; 1 cm = 0.01 m). Table 1 presents some typical lengths, from very small to very large, rounded off to the nearest power of ten. See also Fig. 5. [Note that the abbreviation for inches (in.) is the only one with a period, to distinguish it from the word "in".]

FIGURE 5 Some lengths: (a) viruses (about 10^{-7} m long) attacking a cell; (b) Mt. Everest's height is on the order of 10^4 m (8850 m, to be precise).

Oliver Meckes/Ottawa/Photo Researchers, Inc.



(a)

Douglas C. Giancoli

(b)

Time

The standard unit of **time** is the **second** (s). For many years, the second was defined as 1/86,400 of a mean solar day (24 h/day × 60 min/h × 60 s/min = 86,400 s/day). The standard second is now defined more precisely in terms of the frequency of radiation emitted by cesium atoms when they pass between two particular states. [Specifically, one second is defined as the time required for 9,192,631,770 periods of this radiation.] There are, by definition, 60 s in one minute (min) and 60 minutes in one hour (h). Table 2 presents a range of measured time intervals, rounded off to the nearest power of ten.

Mass

The standard unit of **mass** is the **kilogram** (kg). The standard mass is a particular platinum–iridium cylinder, kept at the International Bureau of Weights and Measures near Paris, France, whose mass is defined as exactly 1 kg. A range of masses is presented in Table 3. [For practical purposes, 1 kg weighs about 2.2 pounds on Earth.]

†Modern measurements of the Earth's circumference reveal that the intended length is off by about one-fiftieth of 1%. Not bad!

‡The new definition of the meter has the effect of giving the speed of light the exact value of 299,792,458 m/s.

TABLE 2 Some Typical Time Intervals

Time Interval	Seconds (approximate)
Lifetime of very unstable subatomic particle	10^{-23} s
Lifetime of radioactive elements	10^{-22} s to 10^{28} s
Lifetime of muon	10^{-6} s
Time between human heartbeats	10^{0} s (= 1 s)
One day	10^{5} s
One year	3×10^{7} s
Human life span	2×10^{9} s
Length of recorded history	10^{11} s
Humans on Earth	10^{14} s
Life on Earth	10^{17} s
Age of Universe	10^{18} s

TABLE 3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

When dealing with atoms and molecules, we usually use the **unified atomic mass unit** (u). In terms of the kilogram,

$$1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}.$$

Unit Prefixes

In the metric system, the larger and smaller units are defined in multiples of 10 from the standard unit, and this makes calculation particularly easy. Thus 1 kilometer (km) is 1000 m, 1 centimeter is $\frac{1}{100}$ m, 1 millimeter (mm) is $\frac{1}{1000}$ m or $\frac{1}{10}$ cm, and so on. The prefixes "centi-," "kilo-," and others are listed in Table 4 and can be applied not only to units of length but to units of volume, mass, or any other metric unit. For example, a centiliter (cL) is $\frac{1}{100}$ liter (L), and a kilogram (kg) is 1000 grams (g).

TABLE 4 Metric (SI) Prefixes

Prefix	Abbreviation	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

† μ is the Greek letter "mu."

Systems of Units

When dealing with the laws and equations of physics it is very important to use a consistent set of units. Several systems of units have been in use over the years. Today the most important is the **Système International** (French for International System), which is abbreviated SI. In SI units, the standard of length is the meter, the standard for time is the second, and the standard for mass is the kilogram. This system used to be called the MKS (meter-kilogram-second) system.

A second metric system is the **cgs system**, in which the centimeter, gram, and second are the standard units of length, mass, and time, as abbreviated in the title. The **British engineering system** has as its standards the foot for length, the pound for force, and the second for time.

We use SI units almost exclusively in this text.

Base versus Derived Quantities

Physical quantities can be divided into two categories: *base quantities* and *derived quantities*. The corresponding units for these quantities are called *base units* and *derived units*. A **base quantity** must be defined in terms of a standard. Scientists, in the interest of simplicity, want the smallest number of base quantities possible consistent with a full description of the physical world. This number turns out to be seven, and those used in the SI are given in Table 5. All other quantities can be defined in terms of these seven base quantities,† and hence are referred to as **derived quantities**. An example of a derived quantity is speed, which is defined as distance divided by the time it takes to travel that distance. To define any quantity, whether base or derived, we can specify a rule or procedure, and this is called an **operational definition**.

†The only exceptions are for angle (radians) and solid angle (steradian). No general agreement has been reached as to whether these are base or derived quantities.

TABLE 5 SI Base Quantities and Units

Quantity	Unit	Unit Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

5 Converting Units

Any quantity we measure, such as a length, a speed, or an electric current, consists of a number *and* a unit. Often we are given a quantity in one set of units, but we want it expressed in another set of units. For example, suppose we measure that a table is 21.5 inches wide, and we want to express this in centimeters. We must use a **conversion factor**, which in this case is (by definition) exactly

$$1\text{ in.} = 2.54\text{ cm}$$

or, written another way,

$$1 = 2.54\text{ cm/in.}$$

Since multiplying by one does not change anything, the width of our table, in cm, is

$$21.5\text{ inches} = (21.5\ \cancel{\text{in.}}) \times \left(2.54\,\frac{\text{cm}}{\cancel{\text{in.}}}\right) = 54.6\text{ cm.}$$

Note how the units (inches in this case) cancelled out. Let's consider some conversion Examples.

PHYSICS APPLIED
The world's tallest peaks

A.Viansson

FIGURE 6 The world's second highest peak, K2, whose summit is considered the most difficult of the "8000-ers." K2 is seen here from the north (China).

TABLE 6 The 8000-m Peaks

Peak	Height (m)
Mt. Everest	8850
K2	8611
Kangchenjunga	8586
Lhotse	8516
Makalu	8462
Cho Oyu	8201
Dhaulagiri	8167
Manaslu	8156
Nanga Parbat	8125
Annapurna	8091
Gasherbrum I	8068
Broad Peak	8047
Gasherbrum II	8035
Shisha Pangma	8013

EXAMPLE 2 The 8000-m peaks. The fourteen tallest peaks in the world (Fig. 6 and Table 6) are referred to as "eight-thousanders," meaning their summits are over 8000 m above sea level. What is the elevation, in feet, of an elevation of 8000 m?

APPROACH We need simply to convert meters to feet, and we can start with the conversion factor $1\text{ in.} = 2.54\text{ cm}$, which is exact. That is, $1\text{ in.} = 2.5400\text{ cm}$ to any number of significant figures, because it is *defined* to be.

SOLUTION One foot is 12 in., so we can write

$$1\text{ ft} = (12\ \cancel{\text{in.}})\left(2.54\,\frac{\text{cm}}{\cancel{\text{in.}}}\right) = 30.48\text{ cm} = 0.3048\text{ m,}$$

which is exact. Note how the units cancel (colored slashes). We can rewrite this equation to find the number of feet in 1 meter:

$$1\text{ m} = \frac{1\text{ ft}}{0.3048} = 3.28084\text{ ft.}$$

We multiply this equation by 8000.0 (to have five significant figures):

$$8000.0\text{ m} = (8000.0\ \cancel{\text{m}})\left(3.28084\,\frac{\text{ft}}{\cancel{\text{m}}}\right) = 26{,}247\text{ ft.}$$

An elevation of 8000 m is 26,247 ft above sea level.

NOTE We could have done the conversion all in one line:

$$8000.0\text{ m} = (8000.0\ \cancel{\text{m}})\left(\frac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}}\right)\left(\frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}}\right)\left(\frac{1\text{ ft}}{12\ \cancel{\text{in.}}}\right) = 26{,}247\text{ ft.}$$

The key is to multiply conversion factors, each equal to one (= 1.0000), and to make sure the units cancel.

EXERCISE E There are only 14 eight-thousand-meter peaks in the world (see Example 2), and their names and elevations are given in Table 6. They are all in the Himalaya mountain range in India, Pakistan, Tibet, and China. Determine the elevation of the world's three highest peaks in feet.

EXAMPLE 3 **Apartment area.** You have seen a nice apartment whose floor area is 880 square feet (ft^2). What is its area in square meters?

APPROACH We use the same conversion factor, $1\text{ in.} = 2.54\text{ cm}$, but this time we have to use it twice.

SOLUTION Because $1\text{ in.} = 2.54\text{ cm} = 0.0254\text{ m}$, then $1\text{ ft}^2 = (12\text{ in.})^2(0.0254\text{ m/in.})^2 = 0.0929\text{ m}^2$. So $880\text{ ft}^2 = (880\text{ ft}^2)(0.0929\text{ m}^2/\text{ft}^2) \approx 82\text{ m}^2$.

NOTE As a rule of thumb, an area given in ft^2 is roughly 10 times the number of square meters (more precisely, about $10.8\times$).

EXAMPLE 4 **Speeds.** Where the posted speed limit is 55 miles per hour (mi/h or mph), what is this speed (*a*) in meters per second (m/s) and (*b*) in kilometers per hour (km/h)?

APPROACH We again use the conversion factor $1\text{ in.} = 2.54\text{ cm}$, and we recall that there are 5280 ft in a mile and 12 inches in a foot; also, one hour contains $(60\text{ min/h}) \times (60\text{ s/min}) = 3600\text{ s/h}$.

SOLUTION (*a*) We can write 1 mile as

$$1\text{ mi} = (5280\,\cancel{\text{ft}})\left(12\,\frac{\cancel{\text{in.}}}{\cancel{\text{ft}}}\right)\left(2.54\,\frac{\cancel{\text{cm}}}{\cancel{\text{in.}}}\right)\left(\frac{1\text{ m}}{100\,\cancel{\text{cm}}}\right) = 1609\text{ m}.$$

We also know that 1 hour contains 3600 s, so

$$55\,\frac{\text{mi}}{\text{h}} = \left(55\,\frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(1609\,\frac{\text{m}}{\cancel{\text{mi}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\text{ s}}\right) = 25\,\frac{\text{m}}{\text{s}},$$

where we rounded off to two significant figures.
(*b*) Now we use $1\text{ mi} = 1609\text{ m} = 1.609\text{ km}$; then

$$55\,\frac{\text{mi}}{\text{h}} = \left(55\,\frac{\cancel{\text{mi}}}{\text{h}}\right)\left(1.609\,\frac{\text{km}}{\cancel{\text{mi}}}\right) = 88\,\frac{\text{km}}{\text{h}}.$$

NOTE Each conversion factor is equal to one.

PROBLEM SOLVING
Conversion factors = 1

EXERCISE F Would a driver traveling at 15 m/s in a 35 mi/h zone be exceeding the speed limit?

PROBLEM SOLVING
Unit conversion is wrong if units do not cancel

When changing units, you can avoid making an error in the use of conversion factors by checking that units cancel out properly. For example, in our conversion of 1 mi to 1609 m in Example 4(*a*), if we had incorrectly used the factor $\left(\frac{100\text{ cm}}{1\text{ m}}\right)$ instead of $\left(\frac{1\text{ m}}{100\text{ cm}}\right)$, the centimeter units would not have cancelled out; we would not have ended up with meters.

6 Order of Magnitude: Rapid Estimating

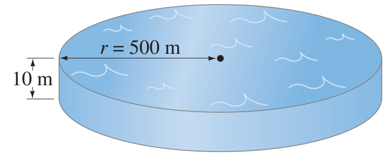
We are sometimes interested only in an approximate value for a quantity. This might be because an accurate calculation would take more time than it is worth or would require additional data that are not available. In other cases, we may want to make a rough estimate in order to check an accurate calculation made on a calculator, to make sure that no blunders were made when the numbers were entered.

PROBLEM SOLVING
How to make a rough estimate

A rough estimate is made by rounding off all numbers to one significant figure and its power of 10, and after the calculation is made, again only one significant figure is kept. Such an estimate is called an **order-of-magnitude estimate** and can be accurate within a factor of 10, and often better. In fact, the phrase "order of magnitude" is sometimes used to refer simply to the power of 10.

(a)

Douglas C. Giancoli



(b)

FIGURE 7 Example 5. (a) How much water is in this lake? (Photo is of one of the Rae Lakes in the Sierra Nevada of California.) (b) Model of the lake as a cylinder. [We could go one step further and estimate the mass or weight of this lake. We will see later that water has a density of $1000\,\mathrm{kg/m^3}$, so this lake has a mass of about $(10^3\,\mathrm{kg/m^3})(10^7\,\mathrm{m^3}) \approx 10^{10}\,\mathrm{kg}$, which is about 10 billion kg or 10 million metric tons. (A metric ton is 1000 kg, about 2200 lbs, slightly larger than a British ton, 2000 lbs.)]

PHYSICS APPLIED
Estimating the volume (or mass) of a lake; see also Fig. 7

EXAMPLE 5 ESTIMATE Volume of a lake. Estimate how much water there is in a particular lake, Fig. 7a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

APPROACH No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate the volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder (Fig. 7b).

SOLUTION The volume V of a cylinder is the product of its height h times the area of its base: $V = h\pi r^2$, where r is the radius of the circular base. The radius r is $\frac{1}{2}\,\mathrm{km} = 500\,\mathrm{m}$, so the volume is approximately

$$V = h\pi r^2 \approx (10\,\mathrm{m}) \times (3) \times (5 \times 10^2\,\mathrm{m})^2 \approx 8 \times 10^6\,\mathrm{m^3} \approx 10^7\,\mathrm{m^3},$$

where π was rounded off to 3. So the volume is on the order of $10^7\,\mathrm{m^3}$, ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate $(10^7\,\mathrm{m^3})$ is probably better to quote than the $8 \times 10^6\,\mathrm{m^3}$ figure.

NOTE To express our result in U.S. gallons, we see in the Table on the inside front cover that $1\,\mathrm{liter} = 10^{-3}\,\mathrm{m^3} \approx \frac{1}{4}\,\mathrm{gallon}$. Hence, the lake contains $(8 \times 10^6\,\mathrm{m^3})(1\,\mathrm{gallon}/4 \times 10^{-3}\,\mathrm{m^3}) \approx 2 \times 10^9$ gallons of water.

EXAMPLE 6 ESTIMATE Thickness of a page. Estimate the thickness of a page of a text.

PROBLEM SOLVING
Use symmetry when possible

APPROACH At first you might think that a special measuring device, a micrometer (Fig. 8), is needed to measure the thickness of one page since an ordinary ruler clearly won't do. But we can use a trick or, to put it in physics terms, make use of a *symmetry*: we can make the reasonable assumption that all the pages of a text are equal in thickness.

SOLUTION We can use a ruler to measure many pages at once. If you measure the thickness of the first 500 pages of a book (page 1 to page 500), you might get something like 1.5 cm. Note that 500 numbered pages, counted front and back, is 250 separate sheets of paper. So one page must have a thickness of about

$$\frac{1.5\text{ cm}}{250\text{ pages}} \approx 6\times10^{-3}\text{ cm} = 6\times10^{-2}\text{ mm},$$

or less than a tenth of a millimeter (0.1 mm).

EXAMPLE 7 ESTIMATE Height by triangulation. Estimate the height of the building shown in Fig. 9, by "triangulation," with the help of a bus-stop pole and a friend.

APPROACH By standing your friend next to the pole, you estimate the height of the pole to be 3 m. You next step away from the pole until the top of the pole is in line with the top of the building, Fig. 9a. You are 5 ft 6 in. tall, so your eyes are about 1.5 m above the ground. Your friend is taller, and when she stretches out her arms, one hand touches you, and the other touches the pole, so you estimate that distance as 2 m (Fig. 9a). You then pace off the distance from the pole to the base of the building with big, 1-m-long steps, and you get a total of 16 steps or 16 m.

SOLUTION Now you draw, to scale, the diagram shown in Fig. 9b using these measurements. You can measure, right on the diagram, the last side of the triangle to be about $x = 13$ m. Alternatively, you can use similar triangles to obtain the height x:

$$\frac{1.5\text{ m}}{2\text{ m}} = \frac{x}{18\text{ m}}, \quad \text{so} \quad x \approx 13\tfrac{1}{2}\text{ m}.$$

Finally you add in your eye height of 1.5 m above the ground to get your final result: the building is about 15 m tall.

EXAMPLE 8 ESTIMATE Estimating the radius of Earth. Believe it or not, you can estimate the radius of the Earth without having to go into space. If you have ever been on the shore of a large lake, you may have noticed that you cannot see the beaches, piers, or rocks at water level across the lake on the opposite shore. The lake seems to bulge out between you and the opposite shore—a good clue that the Earth is round. Suppose you climb a stepladder and discover that when your eyes are 10 ft (3.0 m) above the water, you can just see the rocks at water level on the opposite shore. From a map, you estimate the distance to the opposite shore as $d \approx 6.1$ km. Use Fig. 10 with $h = 3.0$ m to estimate the radius R of the Earth.

APPROACH We use simple geometry, including the theorem of Pythagoras, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse of any right triangle, and a and b are the lengths of the other two sides.

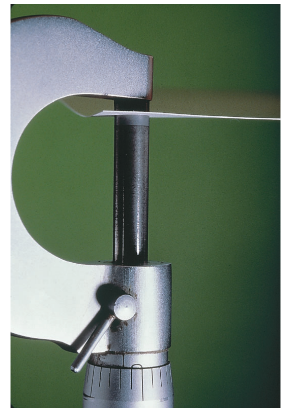
SOLUTION For the right triangle of Fig. 10, the two sides are the radius of the Earth R and the distance $d = 6.1\text{ km} = 6100\text{ m}$. The hypotenuse is approximately the length $R + h$, where $h = 3.0$ m. By the Pythagorean theorem,

$$R^2 + d^2 \approx (R+h)^2$$
$$\approx R^2 + 2hR + h^2.$$

We solve algebraically for R, after cancelling R^2 on both sides:

$$R \approx \frac{d^2 - h^2}{2h} = \frac{(6100\text{ m})^2 - (3.0\text{ m})^2}{6.0\text{ m}} = 6.2\times10^6\text{ m} = 6200\text{ km}.$$

NOTE Precise measurements give 6380 km. But look at your achievement! With a few simple rough measurements and simple geometry, you made a good estimate of the Earth's radius. You did not need to go out in space, nor did you need a very long measuring tape. Now you know the answer to the Chapter-Opening Question.



Larry Voight/Photo Researchers, Inc.

FIGURE 8 Example 6. Micrometer used for measuring small thicknesses.

FIGURE 9 Example 7. Diagrams are really useful!

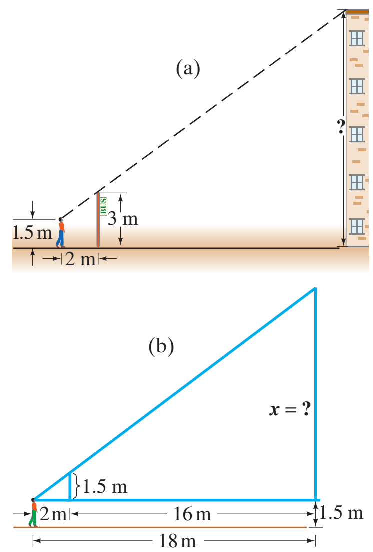
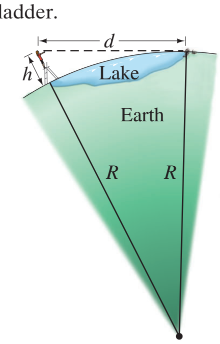


FIGURE 10 Example 8, but not to scale. You can see small rocks at water level on the opposite shore of a lake 6.1 km wide if you stand on a stepladder.



EXAMPLE 9 ESTIMATE Total number of heartbeats. Estimate the total number of beats a typical human heart makes in a lifetime.

APPROACH A typical resting heart rate is 70 beats/min. But during exercise it can be a lot higher. A reasonable average might be 80 beats/min.

SOLUTION One year in terms of seconds is $(24\text{ h})(3600\text{ s/h})(365\text{ d}) \approx 3 \times 10^7\text{ s}$. If an average person lives 70 years $= (70\text{ yr})(3 \times 10^7\text{ s/yr}) \approx 2 \times 10^9\text{ s}$, then the total number of heartbeats would be about

$$\left(80\,\frac{\text{beats}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)(2 \times 10^9\text{ s}) \;\approx\; 3 \times 10^9,$$

or 3 trillion.

Another technique for estimating, this one made famous by Enrico Fermi to his physics students, is to estimate the number of piano tuners in a city, say, Chicago or San Francisco. To get a rough order-of-magnitude estimate of the number of piano tuners today in San Francisco, a city of about 700,000 inhabitants, we can proceed by estimating the number of functioning pianos, how often each piano is tuned, and how many pianos each tuner can tune. To estimate the number of pianos in San Francisco, we note that certainly not everyone has a piano. A guess of 1 family in 3 having a piano would correspond to 1 piano per 12 persons, assuming an average family of 4 persons. As an order of magnitude, let's say 1 piano per 10 people. This is certainly more reasonable than 1 per 100 people, or 1 per every person, so let's proceed with the estimate that 1 person in 10 has a piano, or about 70,000 pianos in San Francisco. Now a piano tuner needs an hour or two to tune a piano. So let's estimate that a tuner can tune 4 or 5 pianos a day. A piano ought to be tuned every 6 months or a year—let's say once each year. A piano tuner tuning 4 pianos a day, 5 days a week, 50 weeks a year can tune about 1000 pianos a year. So San Francisco, with its (very) roughly 70,000 pianos, needs about 70 piano tuners. This is, of course, only a rough estimate.† It tells us that there must be many more than 10 piano tuners, and surely not as many as 1000.

PROBLEM SOLVING
Estimating how many piano tuners there are in a city

*7 Dimensions and Dimensional Analysis

When we speak of the **dimensions** of a quantity, we are referring to the type of base units or base quantities that make it up. The dimensions of area, for example, are always length squared, abbreviated $[L^2]$, using square brackets; the units can be square meters, square feet, cm^2, and so on. Velocity, on the other hand, can be measured in units of km/h, m/s, or mi/h, but the dimensions are always a length $[L]$ divided by a time $[T]$: that is, $[L/T]$.

The formula for a quantity may be different in different cases, but the dimensions remain the same. For example, the area of a triangle of base b and height h is $A = \frac{1}{2}bh$, whereas the area of a circle of radius r is $A = \pi r^2$. The formulas are different in the two cases, but the dimensions of area are always $[L^2]$.

Dimensions can be used as a help in working out relationships, a procedure referred to as **dimensional analysis**. One useful technique is the use of dimensions to check if a relationship is *incorrect*. Note that we add or subtract quantities only if they have the same dimensions (we don't add centimeters and hours); and the quantities on each side of an equals sign must have the same dimensions. (In numerical calculations, the units must also be the same on both sides of an equation.)

For example, suppose you derived the equation $v = v_0 + \frac{1}{2}at^2$, where v is the speed of an object after a time t, v_0 is the object's initial speed, and the object undergoes an acceleration a. Let's do a dimensional check to see if this equation

†A check of the San Francisco Yellow Pages (done after this calculation) reveals about 50 listings. Each of these listings may employ more than one tuner, but on the other hand, each may also do repairs as well as tuning. In any case, our estimate is reasonable.

could be correct or is surely incorrect. Note that numerical factors, like the $\frac{1}{2}$ here, do not affect dimensional checks. We write a dimensional equation as follows, remembering that the dimensions of speed are $[L/T]$ and the dimensions of acceleration are $[L/T^2]$:

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right][T^2] = \left[\frac{L}{T}\right] + [L].$$

The dimensions are incorrect: on the right side, we have the sum of quantities whose dimensions are not the same. Thus we conclude that an error was made in the derivation of the original equation.

A dimensional check can only tell you when a relationship is wrong. It can't tell you if it is completely right. For example, a dimensionless numerical factor (such as $\frac{1}{2}$ or 2π) could be missing.

Dimensional analysis can also be used as a quick check on an equation you are not sure about. For example, suppose that you can't remember whether the equation for the period of a simple pendulum T (the time to make one back-and-forth swing) of length ℓ is $T = 2\pi\sqrt{\ell/g}$ or $T = 2\pi\sqrt{g/\ell}$, where g is the acceleration due to gravity and, like all accelerations, has dimensions $[L/T^2]$. (Do not worry about these formulas; what we are concerned about here is a person's recalling whether it contains ℓ/g or g/ℓ.) A dimensional check shows that the former (ℓ/g) is correct:

$$[T] = \sqrt{\frac{[L]}{[L/T^2]}} = \sqrt{[T^2]} = [T],$$

whereas the latter (g/ℓ) is not:

$$[T] \neq \sqrt{\frac{[L/T^2]}{[L]}} = \sqrt{\frac{1}{[T^2]}} = \frac{1}{[T]}.$$

Note that the constant 2π has no dimensions and so can't be checked using dimensions.

EXAMPLE 10 **Planck length.** The smallest meaningful measure of length is called the "Planck length," and is defined in terms of three fundamental constants in nature, the speed of light $c = 3.00 \times 10^8\,\text{m/s}$, the gravitational constant $G = 6.67 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2$, and Planck's constant $h = 6.63 \times 10^{-34}\,\text{kg}\cdot\text{m}^2/\text{s}$. The Planck length λ_P (λ is the Greek letter "lambda") is given by the following combination of these three constants:

$$\lambda_P = \sqrt{\frac{Gh}{c^3}}.$$

Show that the dimensions of λ_P are length $[L]$, and find the order of magnitude of λ_P.

APPROACH We rewrite the above equation in terms of dimensions. The dimensions of c are $[L/T]$, of G are $[L^3/MT^2]$, and of h are $[ML^2/T]$.

SOLUTION The dimensions of λ_P are

$$\sqrt{\frac{[L^3/MT^2][ML^2/T]}{[L^3/T^3]}} = \sqrt{[L^2]} = [L]$$

which is a length. The value of the Planck length is

$$\lambda_P = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2)(6.63 \times 10^{-34}\,\text{kg}\cdot\text{m}^2/\text{s})}{(3.0 \times 10^8\,\text{m/s})^3}} \approx 4 \times 10^{-35}\,\text{m},$$

which is on the order of 10^{-34} or 10^{-35} m.

NOTE Some recent theories suggest that the smallest particles (quarks, leptons) have sizes on the order of the Planck length, 10^{-35} m. These theories also suggest that the "Big Bang," with which the Universe is believed to have begun, started from an initial size on the order of the Planck length.

Summary

Physics, like other sciences, is a creative endeavor. It is not simply a collection of facts. Important **theories** are created with the idea of explaining **observations**. To be accepted, theories are **tested** by comparing their predictions with the results of actual experiments. Note that, in general, a theory cannot be "proved" in an absolute sense.

Scientists often devise models of physical phenomena. A **model** is a kind of picture or analogy that helps to describe the phenomena in terms of something we already know. A **theory**, often developed from a model, is usually deeper and more complex than a simple model.

A scientific **law** is a concise statement, often expressed in the form of an equation, which quantitatively describes a wide range of phenomena.

Measurements play a crucial role in physics, but can never be perfectly precise. It is important to specify the **uncertainty** of a measurement either by stating it directly using the ± notation, and/or by keeping only the correct number of **significant figures**.

Physical quantities are always specified relative to a particular standard or **unit**, and the unit used should always be stated. The commonly accepted set of units today is the **Système International** (SI), in which the standard units of length, mass, and time are the **meter**, **kilogram**, and **second**.

When converting units, check all **conversion factors** for correct cancellation of units.

Making rough, **order-of-magnitude estimates** is a very useful technique in science as well as in everyday life.

[*The **dimensions** of a quantity refer to the combination of base quantities that comprise it. Velocity, for example, has dimensions of [length/time] or $[L/T]$. **Dimensional analysis** can be used to check a relationship for correct form.]

Answers to Exercises

A: (*d*).

B: No: they have 3 and 2, respectively.

C: All three have three significant figures, although the number of decimal places is (*a*) 2, (*b*) 3, (*c*) 4.

D: (*a*) 2.58×10^{-2}, 3; (*b*) 4.23×10^{4}, 3 (probably); (*c*) 3.4450×10^{2}, 5.

E: Mt. Everest, 29,035 ft; K2, 28,251 ft; Kangchenjunga, 28,169 ft.

F: No: 15 m/s ≈ 34 mi/h.

Introduction, Measurement, Estimating Problem Set

Questions

1. What are the merits and drawbacks of using a person's foot as a standard? Consider both (*a*) a particular person's foot, and (*b*) any person's foot. Keep in mind that it is advantageous that fundamental standards be accessible (easy to compare to), invariable (do not change), indestructible, and reproducible.

2. Why is it incorrect to think that the more digits you represent in your answer, the more accurate it is?

3. When traveling a highway in the mountains, you may see elevation signs that read "914 m (3000 ft)." Critics of the metric system claim that such numbers show the metric system is more complicated. How would you alter such signs to be more consistent with a switch to the metric system?

4. What is wrong with this road sign:

Memphis 7 mi (11.263 km)?

5. For an answer to be complete, the units need to be specified. Why?

6. Discuss how the notion of symmetry could be used to estimate the number of marbles in a 1-liter jar.

7. You measure the radius of a wheel to be 4.16 cm. If you multiply by 2 to get the diameter, should you write the result as 8 cm or as 8.32 cm? Justify your answer.

8. Express the sine of 30.0° with the correct number of significant figures.

9. A recipe for a soufflé specifies that the measured ingredients must be exact, or the soufflé will not rise. The recipe calls for 6 large eggs. The size of "large" eggs can vary by 10%, according to the USDA specifications. What does this tell you about how exactly you need to measure the other ingredients?

10. List assumptions useful to estimate the number of car mechanics in (*a*) San Francisco, (*b*) your hometown, and then make the estimates.

11. Suggest a way to measure the distance from Earth to the Sun.

***12.** Can you set up a complete set of base quantities, as in Table 5, that does not include length as one of them?

TABLE 5
SI Base Quantities and Units

Quantity	Unit	Unit Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Problems

[The Problems in this Section are ranked I, II, or III according to estimated difficulty, with (I) Problems being easiest. Level (III) Problems are meant mainly as a challenge for the best students, for "extra credit." The Problems are arranged by Sections, meaning that the reader should have read up to and including that Section, but this Chapter also has a group of General Problems that are not arranged by Section and not ranked.]

3 Measurement, Uncertainty, Significant Figures

(*Note:* In Problems, assume a number like 6.4 is accurate to ± 0.1; and 950 is ± 10 unless 950 is said to be "precisely" or "very nearly" 950, in which case assume 950 ± 1.)

1. (I) The age of the universe is thought to be about 14 billion years. Assuming two significant figures, write this in powers of ten in (*a*) years, (*b*) seconds.

2. (I) How many significant figures do each of the following numbers have: (*a*) 214, (*b*) 81.60, (*c*) 7.03, (*d*) 0.03, (*e*) 0.0086, (*f*) 3236, and (*g*) 8700?

3. (I) Write the following numbers in powers of ten notation: (*a*) 1.156, (*b*) 21.8, (*c*) 0.0068, (*d*) 328.65, (*e*) 0.219, and (*f*) 444.

4. (I) Write out the following numbers in full with the correct number of zeros: (*a*) 8.69×10^4, (*b*) 9.1×10^3, (*c*) 8.8×10^{-1}, (*d*) 4.76×10^2, and (*e*) 3.62×10^{-5}.

5. (II) What is the percent uncertainty in the measurement 5.48 ± 0.25 m?

6. (II) Time intervals measured with a stopwatch typically have an uncertainty of about 0.2 s, due to human reaction time at the start and stop moments. What is the percent uncertainty of a hand-timed measurement of (*a*) 5 s, (*b*) 50 s, (*c*) 5 min?

7. (II) Add $(9.2 \times 10^3\,\text{s}) + (8.3 \times 10^4\,\text{s}) + (0.008 \times 10^6\,\text{s})$.

8. (II) Multiply 2.079×10^2 m by 0.082×10^{-1}, taking into account significant figures.

9. (III) For small angles θ, the numerical value of $\sin\theta$ is approximately the same as the numerical value of $\tan\theta$. Find the largest angle for which sine and tangent agree to within two significant figures.

10. (III) What, roughly, is the percent uncertainty in the volume of a spherical beach ball whose radius is $r = 0.84 \pm 0.04$ m?

4 and 5 Units, Standards, SI, Converting Units

11. (I) Write the following as full (decimal) numbers with standard units: (*a*) 286.6 mm, (*b*) 85 μV, (*c*) 760 mg, (*d*) 60.0 ps, (*e*) 22.5 fm, (*f*) 2.50 gigavolts.

12. (I) Express the following using the prefixes of Table 4: (*a*) 1×10^6 volts, (*b*) 2×10^{-6} meters, (*c*) 6×10^3 days, (*d*) 18×10^2 bucks, and (*e*) 8×10^{-8} seconds.

From Chapter 1 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

TABLE 4 Metric (SI) Prefixes

Prefix	Abbreviation	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

† μ is the Greek letter "mu."

13. (I) Determine your own height in meters, and your mass in kg.

14. (I) The Sun, on average, is 93 million miles from Earth. How many meters is this? Express (*a*) using powers of ten, and (*b*) using a metric prefix.

15. (II) What is the conversion factor between (*a*) ft^2 and yd^2, (*b*) m^2 and ft^2?

16. (II) An airplane travels at 950 km/h. How long does it take to travel 1.00 km?

17. (II) A typical atom has a diameter of about 1.0×10^{-10} m. (*a*) What is this in inches? (*b*) Approximately how many atoms are there along a 1.0-cm line?

18. (II) Express the following sum with the correct number of significant figures: $1.80\ \text{m} + 142.5\ \text{cm} + 5.34 \times 10^5\ \mu\text{m}$.

19. (II) Determine the conversion factor between (*a*) km/h and mi/h, (*b*) m/s and ft/s, and (*c*) km/h and m/s.

20. (II) How much longer (percentage) is a one-mile race than a 1500-m race ("the metric mile")?

21. (II) A *light-year* is the distance light travels in one year (at speed $= 2.998 \times 10^8$ m/s). (*a*) How many meters are there in 1.00 light-year? (*b*) An astronomical unit (AU) is the average distance from the Sun to Earth, 1.50×10^8 km. How many AU are there in 1.00 light-year? (*c*) What is the speed of light in AU/h?

22. (II) If you used only a keyboard to enter data, how many years would it take to fill up the hard drive in your computer that can store 82 gigabytes (82×10^9 bytes) of data? Assume "normal" eight-hour working days, and that one byte is required to store one keyboard character, and that you can type 180 characters per minute.

23. (III) The diameter of the Moon is 3480 km. (*a*) What is the surface area of the Moon? (*b*) How many times larger is the surface area of the Earth?

6 Order-of-Magnitude Estimating

(*Note*: Remember that for rough estimates, only round numbers are needed both as input to calculations and as final results.)

24. (I) Estimate the order of magnitude (power of ten) of: (*a*) 2800, (*b*) 86.30×10^2, (*c*) 0.0076, and (*d*) 15.0×10^8.

25. (II) Estimate how many books can be shelved in a college library with 3500 m^2 of floor space. Assume 8 shelves high, having books on both sides, with corridors 1.5 m wide. Assume books are about the size of this one, on average.

26. (II) Estimate how many hours it would take a runner to run (at 10 km/h) across the United States from New York to California.

27. (II) Estimate the number of liters of water a human drinks in a lifetime.

28. (II) Estimate how long it would take one person to mow a football field using an ordinary home lawn mower (Fig. 11). Assume the mower moves with a 1-km/h speed, and has a 0.5-m width.

FIGURE 11 Problem 28.

29. (II) Estimate the number of dentists (*a*) in San Francisco and (*b*) in your town or city.

30. (III) The rubber worn from tires mostly enters the atmosphere as *particulate pollution*. Estimate how much rubber (in kg) is put into the air in the United States every year. To get started, a good estimate for a tire tread's depth is 1 cm when new, and rubber has a mass of about 1200 kg per m^3 of volume.

31. (III) You are in a hot air balloon, 200 m above the flat Texas plains. You look out toward the horizon. How far out can you see—that is, how far is your horizon? The Earth's radius is about 6400 km.

32. (III) I agree to hire you for 30 days and you can decide between two possible methods of payment: either (1) \$1000 a day, or (2) one penny on the first day, two pennies on the second day and continue to double your daily pay each day up to day 30. Use quick estimation to make your decision, and justify it.

33. (III) Many sailboats are moored at a marina 4.4 km away on the opposite side of a lake. You stare at one of the sailboats because, when you are lying flat at the water's edge, you can just see its deck but none of the side of the sailboat. You then go to that sailboat on the other side of the lake and measure that the deck is 1.5 m above the level of the water. Using Fig. 12, where $h = 1.5$ m, estimate the radius R of the Earth.

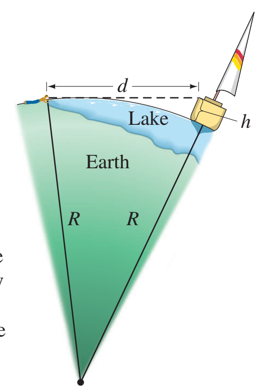


FIGURE 12 Problem 33. You see a sailboat across a lake (not to scale). R is the radius of the Earth. You are a distance $d = 4.4$ km from the sailboat when you can see only its deck and not its side. Because of the curvature of the Earth, the water "bulges out" between you and the boat.

34. (III) Another experiment you can do also uses the radius of the Earth. The Sun sets, fully disappearing over the horizon as you lie on the beach, your eyes 20 cm above the sand. You immediately jump up, your eyes now 150 cm above the sand, and you can again see the top of the Sun. If you count the number of seconds ($= t$) until the Sun fully disappears again, you can estimate the radius of the Earth. But for this Problem, use the known radius of the Earth and calculate the time t.

*7 Dimensions

*35. (I) What are the dimensions of density, which is mass per volume?

*36. (II) The speed v of an object is given by the equation $v = At^3 - Bt$, where t refers to time. (*a*) What are the dimensions of A and B? (*b*) What are the SI units for the constants A and B?

*37. (II) Three students derive the following equations in which x refers to distance traveled, v the speed, a the acceleration $(\mathrm{m/s^2})$, t the time, and the subscript zero $(_0)$ means a quantity at time $t = 0$: (*a*) $x = vt^2 + 2at$, (*b*) $x = v_0 t + \frac{1}{2}at^2$, and (*c*) $x = v_0 t + 2at^2$. Which of these could possibly be correct according to a dimensional check?

*38. (II) Show that the following combination of the three fundamental constants of nature that we used in Example 10 of "Introduction, Measurement, Estimating" (that is G, c, and h) forms a quantity with the dimensions of time:

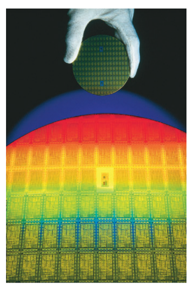
$$t_P = \sqrt{\frac{Gh}{c^5}}.$$

This quantity, t_P, is called the *Planck time* and is thought to be the earliest time, after the creation of the Universe, at which the currently known laws of physics can be applied.

General Problems

39. *Global positioning satellites* (GPS) can be used to determine positions with great accuracy. If one of the satellites is at a distance of 20,000 km from you, what percent uncertainty in the distance does a 2-m uncertainty represent? How many significant figures are needed in the distance?

40. *Computer chips* (Fig. 13) etched on circular silicon wafers of thickness 0.300 mm are sliced from a solid cylindrical silicon crystal of length 25 cm. If each wafer can hold 100 chips, what is the maximum number of chips that can be produced from one entire cylinder?



David Parker/Science Photo Library/Photo Researchers, Inc.

FIGURE 13 Problem 40. The wafer held by the hand (above) is shown below, enlarged and illuminated by colored light. Visible are rows of integrated circuits (chips).

41. (*a*) How many seconds are there in 1.00 year? (*b*) How many nanoseconds are there in 1.00 year? (*c*) How many years are there in 1.00 second?

42. American football uses a field that is 100 yd long, whereas a regulation soccer field is 100 m long. Which field is longer, and by how much (give yards, meters, and percent)?

43. A typical adult human lung contains about 300 million tiny cavities called alveoli. Estimate the average diameter of a single alveolus.

44. One hectare is defined as $1.000 \times 10^4\ \mathrm{m^2}$. One acre is $4.356 \times 10^4\ \mathrm{ft^2}$. How many acres are in one hectare?

45. Estimate the number of gallons of gasoline consumed by the total of all automobile drivers in the United States, per year.

46. Use Table 3 to estimate the total number of protons or neutrons in (*a*) a bacterium, (*b*) a DNA molecule, (*c*) the human body, (*d*) our Galaxy.

47. An average family of four uses roughly 1200 L (about 300 gallons) of water per day $(1\ \mathrm{L} = 1000\ \mathrm{cm^3})$. How much depth would a lake lose per year if it uniformly covered an area of $50\ \mathrm{km^2}$ and supplied a local town with a population of 40,000 people? Consider only population uses, and neglect evaporation and so on.

TABLE 3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

48. Estimate the number of gumballs in the machine of Fig. 14.

The Image Works

FIGURE 14 Problem 48. Estimate the number of gumballs in the machine.

49. Estimate how many kilograms of laundry soap are used in the U.S. in one year (and therefore pumped out of washing machines with the dirty water). Assume each load of laundry takes 0.1 kg of soap.

50. How big is a ton? That is, what is the volume of something that weighs a ton? To be specific, estimate the diameter of a 1-ton rock, but first make a wild guess: will it be 1 ft across, 3 ft, or the size of a car? [*Hint*: Rock has mass per volume about 3 times that of water, which is 1 kg per liter $(10^3\ \mathrm{cm^3})$ or 62 lb per cubic foot.]

51. A certain audio compact disc (CD) contains 783.216 megabytes of digital information. Each byte consists of exactly 8 bits. When played, a CD player reads the CD's digital information at a constant rate of 1.4 megabits per second. How many minutes does it take the player to read the entire CD?

52. Hold a pencil in front of your eye at a position where its blunt end just blocks out the Moon (Fig. 15). Make appropriate measurements to estimate the diameter of the Moon, given that the Earth–Moon distance is 3.8×10^5 km.

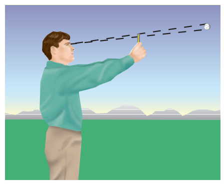


FIGURE 15 Problem 52. How big is the Moon?

53. A heavy rainstorm dumps 1.0 cm of rain on a city 5 km wide and 8 km long in a 2-h period. How many metric tons $\left(1 \text{ metric ton} = 10^3 \text{ kg}\right)$ of water fell on the city? (1 cm^3 of water has a mass of $1 \text{ g} = 10^{-3}$ kg.) How many gallons of water was this?

54. Noah's ark was ordered to be 300 cubits long, 50 cubits wide, and 30 cubits high. The cubit was a unit of measure equal to the length of a human forearm, elbow to the tip of the longest finger. Express the dimensions of Noah's ark in meters, and estimate its volume $\left(m^3\right)$.

55. Estimate how many days it would take to walk around the world, assuming 10 h walking per day at 4 km/h.

56. One liter $\left(1000 \text{ cm}^3\right)$ of oil is spilled onto a smooth lake. If the oil spreads out uniformly until it makes an oil slick just one molecule thick, with adjacent molecules just touching, estimate the diameter of the oil slick. Assume the oil molecules have a diameter of 2×10^{-10} m.

57. Jean camps beside a wide river and wonders how wide it is. She spots a large rock on the bank directly across from her. She then walks upstream until she judges that the angle between her and the rock, which she can still see clearly, is now at an angle of 30° downstream (Fig. 16). Jean measures her stride to be about 1 yard long. The distance back to her camp is 120 strides. About how far across, both in yards and in meters, is the river?

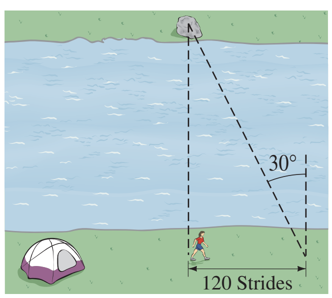


FIGURE 16 Problem 57.

58. A watch manufacturer claims that its watches gain or lose no more than 8 seconds in a year. How accurate is this watch, expressed as a percentage?

59. An angstrom (symbol Å) is a unit of length, defined as 10^{-10} m, which is on the order of the diameter of an atom. (*a*) How many nanometers are in 1.0 angstrom? (*b*) How many femtometers or fermis (the common unit of length in nuclear physics) are in 1.0 angstrom? (*c*) How many angstroms are in 1.0 m? (*d*) How many angstroms are in 1.0 light-year (see Problem 21)?

60. The diameter of the Moon is 3480 km. What is the volume of the Moon? How many Moons would be needed to create a volume equal to that of Earth?

61. Determine the percent uncertainty in θ, and in $\sin\theta$, when (*a*) $\theta = 15.0° \pm 0.5°$, (*b*) $\theta = 75.0° \pm 0.5°$.

62. If you began walking along one of Earth's lines of longitude and walked north until you had changed latitude by 1 minute of arc (there are 60 minutes per degree), how far would you have walked (in miles)? This distance is called a "nautical mile."

63. Make a rough estimate of the volume of your body (in m^3).

64. Estimate the number of bus drivers (*a*) in Washington, D.C., and (*b*) in your town.

65. The American Lung Association gives the following formula for an average person's expected lung capacity V (in liters, where $1 \text{ L} = 10^3 \text{ cm}^3$):

$$V = 4.1H - 0.018A - 2.69,$$

where H and A are the person's height (in meters), and age (in years), respectively. In this formula, what are the units of the numbers 4.1, 0.018, and 2.69?

66. The density of an object is defined as its mass divided by its volume. Suppose the mass and volume of a rock are measured to be 8 g and 2.8325 cm^3. To the correct number of significant figures, determine the rock's density.

67. To the correct number of significant figures, use the information inside the front cover of this book to determine the ratio of (*a*) the surface area of Earth compared to the surface area of the Moon; (*b*) the volume of Earth compared to the volume of the Moon.

68. One mole of atoms consists of 6.02×10^{23} individual atoms. If a mole of atoms were spread uniformly over the surface of the Earth, how many atoms would there be per square meter?

69. Recent findings in astrophysics suggest that the observable Universe can be modeled as a sphere of radius $R = 13.7 \times 10^9$ light-years with an average mass density of about $1 \times 10^{-26} \text{ kg/m}^3$, where only about 4% of the Universe's total mass is due to "ordinary" matter (such as protons, neutrons, and electrons). Use this information to estimate the total mass of ordinary matter in the observable Universe. (1 light-year $= 9.46 \times 10^{15}$ m.)

Answers to Odd-Numbered Problems

1. (*a*) 1.4×10^{10} y;
(*b*) 4.4×10^{17} s.

3. (*a*) 1.156×10^0;
(*b*) 2.18×10^1;
(*c*) 6.8×10^{-3};
(*d*) 3.2865×10^2;
(*e*) 2.19×10^{-1};
(*f*) 4.44×10^2.

5. 4.6%.

7. 1.00×10^5 s.

9. 0.24 rad.

11. (*a*) 0.2866 m;
(*b*) 0.000085 V;
(*c*) 0.00076 kg;
(*d*) 0.0000000000600 s;
(*e*) 0.0000000000000225 m;
(*f*) 2,500,000,000 V.

13. $5'10'' = 1.8$ m, 165 lbs $= 75.2$ kg.
15. (*a*) 1 $ft^2 = 0.111$ yd^2;
(*b*) 1 $m^2 = 10.8$ ft^2.
17. (*a*) 3.9×10^{-9} in.;
(*b*) 1.0×10^8 atoms.
19. (*a*) 1 km/h $= 0.621$ mi/h;
(*b*) 1 m/s $= 3.28$ ft/s;
(*c*) 1 km/h $= 0.278$ m/s.
21. (*a*) 9.46×10^{15} m;
(*b*) 6.31×10^4 AU;
(*c*) 7.20 AU/h.
23. (*a*) 3.80×10^{13} m^2;
(*b*) 13.4.
25. 6×10^5 books.
27. 5×10^4 L.
29. (*a*) 1800.
31. 5×10^4 m.
33. 6.5×10^6 m.
35. $[M/L^3]$.
37. (*a*) Cannot;
(*b*) can;
(*c*) can.
39. $(1 \times 10^{-5})\%$, 8 significant figures.
41. (*a*) 3.16×10^7 s;
(*b*) 3.16×10^{16} ns;
(*c*) 3.17×10^{-8} y.
43. 2×10^{-4} m.
45. 1×10^{11} gal/y.
47. 9 cm/y.
49. 2×10^9 kg/y.
51. 75 min.
53. 4×10^5 metric tons, 1×10^8 gal.
55. 1×10^3 days
57. 210 yd, 190 m.
59. (*a*) 0.10 nm;
(*b*) 1.0×10^5 fm;
(*c*) 1.0×10^{10} Å;
(*d*) 9.5×10^{25} Å.
61. (*a*) 3%, 3%;
(*b*) 0.7%, 0.2%.
63. 8×10^{-2} m^3.
65. L/m, L/y, L.
67. (*a*) 13.4;
(*b*) 49.3.
69. 4×10^{51} kg.

This page intentionally left blank

Describing Motion: Kinematics in One Dimension

A high-speed car has released a parachute to reduce its speed quickly. The directions of the car's velocity and acceleration are shown by the green ($\vec{\mathbf{v}}$) and gold ($\vec{\mathbf{a}}$) arrows.

Motion is described using the concepts of velocity and acceleration. In the case shown here, the acceleration $\vec{\mathbf{a}}$ is in the opposite direction from the velocity $\vec{\mathbf{v}}$, which means the object is slowing down. We examine in detail motion with constant acceleration, including the vertical motion of objects falling under gravity.

George D. Lepp/Corbis/Bettmann

Describing Motion: Kinematics in One Dimension

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Two small heavy balls have the same diameter but one weighs twice as much as the other. The balls are dropped from a second-story balcony at the exact same time. The time to reach the ground below will be:

(a) twice as long for the lighter ball as for the heavier one.
(b) longer for the lighter ball, but not twice as long.
(c) twice as long for the heavier ball as for the lighter one.
(d) longer for the heavier ball, but not twice as long.
(e) nearly the same for both balls.

The motion of objects—baseballs, automobiles, joggers, and even the Sun and Moon—is an obvious part of everyday life. It was not until the sixteenth and seventeenth centuries that our modern understanding of motion was established. Many individuals contributed to this understanding, particularly Galileo Galilei (1564–1642) and Isaac Newton (1642–1727).

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do.

For now we only discuss objects that move without rotating (Fig. 1a). Such motion is called **translational motion**. In this Chapter we will be concerned with describing an object that moves along a straight-line path, which is one-dimensional translational motion.

We will often use the concept, or *model*, of an idealized **particle** which is considered to be a mathematical **point** with no spatial extent (no size). A point particle can undergo only translational motion. The particle model is useful in many real situations where we are interested only in translational motion and the object's size is not significant. For example, we might consider a billiard ball, or even a spacecraft traveling toward the Moon, as a particle for many purposes.

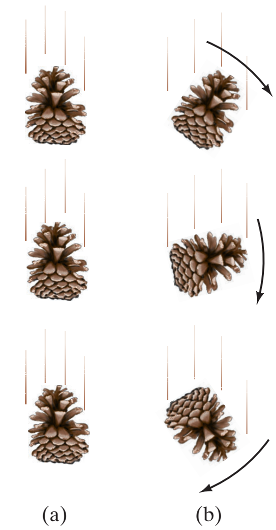


FIGURE 1 The pinecone in (a) undergoes pure translation as it falls, whereas in (b) it is rotating as well as translating.

1 Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a **reference frame**, or **frame of reference**. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of $80\,\mathrm{km/h} + 5\,\mathrm{km/h} = 85\,\mathrm{km/h}$. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.

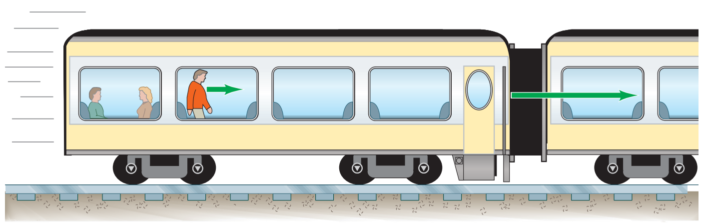
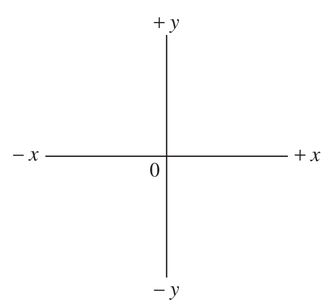


FIGURE 2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using the cardinal points, north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 3, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. The x and y axes are always perpendicular to each other. Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we usually choose to be positive; then points to the left of 0 have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the plane can be specified by giving its x and y coordinates. In three dimensions, a z axis perpendicular to the x and y axes is added.

For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the **position** of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

FIGURE 3 Standard set of xy coordinate axes.



CAUTION

The displacement may not equal the total distance traveled

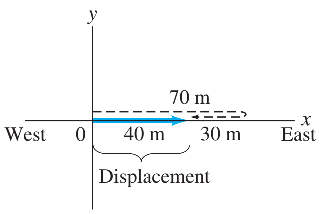


FIGURE 4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is 40 m to the east.

FIGURE 5 The arrow represents the displacement $x_2 - x_1$. Distances are in meters.

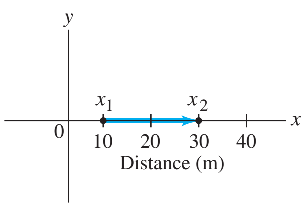
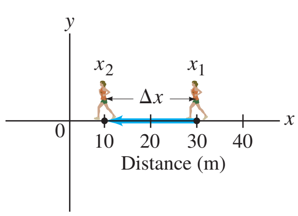


FIGURE 6 For the displacement $\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m}$, the displacement vector points to the left.



We need to make a distinction between the *distance* an object has traveled and its **displacement**, which is defined as the *change in position* of the object. That is, *displacement is how far the object is from its starting point*. To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 4). The total *distance* traveled is 100 m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.

Displacement is a quantity that has both magnitude and direction. Such quantities are called **vectors**, and are represented by arrows in diagrams. For example, in Fig. 4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).

In this chapter, we deal only with motion in one dimension, along a line. In this case, vectors which point in one direction will have a positive sign, whereas vectors that point in the opposite direction will have a negative sign, along with their magnitude.

Consider the motion of an object over a particular time interval. Suppose that at some initial time, call it t_1, the object is on the x axis at the position x_1 in the coordinate system shown in Fig. 5. At some later time, t_2, suppose the object has moved to position x_2. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 5. It is convenient to write

$$\Delta x = x_2 - x_1,$$

where the symbol Δ (Greek letter delta) means "change in." Then Δx means "the change in x," or "change in position," which is the displacement. Note that the "change in" any quantity means the final value of that quantity, minus the initial value.

Suppose $x_1 = 10.0\text{ m}$ and $x_2 = 30.0\text{ m}$. Then

$$\Delta x = x_2 - x_1 = 30.0\text{ m} - 10.0\text{ m} = 20.0\text{ m},$$

so the displacement is 20.0 m in the positive direction, Fig. 5.

Now consider an object moving to the left as shown in Fig. 6. Here the object, say, a person, starts at $x_1 = 30.0\text{ m}$ and walks to the left to the point $x_2 = 10.0\text{ m}$. In this case her displacement is

$$\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m} = -20.0\text{ m},$$

and the blue arrow representing the vector displacement points to the left. For one-dimensional motion along the x axis, a vector pointing to the right has a positive sign, whereas a vector pointing to the left has a negative sign.

EXERCISE A An ant starts at $x = 20\text{ cm}$ on a piece of graph paper and walks along the x axis to $x = -20\text{ cm}$. It then turns around and walks back to $x = -10\text{ cm}$. What is the ant's displacement and total distance traveled?

2 Average Velocity

The most obvious aspect of the motion of a moving object is how fast it is moving—its speed or velocity.

The term "speed" refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the total distance traveled along its path divided by the time it takes to travel this distance*:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}. \tag{1}$$

The terms "velocity" and "speed" are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and also the *direction* in which it is moving. (Velocity is therefore a vector.) There is a second difference between speed and velocity: namely, the **average velocity** is defined in terms of *displacement*, rather than total distance traveled:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}.$$

CAUTION
Average speed is not necessarily equal to the magnitude of the average velocity

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: recall the walk we described earlier, in Fig. 4, where a person walked 70 m east and then 30 m west. The total distance traveled was $70\text{ m} + 30\text{ m} = 100\text{ m}$, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{time elapsed}} = \frac{100\text{ m}}{70\text{ s}} = 1.4\text{ m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{time elapsed}} = \frac{40\text{ m}}{70\text{ s}} = 0.57\text{ m/s}.$$

This difference between the speed and the magnitude of the velocity can occur when we calculate *average* values.

To discuss one-dimensional motion of an object in general, suppose that at some moment in time, call it t_1, the object is on the x axis at position x_1 in a coordinate system, and at some later time, t_2, suppose it is at position x_2. The **elapsed time** is $\Delta t = t_2 - t_1$; during this time interval the displacement of our object is $\Delta x = x_2 - x_1$. Then the average velocity, defined as *the displacement divided by the elapsed time*, can be written

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad \textbf{(2)}$$

where v stands for velocity and the bar $(^{-})$ over the v is a standard symbol meaning "average."

PROBLEM SOLVING
+ or − sign can signify the direction for linear motion

For the usual case of the $+x$ axis to the right, note that if x_2 is less than x_1, the object is moving to the left, and then $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the average velocity, indicates the direction: the average velocity is positive for an object moving to the right along the $+x$ axis and negative when the object moves to the left. The direction of the average velocity is always the same as the direction of the displacement.

Note that it is always important to choose (and state) the *elapsed time*, or *time interval*, $t_2 - t_1$, the time that passes during our chosen period of observation.

EXAMPLE 1 **Runner's average velocity.** The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from $x_1 = 50.0\text{ m}$ to $x_2 = 30.5\text{ m}$, as shown in Fig. 7. What was the runner's average velocity?

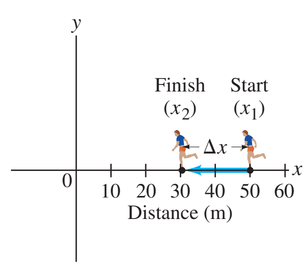
APPROACH We want to find the average velocity, which is the displacement divided by the elapsed time.

SOLUTION The displacement is $\Delta x = x_2 - x_1 = 30.5\text{ m} - 50.0\text{ m} = -19.5\text{ m}$. The elapsed time, or time interval, is $\Delta t = 3.00\text{ s}$. The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\text{ m}}{3.00\text{ s}} = -6.50\text{ m/s}.$$

The displacement and average velocity are negative, which tells us that the runner is moving to the left along the x axis, as indicated by the arrow in Fig. 7. Thus we can say that the runner's average velocity is 6.50 m/s to the left.

FIGURE 7 Example 1. A person runs from $x_1 = 50.0\text{ m}$ to $x_2 = 30.5\text{ m}$. The displacement is -19.5 m.



EXAMPLE 2 **Distance a cyclist travels.** How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

APPROACH We want to find the distance traveled, so we solve Eq. 2 for Δx.

SOLUTION We rewrite Eq. 2 as $\Delta x = \bar{v}\,\Delta t$, and find

$$\Delta x = \bar{v}\,\Delta t = (18\text{ km/h})(2.5\text{ h}) = 45\text{ km}.$$

EXERCISE B A car travels at a constant 50 km/h for 100 km. It then speeds up to 100 km/h and is driven another 100 km. What is the car's average speed for the 200 km trip? (*a*) 67 km/h; (*b*) 75 km/h; (*c*) 81 km/h; (*d*) 50 km/h.

3 Instantaneous Velocity

John E. Gilmore III

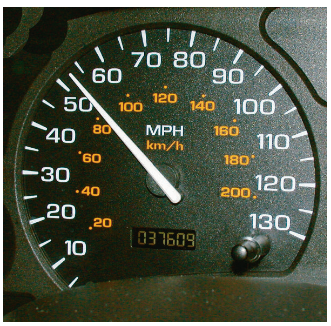


FIGURE 8 Car speedometer showing mi/h in white, and km/h in orange.

If you drive a car along a straight road for 150 km in 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To describe this situation we need the concept of *instantaneous velocity*, which is the velocity at any instant of time. (Its magnitude is the number, with units, indicated by a speedometer, Fig. 8.) More precisely, the **instantaneous velocity** at any moment is defined as *the average velocity over an infinitesimally short time interval*. That is, Eq. 2 is to be evaluated in the limit of Δt becoming extremely small, approaching zero. We can write the definition of instantaneous velocity, v, for one-dimensional motion as

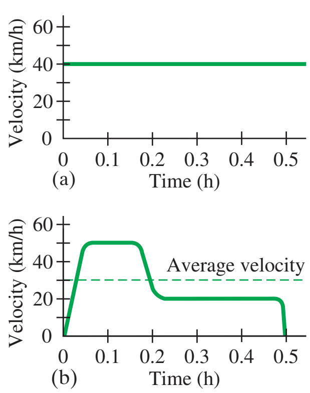
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \tag{3}$$

The notation $\lim_{\Delta t \to 0}$ means the ratio $\Delta x/\Delta t$ is to be evaluated in the limit of Δt approaching zero. But we do not simply set $\Delta t = 0$ in this definition, for then Δx would also be zero, and we would have an undefined number. Rather, we are considering the *ratio* $\Delta x/\Delta t$, as a whole. As we let Δt approach zero, Δx approaches zero as well. But the ratio $\Delta x/\Delta t$ approaches some definite value, which is the instantaneous velocity at a given instant.

In Eq. 3, the limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the *derivative* of x with respect to t:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \tag{4}$$

FIGURE 9 Velocity of a car as a function of time: (a) at constant velocity; (b) with varying velocity.



This equation is the definition of instantaneous velocity for one-dimensional motion.

For instantaneous velocity we use the symbol v, whereas for average velocity we use $\bar{v}$, with a bar above. When we use the term "velocity" it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word "average."

Note that the *instantaneous* speed always equals the magnitude of the instantaneous velocity. Why? Because distance traveled and the magnitude of the displacement become the same when they become infinitesimally small.

If an object moves at a uniform (that is, constant) velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 9a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 9b. Also shown on the graph is the average velocity (dashed line), which is $\bar{v} = \Delta x/\Delta t = 15\text{ km}/0.50\text{ h} = 30\text{ km/h}$.

To better understand instantaneous velocity, let us consider a graph of the position of a particular particle versus time (x vs. t), as shown in Fig. 10. (Note that this is different from showing the "path" of a particle on an x vs. y plot.) The particle is at position x_1 at a time t_1, and at position x_2 at time t_2. P_1 and P_2 represent these two points on the graph. A straight line drawn from point $P_1(x_1, t_1)$ to point $P_2(x_2, t_2)$ forms the hypotenuse of a right triangle whose sides are Δx and Δt. The ratio $\Delta x/\Delta t$ is the **slope** of the straight line P_1P_2. But $\Delta x/\Delta t$ is also the average velocity of the particle during the time interval $\Delta t = t_2 - t_1$. Therefore, we conclude that the average velocity of a particle during any time interval $\Delta t = t_2 - t_1$ is equal to the slope of the straight line (or *chord*) connecting the two points (x_1, t_1) and (x_2, t_2) on an x vs. t graph.

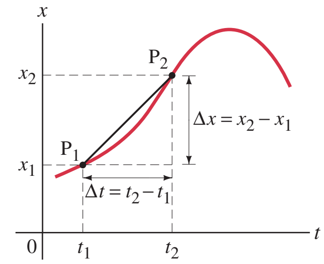
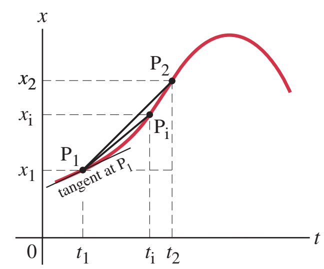


FIGURE 10 Graph of a particle's position x vs. time t. The slope of the straight line P_1P_2 represents the average velocity of the particle during the time interval $\Delta t = t_2 - t_1$.

Consider now a time t_i, intermediate between t_1 and t_2, at which time the particle is at x_i (Fig. 11). The slope of the straight line P_1P_i is less than the slope of P_1P_2 in this case. Thus the average velocity during the time interval $t_i - t_1$ is less than during the time interval $t_2 - t_1$.

Now let us imagine that we take the point P_i in Fig. 11 to be closer and closer to point P_1. That is, we let the interval $t_i - t_1$, which we now call Δt, to become smaller and smaller. The slope of the line connecting the two points becomes closer and closer to the slope of a line tangent to the curve at point P_1. The average velocity (equal to the slope of the chord) thus approaches the slope of the tangent at point P_1. The definition of the instantaneous velocity (Eq. 3) is the limiting value of the average velocity as Δt approaches zero. Thus the *instantaneous velocity equals the slope of the tangent to the curve* at that point (which we can simply call "the slope of the curve" at that point).

FIGURE 11 Same position vs. time curve as in Fig. 10, but note that the average velocity over the time interval $t_i - t_1$ (which is the slope of P_1P_i) is less than the average velocity over the time interval $t_2 - t_1$. The slope of the thin line tangent to the curve at point P_1 equals the instantaneous velocity at time t_1.



Because the velocity at any instant equals the slope of the tangent to the x vs. t graph at that instant, we can obtain the velocity at any instant from such a graph. For example, in Fig. 12 (which shows the same curve as in Figs. 10 and 11), as our object moves from x_1 to x_2, the slope continually increases, so the velocity is increasing. For times after t_2, however, the slope begins to decrease and in fact reaches zero (so $v = 0$) where x has its maximum value, at point P_3 in Fig. 12. Beyond this point, the slope is negative, as for point P_4. The velocity is therefore negative, which makes sense since x is now decreasing—the particle is moving toward decreasing values of x, to the left on a standard xy plot.

If an object moves with constant velocity over a particular time interval, its instantaneous velocity is equal to its average velocity. The graph of x vs. t in this case will be a straight line whose slope equals the velocity. The curve of Fig. 10 has no straight sections, so there are no time intervals when the velocity is constant.

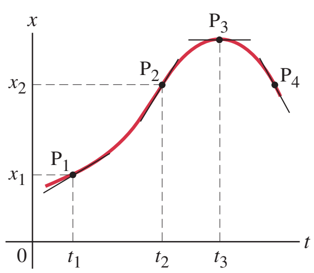


FIGURE 12 Same x vs. t curve as in Figs. 10 and 11, but here showing the slope at four different points: At P_3, the slope is zero, so $v = 0$. At P_4 the slope is negative, so $v < 0$.

EXERCISE C What is your speed at the instant you turn around to move in the opposite direction? (*a*) Depends on how quickly you turn around; (*b*) always zero; (*c*) always negative; (*d*) none of the above.

The derivatives of polynomial functions (which we use a lot) are:

$$\frac{d}{dt}\left(Ct^n\right) = nCt^{n-1} \quad \text{and} \quad \frac{dC}{dt} = 0,$$

where C is any constant.

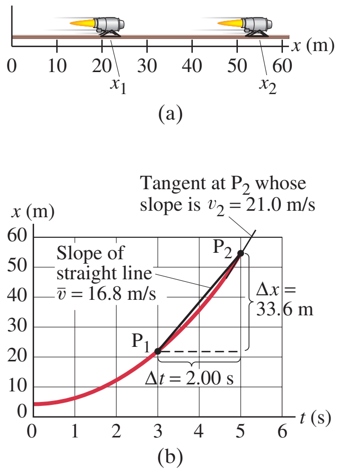


FIGURE 13 Example 3. (a) Engine traveling on a straight track. (b) Graph of x vs. t: $x = At^2 + B$.

EXAMPLE 3 Given x as a function of t. A jet engine moves along an experimental track (which we call the x axis) as shown in Fig. 13a. We will treat the engine as if it were a particle. Its position as a function of time is given by the equation $x = At^2 + B$, where $A = 2.10\,\text{m/s}^2$ and $B = 2.80\,\text{m}$, and this equation is plotted in Fig. 13b. (*a*) Determine the displacement of the engine during the time interval from $t_1 = 3.00\,\text{s}$ to $t_2 = 5.00\,\text{s}$. (*b*) Determine the average velocity during this time interval. (*c*) Determine the magnitude of the instantaneous velocity at $t = 5.00\,\text{s}$.

APPROACH We substitute values for t_1 and t_2 in the given equation for x to obtain x_1 and x_2. The average velocity can be found from Eq. 2. We take the derivative of the given x equation with respect to t to find the instantaneous velocity, using the formulas just given.

SOLUTION (*a*) At $t_1 = 3.00\,\text{s}$, the position (point P_1 in Fig. 13b) is

$$x_1 = At_1^2 + B = (2.10\,\text{m/s}^2)(3.00\,\text{s})^2 + 2.80\,\text{m} = 21.7\,\text{m}.$$

At $t_2 = 5.00\,\text{s}$, the position (P_2 in Fig. 13b) is

$$x_2 = (2.10\,\text{m/s}^2)(5.00\,\text{s})^2 + 2.80\,\text{m} = 55.3\,\text{m}.$$

The displacement is thus

$$x_2 - x_1 = 55.3\,\text{m} - 21.7\,\text{m} = 33.6\,\text{m}.$$

(*b*) The magnitude of the average velocity can then be calculated as

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{33.6\,\text{m}}{2.00\,\text{s}} = 16.8\,\text{m/s}.$$

This equals the slope of the straight line joining points P_1 and P_2 shown in Fig. 13b.

(*c*) The instantaneous velocity at $t = t_2 = 5.00\,\text{s}$ equals the slope of the tangent to the curve at point P_2 shown in Fig. 13b. We could measure this slope off the graph to obtain v_2. But we can calculate v more precisely for any time t, using the given formula

$$x = At^2 + B,$$

which is the engine's position x as a function of time t. We take the derivative of x with respect to time (see formulas at bottom of previous page):

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At.$$

We are given $A = 2.10\,\text{m/s}^2$, so for $t = t_2 = 5.00\,\text{s}$,

$$v_2 = 2At = 2(2.10\,\text{m/s}^2)(5.00\,\text{s}) = 21.0\,\text{m/s}.$$

4 Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how rapidly the velocity of an object is changing.

Average Acceleration

Average acceleration is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

In symbols, the average acceleration over a time interval $\Delta t = t_2 - t_1$ during which the velocity changes by $\Delta v = v_2 - v_1$, is defined as

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \tag{5}$$

Because velocity is a vector, acceleration is a vector too. But for one-dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis.

EXAMPLE 4 **Average acceleration.** A car accelerates along a straight road from rest to 90 km/h in 5.0 s, Fig. 14. What is the magnitude of its average acceleration?

APPROACH Average acceleration is the change in velocity divided by the elapsed time, 5.0 s. The car starts from rest, so $v_1 = 0$. The final velocity is $v_2 = 90\ \text{km/h} = 90 \times 10^3\ \text{m}/3600\ \text{s} = 25\ \text{m/s}$.

SOLUTION From Eq. 5, the average acceleration is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{25\ \text{m/s} - 0\ \text{m/s}}{5.0\ \text{s}} = 5.0\,\frac{\text{m/s}}{\text{s}}.$$

This is read as "five meters per second per second" and means that, on average, the velocity changed by 5.0 m/s during each second. That is, assuming the acceleration was constant, during the first second the car's velocity increased from zero to 5.0 m/s. During the next second its velocity increased by another 5.0 m/s, reaching a velocity of 10.0 m/s at $t = 2.0\ \text{s}$, and so on. See Fig. 14.

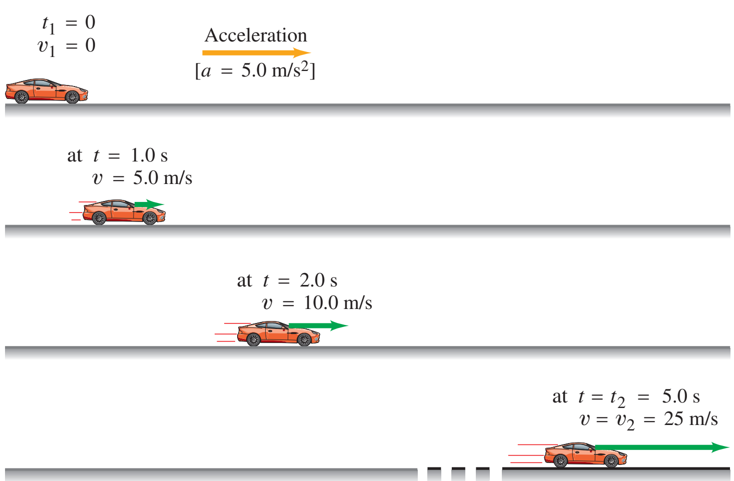


FIGURE 14 Example 4. The car is shown at the start with $v_1 = 0$ at $t_1 = 0$. The car is shown three more times, at $t = 1.0\ \text{s}$, $t = 2.0\ \text{s}$, and at the end of our time interval, $t_2 = 5.0\ \text{s}$. We assume the acceleration is constant and equals $5.0\ \text{m/s}^2$. The green arrows represent the velocity vectors; the length of each arrow represents the magnitude of the velocity at that moment. The acceleration vector is the orange arrow. Distances are not to scale.

We almost always write the units for acceleration as m/s^2 (meters per second squared) instead of m/s/s. This is possible because:

$$\frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}\cdot\text{s}} = \frac{\text{m}}{\text{s}^2}.$$

According to the calculation in Example 4, the velocity changed on average by 5.0 m/s during each second, for a total change of 25 m/s over the 5.0 s; the average acceleration was $5.0\ \text{m/s}^2$.

Note that *acceleration tells us how quickly the velocity changes*, whereas *velocity tells us how quickly the position changes*.

CONCEPTUAL EXAMPLE 5 **Velocity and acceleration.** (*a*) If the velocity of an object is zero, does it mean that the acceleration is zero? (*b*) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

RESPONSE A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (*a*) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but the acceleration is not zero since the velocity of the car changes. (How else could your car start forward if its velocity weren't changing—that is, accelerating?) (*b*) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero: $a = 0, v \neq 0$.

EXERCISE D A powerful car is advertised to go from zero to 60 mi/h in 6.0 s. What does this say about the car: (*a*) it is fast (high speed); or (*b*) it accelerates well?

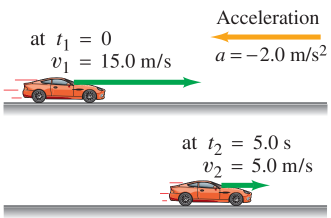


FIGURE 15 Example 6, showing the position of the car at times t_1 and t_2, as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left as the car slows down while moving to the right.

EXAMPLE 6 **Car slowing down.** An automobile is moving to the right along a straight highway, which we choose to be the positive x axis (Fig. 15). Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is $v_1 = 15.0$ m/s, and it takes 5.0 s to slow down to $v_2 = 5.0$ m/s, what was the car's average acceleration?

APPROACH We put the given initial and final velocities, and the elapsed time, into Eq. 5 for $\bar{a}$.

SOLUTION In Eq. 5, we call the initial time $t_1 = 0$, and set $t_2 = 5.0$ s. (Note that our choice of $t_1 = 0$ doesn't affect the calculation of $\bar{a}$ because only $\Delta t = t_2 - t_1$ appears in Eq. 5.) Then

$$\bar{a} = \frac{5.0\text{ m/s} - 15.0\text{ m/s}}{5.0\text{ s}} = -2.0\text{ m/s}^2.$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left (in the negative x direction)—even though the velocity is always pointing to the right. We say that the acceleration is 2.0 m/s^2 to the left, and it is shown in Fig. 15 as an orange arrow.

Deceleration

CAUTION
Deceleration means the magnitude of the velocity is decreasing; a is not *necessarily negative*

When an object is slowing down, we can say it is **decelerating**. But be careful: deceleration does *not* mean that the acceleration is necessarily negative. The velocity of an object moving to the right along the positive x axis is positive; if the object is slowing down (as in Fig. 15), the acceleration *is* negative. But the same car moving to the left (decreasing x), and slowing down, has positive acceleration that points to the right, as shown in Fig. 16. We have a deceleration whenever the magnitude of the velocity is decreasing, and then the velocity and acceleration point in opposite directions.

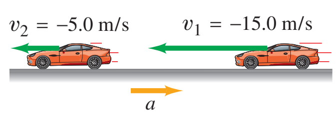


FIGURE 16 The car of Example 6, now moving to the *left* and decelerating. The acceleration is

$$a = \frac{v_2 - v_1}{\Delta t} = \frac{(-5.0\text{ m/s}) - (-15.0\text{ m/s})}{5.0\text{ s}} = \frac{-5.0\text{ m/s} + 15.0\text{ m/s}}{5.0\text{ s}} = +2.0\text{ m/s}.$$

EXERCISE E A car moves along the x axis. What is the sign of the car's acceleration if it is moving in the positive x direction with (*a*) increasing speed or (*b*) decreasing speed? What is the sign of the acceleration if the car moves in the negative direction with (*c*) increasing speed or (*d*) decreasing speed?

Instantaneous Acceleration

The **instantaneous acceleration**, a, is defined as the *limiting value of the average acceleration as we let* Δt *approach zero*:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}. \qquad \textbf{(6)}$$

This limit, dv/dt, is the derivative of v with respect to t. We will use the term "acceleration" to refer to the instantaneous value. If we want to discuss the average acceleration, we will always include the word "average."

If we draw a graph of the velocity, v, vs. time, t, as shown in Fig. 17, then the average acceleration over a time interval $\Delta t = t_2 - t_1$ is represented by the slope of the straight line connecting the two points P_1 and P_2 as shown. [Compare this to the position vs. time graph of Fig. 10 for which the slope of the straight line represents the average velocity.] The instantaneous acceleration at any time, say t_1, is the slope of the tangent to the v vs. t curve at that time, which is also shown in Fig. 17. Let us use this fact for the situation graphed in Fig. 17; as we go from time t_1 to time t_2 the velocity continually increases, but the acceleration (the rate at which the velocity changes) is decreasing since the slope of the curve is decreasing.

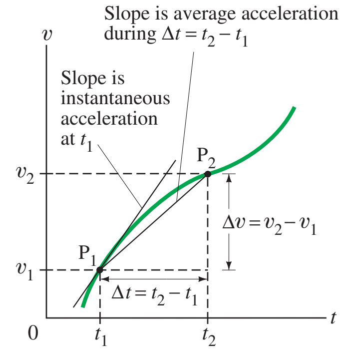


FIGURE 17 A graph of velocity v vs. time t. The average acceleration over a time interval $\Delta t = t_2 - t_1$ is the slope of the straight line P_1P_2: $\bar{a} = \Delta v/\Delta t$. The instantaneous acceleration at time t_1 is the slope of the v vs. t curve at that instant.

EXAMPLE 7 **Acceleration given $x(t)$.** A particle is moving in a straight line so that its position is given by the relation $x = (2.10\,\text{m/s}^2)t^2 + (2.80\,\text{m})$, as in Example 3. Calculate (*a*) its average acceleration during the time interval from $t_1 = 3.00\,\text{s}$ to $t_2 = 5.00\,\text{s}$, and (*b*) its instantaneous acceleration as a function of time.

APPROACH To determine acceleration, we first must find the velocity at t_1 and t_2 by differentiating x: $v = dx/dt$. Then we use Eq. 5 to find the average acceleration, and Eq. 6 to find the instantaneous acceleration.

SOLUTION (*a*) The velocity at any time t is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[(2.10\,\text{m/s}^2)t^2 + 2.80\,\text{m}\right] = (4.20\,\text{m/s}^2)t,$$

as we saw in Example 3c. Therefore, at $t_1 = 3.00\,\text{s}$, $v_1 = (4.20\,\text{m/s}^2)(3.00\,\text{s}) = 12.6\,\text{m/s}$ and at $t_2 = 5.00\,\text{s}$, $v_2 = 21.0\,\text{m/s}$. Therefore,

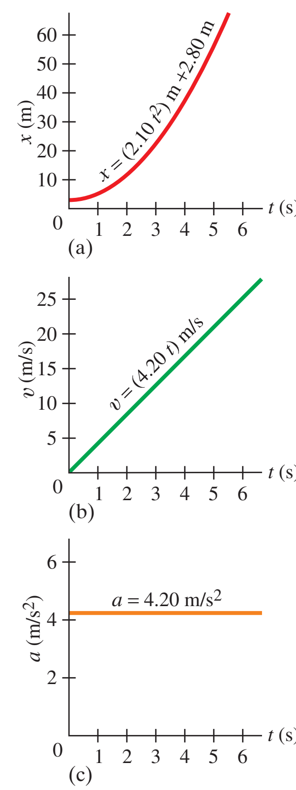
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{21.0\,\text{m/s} - 12.6\,\text{m/s}}{5.00\,\text{s} - 3.00\,\text{s}} = 4.20\,\text{m/s}^2.$$

(*b*) With $v = (4.20\,\text{m/s}^2)t$, the instantaneous acceleration at any time is

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[(4.20\,\text{m/s}^2)t\right] = 4.20\,\text{m/s}^2.$$

The acceleration in this case is constant; it does not depend on time. Figure 18 shows graphs of (a) x vs. t (the same as Fig. 13b), (b) v vs. t, which is linearly increasing as calculated above, and (c) a vs. t, which is a horizontal straight line because a = constant.

FIGURE 18 Example 7. Graphs of (a) x vs. t, (b) v vs. t, and (c) a vs. t for the motion $x = At^2 + B$. Note that v increases linearly with t and that the acceleration a is constant. Also, v is the slope of the x vs. t curve, whereas a is the slope of the v vs. t curve.



Like velocity, acceleration is a rate. The velocity of an object is the rate at which its displacement changes with time; its acceleration, on the other hand, is the rate at which its velocity changes with time. In a sense, acceleration is a "rate of a rate." This can be expressed in equation form as follows: since $a = dv/dt$ and $v = dx/dt$, then

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

Here d^2x/dt^2 is the *second derivative* of x with respect to time: we first take the derivative of x with respect to time (dx/dt), and then we again take the derivative with respect to time, $(d/dt)(dx/dt)$, to get the acceleration.

EXERCISE F The position of a particle is given by the following equation:

$$x = (2.00\,\text{m/s}^3)t^3 + (2.50\,\text{m/s})t.$$

What is the acceleration of the particle at $t = 2.00\,\text{s}$? (*a*) $13.0\,\text{m/s}^2$; (*b*) $22.5\,\text{m/s}^2$; (*c*) $24.0\,\text{m/s}^2$; (*d*) $2.00\,\text{m/s}^2$.

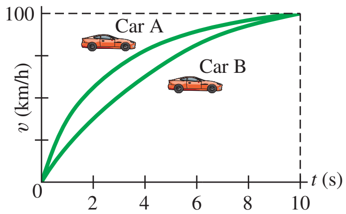


FIGURE 19 Example 8.

CONCEPTUAL EXAMPLE 8 **Analyzing with graphs.** Figure 19 shows the velocity as a function of time for two cars accelerating from 0 to 100 km/h in a time of 10.0 s. Compare (*a*) the average acceleration; (*b*) instantaneous acceleration; and (*c*) total distance traveled for the two cars.

RESPONSE (*a*) Average acceleration is $\Delta v/\Delta t$. Both cars have the same Δv (100 km/h) and the same Δt (10.0 s), so the average acceleration is the same for both cars. (*b*) Instantaneous acceleration is the slope of the tangent to the v vs. t curve. For about the first 4 s, the top curve is steeper than the bottom curve, so car A has a greater acceleration during this interval. The bottom curve is steeper during the last 6 s, so car B has the larger acceleration for this period. (*c*) Except at $t = 0$ and $t = 10.0\,\mathrm{s}$, car A is always going faster than car B. Since it is going faster, it will go farther in the same time.

5 Motion at Constant Acceleration

We now examine the situation when the magnitude of the acceleration is constant and the motion is in a straight line. In this case, the instantaneous and average accelerations are equal. We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate x, v, a, and t when a is constant, allowing us to determine any one of these variables if we know the others.

To simplify our notation, let us take the initial time in any discussion to be zero, and we call it t_0: $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at t_0.) We can then let $t_2 = t$ be the elapsed time. The initial position (x_1) and the initial velocity (v_1) of an object will now be represented by x_0 and v_0, since they represent x and v at $t = 0$. At time t the position and velocity will be called x and v (rather than x_2 and v_2). The average velocity during the time interval $t - t_0$ will be (Eq. 2)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose $t_0 = 0$. The acceleration, assumed constant in time, is (Eq. 5)

$$a = \frac{v - v_0}{t}.$$

A common problem is to determine the velocity of an object after any elapsed time t, when we are given the object's constant acceleration. We can solve such problems by solving for v in the last equation to obtain:

$$v = v_0 + at. \qquad \text{[constant acceleration]} \quad \mathbf{(7)}$$

If an object starts from rest $(v_0 = 0)$ and accelerates at $4.0\,\mathrm{m/s^2}$, after an elapsed time $t = 6.0\,\mathrm{s}$ its velocity will be $v = at = (4.0\,\mathrm{m/s^2})(6.0\,\mathrm{s}) = 24\,\mathrm{m/s}$.

Next, let us see how to calculate the position x of an object after a time t when it undergoes constant acceleration. The definition of average velocity (Eq. 2) is $\bar{v} = (x - x_0)/t$, which we can rewrite as

$$x = x_0 + \bar{v}t. \qquad \mathbf{(8)}$$

Because the velocity increases at a uniform rate, the average velocity, $\bar{v}$, will be midway between the initial and final velocities:

$$\bar{v} = \frac{v_0 + v}{2}. \qquad \text{[constant acceleration]} \quad \mathbf{(9)}$$

CAUTION
Average velocity, but only if a = constant

(Careful: Equation 9 is not necessarily valid if the acceleration is not constant.) We combine the last two Equations with Eq. 7 and find

$$\begin{aligned} x &= x_0 + \bar{v}t \\ &= x_0 + \left(\frac{v_0 + v}{2}\right)t \\ &= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t \end{aligned}$$

or

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2. \qquad \text{[constant acceleration]} \quad \mathbf{(10)}$$

Equations 7, 9, and 10 are three of the four most useful equations for motion at constant acceleration. We now derive the fourth equation, which is useful in situations where the time t is not known. We substitute Eq. 9 into Eq. 8:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v + v_0}{2}\right)t.$$

Next we solve Eq. 7 for t, obtaining

$$t = \frac{v - v_0}{a},$$

and substituting this into the previous equation we have

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

We solve this for v^2 and obtain

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \text{[constant acceleration]} \quad \textbf{(11)}$$

which is the useful equation we sought.

We now have four equations relating position, velocity, acceleration, and time, when the acceleration a is constant. We collect these kinematic equations here in one place for future reference (the tan background screen emphasizes their usefulness):

$$v = v_0 + at \qquad [a = \text{constant}] \quad \textbf{(12a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \quad \textbf{(12b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \quad \textbf{(12c)}$$

$$\bar{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \quad \textbf{(12d)}$$

Kinematic equations for constant acceleration (we'll use them a lot)

These useful equations are not valid unless a is a constant. In many cases we can set $x_0 = 0$, and this simplifies the above equations a bit. Note that x represents position, not distance, that $x - x_0$ is the displacement, and that t is the elapsed time.

PHYSICS APPLIED
Airport design

EXAMPLE 9 **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at $2.00\ \text{m/s}^2$. (*a*) If the runway is 150 m long, can this airplane reach the required speed for takeoff? (*b*) If not, what minimum length must the runway have?

APPROACH The plane's acceleration is constant, so we can use the kinematic equations for constant acceleration. In (*a*), we want to find v, and we are given:

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\ \text{m}$	
$a = 2.00\ \text{m/s}^2$	

SOLUTION (*a*) Of the above four equations, Eq. 12c will give us v when we know v_0, a, x, and x_0:

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$= 0 + 2(2.00\ \text{m/s}^2)(150\ \text{m}) = 600\ \text{m}^2/\text{s}^2$$
$$v = \sqrt{600\ \text{m}^2/\text{s}^2} = 24.5\ \text{m/s}.$$

PROBLEM SOLVING
Equations–12 are valid only when the acceleration is constant, which we assume in this Example

This runway length is *not* sufficient.

(*b*) Now we want to find the minimum length of runway, $x - x_0$, given $v = 27.8\ \text{m/s}$ and $a = 2.00\ \text{m/s}^2$. So we again use Eq. 12c, but rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\ \text{m/s})^2 - 0}{2(2.00\ \text{m/s}^2)} = 193\ \text{m}.$$

A 200-m runway is more appropriate for this plane.

NOTE We did this Example as if the plane were a particle, so we round off our answer to 200 m.

EXERCISE G A car starts from rest and accelerates at a constant $10\ \text{m/s}^2$ during a $\frac{1}{4}$ mile (402 m) race. How fast is the car going at the finish line? (a) 8090 m/s; (b) 90 m/s; (c) 81 m/s; (d) 809 m/s.

6 Solving Problems

Before doing more worked-out Examples, let us look at how to approach problem solving. First, it is important to note that physics is *not* a collection of equations to be memorized. Simply searching for an equation that might work can lead you to a wrong result and will surely not help you understand physics. A better approach is to use the following (rough) procedure, which we put in a special "Problem Solving Strategy."

PROBLEM SOLVING

1. Read and **reread** the whole problem carefully before trying to solve it.
2. Decide what **object** (or objects) you are going to study, and for what **time interval**. You can often choose the initial time to be $t = 0$.
3. **Draw** a **diagram** or picture of the situation, with coordinate axes wherever applicable. [You can place the origin of coordinates and the axes wherever you like to make your calculations easier.]
4. Write down what quantities are "**known**" or "given," and then what you *want* to know. Consider quantities both at the beginning and at the end of the chosen time interval.
5. Think about which **principles of physics** apply in this problem. Use common sense and your own experiences. Then plan an approach.
6. Consider which **equations** (and/or definitions) relate the quantities involved. Before using them, be sure their **range of validity** includes your problem (for example, Eqs. 12 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, **solve** the equation algebraically for the unknown. Sometimes several sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.
7. Carry out the **calculation** if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answer(s) to the correct number of significant figures.
8. Think carefully about the result you obtain: Is it **reasonable**? Does it make sense according to your own intuition and experience? A good check is to do a rough **estimate** using only powers of ten. Often it is preferable to do a rough estimate at the *start* of a numerical problem because it can help you focus your attention on finding a path toward a solution.
9. A very important aspect of doing problems is keeping track of **units**. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has no doubt been made. This can serve as a **check** on your solution (but it only tells you if you're wrong, not if you're right). Always use a consistent set of units.

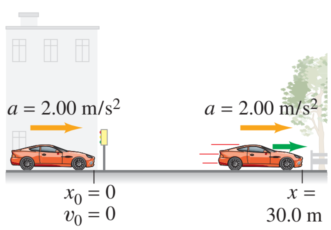
EXAMPLE 10 **Acceleration of a car.** How long does it take a car to cross a 30.0-m-wide intersection after the light turns green, if the car accelerates from rest at a constant $2.00\ \mathrm{m/s^2}$?

APPROACH We follow the Problem Solving Strategy above, step by step.

SOLUTION

1. **Reread** the problem. Be sure you understand what it asks for (here, a time interval).
2. The **object** under study is the car. We choose the **time interval**: $t = 0$, the initial time, is the moment the car starts to accelerate from rest $(v_0 = 0)$; the time t is the instant the car has traveled the full 30.0-m width of the intersection.
3. **Draw** a **diagram**: the situation is shown in Fig. 20, where the car is shown moving along the positive x axis. We choose $x_0 = 0$ at the front bumper of the car before it starts to move.

FIGURE 20 Example 10.



4. The "**knowns**" and the "wanted" are shown in the Table in the margin, and we choose $x_0 = 0$. Note that "starting from rest" means $v = 0$ at $t = 0$; that is, $v_0 = 0$.
5. The **physics**: the motion takes place at constant acceleration, so we can use the kinematic equations, Eqs. 12.
6. **Equations**: we want to find the time, given the distance and acceleration; Eq. 12b is perfect since the only unknown quantity is t. Setting $v_0 = 0$ and $x_0 = 0$ in Eq. 12b $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$, we can solve for t:

$$x = \tfrac{1}{2}at^2,$$

$$t^2 = \frac{2x}{a},$$

so

$$t = \sqrt{\frac{2x}{a}}.$$

7. The **calculation**:

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\,\text{m})}{2.00\,\text{m/s}^2}} = 5.48\,\text{s}.$$

This is our answer. Note that the units come out correctly.
8. We can check the **reasonableness** of the answer by calculating the final velocity $v = at = (2.00\,\text{m/s}^2)(5.48\,\text{s}) = 10.96\,\text{m/s}$, and then finding $x = x_0 + \bar{v}t = 0 + \frac{1}{2}(10.96\,\text{m/s} + 0)(5.48\,\text{s}) = 30.0\,\text{m}$, which is our given distance.
9. We checked the **units**, and they came out perfectly (seconds).

NOTE In steps 6 and 7, when we took the square root, we should have written $t = \pm\sqrt{2x/a} = \pm 5.48\,\text{s}$. Mathematically there are two solutions. But the second solution, $t = -5.48\,\text{s}$, is a time *before* our chosen time interval and makes no sense physically. We say it is "unphysical" and ignore it.

Known	Wanted
$x_0 = 0$	t
$x = 30.0\,\text{m}$	
$a = 2.00\,\text{m/s}^2$	
$v_0 = 0$	

We explicitly followed the steps of the Problem Solving Strategy for Example 10. In upcoming Examples, we will use our usual "Approach" and "Solution" to avoid being wordy.

EXAMPLE 11 **ESTIMATE** **Air bags.** Suppose you want to design an air-bag system that can protect the driver at a speed of 100 km/h (60 mph) if the car hits a brick wall. Estimate how fast the air bag must inflate (Fig. 21) to effectively protect the driver. How does the use of a seat belt help the driver?

PHYSICS APPLIED
Car safety—air bags

APPROACH We assume the acceleration is roughly constant, so we can use Eqs. 12. Both Eqs. 12a and 12b contain t, our desired unknown. They both contain a, so we must first find a, which we can do using Eq. 12c if we know the distance x over which the car crumples. A rough estimate might be about 1 meter. We choose the time interval to start at the instant of impact with the car moving at $v_0 = 100\,\text{km/h}$, and to end when the car comes to rest $(v = 0)$ after traveling 1 m.

SOLUTION We convert the given initial speed to SI units: $100\,\text{km/h} = 100 \times 10^3\,\text{m}/3600\,\text{s} = 28\,\text{m/s}$. We then find the acceleration from Eq. 12c:

$$a = -\frac{v_0^2}{2x} = -\frac{(28\,\text{m/s})^2}{2.0\,\text{m}} = -390\,\text{m/s}^2.$$

This enormous acceleration takes place in a time given by (Eq. 12a):

$$t = \frac{v - v_0}{a} = \frac{0 - 28\,\text{m/s}}{-390\,\text{m/s}^2} = 0.07\,\text{s}.$$

To be effective, the air bag would need to inflate faster than this.

What does the air bag do? It spreads the force over a large area of the chest (to avoid puncture of the chest by the steering wheel). The seat belt keeps the person in a stable position against the expanding air bag.

FIGURE 21 Example 11. An air bag deploying on impact.

SuperStock, Inc.

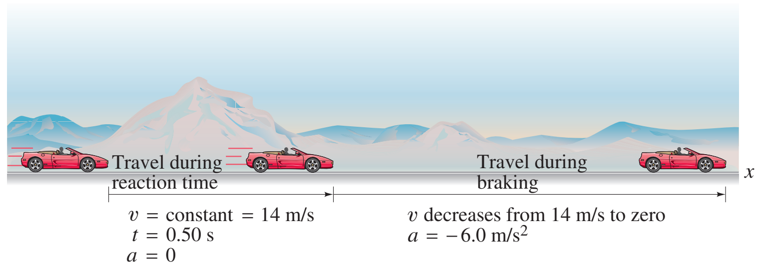


FIGURE 22 Example 12: stopping distance for a braking car.

PHYSICS APPLIED
Braking distances

EXAMPLE 12 **ESTIMATE** **Braking distances.** Estimate the minimum stopping distance for a car, which is important for traffic safety and traffic design. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time" during which the speed is constant, so $a = 0$. (2) The second time interval is the actual braking period when the vehicle slows down ($a \neq 0$) and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. For a dry road and good tires, good brakes can decelerate a car at a rate of about $5\,\text{m/s}^2$ to $8\,\text{m/s}^2$. Calculate the total stopping distance for an initial velocity of 50 km/h ($= 14\,\text{m/s} \approx 31\,\text{mi/h}$) and assume the acceleration of the car is $-6.0\,\text{m/s}^2$ (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing). Reaction time for normal drivers varies from perhaps 0.3 s to about 1.0 s; take it to be 0.50 s.

Part 1: Reaction time

Known	Wanted
$t = 0.50\,\text{s}$	x
$v_0 = 14\,\text{m/s}$	
$v = 14\,\text{m/s}$	
$a = 0$	
$x_0 = 0$	

Part 2: Braking

Known	Wanted
$x_0 = 7.0\,\text{m}$	x
$v_0 = 14\,\text{m/s}$	
$v = 0$	
$a = -6.0\,\text{m/s}^2$	

APPROACH During the "reaction time," part (1), the car moves at constant speed of 14 m/s, so $a = 0$. Once the brakes are applied, part (2), the acceleration is $a = -6.0\,\text{m/s}^2$ and is constant over this time interval. For both parts a is constant, so we can use Eqs. 12.

SOLUTION Part (1). We take $x_0 = 0$ for the first time interval, when the driver is reacting (0.50 s): the car travels at a constant speed of 14 m/s so $a = 0$. See Fig. 22 and the Table in the margin. To find x, the position of the car at $t = 0.50\,\text{s}$ (when the brakes are applied), we cannot use Eq. 12c because x is multiplied by a, which is zero. But Eq. 12b works:

$$x = v_0 t + 0 = (14\,\text{m/s})(0.50\,\text{s}) = 7.0\,\text{m}.$$

Thus the car travels 7.0 m during the driver's reaction time, until the instant the brakes are applied. We will use this result as input to part (2).

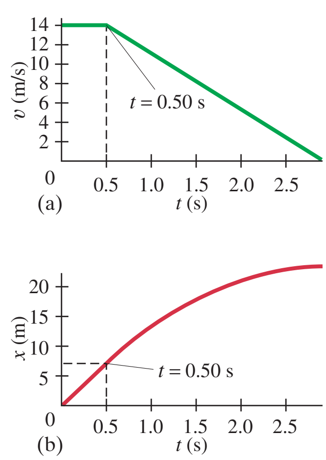
Part (2). During the second time interval, the brakes are applied and the car is brought to rest. The initial position is $x_0 = 7.0\,\text{m}$ (result of part (1)), and other variables are shown in the second Table in the margin. Equation 12a doesn't contain x; Eq. 12b contains x but also the unknown t. Equation 12c, $v^2 - v_0^2 = 2a(x - x_0)$, is what we want; after setting $x_0 = 7.0\,\text{m}$, we solve for x, the final position of the car (when it stops):

$$\begin{aligned} x &= x_0 + \frac{v^2 - v_0^2}{2a} \\ &= 7.0\,\text{m} + \frac{0 - (14\,\text{m/s})^2}{2(-6.0\,\text{m/s}^2)} = 7.0\,\text{m} + \frac{-196\,\text{m}^2/\text{s}^2}{-12\,\text{m/s}^2} \\ &= 7.0\,\text{m} + 16\,\text{m} = 23\,\text{m}. \end{aligned}$$

The car traveled 7.0 m while the driver was reacting and another 16 m during the braking period before coming to a stop, for a total distance traveled of 23 m. Figure 23 shows graphs of (*a*) v vs. t and (*b*) x vs. t.

NOTE From the equation above for x, we see that the stopping distance after the driver hit the brakes $(= x - x_0)$ increases with the *square* of the initial speed, not just linearly with speed. If you are traveling twice as fast, it takes four times the distance to stop.

FIGURE 23 Example 12. Graphs of (*a*) v vs. t and (*b*) x vs. t.



EXAMPLE 13 **ESTIMATE** **Two Moving Objects: Police and Speeder.** A car speeding at 150 km/h passes a still police car which immediately takes off in hot pursuit. Using simple assumptions, such as that the speeder continues at constant speed, estimate how long it takes the police car to overtake the speeder. Then estimate the police car's speed at that moment and decide if the assumptions were reasonable.

APPROACH When the police car takes off, it accelerates, and the simplest assumption is that its acceleration is constant. This may not be reasonable, but let's see what happens. We can estimate the acceleration if we have noticed automobile ads, which claim cars can accelerate from rest to 100 km/h in 5.0 s. So the average acceleration of the police car could be approximately

$$a_{\mathrm{P}} = \frac{100\,\mathrm{km/h}}{5.0\,\mathrm{s}} = 20\,\frac{\mathrm{km/h}}{\mathrm{s}}\left(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}\right)\left(\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right) = 5.6\,\mathrm{m/s^2}.$$

SOLUTION We need to set up the kinematic equations to determine the unknown quantities, and since there are two moving objects, we need two separate sets of equations. We denote the speeding car's position by x_{S} and the police car's position by x_{P}. Because we are interested in solving for the time when the two vehicles arrive at the same position on the road, we use Eq. 12b for each car:

$$x_{\mathrm{S}} = v_{0\mathrm{S}}t + \tfrac{1}{2}a_{\mathrm{S}}t^2 = (150\,\mathrm{km/h})t = (42\,\mathrm{m/s})t$$
$$x_{\mathrm{P}} = v_{0\mathrm{P}}t + \tfrac{1}{2}a_{\mathrm{P}}t^2 = \tfrac{1}{2}(5.6\,\mathrm{m/s^2})t^2,$$

where we have set $v_{0\mathrm{P}} = 0$ and $a_{\mathrm{S}} = 0$ (speeder assumed to move at constant speed). We want the time when the cars meet, so we set $x_{\mathrm{S}} = x_{\mathrm{P}}$ and solve for t:

$$(42\,\mathrm{m/s})t = (2.8\,\mathrm{m/s^2})t^2.$$

The solutions are

$$t = 0 \quad \text{and} \quad t = \frac{42\,\mathrm{m/s}}{2.8\,\mathrm{m/s^2}} = 15\,\mathrm{s}.$$

The first solution corresponds to the instant the speeder passed the police car. The second solution tells us when the police car catches up to the speeder, 15 s later. This is our answer, but is it reasonable? The police car's speed at $t = 15\,\mathrm{s}$ is

$$v_{\mathrm{P}} = v_{0\mathrm{P}} + a_{\mathrm{P}}t = 0 + (5.6\,\mathrm{m/s^2})(15\,\mathrm{s}) = 84\,\mathrm{m/s}$$

or 300 km/h ($\approx$ 190 mi/h). Not reasonable, and highly dangerous.

NOTE More reasonable is to give up the assumption of constant acceleration. The police car surely cannot maintain constant acceleration at those speeds. Also, the speeder, if a reasonable person, would slow down upon hearing the police siren. Figure 24 shows (a) x vs. t and (b) v vs. t graphs, based on the original assumption of a_{P} = constant, whereas (c) shows v vs. t for more reasonable assumptions.

CAUTION
Initial assumptions need to be checked out for reasonableness

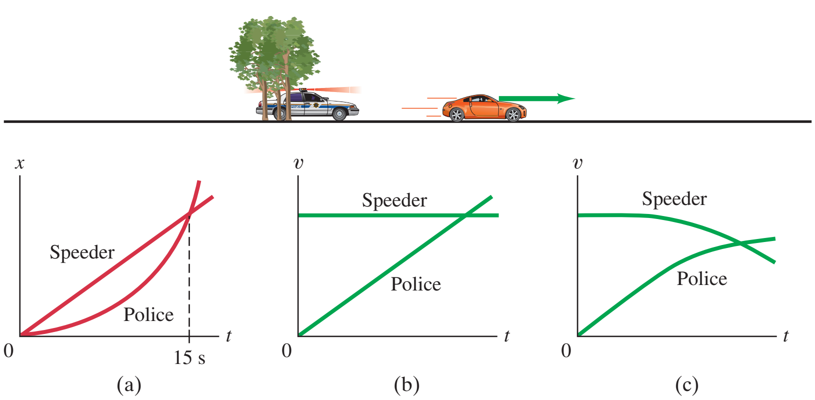


FIGURE 24 Example 13.

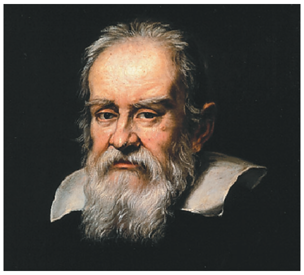


FIGURE 25 Galileo Galilei (1564–1642).

7 Freely Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed before the time of Galileo (Fig. 25), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is.

Galileo made use of his new technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that all objects would fall with the *same constant acceleration* in the absence of air or other resistance. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 26); that is, $d \propto t^2$. We can see this from Eq. 12b; but Galileo was the first to derive this mathematical relation.

⚠ **CAUTION**

A freely falling object increases in speed, but not *in proportion to its mass or weight*

To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

FIGURE 26 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

Galileo claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper horizontally in one hand and a heavier object—say, a baseball—in the other, and release them at the same time as in Fig. 27a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad (see Fig. 27b), you will find that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many ordinary circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 28). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the content of his science (astronomical discoveries, inertia, free fall) but also for his approach to science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

> **at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.**

We call this acceleration the **acceleration due to gravity** on the surface of the Earth, and we give it the symbol g. Its magnitude is approximately

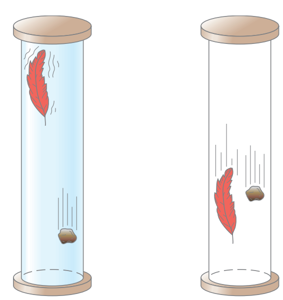
Acceleration due to gravity

$$g = 9.80\ \text{m/s}^2. \qquad \text{[at surface of Earth]}$$

In British units g is about 32 ft/s^2. Actually, g varies slightly according to latitude and elevation, but these variations are so small that we will ignore them for most

(a) (b)

FIGURE 27 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.



(a) (b)

FIGURE 28 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.

purposes. The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.† Acceleration due to gravity is a vector as is any acceleration, and its direction is downward, toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 12, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

PROBLEM SOLVING

You can choose y to be positive either up or down

EXERCISE H Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 14 **Falling from a tower.** Suppose that a ball is dropped $(v_0 = 0)$ from a tower 70.0 m high. How far will it have fallen after a time $t_1 = 1.00\,\text{s}$, $t_2 = 2.00\,\text{s}$, and $t_3 = 3.00\,\text{s}$? Ignore air resistance.

APPROACH Let us take y as positive downward, so the acceleration is $a = g = +9.80\,\text{m/s}^2$. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position y of the ball after three different time intervals. Equation 12b, with x replaced by y, relates the given quantities (t, a, and v_0) to the unknown y.

SOLUTION We set $t = t_1 = 1.00\,\text{s}$ in Eq. 12b:

$$y_1 = v_0 t_1 + \tfrac{1}{2}at_1^2 = 0 + \tfrac{1}{2}at_1^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 4.90\,\text{m}.$$

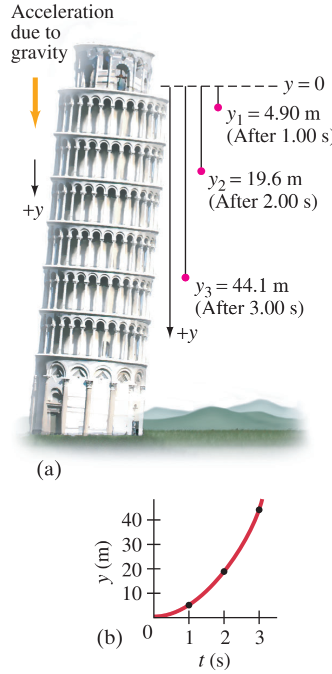
The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\,\text{s}$. Similarly, after 2.00 s $(= t_2)$, the ball's position is

$$y_2 = \tfrac{1}{2}at_2^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 19.6\,\text{m}.$$

Finally, after 3.00 s $(= t_3)$, the ball's position is (see Fig. 29)

$$y_3 = \tfrac{1}{2}at_3^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(3.00\,\text{s})^2 = 44.1\,\text{m}.$$

FIGURE 29 Example 14. (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 26.) (b) Graph of y vs. t.



EXAMPLE 15 **Thrown down from a tower.** Suppose the ball in Example 14 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

APPROACH Again we use Eq. 12b, but now v_0 is not zero, it is $v_0 = 3.00\,\text{m/s}$.

SOLUTION (*a*) At $t = 1.00\,\text{s}$, the position of the ball as given by Eq. 12b is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\text{m/s})(1.00\,\text{s}) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 7.90\,\text{m}.$$

At $t = 2.00\,\text{s}$, (time interval $t = 0$ to $t = 2.00\,\text{s}$), the position is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\text{m/s})(2.00\,\text{s}) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 25.6\,\text{m}.$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.
(*b*) The velocity is obtained from Eq. 12a:

$$\begin{aligned} v &= v_0 + at \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 12.8\,\text{m/s} \quad [\text{at } t_1 = 1.00\,\text{s}] \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 22.6\,\text{m/s}. \quad [\text{at } t_2 = 2.00\,\text{s}] \end{aligned}$$

In Example 14, when the ball was dropped $(v_0 = 0)$, the first term (v_0) in these equations was zero, so

$$\begin{aligned} v &= 0 + at \\ &= (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 9.80\,\text{m/s} \qquad [\text{at } t_1 = 1.00\,\text{s}] \\ &= (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 19.6\,\text{m/s}. \qquad [\text{at } t_2 = 2.00\,\text{s}] \end{aligned}$$

NOTE For both Examples 14 and 15, the speed increases linearly in time by 9.80 m/s during each second. But the speed of the downwardly thrown ball at any instant is always 3.00 m/s (its initial speed) higher than that of a dropped ball.

†The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity** due to air resistance.

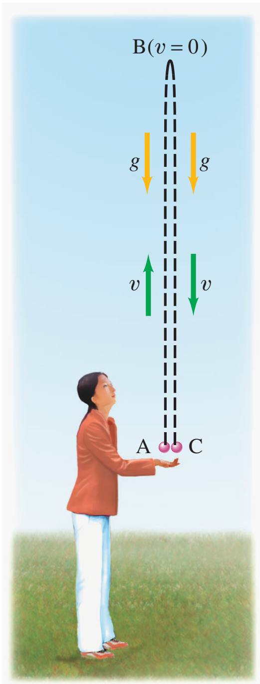


FIGURE 30 An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 16, 17, 18, and 19.

EXAMPLE 16 Ball thrown upward, I. A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate (*a*) how high it goes, and (*b*) how long the ball is in the air before it comes back to the hand. Ignore air resistance.

APPROACH We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 30) and until it comes back to the hand again. Let us choose y to be positive in the upward direction and negative in the downward direction. (This is a different convention from that used in Examples 14 and 15, and so illustrates our options.) The acceleration due to gravity is downward and so will have a negative sign, $a = -g = -9.80\,\text{m/s}^2$. As the ball rises, its speed decreases until it reaches the highest point (B in Fig. 30), where its speed is zero for an instant; then it descends, with increasing speed.

SOLUTION (*a*) We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 30) we have $y_0 = 0$, $v_0 = 15.0\,\text{m/s}$, and $a = -9.80\,\text{m/s}^2$. At time t (maximum height), $v = 0$, $a = -9.80\,\text{m/s}^2$, and we wish to find y. We use Eq. 12c, replacing x with y: $v^2 = v_0^2 + 2ay$. We solve this equation for y:

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\text{m/s})^2}{2(-9.80\,\text{m/s}^2)} = 11.5\,\text{m}.$$

The ball reaches a height of 11.5 m above the hand.

(*b*) Now we need to choose a different time interval to calculate how long the ball is in the air before it returns to the hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 30) in one step and use Eq. 12b. We can do this because y represents position or displacement, and not the total distance traveled. Thus, at both points A and C, $y = 0$. We use Eq. 12b with $a = -9.80\,\text{m/s}^2$ and find

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$0 = 0 + (15.0\,\text{m/s})t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2.$$

This equation is readily factored (we factor out one t):

$$(15.0\,\text{m/s} - 4.90\,\text{m/s}^2\,t)t = 0.$$

There are two solutions:

$$t = 0 \quad \text{and} \quad t = \frac{15.0\,\text{m/s}}{4.90\,\text{m/s}^2} = 3.06\,\text{s}.$$

The first solution ($t = 0$) corresponds to the initial point (A) in Fig. 30, when the ball was first thrown from $y = 0$. The second solution, $t = 3.06\,\text{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

NOTE We have ignored air resistance, which could be significant, so our result is only an approximation to a real, practical situation.

We did not consider the throwing action in this Example. Why? Because during the throw, the thrower's hand is touching the ball and accelerating the ball at a rate unknown to us—the acceleration is *not* g. We consider only the time when the ball is in the air and the acceleration is equal to g.

CAUTION
Quadratic equations have two solutions. Sometimes only one corresponds to reality, sometimes both

Every quadratic equation (where the variable is squared) mathematically produces two solutions. In physics, sometimes only one solution corresponds to the real situation, as in Example 10, in which case we ignore the "unphysical" solution. But in Example 16, both solutions to our equation in t^2 are physically meaningful: $t = 0$ and $t = 3.06\,\text{s}$.

CONCEPTUAL EXAMPLE 17 **Two possible misconceptions.** Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 30).

RESPONSE Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Example 16 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 30), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc points upward, then becomes zero (for zero time) at the highest point, and then points downward. Gravity does not stop acting, so $a = -g = -9.80\,\text{m/s}^2$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration ($=$ rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. In sum, the acceleration of gravity always points down toward the Earth, even when the object is moving up.

CAUTION

(1) Velocity and acceleration are not always in the same direction; the acceleration (of gravity) always points down

(2) $a \neq 0$ even at the highest point of a trajectory

EXAMPLE 18 **Ball thrown upward, II.** Let us consider again the ball thrown upward of Example 16, and make more calculations. Calculate (*a*) how much time it takes for the ball to reach the maximum height (point B in Fig. 30), and (*b*) the velocity of the ball when it returns to the thrower's hand (point C).

APPROACH Again we assume the acceleration is constant, so we can use Eqs. 12. We have the height of 11.5 m from Example 16. Again we take y as positive upward.

SOLUTION (*a*) We consider the time interval between the throw $(t = 0,\ v_0 = 15.0\,\text{m/s})$ and the top of the path $(y = +11.5\,\text{m},\ v = 0)$, and we want to find t. The acceleration is constant at $a = -g = -9.80\,\text{m/s}^2$. Both Eqs. 12a and 12b contain the time t with other quantities known. Let us use Eq. 12a with $a = -9.80\,\text{m/s}^2$, $v_0 = 15.0\,\text{m/s}$, and $v = 0$:

$$v = v_0 + at;$$

setting $v = 0$ and solving for t gives

$$t = -\frac{v_0}{a} = -\frac{15.0\,\text{m/s}}{-9.80\,\text{m/s}^2} = 1.53\,\text{s}.$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in part (*b*) of Example 16]. Thus it takes the same time to reach the maximum height as to fall back to the starting point.

(*b*) Now we consider the time interval from the throw $(t = 0,\ v_0 = 15.0\,\text{m/s})$ until the ball's return to the hand, which occurs at $t = 3.06\,\text{s}$ (as calculated in Example 16), and we want to find v when $t = 3.06\,\text{s}$:

$$v = v_0 + at = 15.0\,\text{m/s} - (9.80\,\text{m/s}^2)(3.06\,\text{s}) = -15.0\,\text{m/s}.$$

NOTE The ball has the same speed (magnitude of velocity) when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). And, as we saw in part (*a*), the time is the same up as down. Thus the motion is *symmetrical* about the maximum height.

The acceleration of objects such as rockets and fast airplanes is often given as a multiple of $g = 9.80\,\text{m/s}^2$. For example, a plane pulling out of a dive and undergoing 3.00 g's would have an acceleration of $(3.00)(9.80\,\text{m/s}^2) = 29.4\,\text{m/s}^2$.

EXERCISE I If a car is said to accelerate at 0.50 g, what is its acceleration in m/s^2?

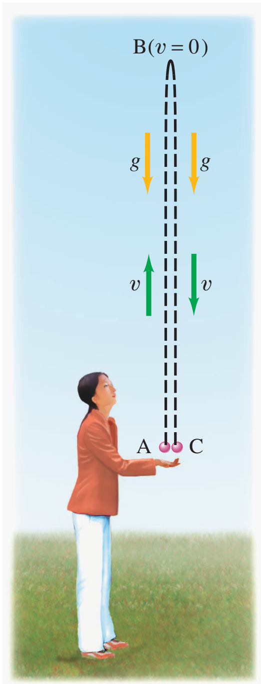


FIGURE 30
(Repeated for Example 19)

EXAMPLE 19 Ball thrown upward, III; the quadratic formula. For the ball in Example 18, calculate at what time t the ball passes a point 8.00 m above the person's hand. (See repeated Fig. 30 here).

APPROACH We choose the time interval from the throw $(t = 0,\ v_0 = 15.0\ \text{m/s})$ until the time t (to be determined) when the ball is at position $y = 8.00\ \text{m}$, using Eq. 12b.

SOLUTION We want to find t, given $y = 8.00\ \text{m}$, $y_0 = 0$, $v_0 = 15.0\ \text{m/s}$, and $a = -9.80\ \text{m/s}^2$. We use Eq. 12b:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

$$8.00\ \text{m} = 0 + (15.0\ \text{m/s})t + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2.$$

To solve any quadratic equation of the form $at^2 + bt + c = 0$, where a, b, and c are constants (a is *not* acceleration here), we use the **quadratic formula**:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our y equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\ \text{m/s}^2)t^2 - (15.0\ \text{m/s})t + (8.00\ \text{m}) = 0.$$

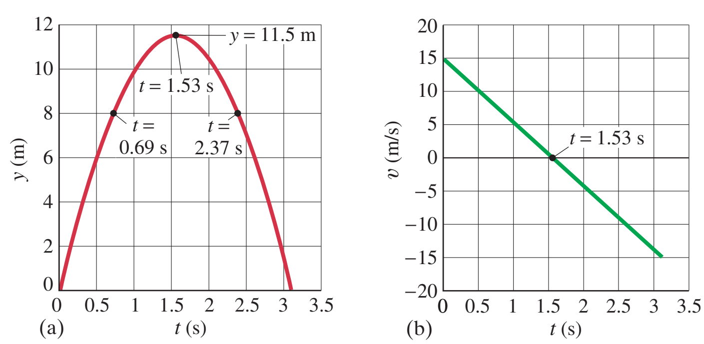
So the coefficient a is $4.90\ \text{m/s}^2$, b is $-15.0\ \text{m/s}$, and c is 8.00 m. Putting these into the quadratic formula, we obtain

$$t = \frac{15.0\ \text{m/s} \pm \sqrt{(15.0\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(8.00\ \text{m})}}{2(4.90\ \text{m/s}^2)},$$

which gives us $t = 0.69\ \text{s}$ and $t = 2.37\ \text{s}$. Are both solutions valid? Yes, because the ball passes $y = 8.00\ \text{m}$ when it goes up $(t = 0.69\ \text{s})$ and again when it comes down $(t = 2.37\ \text{s})$.

NOTE Figure 31 shows graphs of (a) y vs. t and (b) v vs. t for the ball thrown upward in Fig. 30, incorporating the results of Examples 16, 18, and 19.

FIGURE 31 Graphs of (a) y vs. t, (b) v vs. t for a ball thrown upward, Examples 16, 18, and 19.



EXAMPLE 20 Ball thrown upward at edge of cliff. Suppose that the person of Examples 16, 18, and 19 is standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below as in Fig. 32. (a) How long does it take the ball to reach the base of the cliff? (b) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

APPROACH We again use Eq. 12b, but this time we set $y = -50.0\ \text{m}$, the bottom of the cliff, which is 50.0 m below the initial position $(y_0 = 0)$.

SOLUTION (*a*) We use Eq. 12b with $a = -9.80\ \text{m/s}^2$, $v_0 = 15.0\ \text{m/s}$, $y_0 = 0$, and $y = -50.0\ \text{m}$:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$-50.0\ \text{m} = 0 + (15.0\ \text{m/s})t - \tfrac{1}{2}(9.80\ \text{m/s}^2)t^2.$$

Rewriting in the standard form we have

$$(4.90\ \text{m/s}^2)t^2 - (15.0\ \text{m/s})t - (50.0\ \text{m}) = 0.$$

Using the quadratic formula, we find as solutions $t = 5.07\ \text{s}$ and $t = -2.01\ \text{s}$. The first solution, $t = 5.07\ \text{s}$, is the answer we are seeking: the time it takes the ball to rise to its highest point and then fall to the base of the cliff. To rise and fall back to the top of the cliff took 3.06 s (Example 16); so it took an additional 2.01 s to fall to the base. But what is the meaning of the other solution, $t = -2.01\ \text{s}$? This is a time before the throw, when our calculation begins, so it isn't relevant here.†

(*b*) From Example 16, the ball moves up 11.5 m, falls 11.5 m back down to the top of the cliff, and then down another 50.0 m to the base of the cliff, for a total distance traveled of 73.0 m. Note that the *displacement*, however, was −50.0 m. Figure 33 shows the y vs. t graph for this situation.

EXERCISE J Two balls are thrown from a cliff. One is thrown directly up, the other directly down, each with the same initial speed, and both hit the ground below the cliff. Which ball hits the ground at the greater speed: (*a*) the ball thrown upward, (*b*) the ball thrown downward, or (*c*) both the same? Ignore air resistance.

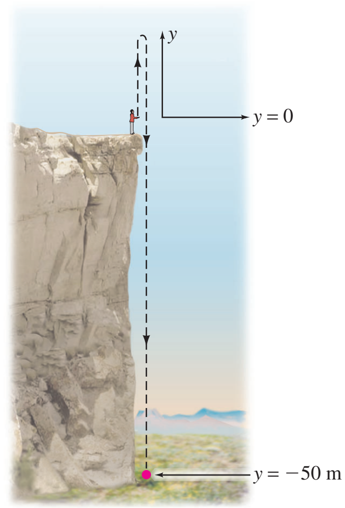
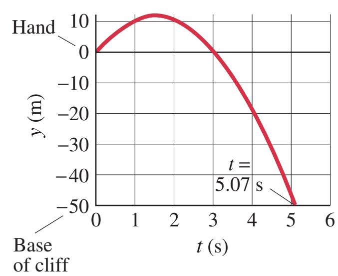


FIGURE 32 Example 20. The person in Fig. 30 stands on the edge of a cliff. The ball falls to the base of the cliff, 50.0 m below.

FIGURE 33 Example 20, the y vs. t graph.



*8 Variable Acceleration; Integral Calculus

In this brief optional Section we use integral calculus to derive the kinematic equations for constant acceleration, Eqs. 12a and b. We also show how calculus can be used when the acceleration is not constant. If you have not yet studied simple integration in your calculus course, you may want to postpone reading this Section until you have.

First we derive Eq. 12a, assuming as we did in Section 5 that an object has velocity v_0 at $t = 0$ and a constant acceleration a. We start with the definition of instantaneous acceleration, $a = dv/dt$, which we rewrite as

$$dv = a\,dt.$$

We take the definite integral of both sides of this equation, using the same notation we did in Section 5:

$$\int_{v=v_0}^{v} dv = \int_{t=0}^{t} a\,dt$$

which gives, since a = constant,

$$v - v_0 = at.$$

This is Eq. 12a, $v = v_0 + at$.

Next we derive Eq. 12b starting with the definition of instantaneous velocity, Eq. 4, $v = dx/dt$. We rewrite this as

$$dx = v\,dt$$

or

$$dx = (v_0 + at)dt$$

where we substituted in Eq. 12a.

†The solution $t = -2.01\ \text{s}$ could be meaningful in a different physical situation. Suppose that a person standing on top of a 50.0-m-high cliff sees a rock pass by him at $t = 0$ moving upward at 15.0 m/s; at what time did the rock leave the base of the cliff, and when did it arrive back at the base of the cliff? The equations will be precisely the same as for our original Example, and the answers $t = -2.01\ \text{s}$ and $t = 5.07\ \text{s}$ will be the correct answers. Note that we cannot put all the information for a problem into the mathematics, so we have to use common sense in interpreting results.

Now we integrate:

$$\int_{x=x_0}^{x} dx = \int_{t=0}^{t} (v_0 + at)\,dt$$

$$x - x_0 = \int_{t=0}^{t} v_0\,dt + \int_{t=0}^{t} at\,dt$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

since v_0 and a are constants. This result is just Eq. 12b, $x = x_0 + v_0 t + \frac{1}{2}at^2$.

Finally let us use calculus to find velocity and displacement, given an acceleration that is not constant but varies in time.

EXAMPLE 21 Integrating a time-varying acceleration. An experimental vehicle starts from rest $(v_0 = 0)$ at $t = 0$ and accelerates at a rate given by $a = (7.00\,\mathrm{m/s^3})t$. What is (*a*) its velocity and (*b*) its displacement 2.00 s later?

APPROACH We cannot use Eqs. 12 because a is not constant. We integrate the acceleration $a = dv/dt$ over time to find v as a function of time; and then integrate $v = dx/dt$ to get the displacement.

SOLUTION From the definition of acceleration, $a = dv/dt$, we have

$$dv = a\,dt.$$

We take the integral of both sides from $v = 0$ at $t = 0$ to velocity v at an arbitrary time t:

$$\int_0^v dv = \int_0^t a\,dt$$

$$v = \int_0^t (7.00\,\mathrm{m/s^3})t\,dt$$

$$= (7.00\,\mathrm{m/s^3})\left(\frac{t^2}{2}\right)\Bigg|_0^t = (7.00\,\mathrm{m/s^3})\left(\frac{t^2}{2} - 0\right) = (3.50\,\mathrm{m/s^3})t^2.$$

At $t = 2.00\,\mathrm{s}$, $v = (3.50\,\mathrm{m/s^3})(2.00\,\mathrm{s})^2 = 14.0\,\mathrm{m/s}$.

(*b*) To get the displacement, we assume $x_0 = 0$ and start with $v = dx/dt$ which we rewrite as $dx = v\,dt$. Then we integrate from $x = 0$ at $t = 0$ to position x at time t:

$$\int_0^x dx = \int_0^t v\,dt$$

$$x = \int_0^{2.00\,\mathrm{s}} (3.50\,\mathrm{m/s^3})t^2\,dt = (3.50\,\mathrm{m/s^3})\frac{t^3}{3}\Bigg|_0^{2.00\,\mathrm{s}} = 9.33\,\mathrm{m}.$$

In sum, at $t = 2.00\,\mathrm{s}$, $v = 14.0\,\mathrm{m/s}$ and $x = 9.33\,\mathrm{m}$.

*9 Graphical Analysis and Numerical Integration

This Section is optional. It discusses how to solve certain Problems numerically, often needing a computer to do the sums.

If we are given the velocity v of an object as a function of time t, we can obtain the displacement, x. Suppose the velocity as a function of time, $v(t)$, is given as a graph (rather than as an equation that could be integrated as discussed in Section 8), as shown in Fig 34a. If we are interested in the time interval from t_1 to t_2, as shown, we divide the time axis into many small subintervals, $\Delta t_1, \Delta t_2, \Delta t_3, \ldots$, which are indicated by the dashed vertical lines. For each subinterval, a horizontal dashed line is drawn to indicate the average velocity during that time interval. The displacement during any subinterval is given by Δx_i, where the subscript i represents the particular subinterval

$(i = 1, 2, 3, \ldots)$. From the definition of average velocity (Eq. 2) we have

$$\Delta x_i = \bar{v}_i \Delta t_i .$$

Thus the displacement during each subinterval equals the product of $\bar{v}_i$ and Δt_i, and equals the area of the dark rectangle in Fig. 34a for that subinterval. The total displacement between times t_1 and t_2 is the sum of the displacements over all the subintervals:

$$x_2 - x_1 = \sum_{t_1}^{t_2} \bar{v}_i \Delta t_i , \qquad \textbf{(13a)}$$

where x_1 is the position at t_1 and x_2 is the position at t_2. This sum equals the area of all the rectangles shown.

It is often difficult to estimate $\bar{v}_i$ with precision for each subinterval from the graph. We can get greater accuracy in our calculation of $x_2 - x_1$ by breaking the interval $t_2 - t_1$ into more, but narrower, subintervals. Ideally, we can let each Δt_i approach zero, so we approach (in principle) an infinite number of subintervals. In this limit the area of all these infinitesimally thin rectangles becomes exactly equal to the area under the curve (Fig. 34b). Thus *the total displacement between any two times is equal to the area between the velocity curve and the t axis between the two times t_1 and t_2*. This limit can be written

$$x_2 - x_1 = \lim_{\Delta t \to 0} \sum_{t_1}^{t_2} \bar{v}_i \Delta t_i$$

or, using standard calculus notation,

$$x_2 - x_1 = \int_{t_1}^{t_2} v(t)\, dt. \qquad \textbf{(13b)}$$

We have let $\Delta t \to 0$and renamed it dt to indicate that it is now infinitesimally small. The average velocity, $\bar{v}$, over an infinitesimal time dt is the instantaneous velocity at that instant, which we have written $v(t)$ to remind us that v is a function of t. The symbol $\int$ is an elongated S and indicates a sum over an infinite number of infinitesimal subintervals. We say that we are taking the *integral* of $v(t)$ over dt from time t_1 to time t_2, and this is equal to the area between the $v(t)$ curve and the t axis between the times t_1 and t_2 (Fig. 34b). The integral in Eq. 13b is a *definite integral*, since the limits t_1 and t_2 are specified.

Similarly, if we know the acceleration as a function of time, we can obtain the velocity by the same process. We use the definition of average acceleration (Eq. 5) and solve for Δv:

$$\Delta v = \bar{a} \Delta t.$$

If a is known as a function of t over some time interval t_1 to t_2, we can subdivide this time interval into many subintervals, Δt_i, just as we did in Fig. 34a. The change in velocity during each subinterval is $\Delta v_i = \bar{a}_i \Delta t_i$. The total change in velocity from time t_1 until time t_2 is

$$v_2 - v_1 = \sum_{t_1}^{t_2} \bar{a}_i \Delta t_i , \qquad \textbf{(14a)}$$

where v_2 represents the velocity at t_2 and v_1 the velocity at t_1. This relation can be written as an integral by letting $\Delta t \to 0$(the number of intervals then approaches infinity)

$$v_2 - v_1 = \lim_{\Delta t \to 0} \sum_{t_1}^{t_2} \bar{a}_i \Delta t_i$$

or

$$v_2 - v_1 = \int_{t_1}^{t_2} a(t)\, dt. \qquad \textbf{(14b)}$$

Equations 14 will allow us to determine the velocity v_2 at some time t_2 if the velocity is known at t_1 and a is known as a function of time.

If the acceleration or velocity is known at discrete intervals of time, we can use the summation forms of the above equations, Eqs. 13a and 14a, to estimate velocity or displacement. This technique is known as **numerical integration**. We now take an Example that can also be evaluated analytically, so we can compare the results.

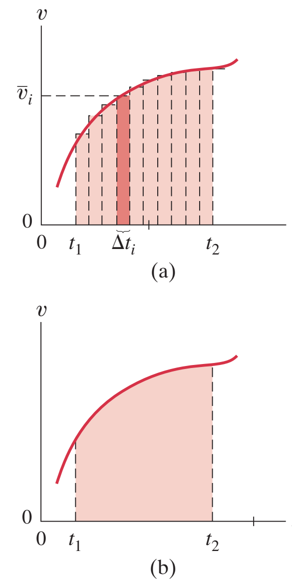


FIGURE 34 Graph of v vs. t for the motion of a particle. In (a), the time axis is broken into subintervals of width Δt_i, the average velocity during each Δt_i is $\bar{v}_i$, and the area of all the rectangles, $\sum \bar{v}_i \Delta t_i$, is numerically equal to the total displacement $(x_2 - x_1)$ during the total time $(t_2 - t_1)$. In (b), $\Delta t_i \to 0$ and the area under the curve is equal to $(x_2 - x_1)$.

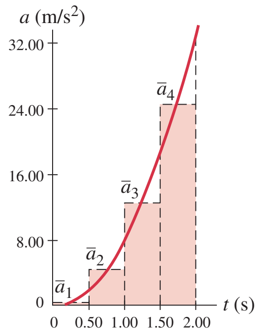


FIGURE 35 Example 22.

EXAMPLE 22 **Numerical integration.** An object starts from rest at $t = 0$ and accelerates at a rate $a(t) = (8.00\,\text{m/s}^4)t^2$. Determine its velocity after 2.00 s using numerical methods.

APPROACH Let us first divide up the interval $t = 0.00\,\text{s}$ to $t = 2.00\,\text{s}$ into four subintervals each of duration $\Delta t_i = 0.50\,\text{s}$ (Fig. 35). We use Eq. 14a with $v_2 = v$, $v_1 = 0$, $t_2 = 2.00\,\text{s}$, and $t_1 = 0$. For each of the subintervals we need to estimate $\bar{a}_i$. There are various ways to do this and we use the simple method of choosing $\bar{a}_i$ to be the acceleration $a(t)$ at the midpoint of each interval (an even simpler but usually less accurate procedure would be to use the value of a at the start of the subinterval). That is, we evaluate $a(t) = (8.00\,\text{m/s}^4)t^2$ at $t = 0.25\,\text{s}$ (which is midway between 0.00 s and 0.50 s), 0.75 s, 1.25 s, and 1.75 s.

SOLUTION The results are as follows:

i	1	2	3	4
$\bar{a}_i(\text{m/s}^2)$	0.50	4.50	12.50	24.50

Now we use Eq. 14a, and note that all Δt_i equal 0.50 s (so they can be factored out):

$$\begin{aligned} v(t = 2.00\,\text{s}) &= \sum_{t=0}^{t=2.00\,\text{s}} \bar{a}_i\,\Delta t_i \\ &= (0.50\,\text{m/s}^2 + 4.50\,\text{m/s}^2 + 12.50\,\text{m/s}^2 + 24.50\,\text{m/s}^2)(0.50\,\text{s}) \\ &= 21.0\,\text{m/s}. \end{aligned}$$

We can compare this result to the analytic solution given by Eq. 14b since the functional form for a is integrable analytically:

$$\begin{aligned} v &= \int_0^{2.00\,\text{s}} (8.00\,\text{m/s}^4)t^2\,dt = \frac{8.00\,\text{m/s}^4}{3}\,t^3\Big|_0^{2.00\,\text{s}} \\ &= \frac{8.00\,\text{m/s}^4}{3}\left[(2.00\,\text{s})^3 - (0)^3\right] = 21.33\,\text{m/s} \end{aligned}$$

or 21.3 m/s to the proper number of significant figures. This analytic solution is precise, and we see that our numerical estimate is not far off even though we only used four Δt intervals. It may not be close enough for purposes requiring high accuracy. If we use more and smaller subintervals, we will get a more accurate result. If we use 10 subintervals, each with $\Delta t = 2.00\,\text{s}/10 = 0.20\,\text{s}$, we have to evaluate $a(t)$ at $t = 0.10\,\text{s}, 0.30\,\text{s}, \dots, 1.90\,\text{s}$ to get the $\bar{a}_i$, and these are as follows:

i	1	2	3	4	5	6	7	8	9	10
$\bar{a}_i(\text{m/s}^2)$	0.08	0.72	2.00	3.92	6.48	9.68	13.52	18.00	23.12	28.88

Then, from Eq. 14a we obtain

$$\begin{aligned} v(t = 2.00\,\text{s}) &= \sum \bar{a}_i\,\Delta t_i = \left(\sum \bar{a}_i\right)(0.200\,\text{s}) \\ &= (106.4\,\text{m/s}^2)(0.200\,\text{s}) = 21.28\,\text{m/s}, \end{aligned}$$

where we have kept an extra significant figure to show that this result is much closer to the (precise) analytic one but still is not quite identical to it. The percentage difference has dropped from 1.4% $(0.3\,\text{m/s}^2/21.3\,\text{m/s}^2)$ for the four-subinterval computation to only 0.2% (0.05/21.3) for the 10-subinterval one.

In the Example above we were given an analytic function that was integrable, so we could compare the accuracy of the numerical calculation to the known precise one. But what do we do if the function is not integrable, so we can't compare our numerical result to an analytic one? That is, how do we know if we've taken enough subintervals so that we can trust our calculated estimate to be accurate to within some desired uncertainty, say 1 percent? What we can do is compare two successive numerical calculations: the first done with n subintervals and the second with, say, twice as many subintervals ($2n$). If the two results are within the desired uncertainty (say 1 percent), we can usually assume that the calculation with more subintervals is within the desired uncertainty of the true value. If the two calculations are not that close, then a third calculation, with more subintervals (maybe double, maybe 10 times as many, depending on how good the previous approximation was) must be done, and compared to the previous one.

The procedure is easy to automate using a computer spreadsheet application.

If we wanted to also obtain the displacement x at some time, we would have to do a second numerical integration over v, which means we would first need to calculate v for many different times. Programmable calculators and computers are very helpful for doing the long sums.

Summary

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame**.

The **displacement** of an object is the change in position of the object.

Average speed is the distance traveled divided by the elapsed time or time interval, Δt, the time period over which we choose to make our observations. An object's **average velocity** over a particular time interval Δt is its displacement Δx during that time interval, divided by Δt:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \quad (2)$$

The **instantaneous velocity**, whose magnitude is the same as the *instantaneous speed*, is defined as the average velocity taken over an infinitesimally short time interval ($\Delta t \rightarrow 0$):

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}, \quad (4)$$

where dx/dt is the derivative of x with respect to t.

On a graph of position vs. time, the *slope* is equal to the instantaneous velocity.

Acceleration is the change of velocity per unit time. An object's **average acceleration** over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t}, \quad (5)$$

where Δv is the change of velocity during the time interval Δt.

Instantaneous acceleration is the average acceleration taken over an infinitesimally short time interval:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}. \quad (6)$$

If an object moves in a straight line with *constant acceleration*, the velocity v and position x are related to the acceleration a, the elapsed time t, the initial position x_0, and the initial velocity v_0 by Eqs. 12:

$$v = v_0 + at, \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2,$$
$$v^2 = v_0^2 + 2a(x - x_0), \qquad \bar{v} = \frac{v + v_0}{2}. \quad (12)$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward **acceleration due to gravity**, whose magnitude is $g = 9.80\ \mathrm{m/s^2}$ if air resistance can be ignored.

[*The kinematic Equations 12 can be derived using integral calculus.]

Answers to Exercises

A: -30 cm; 50 cm.

B: (*a*).

C: (*b*).

D: (*b*).

E: (*a*) +; (*b*) −; (*c*) −; (*d*) +.

F: (*c*).

G: (*b*).

H: (*e*).

I: $4.9\ \mathrm{m/s^2}$.

J: (*c*).

This page intentionally left blank

Describing Motion: Kinematics in One Dimension Problem Set

Questions

1. Does a car speedometer measure speed, velocity, or both?
2. Can an object have a varying speed if its velocity is constant? Can it have varying velocity if its speed is constant? If yes, give examples in each case.
3. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant?
4. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain, using examples.
5. Compare the acceleration of a motorcycle that accelerates from 80 km/h to 90 km/h with the acceleration of a bicycle that accelerates from rest to 10 km/h in the same time.
6. Can an object have a northward velocity and a southward acceleration? Explain.
7. Can the velocity of an object be negative when its acceleration is positive? What about vice versa?
8. Give an example where both the velocity and acceleration are negative.
9. Two cars emerge side by side from a tunnel. Car A is traveling with a speed of 60 km/h and has an acceleration of 40 km/h/min. Car B has a speed of 40 km/h and has an acceleration of 60 km/h/min. Which car is passing the other as they come out of the tunnel? Explain your reasoning.
10. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.
11. A baseball player hits a ball straight up into the air. It leaves the bat with a speed of 120 km/h. In the absence of air resistance, how fast would the ball be traveling when the catcher catches it?
12. As a freely falling object speeds up, what is happening to its acceleration—does it increase, decrease, or stay the same? (*a*) Ignore air resistance. (*b*) Consider air resistance.
13. You travel from point A to point B in a car moving at a constant speed of 70 km/h. Then you travel the same distance from point B to another point C, moving at a constant speed of 90 km/h. Is your average speed for the entire trip from A to C 80 km/h? Explain why or why not.
14. Can an object have zero velocity and nonzero acceleration at the same time? Give examples.
15. Can an object have zero acceleration and nonzero velocity at the same time? Give examples.
16. Which of these motions is *not* at constant acceleration: a rock falling from a cliff, an elevator moving from the second floor to the fifth floor making stops along the way, a dish resting on a table?
17. In a lecture demonstration, a 3.0-m-long vertical string with ten bolts tied to it at equal intervals is dropped from the ceiling of the lecture hall. The string falls on a tin plate, and the class hears the clink of each bolt as it hits the plate. The sounds will not occur at equal time intervals. Why? Will the time between clinks increase or decrease near the end of the fall? How could the bolts be tied so that the clinks occur at equal intervals?
18. Describe in words the motion plotted in Fig. 36 in terms of v, a, etc. [*Hint*: First try to duplicate the motion plotted by walking or moving your hand.]

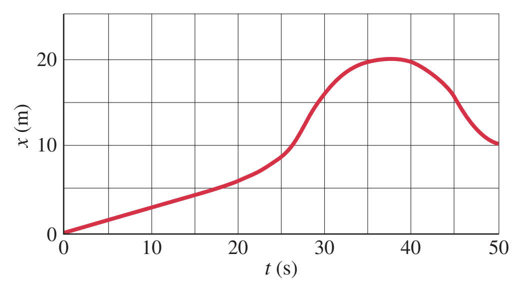


FIGURE 36 Question 18, Problems 9 and 86.

19. Describe in words the motion of the object graphed in Fig. 37.

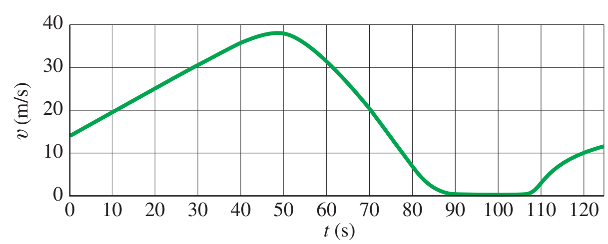


FIGURE 37 Question 19, Problem 23.

Problems

[The Problems in this Section are ranked I, II, or III according to estimated difficulty, with (I) Problems being easiest. Level (III) Problems are meant mainly as a challenge for the best students, for "extra credit." The Problems are arranged by Sections, meaning that the reader should have read up to and including that Section, but this Chapter also has a group of General Problems that are not arranged by Section and not ranked.]

1 to 3 Speed and Velocity

1. (I) If you are driving 110 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?
2. (I) What must your car's average speed be in order to travel 235 km in 3.25 h?

From Chapter 2 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

3. (I) A particle at $t_1 = -2.0\,\text{s}$ is at $x_1 = 4.3\,\text{cm}$ and at $t_2 = 4.5\,\text{s}$ is at $x_2 = 8.5\,\text{cm}$. What is its average velocity? Can you calculate its average speed from these data?

4. (I) A rolling ball moves from $x_1 = 3.4\,\text{cm}$ to $x_2 = -4.2\,\text{cm}$ during the time from $t_1 = 3.0\,\text{s}$ to $t_2 = 5.1\,\text{s}$. What is its average velocity?

5. (II) According to a rule-of-thumb, every five seconds between a lightning flash and the following thunder gives the distance to the flash in miles. Assuming that the flash of light arrives in essentially no time at all, estimate the speed of sound in m/s from this rule. What would be the rule for kilometers?

6. (II) You are driving home from school steadily at 95 km/h for 130 km. It then begins to rain and you slow to 65 km/h. You arrive home after driving 3 hours and 20 minutes. (*a*) How far is your hometown from school? (*b*) What was your average speed?

7. (II) A horse canters away from its trainer in a straight line, moving 116 m away in 14.0 s. It then turns abruptly and gallops halfway back in 4.8 s. Calculate (*a*) its average speed and (*b*) its average velocity for the entire trip, using "away from the trainer" as the positive direction.

8. (II) The position of a small object is given by $x = 34 + 10t - 2t^3$, where t is in seconds and x in meters. (*a*) Plot x as a function of t from $t = 0$ to $t = 3.0\,\text{s}$. (*b*) Find the average velocity of the object between 0 and 3.0 s. (*c*) At what time between 0 and 3.0 s is the instantaneous velocity zero?

9. (II) The position of a rabbit along a straight tunnel as a function of time is plotted in Fig. 36. What is its instantaneous velocity (*a*) at $t = 10.0\,\text{s}$ and (*b*) at $t = 30.0\,\text{s}$? What is its average velocity (*c*) between $t = 0$ and $t = 5.0\,\text{s}$, (*d*) between $t = 25.0\,\text{s}$ and $t = 30.0\,\text{s}$, and (*e*) between $t = 40.0\,\text{s}$ and $t = 50.0\,\text{s}$?

10. (II) On an audio compact disc (CD), digital bits of information are encoded sequentially along a spiral path. Each bit occupies about 0.28 μm. A CD player's readout laser scans along the spiral's sequence of bits at a constant speed of about 1.2 m/s as the CD spins. (*a*) Determine the number N of digital bits that a CD player reads every second. (*b*) The audio information is sent to each of the two loudspeakers 44,100 times per second. Each of these samplings requires 16 bits and so one would (at first glance) think the required bit rate for a CD player is

$$N_0 = 2\left(44{,}100\,\frac{\text{samplings}}{\text{second}}\right)\left(16\,\frac{\text{bits}}{\text{sampling}}\right) = 1.4 \times 10^6\,\frac{\text{bits}}{\text{second}},$$

where the 2 is for the 2 loudspeakers (the 2 stereo channels). Note that N_0 is less than the number N of bits actually read per second by a CD player. The excess number of bits $(= N - N_0)$ is needed for encoding and error-correction. What percentage of the bits on a CD are dedicated to encoding and error-correction?

11. (II) A car traveling 95 km/h is 110 m behind a truck traveling 75 km/h. How long will it take the car to reach the truck?

12. (II) Two locomotives approach each other on parallel tracks. Each has a speed of 95 km/h with respect to the ground. If they are initially 8.5 km apart, how long will it be before they reach each other? (See Fig. 38).

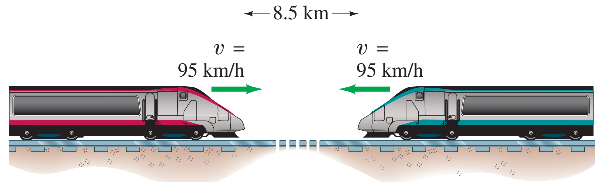


FIGURE 38 Problem 12.

13. (II) Digital bits on a 12.0-cm diameter audio CD are encoded along an outward spiraling path that starts at radius $R_1 = 2.5\,\text{cm}$ and finishes at radius $R_2 = 5.8\,\text{cm}$. The distance between the centers of neighboring spiral-windings is $1.6\,\mu\text{m}\ (= 1.6 \times 10^{-6}\,\text{m})$. (*a*) Determine the total length of the spiraling path. [*Hint*: Imagine "unwinding" the spiral into a straight path of width 1.6 μm, and note that the original spiral and the straight path both occupy the same area.] (*b*) To read information, a CD player adjusts the rotation of the CD so that the player's readout laser moves along the spiral path at a constant speed of 1.25 m/s. Estimate the maximum playing time of such a CD.

14. (II) An airplane travels 3100 km at a speed of 720 km/h, and then encounters a tailwind that boosts its speed to 990 km/h for the next 2800 km. What was the total time for the trip? What was the average speed of the plane for this trip? [*Hint*: Does Eq. 12d apply, or not?]

$$\bar{v} = \frac{v + v_0}{2} \qquad [a = \text{constant}] \qquad \textbf{(12d)}$$

15. (II) Calculate the average speed and average velocity of a complete round trip in which the outgoing 250 km is covered at 95 km/h, followed by a 1.0-h lunch break, and the return 250 km is covered at 55 km/h.

16. (II) The position of a ball rolling in a straight line is given by $x = 2.0 - 3.6\,t + 1.1\,t^2$, where x is in meters and t in seconds. (*a*) Determine the position of the ball at $t = 1.0\,\text{s}$, 2.0 s, and 3.0 s. (*b*) What is the average velocity over the interval $t = 1.0\,\text{s}$ to $t = 3.0\,\text{s}$? (*c*) What is its instantaneous velocity at $t = 2.0\,\text{s}$ and at $t = 3.0\,\text{s}$?

17. (II) A dog runs 120 m away from its master in a straight line in 8.4 s, and then runs halfway back in one-third the time. Calculate (*a*) its average speed and (*b*) its average velocity.

18. (III) An automobile traveling 95 km/h overtakes a 1.10-km-long train traveling in the same direction on a track parallel to the road. If the train's speed is 75 km/h, how long does it take the car to pass it, and how far will the car have traveled in this time? See Fig. 39. What are the results if the car and train are traveling in opposite directions?

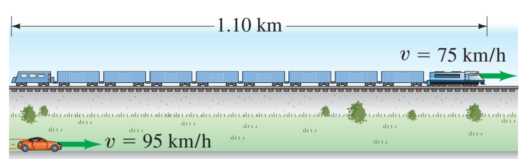


FIGURE 39 Problem 18.

19. (III) A bowling ball traveling with constant speed hits the pins at the end of a bowling lane 16.5 m long. The bowler hears the sound of the ball hitting the pins 2.50 s after the ball is released from his hands. What is the speed of the ball, assuming the speed of sound is 340 m/s?

4 Acceleration

20. (I) A sports car accelerates from rest to 95 km/h in 4.5 s. What is its average acceleration in $\mathrm{m/s^2}$?

21. (I) At highway speeds, a particular automobile is capable of an acceleration of about $1.8\ \mathrm{m/s^2}$. At this rate, how long does it take to accelerate from 80 km/h to 110 km/h?

22. (I) A sprinter accelerates from rest to 9.00 m/s in 1.28 s. What is her acceleration in (*a*) $\mathrm{m/s^2}$; (*b*) $\mathrm{km/h^2}$?

23. (I) Figure 37 shows the velocity of a train as a function of time. (*a*) At what time was its velocity greatest? (*b*) During what periods, if any, was the velocity constant? (*c*) During what periods, if any, was the acceleration constant? (*d*) When was the magnitude of the acceleration greatest?

24. (II) A sports car moving at constant speed travels 110 m in 5.0 s. If it then brakes and comes to a stop in 4.0 s, what is the magnitude of its acceleration in $\mathrm{m/s^2}$, and in g's $(g = 9.80\ \mathrm{m/s^2})$?

25. (II) A car moving in a straight line starts at $x = 0$ at $t = 0$. It passes the point $x = 25.0\ \mathrm{m}$ with a speed of 11.0 m/s at $t = 3.00\ \mathrm{s}$. It passes the point $x = 385\ \mathrm{m}$ with a speed of 45.0 m/s at $t = 20.0\ \mathrm{s}$. Find (*a*) the average velocity and (*b*) the average acceleration between $t = 3.00\ \mathrm{s}$ and $t = 20.0\ \mathrm{s}$.

26. (II) A particular automobile can accelerate approximately as shown in the velocity vs. time graph of Fig. 40. (The short flat spots in the curve represent shifting of the gears.) Estimate the average acceleration of the car in (*a*) second gear; and (*b*) fourth gear. (*c*) What is its average acceleration through the first four gears?

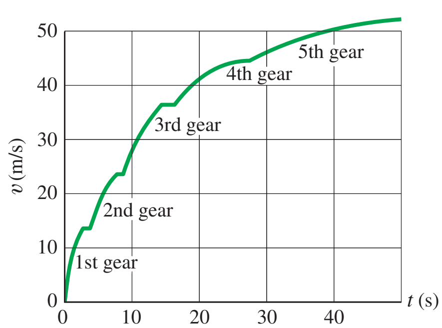


FIGURE 40 Problem 26. The velocity of a high-performance automobile as a function of time, starting from a dead stop. The flat spots in the curve represent gear shifts.

27. (II) A particle moves along the x axis. Its position as a function of time is given by $x = 6.8\,t + 8.5\,t^2$, where t is in seconds and x is in meters. What is the acceleration as a function of time?

28. (II) The position of a racing car, which starts from rest at $t = 0$ and moves in a straight line, is given as a function of time in the following Table. Estimate (*a*) its velocity and (*b*) its acceleration as a function of time. Display each in a Table and on a graph.

t(s)	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50
x(m)	0	0.11	0.46	1.06	1.94	4.62	8.55	13.79
t(s)	3.00	3.50	4.00	4.50	5.00	5.50	6.00	
x(m)	20.36	28.31	37.65	48.37	60.30	73.26	87.16	

29. (II) The position of an object is given by $x = At + Bt^2$, where x is in meters and t is in seconds. (*a*) What are the units of A and B? (*b*) What is the acceleration as a function of time? (*c*) What are the velocity and acceleration at $t = 5.0\ \mathrm{s}$? (*d*) What is the velocity as a function of time if $x = At + Bt^{-3}$?

5 and 6 Motion at Constant Acceleration

30. (I) A car slows down from 25 m/s to rest in a distance of 85 m. What was its acceleration, assumed constant?

31. (I) A car accelerates from 12 m/s to 21 m/s in 6.0 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

32. (I) A light plane must reach a speed of 32 m/s for takeoff. How long a runway is needed if the (constant) acceleration is $3.0\ \mathrm{m/s^2}$?

33. (II) A baseball pitcher throws a baseball with a speed of 41 m/s. Estimate the average acceleration of the ball during the throwing motion. In throwing the baseball, the pitcher accelerates the ball through a displacement of about 3.5 m, from behind the body to the point where it is released (Fig. 41).

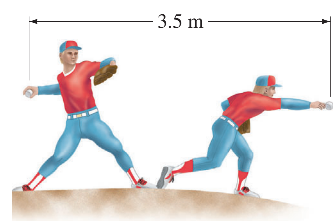


FIGURE 41 Problem 33.

34. (II) Show that $\bar{v} = (v + v_0)/2$ (see Eq. 12d) is not valid when the acceleration $a = A + Bt$, where A and B are constants.

35. (II) A world-class sprinter can reach a top speed (of about 11.5 m/s) in the first 15.0 m of a race. What is the average acceleration of this sprinter and how long does it take her to reach that speed?

36. (II) An inattentive driver is traveling 18.0 m/s when he notices a red light ahead. His car is capable of decelerating at a rate of $3.65\ \mathrm{m/s^2}$. If it takes him 0.200 s to get the brakes on and he is 20.0 m from the intersection when he sees the light, will he be able to stop in time?

37. (II) A car slows down uniformly from a speed of 18.0 m/s to rest in 5.00 s. How far did it travel in that time?

38. (II) In coming to a stop, a car leaves skid marks 85 m long on the highway. Assuming a deceleration of $4.00\ \mathrm{m/s^2}$, estimate the speed of the car just before braking.

39. (II) A car traveling 85 km/h slows down at a constant $0.50\ \mathrm{m/s^2}$ just by "letting up on the gas." Calculate (*a*) the distance the car coasts before it stops, (*b*) the time it takes to stop, and (*c*) the distance it travels during the first and fifth seconds.

40. (II) A car traveling at 105 km/h strikes a tree. The front end of the car compresses and the driver comes to rest after traveling 0.80 m. What was the magnitude of the average acceleration of the driver during the collision? Express the answer in terms of "g's," where $1.00\ g = 9.80\ \mathrm{m/s^2}$.

41. (II) Determine the stopping distances for an automobile with an initial speed of 95 km/h and human reaction time of 1.0 s: (*a*) for an acceleration $a = -5.0\ \mathrm{m/s^2}$; (*b*) for $a = -7.0\ \mathrm{m/s^2}$.

42. (II) A space vehicle accelerates uniformly from 65 m/s at $t = 0$ to 162 m/s at $t = 10.0$ s. How far did it move between $t = 2.0$ s and $t = 6.0$ s?

43. (II) A 75-m-long train begins uniform acceleration from rest. The front of the train has a speed of 23 m/s when it passes a railway worker who is standing 180 m from where the front of the train started. What will be the speed of the last car as it passes the worker? (See Fig. 42.)

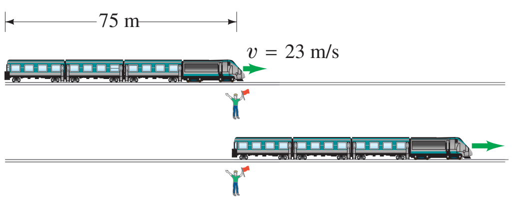


FIGURE 42 Problem 43.

44. (II) An unmarked police car traveling a constant 95 km/h is passed by a speeder traveling 135 km/h. Precisely 1.00 s after the speeder passes, the police officer steps on the accelerator; if the police car's acceleration is $2.00\ \mathrm{m/s^2}$, how much time passes before the police car overtakes the speeder (assumed moving at constant speed)?

45. (III) Assume in Problem 44 that the speeder's speed is not known. If the police car accelerates uniformly as given above and overtakes the speeder after accelerating for 7.00 s, what was the speeder's speed?

46. (III) A runner hopes to complete the 10,000-m run in less than 30.0 min. After running at constant speed for exactly 27.0 min, there are still 1100 m to go. The runner must then accelerate at $0.20\ \mathrm{m/s^2}$ for how many seconds in order to achieve the desired time?

47. (III) Mary and Sally are in a foot race (Fig. 43). When Mary is 22 m from the finish line, she has a speed of 4.0 m/s and is 5.0 m behind Sally, who has a speed of 5.0 m/s. Sally thinks she has an easy win and so, during the remaining portion of the race, decelerates at a constant rate of $0.50\ \mathrm{m/s^2}$ to the finish line. What constant acceleration does Mary now need during the remaining portion of the race, if she wishes to cross the finish line side-by-side with Sally?

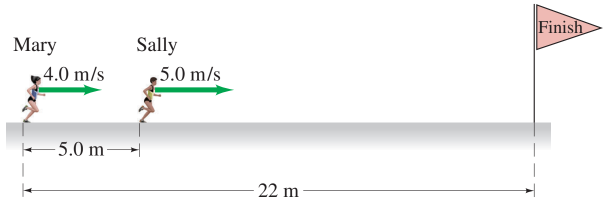


FIGURE 43 Problem 47.

7 Freely Falling Objects

[Neglect air resistance.]

48. (I) A stone is dropped from the top of a cliff. It is seen to hit the ground below after 3.75 s. How high is the cliff?

49. (I) If a car rolls gently $(v_0 = 0)$ off a vertical cliff, how long does it take to reach 55 km/h?

50. (I) Estimate (*a*) how long it took King Kong to fall straight down from the top of the Empire State Building (380 m high), and (*b*) his velocity just before "landing."

51. (II) A baseball is hit almost straight up into the air with a speed of about 20 m/s. (*a*) How high does it go? (*b*) How long is it in the air?

52. (II) A ball player catches a ball 3.2 s after throwing it vertically upward. With what speed did he throw it, and what height did it reach?

53. (II) A kangaroo jumps to a vertical height of 1.65 m. How long was it in the air before returning to Earth?

54. (II) The best rebounders in basketball have a vertical leap (that is, the vertical movement of a fixed point on their body) of about 120 cm. (*a*) What is their initial "launch" speed off the ground? (*b*) How long are they in the air?

55. (II) A helicopter is ascending vertically with a speed of 5.10 m/s. At a height of 105 m above the Earth, a package is dropped from a window. How much time does it take for the package to reach the ground? [*Hint*: v_0 for the package equals the speed of the helicopter.]

56. (II) For an object falling freely from rest, show that the distance traveled during each successive second increases in the ratio of successive odd integers (1, 3, 5, etc.). (This was first shown by Galileo.) See Figs. 26 and 29.

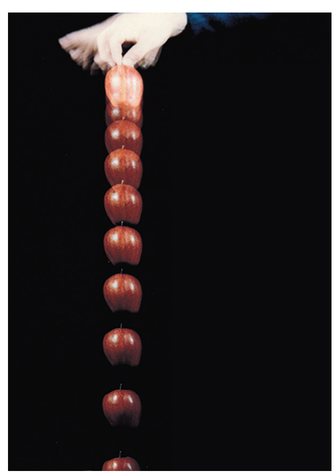


FIGURE 26 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

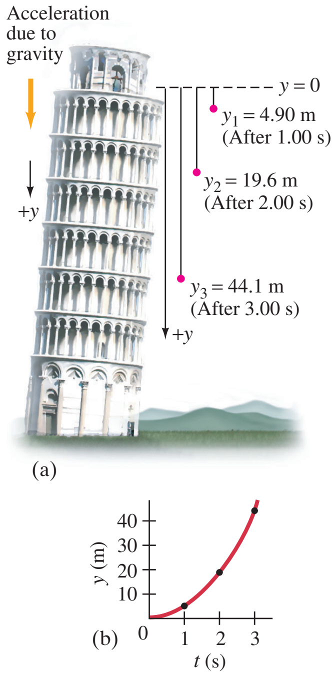


FIGURE 29 See Example 14 of "Describing Motion: Kinematics in One Dimension." (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 26.) (b) Graph of y vs. t.

57. (II) A baseball is seen to pass upward by a window 23 m above the street with a vertical speed of 14 m/s. If the ball was thrown from the street, (*a*) what was its initial speed, (*b*) what altitude does it reach, (*c*) when was it thrown, and (*d*) when does it reach the street again?

58. (II) A rocket rises vertically, from rest, with an acceleration of 3.2 m/s^2 until it runs out of fuel at an altitude of 950 m. After this point, its acceleration is that of gravity, downward. (*a*) What is the velocity of the rocket when it runs out of fuel? (*b*) How long does it take to reach this point? (*c*) What maximum altitude does the rocket reach? (*d*) How much time (total) does it take to reach maximum altitude? (*e*) With what velocity does it strike the Earth? (*f*) How long (total) is it in the air?

59. (II) Roger sees water balloons fall past his window. He notices that each balloon strikes the sidewalk 0.83 s after passing his window. Roger's room is on the third floor, 15 m above the sidewalk. (*a*) How fast are the balloons traveling when they pass Roger's window? (*b*) Assuming the balloons are being released from rest, from what floor are they being released? Each floor of the dorm is 5.0 m high.

60. (II) A stone is thrown vertically upward with a speed of 24.0 m/s. (*a*) How fast is it moving when it reaches a height of 13.0 m? (*b*) How much time is required to reach this height? (*c*) Why are there two answers to (*b*)?

61. (II) A falling stone takes 0.33 s to travel past a window 2.2 m tall (Fig. 44). From what height above the top of the window did the stone fall?

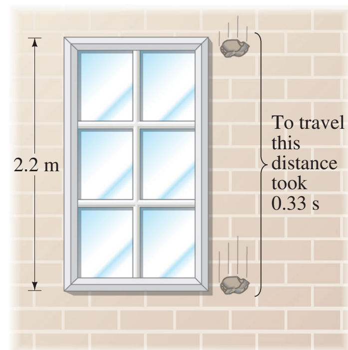


FIGURE 44 Problem 61.

62. (II) Suppose you adjust your garden hose nozzle for a hard stream of water. You point the nozzle vertically upward at a height of 1.5 m above the ground (Fig. 45). When you quickly turn off the nozzle, you hear the water striking the ground next to you for another 2.0 s. What is the water speed as it leaves the nozzle?

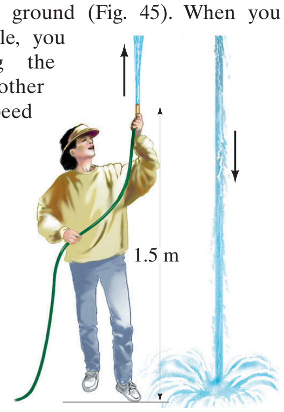


FIGURE 45 Problem 62.

63. (III) A toy rocket moving vertically upward passes by a 2.0-m-high window whose sill is 8.0 m above the ground. The rocket takes 0.15 s to travel the 2.0 m height of the window. What was the launch speed of the rocket, and how high will it go? Assume the propellant is burned very quickly at blastoff.

64. (III) A ball is dropped from the top of a 50.0-m-high cliff. At the same time, a carefully aimed stone is thrown straight up from the bottom of the cliff with a speed of 24.0 m/s. The stone and ball collide part way up. How far above the base of the cliff does this happen?

65. (III) A rock is dropped from a sea cliff and the sound of it striking the ocean is heard 3.4 s later. If the speed of sound is 340 m/s, how high is the cliff?

66. (III) A rock is thrown vertically upward with a speed of 12.0 m/s. Exactly 1.00 s later, a ball is thrown up vertically along the same path with a speed of 18.0 m/s. (*a*) At what time will they strike each other? (*b*) At what height will the collision occur? (*c*) Answer (*a*) and (*b*) assuming that the order is reversed: the ball is thrown 1.00 s before the rock.

*8 Variable Acceleration; Calculus

*67. (II) Given $v(t) = 25 + 18t$, where v is in m/s and t is in s, use calculus to determine the total displacement from $t_1 = 1.5$ s to $t_2 = 3.1$ s.

*68. (III) The acceleration of a particle is given by $a = A\sqrt{t}$ where $A = 2.0\,\text{m/s}^{5/2}$. At $t = 0$, $v = 7.5$ m/s and $x = 0$. (*a*) What is the speed as a function of time? (*b*) What is the displacement as a function of time? (*c*) What are the acceleration, speed and displacement at $t = 5.0$ s?

*69. (III) Air resistance acting on a falling body can be taken into account by the approximate relation for the acceleration:

$$a = \frac{dv}{dt} = g - kv,$$

where k is a constant. (*a*) Derive a formula for the velocity of the body as a function of time assuming it starts from rest ($v = 0$ at $t = 0$). [*Hint*: Change variables by setting $u = g - kv$.] (*b*) Determine an expression for the terminal velocity, which is the maximum value the velocity reaches.

*9 Graphical Analysis and Numerical Integration

[See Problems 95–97 at the end of this Chapter.]

General Problems

70. A fugitive tries to hop on a freight train traveling at a constant speed of 5.0 m/s. Just as an empty box car passes him, the fugitive starts from rest and accelerates at $a = 1.2\,\text{m/s}^2$ to his maximum speed of 6.0 m/s. (*a*) How long does it take him to catch up to the empty box car? (*b*) What is the distance traveled to reach the box car?

71. The acceleration due to gravity on the Moon is about one-sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

72. A person jumps from a fourth-story window 15.0 m above a firefighter's safety net. The survivor stretches the net 1.0 m before coming to rest, Fig. 46. (*a*) What was the average deceleration experienced by the survivor when she was slowed to rest by the net? (*b*) What would you do to make it "safer" (that is, to generate a smaller deceleration): would you stiffen or loosen the net? Explain.

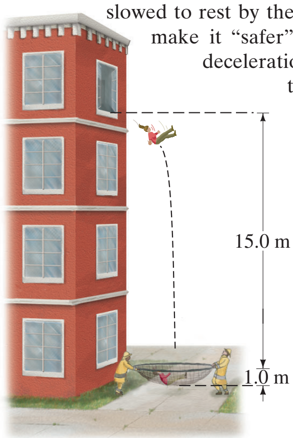


FIGURE 46 Problem 72.

73. A person who is properly restrained by an over-the-shoulder seat belt has a good chance of surviving a car collision if the deceleration does not exceed 30 "g's" ($1.00\,g = 9.80\,\text{m/s}^2$). Assuming uniform deceleration of this value, calculate the distance over which the front end of the car must be designed to collapse if a crash brings the car to rest from 100 km/h.

74. Pelicans tuck their wings and free-fall straight down when diving for fish. Suppose a pelican starts its dive from a height of 16.0 m and cannot change its path once committed. If it takes a fish 0.20 s to perform evasive action, at what minimum height must it spot the pelican to escape? Assume the fish is at the surface of the water.

75. Suppose a car manufacturer tested its cars for front-end collisions by hauling them up on a crane and dropping them from a certain height. (*a*) Show that the speed just before a car hits the ground, after falling from rest a vertical distance H, is given by $\sqrt{2gH}$. What height corresponds to a collision at (*b*) 50 km/h? (*c*) 100 km/h?

76. A stone is dropped from the roof of a high building. A second stone is dropped 1.50 s later. How far apart are the stones when the second one has reached a speed of 12.0 m/s?

77. A bicyclist in the Tour de France crests a mountain pass as he moves at 15 km/h. At the bottom, 4.0 km farther, his speed is 75 km/h. What was his average acceleration (in m/s^2) while riding down the mountain?

78. Consider the street pattern shown in Fig. 47. Each intersection has a traffic signal, and the speed limit is 50 km/h. Suppose you are driving from the west at the speed limit. When you are 10.0 m from the first intersection, all the lights turn green. The lights are green for 13.0 s each. (*a*) Calculate the time needed to reach the third stoplight. Can you make it through all three lights without stopping? (*b*) Another car was stopped at the first light when all the lights turned green. It can accelerate at the rate of $2.00\,\text{m/s}^2$ to the speed limit. Can the second car make it through all three lights without stopping? By how many seconds would it make it or not?

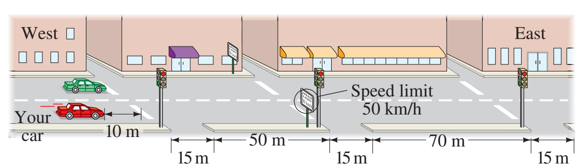


FIGURE 47 Problem 78.

79. In putting, the force with which a golfer strikes a ball is planned so that the ball will stop within some small distance of the cup, say 1.0 m long or short, in case the putt is missed. Accomplishing this from an uphill lie (that is, putting the ball downhill, see Fig. 48) is more difficult than from a downhill lie. To see why, assume that on a particular green

the ball decelerates constantly at $1.8\,m/s^2$ going downhill, and constantly at $2.8\,m/s^2$ going uphill. Suppose we have an uphill lie 7.0 m from the cup. Calculate the allowable range of initial velocities we may impart to the ball so that it stops in the range 1.0 m short to 1.0 m long of the cup. Do the same for a downhill lie 7.0 m from the cup. What in your results suggests that the downhill putt is more difficult?

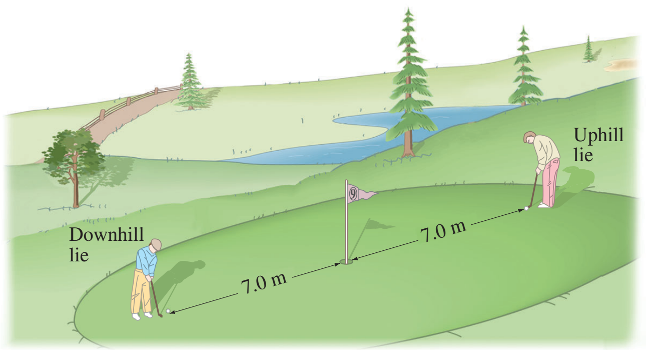


FIGURE 48 Problem 79.

80. A robot used in a pharmacy picks up a medicine bottle at $t = 0$. It accelerates at $0.20\,m/s^2$ for 5.0 s, then travels without acceleration for 68 s and finally decelerates at $-0.40\,m/s^2$ for 2.5 s to reach the counter where the pharmacist will take the medicine from the robot. From how far away did the robot fetch the medicine?

81. A stone is thrown vertically upward with a speed of 12.5 m/s from the edge of a cliff 75.0 m high (Fig. 49). (*a*) How much later does it reach the bottom of the cliff? (*b*) What is its speed just before hitting? (*c*) What total distance did it travel?

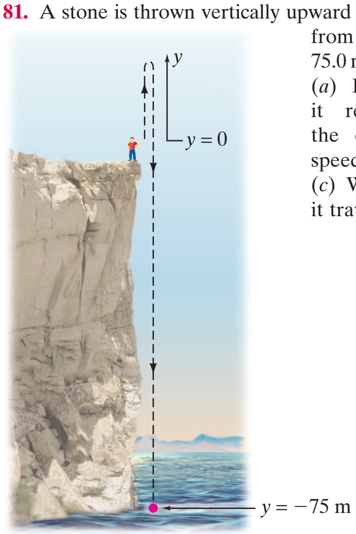


FIGURE 49 Problem 81.

82. Figure 50 is a position versus time graph for the motion of an object along the x axis. Consider the time interval from A to B. (*a*) Is the object moving in the positive or negative direction? (*b*) Is the object speeding up or slowing down? (*c*) Is the acceleration of the object positive or negative? Next, consider the time interval from D to E. (*d*) Is the object moving in the positive or negative direction? (*e*) Is the object speeding up or slowing down? (*f*) Is the acceleration of the object positive or negative? (*g*) Finally, answer these same three questions for the time interval from C to D.

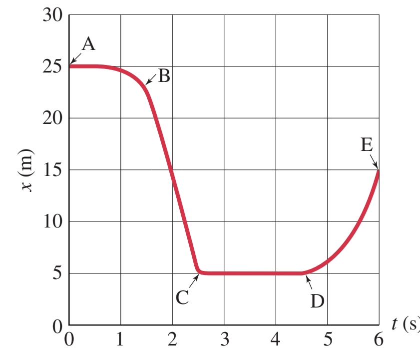


FIGURE 50 Problem 82.

83. In the design of a *rapid transit system*, it is necessary to balance the average speed of a train against the distance between stops. The more stops there are, the slower the train's average speed. To get an idea of this problem, calculate the time it takes a train to make a 9.0-km trip in two situations: (*a*) the stations at which the trains must stop are 1.8 km apart (a total of 6 stations, including those at the ends); and (*b*) the stations are 3.0 km apart (4 stations total). Assume that at each station the train accelerates at a rate of $1.1\,m/s^2$ until it reaches 95 km/h, then stays at this speed until its brakes are applied for arrival at the next station, at which time it decelerates at $-2.0\,m/s^2$. Assume it stops at each intermediate station for 22 s.

84. A person jumps off a diving board 4.0 m above the water's surface into a deep pool. The person's downward motion stops 2.0 m below the surface of the water. Estimate the average deceleration of the person while under the water.

85. Bill can throw a ball vertically at a speed 1.5 times faster than Joe can. How many times higher will Bill's ball go than Joe's?

86. Sketch the v vs. t graph for the object whose displacement as a function of time is given by Fig. 36.

87. A person driving her car at 45 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning to red, and she is 28 m away from the near side of the intersection (Fig. 51). Should she try to stop, or should she speed up to cross the intersection before the light turns red? The intersection is 15 m wide. Her car's maximum deceleration is $-5.8\,m/s^2$, whereas it can accelerate from 45 km/h to 65 km/h in 6.0 s. Ignore the length of her car and her reaction time.

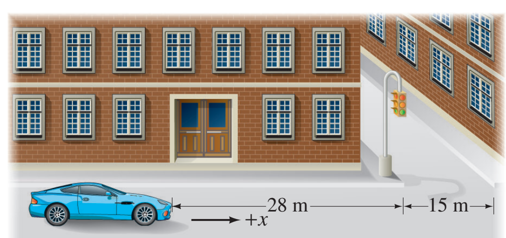


FIGURE 51 Problem 87.