Pearson New International Edition

Physics for Scientists & Engineers with Modern Physics

Douglas C. Giancoli

Fourth Edition

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Preface

Preface

I was motivated from the beginning to write a textbook different from others that present physics as a sequence of facts, like a Sears catalog: "here are the facts and you better learn them." Instead of that approach in which topics are begun formally and dogmatically, I have sought to begin each topic with concrete observations and experiences students can relate to: start with specifics and only then go to the great generalizations and the more formal aspects of a topic, showing *why* we believe what we believe. This approach reflects how science is actually practiced.

Why a Fourth Edition?

Two recent trends in physics texbooks are disturbing: (1) their revision cycles have become short—they are being revised every 3 or 4 years; (2) the books are getting larger, some over 1500 pages. I don't see how either trend can be of benefit to students. My response: (1) It has been 8 years since the previous edition of this book. (2) This book makes use of physics education research, although it avoids the detail a Professor may need to say in class but in a book shuts down the reader. And this book still remains among the shortest.

This new edition introduces some important new pedagogic tools. It contains new physics (such as in cosmology) and many new appealing applications (list on previous page). Pages and page breaks have been carefully formatted to make the physics easier to follow: no turning a page in the middle of a derivation or Example. Great efforts were made to make the book attractive so students will want to *read* it.

Some of the new features are listed below.

What's New

Chapter-Opening Questions: Each Chapter begins with a multiple-choice question, whose responses include common misconceptions. Students are asked to answer before starting the Chapter, to get them involved in the material and to get any preconceived notions out on the table. The issues reappear later in the Chapter, usually as Exercises, after the material has been covered. The Chapter-Opening Questions also show students the power and usefulness of Physics.

APPROACH paragraph in worked-out numerical Examples: A short introductory paragraph before the Solution, outlining an approach and the steps we can take to get started. Brief NOTES after the Solution may remark on the Solution, may give an alternate approach, or mention an application.

Step-by-Step Examples: After many Problem Solving Strategies (more than 20 in the book), the next Example is done step-by-step following precisely the steps just seen.

Exercises within the text, after an Example or derivation, give students a chance to see if they have understood enough to answer a simple question or do a simple calculation. Many are multiple choice.

Greater clarity: No topic, no paragraph in this book was overlooked in the search to improve the clarity and conciseness of the presentation. Phrases and sentences that may slow down the principal argument have been eliminated: keep to the essentials at first, give the elaborations later.

$\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{B}}$ ***Vector notation, arrows***: The symbols for vector quantities in the text and Figures now have a tiny arrow over them, so they are similar to what we write by hand.

Cosmological Revolution: With generous help from top experts in the field, readers have the latest results.

Page layout: more than in the previous edition, serious attention has been paid to how each page is formatted. Examples and all important derivations and arguments are on facing pages. Students then don't have to turn back and forth. Throughout, readers see, on two facing pages, an important slice of physics.

New Applications: LCDs, digital cameras and electronic sensors (CCD, CMOS), electric hazards, GFCIs, photocopiers, inkjet and laser printers, metal detectors, underwater vision, curve balls, airplane wings, DNA, how we actually *see* images. (Turn back a page to see a longer list.)

Examples modified: more math steps are spelled out, and many new Examples added. About 10% of all Examples are Estimation Examples.

This Book is Shorter than other complete full-service books at this level. Shorter explanations are easier to understand and more likely to be read.

Content and Organizational Changes

- **Rotational Motion**: Chapters 10 and 11 have been reorganized. All of angular momentum is now in Chapter 11.
- **First law of thermodynamics**, in Chapter 19, has been rewritten and extended. The full form is given: $\Delta K + \Delta U + \Delta E_{\text{int}} = Q - W$, where internal energy is E_{int}, and U is potential energy; the form $Q - W$ is kept so that $dW = P\,dV$.
- Kinematics and Dynamics of Circular Motion are now treated together in Chapter 5.
- Work and Energy, Chapters 7 and 8, have been carefully revised.
- Work done by friction is discussed now with energy conservation (energy terms due to friction).
- Chapters on Inductance and AC Circuits have been combined into one: Chapter 30.
- Graphical Analysis and Numerical Integration is a new optional Section 2–9. Problems requiring a computer or graphing calculator are found at the end of most Chapters.
- Length of an object is a script ℓ rather than normal l, which looks like 1 or I (moment of inertia, current), as in $F = I\ell B$. Capital L is for angular momentum, latent heat, inductance, dimensions of length $[L]$.
- Newton's law of gravitation remains in Chapter 6. Why? Because the $1/r^2$ law is too important to relegate to a late chapter that might not be covered at all late in the semester; furthermore, it is one of the basic forces in nature. In Chapter 8 we can treat real gravitational potential energy and have a fine instance of using $U = -\int \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}}$.
- New Appendices include the differential form of Maxwell's equations and more on dimensional analysis.
- Problem Solving Strategies are found on pages 30, 58, 64, 96, 102, 125, 166, 198, 229, 261, 314, 504, 551, 571, 600, 685, 716, 740, 763, 849, 871, and 913.

Organization

Some instructors may find that this book contains more material than can be covered in their courses. The text offers great flexibility. Sections marked with a star * are considered optional. These contain slightly more advanced physics material, or material not usually covered in typical courses and/or interesting applications; they contain no material needed in later Chapters (except perhaps in later optional Sections). For a brief course, all optional material could be dropped as well as major parts of Chapters 1, 13, 16, 26, 30, and 35, and selected parts of Chapters 9, 12, 19, 20, 33, and the modern physics Chapters. Topics not covered in class can be a valuable resource for later study by students. Indeed, this text can serve as a useful reference for years because of its wide range of coverage.

Versions of this Book

Complete version: 44 Chapters including 9 Chapters of modern physics.

Classic version: 37 Chapters including one each on relativity and quantum theory.

3 Volume version: Available separately or packaged together (Vols. 1 & 2 or all 3 Volumes):

Volume 1: Chapters 1–20 on mechanics, including fluids, oscillations, waves, plus heat and thermodynamics.

Volume 2: Chapters 21–35 on electricity and magnetism, plus light and optics.

Volume 3: Chapters 36–44 on modern physics: relativity, quantum theory, atomic physics, condensed matter, nuclear physics, elementary particles, cosmology and astrophysics.

Thanks

Many physics professors provided input or direct feedback on every aspect of this textbook. They are listed below, and I owe each a debt of gratitude.

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The final responsibility for all errors lies with me. I welcome comments, corrections, and suggestions as soon as possible to benefit students for the next reprint.

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About the Author

Douglas C. Giancoli obtained his BA in physics (summa cum laude) from the University of California, Berkeley, his MS in physics at the Massachusetts Institute of Technology, and his PhD in elementary particle physics at the University of California, Berkeley. He spent 2 years as a post-doctoral fellow at UC Berkeley's Virus lab developing skills in molecular biology and biophysics. His mentors include Nobel winners Emilio Segrè and Donald Glaser.

He has taught a wide range of undergraduate courses, traditional as well as innovative ones, and continues to update his textbooks meticulously, seeking ways to better provide an understanding of physics for students.

Doug's favorite spare-time activity is the outdoors, especially climbing peaks (here on a dolomite summit, Italy). He says climbing peaks is like learning physics: it takes effort and the rewards are great.

Online Supplements (partial list)

MasteringPhysics™ (www.masteringphysics.com)
is a sophisticated online tutoring and homework system developed specially for courses using calculus-based physics. Originally developed by David Pritchard and collaborators at MIT, MasteringPhysics provides **students** with individualized online tutoring by responding to their wrong answers and providing hints for solving multi-step problems when they get stuck. It gives them immediate and up-to-date assessment of their progress, and shows where they need to practice more. MasteringPhysics provides **instructors** with a fast and effective way to assign tried-and-tested online homework assignments that comprise a range of problem types. The powerful post-assignment diagnostics allow instructors to assess the progress of their class as a whole as well as individual students, and quickly identify areas of difficulty.

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Student Pocket Companion (0-13-227326-8) by Biman Das

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by Lillian C. McDermott, Peter S. Schaffer, and the Physics Education Group at the University of Washington

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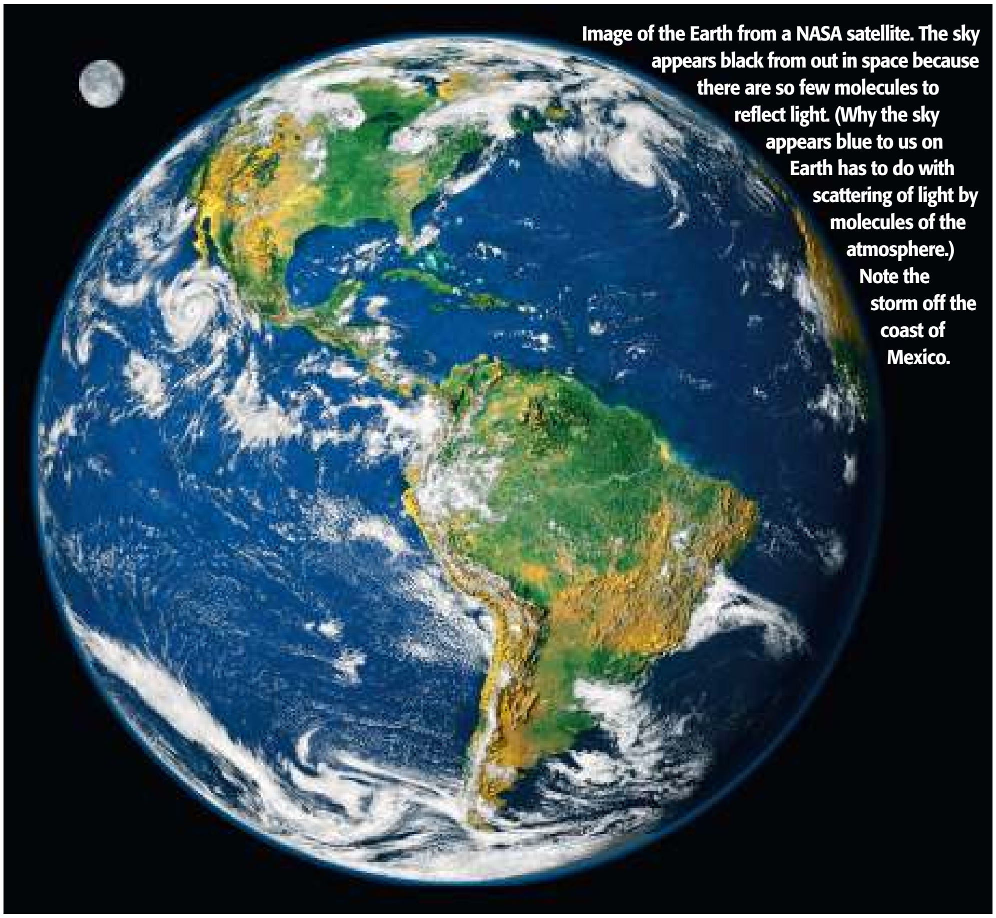
To Students

HOW TO STUDY

1. Read the Chapter. Learn new vocabulary and notation. Try to respond to questions and exercises as they occur.
2. Attend all class meetings. Listen. Take notes, especially about aspects you do not remember seeing in the book. Ask questions (everyone else wants to, but maybe you will have the courage). You will get more out of class if you read the Chapter first.
3. Read the Chapter again, paying attention to details. Follow derivations and worked-out Examples. Absorb their logic. Answer Exercises and as many of the end of Chapter Questions as you can.
4. Solve 10 to 20 end of Chapter Problems (or more), especially those assigned. In doing Problems you find out what you learned and what you didn't. Discuss them with other students. Problem solving is one of the great learning tools. Don't just look for a formula—it won't cut it.

NOTES ON THE FORMAT AND PROBLEM SOLVING

1. Sections marked with a star (*) are considered **optional**. They can be omitted without interrupting the main flow of topics. No later material depends on them except possibly later starred Sections. They may be fun to read, though.
2. The customary **conventions** are used: symbols for quantities (such as m for mass) are italicized, whereas units (such as m for meter) are not italicized. Symbols for vectors are shown in boldface with a small arrow above: $\vec{\mathbf{F}}$.
3. Few equations are valid in all situations. Where practical, the **limitations** of important equations are stated in square brackets next to the equation. The equations that represent the great laws of physics are displayed with a tan background, as are a few other indispensable equations.
4. At the end of each Chapter is a set of **Problems** which are ranked as Level I, II, or III, according to estimated difficulty. Level I Problems are easiest, Level II are standard Problems, and Level III are "challenge problems." These ranked Problems are arranged by Section, but Problems for a given Section may depend on earlier material too. There follows a group of General Problems, which are not arranged by Section nor ranked as to difficulty. Problems that relate to optional Sections are starred (*). Most Chapters have 1 or 2 Computer/Numerical Problems at the end, requiring a computer or graphing calculator. Answers to odd-numbered Problems are given at the end of the book.
5. Being able to solve **Problems** is a crucial part of learning physics, and provides a powerful means for understanding the concepts and principles. This book contains many aids to problem solving: (a) worked-out **Examples** and their solutions in the text, which should be studied as an integral part of the text; (b) some of the worked-out Examples are **Estimation Examples**, which show how rough or approximate results can be obtained even if the given data are sparse (see Section 1–6); (c) special **Problem Solving Strategies** placed throughout the text to suggest a step-by-step approach to problem solving for a particular topic—but remember that the basics remain the same; most of these "Strategies" are followed by an Example that is solved by explicitly following the suggested steps; (d) special problem-solving Sections; (e) "Problem Solving" marginal notes which refer to hints within the text for solving Problems; (f) **Exercises** within the text that you should work out immediately, and then check your response against the answer given at the bottom of the last page of that Chapter; (g) the Problems themselves at the end of each Chapter (point 4 above).
6. **Conceptual Examples** pose a question which hopefully starts you to think and come up with a response. Give yourself a little time to come up with your own response before reading the Response given.
7. **Math** review, plus some additional topics, are found in Appendices. Useful data, conversion factors, and math formulas are found inside the front and back covers.



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Introduction, Measurement, Estimating

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Suppose you wanted to actually measure the radius of the Earth, at least roughly, rather than taking other people's word for what it is. Which response below describes the best approach?

(a) Give up; it is impossible using ordinary means.
(b) Use an extremely long measuring tape.
(c) It is only possible by flying high enough to see the actual curvature of the Earth.
(d) Use a standard measuring tape, a step ladder, and a large smooth lake.
(e) Use a laser and a mirror on the Moon or on a satellite.

CONTENTS

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

From Chapter 1 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

Physics is the most basic of the sciences. It deals with the behavior and structure of matter. The field of physics is usually divided into *classical physics* which includes motion, fluids, heat, sound, light, electricity and magnetism; and *modern physics* which includes the topics of relativity, atomic structure, condensed matter, nuclear physics, elementary particles, and cosmology and astrophysics.

An understanding of physics is crucial for anyone making a career in science or technology. Engineers, for example, must know how to calculate the forces within a structure to design it so that it remains standing (Fig. 1a). Indeed, a simple physics calculation—or even intuition based on understanding the physics of forces—can save hundreds of lives (Fig. 1b). Physics is useful in many fields, and in everyday life.

Philip H. Coblentz/World Travel Images, Inc.

(a)

Antranig M. Ouzoonian, P.E./Weidlinger Associates, Inc.

(b)

FIGURE 1 (a) This Roman aqueduct was built 2000 years ago and still stands. (b) The Hartford Civic Center collapsed in 1978, just two years after it was built.

1 The Nature of Science

The principal aim of all sciences, including physics, is generally considered to be the search for order in our observations of the world around us. Many people think that science is a mechanical process of collecting facts and devising theories. But it is not so simple. Science is a creative activity that in many respects resembles other creative activities of the human mind.

One important aspect of science is **observation** of events, which includes the design and carrying out of experiments. But observation and experiment require imagination, for scientists can never include everything in a description of what they observe. Hence, scientists must make judgments about what is relevant in their observations and experiments.

Consider, for example, how two great minds, Aristotle (384–322 B.C.) and Galileo (1564–1642), interpreted motion along a horizontal surface. Aristotle noted that objects given an initial push along the ground (or on a tabletop) always slow down and stop. Consequently, Aristotle argued that the natural state of an object is to be at rest. Galileo, in his reexamination of horizontal motion in the 1600s, imagined that if friction could be eliminated, an object given an initial push along a horizontal surface would continue to move indefinitely without stopping. He concluded that for an object to be in motion was just as natural as for it to be at rest. By inventing a new approach, Galileo founded our modern view of motion, and he did so with a leap of the imagination. Galileo made this leap conceptually, without actually eliminating friction.

Observation, with careful experimentation and measurement, is one side of the scientific process. The other side is the invention or creation of theories to explain and order the observations. Theories are never derived directly from observations. Observations may help inspire a theory, and theories are accepted or rejected based on the results of observation and experiment.

The great theories of science may be compared, as creative achievements, with great works of art or literature. But how does science differ from these other creative activities? One important difference is that science requires **testing** of its ideas or theories to see if their predictions are borne out by experiment.

Although the testing of theories distinguishes science from other creative fields, it should not be assumed that a theory is "proved" by testing. First of all, no measuring instrument is perfect, so exact confirmation is not possible. Furthermore, it is not possible to test a theory in every single possible circumstance. Hence a theory cannot be absolutely verified. Indeed, the history of science tells us that long-held theories can be replaced by new ones.

2 Models, Theories, and Laws

When scientists are trying to understand a particular set of phenomena, they often make use of a **model**. A model, in the scientist's sense, is a kind of analogy or mental image of the phenomena in terms of something we are familiar with. One example is the wave model of light. We cannot see waves of light as we can water waves. But it is valuable to think of light as made up of waves because experiments indicate that light behaves in many respects as water waves do.

The purpose of a model is to give us an approximate mental or visual picture—something to hold on to—when we cannot see what actually is happening. Models often give us a deeper understanding: the analogy to a known system (for instance, water waves in the above example) can suggest new experiments to perform and can provide ideas about what other related phenomena might occur.

You may wonder what the difference is between a theory and a model. Usually a model is relatively simple and provides a structural similarity to the phenomena being studied. A **theory** is broader, more detailed, and can give quantitatively testable predictions, often with great precision.

It is important, however, not to confuse a model or a theory with the real system or the phenomena themselves.

Scientists give the title **law** to certain concise but general statements about how nature behaves (that energy is conserved, for example). Sometimes the statement takes the form of a relationship or equation between quantities (such as Newton's second law, $F = ma$).

To be called a law, a statement must be found experimentally valid over a wide range of observed phenomena. For less general statements, the term **principle** is often used (such as Archimedes' principle).

Scientific laws are different from political laws in that the latter are *prescriptive*: they tell us how we ought to behave. Scientific laws are *descriptive*: they do not say how nature *should* behave, but rather are meant to describe how nature *does* behave. As with theories, laws cannot be tested in the infinite variety of cases possible. So we cannot be sure that any law is absolutely true. We use the term "law" when its validity has been tested over a wide range of cases, and when any limitations and the range of validity are clearly understood.

Scientists normally do their research as if the accepted laws and theories were true. But they are obliged to keep an open mind in case new information should alter the validity of any given law or theory.

3 Measurement and Uncertainty; Significant Figures

In the quest to understand the world around us, scientists seek to find relationships among physical quantities that can be measured.

Uncertainty

Reliable measurements are an important part of physics. But no measurement is absolutely precise. There is an uncertainty associated with every measurement. Among the most important sources of uncertainty, other than blunders, are the limited accuracy of every measuring instrument and the inability to read an instrument beyond some fraction of the smallest division shown. For example, if you were to use a centimeter ruler to measure the width of a board (Fig. 2), the result could be claimed to be precise to about 0.1 cm (1 mm), the smallest division on the ruler, although half of this value might be a valid claim as well. The reason is that it is difficult for the observer to estimate (or interpolate) between the smallest divisions. Furthermore, the ruler itself may not have been manufactured to an accuracy very much better than this.

FIGURE 2 Measuring the width of a board with a centimeter ruler. The uncertainty is about ± 1mm.

Mary Teresa Giancoli

When giving the result of a measurement, it is important to state the **estimated uncertainty** in the measurement. For example, the width of a board might be written as 8.8 ± 0.1 cm. The ± 0.1 cm ("plus or minus 0.1 cm") represents the estimated uncertainty in the measurement, so that the actual width most likely lies between 8.7 and 8.9 cm. The **percent uncertainty** is the ratio of the uncertainty to the measured value, multiplied by 100. For example, if the measurement is 8.8 and the uncertainty about 0.1 cm, the percent uncertainty is

$$\frac{0.1}{8.8} \times 100\% \approx 1\%,$$

where $\approx$ means "is approximately equal to."

Often the uncertainty in a measured value is not specified explicitly. In such cases, the uncertainty is generally assumed to be one or a few units in the last digit specified. For example, if a length is given as 8.8 cm, the uncertainty is assumed to be about 0.1 cm or 0.2 cm. It is important in this case that you do not write 8.80 cm, for this implies an uncertainty on the order of 0.01 cm; it assumes that the length is probably between 8.79 cm and 8.81 cm, when actually you believe it is between 8.7 and 8.9 cm.

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(a) (b)

FIGURE 3 These two calculators show the wrong number of significant figures. In (a), 2.0 was divided by 3.0. The correct final result would be 0.67. In (b), 2.5 was multiplied by 3.2. The correct result is 8.0.

Significant Figures

The number of reliably known digits in a number is called the number of **significant figures**. Thus there are four significant figures in the number 23.21 cm and two in the number 0.062 cm (the zeros in the latter are merely place holders that show where the decimal point goes). The number of significant figures may not always be clear. Take, for example, the number 80. Are there one or two significant figures? We need words here: If we say it is *roughly* 80 km between two cities, there is only one significant figure (the 8) since the zero is merely a place holder. If there is no suggestion that the 80 is a rough approximation, then we can often assume that it is 80 km within an accuracy of about 1 or 2 km, and then the 80 has two significant figures. If it is precisely 80 km, to within ± 0.1 km, then we write 80.0 km (three significant figures).

When making measurements, or when doing calculations, you should avoid the temptation to keep more digits in the final answer than is justified. For example, to calculate the area of a rectangle 11.3 cm by 6.8 cm, the result of multiplication would be 76.84 cm^2. But this answer is clearly not accurate to 0.01 cm^2, since (using the outer limits of the assumed uncertainty for each measurement) the result could be between $11.2\text{ cm} \times 6.7\text{ cm} = 75.04\text{ cm}^2$ and $11.4\text{ cm} \times 6.9\text{ cm} = 78.66\text{ cm}^2$. At best, we can quote the answer as 77 cm^2, which implies an uncertainty of about 1 or 2 cm^2. The other two digits (in the number 76.84 cm^2) must be dropped because they are not significant. As a rough general rule (i.e., in the absence of a detailed consideration of uncertainties), we can say that *the final result of a multiplication or division should have only as many digits as the number with the least number of significant figures used in the calculation*. In our example, 6.8 cm has the least number of significant figures, namely two. Thus the result 76.84 cm^2 needs to be rounded off to 77 cm^2.

PROBLEM SOLVING

Significant figure rule: Number of significant figures in final result should be same as the least significant input value

EXERCISE A The area of a rectangle 4.5 cm by 3.25 cm is correctly given by (*a*) 14.625 cm^2; (*b*) 14.63 cm^2; (*c*) 14.6 cm^2; (*d*) 15 cm^2.

When adding or subtracting numbers, the final result is no more precise than the least precise number used. For example, the result of subtracting 0.57 from 3.6 is 3.0 (and not 3.03).

CAUTION

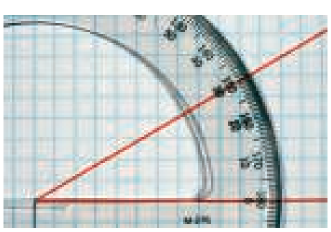
Calculators err with significant figures

PROBLEM SOLVING

Report only the proper number of significant figures in the final result. Keep extra digits during the calculation

Keep in mind when you use a calculator that all the digits it produces may not be significant. When you divide 2.0 by 3.0, the proper answer is 0.67, and not some such thing as 0.666666666. Digits should not be quoted in a result, unless they are truly significant figures. However, to obtain the most accurate result, you should normally *keep one or more extra significant figures throughout a calculation, and round off only in the final result*. (With a calculator, you can keep all its digits in intermediate results.) Note also that calculators sometimes give too few significant figures. For example, when you multiply 2.5×3.2, a calculator may give the answer as simply 8. But the answer is accurate to two significant figures, so the proper answer is 8.0. See Fig. 3.

FIGURE 4 Example 1. A protractor used to measure an angle.



Paul Silverman/Fundamental Photographs, NYC

CONCEPTUAL EXAMPLE 1 **Significant figures.** Using a protractor (Fig. 4), you measure an angle to be 30°. (*a*) How many significant figures should you quote in this measurement? (*b*) Use a calculator to find the cosine of the angle you measured.

RESPONSE (*a*) If you look at a protractor, you will see that the precision with which you can measure an angle is about one degree (certainly not 0.1°). So you can quote two significant figures, namely, 30° (not 30.0°). (*b*) If you enter cos 30° in your calculator, you will get a number like 0.866025403. However, the angle you entered is known only to two significant figures, so its cosine is correctly given by 0.87; you must round your answer to two significant figures.

EXERCISE B Do 0.00324 and 0.00056 have the same number of significant figures?

Be careful not to confuse significant figures with the number of decimal places.

EXERCISE C For each of the following numbers, state the number of significant figures and the number of decimal places: (*a*) 1.23; (*b*) 0.123; (*c*) 0.0123.

Scientific Notation

We commonly write numbers in "powers of ten," or "scientific" notation—for instance 36,900 as 3.69×10^4, or 0.0021 as 2.1×10^{-3}. One advantage of scientific notation is that it allows the number of significant figures to be clearly expressed. For example, it is not clear whether 36,900 has three, four, or five significant figures. With powers of ten notation the ambiguity can be avoided: if the number is known to three significant figures, we write 3.69×10^4, but if it is known to four, we write 3.690×10^4.

EXERCISE D Write each of the following in scientific notation and state the number of significant figures for each: (*a*) 0.0258, (*b*) 42,300, (*c*) 344.50.

Percent Uncertainty versus Significant Figures

The significant figures rule is only approximate, and in some cases may underestimate the accuracy (or uncertainty) of the answer. Suppose for example we divide 97 by 92:

$$\frac{97}{92} = 1.05 \approx 1.1.$$

Both 97 and 92 have two significant figures, so the rule says to give the answer as 1.1. Yet the numbers 97 and 92 both imply an uncertainty of ± 1 if no other uncertainty is stated. Now 92 ± 1 and 97 ± 1 both imply an uncertainty of about 1% ($1/92 \approx 0.01 = 1\%$). But the final result to two significant figures is 1.1, with an implied uncertainty of ± 0.1, which is an uncertainty of $0.1/1.1 \approx 0.1 \approx 10\%$. In this case it is better to give the answer as 1.05 (which is three significant figures). Why? Because 1.05 implies an uncertainty of ± 0.01 which is $0.01/1.05 \approx 0.01 \approx 1\%$, just like the uncertainty in the original numbers 92 and 97.

SUGGESTION: Use the significant figures rule, but consider the % uncertainty too, and add an extra digit if it gives a more realistic estimate of uncertainty.

Approximations

Much of physics involves approximations, often because we do not have the means to solve a problem precisely. For example, we may choose to ignore air resistance or friction in doing a Problem even though they are present in the real world, and then our calculation is only an approximation. In doing Problems, we should be aware of what approximations we are making, and be aware that the precision of our answer may not be nearly as good as the number of significant figures given in the result.

Accuracy versus Precision

There is a technical difference between "precision" and "accuracy." **Precision** in a strict sense refers to the repeatability of the measurement using a given instrument. For example, if you measure the width of a board many times, getting results like 8.81 cm, 8.85 cm, 8.78 cm, 8.82 cm (interpolating between the 0.1 cm marks as best as possible each time), you could say the measurements give a *precision* a bit better than 0.1 cm. **Accuracy** refers to how close a measurement is to the true value. For example, if the ruler shown in Fig. 2 was manufactured with a 2% error, the accuracy of its measurement of the board's width (about 8.8 cm) would be about 2% of 8.8 cm or about ± 0.2 cm. Estimated uncertainty is meant to take both accuracy and precision into account.

4 Units, Standards, and the SI System

TABLE 1 Some Typical Lengths or Distances (order of magnitude)

Length (or Distance)	Meters (approximate)
Neutron or proton (diameter)	10^{-15} m
Atom (diameter)	10^{-10} m
Virus [see Fig. 5a]	10^{-7} m
Sheet of paper (thickness)	10^{-4} m
Finger width	10^{-2} m
Football field length	10^{2} m
Height of Mt. Everest [see Fig. 5b]	10^{4} m
Earth diameter	10^{7} m
Earth to Sun	10^{11} m
Earth to nearest star	10^{16} m
Earth to nearest galaxy	10^{22} m
Earth to farthest galaxy visible	10^{26} m

The measurement of any quantity is made relative to a particular standard or **unit**, and this unit must be specified along with the numerical value of the quantity. For example, we can measure length in British units such as inches, feet, or miles, or in the metric system in centimeters, meters, or kilometers. To specify that the length of a particular object is 18.6 is meaningless. The unit *must* be given; for clearly, 18.6 meters is very different from 18.6 inches or 18.6 millimeters.

For any unit we use, such as the meter for distance or the second for time, we need to define a **standard** which defines exactly how long one meter or one second is. It is important that standards be chosen that are readily reproducible so that anyone needing to make a very accurate measurement can refer to the standard in the laboratory.

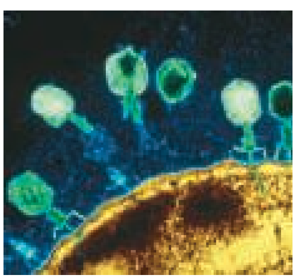
Length

The first truly international standard was the **meter** (abbreviated m) established as the standard of **length** by the French Academy of Sciences in the 1790s. The standard meter was originally chosen to be one ten-millionth of the distance from the Earth's equator to either pole,† and a platinum rod to represent this length was made. (One meter is, very roughly, the distance from the tip of your nose to the tip of your finger, with arm and hand stretched out to the side.) In 1889, the meter was defined more precisely as the distance between two finely engraved marks on a particular bar of platinum–iridium alloy. In 1960, to provide greater precision and reproducibility, the meter was redefined as 1,650,763.73 wavelengths of a particular orange light emitted by the gas krypton-86. In 1983 the meter was again redefined, this time in terms of the speed of light (whose best measured value in terms of the older definition of the meter was 299,792,458 m/s, with an uncertainty of 1 m/s). The new definition reads: "The meter is the length of path traveled by light in vacuum during a time interval of 1/299,792,458 of a second."‡

British units of length (inch, foot, mile) are now defined in terms of the meter. The inch (in.) is defined as precisely 2.54 centimeters (cm; $1\text{ cm} = 0.01\text{ m}$). Table 1 presents some typical lengths, from very small to very large, rounded off to the nearest power of ten. See also Fig. 5. [Note that the abbreviation for inches (in.) is the only one with a period, to distinguish it from the word "in".]

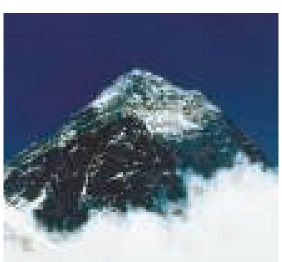
FIGURE 5 Some lengths: (a) viruses (about 10^{-7} m long) attacking a cell; (b) Mt. Everest's height is on the order of 10^4 m (8850 m, to be precise).

Oliver Meckes/Ottawa/Photo Researchers, Inc.



(a)

Douglas C. Giancoli



(b)

Time

The standard unit of **time** is the **second** (s). For many years, the second was defined as 1/86,400 of a mean solar day ($24\text{ h/day} \times 60\text{ min/h} \times 60\text{ s/min} = 86{,}400\text{ s/day}$). The standard second is now defined more precisely in terms of the frequency of radiation emitted by cesium atoms when they pass between two particular states. [Specifically, one second is defined as the time required for 9,192,631,770 periods of this radiation.] There are, by definition, 60 s in one minute (min) and 60 minutes in one hour (h). Table 2 presents a range of measured time intervals, rounded off to the nearest power of ten.

Mass

The standard unit of **mass** is the **kilogram** (kg). The standard mass is a particular platinum–iridium cylinder, kept at the International Bureau of Weights and Measures near Paris, France, whose mass is defined as exactly 1 kg. A range of masses is presented in Table 3. [For practical purposes, 1 kg weighs about 2.2 pounds on Earth.]

†Modern measurements of the Earth's circumference reveal that the intended length is off by about one-fiftieth of 1%. Not bad!

‡The new definition of the meter has the effect of giving the speed of light the exact value of 299,792,458 m/s.

TABLE 2 Some Typical Time Intervals

Time Interval	Seconds (approximate)
Lifetime of very unstable subatomic particle	10^{-23} s
Lifetime of radioactive elements	10^{-22} s to 10^{28} s
Lifetime of muon	10^{-6} s
Time between human heartbeats	10^{0} s (= 1 s)
One day	10^{5} s
One year	3×10^{7} s
Human life span	2×10^{9} s
Length of recorded history	10^{11} s
Humans on Earth	10^{14} s
Life on Earth	10^{17} s
Age of Universe	10^{18} s

TABLE 3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

When dealing with atoms and molecules, we usually use the **unified atomic mass unit** (u). In terms of the kilogram,

$$1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}.$$

Unit Prefixes

In the metric system, the larger and smaller units are defined in multiples of 10 from the standard unit, and this makes calculation particularly easy. Thus 1 kilometer (km) is 1000 m, 1 centimeter is $\frac{1}{100}$ m, 1 millimeter (mm) is $\frac{1}{1000}$ m or $\frac{1}{10}$ cm, and so on. The prefixes "centi-," "kilo-," and others are listed in Table 4 and can be applied not only to units of length but to units of volume, mass, or any other metric unit. For example, a centiliter (cL) is $\frac{1}{100}$ liter (L), and a kilogram (kg) is 1000 grams (g).

TABLE 4 Metric (SI) Prefixes

Prefix	Abbreviation	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

† μ is the Greek letter "mu."

Systems of Units

When dealing with the laws and equations of physics it is very important to use a consistent set of units. Several systems of units have been in use over the years. Today the most important is the **Système International** (French for International System), which is abbreviated SI. In SI units, the standard of length is the meter, the standard for time is the second, and the standard for mass is the kilogram. This system used to be called the MKS (meter-kilogram-second) system.

A second metric system is the **cgs system**, in which the centimeter, gram, and second are the standard units of length, mass, and time, as abbreviated in the title. The **British engineering system** has as its standards the foot for length, the pound for force, and the second for time.

We use SI units almost exclusively in this text.

Base versus Derived Quantities

Physical quantities can be divided into two categories: *base quantities* and *derived quantities*. The corresponding units for these quantities are called *base units* and *derived units*. A **base quantity** must be defined in terms of a standard. Scientists, in the interest of simplicity, want the smallest number of base quantities possible consistent with a full description of the physical world. This number turns out to be seven, and those used in the SI are given in Table 5. All other quantities can be defined in terms of these seven base quantities,† and hence are referred to as **derived quantities**. An example of a derived quantity is speed, which is defined as distance divided by the time it takes to travel that distance. To define any quantity, whether base or derived, we can specify a rule or procedure, and this is called an **operational definition**.

TABLE 5 SI Base Quantities and Units

Quantity	Unit	Unit Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

†The only exceptions are for angle (radians) and solid angle (steradian). No general agreement has been reached as to whether these are base or derived quantities.

5 Converting Units

Any quantity we measure, such as a length, a speed, or an electric current, consists of a number *and* a unit. Often we are given a quantity in one set of units, but we want it expressed in another set of units. For example, suppose we measure that a table is 21.5 inches wide, and we want to express this in centimeters. We must use a **conversion factor**, which in this case is (by definition) exactly

$$1\text{ in.} = 2.54\text{ cm}$$

or, written another way,

$$1 = 2.54\text{ cm/in.}$$

Since multiplying by one does not change anything, the width of our table, in cm, is

$$21.5\text{ inches} = (21.5\,\cancel{\text{in.}}) \times \left(2.54\,\frac{\text{cm}}{\cancel{\text{in.}}}\right) = 54.6\text{ cm.}$$

Note how the units (inches in this case) cancelled out. Let's consider some conversion Examples.

PHYSICS APPLIED
The world's tallest peaks

A.Viansson

FIGURE 6 The world's second highest peak, K2, whose summit is considered the most difficult of the "8000-ers." K2 is seen here from the north (China).

TABLE 6
The 8000-m Peaks

Peak	Height (m)
Mt. Everest	8850
K2	8611
Kangchenjunga	8586
Lhotse	8516
Makalu	8462
Cho Oyu	8201
Dhaulagiri	8167
Manaslu	8156
Nanga Parbat	8125
Annapurna	8091
Gasherbrum I	8068
Broad Peak	8047
Gasherbrum II	8035
Shisha Pangma	8013

EXAMPLE 2 **The 8000-m peaks.** The fourteen tallest peaks in the world (Fig. 6 and Table 6) are referred to as "eight-thousanders," meaning their summits are over 8000 m above sea level. What is the elevation, in feet, of an elevation of 8000 m?

APPROACH We need simply to convert meters to feet, and we can start with the conversion factor 1 in. = 2.54 cm, which is exact. That is, 1 in. = 2.5400 cm to any number of significant figures, because it is *defined* to be.

SOLUTION One foot is 12 in., so we can write

$$1\text{ ft} = (12\,\cancel{\text{in.}})\left(2.54\,\frac{\text{cm}}{\cancel{\text{in.}}}\right) = 30.48\text{ cm} = 0.3048\text{ m,}$$

which is exact. Note how the units cancel (colored slashes). We can rewrite this equation to find the number of feet in 1 meter:

$$1\text{ m} = \frac{1\text{ ft}}{0.3048} = 3.28084\text{ ft.}$$

We multiply this equation by 8000.0 (to have five significant figures):

$$8000.0\text{ m} = (8000.0\,\cancel{\text{m}})\left(3.28084\,\frac{\text{ft}}{\cancel{\text{m}}}\right) = 26{,}247\text{ ft.}$$

An elevation of 8000 m is 26,247 ft above sea level.

NOTE We could have done the conversion all in one line:

$$8000.0\text{ m} = (8000.0\,\cancel{\text{m}})\left(\frac{100\,\cancel{\text{cm}}}{1\,\cancel{\text{m}}}\right)\left(\frac{1\,\cancel{\text{in.}}}{2.54\,\cancel{\text{cm}}}\right)\left(\frac{1\text{ ft}}{12\,\cancel{\text{in.}}}\right) = 26{,}247\text{ ft.}$$

The key is to multiply conversion factors, each equal to one (= 1.0000), and to make sure the units cancel.

EXERCISE E There are only 14 eight-thousand-meter peaks in the world (see Example 2), and their names and elevations are given in Table 6. They are all in the Himalaya mountain range in India, Pakistan, Tibet, and China. Determine the elevation of the world's three highest peaks in feet.

EXAMPLE 3 **Apartment area.** You have seen a nice apartment whose floor area is 880 square feet (ft^2). What is its area in square meters?

APPROACH We use the same conversion factor, $1\text{ in.} = 2.54\text{ cm}$, but this time we have to use it twice.

SOLUTION Because $1\text{ in.} = 2.54\text{ cm} = 0.0254\text{ m}$, then $1\text{ ft}^2 = (12\text{ in.})^2(0.0254\text{ m/in.})^2 = 0.0929\text{ m}^2$. So $880\text{ ft}^2 = (880\text{ ft}^2)(0.0929\text{ m}^2/\text{ft}^2) \approx 82\text{ m}^2$.

NOTE As a rule of thumb, an area given in ft^2 is roughly 10 times the number of square meters (more precisely, about $10.8\times$).

EXAMPLE 4 **Speeds.** Where the posted speed limit is 55 miles per hour (mi/h or mph), what is this speed (*a*) in meters per second (m/s) and (*b*) in kilometers per hour (km/h)?

APPROACH We again use the conversion factor $1\text{ in.} = 2.54\text{ cm}$, and we recall that there are 5280 ft in a mile and 12 inches in a foot; also, one hour contains $(60\text{ min/h}) \times (60\text{ s/min}) = 3600\text{ s/h}$.

SOLUTION (*a*) We can write 1 mile as

$$1\text{ mi} = (5280\ \cancel{\text{ft}})\left(12\frac{\cancel{\text{in.}}}{\cancel{\text{ft}}}\right)\left(2.54\frac{\cancel{\text{cm}}}{\cancel{\text{in.}}}\right)\left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right) = 1609\text{ m}.$$

We also know that 1 hour contains 3600 s, so

$$55\frac{\text{mi}}{\text{h}} = \left(55\frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(1609\frac{\text{m}}{\cancel{\text{mi}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\text{ s}}\right) = 25\frac{\text{m}}{\text{s}},$$

where we rounded off to two significant figures.
(*b*) Now we use $1\text{ mi} = 1609\text{ m} = 1.609\text{ km}$; then

$$55\frac{\text{mi}}{\text{h}} = \left(55\frac{\cancel{\text{mi}}}{\text{h}}\right)\left(1.609\frac{\text{km}}{\cancel{\text{mi}}}\right) = 88\frac{\text{km}}{\text{h}}.$$

NOTE Each conversion factor is equal to one.

PROBLEM SOLVING
Conversion factors = 1

EXERCISE F Would a driver traveling at 15 m/s in a 35 mi/h zone be exceeding the speed limit?

PROBLEM SOLVING
Unit conversion is wrong if units do not cancel

When changing units, you can avoid making an error in the use of conversion factors by checking that units cancel out properly. For example, in our conversion of 1 mi to 1609 m in Example 4(*a*), if we had incorrectly used the factor $\left(\frac{100\text{ cm}}{1\text{ m}}\right)$ instead of $\left(\frac{1\text{ m}}{100\text{ cm}}\right)$, the centimeter units would not have cancelled out; we would not have ended up with meters.

6 Order of Magnitude: Rapid Estimating

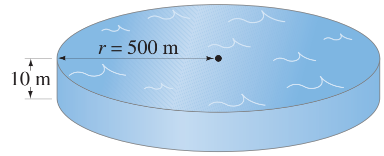
We are sometimes interested only in an approximate value for a quantity. This might be because an accurate calculation would take more time than it is worth or would require additional data that are not available. In other cases, we may want to make a rough estimate in order to check an accurate calculation made on a calculator, to make sure that no blunders were made when the numbers were entered.

PROBLEM SOLVING
How to make a rough estimate

A rough estimate is made by rounding off all numbers to one significant figure and its power of 10, and after the calculation is made, again only one significant figure is kept. Such an estimate is called an **order-of-magnitude estimate** and can be accurate within a factor of 10, and often better. In fact, the phrase "order of magnitude" is sometimes used to refer simply to the power of 10.

(a)

Douglas C. Giancoli



(b)

FIGURE 7 Example 5. (a) How much water is in this lake? (Photo is of one of the Rae Lakes in the Sierra Nevada of California.) (b) Model of the lake as a cylinder. [We could go one step further and estimate the mass or weight of this lake. We will see later that water has a density of $1000\ \mathrm{kg/m^3}$, so this lake has a mass of about $\left(10^3\ \mathrm{kg/m^3}\right)\left(10^7\ \mathrm{m^3}\right) \approx 10^{10}\ \mathrm{kg}$, which is about 10 billion kg or 10 million metric tons. (A metric ton is 1000 kg, about 2200 lbs, slightly larger than a British ton, 2000 lbs.)]

PHYSICS APPLIED
Estimating the volume (or mass) of a lake; see also Fig. 7

EXAMPLE 5 **ESTIMATE** **Volume of a lake.** Estimate how much water there is in a particular lake, Fig. 7a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

APPROACH No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate the volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder (Fig. 7b).

SOLUTION The volume V of a cylinder is the product of its height h times the area of its base: $V = h\pi r^2$, where r is the radius of the circular base. The radius r is $\frac{1}{2}\ \mathrm{km} = 500\ \mathrm{m}$, so the volume is approximately

$$V = h\pi r^2 \approx (10\ \mathrm{m}) \times (3) \times \left(5 \times 10^2\ \mathrm{m}\right)^2 \approx 8 \times 10^6\ \mathrm{m^3} \approx 10^7\ \mathrm{m^3},$$

where π was rounded off to 3. So the volume is on the order of $10^7\ \mathrm{m^3}$, ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate $\left(10^7\ \mathrm{m^3}\right)$ is probably better to quote than the $8 \times 10^6\ \mathrm{m^3}$ figure.

NOTE To express our result in U.S. gallons, we see in the Table on the inside front cover that $1\ \text{liter} = 10^{-3}\ \mathrm{m^3} \approx \frac{1}{4}\ \text{gallon}$. Hence, the lake contains $\left(8 \times 10^6\ \mathrm{m^3}\right)\left(1\ \text{gallon}/4 \times 10^{-3}\ \mathrm{m^3}\right) \approx 2 \times 10^9$ gallons of water.

EXAMPLE 6 **ESTIMATE** **Thickness of a page.** Estimate the thickness of a page of a text.

PROBLEM SOLVING
Use symmetry when possible

APPROACH At first you might think that a special measuring device, a micrometer (Fig. 8), is needed to measure the thickness of one page since an ordinary ruler clearly won't do. But we can use a trick or, to put it in physics terms, make use of a *symmetry*: we can make the reasonable assumption that all the pages of a text are equal in thickness.

SOLUTION We can use a ruler to measure many pages at once. If you measure the thickness of the first 500 pages of a book (page 1 to page 500), you might get something like 1.5 cm. Note that 500 numbered pages, counted front and back, is 250 separate sheets of paper. So one page must have a thickness of about

$$\frac{1.5\text{ cm}}{250\text{ pages}} \approx 6 \times 10^{-3}\text{ cm} = 6 \times 10^{-2}\text{ mm},$$

or less than a tenth of a millimeter (0.1 mm).

FIGURE 8 Example 6. Micrometer used for measuring small thicknesses.

EXAMPLE 7 **ESTIMATE** **Height by triangulation.** Estimate the height of the building shown in Fig. 9, by "triangulation," with the help of a bus-stop pole and a friend.

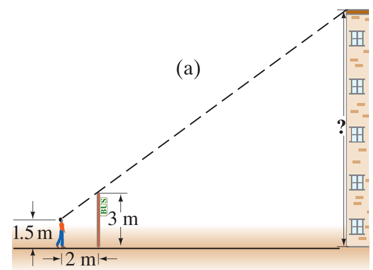
APPROACH By standing your friend next to the pole, you estimate the height of the pole to be 3 m. You next step away from the pole until the top of the pole is in line with the top of the building, Fig. 9a. You are 5 ft 6 in. tall, so your eyes are about 1.5 m above the ground. Your friend is taller, and when she stretches out her arms, one hand touches you, and the other touches the pole, so you estimate that distance as 2 m (Fig. 9a). You then pace off the distance from the pole to the base of the building with big, 1-m-long steps, and you get a total of 16 steps or 16 m.

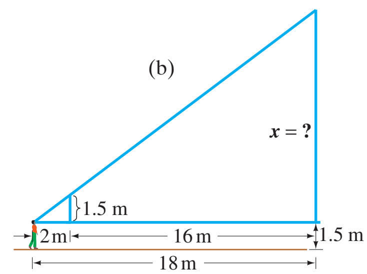
SOLUTION Now you draw, to scale, the diagram shown in Fig. 9b using these measurements. You can measure, right on the diagram, the last side of the triangle to be about $x = 13$ m. Alternatively, you can use similar triangles to obtain the height x:

$$\frac{1.5\text{ m}}{2\text{ m}} = \frac{x}{18\text{ m}}, \quad \text{so} \quad x \approx 13\tfrac{1}{2}\text{ m}.$$

Finally you add in your eye height of 1.5 m above the ground to get your final result: the building is about 15 m tall.

FIGURE 9 Example 7. Diagrams are really useful!





EXAMPLE 8 **ESTIMATE** **Estimating the radius of Earth.** Believe it or not, you can estimate the radius of the Earth without having to go into space. If you have ever been on the shore of a large lake, you may have noticed that you cannot see the beaches, piers, or rocks at water level across the lake on the opposite shore. The lake seems to bulge out between you and the opposite shore—a good clue that the Earth is round. Suppose you climb a stepladder and discover that when your eyes are 10 ft (3.0 m) above the water, you can just see the rocks at water level on the opposite shore. From a map, you estimate the distance to the opposite shore as $d \approx 6.1$ km. Use Fig. 10 with $h = 3.0$ m to estimate the radius R of the Earth.

APPROACH We use simple geometry, including the theorem of Pythagoras, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse of any right triangle, and a and b are the lengths of the other two sides.

SOLUTION For the right triangle of Fig. 10, the two sides are the radius of the Earth R and the distance $d = 6.1\text{ km} = 6100$ m. The hypotenuse is approximately the length $R + h$, where $h = 3.0$ m. By the Pythagorean theorem,

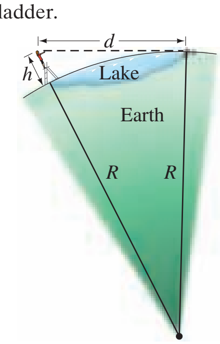
$$R^2 + d^2 \approx (R + h)^2$$
$$\approx R^2 + 2hR + h^2.$$

We solve algebraically for R, after cancelling R^2 on both sides:

$$R \approx \frac{d^2 - h^2}{2h} = \frac{(6100\text{ m})^2 - (3.0\text{ m})^2}{6.0\text{ m}} = 6.2 \times 10^6\text{ m} = 6200\text{ km}.$$

NOTE Precise measurements give 6380 km. But look at your achievement! With a few simple rough measurements and simple geometry, you made a good estimate of the Earth's radius. You did not need to go out in space, nor did you need a very long measuring tape. Now you know the answer to the Chapter-Opening Question.

FIGURE 10 Example 8, but not to scale. You can see small rocks at water level on the opposite shore of a lake 6.1 km wide if you stand on a stepladder.



EXAMPLE 9 ESTIMATE Total number of heartbeats. Estimate the total number of beats a typical human heart makes in a lifetime.

APPROACH A typical resting heart rate is 70 beats/min. But during exercise it can be a lot higher. A reasonable average might be 80 beats/min.

SOLUTION One year in terms of seconds is $(24\,\mathrm{h})(3600\,\mathrm{s/h})(365\,\mathrm{d}) \approx 3 \times 10^7\,\mathrm{s}$. If an average person lives $70\text{ years} = (70\,\mathrm{yr})(3 \times 10^7\,\mathrm{s/yr}) \approx 2 \times 10^9\,\mathrm{s}$, then the total number of heartbeats would be about

$$\left(80\,\frac{\text{beats}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)(2 \times 10^9\,\mathrm{s}) \;\approx\; 3 \times 10^9,$$

or 3 trillion.

PROBLEM SOLVING
Estimating how many piano tuners there are in a city

Another technique for estimating, this one made famous by Enrico Fermi to his physics students, is to estimate the number of piano tuners in a city, say, Chicago or San Francisco. To get a rough order-of-magnitude estimate of the number of piano tuners today in San Francisco, a city of about 700,000 inhabitants, we can proceed by estimating the number of functioning pianos, how often each piano is tuned, and how many pianos each tuner can tune. To estimate the number of pianos in San Francisco, we note that certainly not everyone has a piano. A guess of 1 family in 3 having a piano would correspond to 1 piano per 12 persons, assuming an average family of 4 persons. As an order of magnitude, let's say 1 piano per 10 people. This is certainly more reasonable than 1 per 100 people, or 1 per every person, so let's proceed with the estimate that 1 person in 10 has a piano, or about 70,000 pianos in San Francisco. Now a piano tuner needs an hour or two to tune a piano. So let's estimate that a tuner can tune 4 or 5 pianos a day. A piano ought to be tuned every 6 months or a year—let's say once each year. A piano tuner tuning 4 pianos a day, 5 days a week, 50 weeks a year can tune about 1000 pianos a year. So San Francisco, with its (very) roughly 70,000 pianos, needs about 70 piano tuners. This is, of course, only a rough estimate.[†] It tells us that there must be many more than 10 piano tuners, and surely not as many as 1000.

*7 Dimensions and Dimensional Analysis

When we speak of the **dimensions** of a quantity, we are referring to the type of base units or base quantities that make it up. The dimensions of area, for example, are always length squared, abbreviated $[L^2]$, using square brackets; the units can be square meters, square feet, cm^2, and so on. Velocity, on the other hand, can be measured in units of km/h, m/s, or mi/h, but the dimensions are always a length $[L]$ divided by a time $[T]$: that is, $[L/T]$.

The formula for a quantity may be different in different cases, but the dimensions remain the same. For example, the area of a triangle of base b and height h is $A = \frac{1}{2}bh$, whereas the area of a circle of radius r is $A = \pi r^2$. The formulas are different in the two cases, but the dimensions of area are always $[L^2]$.

Dimensions can be used as a help in working out relationships, a procedure referred to as **dimensional analysis**. One useful technique is the use of dimensions to check if a relationship is *incorrect*. Note that we add or subtract quantities only if they have the same dimensions (we don't add centimeters and hours); and the quantities on each side of an equals sign must have the same dimensions. (In numerical calculations, the units must also be the same on both sides of an equation.)

For example, suppose you derived the equation $v = v_0 + \frac{1}{2}at^2$, where v is the speed of an object after a time t, v_0 is the object's initial speed, and the object undergoes an acceleration a. Let's do a dimensional check to see if this equation

[†] A check of the San Francisco Yellow Pages (done after this calculation) reveals about 50 listings. Each of these listings may employ more than one tuner, but on the other hand, each may also do repairs as well as tuning. In any case, our estimate is reasonable.

could be correct or is surely incorrect. Note that numerical factors, like the $\frac{1}{2}$ here, do not affect dimensional checks. We write a dimensional equation as follows, remembering that the dimensions of speed are $[L/T]$ and the dimensions of acceleration are $[L/T^2]$:

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right][T^2] = \left[\frac{L}{T}\right] + [L].$$

The dimensions are incorrect: on the right side, we have the sum of quantities whose dimensions are not the same. Thus we conclude that an error was made in the derivation of the original equation.

A dimensional check can only tell you when a relationship is wrong. It can't tell you if it is completely right. For example, a dimensionless numerical factor (such as $\frac{1}{2}$ or 2π) could be missing.

Dimensional analysis can also be used as a quick check on an equation you are not sure about. For example, suppose that you can't remember whether the equation for the period of a simple pendulum T (the time to make one back-and-forth swing) of length ℓ is $T = 2\pi\sqrt{\ell/g}$ or $T = 2\pi\sqrt{g/\ell}$, where g is the acceleration due to gravity and, like all accelerations, has dimensions $[L/T^2]$. (Do not worry about these formulas; what we are concerned about here is a person's recalling whether it contains ℓ/g or g/ℓ.) A dimensional check shows that the former (ℓ/g) is correct:

$$[T] = \sqrt{\frac{[L]}{[L/T^2]}} = \sqrt{[T^2]} = [T],$$

whereas the latter (g/ℓ) is not:

$$[T] \neq \sqrt{\frac{[L/T^2]}{[L]}} = \sqrt{\frac{1}{[T^2]}} = \frac{1}{[T]}.$$

Note that the constant 2π has no dimensions and so can't be checked using dimensions.

EXAMPLE 10 **Planck length.** The smallest meaningful measure of length is called the "Planck length," and is defined in terms of three fundamental constants in nature, the speed of light $c = 3.00 \times 10^8\,\mathrm{m/s}$, the gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{m^3/kg\cdot s^2}$, and Planck's constant $h = 6.63 \times 10^{-34}\,\mathrm{kg\cdot m^2/s}$. The Planck length λ_P (λ is the Greek letter "lambda") is given by the following combination of these three constants:

$$\lambda_P = \sqrt{\frac{Gh}{c^3}}.$$

Show that the dimensions of λ_P are length $[L]$, and find the order of magnitude of λ_P.

APPROACH We rewrite the above equation in terms of dimensions. The dimensions of c are $[L/T]$, of G are $[L^3/MT^2]$, and of h are $[ML^2/T]$.

SOLUTION The dimensions of λ_P are

$$\sqrt{\frac{[L^3/MT^2][ML^2/T]}{[L^3/T^3]}} = \sqrt{[L^2]} = [L]$$

which is a length. The value of the Planck length is

$$\lambda_P = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{m^3/kg\cdot s^2})(6.63 \times 10^{-34}\,\mathrm{kg\cdot m^2/s})}{(3.0 \times 10^8\,\mathrm{m/s})^3}} \approx 4 \times 10^{-35}\,\mathrm{m},$$

which is on the order of 10^{-34} or 10^{-35} m.

NOTE Some recent theories suggest that the smallest particles (quarks, leptons) have sizes on the order of the Planck length, 10^{-35} m. These theories also suggest that the "Big Bang," with which the Universe is believed to have begun, started from an initial size on the order of the Planck length.

Summary

Physics, like other sciences, is a creative endeavor. It is not simply a collection of facts. Important **theories** are created with the idea of explaining **observations**. To be accepted, theories are **tested** by comparing their predictions with the results of actual experiments. Note that, in general, a theory cannot be "proved" in an absolute sense.

Scientists often devise models of physical phenomena. A **model** is a kind of picture or analogy that helps to describe the phenomena in terms of something we already know. A **theory**, often developed from a model, is usually deeper and more complex than a simple model.

A scientific **law** is a concise statement, often expressed in the form of an equation, which quantitatively describes a wide range of phenomena.

Measurements play a crucial role in physics, but can never be perfectly precise. It is important to specify the **uncertainty** of a measurement either by stating it directly using the $\pm$ notation, and/or by keeping only the correct number of **significant figures**.

Physical quantities are always specified relative to a particular standard or **unit**, and the unit used should always be stated. The commonly accepted set of units today is the **Système International** (SI), in which the standard units of length, mass, and time are the **meter**, **kilogram**, and **second**.

When converting units, check all **conversion factors** for correct cancellation of units.

Making rough, **order-of-magnitude estimates** is a very useful technique in science as well as in everyday life.

[*The **dimensions** of a quantity refer to the combination of base quantities that comprise it. Velocity, for example, has dimensions of [length/time] or $[L/T]$. **Dimensional analysis** can be used to check a relationship for correct form.]

Answers to Exercises

A: (*d*).

B: No: they have 3 and 2, respectively.

C: All three have three significant figures, although the number of decimal places is (*a*) 2, (*b*) 3, (*c*) 4.

D: (*a*) 2.58×10^{-2}, 3; (*b*) 4.23×10^{4}, 3 (probably); (*c*) 3.4450×10^{2}, 5.

E: Mt. Everest, 29,035 ft; K2, 28,251 ft; Kangchenjunga, 28,169 ft.

F: No: 15 m/s $\approx$ 34 mi/h.

Introduction, Measurement, Estimating Problem Set

Questions

1. What are the merits and drawbacks of using a person's foot as a standard? Consider both (*a*) a particular person's foot, and (*b*) any person's foot. Keep in mind that it is advantageous that fundamental standards be accessible (easy to compare to), invariable (do not change), indestructible, and reproducible.

2. Why is it incorrect to think that the more digits you represent in your answer, the more accurate it is?

3. When traveling a highway in the mountains, you may see elevation signs that read "914 m (3000 ft)." Critics of the metric system claim that such numbers show the metric system is more complicated. How would you alter such signs to be more consistent with a switch to the metric system?

4. What is wrong with this road sign:

Memphis 7 mi (11.263 km)?

5. For an answer to be complete, the units need to be specified. Why?

6. Discuss how the notion of symmetry could be used to estimate the number of marbles in a 1-liter jar.

7. You measure the radius of a wheel to be 4.16 cm. If you multiply by 2 to get the diameter, should you write the result as 8 cm or as 8.32 cm? Justify your answer.

8. Express the sine of 30.0° with the correct number of significant figures.

9. A recipe for a soufflé specifies that the measured ingredients must be exact, or the soufflé will not rise. The recipe calls for 6 large eggs. The size of "large" eggs can vary by 10%, according to the USDA specifications. What does this tell you about how exactly you need to measure the other ingredients?

10. List assumptions useful to estimate the number of car mechanics in (*a*) San Francisco, (*b*) your hometown, and then make the estimates.

11. Suggest a way to measure the distance from Earth to the Sun.

***12.** Can you set up a complete set of base quantities, as in Table 5, that does not include length as one of them?

TABLE 5
SI Base Quantities and Units

Quantity	Unit	Unit Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Problems

[The Problems in this Section are ranked I, II, or III according to estimated difficulty, with (I) Problems being easiest. Level (III) Problems are meant mainly as a challenge for the best students, for "extra credit." The Problems are arranged by Sections, meaning that the reader should have read up to and including that Section, but this Chapter also has a group of General Problems that are not arranged by Section and not ranked.]

3 Measurement, Uncertainty, Significant Figures

(*Note:* In Problems, assume a number like 6.4 is accurate to ± 0.1; and 950 is ± 10 unless 950 is said to be "precisely" or "very nearly" 950, in which case assume 950 ± 1.)

1. (I) The age of the universe is thought to be about 14 billion years. Assuming two significant figures, write this in powers of ten in (*a*) years, (*b*) seconds.

2. (I) How many significant figures do each of the following numbers have: (*a*) 214, (*b*) 81.60, (*c*) 7.03, (*d*) 0.03, (*e*) 0.0086, (*f*) 3236, and (*g*) 8700?

3. (I) Write the following numbers in powers of ten notation: (*a*) 1.156, (*b*) 21.8, (*c*) 0.0068, (*d*) 328.65, (*e*) 0.219, and (*f*) 444.

4. (I) Write out the following numbers in full with the correct number of zeros: (*a*) 8.69×10^4, (*b*) 9.1×10^3, (*c*) 8.8×10^{-1}, (*d*) 4.76×10^2, and (*e*) 3.62×10^{-5}.

5. (II) What is the percent uncertainty in the measurement 5.48 ± 0.25 m?

6. (II) Time intervals measured with a stopwatch typically have an uncertainty of about 0.2 s, due to human reaction time at the start and stop moments. What is the percent uncertainty of a hand-timed measurement of (*a*) 5 s, (*b*) 50 s, (*c*) 5 min?

7. (II) Add $(9.2 \times 10^3\ \text{s}) + (8.3 \times 10^4\ \text{s}) + (0.008 \times 10^6\ \text{s})$.

8. (II) Multiply 2.079×10^2 m by 0.082×10^{-1}, taking into account significant figures.

9. (III) For small angles θ, the numerical value of $\sin\theta$ is approximately the same as the numerical value of $\tan\theta$. Find the largest angle for which sine and tangent agree to within two significant figures.

10. (III) What, roughly, is the percent uncertainty in the volume of a spherical beach ball whose radius is $r = 0.84 \pm 0.04$ m?

4 and 5 Units, Standards, SI, Converting Units

11. (I) Write the following as full (decimal) numbers with standard units: (*a*) 286.6 mm, (*b*) 85 μV, (*c*) 760 mg, (*d*) 60.0 ps, (*e*) 22.5 fm, (*f*) 2.50 gigavolts.

12. (I) Express the following using the prefixes of Table 4: (*a*) 1×10^6 volts, (*b*) 2×10^{-6} meters, (*c*) 6×10^3 days, (*d*) 18×10^2 bucks, and (*e*) 8×10^{-8} seconds.

TABLE 4 Metric (SI) Prefixes

Prefix	Abbreviation	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

† μ is the Greek letter "mu."

13. (I) Determine your own height in meters, and your mass in kg.

14. (I) The Sun, on average, is 93 million miles from Earth. How many meters is this? Express (*a*) using powers of ten, and (*b*) using a metric prefix.

15. (II) What is the conversion factor between (*a*) ft^2 and yd^2, (*b*) m^2 and ft^2?

16. (II) An airplane travels at 950 km/h. How long does it take to travel 1.00 km?

17. (II) A typical atom has a diameter of about 1.0×10^{-10} m. (*a*) What is this in inches? (*b*) Approximately how many atoms are there along a 1.0-cm line?

18. (II) Express the following sum with the correct number of significant figures: 1.80 m + 142.5 cm + $5.34 \times 10^5\ \mu$m.

19. (II) Determine the conversion factor between (*a*) km/h and mi/h, (*b*) m/s and ft/s, and (*c*) km/h and m/s.

20. (II) How much longer (percentage) is a one-mile race than a 1500-m race ("the metric mile")?

21. (II) A *light-year* is the distance light travels in one year (at speed = 2.998×10^8 m/s). (*a*) How many meters are there in 1.00 light-year? (*b*) An astronomical unit (AU) is the average distance from the Sun to Earth, 1.50×10^8 km. How many AU are there in 1.00 light-year? (*c*) What is the speed of light in AU/h?

22. (II) If you used only a keyboard to enter data, how many years would it take to fill up the hard drive in your computer that can store 82 gigabytes (82×10^9 bytes) of data? Assume "normal" eight-hour working days, and that one byte is required to store one keyboard character, and that you can type 180 characters per minute.

23. (III) The diameter of the Moon is 3480 km. (*a*) What is the surface area of the Moon? (*b*) How many times larger is the surface area of the Earth?

6 Order-of-Magnitude Estimating

(*Note*: Remember that for rough estimates, only round numbers are needed both as input to calculations and as final results.)

24. (I) Estimate the order of magnitude (power of ten) of: (*a*) 2800, (*b*) 86.30×10^2, (*c*) 0.0076, and (*d*) 15.0×10^8.

25. (II) Estimate how many books can be shelved in a college library with 3500 m^2 of floor space. Assume 8 shelves high, having books on both sides, with corridors 1.5 m wide. Assume books are about the size of this one, on average.

26. (II) Estimate how many hours it would take a runner to run (at 10 km/h) across the United States from New York to California.

27. (II) Estimate the number of liters of water a human drinks in a lifetime.

28. (II) Estimate how long it would take one person to mow a football field using an ordinary home lawn mower (Fig. 11). Assume the mower moves with a 1-km/h speed, and has a 0.5-m width.

FIGURE 11 Problem 28.

29. (II) Estimate the number of dentists (*a*) in San Francisco and (*b*) in your town or city.

30. (III) The rubber worn from tires mostly enters the atmosphere as *particulate pollution*. Estimate how much rubber (in kg) is put into the air in the United States every year. To get started, a good estimate for a tire tread's depth is 1 cm when new, and rubber has a mass of about 1200 kg per m^3 of volume.

31. (III) You are in a hot air balloon, 200 m above the flat Texas plains. You look out toward the horizon. How far out can you see—that is, how far is your horizon? The Earth's radius is about 6400 km.

32. (III) I agree to hire you for 30 days and you can decide between two possible methods of payment: either (1) \$1000 a day, or (2) one penny on the first day, two pennies on the second day and continue to double your daily pay each day up to day 30. Use quick estimation to make your decision, and justify it.

33. (III) Many sailboats are moored at a marina 4.4 km away on the opposite side of a lake. You stare at one of the sailboats because, when you are lying flat at the water's edge, you can just see its deck but none of the side of the sailboat. You then go to that sailboat on the other side of the lake and measure that the deck is 1.5 m above the level of the water. Using Fig. 12, where $h = 1.5$ m, estimate the radius R of the Earth.

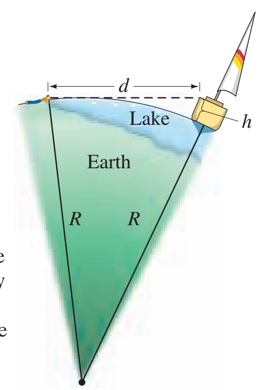


FIGURE 12 Problem 33. You see a sailboat across a lake (not to scale). R is the radius of the Earth. You are a distance $d = 4.4$ km from the sailboat when you can see only its deck and not its side. Because of the curvature of the Earth, the water "bulges out" between you and the boat.

34. (III) Another experiment you can do also uses the radius of the Earth. The Sun sets, fully disappearing over the horizon as you lie on the beach, your eyes 20 cm above the sand. You immediately jump up, your eyes now 150 cm above the sand, and you can again see the top of the Sun. If you count the number of seconds (= t) until the Sun fully disappears again, you can estimate the radius of the Earth. But for this Problem, use the known radius of the Earth and calculate the time t.

*7 Dimensions

*35. (I) What are the dimensions of density, which is mass per volume?

*36. (II) The speed v of an object is given by the equation $v = At^3 - Bt$, where t refers to time. (*a*) What are the dimensions of A and B? (*b*) What are the SI units for the constants A and B?

*37. (II) Three students derive the following equations in which x refers to distance traveled, v the speed, a the acceleration $(\mathrm{m/s^2})$, t the time, and the subscript zero $(_0)$ means a quantity at time $t = 0$: (*a*) $x = vt^2 + 2at$, (*b*) $x = v_0 t + \frac{1}{2}at^2$, and (*c*) $x = v_0 t + 2at^2$. Which of these could possibly be correct according to a dimensional check?

*38. (II) Show that the following combination of the three fundamental constants of nature that we used in Example 10 of "Introduction, Measurement, Estimating" (that is G, c, and h) forms a quantity with the dimensions of time:

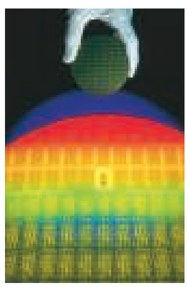
$$t_P = \sqrt{\frac{Gh}{c^5}}.$$

This quantity, t_P, is called the *Planck time* and is thought to be the earliest time, after the creation of the Universe, at which the currently known laws of physics can be applied.

General Problems

39. *Global positioning satellites* (GPS) can be used to determine positions with great accuracy. If one of the satellites is at a distance of 20,000 km from you, what percent uncertainty in the distance does a 2-m uncertainty represent? How many significant figures are needed in the distance?

40. *Computer chips* (Fig. 13) etched on circular silicon wafers of thickness 0.300 mm are sliced from a solid cylindrical silicon crystal of length 25 cm. If each wafer can hold 100 chips, what is the maximum number of chips that can be produced from one entire cylinder?



David Parker/Science Photo Library/Photo Researchers, Inc.

FIGURE 13 Problem 40. The wafer held by the hand (above) is shown below, enlarged and illuminated by colored light. Visible are rows of integrated circuits (chips).

41. (*a*) How many seconds are there in 1.00 year? (*b*) How many nanoseconds are there in 1.00 year? (*c*) How many years are there in 1.00 second?

42. American football uses a field that is 100 yd long, whereas a regulation soccer field is 100 m long. Which field is longer, and by how much (give yards, meters, and percent)?

43. A typical adult human lung contains about 300 million tiny cavities called alveoli. Estimate the average diameter of a single alveolus.

44. One hectare is defined as $1.000 \times 10^4\ \mathrm{m^2}$. One acre is $4.356 \times 10^4\ \mathrm{ft^2}$. How many acres are in one hectare?

45. Estimate the number of gallons of gasoline consumed by the total of all automobile drivers in the United States, per year.

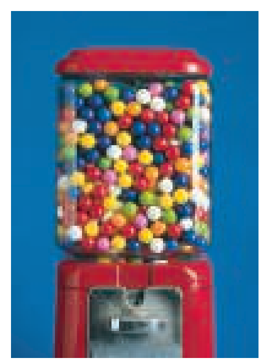
46. Use Table 3 to estimate the total number of protons or neutrons in (*a*) a bacterium, (*b*) a DNA molecule, (*c*) the human body, (*d*) our Galaxy.

47. An average family of four uses roughly 1200 L (about 300 gallons) of water per day $(1\ \mathrm{L} = 1000\ \mathrm{cm^3})$. How much depth would a lake lose per year if it uniformly covered an area of $50\ \mathrm{km^2}$ and supplied a local town with a population of 40,000 people? Consider only population uses, and neglect evaporation and so on.

TABLE 3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

48. Estimate the number of gumballs in the machine of Fig. 14.



The Image Works

FIGURE 14 Problem 48. Estimate the number of gumballs in the machine.

49. Estimate how many kilograms of laundry soap are used in the U.S. in one year (and therefore pumped out of washing machines with the dirty water). Assume each load of laundry takes 0.1 kg of soap.

50. How big is a ton? That is, what is the volume of something that weighs a ton? To be specific, estimate the diameter of a 1-ton rock, but first make a wild guess: will it be 1 ft across, 3 ft, or the size of a car? [*Hint*: Rock has mass per volume about 3 times that of water, which is 1 kg per liter $(10^3\ \mathrm{cm^3})$ or 62 lb per cubic foot.]

51. A certain audio compact disc (CD) contains 783.216 megabytes of digital information. Each byte consists of exactly 8 bits. When played, a CD player reads the CD's digital information at a constant rate of 1.4 megabits per second. How many minutes does it take the player to read the entire CD?

52. Hold a pencil in front of your eye at a position where its blunt end just blocks out the Moon (Fig. 15). Make appropriate measurements to estimate the diameter of the Moon, given that the Earth–Moon distance is 3.8×10^5 km.

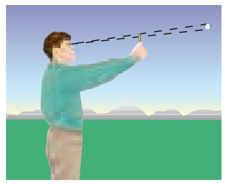


FIGURE 15 Problem 52. How big is the Moon?

53. A heavy rainstorm dumps 1.0 cm of rain on a city 5 km wide and 8 km long in a 2-h period. How many metric tons $(1 \text{ metric ton} = 10^3 \text{ kg})$ of water fell on the city? (1 cm^3 of water has a mass of $1 \text{ g} = 10^{-3} \text{ kg}$.) How many gallons of water was this?

54. Noah's ark was ordered to be 300 cubits long, 50 cubits wide, and 30 cubits high. The cubit was a unit of measure equal to the length of a human forearm, elbow to the tip of the longest finger. Express the dimensions of Noah's ark in meters, and estimate its volume (m^3).

55. Estimate how many days it would take to walk around the world, assuming 10 h walking per day at 4 km/h.

56. One liter (1000 cm^3) of oil is spilled onto a smooth lake. If the oil spreads out uniformly until it makes an oil slick just one molecule thick, with adjacent molecules just touching, estimate the diameter of the oil slick. Assume the oil molecules have a diameter of 2×10^{-10} m.

57. Jean camps beside a wide river and wonders how wide it is. She spots a large rock on the bank directly across from her. She then walks upstream until she judges that the angle between her and the rock, which she can still see clearly, is now at an angle of 30° downstream (Fig. 16). Jean measures her stride to be about 1 yard long. The distance back to her camp is 120 strides. About how far across, both in yards and in meters, is the river?

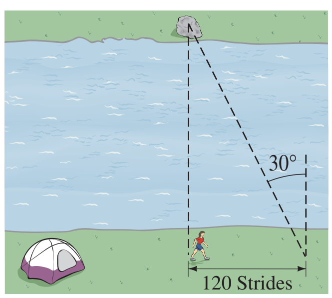


FIGURE 16 Problem 57.

58. A watch manufacturer claims that its watches gain or lose no more than 8 seconds in a year. How accurate is this watch, expressed as a percentage?

59. An angstrom (symbol Å) is a unit of length, defined as 10^{-10} m, which is on the order of the diameter of an atom. (*a*) How many nanometers are in 1.0 angstrom? (*b*) How many femtometers or fermis (the common unit of length in nuclear physics) are in 1.0 angstrom? (*c*) How many angstroms are in 1.0 m? (*d*) How many angstroms are in 1.0 light-year (see Problem 21)?

60. The diameter of the Moon is 3480 km. What is the volume of the Moon? How many Moons would be needed to create a volume equal to that of Earth?

61. Determine the percent uncertainty in θ, and in $\sin\theta$, when (*a*) $\theta = 15.0° \pm 0.5°$, (*b*) $\theta = 75.0° \pm 0.5°$.

62. If you began walking along one of Earth's lines of longitude and walked north until you had changed latitude by 1 minute of arc (there are 60 minutes per degree), how far would you have walked (in miles)? This distance is called a "nautical mile."

63. Make a rough estimate of the volume of your body (in m^3).

64. Estimate the number of bus drivers (*a*) in Washington, D.C., and (*b*) in your town.

65. The American Lung Association gives the following formula for an average person's expected lung capacity V (in liters, where $1 \text{ L} = 10^3 \text{ cm}^3$):

$$V = 4.1H - 0.018A - 2.69,$$

where H and A are the person's height (in meters), and age (in years), respectively. In this formula, what are the units of the numbers 4.1, 0.018, and 2.69?

66. The density of an object is defined as its mass divided by its volume. Suppose the mass and volume of a rock are measured to be 8 g and 2.8325 cm^3. To the correct number of significant figures, determine the rock's density.

67. To the correct number of significant figures, use the information inside the front cover of this book to determine the ratio of (*a*) the surface area of Earth compared to the surface area of the Moon; (*b*) the volume of Earth compared to the volume of the Moon.

68. One mole of atoms consists of 6.02×10^{23} individual atoms. If a mole of atoms were spread uniformly over the surface of the Earth, how many atoms would there be per square meter?

69. Recent findings in astrophysics suggest that the observable Universe can be modeled as a sphere of radius $R = 13.7 \times 10^9$ light-years with an average mass density of about $1 \times 10^{-26} \text{ kg/m}^3$, where only about 4% of the Universe's total mass is due to "ordinary" matter (such as protons, neutrons, and electrons). Use this information to estimate the total mass of ordinary matter in the observable Universe. (1 light-year $= 9.46 \times 10^{15}$ m.)

Answers to Odd-Numbered Problems

1. (*a*) 1.4×10^{10} y;
(*b*) 4.4×10^{17} s.

3. (*a*) 1.156×10^0;
(*b*) 2.18×10^1;
(*c*) 6.8×10^{-3};
(*d*) 3.2865×10^2;
(*e*) 2.19×10^{-1};
(*f*) 4.44×10^2.

5. 4.6%.

7. 1.00×10^5 s.

9. 0.24 rad.

11. (*a*) 0.2866 m;
(*b*) 0.000085 V;
(*c*) 0.00076 kg;
(*d*) 0.0000000000600 s;
(*e*) 0.0000000000000225 m;
(*f*) 2,500,000,000 V.

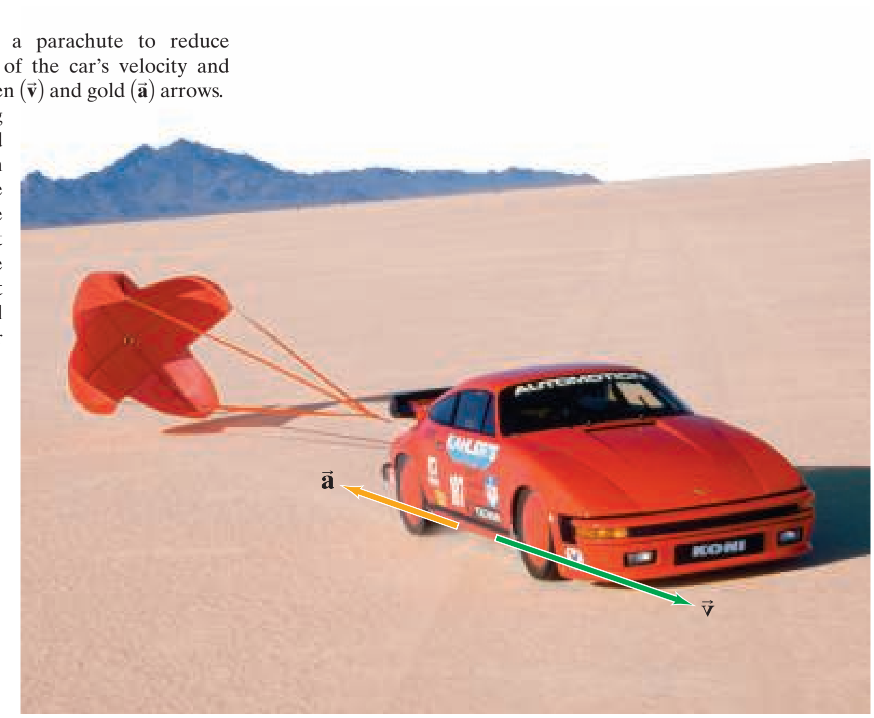
13. $5'10'' = 1.8$ m, 165 lbs $= 75.2$ kg.
15. (*a*) 1 $\text{ft}^2 = 0.111\ \text{yd}^2$;
(*b*) 1 $\text{m}^2 = 10.8\ \text{ft}^2$.
17. (*a*) 3.9×10^{-9} in.;
(*b*) 1.0×10^8 atoms.
19. (*a*) 1 km/h $= 0.621$ mi/h;
(*b*) 1 m/s $= 3.28$ ft/s;
(*c*) 1 km/h $= 0.278$ m/s.
21. (*a*) 9.46×10^{15} m;
(*b*) 6.31×10^4 AU;
(*c*) 7.20 AU/h.
23. (*a*) $3.80 \times 10^{13}\ \text{m}^2$;
(*b*) 13.4.
25. 6×10^5 books.
27. 5×10^4 L.
29. (*a*) 1800.
31. 5×10^4 m.
33. 6.5×10^6 m.
35. $[M/L^3]$.
37. (*a*) Cannot;
(*b*) can;
(*c*) can.
39. $(1 \times 10^{-5})\%$, 8 significant figures.
41. (*a*) 3.16×10^7 s;
(*b*) 3.16×10^{16} ns;
(*c*) 3.17×10^{-8} y.
43. 2×10^{-4} m.
45. 1×10^{11} gal/y.
47. 9 cm/y.
49. 2×10^9 kg/y.
51. 75 min.
53. 4×10^5 metric tons, 1×10^8 gal.
55. 1×10^3 days
57. 210 yd, 190 m.
59. (*a*) 0.10 nm;
(*b*) 1.0×10^5 fm;
(*c*) 1.0×10^{10} Å;
(*d*) 9.5×10^{25} Å.
61. (*a*) 3%, 3%;
(*b*) 0.7%, 0.2%.
63. $8 \times 10^{-2}\ \text{m}^3$.
65. L/m, L/y, L.
67. (*a*) 13.4;
(*b*) 49.3.
69. 4×10^{51} kg.

Describing Motion: Kinematics in One Dimension

From Chapter 2 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

A high-speed car has released a parachute to reduce its speed quickly. The directions of the car's velocity and acceleration are shown by the green $(\vec{\mathbf{v}})$ and gold $(\vec{\mathbf{a}})$ arrows.

Motion is described using the concepts of velocity and acceleration. In the case shown here, the acceleration $\vec{\mathbf{a}}$ is in the opposite direction from the velocity $\vec{\mathbf{v}}$, which means the object is slowing down. We examine in detail motion with constant acceleration, including the vertical motion of objects falling under gravity.



George D. Lepp/Corbis/Bettmann

Describing Motion: Kinematics in One Dimension

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Two small heavy balls have the same diameter but one weighs twice as much as the other. The balls are dropped from a second-story balcony at the exact same time. The time to reach the ground below will be:

(a) twice as long for the lighter ball as for the heavier one.
(b) longer for the lighter ball, but not twice as long.
(c) twice as long for the heavier ball as for the lighter one.
(d) longer for the heavier ball, but not twice as long.
(e) nearly the same for both balls.

The motion of objects—baseballs, automobiles, joggers, and even the Sun and Moon—is an obvious part of everyday life. It was not until the sixteenth and seventeenth centuries that our modern understanding of motion was established. Many individuals contributed to this understanding, particularly Galileo Galilei (1564–1642) and Isaac Newton (1642–1727).

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do.

For now we only discuss objects that move without rotating (Fig. 1a). Such motion is called **translational motion**. In this Chapter we will be concerned with describing an object that moves along a straight-line path, which is one-dimensional translational motion.

We will often use the concept, or *model*, of an idealized **particle** which is considered to be a mathematical **point** with no spatial extent (no size). A point particle can undergo only translational motion. The particle model is useful in many real situations where we are interested only in translational motion and the object's size is not significant. For example, we might consider a billiard ball, or even a spacecraft traveling toward the Moon, as a particle for many purposes.

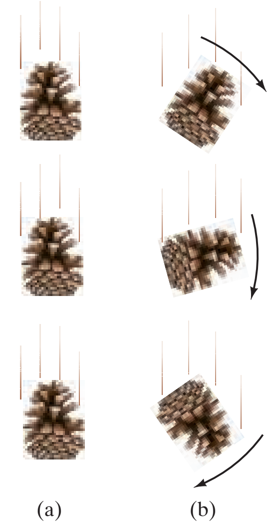


FIGURE 1 The pinecone in (a) undergoes pure translation as it falls, whereas in (b) it is rotating as well as translating.

1 Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a **reference frame**, or **frame of reference**. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of $80\,\mathrm{km/h} + 5\,\mathrm{km/h} = 85\,\mathrm{km/h}$. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.

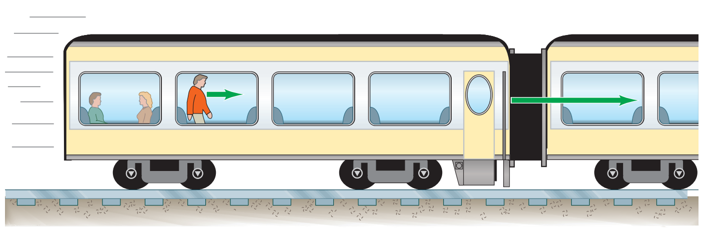
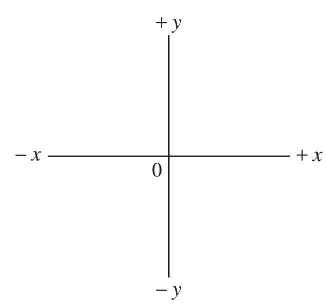


FIGURE 2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using the cardinal points, north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 3, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. The x and y axes are always perpendicular to each other. Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we usually choose to be positive; then points to the left of 0 have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the plane can be specified by giving its x and y coordinates. In three dimensions, a z axis perpendicular to the x and y axes is added.

FIGURE 3 Standard set of xy coordinate axes.



For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the **position** of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

CAUTION

The displacement may not equal the total distance traveled

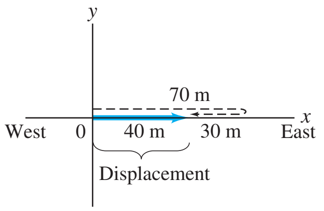


FIGURE 4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is 40 m to the east.

FIGURE 5 The arrow represents the displacement $x_2 - x_1$. Distances are in meters.

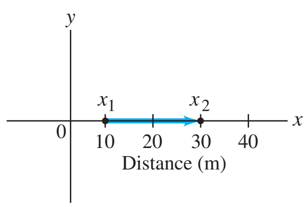
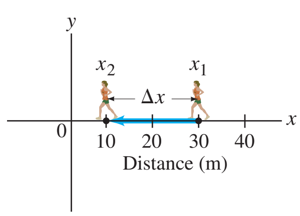


FIGURE 6 For the displacement $\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m}$, the displacement vector points to the left.



We need to make a distinction between the *distance* an object has traveled and its **displacement**, which is defined as the *change in position* of the object. That is, *displacement is how far the object is from its starting point*. To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 4). The total *distance* traveled is 100 m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.

Displacement is a quantity that has both magnitude and direction. Such quantities are called **vectors**, and are represented by arrows in diagrams. For example, in Fig. 4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).

In this chapter, we deal only with motion in one dimension, along a line. In this case, vectors which point in one direction will have a positive sign, whereas vectors that point in the opposite direction will have a negative sign, along with their magnitude.

Consider the motion of an object over a particular time interval. Suppose that at some initial time, call it t_1, the object is on the x axis at the position x_1 in the coordinate system shown in Fig. 5. At some later time, t_2, suppose the object has moved to position x_2. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 5. It is convenient to write

$$\Delta x = x_2 - x_1,$$

where the symbol Δ (Greek letter delta) means "change in." Then Δx means "the change in x," or "change in position," which is the displacement. Note that the "change in" any quantity means the final value of that quantity, minus the initial value.

Suppose $x_1 = 10.0\text{ m}$ and $x_2 = 30.0\text{ m}$. Then

$$\Delta x = x_2 - x_1 = 30.0\text{ m} - 10.0\text{ m} = 20.0\text{ m},$$

so the displacement is 20.0 m in the positive direction, Fig. 5.

Now consider an object moving to the left as shown in Fig. 6. Here the object, say, a person, starts at $x_1 = 30.0\text{ m}$ and walks to the left to the point $x_2 = 10.0\text{ m}$. In this case her displacement is

$$\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m} = -20.0\text{ m},$$

and the blue arrow representing the vector displacement points to the left. For one-dimensional motion along the x axis, a vector pointing to the right has a positive sign, whereas a vector pointing to the left has a negative sign.

EXERCISE A An ant starts at $x = 20\text{ cm}$ on a piece of graph paper and walks along the x axis to $x = -20\text{ cm}$. It then turns around and walks back to $x = -10\text{ cm}$. What is the ant's displacement and total distance traveled?

2 Average Velocity

The most obvious aspect of the motion of a moving object is how fast it is moving—its speed or velocity.

The term "speed" refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the total distance traveled along its path divided by the time it takes to travel this distance*:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}. \qquad \textbf{(1)}$$

The terms "velocity" and "speed" are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and also the *direction* in which it is moving. (Velocity is therefore a vector.) There is a second difference between speed and velocity: namely, the **average velocity** is defined in terms of *displacement*, rather than total distance traveled:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}.$$

CAUTION

Average speed is not necessarily equal to the magnitude of the average velocity

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: recall the walk we described earlier, in Fig. 4, where a person walked 70 m east and then 30 m west. The total distance traveled was $70\text{ m} + 30\text{ m} = 100\text{ m}$, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{time elapsed}} = \frac{100\text{ m}}{70\text{ s}} = 1.4\text{ m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{time elapsed}} = \frac{40\text{ m}}{70\text{ s}} = 0.57\text{ m/s}.$$

This difference between the speed and the magnitude of the velocity can occur when we calculate *average* values.

To discuss one-dimensional motion of an object in general, suppose that at some moment in time, call it t_1, the object is on the x axis at position x_1 in a coordinate system, and at some later time, t_2, suppose it is at position x_2. The **elapsed time** is $\Delta t = t_2 - t_1$; during this time interval the displacement of our object is $\Delta x = x_2 - x_1$. Then the average velocity, defined as *the displacement divided by the elapsed time*, can be written

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad \textbf{(2)}$$

where v stands for velocity and the bar $(^{-})$ over the v is a standard symbol meaning "average."

PROBLEM SOLVING

+ or − sign can signify the direction for linear motion

For the usual case of the $+x$ axis to the right, note that if x_2 is less than x_1, the object is moving to the left, and then $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the average velocity, indicates the direction: the average velocity is positive for an object moving to the right along the $+x$ axis and negative when the object moves to the left. The direction of the average velocity is always the same as the direction of the displacement.

Note that it is always important to choose (and state) the *elapsed time*, or *time interval*, $t_2 - t_1$, the time that passes during our chosen period of observation.

EXAMPLE 1 **Runner's average velocity.** The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from $x_1 = 50.0\text{ m}$ to $x_2 = 30.5\text{ m}$, as shown in Fig. 7. What was the runner's average velocity?

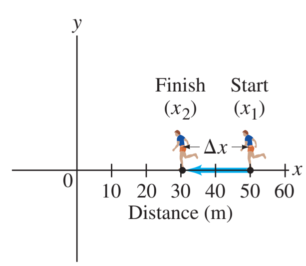
APPROACH We want to find the average velocity, which is the displacement divided by the elapsed time.

SOLUTION The displacement is $\Delta x = x_2 - x_1 = 30.5\text{ m} - 50.0\text{ m} = -19.5\text{ m}$. The elapsed time, or time interval, is $\Delta t = 3.00\text{ s}$. The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\text{ m}}{3.00\text{ s}} = -6.50\text{ m/s}.$$

The displacement and average velocity are negative, which tells us that the runner is moving to the left along the x axis, as indicated by the arrow in Fig. 7. Thus we can say that the runner's average velocity is 6.50 m/s to the left.

FIGURE 7 Example 1. A person runs from $x_1 = 50.0\text{ m}$ to $x_2 = 30.5\text{ m}$. The displacement is -19.5 m.



EXAMPLE 2 **Distance a cyclist travels.** How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

APPROACH We want to find the distance traveled, so we solve Eq. 2 for Δx.

SOLUTION We rewrite Eq. 2 as $\Delta x = \bar{v}\,\Delta t$, and find

$$\Delta x = \bar{v}\,\Delta t = (18\text{ km/h})(2.5\text{ h}) = 45\text{ km}.$$

EXERCISE B A car travels at a constant 50 km/h for 100 km. It then speeds up to 100 km/h and is driven another 100 km. What is the car's average speed for the 200 km trip? (*a*) 67 km/h; (*b*) 75 km/h; (*c*) 81 km/h; (*d*) 50 km/h.

3 Instantaneous Velocity

John E. Gilmore III

FIGURE 8 Car speedometer showing mi/h in white, and km/h in orange.

If you drive a car along a straight road for 150 km in 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To describe this situation we need the concept of *instantaneous velocity*, which is the velocity at any instant of time. (Its magnitude is the number, with units, indicated by a speedometer, Fig. 8.) More precisely, the **instantaneous velocity** at any moment is defined as *the average velocity over an infinitesimally short time interval*. That is, Eq. 2 is to be evaluated in the limit of Δt becoming extremely small, approaching zero. We can write the definition of instantaneous velocity, v, for one-dimensional motion as

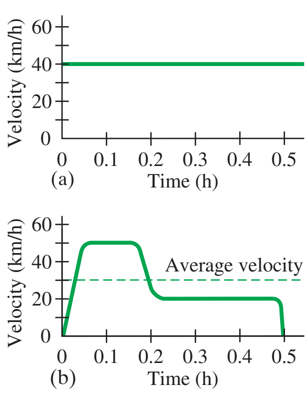
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \qquad (3)$$

The notation $\lim_{\Delta t \to 0}$ means the ratio $\Delta x/\Delta t$ is to be evaluated in the limit of Δt approaching zero. But we do not simply set $\Delta t = 0$ in this definition, for then Δx would also be zero, and we would have an undefined number. Rather, we are considering the *ratio* $\Delta x/\Delta t$, as a whole. As we let Δt approach zero, Δx approaches zero as well. But the ratio $\Delta x/\Delta t$ approaches some definite value, which is the instantaneous velocity at a given instant.

In Eq. 3, the limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the *derivative* of x with respect to t:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \qquad (4)$$

FIGURE 9 Velocity of a car as a function of time: (a) at constant velocity; (b) with varying velocity.



This equation is the definition of instantaneous velocity for one-dimensional motion.

For instantaneous velocity we use the symbol v, whereas for average velocity we use $\bar{v}$, with a bar above. When we use the term "velocity" it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word "average."

Note that the *instantaneous* speed always equals the magnitude of the instantaneous velocity. Why? Because distance traveled and the magnitude of the displacement become the same when they become infinitesimally small.

If an object moves at a uniform (that is, constant) velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 9a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 9b. Also shown on the graph is the average velocity (dashed line), which is $\bar{v} = \Delta x/\Delta t = 15\text{ km}/0.50\text{ h} = 30\text{ km/h}$.

To better understand instantaneous velocity, let us consider a graph of the position of a particular particle versus time (x vs. t), as shown in Fig. 10. (Note that this is different from showing the "path" of a particle on an x vs. y plot.) The particle is at position x_1 at a time t_1, and at position x_2 at time t_2. P_1 and P_2 represent these two points on the graph. A straight line drawn from point $P_1(x_1, t_1)$ to point $P_2(x_2, t_2)$ forms the hypotenuse of a right triangle whose sides are Δx and Δt. The ratio $\Delta x/\Delta t$ is the **slope** of the straight line P_1P_2. But $\Delta x/\Delta t$ is also the average velocity of the particle during the time interval $\Delta t = t_2 - t_1$. Therefore, we conclude that the average velocity of a particle during any time interval $\Delta t = t_2 - t_1$ is equal to the slope of the straight line (or *chord*) connecting the two points (x_1, t_1) and (x_2, t_2) on an x vs. t graph.

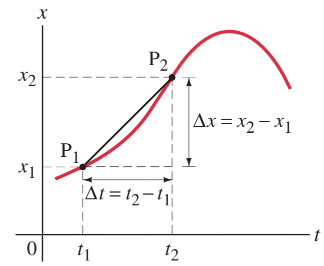
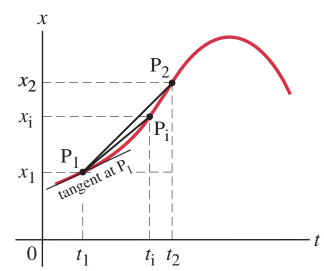


FIGURE 10 Graph of a particle's position x vs. time t. The slope of the straight line P_1P_2 represents the average velocity of the particle during the time interval $\Delta t = t_2 - t_1$.

Consider now a time t_i, intermediate between t_1 and t_2, at which time the particle is at x_i (Fig. 11). The slope of the straight line P_1P_i is less than the slope of P_1P_2 in this case. Thus the average velocity during the time interval $t_i - t_1$ is less than during the time interval $t_2 - t_1$.

Now let us imagine that we take the point P_i in Fig. 11 to be closer and closer to point P_1. That is, we let the interval $t_i - t_1$, which we now call Δt, to become smaller and smaller. The slope of the line connecting the two points becomes closer and closer to the slope of a line tangent to the curve at point P_1. The average velocity (equal to the slope of the chord) thus approaches the slope of the tangent at point P_1. The definition of the instantaneous velocity (Eq. 3) is the limiting value of the average velocity as Δt approaches zero. Thus the *instantaneous velocity equals the slope of the tangent to the curve* at that point (which we can simply call "the slope of the curve" at that point).

FIGURE 11 Same position vs. time curve as in Fig. 10, but note that the average velocity over the time interval $t_i - t_1$ (which is the slope of P_1P_i) is less than the average velocity over the time interval $t_2 - t_1$. The slope of the thin line tangent to the curve at point P_1 equals the instantaneous velocity at time t_1.



Because the velocity at any instant equals the slope of the tangent to the x vs. t graph at that instant, we can obtain the velocity at any instant from such a graph. For example, in Fig. 12 (which shows the same curve as in Figs. 10 and 11), as our object moves from x_1 to x_2, the slope continually increases, so the velocity is increasing. For times after t_2, however, the slope begins to decrease and in fact reaches zero (so $v = 0$) where x has its maximum value, at point P_3 in Fig. 12. Beyond this point, the slope is negative, as for point P_4. The velocity is therefore negative, which makes sense since x is now decreasing—the particle is moving toward decreasing values of x, to the left on a standard xy plot.

If an object moves with constant velocity over a particular time interval, its instantaneous velocity is equal to its average velocity. The graph of x vs. t in this case will be a straight line whose slope equals the velocity. The curve of Fig. 10 has no straight sections, so there are no time intervals when the velocity is constant.

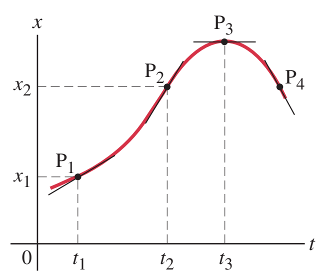


FIGURE 12 Same x vs. t curve as in Figs. 10 and 11, but here showing the slope at four different points: At P_3, the slope is zero, so $v = 0$. At P_4 the slope is negative, so $v < 0$.

EXERCISE C What is your speed at the instant you turn around to move in the opposite direction? (*a*) Depends on how quickly you turn around; (*b*) always zero; (*c*) always negative; (*d*) none of the above.

The derivatives of polynomial functions (which we use a lot) are:

$$\frac{d}{dt}(Ct^n) = nCt^{n-1} \quad \text{and} \quad \frac{dC}{dt} = 0,$$

where C is any constant.

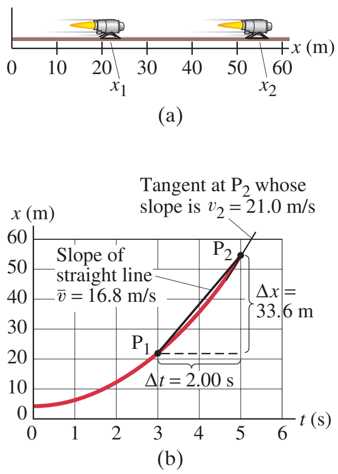


FIGURE 13 Example 3.
(a) Engine traveling on a straight track.
(b) Graph of x vs. t: $x = At^2 + B$.

EXAMPLE 3 **Given x as a function of t.** A jet engine moves along an experimental track (which we call the x axis) as shown in Fig. 13a. We will treat the engine as if it were a particle. Its position as a function of time is given by the equation $x = At^2 + B$, where $A = 2.10\,\text{m/s}^2$ and $B = 2.80\,\text{m}$, and this equation is plotted in Fig. 13b. (a) Determine the displacement of the engine during the time interval from $t_1 = 3.00\,\text{s}$ to $t_2 = 5.00\,\text{s}$. (b) Determine the average velocity during this time interval. (c) Determine the magnitude of the instantaneous velocity at $t = 5.00\,\text{s}$.

APPROACH We substitute values for t_1 and t_2 in the given equation for x to obtain x_1 and x_2. The average velocity can be found from Eq. 2. We take the derivative of the given x equation with respect to t to find the instantaneous velocity, using the formulas just given.

SOLUTION (a) At $t_1 = 3.00\,\text{s}$, the position (point P_1 in Fig. 13b) is

$$x_1 = At_1^2 + B = (2.10\,\text{m/s}^2)(3.00\,\text{s})^2 + 2.80\,\text{m} = 21.7\,\text{m}.$$

At $t_2 = 5.00\,\text{s}$, the position (P_2 in Fig. 13b) is

$$x_2 = (2.10\,\text{m/s}^2)(5.00\,\text{s})^2 + 2.80\,\text{m} = 55.3\,\text{m}.$$

The displacement is thus

$$x_2 - x_1 = 55.3\,\text{m} - 21.7\,\text{m} = 33.6\,\text{m}.$$

(b) The magnitude of the average velocity can then be calculated as

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{33.6\,\text{m}}{2.00\,\text{s}} = 16.8\,\text{m/s}.$$

This equals the slope of the straight line joining points P_1 and P_2 shown in Fig. 13b.

(c) The instantaneous velocity at $t = t_2 = 5.00\,\text{s}$ equals the slope of the tangent to the curve at point P_2 shown in Fig. 13b. We could measure this slope off the graph to obtain v_2. But we can calculate v more precisely for any time t, using the given formula

$$x = At^2 + B,$$

which is the engine's position x as a function of time t. We take the derivative of x with respect to time (see formulas at bottom of previous page):

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At.$$

We are given $A = 2.10\,\text{m/s}^2$, so for $t = t_2 = 5.00\,\text{s}$,

$$v_2 = 2At = 2(2.10\,\text{m/s}^2)(5.00\,\text{s}) = 21.0\,\text{m/s}.$$

4 Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how rapidly the velocity of an object is changing.

Average Acceleration

Average acceleration is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

In symbols, the average acceleration over a time interval $\Delta t = t_2 - t_1$ during which the velocity changes by $\Delta v = v_2 - v_1$, is defined as

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \quad \textbf{(5)}$$

Because velocity is a vector, acceleration is a vector too. But for one-dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis.

EXAMPLE 4 **Average acceleration.** A car accelerates along a straight road from rest to 90 km/h in 5.0 s, Fig. 14. What is the magnitude of its average acceleration?

APPROACH Average acceleration is the change in velocity divided by the elapsed time, 5.0 s. The car starts from rest, so $v_1 = 0$. The final velocity is $v_2 = 90\text{ km/h} = 90 \times 10^3\text{ m}/3600\text{ s} = 25\text{ m/s}$.

SOLUTION From Eq. 5, the average acceleration is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{25\text{ m/s} - 0\text{ m/s}}{5.0\text{ s}} = 5.0\,\frac{\text{m/s}}{\text{s}}.$$

This is read as “five meters per second per second” and means that, on average, the velocity changed by 5.0 m/s during each second. That is, assuming the acceleration was constant, during the first second the car’s velocity increased from zero to 5.0 m/s. During the next second its velocity increased by another 5.0 m/s, reaching a velocity of 10.0 m/s at $t = 2.0\text{ s}$, and so on. See Fig. 14.

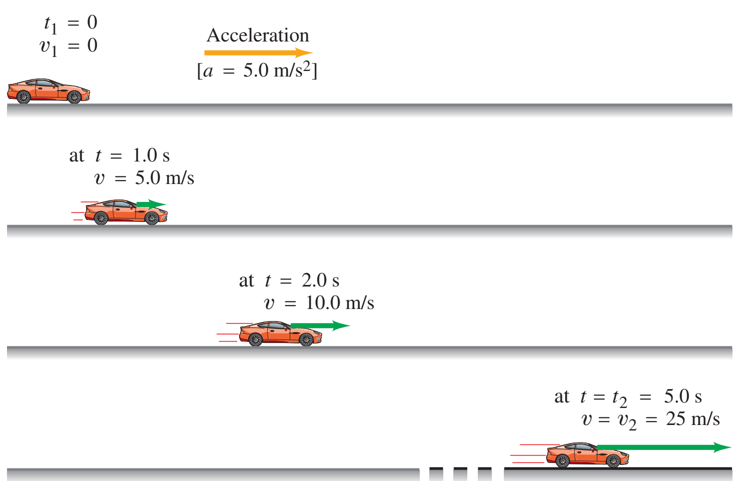


FIGURE 14 Example 4. The car is shown at the start with $v_1 = 0$ at $t_1 = 0$. The car is shown three more times, at $t = 1.0\text{ s}$, $t = 2.0\text{ s}$, and at the end of our time interval, $t_2 = 5.0\text{ s}$. We assume the acceleration is constant and equals 5.0 m/s^2. The green arrows represent the velocity vectors; the length of each arrow represents the magnitude of the velocity at that moment. The acceleration vector is the orange arrow. Distances are not to scale.

We almost always write the units for acceleration as m/s^2 (meters per second squared) instead of m/s/s. This is possible because:

$$\frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}\cdot\text{s}} = \frac{\text{m}}{\text{s}^2}.$$

According to the calculation in Example 4, the velocity changed on average by 5.0 m/s during each second, for a total change of 25 m/s over the 5.0 s; the average acceleration was 5.0 m/s^2.

Note that *acceleration tells us how quickly the velocity changes*, whereas *velocity tells us how quickly the position changes*.

CONCEPTUAL EXAMPLE 5 **Velocity and acceleration.** (*a*) If the velocity of an object is zero, does it mean that the acceleration is zero? (*b*) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

RESPONSE A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (*a*) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but the acceleration is not zero since the velocity of the car changes. (How else could your car start forward if its velocity weren't changing—that is, accelerating?) (*b*) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero: $a = 0,\ v \neq 0$.

EXERCISE D A powerful car is advertised to go from zero to 60 mi/h in 6.0 s. What does this say about the car: (*a*) it is fast (high speed); or (*b*) it accelerates well?

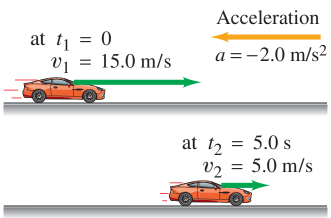


FIGURE 15 Example 6, showing the position of the car at times t_1 and t_2, as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left as the car slows down while moving to the right.

EXAMPLE 6 **Car slowing down.** An automobile is moving to the right along a straight highway, which we choose to be the positive x axis (Fig. 15). Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is $v_1 = 15.0\ \mathrm{m/s}$, and it takes 5.0 s to slow down to $v_2 = 5.0\ \mathrm{m/s}$, what was the car's average acceleration?

APPROACH We put the given initial and final velocities, and the elapsed time, into Eq. 5 for $\bar{a}$.

SOLUTION In Eq. 5, we call the initial time $t_1 = 0$, and set $t_2 = 5.0\ \mathrm{s}$. (Note that our choice of $t_1 = 0$ doesn't affect the calculation of $\bar{a}$ because only $\Delta t = t_2 - t_1$ appears in Eq. 5.) Then

$$\bar{a} = \frac{5.0\ \mathrm{m/s} - 15.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = -2.0\ \mathrm{m/s^2}.$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left (in the negative x direction)—even though the velocity is always pointing to the right. We say that the acceleration is $2.0\ \mathrm{m/s^2}$ to the left, and it is shown in Fig. 15 as an orange arrow.

Deceleration

CAUTION

Deceleration means the magnitude of the velocity is decreasing; a is not *necessarily negative*

When an object is slowing down, we can say it is **decelerating**. But be careful: deceleration does *not* mean that the acceleration is necessarily negative. The velocity of an object moving to the right along the positive x axis is positive; if the object is slowing down (as in Fig. 15), the acceleration *is* negative. But the same car moving to the left (decreasing x), and slowing down, has positive acceleration that points to the right, as shown in Fig. 16. We have a deceleration whenever the magnitude of the velocity is decreasing, and then the velocity and acceleration point in opposite directions.

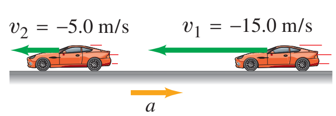


FIGURE 16 The car of Example 6, now moving to the *left* and decelerating. The acceleration is

$$a = \frac{v_2 - v_1}{\Delta t} = \frac{(-5.0\ \mathrm{m/s}) - (-15.0\ \mathrm{m/s})}{5.0\ \mathrm{s}} = \frac{-5.0\ \mathrm{m/s} + 15.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = +2.0\ \mathrm{m/s}.$$

EXERCISE E A car moves along the x axis. What is the sign of the car's acceleration if it is moving in the positive x direction with (*a*) increasing speed or (*b*) decreasing speed? What is the sign of the acceleration if the car moves in the negative direction with (*c*) increasing speed or (*d*) decreasing speed?

Instantaneous Acceleration

The **instantaneous acceleration**, a, is defined as the *limiting value of the average acceleration as we let* Δt *approach zero*:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}. \quad (6)$$

This limit, dv/dt, is the derivative of v with respect to t. We will use the term "acceleration" to refer to the instantaneous value. If we want to discuss the average acceleration, we will always include the word "average."

If we draw a graph of the velocity, v, vs. time, t, as shown in Fig. 17, then the average acceleration over a time interval $\Delta t = t_2 - t_1$ is represented by the slope of the straight line connecting the two points P_1 and P_2 as shown. [Compare this to the position vs. time graph of Fig. 10 for which the slope of the straight line represents the average velocity.] The instantaneous acceleration at any time, say t_1, is the slope of the tangent to the v vs. t curve at that time, which is also shown in Fig. 17. Let us use this fact for the situation graphed in Fig. 17; as we go from time t_1 to time t_2 the velocity continually increases, but the acceleration (the rate at which the velocity changes) is decreasing since the slope of the curve is decreasing.

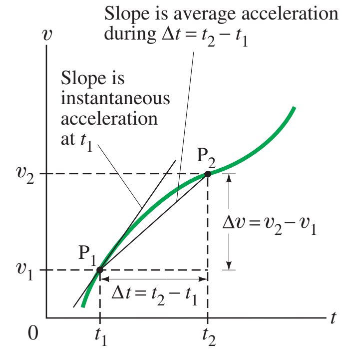


FIGURE 17 A graph of velocity v vs. time t. The average acceleration over a time interval $\Delta t = t_2 - t_1$ is the slope of the straight line P_1P_2: $\bar{a} = \Delta v/\Delta t$. The instantaneous acceleration at time t_1 is the slope of the v vs. t curve at that instant.

EXAMPLE 7 **Acceleration given $x(t)$.** A particle is moving in a straight line so that its position is given by the relation $x = (2.10\,\text{m/s}^2)t^2 + (2.80\,\text{m})$, as in Example 3. Calculate (*a*) its average acceleration during the time interval from $t_1 = 3.00\,\text{s}$ to $t_2 = 5.00\,\text{s}$, and (*b*) its instantaneous acceleration as a function of time.

APPROACH To determine acceleration, we first must find the velocity at t_1 and t_2 by differentiating x: $v = dx/dt$. Then we use Eq. 5 to find the average acceleration, and Eq. 6 to find the instantaneous acceleration.

SOLUTION (*a*) The velocity at any time t is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[(2.10\,\text{m/s}^2)t^2 + 2.80\,\text{m}\right] = (4.20\,\text{m/s}^2)t,$$

as we saw in Example 3*c*. Therefore, at $t_1 = 3.00\,\text{s}$, $v_1 = (4.20\,\text{m/s}^2)(3.00\,\text{s}) = 12.6\,\text{m/s}$ and at $t_2 = 5.00\,\text{s}$, $v_2 = 21.0\,\text{m/s}$. Therefore,

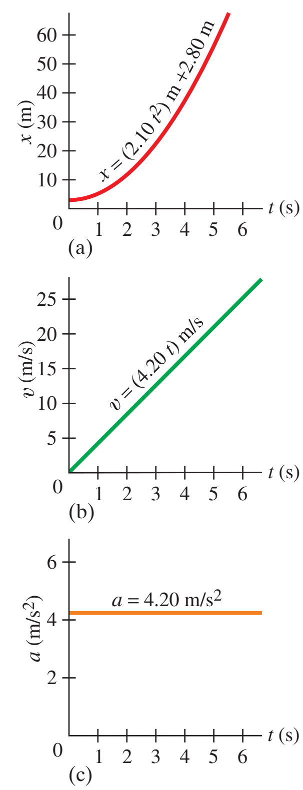
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{21.0\,\text{m/s} - 12.6\,\text{m/s}}{5.00\,\text{s} - 3.00\,\text{s}} = 4.20\,\text{m/s}^2.$$

(*b*) With $v = (4.20\,\text{m/s}^2)t$, the instantaneous acceleration at any time is

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[(4.20\,\text{m/s}^2)t\right] = 4.20\,\text{m/s}^2.$$

The acceleration in this case is constant; it does not depend on time. Figure 18 shows graphs of (a) x vs. t (the same as Fig. 13b), (b) v vs. t, which is linearly increasing as calculated above, and (c) a vs. t, which is a horizontal straight line because a = constant.

FIGURE 18 Example 7. Graphs of (a) x vs. t, (b) v vs. t, and (c) a vs. t for the motion $x = At^2 + B$. Note that v increases linearly with t and that the acceleration a is constant. Also, v is the slope of the x vs. t curve, whereas a is the slope of the v vs. t curve.



Like velocity, acceleration is a rate. The velocity of an object is the rate at which its displacement changes with time; its acceleration, on the other hand, is the rate at which its velocity changes with time. In a sense, acceleration is a "rate of a rate." This can be expressed in equation form as follows: since $a = dv/dt$ and $v = dx/dt$, then

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

Here d^2x/dt^2 is the *second derivative* of x with respect to time: we first take the derivative of x with respect to time (dx/dt), and then we again take the derivative with respect to time, $(d/dt)(dx/dt)$, to get the acceleration.

EXERCISE F The position of a particle is given by the following equation:

$$x = (2.00\,\text{m/s}^3)t^3 + (2.50\,\text{m/s})t.$$

What is the acceleration of the particle at $t = 2.00\,\text{s}$? (*a*) $13.0\,\text{m/s}^2$; (*b*) $22.5\,\text{m/s}^2$; (*c*) $24.0\,\text{m/s}^2$; (*d*) $2.00\,\text{m/s}^2$.

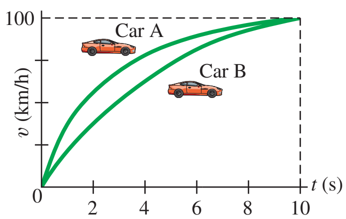


FIGURE 19 Example 8.

CONCEPTUAL EXAMPLE 8 **Analyzing with graphs.** Figure 19 shows the velocity as a function of time for two cars accelerating from 0 to 100 km/h in a time of 10.0 s. Compare (*a*) the average acceleration; (*b*) instantaneous acceleration; and (*c*) total distance traveled for the two cars.

RESPONSE (*a*) Average acceleration is $\Delta v/\Delta t$. Both cars have the same Δv (100 km/h) and the same Δt (10.0 s), so the average acceleration is the same for both cars. (*b*) Instantaneous acceleration is the slope of the tangent to the v vs. t curve. For about the first 4 s, the top curve is steeper than the bottom curve, so car A has a greater acceleration during this interval. The bottom curve is steeper during the last 6 s, so car B has the larger acceleration for this period. (*c*) Except at $t = 0$ and $t = 10.0\,\text{s}$, car A is always going faster than car B. Since it is going faster, it will go farther in the same time.

5 Motion at Constant Acceleration

We now examine the situation when the magnitude of the acceleration is constant and the motion is in a straight line. In this case, the instantaneous and average accelerations are equal. We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate x, v, a, and t when a is constant, allowing us to determine any one of these variables if we know the others.

To simplify our notation, let us take the initial time in any discussion to be zero, and we call it t_0: $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at t_0.) We can then let $t_2 = t$ be the elapsed time. The initial position (x_1) and the initial velocity (v_1) of an object will now be represented by x_0 and v_0, since they represent x and v at $t = 0$. At time t the position and velocity will be called x and v (rather than x_2 and v_2). The average velocity during the time interval $t - t_0$ will be (Eq. 2)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose $t_0 = 0$. The acceleration, assumed constant in time, is (Eq. 5)

$$a = \frac{v - v_0}{t}.$$

A common problem is to determine the velocity of an object after any elapsed time t, when we are given the object's constant acceleration. We can solve such problems by solving for v in the last equation to obtain:

$$v = v_0 + at. \qquad \text{[constant acceleration]} \quad \mathbf{(7)}$$

If an object starts from rest $(v_0 = 0)$ and accelerates at $4.0\,\text{m/s}^2$, after an elapsed time $t = 6.0\,\text{s}$ its velocity will be $v = at = (4.0\,\text{m/s}^2)(6.0\,\text{s}) = 24\,\text{m/s}$.

Next, let us see how to calculate the position x of an object after a time t when it undergoes constant acceleration. The definition of average velocity (Eq. 2) is $\bar{v} = (x - x_0)/t$, which we can rewrite as

$$x = x_0 + \bar{v}t. \qquad \mathbf{(8)}$$

Because the velocity increases at a uniform rate, the average velocity, $\bar{v}$, will be midway between the initial and final velocities:

CAUTION
Average velocity, but only if a = constant

$$\bar{v} = \frac{v_0 + v}{2}. \qquad \text{[constant acceleration]} \quad \mathbf{(9)}$$

(Careful: Equation 9 is not necessarily valid if the acceleration is not constant.) We combine the last two Equations with Eq. 7 and find

$$\begin{aligned} x &= x_0 + \bar{v}t \\ &= x_0 + \left(\frac{v_0 + v}{2}\right)t \\ &= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t \end{aligned}$$

or

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2. \qquad \text{[constant acceleration]} \quad \mathbf{(10)}$$

Equations 7, 9, and 10 are three of the four most useful equations for motion at constant acceleration. We now derive the fourth equation, which is useful in situations where the time t is not known. We substitute Eq. 9 into Eq. 8:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v + v_0}{2}\right)t.$$

Next we solve Eq. 7 for t, obtaining

$$t = \frac{v - v_0}{a},$$

and substituting this into the previous equation we have

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

We solve this for v^2 and obtain

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \text{[constant acceleration]} \quad \textbf{(11)}$$

which is the useful equation we sought.

We now have four equations relating position, velocity, acceleration, and time, when the acceleration a is constant. We collect these kinematic equations here in one place for future reference (the tan background screen emphasizes their usefulness):

$$v = v_0 + at \qquad [a = \text{constant}] \quad \textbf{(12a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \quad \textbf{(12b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \quad \textbf{(12c)}$$

$$\bar{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \quad \textbf{(12d)}$$

Kinematic equations for constant acceleration (we'll use them a lot)

These useful equations are not valid unless a is a constant. In many cases we can set $x_0 = 0$, and this simplifies the above equations a bit. Note that x represents position, not distance, that $x - x_0$ is the displacement, and that t is the elapsed time.

PHYSICS APPLIED
Airport design

EXAMPLE 9 **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at $2.00\ \text{m/s}^2$. (*a*) If the runway is 150 m long, can this airplane reach the required speed for takeoff? (*b*) If not, what minimum length must the runway have?

APPROACH The plane's acceleration is constant, so we can use the kinematic equations for constant acceleration. In (*a*), we want to find v, and we are given:

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\ \text{m}$	
$a = 2.00\ \text{m/s}^2$	

PROBLEM SOLVING
Equations–12 are valid only when the acceleration is constant, which we assume in this Example

SOLUTION (*a*) Of the above four equations, Eq. 12c will give us v when we know v_0, a, x, and x_0:

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= 0 + 2(2.00\ \text{m/s}^2)(150\ \text{m}) = 600\ \text{m}^2/\text{s}^2 \\ v &= \sqrt{600\ \text{m}^2/\text{s}^2} = 24.5\ \text{m/s}. \end{aligned}$$

This runway length is *not* sufficient.

(*b*) Now we want to find the minimum length of runway, $x - x_0$, given $v = 27.8\ \text{m/s}$ and $a = 2.00\ \text{m/s}^2$. So we again use Eq. 12c, but rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\ \text{m/s})^2 - 0}{2(2.00\ \text{m/s}^2)} = 193\ \text{m}.$$

A 200-m runway is more appropriate for this plane.

NOTE We did this Example as if the plane were a particle, so we round off our answer to 200 m.

EXERCISE G A car starts from rest and accelerates at a constant $10\ \text{m/s}^2$ during a $\frac{1}{4}$ mile (402 m) race. How fast is the car going at the finish line? (a) 8090 m/s; (b) 90 m/s; (c) 81 m/s; (d) 809 m/s.

6 Solving Problems

Before doing more worked-out Examples, let us look at how to approach problem solving. First, it is important to note that physics is *not* a collection of equations to be memorized. Simply searching for an equation that might work can lead you to a wrong result and will surely not help you understand physics. A better approach is to use the following (rough) procedure, which we put in a special "Problem Solving Strategy."

PROBLEM SOLVING

1. Read and **reread** the whole problem carefully before trying to solve it.
2. Decide what **object** (or objects) you are going to study, and for what **time interval**. You can often choose the initial time to be $t = 0$.
3. **Draw** a **diagram** or picture of the situation, with coordinate axes wherever applicable. [You can place the origin of coordinates and the axes wherever you like to make your calculations easier.]
4. Write down what quantities are "**known**" or "given," and then what you *want* to know. Consider quantities both at the beginning and at the end of the chosen time interval.
5. Think about which **principles of physics** apply in this problem. Use common sense and your own experiences. Then plan an approach.
6. Consider which **equations** (and/or definitions) relate the quantities involved. Before using them, be sure their **range of validity** includes your problem (for example, Eqs. 12 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, **solve** the equation algebraically for the unknown. Sometimes several sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.
7. Carry out the **calculation** if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answer(s) to the correct number of significant figures.
8. Think carefully about the result you obtain: Is it **reasonable**? Does it make sense according to your own intuition and experience? A good check is to do a rough **estimate** using only powers of ten. Often it is preferable to do a rough estimate at the *start* of a numerical problem because it can help you focus your attention on finding a path toward a solution.
9. A very important aspect of doing problems is keeping track of **units**. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has no doubt been made. This can serve as a **check** on your solution (but it only tells you if you're wrong, not if you're right). Always use a consistent set of units.

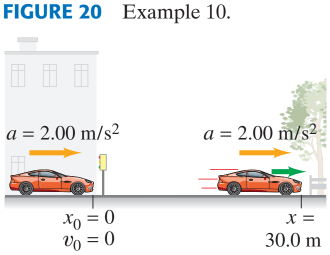
EXAMPLE 10 **Acceleration of a car.** How long does it take a car to cross a 30.0-m-wide intersection after the light turns green, if the car accelerates from rest at a constant $2.00\ \mathrm{m/s^2}$?

APPROACH We follow the Problem Solving Strategy above, step by step.

SOLUTION

1. **Reread** the problem. Be sure you understand what it asks for (here, a time interval).
2. The **object** under study is the car. We choose the **time interval**: $t = 0$, the initial time, is the moment the car starts to accelerate from rest $(v_0 = 0)$; the time t is the instant the car has traveled the full 30.0-m width of the intersection.
3. **Draw** a **diagram**: the situation is shown in Fig. 20, where the car is shown moving along the positive x axis. We choose $x_0 = 0$ at the front bumper of the car before it starts to move.

FIGURE 20 Example 10.



4. The "**knowns**" and the "wanted" are shown in the Table in the margin, and we choose $x_0 = 0$. Note that "starting from rest" means $v = 0$ at $t = 0$; that is, $v_0 = 0$.
5. The **physics**: the motion takes place at constant acceleration, so we can use the kinematic equations, Eqs. 12.
6. **Equations**: we want to find the time, given the distance and acceleration; Eq. 12b is perfect since the only unknown quantity is t. Setting $v_0 = 0$ and $x_0 = 0$ in Eq. 12b $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$, we can solve for t:
$$x = \tfrac{1}{2}at^2,$$
$$t^2 = \frac{2x}{a},$$
so
$$t = \sqrt{\frac{2x}{a}}.$$
7. The **calculation**:
$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\,\text{m})}{2.00\,\text{m/s}^2}} = 5.48\,\text{s}.$$
This is our answer. Note that the units come out correctly.
8. We can check the **reasonableness** of the answer by calculating the final velocity $v = at = (2.00\,\text{m/s}^2)(5.48\,\text{s}) = 10.96\,\text{m/s}$, and then finding $x = x_0 + \bar{v}t = 0 + \frac{1}{2}(10.96\,\text{m/s} + 0)(5.48\,\text{s}) = 30.0\,\text{m}$, which is our given distance.
9. We checked the **units**, and they came out perfectly (seconds).

NOTE In steps 6 and 7, when we took the square root, we should have written $t = \pm\sqrt{2x/a} = \pm 5.48\,\text{s}$. Mathematically there are two solutions. But the second solution, $t = -5.48\,\text{s}$, is a time *before* our chosen time interval and makes no sense physically. We say it is "unphysical" and ignore it.

Known	Wanted
$x_0 = 0$	t
$x = 30.0\,\text{m}$	
$a = 2.00\,\text{m/s}^2$	
$v_0 = 0$	

We explicitly followed the steps of the Problem Solving Strategy for Example 10. In upcoming Examples, we will use our usual "Approach" and "Solution" to avoid being wordy.

EXAMPLE 11 **ESTIMATE** **Air bags.** Suppose you want to design an air-bag system that can protect the driver at a speed of 100 km/h (60 mph) if the car hits a brick wall. Estimate how fast the air bag must inflate (Fig. 21) to effectively protect the driver. How does the use of a seat belt help the driver?

APPROACH We assume the acceleration is roughly constant, so we can use Eqs. 12. Both Eqs. 12a and 12b contain t, our desired unknown. They both contain a, so we must first find a, which we can do using Eq. 12c if we know the distance x over which the car crumples. A rough estimate might be about 1 meter. We choose the time interval to start at the instant of impact with the car moving at $v_0 = 100\,\text{km/h}$, and to end when the car comes to rest $(v = 0)$ after traveling 1 m.

SOLUTION We convert the given initial speed to SI units: $100\,\text{km/h} = 100 \times 10^3\,\text{m}/3600\,\text{s} = 28\,\text{m/s}$. We then find the acceleration from Eq. 12c:
$$a = -\frac{v_0^2}{2x} = -\frac{(28\,\text{m/s})^2}{2.0\,\text{m}} = -390\,\text{m/s}^2.$$
This enormous acceleration takes place in a time given by (Eq. 12a):
$$t = \frac{v - v_0}{a} = \frac{0 - 28\,\text{m/s}}{-390\,\text{m/s}^2} = 0.07\,\text{s}.$$
To be effective, the air bag would need to inflate faster than this.

What does the air bag do? It spreads the force over a large area of the chest (to avoid puncture of the chest by the steering wheel). The seat belt keeps the person in a stable position against the expanding air bag.

PHYSICS APPLIED
Car safety—air bags

FIGURE 21 Example 11. An air bag deploying on impact.

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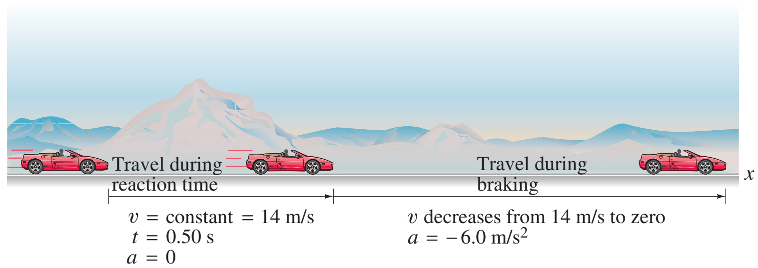


FIGURE 22 Example 12: stopping distance for a braking car.

PHYSICS APPLIED
Braking distances

EXAMPLE 12 **ESTIMATE** **Braking distances.** Estimate the minimum stopping distance for a car, which is important for traffic safety and traffic design. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time" during which the speed is constant, so $a = 0$. (2) The second time interval is the actual braking period when the vehicle slows down $(a \neq 0)$ and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. For a dry road and good tires, good brakes can decelerate a car at a rate of about $5\ \mathrm{m/s^2}$ to $8\ \mathrm{m/s^2}$. Calculate the total stopping distance for an initial velocity of 50 km/h (= 14 m/s ≈ 31 mi/h) and assume the acceleration of the car is $-6.0\ \mathrm{m/s^2}$ (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing). Reaction time for normal drivers varies from perhaps 0.3 s to about 1.0 s; take it to be 0.50 s.

Part 1: Reaction time

Known	Wanted
$t = 0.50\ \mathrm{s}$	x
$v_0 = 14\ \mathrm{m/s}$	
$v = 14\ \mathrm{m/s}$	
$a = 0$	
$x_0 = 0$	

Part 2: Braking

Known	Wanted
$x_0 = 7.0\ \mathrm{m}$	x
$v_0 = 14\ \mathrm{m/s}$	
$v = 0$	
$a = -6.0\ \mathrm{m/s^2}$	

APPROACH During the "reaction time," part (1), the car moves at constant speed of 14 m/s, so $a = 0$. Once the brakes are applied, part (2), the acceleration is $a = -6.0\ \mathrm{m/s^2}$ and is constant over this time interval. For both parts a is constant, so we can use Eqs. 12.

SOLUTION Part (1). We take $x_0 = 0$ for the first time interval, when the driver is reacting (0.50 s): the car travels at a constant speed of 14 m/s so $a = 0$. See Fig. 22 and the Table in the margin. To find x, the position of the car at $t = 0.50\ \mathrm{s}$ (when the brakes are applied), we cannot use Eq. 12c because x is multiplied by a, which is zero. But Eq. 12b works:

$$x = v_0 t + 0 = (14\ \mathrm{m/s})(0.50\ \mathrm{s}) = 7.0\ \mathrm{m}.$$

Thus the car travels 7.0 m during the driver's reaction time, until the instant the brakes are applied. We will use this result as input to part (2).

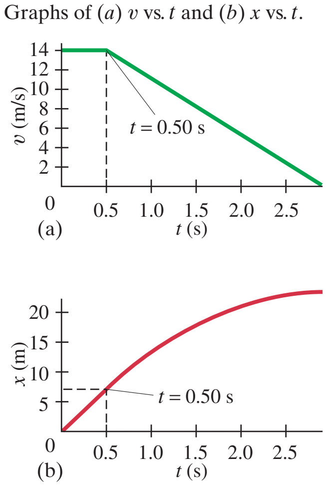
Part (2). During the second time interval, the brakes are applied and the car is brought to rest. The initial position is $x_0 = 7.0\ \mathrm{m}$ (result of part (1)), and other variables are shown in the second Table in the margin. Equation 12a doesn't contain x; Eq. 12b contains x but also the unknown t. Equation 12c, $v^2 - v_0^2 = 2a(x - x_0)$, is what we want; after setting $x_0 = 7.0\ \mathrm{m}$, we solve for x, the final position of the car (when it stops):

$$\begin{aligned} x &= x_0 + \frac{v^2 - v_0^2}{2a} \\ &= 7.0\ \mathrm{m} + \frac{0 - (14\ \mathrm{m/s})^2}{2(-6.0\ \mathrm{m/s^2})} = 7.0\ \mathrm{m} + \frac{-196\ \mathrm{m^2/s^2}}{-12\ \mathrm{m/s^2}} \\ &= 7.0\ \mathrm{m} + 16\ \mathrm{m} = 23\ \mathrm{m}. \end{aligned}$$

The car traveled 7.0 m while the driver was reacting and another 16 m during the braking period before coming to a stop, for a total distance traveled of 23 m. Figure 23 shows graphs of (*a*) v vs. t and (*b*) x vs. t.

NOTE From the equation above for x, we see that the stopping distance after the driver hit the brakes $(= x - x_0)$ increases with the *square* of the initial speed, not just linearly with speed. If you are traveling twice as fast, it takes four times the distance to stop.

FIGURE 23 Example 12. Graphs of (*a*) v vs. t and (*b*) x vs. t.



EXAMPLE 13 **ESTIMATE** **Two Moving Objects: Police and Speeder.** A car speeding at 150 km/h passes a still police car which immediately takes off in hot pursuit. Using simple assumptions, such as that the speeder continues at constant speed, estimate how long it takes the police car to overtake the speeder. Then estimate the police car's speed at that moment and decide if the assumptions were reasonable.

APPROACH When the police car takes off, it accelerates, and the simplest assumption is that its acceleration is constant. This may not be reasonable, but let's see what happens. We can estimate the acceleration if we have noticed automobile ads, which claim cars can accelerate from rest to 100 km/h in 5.0 s. So the average acceleration of the police car could be approximately

$$a_{\mathrm{P}} = \frac{100\,\mathrm{km/h}}{5.0\,\mathrm{s}} = 20\,\frac{\mathrm{km/h}}{\mathrm{s}}\left(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}\right)\left(\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right) = 5.6\,\mathrm{m/s^2}.$$

SOLUTION We need to set up the kinematic equations to determine the unknown quantities, and since there are two moving objects, we need two separate sets of equations. We denote the speeding car's position by x_{S} and the police car's position by x_{P}. Because we are interested in solving for the time when the two vehicles arrive at the same position on the road, we use Eq. 12b for each car:

$$x_{\mathrm{S}} = v_{0\mathrm{S}}t + \tfrac{1}{2}a_{\mathrm{S}}t^2 = (150\,\mathrm{km/h})t = (42\,\mathrm{m/s})t$$
$$x_{\mathrm{P}} = v_{0\mathrm{P}}t + \tfrac{1}{2}a_{\mathrm{P}}t^2 = \tfrac{1}{2}(5.6\,\mathrm{m/s^2})t^2,$$

where we have set $v_{0\mathrm{P}} = 0$ and $a_{\mathrm{S}} = 0$ (speeder assumed to move at constant speed). We want the time when the cars meet, so we set $x_{\mathrm{S}} = x_{\mathrm{P}}$ and solve for t:

$$(42\,\mathrm{m/s})t = (2.8\,\mathrm{m/s^2})t^2.$$

The solutions are

$$t = 0 \quad \text{and} \quad t = \frac{42\,\mathrm{m/s}}{2.8\,\mathrm{m/s^2}} = 15\,\mathrm{s}.$$

The first solution corresponds to the instant the speeder passed the police car. The second solution tells us when the police car catches up to the speeder, 15 s later. This is our answer, but is it reasonable? The police car's speed at $t = 15\,\mathrm{s}$ is

$$v_{\mathrm{P}} = v_{0\mathrm{P}} + a_{\mathrm{P}}t = 0 + (5.6\,\mathrm{m/s^2})(15\,\mathrm{s}) = 84\,\mathrm{m/s}$$

or 300 km/h ($\approx$ 190 mi/h). Not reasonable, and highly dangerous.

NOTE More reasonable is to give up the assumption of constant acceleration. The police car surely cannot maintain constant acceleration at those speeds. Also, the speeder, if a reasonable person, would slow down upon hearing the police siren. Figure 24 shows (a) x vs. t and (b) v vs. t graphs, based on the original assumption of $a_{\mathrm{P}} =$ constant, whereas (c) shows v vs. t for more reasonable assumptions.

CAUTION

Initial assumptions need to be checked out for reasonableness

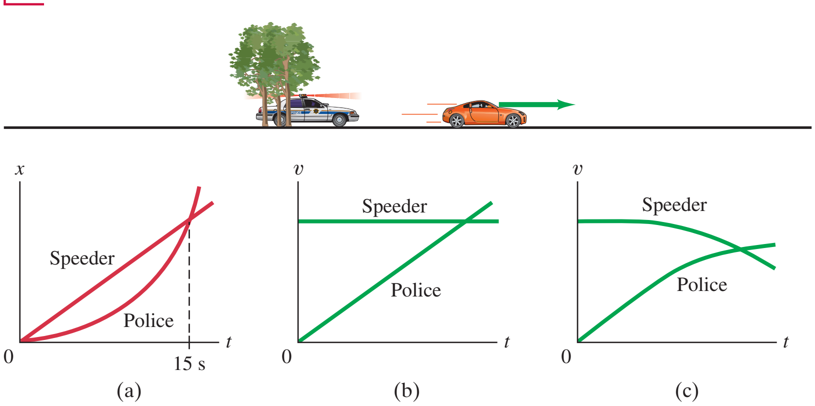
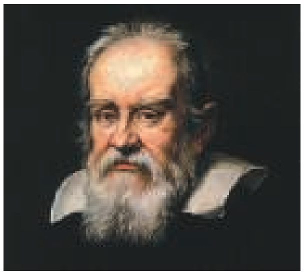


FIGURE 24 Example 13.

7 Freely Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed before the time of Galileo (Fig. 25), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is.



Justus Sustermans (1597–1681), "Portrait of Galileo Galilei," Galleria Palatina, Palazzo Pitti, Florence, Italy. Nimatallah/Art Resource, NY

FIGURE 25 Galileo Galilei (1564–1642).

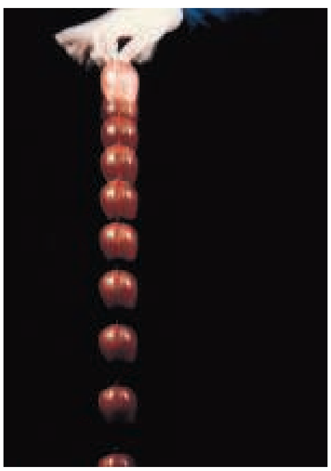
Galileo made use of his new technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that all objects would fall with the *same constant acceleration* in the absence of air or other resistance. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 26); that is, $d \propto t^2$. We can see this from Eq. 12b; but Galileo was the first to derive this mathematical relation.

CAUTION

A freely falling object increases in speed, but not *in proportion to its mass or weight*

To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

FIGURE 26 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.



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Galileo claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper horizontally in one hand and a heavier object—say, a baseball—in the other, and release them at the same time as in Fig. 27a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad (see Fig. 27b), you will find that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many ordinary circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 28). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the content of his science (astronomical discoveries, inertia, free fall) but also for his approach to science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

> **at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.**

We call this acceleration the **acceleration due to gravity** on the surface of the Earth, and we give it the symbol g. Its magnitude is approximately

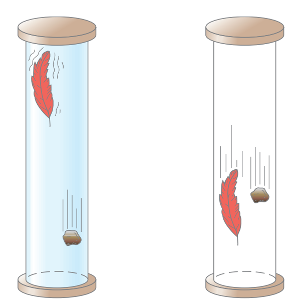
Acceleration due to gravity

$$g = 9.80\ \mathrm{m/s^2}. \qquad \text{[at surface of Earth]}$$

In British units g is about 32 ft/s^2. Actually, g varies slightly according to latitude and elevation, but these variations are so small that we will ignore them for most

(a) (b)

FIGURE 27 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.



Air-filled tube (a)

Evacuated tube (b)

FIGURE 28 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.

purposes. The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.† Acceleration due to gravity is a vector as is any acceleration, and its direction is downward, toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 12, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

PROBLEM SOLVING
You can choose y to be positive either up or down

EXERCISE H Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 14 Falling from a tower. Suppose that a ball is dropped $(v_0 = 0)$ from a tower 70.0 m high. How far will it have fallen after a time $t_1 = 1.00\,\text{s}$, $t_2 = 2.00\,\text{s}$, and $t_3 = 3.00\,\text{s}$? Ignore air resistance.

APPROACH Let us take y as positive downward, so the acceleration is $a = g = +9.80\,\text{m/s}^2$. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position y of the ball after three different time intervals. Equation 12b, with x replaced by y, relates the given quantities (t, a, and v_0) to the unknown y.

SOLUTION We set $t = t_1 = 1.00\,\text{s}$ in Eq. 12b:

$$y_1 = v_0 t_1 + \tfrac{1}{2}at_1^2 = 0 + \tfrac{1}{2}at_1^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 4.90\,\text{m}.$$

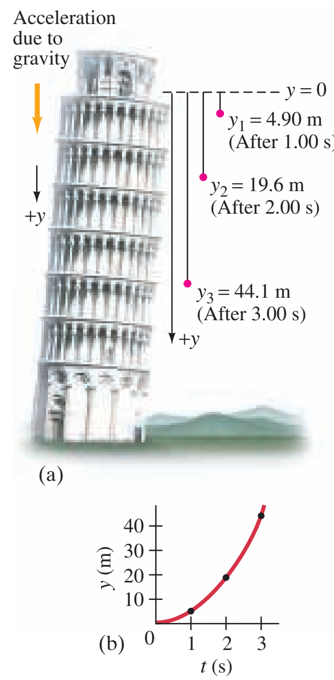
The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\,\text{s}$. Similarly, after 2.00 s $(= t_2)$, the ball's position is

$$y_2 = \tfrac{1}{2}at_2^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 19.6\,\text{m}.$$

Finally, after 3.00 s $(= t_3)$, the ball's position is (see Fig. 29)

$$y_3 = \tfrac{1}{2}at_3^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(3.00\,\text{s})^2 = 44.1\,\text{m}.$$

FIGURE 29 Example 14. (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 26.) (b) Graph of y vs. t.



EXAMPLE 15 Thrown down from a tower. Suppose the ball in Example 14 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

APPROACH Again we use Eq. 12b, but now v_0 is not zero, it is $v_0 = 3.00\,\text{m/s}$.

SOLUTION (*a*) At $t = 1.00\,\text{s}$, the position of the ball as given by Eq. 12b is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\text{m/s})(1.00\,\text{s}) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 7.90\,\text{m}.$$

At $t = 2.00\,\text{s}$, (time interval $t = 0$ to $t = 2.00\,\text{s}$), the position is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\text{m/s})(2.00\,\text{s}) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 25.6\,\text{m}.$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.
(*b*) The velocity is obtained from Eq. 12a:

$$\begin{aligned} v &= v_0 + at \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 12.8\,\text{m/s} \quad [\text{at } t_1 = 1.00\,\text{s}] \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 22.6\,\text{m/s}. \quad [\text{at } t_2 = 2.00\,\text{s}] \end{aligned}$$

In Example 14, when the ball was dropped $(v_0 = 0)$, the first term (v_0) in these equations was zero, so

$$\begin{aligned} v &= 0 + at \\ &= (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 9.80\,\text{m/s} \qquad [\text{at } t_1 = 1.00\,\text{s}] \\ &= (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 19.6\,\text{m/s}. \qquad [\text{at } t_2 = 2.00\,\text{s}] \end{aligned}$$

NOTE For both Examples 14 and 15, the speed increases linearly in time by 9.80 m/s during each second. But the speed of the downwardly thrown ball at any instant is always 3.00 m/s (its initial speed) higher than that of a dropped ball.

†The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity** due to air resistance.

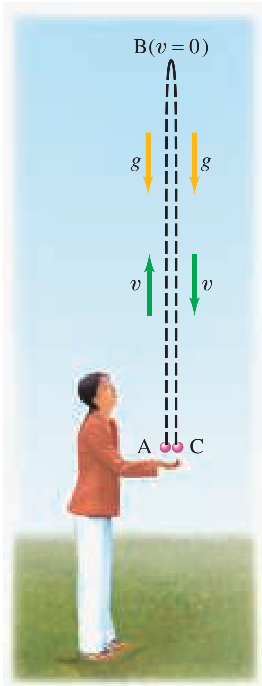


FIGURE 30 An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 16, 17, 18, and 19.

EXAMPLE 16 Ball thrown upward, I. A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate (a) how high it goes, and (b) how long the ball is in the air before it comes back to the hand. Ignore air resistance.

APPROACH We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 30) and until it comes back to the hand again. Let us choose y to be positive in the upward direction and negative in the downward direction. (This is a different convention from that used in Examples 14 and 15, and so illustrates our options.) The acceleration due to gravity is downward and so will have a negative sign, $a = -g = -9.80\ \mathrm{m/s^2}$. As the ball rises, its speed decreases until it reaches the highest point (B in Fig. 30), where its speed is zero for an instant; then it descends, with increasing speed.

SOLUTION (a) We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 30) we have $y_0 = 0$, $v_0 = 15.0\ \mathrm{m/s}$, and $a = -9.80\ \mathrm{m/s^2}$. At time t (maximum height), $v = 0$, $a = -9.80\ \mathrm{m/s^2}$, and we wish to find y. We use Eq. 12c, replacing x with y: $v^2 = v_0^2 + 2ay$. We solve this equation for y:

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\ \mathrm{m/s})^2}{2(-9.80\ \mathrm{m/s^2})} = 11.5\ \mathrm{m}.$$

The ball reaches a height of 11.5 m above the hand.

(b) Now we need to choose a different time interval to calculate how long the ball is in the air before it returns to the hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 30) in one step and use Eq. 12b. We can do this because y represents position or displacement, and not the total distance traveled. Thus, at both points A and C, $y = 0$. We use Eq. 12b with $a = -9.80\ \mathrm{m/s^2}$ and find

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$0 = 0 + (15.0\ \mathrm{m/s})t + \tfrac{1}{2}(-9.80\ \mathrm{m/s^2})t^2.$$

This equation is readily factored (we factor out one t):

$$(15.0\ \mathrm{m/s} - 4.90\ \mathrm{m/s^2}\, t)t = 0.$$

There are two solutions:

$$t = 0 \quad \text{and} \quad t = \frac{15.0\ \mathrm{m/s}}{4.90\ \mathrm{m/s^2}} = 3.06\ \mathrm{s}.$$

The first solution ($t = 0$) corresponds to the initial point (A) in Fig. 30, when the ball was first thrown from $y = 0$. The second solution, $t = 3.06\ \mathrm{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

NOTE We have ignored air resistance, which could be significant, so our result is only an approximation to a real, practical situation.

We did not consider the throwing action in this Example. Why? Because during the throw, the thrower's hand is touching the ball and accelerating the ball at a rate unknown to us—the acceleration is *not* g. We consider only the time when the ball is in the air and the acceleration is equal to g.

CAUTION

Quadratic equations have two solutions. Sometimes only one corresponds to reality, sometimes both

Every quadratic equation (where the variable is squared) mathematically produces two solutions. In physics, sometimes only one solution corresponds to the real situation, as in Example 10, in which case we ignore the "unphysical" solution. But in Example 16, both solutions to our equation in t^2 are physically meaningful: $t = 0$ and $t = 3.06\ \mathrm{s}$.

CONCEPTUAL EXAMPLE 17 **Two possible misconceptions.** Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 30).

RESPONSE Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Example 16 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 30), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc points upward, then becomes zero (for zero time) at the highest point, and then points downward. Gravity does not stop acting, so $a = -g = -9.80\,\mathrm{m/s^2}$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration (= rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. In sum, the acceleration of gravity always points down toward the Earth, even when the object is moving up.

CAUTION

(1) Velocity and acceleration are not always in the same direction; the acceleration (of gravity) always points down

(2) $a \neq 0$ even at the highest point of a trajectory

EXAMPLE 18 **Ball thrown upward, II.** Let us consider again the ball thrown upward of Example 16, and make more calculations. Calculate (*a*) how much time it takes for the ball to reach the maximum height (point B in Fig. 30), and (*b*) the velocity of the ball when it returns to the thrower's hand (point C).

APPROACH Again we assume the acceleration is constant, so we can use Eqs. 12. We have the height of 11.5 m from Example 16. Again we take y as positive upward.

SOLUTION (*a*) We consider the time interval between the throw $(t = 0,$ $v_0 = 15.0\,\mathrm{m/s})$ and the top of the path $(y = +11.5\,\mathrm{m},\ v = 0)$, and we want to find t. The acceleration is constant at $a = -g = -9.80\,\mathrm{m/s^2}$. Both Eqs. 12a and 12b contain the time t with other quantities known. Let us use Eq. 12a with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, and $v = 0$:

$$v = v_0 + at;$$

setting $v = 0$ and solving for t gives

$$t = -\frac{v_0}{a} = -\frac{15.0\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 1.53\,\mathrm{s}.$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in part (*b*) of Example 16]. Thus it takes the same time to reach the maximum height as to fall back to the starting point.

(*b*) Now we consider the time interval from the throw $(t = 0,\ v_0 = 15.0\,\mathrm{m/s})$ until the ball's return to the hand, which occurs at $t = 3.06\,\mathrm{s}$ (as calculated in Example 16), and we want to find v when $t = 3.06\,\mathrm{s}$:

$$v = v_0 + at = 15.0\,\mathrm{m/s} - (9.80\,\mathrm{m/s^2})(3.06\,\mathrm{s}) = -15.0\,\mathrm{m/s}.$$

NOTE The ball has the same speed (magnitude of velocity) when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). And, as we saw in part (*a*), the time is the same up as down. Thus the motion is *symmetrical* about the maximum height.

The acceleration of objects such as rockets and fast airplanes is often given as a multiple of $g = 9.80\,\mathrm{m/s^2}$. For example, a plane pulling out of a dive and undergoing 3.00 g's would have an acceleration of $(3.00)(9.80\,\mathrm{m/s^2}) = 29.4\,\mathrm{m/s^2}$.

EXERCISE I If a car is said to accelerate at 0.50 g, what is its acceleration in $\mathrm{m/s^2}$?

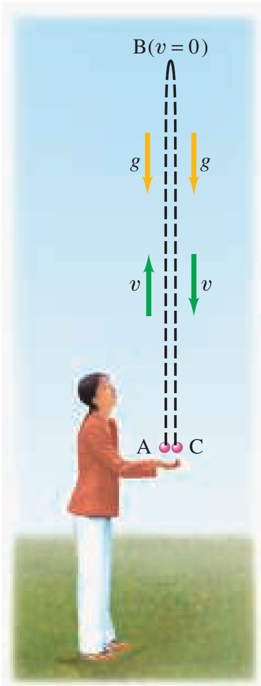


FIGURE 30
(Repeated for Example 19)

EXAMPLE 19 **Ball thrown upward, III; the quadratic formula.** For the ball in Example 18, calculate at what time t the ball passes a point 8.00 m above the person's hand. (See repeated Fig. 30 here).

APPROACH We choose the time interval from the throw $(t = 0,\ v_0 = 15.0\ \text{m/s})$ until the time t (to be determined) when the ball is at position $y = 8.00\ \text{m}$, using Eq. 12b.

SOLUTION We want to find t, given $y = 8.00\ \text{m}$, $y_0 = 0$, $v_0 = 15.0\ \text{m/s}$, and $a = -9.80\ \text{m/s}^2$. We use Eq. 12b:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

$$8.00\ \text{m} = 0 + (15.0\ \text{m/s})t + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2.$$

To solve any quadratic equation of the form $at^2 + bt + c = 0$, where a, b, and c are constants (a is *not* acceleration here), we use the **quadratic formula**:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our y equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\ \text{m/s}^2)t^2 - (15.0\ \text{m/s})t + (8.00\ \text{m}) = 0.$$

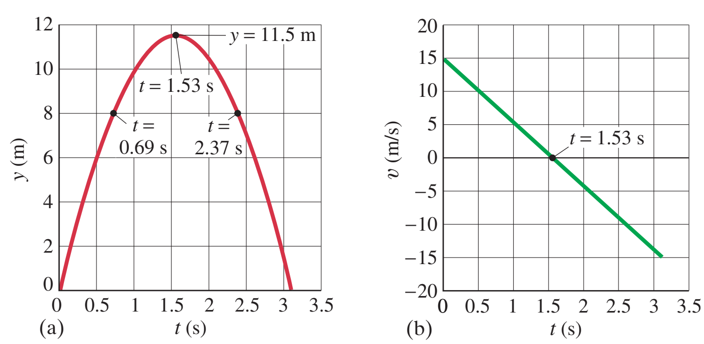
So the coefficient a is $4.90\ \text{m/s}^2$, b is $-15.0\ \text{m/s}$, and c is 8.00 m. Putting these into the quadratic formula, we obtain

$$t = \frac{15.0\ \text{m/s} \pm \sqrt{(15.0\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(8.00\ \text{m})}}{2(4.90\ \text{m/s}^2)},$$

which gives us $t = 0.69\ \text{s}$ and $t = 2.37\ \text{s}$. Are both solutions valid? Yes, because the ball passes $y = 8.00\ \text{m}$ when it goes up $(t = 0.69\ \text{s})$ and again when it comes down $(t = 2.37\ \text{s})$.

NOTE Figure 31 shows graphs of (a) y vs. t and (b) v vs. t for the ball thrown upward in Fig. 30, incorporating the results of Examples 16, 18, and 19.

FIGURE 31 Graphs of (a) y vs. t, (b) v vs. t for a ball thrown upward, Examples 16, 18, and 19.



EXAMPLE 20 **Ball thrown upward at edge of cliff.** Suppose that the person of Examples 16, 18, and 19 is standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below as in Fig. 32. (a) How long does it take the ball to reach the base of the cliff? (b) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

APPROACH We again use Eq. 12b, but this time we set $y = -50.0\ \text{m}$, the bottom of the cliff, which is 50.0 m below the initial position $(y_0 = 0)$.

SOLUTION (*a*) We use Eq. 12b with $a = -9.80\,\text{m/s}^2$, $v_0 = 15.0\,\text{m/s}$, $y_0 = 0$, and $y = -50.0\,\text{m}$:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$-50.0\,\text{m} = 0 + (15.0\,\text{m/s})t - \tfrac{1}{2}(9.80\,\text{m/s}^2)t^2.$$

Rewriting in the standard form we have

$$(4.90\,\text{m/s}^2)t^2 - (15.0\,\text{m/s})t - (50.0\,\text{m}) = 0.$$

Using the quadratic formula, we find as solutions $t = 5.07\,\text{s}$ and $t = -2.01\,\text{s}$. The first solution, $t = 5.07\,\text{s}$, is the answer we are seeking: the time it takes the ball to rise to its highest point and then fall to the base of the cliff. To rise and fall back to the top of the cliff took 3.06 s (Example 16); so it took an additional 2.01 s to fall to the base. But what is the meaning of the other solution, $t = -2.01\,\text{s}$? This is a time before the throw, when our calculation begins, so it isn't relevant here.†

(*b*) From Example 16, the ball moves up 11.5 m, falls 11.5 m back down to the top of the cliff, and then down another 50.0 m to the base of the cliff, for a total distance traveled of 73.0 m. Note that the *displacement*, however, was −50.0 m. Figure 33 shows the y vs. t graph for this situation.

EXERCISE J Two balls are thrown from a cliff. One is thrown directly up, the other directly down, each with the same initial speed, and both hit the ground below the cliff. Which ball hits the ground at the greater speed: (*a*) the ball thrown upward, (*b*) the ball thrown downward, or (*c*) both the same? Ignore air resistance.

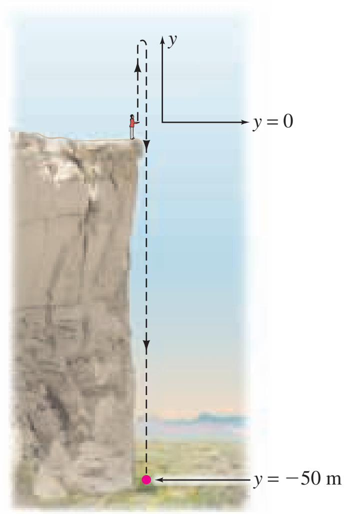
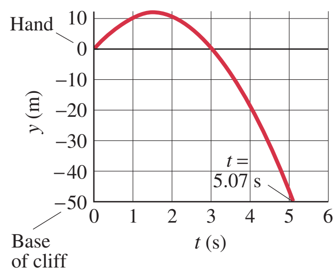


FIGURE 32 Example 20. The person in Fig. 30 stands on the edge of a cliff. The ball falls to the base of the cliff, 50.0 m below.

FIGURE 33 Example 20, the y vs. t graph.



*8 Variable Acceleration; Integral Calculus

In this brief optional Section we use integral calculus to derive the kinematic equations for constant acceleration, Eqs. 12a and b. We also show how calculus can be used when the acceleration is not constant. If you have not yet studied simple integration in your calculus course, you may want to postpone reading this Section until you have.

First we derive Eq. 12a, assuming as we did in Section 5 that an object has velocity v_0 at $t = 0$ and a constant acceleration a. We start with the definition of instantaneous acceleration, $a = dv/dt$, which we rewrite as

$$dv = a\,dt.$$

We take the definite integral of both sides of this equation, using the same notation we did in Section 5:

$$\int_{v=v_0}^{v} dv = \int_{t=0}^{t} a\,dt$$

which gives, since a = constant,

$$v - v_0 = at.$$

This is Eq. 12a, $v = v_0 + at$.

Next we derive Eq. 12b starting with the definition of instantaneous velocity, Eq. 4, $v = dx/dt$. We rewrite this as

$$dx = v\,dt$$

or

$$dx = (v_0 + at)\,dt$$

where we substituted in Eq. 12a.

†The solution $t = -2.01\,\text{s}$ could be meaningful in a different physical situation. Suppose that a person standing on top of a 50.0-m-high cliff sees a rock pass by him at $t = 0$ moving upward at 15.0 m/s; at what time did the rock leave the base of the cliff, and when did it arrive back at the base of the cliff? The equations will be precisely the same as for our original Example, and the answers $t = -2.01\,\text{s}$ and $t = 5.07\,\text{s}$ will be the correct answers. Note that we cannot put all the information for a problem into the mathematics, so we have to use common sense in interpreting results.

Now we integrate:

$$\int_{x=x_0}^{x} dx = \int_{t=0}^{t} (v_0 + at)\,dt$$

$$x - x_0 = \int_{t=0}^{t} v_0\,dt + \int_{t=0}^{t} at\,dt$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

since v_0 and a are constants. This result is just Eq. 12b, $x = x_0 + v_0 t + \frac{1}{2}at^2$.

Finally let us use calculus to find velocity and displacement, given an acceleration that is not constant but varies in time.

EXAMPLE 21 **Integrating a time-varying acceleration.** An experimental vehicle starts from rest $(v_0 = 0)$ at $t = 0$ and accelerates at a rate given by $a = (7.00\,\mathrm{m/s^3})t$. What is (*a*) its velocity and (*b*) its displacement 2.00 s later?

APPROACH We cannot use Eqs. 12 because a is not constant. We integrate the acceleration $a = dv/dt$ over time to find v as a function of time; and then integrate $v = dx/dt$ to get the displacement.

SOLUTION From the definition of acceleration, $a = dv/dt$, we have

$$dv = a\,dt.$$

We take the integral of both sides from $v = 0$ at $t = 0$ to velocity v at an arbitrary time t:

$$\int_0^v dv = \int_0^t a\,dt$$

$$v = \int_0^t (7.00\,\mathrm{m/s^3})t\,dt$$

$$= (7.00\,\mathrm{m/s^3})\left(\frac{t^2}{2}\right)\Bigg|_0^t = (7.00\,\mathrm{m/s^3})\left(\frac{t^2}{2} - 0\right) = (3.50\,\mathrm{m/s^3})t^2.$$

At $t = 2.00\,\mathrm{s}$, $v = (3.50\,\mathrm{m/s^3})(2.00\,\mathrm{s})^2 = 14.0\,\mathrm{m/s}$.

(*b*) To get the displacement, we assume $x_0 = 0$ and start with $v = dx/dt$ which we rewrite as $dx = v\,dt$. Then we integrate from $x = 0$ at $t = 0$ to position x at time t:

$$\int_0^x dx = \int_0^t v\,dt$$

$$x = \int_0^{2.00\,\mathrm{s}} (3.50\,\mathrm{m/s^3})t^2\,dt = (3.50\,\mathrm{m/s^3})\frac{t^3}{3}\Bigg|_0^{2.00\,\mathrm{s}} = 9.33\,\mathrm{m}.$$

In sum, at $t = 2.00\,\mathrm{s}$, $v = 14.0\,\mathrm{m/s}$ and $x = 9.33\,\mathrm{m}$.

*9 Graphical Analysis and Numerical Integration

This Section is optional. It discusses how to solve certain Problems numerically, often needing a computer to do the sums.

If we are given the velocity v of an object as a function of time t, we can obtain the displacement, x. Suppose the velocity as a function of time, $v(t)$, is given as a graph (rather than as an equation that could be integrated as discussed in Section 8), as shown in Fig 34a. If we are interested in the time interval from t_1 to t_2, as shown, we divide the time axis into many small subintervals, $\Delta t_1, \Delta t_2, \Delta t_3, \ldots$, which are indicated by the dashed vertical lines. For each subinterval, a horizontal dashed line is drawn to indicate the average velocity during that time interval. The displacement during any subinterval is given by Δx_i, where the subscript i represents the particular subinterval

$(i = 1, 2, 3, \ldots)$. From the definition of average velocity (Eq. 2) we have

$$\Delta x_i = \bar{v}_i \, \Delta t_i .$$

Thus the displacement during each subinterval equals the product of $\bar{v}_i$ and Δt_i, and equals the area of the dark rectangle in Fig. 34a for that subinterval. The total displacement between times t_1 and t_2 is the sum of the displacements over all the subintervals:

$$x_2 - x_1 = \sum_{t_1}^{t_2} \bar{v}_i \, \Delta t_i , \qquad \textbf{(13a)}$$

where x_1 is the position at t_1 and x_2 is the position at t_2. This sum equals the area of all the rectangles shown.

It is often difficult to estimate $\bar{v}_i$ with precision for each subinterval from the graph. We can get greater accuracy in our calculation of $x_2 - x_1$ by breaking the interval $t_2 - t_1$ into more, but narrower, subintervals. Ideally, we can let each Δt_i approach zero, so we approach (in principle) an infinite number of subintervals. In this limit the area of all these infinitesimally thin rectangles becomes exactly equal to the area under the curve (Fig. 34b). Thus *the total displacement between any two times is equal to the area between the velocity curve and the t axis between the two times t_1 and t_2*. This limit can be written

$$x_2 - x_1 = \lim_{\Delta t \to 0} \sum_{t_1}^{t_2} \bar{v}_i \, \Delta t_i$$

or, using standard calculus notation,

$$x_2 - x_1 = \int_{t_1}^{t_2} v(t) \, dt . \qquad \textbf{(13b)}$$

We have let $\Delta t \rightarrow 0$and renamed it *dt* to indicate that it is now infinitesimally small. The average velocity, $\bar{v}$, over an infinitesimal time *dt* is the instantaneous velocity at that instant, which we have written $v(t)$ to remind us that v is a function of t. The symbol $\int$ is an elongated S and indicates a sum over an infinite number of infinitesimal subintervals. We say that we are taking the *integral* of $v(t)$ over *dt* from time t_1 to time t_2, and this is equal to the area between the $v(t)$ curve and the t axis between the times t_1 and t_2 (Fig. 34b). The integral in Eq. 13b is a *definite integral*, since the limits t_1 and t_2 are specified.

Similarly, if we know the acceleration as a function of time, we can obtain the velocity by the same process. We use the definition of average acceleration (Eq. 5) and solve for Δv:

$$\Delta v = \bar{a} \, \Delta t .$$

If a is known as a function of t over some time interval t_1 to t_2, we can subdivide this time interval into many subintervals, Δt_i, just as we did in Fig. 34a. The change in velocity during each subinterval is $\Delta v_i = \bar{a}_i \, \Delta t_i$. The total change in velocity from time t_1 until time t_2 is

$$v_2 - v_1 = \sum_{t_1}^{t_2} \bar{a}_i \, \Delta t_i , \qquad \textbf{(14a)}$$

where v_2 represents the velocity at t_2 and v_1 the velocity at t_1. This relation can be written as an integral by letting $\Delta t \rightarrow 0$(the number of intervals then approaches infinity)

$$v_2 - v_1 = \lim_{\Delta t \to 0} \sum_{t_1}^{t_2} \bar{a}_i \, \Delta t_i$$

or

$$v_2 - v_1 = \int_{t_1}^{t_2} a(t) \, dt . \qquad \textbf{(14b)}$$

Equations 14 will allow us to determine the velocity v_2 at some time t_2 if the velocity is known at t_1 and a is known as a function of time.

If the acceleration or velocity is known at discrete intervals of time, we can use the summation forms of the above equations, Eqs. 13a and 14a, to estimate velocity or displacement. This technique is known as **numerical integration**. We now take an Example that can also be evaluated analytically, so we can compare the results.

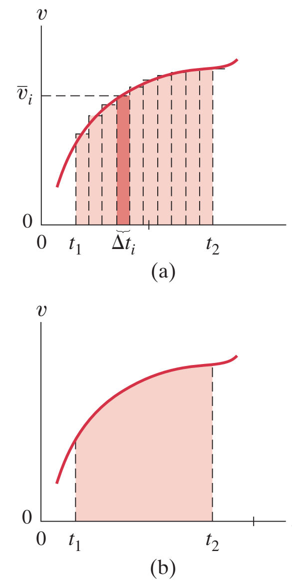


FIGURE 34 Graph of v vs. t for the motion of a particle. In (a), the time axis is broken into subintervals of width Δt_i, the average velocity during each Δt_i is $\bar{v}_i$, and the area of all the rectangles, $\Sigma \bar{v}_i \, \Delta t_i$, is numerically equal to the total displacement $(x_2 - x_1)$ during the total time $(t_2 - t_1)$. In (b), $\Delta t_i \rightarrow 0$ and the area under the curve is equal to $(x_2 - x_1)$.

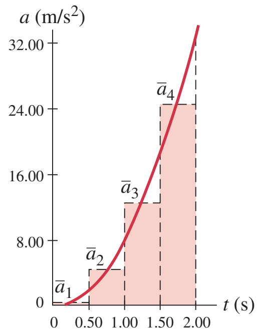


FIGURE 35 Example 22.

EXAMPLE 22 **Numerical integration.** An object starts from rest at $t = 0$ and accelerates at a rate $a(t) = (8.00\,\text{m/s}^4)t^2$. Determine its velocity after 2.00 s using numerical methods.

APPROACH Let us first divide up the interval $t = 0.00\,\text{s}$ to $t = 2.00\,\text{s}$ into four subintervals each of duration $\Delta t_i = 0.50\,\text{s}$ (Fig. 35). We use Eq. 14a with $v_2 = v$, $v_1 = 0$, $t_2 = 2.00\,\text{s}$, and $t_1 = 0$. For each of the subintervals we need to estimate $\bar{a}_i$. There are various ways to do this and we use the simple method of choosing $\bar{a}_i$ to be the acceleration $a(t)$ at the midpoint of each interval (an even simpler but usually less accurate procedure would be to use the value of a at the start of the subinterval). That is, we evaluate $a(t) = (8.00\,\text{m/s}^4)t^2$ at $t = 0.25\,\text{s}$ (which is midway between 0.00 s and 0.50 s), 0.75 s, 1.25 s, and 1.75 s.

SOLUTION The results are as follows:

i	1	2	3	4
$\bar{a}_i(\text{m/s}^2)$	0.50	4.50	12.50	24.50

Now we use Eq. 14a, and note that all Δt_i equal 0.50 s (so they can be factored out):

$$\begin{aligned} v(t = 2.00\,\text{s}) &= \sum_{t=0}^{t=2.00\,\text{s}} \bar{a}_i\,\Delta t_i \\ &= (0.50\,\text{m/s}^2 + 4.50\,\text{m/s}^2 + 12.50\,\text{m/s}^2 + 24.50\,\text{m/s}^2)(0.50\,\text{s}) \\ &= 21.0\,\text{m/s}. \end{aligned}$$

We can compare this result to the analytic solution given by Eq. 14b since the functional form for a is integrable analytically:

$$\begin{aligned} v &= \int_0^{2.00\,\text{s}} (8.00\,\text{m/s}^4)t^2\,dt = \frac{8.00\,\text{m/s}^4}{3}\,t^3\bigg|_0^{2.00\,\text{s}} \\ &= \frac{8.00\,\text{m/s}^4}{3}\left[(2.00\,\text{s})^3 - (0)^3\right] = 21.33\,\text{m/s} \end{aligned}$$

or 21.3 m/s to the proper number of significant figures. This analytic solution is precise, and we see that our numerical estimate is not far off even though we only used four Δt intervals. It may not be close enough for purposes requiring high accuracy. If we use more and smaller subintervals, we will get a more accurate result. If we use 10 subintervals, each with $\Delta t = 2.00\,\text{s}/10 = 0.20\,\text{s}$, we have to evaluate $a(t)$ at $t = 0.10\,\text{s}, 0.30\,\text{s}, \ldots, 1.90\,\text{s}$ to get the $\bar{a}_i$, and these are as follows:

i	1	2	3	4	5	6	7	8	9	10
$\bar{a}_i(\text{m/s}^2)$	0.08	0.72	2.00	3.92	6.48	9.68	13.52	18.00	23.12	28.88

Then, from Eq. 14a we obtain

$$\begin{aligned} v(t = 2.00\,\text{s}) &= \sum \bar{a}_i\,\Delta t_i = \left(\sum \bar{a}_i\right)(0.200\,\text{s}) \\ &= (106.4\,\text{m/s}^2)(0.200\,\text{s}) = 21.28\,\text{m/s}, \end{aligned}$$

where we have kept an extra significant figure to show that this result is much closer to the (precise) analytic one but still is not quite identical to it. The percentage difference has dropped from 1.4% $(0.3\,\text{m/s}^2/21.3\,\text{m/s}^2)$ for the four-subinterval computation to only 0.2% (0.05/21.3) for the 10-subinterval one.

In the Example above we were given an analytic function that was integrable, so we could compare the accuracy of the numerical calculation to the known precise one. But what do we do if the function is not integrable, so we can't compare our numerical result to an analytic one? That is, how do we know if we've taken enough subintervals so that we can trust our calculated estimate to be accurate to within some desired uncertainty, say 1 percent? What we can do is compare two successive numerical calculations: the first done with n subintervals and the second with, say, twice as many subintervals ($2n$). If the two results are within the desired uncertainty (say 1 percent), we can usually assume that the calculation with more subintervals is within the desired uncertainty of the true value. If the two calculations are not that close, then a third calculation, with more subintervals (maybe double, maybe 10 times as many, depending on how good the previous approximation was) must be done, and compared to the previous one.

The procedure is easy to automate using a computer spreadsheet application.

If we wanted to also obtain the displacement x at some time, we would have to do a second numerical integration over v, which means we would first need to calculate v for many different times. Programmable calculators and computers are very helpful for doing the long sums.

Summary

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame**.

The **displacement** of an object is the change in position of the object.

Average speed is the distance traveled divided by the elapsed time or time interval, Δt, the time period over which we choose to make our observations. An object's **average velocity** over a particular time interval Δt is its displacement Δx during that time interval, divided by Δt:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \qquad (2)$$

The **instantaneous velocity**, whose magnitude is the same as the *instantaneous speed*, is defined as the average velocity taken over an infinitesimally short time interval $(\Delta t \rightarrow 0)$:

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}, \qquad (4)$$

where dx/dt is the derivative of x with respect to t.

On a graph of position vs. time, the *slope* is equal to the instantaneous velocity.

Acceleration is the change of velocity per unit time. An object's **average acceleration** over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t}, \qquad (5)$$

where Δv is the change of velocity during the time interval Δt.

Instantaneous acceleration is the average acceleration taken over an infinitesimally short time interval:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}. \qquad (6)$$

If an object moves in a straight line with *constant acceleration*, the velocity v and position x are related to the acceleration a, the elapsed time t, the initial position x_0, and the initial velocity v_0 by Eqs. 12:

$$v = v_0 + at, \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2,$$
$$v^2 = v_0^2 + 2a(x - x_0), \qquad \bar{v} = \frac{v + v_0}{2}. \qquad (12)$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward **acceleration due to gravity**, whose magnitude is $g = 9.80\ \mathrm{m/s^2}$ if air resistance can be ignored.

[*The kinematic Equations 12 can be derived using integral calculus.]

Answers to Exercises

A: -30 cm; 50 cm.

B: (*a*).

C: (*b*).

D: (*b*).

E: (*a*) +; (*b*) −; (*c*) −; (*d*) +.

F: (*c*).

G: (*b*).

H: (*e*).

I: $4.9\ \mathrm{m/s^2}$.

J: (*c*).

Describing Motion: Kinematics in One Dimension Problem Set

Questions

1. Does a car speedometer measure speed, velocity, or both?
2. Can an object have a varying speed if its velocity is constant? Can it have varying velocity if its speed is constant? If yes, give examples in each case.
3. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant?
4. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain, using examples.
5. Compare the acceleration of a motorcycle that accelerates from 80 km/h to 90 km/h with the acceleration of a bicycle that accelerates from rest to 10 km/h in the same time.
6. Can an object have a northward velocity and a southward acceleration? Explain.
7. Can the velocity of an object be negative when its acceleration is positive? What about vice versa?
8. Give an example where both the velocity and acceleration are negative.
9. Two cars emerge side by side from a tunnel. Car A is traveling with a speed of 60 km/h and has an acceleration of 40 km/h/min. Car B has a speed of 40 km/h and has an acceleration of 60 km/h/min. Which car is passing the other as they come out of the tunnel? Explain your reasoning.
10. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.
11. A baseball player hits a ball straight up into the air. It leaves the bat with a speed of 120 km/h. In the absence of air resistance, how fast would the ball be traveling when the catcher catches it?
12. As a freely falling object speeds up, what is happening to its acceleration—does it increase, decrease, or stay the same? (*a*) Ignore air resistance. (*b*) Consider air resistance.
13. You travel from point A to point B in a car moving at a constant speed of 70 km/h. Then you travel the same distance from point B to another point C, moving at a constant speed of 90 km/h. Is your average speed for the entire trip from A to C 80 km/h? Explain why or why not.
14. Can an object have zero velocity and nonzero acceleration at the same time? Give examples.
15. Can an object have zero acceleration and nonzero velocity at the same time? Give examples.
16. Which of these motions is *not* at constant acceleration: a rock falling from a cliff, an elevator moving from the second floor to the fifth floor making stops along the way, a dish resting on a table?
17. In a lecture demonstration, a 3.0-m-long vertical string with ten bolts tied to it at equal intervals is dropped from the ceiling of the lecture hall. The string falls on a tin plate, and the class hears the clink of each bolt as it hits the plate. The sounds will not occur at equal time intervals. Why? Will the time between clinks increase or decrease near the end of the fall? How could the bolts be tied so that the clinks occur at equal intervals?
18. Describe in words the motion plotted in Fig. 36 in terms of v, a, etc. [*Hint*: First try to duplicate the motion plotted by walking or moving your hand.]

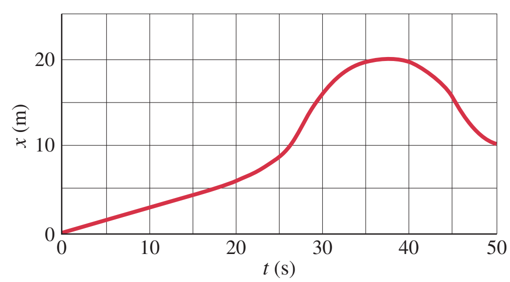


FIGURE 36 Question 18, Problems 9 and 86.

19. Describe in words the motion of the object graphed in Fig. 37.

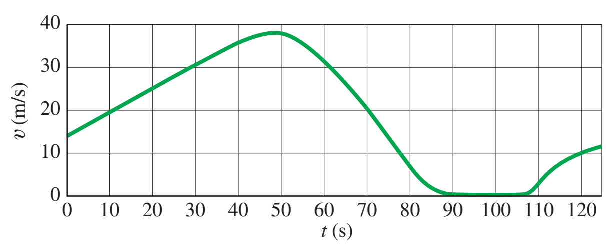


FIGURE 37 Question 19, Problem 23.

Problems

[The Problems in this Section are ranked I, II, or III according to estimated difficulty, with (I) Problems being easiest. Level (III) Problems are meant mainly as a challenge for the best students, for "extra credit." The Problems are arranged by Sections, meaning that the reader should have read up to and including that Section, but this Chapter also has a group of General Problems that are not arranged by Section and not ranked.]

1 to 3 Speed and Velocity

1. (I) If you are driving 110 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?
2. (I) What must your car's average speed be in order to travel 235 km in 3.25 h?

3. (I) A particle at $t_1 = -2.0\,\text{s}$ is at $x_1 = 4.3\,\text{cm}$ and at $t_2 = 4.5\,\text{s}$ is at $x_2 = 8.5\,\text{cm}$. What is its average velocity? Can you calculate its average speed from these data?

4. (I) A rolling ball moves from $x_1 = 3.4\,\text{cm}$ to $x_2 = -4.2\,\text{cm}$ during the time from $t_1 = 3.0\,\text{s}$ to $t_2 = 5.1\,\text{s}$. What is its average velocity?

5. (II) According to a rule-of-thumb, every five seconds between a lightning flash and the following thunder gives the distance to the flash in miles. Assuming that the flash of light arrives in essentially no time at all, estimate the speed of sound in m/s from this rule. What would be the rule for kilometers?

6. (II) You are driving home from school steadily at 95 km/h for 130 km. It then begins to rain and you slow to 65 km/h. You arrive home after driving 3 hours and 20 minutes. (*a*) How far is your hometown from school? (*b*) What was your average speed?

7. (II) A horse canters away from its trainer in a straight line, moving 116 m away in 14.0 s. It then turns abruptly and gallops halfway back in 4.8 s. Calculate (*a*) its average speed and (*b*) its average velocity for the entire trip, using "away from the trainer" as the positive direction.

8. (II) The position of a small object is given by $x = 34 + 10t - 2t^3$, where t is in seconds and x in meters. (*a*) Plot x as a function of t from $t = 0$ to $t = 3.0\,\text{s}$. (*b*) Find the average velocity of the object between 0 and 3.0 s. (*c*) At what time between 0 and 3.0 s is the instantaneous velocity zero?

9. (II) The position of a rabbit along a straight tunnel as a function of time is plotted in Fig. 36. What is its instantaneous velocity (*a*) at $t = 10.0\,\text{s}$ and (*b*) at $t = 30.0\,\text{s}$? What is its average velocity (*c*) between $t = 0$ and $t = 5.0\,\text{s}$, (*d*) between $t = 25.0\,\text{s}$ and $t = 30.0\,\text{s}$, and (*e*) between $t = 40.0\,\text{s}$ and $t = 50.0\,\text{s}$?

10. (II) On an audio compact disc (CD), digital bits of information are encoded sequentially along a spiral path. Each bit occupies about 0.28 μm. A CD player's readout laser scans along the spiral's sequence of bits at a constant speed of about 1.2 m/s as the CD spins. (*a*) Determine the number N of digital bits that a CD player reads every second. (*b*) The audio information is sent to each of the two loudspeakers 44,100 times per second. Each of these samplings requires 16 bits and so one would (at first glance) think the required bit rate for a CD player is

$$N_0 = 2\left(44{,}100\,\frac{\text{samplings}}{\text{second}}\right)\left(16\,\frac{\text{bits}}{\text{sampling}}\right) = 1.4 \times 10^6\,\frac{\text{bits}}{\text{second}},$$

where the 2 is for the 2 loudspeakers (the 2 stereo channels). Note that N_0 is less than the number N of bits actually read per second by a CD player. The excess number of bits $(= N - N_0)$ is needed for encoding and error-correction. What percentage of the bits on a CD are dedicated to encoding and error-correction?

11. (II) A car traveling 95 km/h is 110 m behind a truck traveling 75 km/h. How long will it take the car to reach the truck?

12. (II) Two locomotives approach each other on parallel tracks. Each has a speed of 95 km/h with respect to the ground. If they are initially 8.5 km apart, how long will it be before they reach each other? (See Fig. 38.)

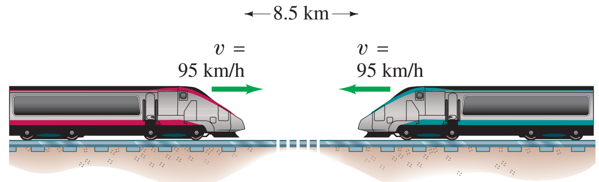


FIGURE 38 Problem 12.

13. (II) Digital bits on a 12.0-cm diameter audio CD are encoded along an outward spiraling path that starts at radius $R_1 = 2.5\,\text{cm}$ and finishes at radius $R_2 = 5.8\,\text{cm}$. The distance between the centers of neighboring spiral-windings is $1.6\,\mu\text{m}$ $(= 1.6 \times 10^{-6}\,\text{m})$. (*a*) Determine the total length of the spiraling path. [*Hint*: Imagine "unwinding" the spiral into a straight path of width 1.6 μm, and note that the original spiral and the straight path both occupy the same area.] (*b*) To read information, a CD player adjusts the rotation of the CD so that the player's readout laser moves along the spiral path at a constant speed of 1.25 m/s. Estimate the maximum playing time of such a CD.

14. (II) An airplane travels 3100 km at a speed of 720 km/h, and then encounters a tailwind that boosts its speed to 990 km/h for the next 2800 km. What was the total time for the trip? What was the average speed of the plane for this trip? [*Hint*: Does Eq. 12d apply, or not?]

$$\bar{v} = \frac{v + v_0}{2} \qquad [a = \text{constant}] \qquad \textbf{(12d)}$$

15. (II) Calculate the average speed and average velocity of a complete round trip in which the outgoing 250 km is covered at 95 km/h, followed by a 1.0-h lunch break, and the return 250 km is covered at 55 km/h.

16. (II) The position of a ball rolling in a straight line is given by $x = 2.0 - 3.6\,t + 1.1\,t^2$, where x is in meters and t in seconds. (*a*) Determine the position of the ball at $t = 1.0\,\text{s}$, 2.0 s, and 3.0 s. (*b*) What is the average velocity over the interval $t = 1.0\,\text{s}$ to $t = 3.0\,\text{s}$? (*c*) What is its instantaneous velocity at $t = 2.0\,\text{s}$ and at $t = 3.0\,\text{s}$?

17. (II) A dog runs 120 m away from its master in a straight line in 8.4 s, and then runs halfway back in one-third the time. Calculate (*a*) its average speed and (*b*) its average velocity.

18. (III) An automobile traveling 95 km/h overtakes a 1.10-km-long train traveling in the same direction on a track parallel to the road. If the train's speed is 75 km/h, how long does it take the car to pass it, and how far will the car have traveled in this time? See Fig. 39. What are the results if the car and train are traveling in opposite directions?

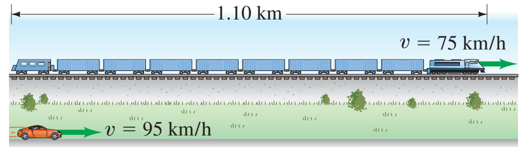


FIGURE 39 Problem 18.

19. (III) A bowling ball traveling with constant speed hits the pins at the end of a bowling lane 16.5 m long. The bowler hears the sound of the ball hitting the pins 2.50 s after the ball is released from his hands. What is the speed of the ball, assuming the speed of sound is 340 m/s?

4 Acceleration

20. (I) A sports car accelerates from rest to 95 km/h in 4.5 s. What is its average acceleration in m/s^2?

21. (I) At highway speeds, a particular automobile is capable of an acceleration of about $1.8\ m/s^2$. At this rate, how long does it take to accelerate from 80 km/h to 110 km/h?

22. (I) A sprinter accelerates from rest to 9.00 m/s in 1.28 s. What is her acceleration in (*a*) m/s^2; (*b*) km/h^2?

23. (I) Figure 37 shows the velocity of a train as a function of time. (*a*) At what time was its velocity greatest? (*b*) During what periods, if any, was the velocity constant? (*c*) During what periods, if any, was the acceleration constant? (*d*) When was the magnitude of the acceleration greatest?

24. (II) A sports car moving at constant speed travels 110 m in 5.0 s. If it then brakes and comes to a stop in 4.0 s, what is the magnitude of its acceleration in m/s^2, and in g's $(g = 9.80\ m/s^2)$?

25. (II) A car moving in a straight line starts at $x = 0$ at $t = 0$. It passes the point $x = 25.0$ m with a speed of 11.0 m/s at $t = 3.00$ s. It passes the point $x = 385$ m with a speed of 45.0 m/s at $t = 20.0$ s. Find (*a*) the average velocity and (*b*) the average acceleration between $t = 3.00$ s and $t = 20.0$ s.

26. (II) A particular automobile can accelerate approximately as shown in the velocity vs. time graph of Fig. 40. (The short flat spots in the curve represent shifting of the gears.) Estimate the average acceleration of the car in (*a*) second gear; and (*b*) fourth gear. (*c*) What is its average acceleration through the first four gears?

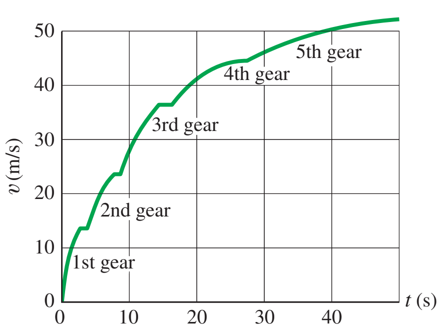


FIGURE 40 Problem 26. The velocity of a high-performance automobile as a function of time, starting from a dead stop. The flat spots in the curve represent gear shifts.

27. (II) A particle moves along the x axis. Its position as a function of time is given by $x = 6.8\,t + 8.5\,t^2$, where t is in seconds and x is in meters. What is the acceleration as a function of time?

28. (II) The position of a racing car, which starts from rest at $t = 0$ and moves in a straight line, is given as a function of time in the following Table. Estimate (*a*) its velocity and (*b*) its acceleration as a function of time. Display each in a Table and on a graph.

t(s)	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50
x(m)	0	0.11	0.46	1.06	1.94	4.62	8.55	13.79
t(s)	3.00	3.50	4.00	4.50	5.00	5.50	6.00	
x(m)	20.36	28.31	37.65	48.37	60.30	73.26	87.16	

29. (II) The position of an object is given by $x = At + Bt^2$, where x is in meters and t is in seconds. (*a*) What are the units of A and B? (*b*) What is the acceleration as a function of time? (*c*) What are the velocity and acceleration at $t = 5.0$ s? (*d*) What is the velocity as a function of time if $x = At + Bt^{-3}$?

5 and 6 Motion at Constant Acceleration

30. (I) A car slows down from 25 m/s to rest in a distance of 85 m. What was its acceleration, assumed constant?

31. (I) A car accelerates from 12 m/s to 21 m/s in 6.0 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

32. (I) A light plane must reach a speed of 32 m/s for takeoff. How long a runway is needed if the (constant) acceleration is $3.0\ m/s^2$?

33. (II) A baseball pitcher throws a baseball with a speed of 41 m/s. Estimate the average acceleration of the ball during the throwing motion. In throwing the baseball, the pitcher accelerates the ball through a displacement of about 3.5 m, from behind the body to the point where it is released (Fig. 41).

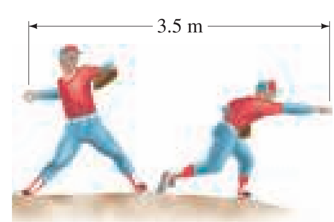


FIGURE 41 Problem 33.

34. (II) Show that $\bar{v} = (v + v_0)/2$ (see Eq. 12d) is not valid when the acceleration $a = A + Bt$, where A and B are constants.

35. (II) A world-class sprinter can reach a top speed (of about 11.5 m/s) in the first 15.0 m of a race. What is the average acceleration of this sprinter and how long does it take her to reach that speed?

36. (II) An inattentive driver is traveling 18.0 m/s when he notices a red light ahead. His car is capable of decelerating at a rate of $3.65\ m/s^2$. If it takes him 0.200 s to get the brakes on and he is 20.0 m from the intersection when he sees the light, will he be able to stop in time?

37. (II) A car slows down uniformly from a speed of 18.0 m/s to rest in 5.00 s. How far did it travel in that time?

38. (II) In coming to a stop, a car leaves skid marks 85 m long on the highway. Assuming a deceleration of $4.00\ m/s^2$, estimate the speed of the car just before braking.

39. (II) A car traveling 85 km/h slows down at a constant $0.50\,\mathrm{m/s^2}$ just by "letting up on the gas." Calculate (*a*) the distance the car coasts before it stops, (*b*) the time it takes to stop, and (*c*) the distance it travels during the first and fifth seconds.

40. (II) A car traveling at 105 km/h strikes a tree. The front end of the car compresses and the driver comes to rest after traveling 0.80 m. What was the magnitude of the average acceleration of the driver during the collision? Express the answer in terms of "g's," where $1.00\,g = 9.80\,\mathrm{m/s^2}$.

41. (II) Determine the stopping distances for an automobile with an initial speed of 95 km/h and human reaction time of 1.0 s: (*a*) for an acceleration $a = -5.0\,\mathrm{m/s^2}$; (*b*) for $a = -7.0\,\mathrm{m/s^2}$.

42. (II) A space vehicle accelerates uniformly from 65 m/s at $t = 0$ to 162 m/s at $t = 10.0\,\mathrm{s}$. How far did it move between $t = 2.0\,\mathrm{s}$ and $t = 6.0\,\mathrm{s}$?

43. (II) A 75-m-long train begins uniform acceleration from rest. The front of the train has a speed of 23 m/s when it passes a railway worker who is standing 180 m from where the front of the train started. What will be the speed of the last car as it passes the worker? (See Fig. 42.)

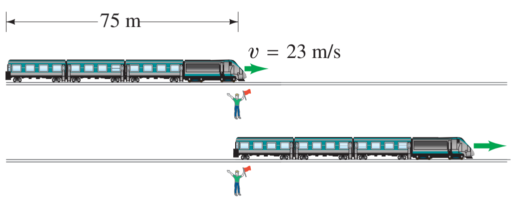


FIGURE 42 Problem 43.

44. (II) An unmarked police car traveling a constant 95 km/h is passed by a speeder traveling 135 km/h. Precisely 1.00 s after the speeder passes, the police officer steps on the accelerator; if the police car's acceleration is $2.00\,\mathrm{m/s^2}$, how much time passes before the police car overtakes the speeder (assumed moving at constant speed)?

45. (III) Assume in Problem 44 that the speeder's speed is not known. If the police car accelerates uniformly as given above and overtakes the speeder after accelerating for 7.00 s, what was the speeder's speed?

46. (III) A runner hopes to complete the 10,000-m run in less than 30.0 min. After running at constant speed for exactly 27.0 min, there are still 1100 m to go. The runner must then accelerate at $0.20\,\mathrm{m/s^2}$ for how many seconds in order to achieve the desired time?

47. (III) Mary and Sally are in a foot race (Fig. 43). When Mary is 22 m from the finish line, she has a speed of 4.0 m/s and is 5.0 m behind Sally, who has a speed of 5.0 m/s. Sally thinks she has an easy win and so, during the remaining portion of the race, decelerates at a constant rate of $0.50\,\mathrm{m/s^2}$ to the finish line. What constant acceleration does Mary now need during the remaining portion of the race, if she wishes to cross the finish line side-by-side with Sally?

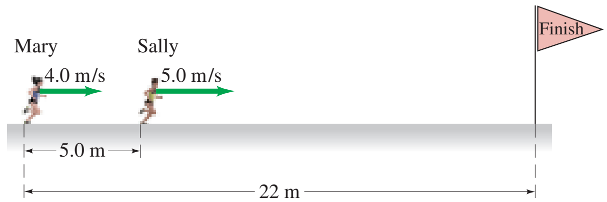


FIGURE 43 Problem 47.

7 Freely Falling Objects

[Neglect air resistance.]

48. (I) A stone is dropped from the top of a cliff. It is seen to hit the ground below after 3.75 s. How high is the cliff?

49. (I) If a car rolls gently $(v_0 = 0)$ off a vertical cliff, how long does it take to reach 55 km/h?

50. (I) Estimate (*a*) how long it took King Kong to fall straight down from the top of the Empire State Building (380 m high), and (*b*) his velocity just before "landing."

51. (II) A baseball is hit almost straight up into the air with a speed of about 20 m/s. (*a*) How high does it go? (*b*) How long is it in the air?

52. (II) A ball player catches a ball 3.2 s after throwing it vertically upward. With what speed did he throw it, and what height did it reach?

53. (II) A kangaroo jumps to a vertical height of 1.65 m. How long was it in the air before returning to Earth?

54. (II) The best rebounders in basketball have a vertical leap (that is, the vertical movement of a fixed point on their body) of about 120 cm. (*a*) What is their initial "launch" speed off the ground? (*b*) How long are they in the air?

55. (II) A helicopter is ascending vertically with a speed of 5.10 m/s. At a height of 105 m above the Earth, a package is dropped from a window. How much time does it take for the package to reach the ground? [*Hint*: v_0 for the package equals the speed of the helicopter.]

56. (II) For an object falling freely from rest, show that the distance traveled during each successive second increases in the ratio of successive odd integers (1, 3, 5, etc.). (This was first shown by Galileo.) See Figs. 26 and 29.

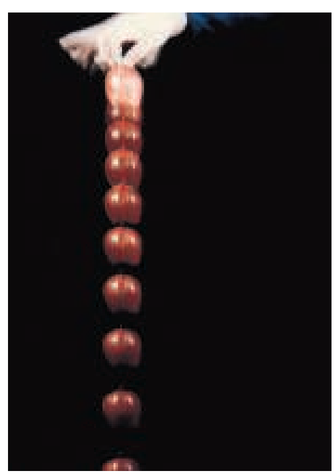


FIGURE 26 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

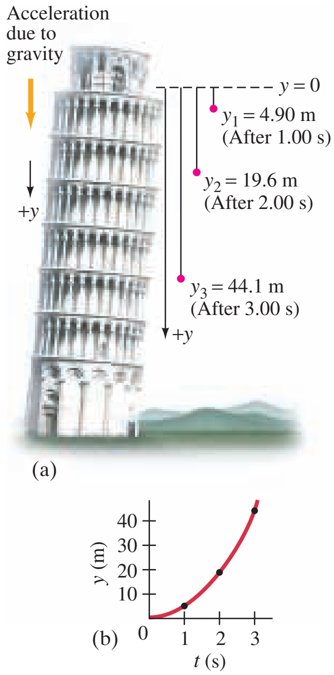


FIGURE 29 See Example 14 of "Describing Motion: Kinematics in One Dimension." (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 26.) (b) Graph of y vs. t.

57. (II) A baseball is seen to pass upward by a window 23 m above the street with a vertical speed of 14 m/s. If the ball was thrown from the street, (*a*) what was its initial speed, (*b*) what altitude does it reach, (*c*) when was it thrown, and (*d*) when does it reach the street again?

58. (II) A rocket rises vertically, from rest, with an acceleration of $3.2\ \mathrm{m/s^2}$ until it runs out of fuel at an altitude of 950 m. After this point, its acceleration is that of gravity, downward. (*a*) What is the velocity of the rocket when it runs out of fuel? (*b*) How long does it take to reach this point? (*c*) What maximum altitude does the rocket reach? (*d*) How much time (total) does it take to reach maximum altitude? (*e*) With what velocity does it strike the Earth? (*f*) How long (total) is it in the air?

59. (II) Roger sees water balloons fall past his window. He notices that each balloon strikes the sidewalk 0.83 s after passing his window. Roger's room is on the third floor, 15 m above the sidewalk. (*a*) How fast are the balloons traveling when they pass Roger's window? (*b*) Assuming the balloons are being released from rest, from what floor are they being released? Each floor of the dorm is 5.0 m high.

60. (II) A stone is thrown vertically upward with a speed of 24.0 m/s. (*a*) How fast is it moving when it reaches a height of 13.0 m? (*b*) How much time is required to reach this height? (*c*) Why are there two answers to (*b*)?

61. (II) A falling stone takes 0.33 s to travel past a window 2.2 m tall (Fig. 44). From what height above the top of the window did the stone fall?

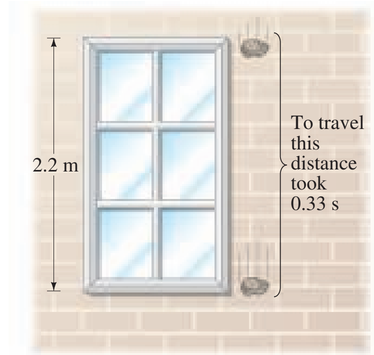


FIGURE 44 Problem 61.

62. (II) Suppose you adjust your garden hose nozzle for a hard stream of water. You point the nozzle vertically upward at a height of 1.5 m above the ground (Fig. 45). When you quickly turn off the nozzle, you hear the water striking the ground next to you for another 2.0 s. What is the water speed as it leaves the nozzle?

FIGURE 45 Problem 62.

63. (III) A toy rocket moving vertically upward passes by a 2.0-m-high window whose sill is 8.0 m above the ground. The rocket takes 0.15 s to travel the 2.0 m height of the window. What was the launch speed of the rocket, and how high will it go? Assume the propellant is burned very quickly at blastoff.

64. (III) A ball is dropped from the top of a 50.0-m-high cliff. At the same time, a carefully aimed stone is thrown straight up from the bottom of the cliff with a speed of 24.0 m/s. The stone and ball collide part way up. How far above the base of the cliff does this happen?

65. (III) A rock is dropped from a sea cliff and the sound of it striking the ocean is heard 3.4 s later. If the speed of sound is 340 m/s, how high is the cliff?

66. (III) A rock is thrown vertically upward with a speed of 12.0 m/s. Exactly 1.00 s later, a ball is thrown up vertically along the same path with a speed of 18.0 m/s. (*a*) At what time will they strike each other? (*b*) At what height will the collision occur? (*c*) Answer (*a*) and (*b*) assuming that the order is reversed: the ball is thrown 1.00 s before the rock.

*8 Variable Acceleration; Calculus

*67. (II) Given $v(t) = 25 + 18t$, where v is in m/s and t is in s, use calculus to determine the total displacement from $t_1 = 1.5\text{ s}$ to $t_2 = 3.1\text{ s}$.

*68. (III) The acceleration of a particle is given by $a = A\sqrt{t}$ where $A = 2.0\text{ m/s}^{5/2}$. At $t = 0$, $v = 7.5\text{ m/s}$ and $x = 0$. (*a*) What is the speed as a function of time? (*b*) What is the displacement as a function of time? (*c*) What are the acceleration, speed and displacement at $t = 5.0\text{ s}$?

*69. (III) Air resistance acting on a falling body can be taken into account by the approximate relation for the acceleration:

$$a = \frac{dv}{dt} = g - kv,$$

where k is a constant. (*a*) Derive a formula for the velocity of the body as a function of time assuming it starts from rest ($v = 0$ at $t = 0$). [*Hint*: Change variables by setting $u = g - kv$.] (*b*) Determine an expression for the terminal velocity, which is the maximum value the velocity reaches.

*9 Graphical Analysis and Numerical Integration

[See Problems 95–97 at the end of this Chapter.]

General Problems

70. A fugitive tries to hop on a freight train traveling at a constant speed of 5.0 m/s. Just as an empty box car passes him, the fugitive starts from rest and accelerates at $a = 1.2\text{ m/s}^2$ to his maximum speed of 6.0 m/s. (*a*) How long does it take him to catch up to the empty box car? (*b*) What is the distance traveled to reach the box car?

71. The acceleration due to gravity on the Moon is about one-sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

72. A person jumps from a fourth-story window 15.0 m above a firefighter's safety net. The survivor stretches the net 1.0 m before coming to rest, Fig. 46. (*a*) What was the average deceleration experienced by the survivor when she was slowed to rest by the net? (*b*) What would you do to make it "safer" (that is, to generate a smaller deceleration): would you stiffen or loosen the net? Explain.

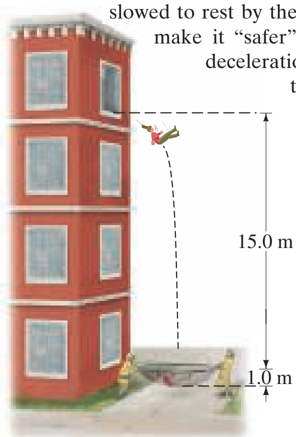


FIGURE 46 Problem 72.

73. A person who is properly restrained by an over-the-shoulder seat belt has a good chance of surviving a car collision if the deceleration does not exceed 30 "g's" ($1.00\ g = 9.80\text{ m/s}^2$). Assuming uniform deceleration of this value, calculate the distance over which the front end of the car must be designed to collapse if a crash brings the car to rest from 100 km/h.

74. Pelicans tuck their wings and free-fall straight down when diving for fish. Suppose a pelican starts its dive from a height of 16.0 m and cannot change its path once committed. If it takes a fish 0.20 s to perform evasive action, at what minimum height must it spot the pelican to escape? Assume the fish is at the surface of the water.

75. Suppose a car manufacturer tested its cars for front-end collisions by hauling them up on a crane and dropping them from a certain height. (*a*) Show that the speed just before a car hits the ground, after falling from rest a vertical distance H, is given by $\sqrt{2gH}$. What height corresponds to a collision at (*b*) 50 km/h? (*c*) 100 km/h?

76. A stone is dropped from the roof of a high building. A second stone is dropped 1.50 s later. How far apart are the stones when the second one has reached a speed of 12.0 m/s?

77. A bicyclist in the Tour de France crests a mountain pass as he moves at 15 km/h. At the bottom, 4.0 km farther, his speed is 75 km/h. What was his average acceleration (in m/s^2) while riding down the mountain?

78. Consider the street pattern shown in Fig. 47. Each intersection has a traffic signal, and the speed limit is 50 km/h. Suppose you are driving from the west at the speed limit. When you are 10.0 m from the first intersection, all the lights turn green. The lights are green for 13.0 s each. (*a*) Calculate the time needed to reach the third stoplight. Can you make it through all three lights without stopping? (*b*) Another car was stopped at the first light when all the lights turned green. It can accelerate at the rate of 2.00 m/s^2 to the speed limit. Can the second car make it through all three lights without stopping? By how many seconds would it make it or not?

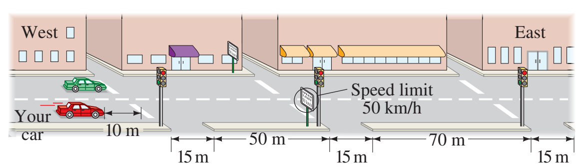


FIGURE 47 Problem 78.

79. In putting, the force with which a golfer strikes a ball is planned so that the ball will stop within some small distance of the cup, say 1.0 m long or short, in case the putt is missed. Accomplishing this from an uphill lie (that is, putting the ball downhill, see Fig. 48) is more difficult than from a downhill lie. To see why, assume that on a particular green

the ball decelerates constantly at $1.8\,\mathrm{m/s^2}$ going downhill, and constantly at $2.8\,\mathrm{m/s^2}$ going uphill. Suppose we have an uphill lie 7.0 m from the cup. Calculate the allowable range of initial velocities we may impart to the ball so that it stops in the range 1.0 m short to 1.0 m long of the cup. Do the same for a downhill lie 7.0 m from the cup. What in your results suggests that the downhill putt is more difficult?

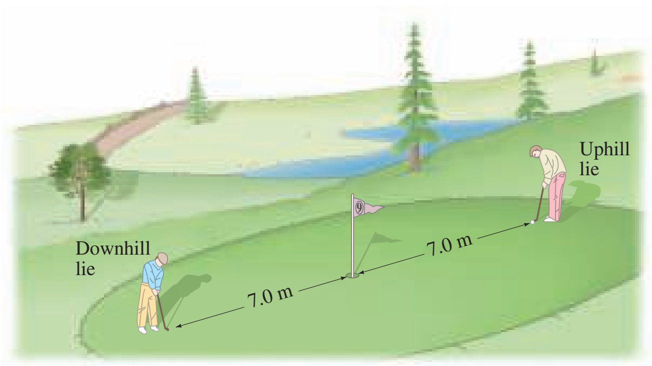


FIGURE 48 Problem 79.

80. A robot used in a pharmacy picks up a medicine bottle at $t = 0$. It accelerates at $0.20\,\mathrm{m/s^2}$ for 5.0 s, then travels without acceleration for 68 s and finally decelerates at $-0.40\,\mathrm{m/s^2}$ for 2.5 s to reach the counter where the pharmacist will take the medicine from the robot. From how far away did the robot fetch the medicine?

81. A stone is thrown vertically upward with a speed of 12.5 m/s from the edge of a cliff 75.0 m high (Fig. 49). (*a*) How much later does it reach the bottom of the cliff? (*b*) What is its speed just before hitting? (*c*) What total distance did it travel?

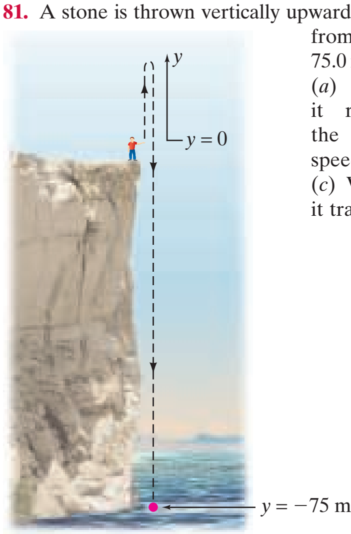


FIGURE 49 Problem 81.

82. Figure 50 is a position versus time graph for the motion of an object along the x axis. Consider the time interval from A to B. (*a*) Is the object moving in the positive or negative direction? (*b*) Is the object speeding up or slowing down? (*c*) Is the acceleration of the object positive or negative? Next, consider the time interval from D to E. (*d*) Is the object moving in the positive or negative direction? (*e*) Is the object speeding up or slowing down? (*f*) Is the acceleration of the object positive or negative? (*g*) Finally, answer these same three questions for the time interval from C to D.

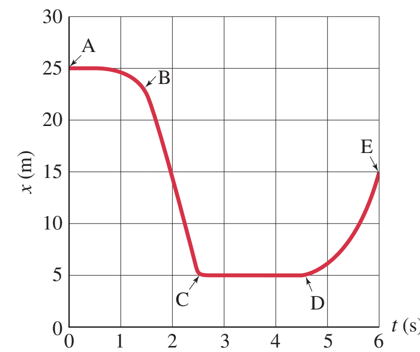


FIGURE 50 Problem 82.

83. In the design of a *rapid transit system*, it is necessary to balance the average speed of a train against the distance between stops. The more stops there are, the slower the train's average speed. To get an idea of this problem, calculate the time it takes a train to make a 9.0-km trip in two situations: (*a*) the stations at which the trains must stop are 1.8 km apart (a total of 6 stations, including those at the ends); and (*b*) the stations are 3.0 km apart (4 stations total). Assume that at each station the train accelerates at a rate of $1.1\,\mathrm{m/s^2}$ until it reaches 95 km/h, then stays at this speed until its brakes are applied for arrival at the next station, at which time it decelerates at $-2.0\,\mathrm{m/s^2}$. Assume it stops at each intermediate station for 22 s.

84. A person jumps off a diving board 4.0 m above the water's surface into a deep pool. The person's downward motion stops 2.0 m below the surface of the water. Estimate the average deceleration of the person while under the water.

85. Bill can throw a ball vertically at a speed 1.5 times faster than Joe can. How many times higher will Bill's ball go than Joe's?

86. Sketch the v vs. t graph for the object whose displacement as a function of time is given by Fig. 36.

87. A person driving her car at 45 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning to red, and she is 28 m away from the near side of the intersection (Fig. 51). Should she try to stop, or should she speed up to cross the intersection before the light turns red? The intersection is 15 m wide. Her car's maximum deceleration is $-5.8\,\mathrm{m/s^2}$, whereas it can accelerate from 45 km/h to 65 km/h in 6.0 s. Ignore the length of her car and her reaction time.

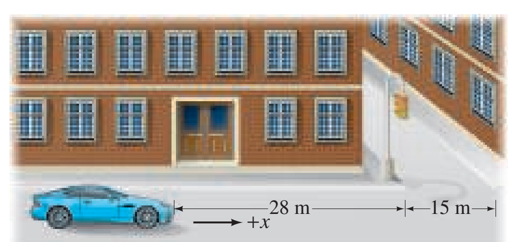


FIGURE 51 Problem 87.

88. A car is behind a truck going 25 m/s on the highway. The driver looks for an opportunity to pass, guessing that his car can accelerate at $1.0\ \mathrm{m/s^2}$, and he gauges that he has to cover the 20-m length of the truck, plus 10-m clear room at the rear of the truck and 10 m more at the front of it. In the oncoming lane, he sees a car approaching, probably also traveling at 25 m/s. He estimates that the car is about 400 m away. Should he attempt the pass? Give details.

89. Agent Bond is standing on a bridge, 13 m above the road below, and his pursuers are getting too close for comfort. He spots a flatbed truck approaching at 25 m/s, which he measures by knowing that the telephone poles the truck is passing are 25 m apart in this country. The bed of the truck is 1.5 m above the road, and Bond quickly calculates how many poles away the truck should be when he jumps down from the bridge onto the truck, making his getaway. How many poles is it?

90. A police car at rest, passed by a speeder traveling at a constant 130 km/h, takes off in hot pursuit. The police officer catches up to the speeder in 750 m, maintaining a constant acceleration. (*a*) Qualitatively plot the position vs. time graph for both cars from the police car's start to the catch-up point. Calculate (*b*) how long it took the police officer to overtake the speeder, (*c*) the required police car acceleration, and (*d*) the speed of the police car at the overtaking point.

91. A fast-food restaurant uses a conveyor belt to send the burgers through a grilling machine. If the grilling machine is 1.1 m long and the burgers require 2.5 min to cook, how fast must the conveyor belt travel? If the burgers are spaced 15 cm apart, what is the rate of burger production (in burgers/min)?

92. Two students are asked to find the height of a particular building using a barometer. Instead of using the barometer as an altitude-measuring device, they take it to the roof of the building and drop it off, timing its fall. One student reports a fall time of 2.0 s, and the other, 2.3 s. What % difference does the 0.3 s make for the estimates of the building's height?

93. Figure 52 shows the position vs. time graph for two bicycles, A and B. (*a*) Is there any instant at which the two bicycles have the same velocity? (*b*) Which bicycle has the larger acceleration? (*c*) At which instant(s) are the bicycles passing each other? Which bicycle is passing the other? (*d*) Which bicycle has the highest instantaneous velocity? (*e*) Which bicycle has the higher average velocity?

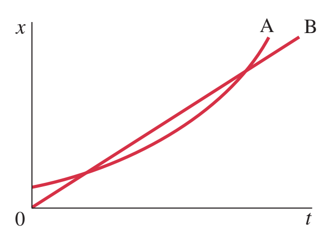


FIGURE 52 Problem 93.

94. You are traveling at a constant speed v_M, and there is a car in front of you traveling with a speed v_A. You notice that $v_M > v_A$, so you start slowing down with a constant acceleration a when the distance between you and the other car is x. What relationship between a and x determines whether or not you run into the car in front of you?

*Numerical/Computer

***95.** (II) The Table below gives the speed of a particular drag racer as a function of time. (*a*) Calculate the average acceleration $(\mathrm{m/s^2})$ during each time interval. (*b*) Using numerical integration (see Section 9 of "Describing Motion: Kinematics in One Dimension") estimate the total distance traveled (m) as a function of time. [*Hint*: for $\bar{v}$ in each interval sum the velocities at the beginning and end of the interval and divide by 2; for example, in the second interval use $\bar{v} = (6.0 + 13.2)/2 = 9.6$] (*c*) Graph each of these.

t(s)	0	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
v(km/h)	0.0	6.0	13.2	22.3	32.2	43.0	53.5	62.6	70.6	78.4	85.1

***96.** (III) The acceleration of an object (in $\mathrm{m/s^2}$) is measured at 1.00-s intervals starting at $t = 0$ to be as follows: 1.25, 1.58, 1.96, 2.40, 2.66, 2.70, 2.74, 2.72, 2.60, 2.30, 2.04, 1.76, 1.41, 1.09, 0.86, 0.51, 0.28, 0.10. Use numerical integration (see Section 9 of "Describing Motion: Kinematics in One Dimension") to estimate (*a*) the velocity (assume that $v = 0$ at $t = 0$) and (*b*) the displacement at $t = 17.00$ s.

***97.** (III) A lifeguard standing at the side of a swimming pool spots a child in distress, Fig. 53. The lifeguard runs with average speed v_R along the pool's edge for a distance x, then jumps into the pool and swims with average speed v_S on a straight path to the child. (*a*) Show that the total time t it takes the lifeguard to get to the child is given by

$$t = \frac{x}{v_R} + \frac{\sqrt{D^2 + (d - x)^2}}{v_S}.$$

(*b*) Assume $v_R = 4.0$ m/s and $v_S = 1.5$ m/s. Use a graphing calculator or computer to plot t vs. x in part (*a*), and from this plot determine the optimal distance x the lifeguard should run before jumping into the pool (that is, find the value of x that minimizes the time t to get to the child).

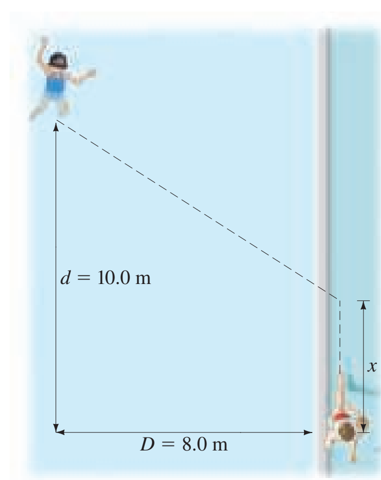


FIGURE 53 Problem 97.

Answers to Odd-Numbered Problems

1. 61 m.

3. 0.65 cm/s, no.

5. 300 m/s, 1 km every 3 sec.

7. (*a*) 9.26 m/s;
(*b*) 3.1 m/s.

9. (*a*) 0.3 m/s;
(*b*) 1.2 m/s;
(*c*) 0.30 m/s;
(*d*) 1.4 m/s;
(*e*) −0.95 m/s.

11. 2.0×10^1 s.

13. (*a*) 5.4×10^3 m;
(*b*) 72 min.

15. (*a*) 61 km/h;
(*b*) 0.

17. (*a*) 16 m/s;
(*b*) +5 m/s.

19. 6.73 m/s.

21. 5 s.

23. (*a*) 48 s;
(*b*) 90 s to 108 s;
(*c*) 0 to 42 s, 65 s to 83 s, 90 s to 108 s;
(*d*) 65 s to 83 s.

25. (*a*) 21.2 m/s;
(*b*) $2.00\ \mathrm{m/s^2}$.

27. $17.0\ \mathrm{m/s^2}$.

29. (*a*) m/s, $\mathrm{m/s^2}$;
(*b*) $2B\ \mathrm{m/s^2}$;
(*c*) $(A + 10B)$ m/s, $2B\ \mathrm{m/s^2}$;
(*d*) $A - 3Bt^{-4}$.

31. $1.5\ \mathrm{m/s^2}$, 99 m.

33. $240\ \mathrm{m/s^2}$.

35. $4.41\ \mathrm{m/s^2}$, 2.61 s.

37. 45.0 m.

39. (*a*) 560 m;
(*b*) 47 s;
(*c*) 23 m, 21 m.

41. (*a*) 96 m;
(*b*) 76 m.

43. 27 m/s.

45. 117 km/h.

47. $0.49\ \mathrm{m/s^2}$.

49. 1.6 s.

51. (*a*) 20 m;
(*b*) 4 s.

53. 1.16 s.

55. 5.18 s.

57. (*a*) 25 m/s;
(*b*) 33 m;
(*c*) 1.2 s;
(*d*) 5.2 s.

59. (*a*) 14 m/s;
(*b*) fifth floor.

61. 1.3 m.

63. 18.8 m/s, 18.1 m.

65. 52 m.

67. 106 m.

69. (*a*) $\frac{g}{k}(1 - e^{-kt})$;
(*b*) $\frac{g}{k}$.

71. 6.

73. 1.3 m.

75. (*b*) 10 m;
(*c*) 40 m.

77. $5.2 \times 10^{-2}\ \mathrm{m/s^2}$.

79. 4.6 m/s to 5.4 m/s, 5.8 m/s to 6.7 m/s, smaller range of velocities.

81. (*a*) 5.39 s;
(*b*) 40.3 m/s;
(*c*) 90.9 m.

83. (*a*) 8.7 min;
(*b*) 7.3 min.

85. 2.3.

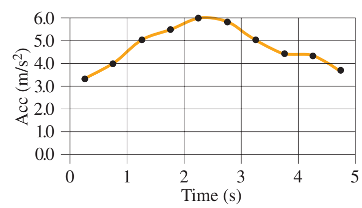
87. Stop.

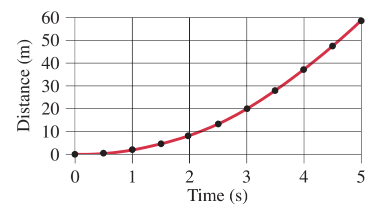
89. 1.5 poles.

91. 0.44 m/min, 2.9 burgers/min.

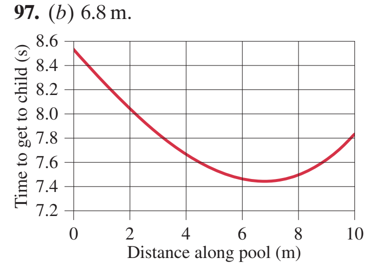
93. (*a*) Where the slopes are the same;
(*b*) bicycle A;
(*c*) when the two graphs cross; first crossing, B passing A; second crossing, A passing B;
(*d*) B until the slopes are equal, A after that;
(*e*) same.

95. (*c*)





97. (*b*) 6.8 m.



Lucas Kane Photography, LLC

This snowboarder flying through the air shows an example of motion in two dimensions. In the absence of air resistance, the path would be a perfect parabola. The gold arrow represents the downward acceleration of gravity, $\vec{\mathbf{g}}$. Galileo analyzed the motion of objects in 2 dimensions under the action of gravity near the Earth's surface (now called "projectile motion") into its horizontal and vertical components.

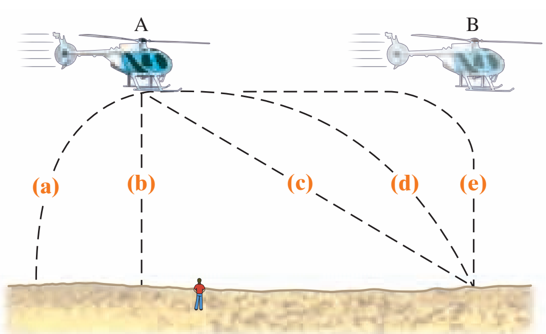
We will discuss how to manipulate vectors and how to add them. Besides analyzing projectile motion, we will also see how to work with relative velocity.

Kinematics in Two or Three Dimensions; Vectors

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies along in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting air resistance) as seen by a person standing on the ground?



We now consider the description of the motion of objects that move in paths in two (or three) dimensions. To do so, we first need to discuss vectors and how they are added. We will examine the description of motion in general, followed by an interesting special case, the motion of projectiles near the Earth's surface. We also discuss how to determine the relative velocity of an object as measured in different reference frames.

CONTENTS

1 Vectors and Scalars

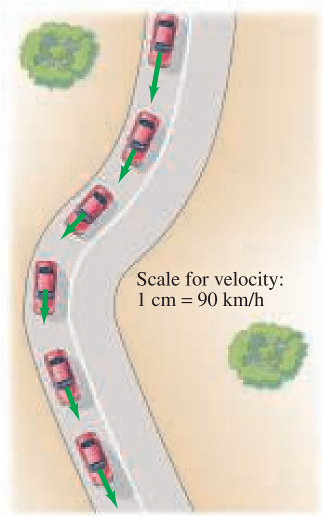


FIGURE 1 Car traveling on a road, slowing down to round the curve. The green arrows represent the velocity vector at each position.

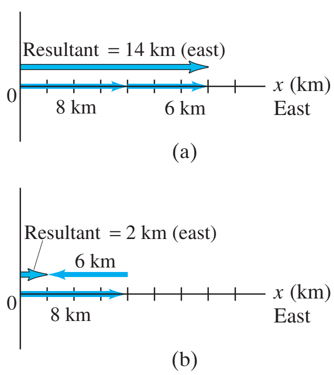
The term *velocity* refers not only to how fast an object is moving but also to its direction. A quantity such as velocity, which has *direction* as well as *magnitude*, is a **vector** quantity. Other quantities that are also vectors are displacement, force, and momentum. However, many quantities have no direction associated with them, such as mass, time, and temperature. They are specified completely by a number and units. Such quantities are called **scalar** quantities.

Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, each vector is represented by an arrow. The arrow is always drawn so that it points in the direction of the vector quantity it represents. The length of the arrow is drawn proportional to the magnitude of the vector quantity. For example, in Fig. 1, green arrows have been drawn representing the velocity of a car at various places as it rounds a curve. The magnitude of the velocity at each point can be read off Fig. 1 by measuring the length of the corresponding arrow and using the scale shown ($1\text{ cm} = 90\text{ km/h}$).

When we write the symbol for a vector, we will always use boldface type, with a tiny arrow over the symbol. Thus for velocity we write $\vec{\mathbf{v}}$. If we are concerned only with the magnitude of the vector, we will write simply v, in italics, as we do for other symbols.

2 Addition of Vectors—Graphical Methods

FIGURE 2 Combining vectors in one dimension.

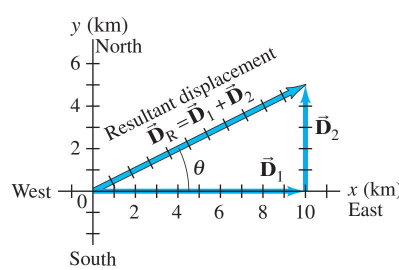


Because vectors are quantities that have direction as well as magnitude, they must be added in a special way. In this Chapter, we will deal mainly with displacement vectors, for which we now use the symbol $\vec{\mathbf{D}}$, and velocity vectors, $\vec{\mathbf{v}}$. But the results will apply for other vectors we encounter later.

We use simple arithmetic for adding scalars. Simple arithmetic can also be used for adding vectors if they are in the same direction. For example, if a person walks 8 km east one day, and 6 km east the next day, the person will be $8\text{ km} + 6\text{ km} = 14\text{ km}$ east of the point of origin. We say that the *net* or *resultant* displacement is 14 km to the east (Fig. 2a). If, on the other hand, the person walks 8 km east on the first day, and 6 km west (in the reverse direction) on the second day, then the person will end up 2 km from the origin (Fig. 2b), so the resultant displacement is 2 km to the east. In this case, the resultant displacement is obtained by subtraction: $8\text{ km} - 6\text{ km} = 2\text{ km}$.

But simple arithmetic cannot be used if the two vectors are not along the same line. For example, suppose a person walks 10.0 km east and then walks 5.0 km north. These displacements can be represented on a graph in which the positive y axis points north and the positive x axis points east, Fig. 3. On this graph, we draw an arrow, labeled $\vec{\mathbf{D}}_1$, to represent the 10.0-km displacement to the east. Then we draw a second arrow, $\vec{\mathbf{D}}_2$, to represent the 5.0-km displacement to the north. Both vectors are drawn to scale, as in Fig. 3.

FIGURE 3 A person walks 10.0 km east and then 5.0 km north. These two displacements are represented by the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$, which are shown as arrows. The resultant displacement vector, $\vec{\mathbf{D}}_R$, which is the vector sum of $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$, is also shown. Measurement on the graph with ruler and protractor shows that $\vec{\mathbf{D}}_R$ has a magnitude of 11.2 km and points at an angle $\theta = 27°$ north of east.



After taking this walk, the person is now 10.0 km east and 5.0 km north of the point of origin. The **resultant displacement** is represented by the arrow labeled $\vec{\mathbf{D}}_R$ in Fig. 3. Using a ruler and a protractor, you can measure on this diagram that the person is 11.2 km from the origin at an angle $\theta = 27°$ north of east. In other words, the resultant displacement vector has a magnitude of 11.2 km and makes an angle $\theta = 27°$ with the positive x axis. The magnitude (length) of $\vec{\mathbf{D}}_R$ can also be obtained using the theorem of Pythagoras in this case, since D_1, D_2, and D_R form a right triangle with D_R as the hypotenuse. Thus

$$\begin{aligned} D_R &= \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0\text{ km})^2 + (5.0\text{ km})^2} \\ &= \sqrt{125\text{ km}^2} = 11.2\text{ km}. \end{aligned}$$

You can use the Pythagorean theorem, of course, only when the vectors are *perpendicular* to each other.

The resultant displacement vector, $\vec{\mathbf{D}}_R$, is the sum of the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. That is,

$$\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2.$$

This is a *vector* equation. An important feature of adding two vectors that are not along the same line is that the magnitude of the resultant vector is not equal to the sum of the magnitudes of the two separate vectors, but is smaller than their sum. That is,

$$D_R \leq D_1 + D_2,$$

where the equals sign applies only if the two vectors point in the same direction. In our example (Fig. 3), $D_R = 11.2$ km, whereas $D_1 + D_2$ equals 15 km, which is the total distance traveled. Note also that we cannot set $\vec{\mathbf{D}}_R$ equal to 11.2 km, because we have a vector equation and 11.2 km is only a part of the resultant vector, its magnitude. We could write something like this, though: $\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 = (11.2\text{ km}, 27°\text{ N of E})$.

EXERCISE A Under what conditions can the magnitude of the resultant vector above be $D_R = D_1 + D_2$?

Figure 3 illustrates the general rules for graphically adding two vectors together, no matter what angles they make, to get their sum. The rules are as follows:

1. On a diagram, draw one of the vectors—call it $\vec{\mathbf{D}}_1$—to scale.
2. Next draw the second vector, $\vec{\mathbf{D}}_2$, to scale, placing its tail at the tip of the first vector and being sure its direction is correct.
3. The arrow drawn from the tail of the first vector to the tip of the second vector represents the *sum*, or **resultant**, of the two vectors.

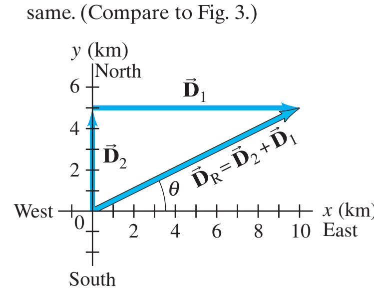
The length of the resultant vector represents its magnitude. Note that vectors can be translated parallel to themselves (maintaining the same length and angle) to accomplish these manipulations. The length of the resultant can be measured with a ruler and compared to the scale. Angles can be measured with a protractor. This method is known as the **tail-to-tip method of adding vectors**.

The resultant is not affected by the order in which the vectors are added. For example, a displacement of 5.0 km north, to which is added a displacement of 10.0 km east, yields a resultant of 11.2 km and angle $\theta = 27°$ (see Fig. 4), the same as when they were added in reverse order (Fig. 3). That is, now using $\vec{\mathbf{V}}$ to represent any type of vector,

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1, \qquad \text{[commutative property]} \quad \textbf{(1a)}$$

which is known as the *commutative* property of vector addition.

FIGURE 4 If the vectors are added in reverse order, the resultant is the same. (Compare to Fig. 3.)



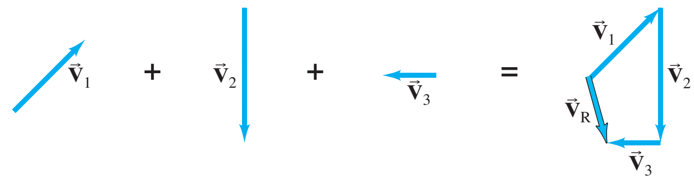


FIGURE 5 The resultant of three vectors: $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3$.

The tail-to-tip method of adding vectors can be extended to three or more vectors. The resultant is drawn from the tail of the first vector to the tip of the last one added. An example is shown in Fig. 5; the three vectors could represent displacements (northeast, south, west) or perhaps three forces. Check for yourself that you get the same resultant no matter in which order you add the three vectors; that is,

$$\left(\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\right) + \vec{\mathbf{V}}_3 = \vec{\mathbf{V}}_1 + \left(\vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3\right), \qquad \text{[associative property]} \quad \textbf{(1b)}$$

which is known as the *associative* property of vector addition.

A second way to add two vectors is the **parallelogram method**. It is fully equivalent to the tail-to-tip method. In this method, the two vectors are drawn starting from a common origin, and a parallelogram is constructed using these two vectors as adjacent sides as shown in Fig. 6b. The resultant is the diagonal drawn from the common origin. In Fig. 6a, the tail-to-tip method is shown, and it is clear that both methods yield the same result.

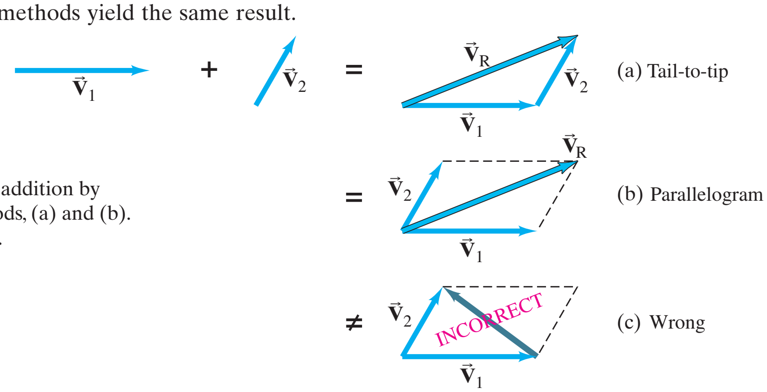


FIGURE 6 Vector addition by two different methods, (a) and (b). Part (c) is incorrect.

CAUTION

Be sure to use the correct diagonal on parallelogram to get the resultant

It is a common error to draw the sum vector as the diagonal running between the tips of the two vectors, as in Fig. 6c. *This is incorrect*: it does not represent the sum of the two vectors. (In fact, it represents their difference, $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$, as we will see in the next Section.)

CONCEPTUAL EXAMPLE 1 **Range of vector lengths.** Suppose two vectors each have length 3.0 units. What is the range of possible lengths for the vector representing the sum of the two?

RESPONSE The sum can take on any value from 6.0 $(= 3.0 + 3.0)$ where the vectors point in the same direction, to 0 $(= 3.0 - 3.0)$ when the vectors are antiparallel.

EXERCISE B If the two vectors of Example 1 are perpendicular to each other, what is the resultant vector length?

3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

Given a vector $\vec{\mathbf{V}}$, we define the *negative* of this vector $(-\vec{\mathbf{V}})$ to be a vector with the same magnitude as $\vec{\mathbf{V}}$ but opposite in direction, Fig. 7. Note, however, that no vector is ever negative in the sense of its magnitude: the magnitude of every vector is positive. Rather, a minus sign tells us about its direction.

FIGURE 7 The negative of a vector is a vector having the same length but opposite direction.

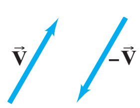


FIGURE 8 Subtracting two vectors: $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$.

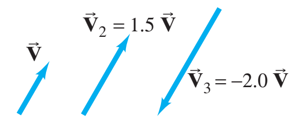
We can now define the subtraction of one vector from another: the difference between two vectors $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$ is defined as

$$\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = \vec{\mathbf{V}}_2 + (-\vec{\mathbf{V}}_1).$$

That is, the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus our rules for addition of vectors can be applied as shown in Fig. 8 using the tail-to-tip method.

A vector $\vec{\mathbf{V}}$ can be multiplied by a scalar c. We define their product so that $c\vec{\mathbf{V}}$ has the same direction as $\vec{\mathbf{V}}$ and has magnitude cV. That is, multiplication of a vector by a positive scalar c changes the magnitude of the vector by a factor c but doesn't alter the direction. If c is a negative scalar, the magnitude of the product $c\vec{\mathbf{V}}$ is still $|c|V$ (where $|c|$ means the magnitude of c), but the direction is precisely opposite to that of $\vec{\mathbf{V}}$. See Fig. 9.

FIGURE 9 Multiplying a vector $\vec{\mathbf{V}}$ by a scalar c gives a vector whose magnitude is c times greater and in the same direction as $\vec{\mathbf{V}}$ (or opposite direction if c is negative).



EXERCISE C What does the "incorrect" vector in Fig. 6c represent? (a) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$, (b) $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$, (c) something else (specify).

4 Adding Vectors by Components

Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors. But do not forget graphical methods—they are useful for visualizing, for checking your math, and thus for getting the correct result.

Consider first a vector $\vec{\mathbf{V}}$ that lies in a particular plane. It can be expressed as the sum of two other vectors, called the **components** of the original vector. The components are usually chosen to be along two perpendicular directions, such as the x and y axes. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 10; the vector $\vec{\mathbf{V}}$ could be a displacement vector that points at an angle $\theta = 30°$ north of east, where we have chosen the positive x axis to be to the east and the positive y axis north. This vector $\vec{\mathbf{V}}$ is resolved into its x and y components by drawing dashed lines out from the tip (A) of the vector (lines AB and AC) making them perpendicular to the x and y axes. Then the lines OB and OC represent the x and y components of $\vec{\mathbf{V}}$, respectively, as shown in Fig. 10b. These *vector components* are written $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$. We generally show vector components as arrows, like vectors, but dashed. The *scalar components*, V_x and V_y, are the magnitudes of the vector components, with units, accompanied by a positive or negative sign depending on whether they point along the positive or negative x or y axis. As can be seen in Fig. 10, $\vec{\mathbf{V}}_x + \vec{\mathbf{V}}_y = \vec{\mathbf{V}}$ by the parallelogram method of adding vectors.

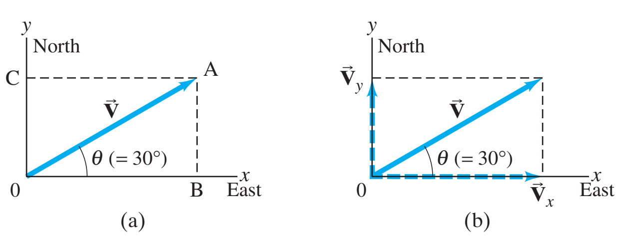
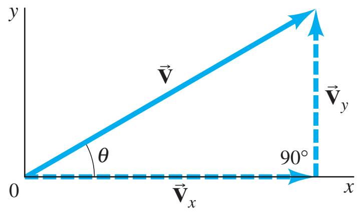


FIGURE 10 Resolving a vector $\vec{\mathbf{V}}$ into its components along an arbitrarily chosen set of x and y axes. The components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.



$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

FIGURE 11 Finding the components of a vector using trigonometric functions.

Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are $\vec{\mathbf{V}}_x$, $\vec{\mathbf{V}}_y$, and $\vec{\mathbf{V}}_z$. Resolution of a vector in three dimensions is merely an extension of the above technique.

The use of trigonometric functions for finding the components of a vector is illustrated in Fig. 11, where a vector and its two components are thought of as making up a right triangle. We then see that the sine, cosine, and tangent are as given in Fig. 11. If we multiply the definition of $\sin\theta = V_y/V$ by V on both sides, we get

$$V_y = V\sin\theta. \tag{2a}$$

Similarly, from the definition of $\cos\theta$, we obtain

$$V_x = V\cos\theta. \tag{2b}$$

Note that θ is chosen (by convention) to be the angle that the vector makes with the positive x axis, measured positive counterclockwise.

The components of a given vector will be different for different choices of coordinate axes. It is therefore crucial to specify the choice of coordinate system when giving the components.

There are two ways to specify a vector in a given coordinate system:

1. We can give its components, V_x and V_y.
2. We can give its magnitude V and the angle θ it makes with the positive x axis.

We can shift from one description to the other using Eqs. 2, and, for the reverse, by using the theorem of Pythagoras† and the definition of tangent:

$$V = \sqrt{V_x^2 + V_y^2} \tag{3a}$$

$$\tan\theta = \frac{V_y}{V_x} \tag{3b}$$

as can be seen in Fig. 11.

We can now discuss how to add vectors using components. The first step is to resolve each vector into its components. Next we can see, using Fig. 12, that the addition of any two vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ to give a resultant, $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, implies that

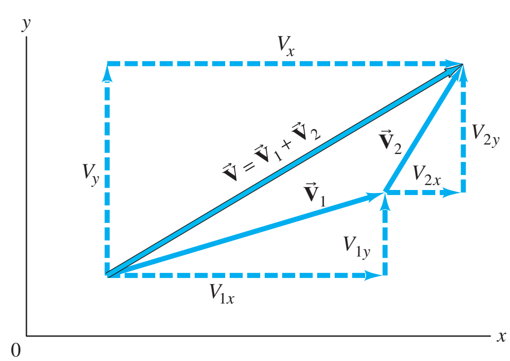
$$\begin{aligned} V_x &= V_{1x} + V_{2x} \\ V_y &= V_{1y} + V_{2y}. \end{aligned} \tag{4}$$

That is, the sum of the x components equals the x component of the resultant, and the sum of the y components equals the y component of the resultant, as can be verified by a careful examination of Fig. 12. Note that we do *not* add x components to y components.

†In three dimensions, the theorem of Pythagoras becomes $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$, where V_z is the component along the third, or z, axis.

FIGURE 12 The components of $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ are

$$V_x = V_{1x} + V_{2x}$$
$$V_y = V_{1y} + V_{2y}.$$



If the magnitude and direction of the resultant vector are desired, they can be obtained using Eqs. 3.

The components of a given vector depend on the choice of coordinate axes. You can often reduce the work involved in adding vectors by a good choice of axes—for example, by choosing one of the axes to be in the same direction as one of the vectors. Then that vector will have only one nonzero component.

EXAMPLE 2 **Mail carrier's displacement.** A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km (Fig. 13a). What is her displacement from the post office?

APPROACH We choose the positive x axis to be east and the positive y axis to be north, since those are the compass directions used on most maps. The origin of the xy coordinate system is at the post office. We resolve each vector into its x and y components. We add the x components together, and then the y components together, giving us the x and y components of the resultant.

SOLUTION Resolve each displacement vector into its components, as shown in Fig. 13b. Since $\vec{\mathbf{D}}_1$ has magnitude 22.0 km and points north, it has only a y component:

$$D_{1x} = 0, \qquad D_{1y} = 22.0\text{ km}.$$

$\vec{\mathbf{D}}_2$ has both x and y components:

$$D_{2x} = +(47.0\text{ km})(\cos 60°) = +(47.0\text{ km})(0.500) = +23.5\text{ km}$$
$$D_{2y} = -(47.0\text{ km})(\sin 60°) = -(47.0\text{ km})(0.866) = -40.7\text{ km}.$$

Notice that D_{2y} is negative because this vector component points along the negative y axis. The resultant vector, $\vec{\mathbf{D}}$, has components:

$$D_x = D_{1x} + D_{2x} = 0\text{ km} + 23.5\text{ km} = +23.5\text{ km}$$
$$D_y = D_{1y} + D_{2y} = 22.0\text{ km} + (-40.7\text{ km}) = -18.7\text{ km}.$$

This specifies the resultant vector completely:

$$D_x = 23.5\text{ km}, \qquad D_y = -18.7\text{ km}.$$

We can also specify the resultant vector by giving its magnitude and angle using Eqs. 3:

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(23.5\text{ km})^2 + (-18.7\text{ km})^2} = 30.0\text{ km}$$
$$\tan\theta = \frac{D_y}{D_x} = \frac{-18.7\text{ km}}{23.5\text{ km}} = -0.796.$$

A calculator with an INV TAN, an ARC TAN, or a TAN^{-1} key gives $\theta = \tan^{-1}(-0.796) = -38.5°$. The negative sign means $\theta = 38.5°$ below the x axis, Fig. 13c. So, the resultant displacement is 30.0 km directed at 38.5° in a southeasterly direction.

NOTE Always be attentive about the quadrant in which the resultant vector lies. An electronic calculator does not fully give this information, but a good diagram does.

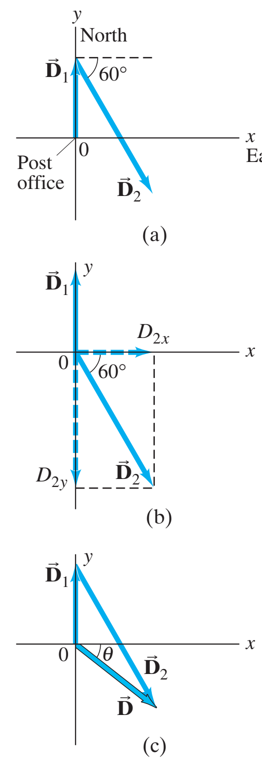


FIGURE 13 Example 2. (a) The two displacement vectors, $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. (b) $\vec{\mathbf{D}}_2$ is resolved into its components. (c) $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$ are added graphically to obtain the resultant $\vec{\mathbf{D}}$. The component method of adding the vectors is explained in the Example.

The signs of trigonometric functions depend on which "quadrant" the angle falls in: for example, the tangent is positive in the first and third quadrants (from 0° to 90°, and 180° to 270°), but negative in the second and fourth quadrants. The best way to keep track of angles, and to check any vector result, is always to draw a vector diagram. A vector diagram gives you something tangible to look at when analyzing a problem, and provides a check on the results.

PROBLEM SOLVING
Identify the correct quadrant by drawing a careful diagram

The following Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do to get you thinking and involved in the problem at hand.

PROBLEM SOLVING

Adding Vectors

Here is a brief summary of how to add two or more vectors using components:

1. **Draw a diagram**, adding the vectors graphically by either the parallelogram or tail-to-tip method.
2. **Choose** x and y **axes**. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors so that vector will have only one component.)
3. **Resolve** each vector into its x and y **components**, showing each component along its appropriate (x or y) axis as a (dashed) arrow.
4. **Calculate each component** (when not given) using sines and cosines. If θ_1 is the angle that vector $\vec{\mathbf{V}}_1$ makes with the positive x axis, then:
$$V_{1x} = V_1 \cos\theta_1, \qquad V_{1y} = V_1 \sin\theta_1.$$
Pay careful attention to **signs**: any component that points along the negative x or y axis gets a minus sign.
5. **Add** the x **components** together to get the x component of the resultant. Ditto for y:
$$V_x = V_{1x} + V_{2x} + \text{any others}$$
$$V_y = V_{1y} + V_{2y} + \text{any others.}$$
This is the answer: the components of the resultant vector. Check signs to see if they fit the quadrant shown in your diagram (point 1 above).
6. If you want to know the **magnitude and direction** of the resultant vector, use Eqs. 3:
$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}.$$
The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle θ.

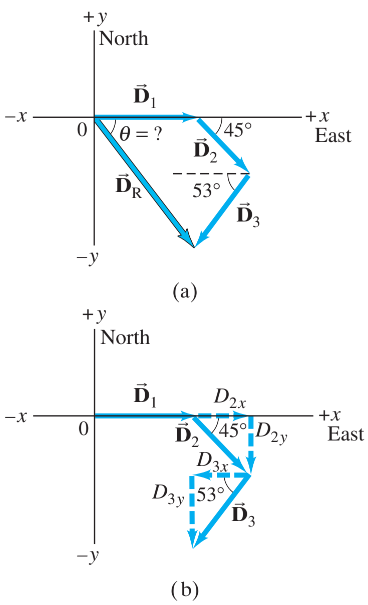


FIGURE 14 Example 3.

Vector	Components x (km)	y (km)
$\vec{\mathbf{D}}_1$	620	0
$\vec{\mathbf{D}}_2$	311	−311
$\vec{\mathbf{D}}_3$	−331	−439
$\vec{\mathbf{D}}_R$	600	−750

EXAMPLE 3 **Three short trips.** An airplane trip involves three legs, with two stopovers, as shown in Fig. 14a. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?

APPROACH We follow the steps in the Problem Solving Strategy above.

SOLUTION

1. **Draw a diagram** such as Fig. 14a, where $\vec{\mathbf{D}}_1$, $\vec{\mathbf{D}}_2$, and $\vec{\mathbf{D}}_3$ represent the three legs of the trip, and $\vec{\mathbf{D}}_R$ is the plane's total displacement.
2. **Choose axes**: Axes are also shown in Fig. 14a: x is east, y north.
3. **Resolve components**: It is imperative to draw a good diagram. The components are drawn in Fig. 14b. Instead of drawing all the vectors starting from a common origin, as we did in Fig. 13b, here we draw them "tail-to-tip" style, which is just as valid and may make it easier to see.
4. **Calculate the components**:
$$\begin{aligned}
\vec{\mathbf{D}}_1: D_{1x} &= +D_1\cos 0^\circ = D_1 = 620\text{ km}\\
D_{1y} &= +D_1 \sin 0^\circ = 0\text{ km}\\
\vec{\mathbf{D}}_2: D_{2x} &= +D_2\cos 45^\circ = +(440\text{ km})(0.707) = +311\text{ km}\\
D_{2y} &= -D_2\sin 45^\circ = -(440\text{ km})(0.707) = -311\text{ km}\\
\vec{\mathbf{D}}_3: D_{3x} &= -D_3\cos 53^\circ = -(550\text{ km})(0.602) = -331\text{ km}\\
D_{3y} &= -D_3\sin 53^\circ = -(550\text{ km})(0.799) = -439\text{ km}.
\end{aligned}$$
We have given a minus sign to each component that in Fig. 14b points in the $-x$ or $-y$ direction. The components are shown in the Table in the margin.
5. **Add the components**: We add the x components together, and we add the y components together to obtain the x and y components of the resultant:
$$D_x = D_{1x} + D_{2x} + D_{3x} = 620\text{ km} + 311\text{ km} - 331\text{ km} = 600\text{ km}$$
$$D_y = D_{1y} + D_{2y} + D_{3y} = 0\text{ km} - 311\text{ km} - 439\text{ km} = -750\text{ km}.$$
The x and y components are 600 km and −750 km, and point respectively to the east and south. This is one way to give the answer.
6. **Magnitude and direction**: We can also give the answer as
$$D_R = \sqrt{D_x^2 + D_y^2} = \sqrt{(600)^2 + (-750)^2}\text{ km} = 960\text{ km}$$
$$\tan\theta = \frac{D_y}{D_x} = \frac{-750\text{ km}}{600\text{ km}} = -1.25, \qquad \text{so } \theta = -51^\circ.$$
Thus, the total displacement has magnitude 960 km and points 51° below the x axis (south of east), as was shown in our original sketch, Fig. 14a.

5 Unit Vectors

Vectors can be conveniently written in terms of *unit vectors*. A **unit vector** is defined to have a magnitude exactly equal to one (1). It is useful to define unit vectors that point along coordinate axes, and in an x, y, z rectangular coordinate system these unit vectors are called $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$. They point, respectively, along the positive x, y, and z axes as shown in Fig. 15. Like other vectors, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ do not have to be placed at the origin, but can be placed elsewhere as long as the direction and unit length remain unchanged. It is common to write unit vectors with a "hat": $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ (and we will do so in this text) as a reminder that each is a unit vector.

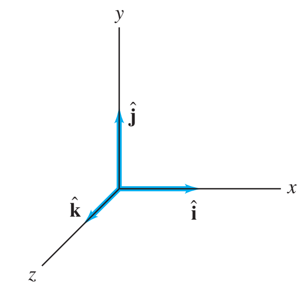


FIGURE 15 Unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ along the x, y, and z axes.

Because of the definition of multiplication of a vector by a scalar (Section 3), the components of a vector $\vec{\mathbf{V}}$ can be written $\vec{\mathbf{V}}_x = V_x\hat{\mathbf{i}}$, $\vec{\mathbf{V}}_y = V_y\hat{\mathbf{j}}$, and $\vec{\mathbf{V}}_z = V_z\hat{\mathbf{k}}$. Hence any vector $\vec{\mathbf{V}}$ can be written in terms of its components as

$$\vec{\mathbf{V}} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}}. \tag{5}$$

Unit vectors are helpful when adding vectors analytically by components. For example, Eq. 4 can be seen to be true by using unit vector notation for each vector (which we write for the two-dimensional case, with the extension to three dimensions being straightforward):

$$\begin{aligned}\vec{\mathbf{V}} = (V_x)\hat{\mathbf{i}} + (V_y)\hat{\mathbf{j}} &= \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 \\ &= (V_{1x}\hat{\mathbf{i}} + V_{1y}\hat{\mathbf{j}}) + (V_{2x}\hat{\mathbf{i}} + V_{2y}\hat{\mathbf{j}}) \\ &= (V_{1x} + V_{2x})\hat{\mathbf{i}} + (V_{1y} + V_{2y})\hat{\mathbf{j}}.\end{aligned}$$

Comparing the first line to the third line, we get Eq. 4.

EXAMPLE 4 **Using unit vectors.** Write the vectors of Example 2 in unit vector notation, and perform the addition.

APPROACH We use the components we found in Example 2,

$$D_{1x} = 0, \quad D_{1y} = 22.0\,\text{km}, \text{ and } \quad D_{2x} = 23.5\,\text{km}, \quad D_{2y} = -40.7\,\text{km},$$

and we now write them in the form of Eq. 5.

SOLUTION We have

$$\begin{aligned}\vec{\mathbf{D}}_1 &= 0\hat{\mathbf{i}} + 22.0\,\text{km}\,\hat{\mathbf{j}} \\ \vec{\mathbf{D}}_2 &= 23.5\,\text{km}\,\hat{\mathbf{i}} - 40.7\,\text{km}\,\hat{\mathbf{j}}.\end{aligned}$$

Then

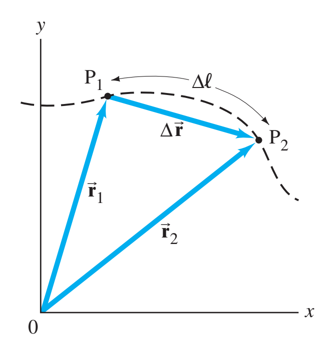
$$\begin{aligned}\vec{\mathbf{D}} = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 &= (0 + 23.5)\,\text{km}\,\hat{\mathbf{i}} + (22.0 - 40.7)\,\text{km}\,\hat{\mathbf{j}} \\ &= 23.5\,\text{km}\,\hat{\mathbf{i}} - 18.7\,\text{km}\,\hat{\mathbf{j}}.\end{aligned}$$

The components of the resultant displacement, $\vec{\mathbf{D}}$, are $D_x = 23.5\,\text{km}$ and $D_y = -18.7\,\text{km}$. The magnitude of $\vec{\mathbf{D}}$ is $D = \sqrt{(23.5\,\text{km})^2 + (18.7\,\text{km})^2} = 30.0\,\text{km}$, just as in Example 2.

6 Vector Kinematics

We can now extend our definitions of velocity and acceleration in a formal way to two- and three-dimensional motion. Suppose a particle follows a path in the xy plane as shown in Fig. 16. At time t_1, the particle is at point P_1, and at time t_2, it is at point P_2. The vector $\vec{\mathbf{r}}_1$ is the position vector of the particle at time t_1 (it represents the displacement of the particle from the origin of the coordinate system). And $\vec{\mathbf{r}}_2$ is the position vector at time t_2.

FIGURE 16 Path of a particle in the xy plane. At time t_1 the particle is at point P_1 given by the position vector $\vec{\mathbf{r}}_1$; at t_2 the particle is at point P_2 given by the position vector $\vec{\mathbf{r}}_2$. The displacement vector for the time interval $t_2 - t_1$ is $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$.



In one dimension, we defined displacement as the *change in position* of the particle. In the more general case of two or three dimensions, the **displacement vector** is defined as the vector representing change in position. We call it $\Delta\vec{\mathbf{r}}$,[†] where

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1.$$

This represents the displacement during the time interval $\Delta t = t_2 - t_1$.

[†]We used $\vec{\mathbf{D}}$ for the displacement vector earlier in the Chapter for illustrating vector addition. The new notation here, $\Delta\vec{\mathbf{r}}$, emphasizes that it is the difference between two position vectors.

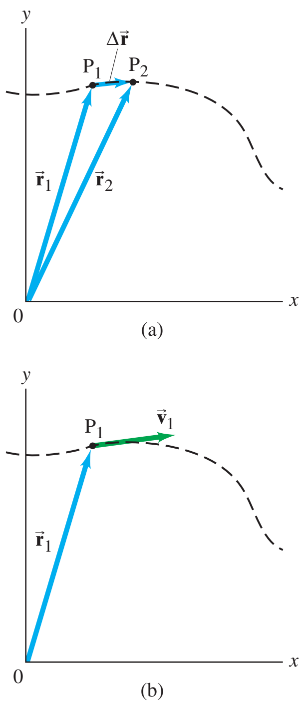
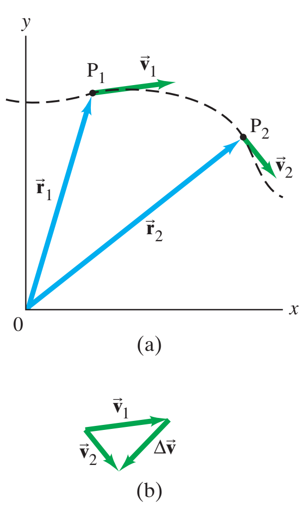


FIGURE 17 (a) As we take Δt and $\Delta\vec{\mathbf{r}}$ smaller and smaller [compare to Fig. 16] we see that the direction of $\Delta\vec{\mathbf{r}}$ and of the instantaneous velocity ($\Delta\vec{\mathbf{r}}/\Delta t$, where $\Delta t \rightarrow 0$) is (b) tangent to the curve at P_1.

FIGURE 18 (a) Velocity vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ at instants t_1 and t_2 for a particle at points P_1 and P_2, as in Fig. 16. (b) The direction of the average acceleration is in the direction of $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$.



In unit vector notation, we can write

$$\vec{\mathbf{r}}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}, \tag{6a}$$

where x_1, y_1, and z_1 are the coordinates of point P_1. Similarly,

$$\vec{\mathbf{r}}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}.$$

Hence

$$\Delta\vec{\mathbf{r}} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}. \tag{6b}$$

If the motion is along the x axis only, then $y_2 - y_1 = 0$, $z_2 - z_1 = 0$, and the magnitude of the displacement is $\Delta r = x_2 - x_1$, which is consistent with a one-dimensional equation. Even in one dimension, displacement is a vector, as are velocity and acceleration.

The **average velocity vector** over the time interval $\Delta t = t_2 - t_1$ is defined as

$$\text{average velocity} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}. \tag{7}$$

Now let us consider shorter and shorter time intervals—that is, we let Δt approach zero so that the distance between points P_2 and P_1 also approaches zero, Fig. 17. We define the **instantaneous velocity vector** as the limit of the average velocity as Δt approaches zero:

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}. \tag{8}$$

The direction of $\vec{\mathbf{v}}$ at any moment is along the line tangent to the path at that moment (Fig. 17).

Note that the magnitude of the average velocity in Fig. 16 is not equal to the average speed, which is the actual distance traveled along the path, $\Delta\ell$, divided by Δt. In some special cases, the average speed and average velocity are equal (such as motion along a straight line in one direction), but in general they are not. However, in the limit $\Delta t \rightarrow 0$, Δr always approaches $\Delta\ell$, so the instantaneous speed *always* equals the magnitude of the instantaneous velocity at any time.

The instantaneous velocity (Eq. 8) is equal to the derivative of the position vector with respect to time. Equation 8 can be written in terms of components starting with Eq. 6a as:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}, \tag{9}$$

where $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$ are the x, y, and z components of the velocity. Note that $d\hat{\mathbf{i}}/dt = d\hat{\mathbf{j}}/dt = d\hat{\mathbf{k}}/dt = 0$ since these unit vectors are constant in both magnitude and direction.

Acceleration in two or three dimensions is treated in a similar way. The **average acceleration vector**, over a time interval $\Delta t = t_2 - t_1$ is defined as

$$\text{average acceleration} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1}, \tag{10}$$

where $\Delta\vec{\mathbf{v}}$ is the change in the instantaneous velocity vector during that time interval: $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$. Note that $\vec{\mathbf{v}}_2$ in many cases, such as in Fig. 18a, may not be in the same direction as $\vec{\mathbf{v}}_1$. Hence the average acceleration vector may be in a different direction from either $\vec{\mathbf{v}}_1$ or $\vec{\mathbf{v}}_2$ (Fig. 18b). Furthermore, $\vec{\mathbf{v}}_2$ and $\vec{\mathbf{v}}_1$ may have the same magnitude but different directions, and the difference of two such vectors will not be zero. Hence acceleration can result from either a change in the magnitude of the velocity, or from a change in direction of the velocity, or from a change in both.

The **instantaneous acceleration vector** is defined as the limit of the average acceleration vector as the time interval Δt is allowed to approach zero:

$$\vec{\mathbf{a}} = \lim_{\Delta \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}, \tag{11}$$

and is thus the derivative of $\vec{\mathbf{v}}$ with respect to t.

We can write $\vec{\mathbf{a}}$ using components:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}} + \frac{dv_z}{dt}\hat{\mathbf{k}}$$
$$= a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}, \qquad \textbf{(12)}$$

where $a_x = dv_x/dt$, etc. Because $v_x = dx/dt$, then $a_x = dv_x/dt = d^2x/dt^2$. Thus we can also write the acceleration as

$$\vec{\mathbf{a}} = \frac{d^2x}{dt^2}\hat{\mathbf{i}} + \frac{d^2y}{dt^2}\hat{\mathbf{j}} + \frac{d^2z}{dt^2}\hat{\mathbf{k}}. \qquad \textbf{(12c)}$$

The instantaneous acceleration will be nonzero not only when the magnitude of the velocity changes but also if its direction changes. For example, a person riding in a car traveling at constant speed around a curve, or a child riding on a merry-go-round, will both experience an acceleration because of a change in the direction of the velocity, even though the speed may be constant.

In general, we will use the terms "velocity" and "acceleration" to mean the instantaneous values. If we want to discuss average values, we will use the word "average."

EXAMPLE 5 **Position given as a function of time.** The position of a particle as a function of time is given by

$$\vec{\mathbf{r}} = \left[(5.0\,\mathrm{m/s})t + (6.0\,\mathrm{m/s^2})t^2\right]\hat{\mathbf{i}} + \left[(7.0\,\mathrm{m}) - (3.0\,\mathrm{m/s^3})t^3\right]\hat{\mathbf{j}},$$

where r is in meters and t is in seconds. (a) What is the particle's displacement between $t_1 = 2.0\,\mathrm{s}$ and $t_2 = 3.0\,\mathrm{s}$? (b) Determine the particle's instantaneous velocity and acceleration as a function of time. (c) Evaluate $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ at $t = 3.0\,\mathrm{s}$.

APPROACH For (a), we find $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$, inserting $t_1 = 2.0\,\mathrm{s}$ for finding $\vec{\mathbf{r}}_1$, and $t_2 = 3.0\,\mathrm{s}$ for $\vec{\mathbf{r}}_2$. For (b), we take derivatives (Eqs. 9 and 11), and for (c) we substitute $t = 3.0\,\mathrm{s}$ into our results in (b).

SOLUTION (a) At $t_1 = 2.0\,\mathrm{s}$,

$$\vec{\mathbf{r}}_1 = \left[(5.0\,\mathrm{m/s})(2.0\,\mathrm{s}) + (6.0\,\mathrm{m/s^2})(2.0\,\mathrm{s})^2\right]\hat{\mathbf{i}} + \left[(7.0\,\mathrm{m}) - (3.0\,\mathrm{m/s^3})(2.0\,\mathrm{s})^3\right]\hat{\mathbf{j}}$$
$$= (34\,\mathrm{m})\hat{\mathbf{i}} - (17\,\mathrm{m})\hat{\mathbf{j}}.$$

Similarly, at $t_2 = 3.0\,\mathrm{s}$,

$$\vec{\mathbf{r}}_2 = (15\,\mathrm{m} + 54\,\mathrm{m})\hat{\mathbf{i}} + (7.0\,\mathrm{m} - 81\,\mathrm{m})\hat{\mathbf{j}} = (69\,\mathrm{m})\hat{\mathbf{i}} - (74\,\mathrm{m})\hat{\mathbf{j}}.$$

Thus

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = (69\,\mathrm{m} - 34\,\mathrm{m})\hat{\mathbf{i}} + (-74\,\mathrm{m} + 17\,\mathrm{m})\hat{\mathbf{j}} = (35\,\mathrm{m})\hat{\mathbf{i}} - (57\,\mathrm{m})\hat{\mathbf{j}}.$$

That is, $\Delta x = 35\,\mathrm{m}$, and $\Delta y = -57\,\mathrm{m}$.

(b) To find velocity, we take the derivative of the given $\vec{\mathbf{r}}$ with respect to time, noting that $d(t^2)/dt = 2t$, and $d(t^3)/dt = 3t^2$:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \left[5.0\,\mathrm{m/s} + (12\,\mathrm{m/s^2})t\right]\hat{\mathbf{i}} + \left[0 - (9.0\,\mathrm{m/s^3})t^2\right]\hat{\mathbf{j}}.$$

The acceleration is (keeping only two significant figures):

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = (12\,\mathrm{m/s^2})\hat{\mathbf{i}} - (18\,\mathrm{m/s^3})t\,\hat{\mathbf{j}}.$$

Thus $a_x = 12\,\mathrm{m/s^2}$ is constant; but $a_y = -(18\,\mathrm{m/s^3})t$ depends linearly on time, increasing in magnitude with time in the negative y direction.

(c) We substitute $t = 3.0\,\mathrm{s}$ into the equations we just derived for $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$:

$$\vec{\mathbf{v}} = (5.0\,\mathrm{m/s} + 36\,\mathrm{m/s})\hat{\mathbf{i}} - (81\,\mathrm{m/s})\hat{\mathbf{j}} = (41\,\mathrm{m/s})\hat{\mathbf{i}} - (81\,\mathrm{m/s})\hat{\mathbf{j}}$$
$$\vec{\mathbf{a}} = (12\,\mathrm{m/s^2})\hat{\mathbf{i}} - (54\,\mathrm{m/s^2})\hat{\mathbf{j}}.$$

Their magnitudes at $t = 3.0\,\mathrm{s}$ are $v = \sqrt{(41\,\mathrm{m/s})^2 + (81\,\mathrm{m/s})^2} = 91\,\mathrm{m/s}$, and $a = \sqrt{(12\,\mathrm{m/s^2})^2 + (54\,\mathrm{m/s^2})^2} = 55\,\mathrm{m/s^2}$.

Constant Acceleration

In two or three dimensions, if the acceleration vector, $\vec{\mathbf{a}}$, is constant in magnitude and direction, then $a_x = \text{constant}$, $a_y = \text{constant}$, $a_z = \text{constant}$. The average acceleration in this case is equal to the instantaneous acceleration at any moment. In two dimensions we let $\vec{\mathbf{v}}_0 = v_{x0}\hat{\mathbf{i}} + v_{y0}\hat{\mathbf{j}}$ be the initial velocity, and we apply Eqs. 6a, 9, and 12b for the position vector, $\vec{\mathbf{r}}$, velocity, $\vec{\mathbf{v}}$, and acceleration, $\vec{\mathbf{a}}$.

TABLE 1 Kinematic Equations for Constant Acceleration in 2 Dimensions

x component (horizontal)	*y* component (vertical)
$v_x = v_{x0} + a_x t$	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$	$v_y^2 = v_{y0}^2 + 2a_y (y - y_0)$

The first two of the equations in Table 1 can be written more formally in vector notation.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t \qquad [\vec{\mathbf{a}} = \text{constant}] \qquad \textbf{(13a)}$$

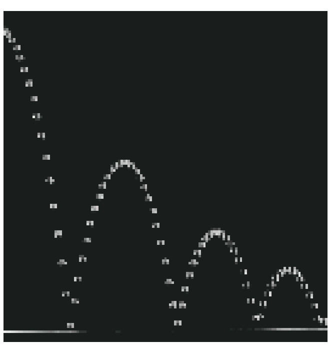
$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2. \qquad [\vec{\mathbf{a}} = \text{constant}] \qquad \textbf{(13b)}$$

Here, $\vec{\mathbf{r}}$ is the position vector at any time, and $\vec{\mathbf{r}}_0$ is the position vector at $t = 0$. These equations are the vector equivalent of. In practical situations, we usually use the component form given in Table 1.

7 Projectile Motion

Recall one-dimensional motion of an object in terms of displacement, velocity, and acceleration, including purely vertical motion of a falling object undergoing acceleration due to gravity. Now we examine the more general translational motion of objects moving through the air in two dimensions near the Earth's surface, such as a golf ball, a thrown or batted baseball, kicked footballs, and speeding bullets. These are all examples of **projectile motion** (see Fig. 19), which we can describe as taking place in two dimensions.

FIGURE 19 This strobe photograph of a ball making a series of bounces shows the characteristic "parabolic" path of projectile motion.



Although air resistance is often important, in many cases its effect can be ignored, and we will ignore it in the following analysis. We will not be concerned now with the process by which the object is thrown or projected. We consider only its motion *after* it has been projected, and *before* it lands or is caught—that is, we analyze our projected object only when it is moving freely through the air under the action of gravity alone. Then the acceleration of the object is that due to gravity, which acts downward with magnitude $g = 9.80\,\text{m/s}^2$, and we assume it is constant.†

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time $t = 0$ at the origin of an *xy* coordinate system (so $x_0 = y_0 = 0$).

Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal (x) direction, v_{x0}. See Fig. 20, where an object falling vertically is also shown for comparison. The velocity vector $\vec{\mathbf{v}}$ at each instant points in the direction of the ball's motion at that instant and is always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of the velocity, v_x and v_y, separately, and we can apply the kinematic equations to the x and y components of the motion.

First we examine the vertical (y) component of the motion. At the instant the ball leaves the table's top ($t = 0$), it has only an x component of velocity. Once the

†This restricts us to objects whose distance traveled and maximum height above the Earth are small compared to the Earth's radius (6400 km).

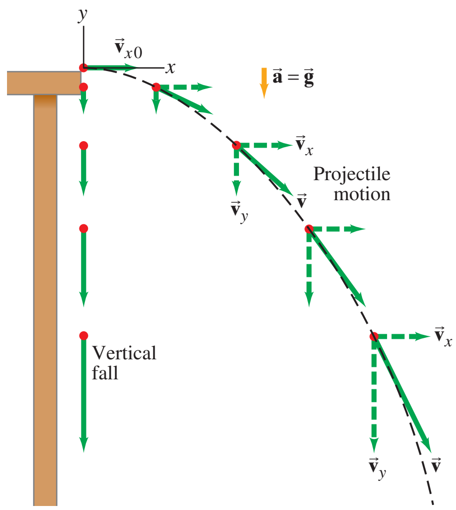


FIGURE 20 Projectile motion of a small ball projected horizontally. The dashed black line represents the path of the object. The velocity vector $\vec{\mathbf{v}}$ at each point is in the direction of motion and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting at the same point is shown at the left for comparison; v_y is the same for the falling object and the projectile.)

ball leaves the table (at $t = 0$), it experiences a vertically downward acceleration g, the acceleration due to gravity. Thus v_y is initially zero $(v_{y0} = 0)$ but increases continually in the downward direction (until the ball hits the ground). Let us take y to be positive upward. Then $a_y = -g$, and we can write $v_y = -gt$ since we set $v_{y0} = 0$. The vertical displacement is given by $y = -\frac{1}{2}gt^2$.

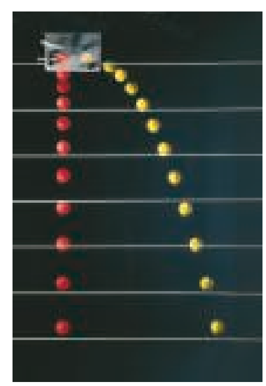
In the horizontal direction, on the other hand, the acceleration is zero (we are ignoring air resistance). With $a_x = 0$, the horizontal component of velocity, v_x, remains constant, equal to its initial value, v_{x0}, and thus has the same magnitude at each point on the path. The horizontal displacement is then given by $x = v_{x0}t$. The two vector components, $\vec{\mathbf{v}}_x$ and $\vec{\mathbf{v}}_y$, can be added vectorially at any instant to obtain the velocity $\vec{\mathbf{v}}$ at that time (that is, for each point on the path), as shown in Fig. 20.

One result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. This is because the vertical motions are the same in both cases, as shown in Fig. 20. Figure 21 is a multiple-exposure photograph of an experiment that confirms this.

EXERCISE D Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

If an object is projected at an upward angle, as in Fig. 22, the analysis is similar, except that now there is an initial vertical component of velocity, v_{y0}. Because of the downward acceleration of gravity, the upward component of velocity v_y gradually decreases with time until the object reaches the highest point on its path, at which point $v_y = 0$. Subsequently the object moves downward (Fig. 22) and v_y increases in the downward direction, as shown (that is, becoming more negative). As before, v_x remains constant.

FIGURE 21 Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.



Richard Megna/Fundamental Photographs, NYC

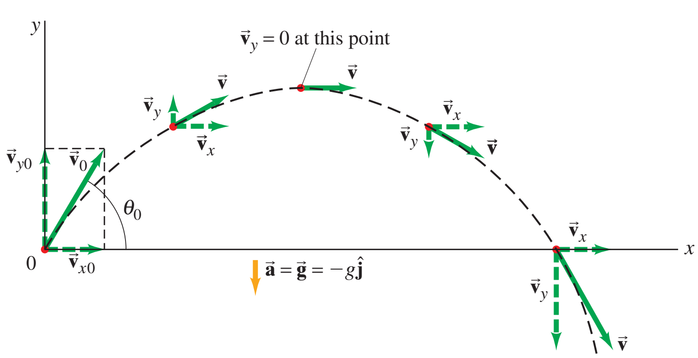


FIGURE 22 Path of a projectile fired with initial velocity $\vec{\mathbf{v}}_0$ at angle θ_0 to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The acceleration $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$ is downward. That is, $\vec{\mathbf{a}} = \vec{\mathbf{g}} = -g\hat{\mathbf{j}}$ where $\hat{\mathbf{j}}$ is the unit vector in the positive y direction.

8 Solving Problems Involving Projectile Motion

We now work through several Examples of projectile motion quantitatively.

We have simplified Equations (Table 1) for the case of projectile motion because we can set $a_x = 0$. See Table 2, which assumes y is positive upward, so $a_y = -g = -9.80\ \mathrm{m/s^2}$. Note that if θ is chosen relative to the $+x$ axis, as in Fig. 22, then

$$v_{x0} = v_0 \cos\theta_0,$$

$$v_{y0} = v_0 \sin\theta_0.$$

PROBLEM SOLVING
Choice of time interval

In doing Problems involving projectile motion, we must consider a time interval for which our chosen object is in the air, influenced only by gravity. We do not consider the throwing (or projecting) process, nor the time after the object lands or is caught, because then other influences act on the object, and we can no longer set $\vec{\mathbf{a}} = \vec{\mathbf{g}}$.

TABLE 2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\ \mathrm{m/s^2}$)

Horizontal Motion ($a_x = 0, v_x =$ constant)	Vertical Motion[†] ($a_y = -g =$ constant)
$v_x = v_{x0}$	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

[†] If y is taken positive downward, the minus ($-$) signs in front of g become plus ($+$) signs.

PROBLEM SOLVING

Projectile Motion

Solving problems involving projectile motion can require creativity, and cannot be done just by following some rules. Certainly you must avoid just plugging numbers into equations that seem to "work."

1. As always, **read** carefully; **choose** the **object** (or objects) you are going to analyze.
2. **Draw** a careful **diagram** showing what is happening to the object.
3. **Choose** an origin and an xy **coordinate system**.
4. Decide on the **time interval**, which for projectile motion can only include motion under the effect of gravity alone, not throwing or landing. The time interval must be the same for the x and y analyses. The x and y motions are connected by the common time.
5. **Examine** the horizontal (x) and vertical (y) **motions** separately. If you are given the initial velocity, you may want to resolve it into its x and y components.
6. List the **known** and **unknown** quantities, choosing $a_x = 0$ and $a_y = -g$ or $+g$, where $g = 9.80\ \mathrm{m/s^2}$, and using the $+$ or $-$ sign, depending on whether you choose y positive down or up. Remember that v_x never changes throughout the trajectory, and that $v_y = 0$ at the highest point of any trajectory that returns downward. The velocity just before landing is generally not zero.
7. Think for a minute before jumping into the equations. A little planning goes a long way. **Apply** the **relevant equations** (Table 2), combining equations if necessary. You may need to combine components of a vector to get magnitude and direction (Eqs. 3).

EXAMPLE 6 **Driving off a cliff.** A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.

APPROACH We explicitly follow the steps of the Problem Solving Strategy above.

SOLUTION

1. and **2. Read, choose the object, and draw a diagram**. Our object is the motorcycle and driver, taken as a single unit. The diagram is shown in Fig. 23.

3. Choose a coordinate system. We choose the y direction to be positive upward, with the top of the cliff as $y_0 = 0$. The x direction is horizontal with $x_0 = 0$ at the point where the motorcycle leaves the cliff.

4. Choose a time interval. We choose our time interval to begin $(t = 0)$ just as the motorcycle leaves the cliff top at position $x_0 = 0, y_0 = 0;$ our time interval ends just before the motorcycle hits the ground below.

5. Examine x and y motions. In the horizontal (x) direction, the acceleration $a_x = 0,$ so the velocity is constant. The value of x when the motorcycle reaches the ground is $x = +90.0\text{ m}.$ In the vertical direction, the acceleration is the acceleration due to gravity, $a_y = -g = -9.80\text{ m/s}^2.$ The value of y when the motorcycle reaches the ground is $y = -50.0\text{ m}.$ The initial velocity is horizontal and is our unknown, v_{x0}; the initial vertical velocity is zero, $v_{y0} = 0.$

6. List knowns and unknowns. See the Table in the margin. Note that in addition to not knowing the initial horizontal velocity v_{x0} (which stays constant until landing), we also do not know the time t when the motorcycle reaches the ground.

7. Apply relevant equations. The motorcycle maintains constant v_x as long as it is in the air. The time it stays in the air is determined by the y motion—when it hits the ground. So we first find the time using the y motion, and then use this time value in the x equations. To find out how long it takes the motorcycle to reach the ground below, we use Equation (Table 2) for the vertical (y) direction with $y_0 = 0$ and $v_{y0} = 0$:

$$\begin{aligned} y &= y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \\ &= 0 + 0 + \tfrac{1}{2}(-g)t^2 \end{aligned}$$

or

$$y = -\tfrac{1}{2}gt^2.$$

We solve for t and set $y = -50.0\text{ m}$:

$$t = \sqrt{\frac{2y}{-g}} = \sqrt{\frac{2(-50.0\text{ m})}{-9.80\text{ m/s}^2}} = 3.19\text{ s}.$$

To calculate the initial velocity, v_{x0}, we again use Equation, but this time for the horizontal (x) direction, with $a_x = 0$ and $x_0 = 0$:

$$\begin{aligned} x &= x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 \\ &= 0 + v_{x0}t + 0 \end{aligned}$$

or

$$x = v_{x0}t.$$

Then

$$v_{x0} = \frac{x}{t} = \frac{90.0\text{ m}}{3.19\text{ s}} = 28.2\text{ m/s},$$

which is about 100 km/h (roughly 60 mi/h).

NOTE In the time interval of the projectile motion, the only acceleration is g in the negative y direction. The acceleration in the x direction is zero.

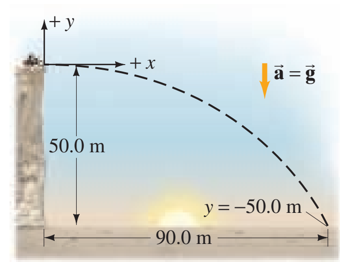


FIGURE 23 Example 6.

Known	Unknown
$x_0 = y_0 = 0$	v_{x0}
$x = 90.0\text{ m}$	t
$y = -50.0\text{ m}$	
$a_x = 0$	
$a_y = -g = -9.80\text{ m/s}^2$	
$v_{y0} = 0$	

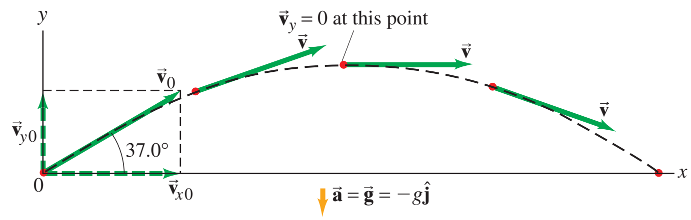


FIGURE 24 Example 7.

PHYSICS APPLIED
Sports

EXAMPLE 7 **A kicked football.** A football is kicked at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown in Fig. 24. Calculate (*a*) the maximum height, (*b*) the time of travel before the football hits the ground, (*c*) how far away it hits the ground, (*d*) the velocity vector at the maximum height, and (*e*) the acceleration vector at maximum height. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

APPROACH This may seem difficult at first because there are so many questions. But we can deal with them one at a time. We take the y direction as positive upward, and treat the x and y motions separately. The total time in the air is again determined by the y motion. The x motion occurs at constant velocity. The y component of velocity varies, being positive (upward) initially, decreasing to zero at the highest point, and then becoming negative as the football falls.

SOLUTION We resolve the initial velocity into its components (Fig. 24):

$$v_{x0} = v_0 \cos 37.0° = (20.0\,\text{m/s})(0.799) = 16.0\,\text{m/s}$$
$$v_{y0} = v_0 \sin 37.0° = (20.0\,\text{m/s})(0.602) = 12.0\,\text{m/s}.$$

(*a*) We consider a time interval that begins just after the football loses contact with the foot until it reaches its maximum height. During this time interval, the acceleration is g downward. At the maximum height, the velocity is horizontal (Fig. 24), so $v_y = 0$; and this occurs at a time given by $v_y = v_{y0} - gt$ with $v_y = 0$ (see Table 2). Thus

$$t = \frac{v_{y0}}{g} = \frac{(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 1.224\,\text{s} \approx 1.22\,\text{s}.$$

With $y_0 = 0$, we have

$$y = v_{y0}t - \tfrac{1}{2}gt^2$$
$$= (12.0\,\text{m/s})(1.224\,\text{s}) - \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.224\,\text{s})^2 = 7.35\,\text{m}.$$

Alternatively, we could have solved for y, and found

$$y = \frac{v_{y0}^2 - v_y^2}{2g} = \frac{(12.0\,\text{m/s})^2 - (0\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = 7.35\,\text{m}.$$

The maximum height is 7.35 m.

(*b*) To find the time it takes for the ball to return to the ground, we consider a different time interval, starting at the moment the ball leaves the foot $(t = 0,\ y_0 = 0)$ and ending just before the ball touches the ground ($y = 0$ again). With $y_0 = 0$ and also set $y = 0$ (ground level):

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$
$$0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2.$$

This equation can be easily factored:

$$t\left(\tfrac{1}{2}gt - v_{y0}\right) = 0.$$

There are two solutions, $t = 0$ (which corresponds to the initial point, y_0), and

$$t = \frac{2v_{y0}}{g} = \frac{2(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 2.45\,\text{s},$$

which is the total travel time of the football.

NOTE The time needed for the whole trip, $t = 2v_{y0}/g = 2.45\text{ s}$, is double the time to reach the highest point, calculated in (*a*). That is, the time to go up equals the time to come back down to the same level (ignoring air resistance).

(*c*) The total distance traveled in the x direction is found (see Table 2) with $x_0 = 0$, $a_x = 0$, $v_{x0} = 16.0\text{ m/s}$:

$$x = v_{x0}t = (16.0\text{ m/s})(2.45\text{ s}) = 39.2\text{ m}.$$

(*d*) At the highest point, there is no vertical component to the velocity. There is only the horizontal component (which remains constant throughout the flight), so $v = v_{x0} = v_0 \cos 37.0° = 16.0\text{ m/s}$.

(*e*) The acceleration vector is the same at the highest point as it is throughout the flight, which is 9.80 m/s^2 downward.

NOTE We treated the football as if it were a particle, ignoring its rotation. We also ignored air resistance. Because air resistance is significant on a football, our results are only estimates.

EXERCISE E Two balls are thrown in the air at different angles, but each reaches the same height. Which ball remains in the air longer: the one thrown at the steeper angle or the one thrown at a shallower angle?

CONCEPTUAL EXAMPLE 8 **Where does the apple land?** A child sits upright in a wagon which is moving to the right at constant speed as shown in Fig. 25. The child extends her hand and throws an apple straight upward (from her own point of view, Fig. 25a), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (*a*) behind the wagon, (*b*) in the wagon, or (*c*) in front of the wagon?

RESPONSE The child throws the apple straight up from her own reference frame with initial velocity $\vec{\mathbf{v}}_{y0}$ (Fig. 25a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the speed of the wagon, $\vec{\mathbf{v}}_{x0}$. Thus, to a person on the ground, the apple will follow the path of a projectile as shown in Fig. 25b. The apple experiences no horizontal acceleration, so $\vec{\mathbf{v}}_{x0}$ will stay constant and equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal velocity. When the apple comes down, it will drop right into the outstretched hand of the child. The answer is (*b*).

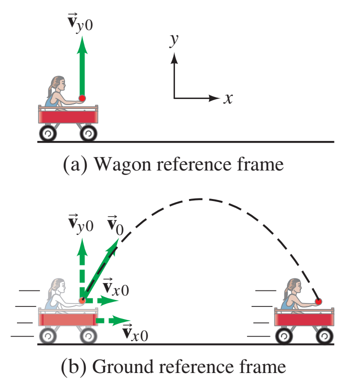


FIGURE 25 Example 8.

CONCEPTUAL EXAMPLE 9 **The wrong strategy.** A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away, Fig. 26. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.

RESPONSE Both the water balloon and the boy in the tree start falling at the same instant, and in a time t they each fall the same vertical distance $y = \frac{1}{2}gt^2$, much like Fig. 21. In the time it takes the water balloon to travel the horizontal distance d, the balloon will have the same y position as the falling boy. Splat. If the boy had stayed in the tree, he would have avoided the humiliation.

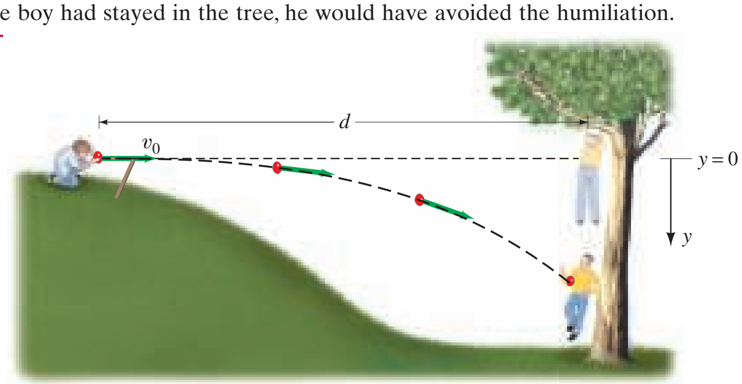


FIGURE 26 Example 9.

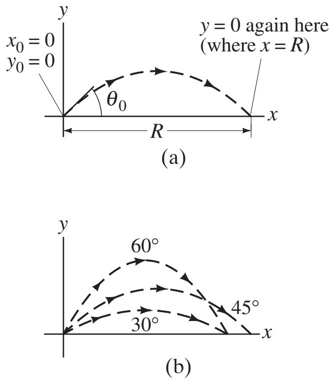


FIGURE 27 Example 10.
(a) The range R of a projectile;
(b) there are generally two angles θ_0 that will give the same range. Can you show that if one angle is θ_{01}, the other is $\theta_{02} = 90° - \theta_{01}$?

EXAMPLE 10 **Level horizontal range.** (*a*) Derive a formula for the horizontal range R of a projectile in terms of its initial speed v_0 and angle θ_0. The horizontal *range* is defined as the horizontal distance the projectile travels before returning to its original height (which is typically the ground); that is, $y\,(\text{final}) = y_0$. See Fig. 27a. (*b*) Suppose one of Napoleon's cannons had a muzzle speed, v_0, of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?

APPROACH The situation is the same as in Example 7, except we are now not given numbers in (*a*). We will algebraically manipulate equations to obtain our result.

SOLUTION (*a*) We set $x_0 = 0$ and $y_0 = 0$ at $t = 0$. After the projectile travels a horizontal distance R, it returns to the same level, $y = 0$, the final point. We choose our time interval to start $(t = 0)$ just after the projectile is fired and to end when it returns to the same vertical height. To find a general expression for R, we set both $y = 0$ and $y_0 = 0$ (see Table 2) for the vertical motion, and obtain

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2$$

so

$$0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2.$$

We solve for t, which gives two solutions: $t = 0$ and $t = 2v_{y0}/g$. The first solution corresponds to the initial instant of projection and the second is the time when the projectile returns to $y = 0$. Then the range, R, will be equal to x at the moment t has this value, (see Table 2) for the *horizontal* motion ($x = v_{x0}t$, with $x_0 = 0$). Thus we have:

$$R = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}, \qquad [y = y_0]$$

where we have written $v_{x0} = v_0\cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$. This is the result we sought. It can be rewritten, using the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$:

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \qquad [\text{only if } y\,(\text{final}) = y_0]$$

We see that the maximum range, for a given initial velocity v_0, is obtained when $\sin 2\theta$ takes on its maximum value of 1.0, which occurs for $2\theta_0 = 90°$; so

$$\theta_0 = 45° \text{ for maximum range, and } R_{\max} = v_0^2/g.$$

[When air resistance is important, the range is less for a given v_0, and the maximum range is obtained at an angle smaller than 45°.]

NOTE The maximum range increases by the square of v_0, so doubling the muzzle velocity of a cannon increases its maximum range by a factor of 4.

(*b*) We put $R = 320\text{ m}$ into the equation we just derived, and (assuming, unrealistically, no air resistance) we solve it to find

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(320\text{ m})(9.80\text{ m/s}^2)}{(60.0\text{ m/s})^2} = 0.871.$$

We want to solve for an angle θ_0 that is between 0° and 90°, which means $2\theta_0$ in this equation can be as large as 180°. Thus, $2\theta_0 = 60.6°$ is a solution, but $2\theta_0 = 180° - 60.6° = 119.4°$ is also a solution. In general we will have two solutions (see Fig. 27b), which in the present case are given by

$$\theta_0 = 30.3° \quad \text{or} \quad 59.7°.$$

Either angle gives the same range. Only when $\sin 2\theta_0 = 1$ (so $\theta_0 = 45°$) is there a single solution (that is, both solutions are the same).

EXERCISE F The maximum range of a projectile is found to be 100 m. If the projectile strikes the ground a distance of 82 m away, what was the angle of launch? (*a*) 35° or 55°; (*b*) 30° or 60°; (*c*) 27.5° or 62.5°; (*d*) 13.75° or 76.25°.

The level range formula derived in Example 10 applies only if takeoff and landing are at the same height $(y = y_0)$. Example 11 below considers a case where they are not equal heights $(y \neq y_0)$.

EXAMPLE 11 **A punt.** Suppose the football in Example 7 was punted and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0,\ y_0 = 0$.

PHYSICS APPLIED
Sports

PROBLEM SOLVING
Do not use any formula unless you are sure its range of validity fits the problem; the range formula does not apply here because $y \neq y_0$

APPROACH The x and y motions are again treated separately. But we cannot use the range formula from Example 10 because it is valid only if $y\,(\text{final}) = y_0$, which is not the case here. Now we have $y_0 = 0$, and the football hits the ground where $y = -1.00\ \text{m}$ (see Fig. 28). We choose our time interval to start when the ball leaves his foot $(t = 0,\ y_0 = 0,\ x_0 = 0)$ and end just before the ball hits the ground $(y = -1.00\ \text{m})$. We can get x (see Table 3) where, $x = v_{x0}t$, since we know that $v_{x0} = 16.0\ \text{m/s}$ from Example 7. But first we must find t, the time at which the ball hits the ground, which we obtain from the y motion.

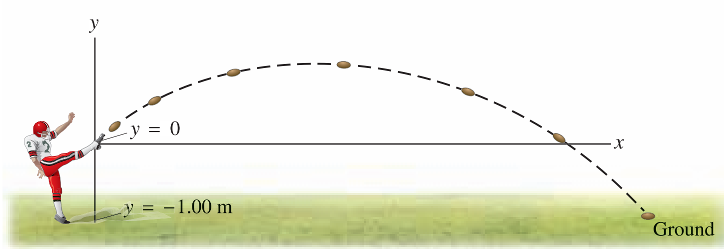


FIGURE 28 Example 11: the football leaves the punter's foot at $y = 0$, and reaches the ground where $y = -1.00\ \text{m}$.

SOLUTION With $y = -1.00\ \text{m}$ and $v_{y0} = 12.0\ \text{m/s}$ (see Example 7), we use the equation

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2,$$

and obtain

$$-1.00\ \text{m} = 0 + (12.0\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2.$$

We rearrange this equation into standard form $(ax^2 + bx + c = 0)$ so we can use the quadratic formula:

$$(4.90\ \text{m/s}^2)t^2 - (12.0\ \text{m/s})t - (1.00\ \text{m}) = 0.$$

The quadratic formula gives

$$t = \frac{12.0\ \text{m/s} \pm \sqrt{(-12.0\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(-1.00\ \text{m})}}{2(4.90\ \text{m/s}^2)}$$

$$= 2.53\ \text{s} \quad \text{or} \quad -0.081\ \text{s}.$$

The second solution would correspond to a time prior to our chosen time interval that begins at the kick, so it doesn't apply. With $t = 2.53\ \text{s}$ for the time at which the ball touches the ground, the horizontal distance the ball traveled is (using $v_{x0} = 16.0\ \text{m/s}$ from Example 7):

$$x = v_{x0}t = (16.0\ \text{m/s})(2.53\ \text{s}) = 40.5\ \text{m}.$$

Our assumption in Example 7 that the ball leaves the foot at ground level would result in an underestimate of about 1.3 m in the distance our punt traveled.

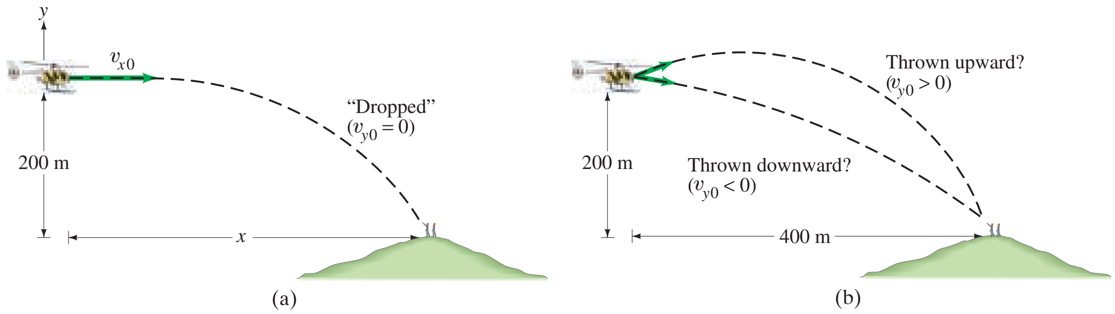


FIGURE 29 Example 12.

PHYSICS APPLIED
Reaching a target from a moving helicopter

EXAMPLE 12 Rescue helicopter drops supplies. A rescue helicopter wants to drop a package of supplies to isolated mountain climbers on a rocky ridge 200 m below. If the helicopter is traveling horizontally with a speed of 70 m/s (250 km/h), (*a*) how far in advance of the recipients (horizontal distance) must the package be dropped (Fig. 29a)? (*b*) Suppose, instead, that the helicopter releases the package a horizontal distance of 400 m in advance of the mountain climbers. What vertical velocity should the package be given (up or down) so that it arrives precisely at the climbers' position (Fig. 29b)? (*c*) With what speed does the package land in the latter case?

APPROACH We choose the origin of our xy coordinate system at the initial position of the helicopter, taking $+y$ upward, and use the kinematic equations (Table 2).

SOLUTION (*a*) We can find the time to reach the climbers using the vertical distance of 200 m. The package is "dropped" so initially it has the velocity of the helicopter, $v_{x0} = 70\,\mathrm{m/s}$, $v_{y0} = 0$. Then, since $y = -\frac{1}{2}gt^2$, we have

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-200\,\mathrm{m})}{9.80\,\mathrm{m/s^2}}} = 6.39\,\mathrm{s}.$$

The horizontal motion of the falling package is at constant speed of 70 m/s. So

$$x = v_{x0}t = (70\,\mathrm{m/s})(6.39\,\mathrm{s}) = 447\,\mathrm{m} \approx 450\,\mathrm{m},$$

assuming the given numbers were good to two significant figures.

(*b*) We are given $x = 400\,\mathrm{m}$, $v_{x0} = 70\,\mathrm{m/s}$, $y = -200\,\mathrm{m}$, and we want to find v_{y0} (see Fig. 29b). Like most problems, this one can be approached in various ways. Instead of searching for a formula or two, let's try to reason it out in a simple way, based on what we did in part (*a*). If we know t, perhaps we can get v_{y0}. Since the horizontal motion of the package is at constant speed (once it is released we don't care what the helicopter does), we have $x = v_{x0}t$, so

$$t = \frac{x}{v_{x0}} = \frac{400\,\mathrm{m}}{70\,\mathrm{m/s}} = 5.71\,\mathrm{s}.$$

Now let's try to use the vertical motion to get v_{y0}: $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$. Since $y_0 = 0$ and $y = -200\,\mathrm{m}$, we can solve for v_{y0}:

$$v_{y0} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{-200\,\mathrm{m} + \frac{1}{2}(9.80\,\mathrm{m/s^2})(5.71\,\mathrm{s})^2}{5.71\,\mathrm{s}} = -7.0\,\mathrm{m/s}.$$

Thus, in order to arrive at precisely the mountain climbers' position, the package must be thrown *downward* from the helicopter with a speed of 7.0 m/s.

(*c*) We want to know v of the package at $t = 5.71\,\mathrm{s}$. The components are:

$$v_x = v_{x0} = 70\,\mathrm{m/s}$$
$$v_y = v_{y0} - gt = -7.0\,\mathrm{m/s} - (9.80\,\mathrm{m/s^2})(5.71\,\mathrm{s}) = -63\,\mathrm{m/s}.$$

So $v = \sqrt{(70\,\mathrm{m/s})^2 + (-63\,\mathrm{m/s})^2} = 94\,\mathrm{m/s}$. (Better not to release the package from such an altitude, or use a parachute.)

Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a *parabola*, if we can ignore air resistance and can assume that $\vec{\mathbf{g}}$ is constant. To do so, we need to find y as a function of x by eliminating t between the two equations for horizontal and vertical motion (Table 2), and for simplicity we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$

$$y = v_{y0}t - \tfrac{1}{2}gt^2$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2. \tag{14}$$

We see that y as a function of x has the form

$$y = Ax - Bx^2,$$

where A and B are constants for any specific projectile motion. This is the well-known equation for a parabola. See Figs. 19 and 30.

The idea that projectile motion is parabolic was, in Galileo's day, at the forefront of physics research. Today we discuss it in this Chapter of introductory physics!

FIGURE 30 Examples of projectile motion—sparks (small hot glowing pieces of metal), water, and fireworks. The parabolic path characteristic of projectile motion is affected by air resistance.

Don Farrall/PhotoDisc/Getty Images

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Richard Megna/Fundamental Photographs, NYC

9 Relative Velocity

We now consider how observations made in different frames of reference are related to each other. For example, consider two trains approaching one another, each with a speed of 80 km/h with respect to the Earth. Observers on the Earth beside the train tracks will measure 80 km/hr for the speed of each of the trains. Observers on either one of the trains (a different frame of reference) will measure a speed of 160 km/h for the train approaching them.

Similarly, when one car traveling 90 km/h passes a second car traveling in the same direction at 75 km/h, the first car has a speed relative to the second car of $90\,\text{km/h} - 75\,\text{km/h} = 15\,\text{km/h}$.

When the velocities are along the same line, simple addition or subtraction is sufficient to obtain the relative velocity. But if they are not along the same line, we must make use of vector addition. We emphasize that when specifying a velocity, it is important to specify what the reference frame is.

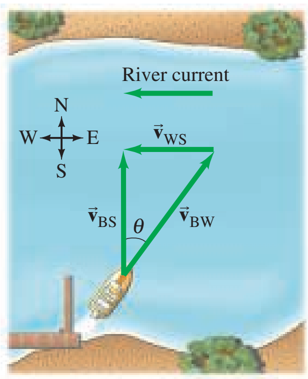


FIGURE 31 To move directly across the river, the boat must head upstream at an angle θ. Velocity vectors are shown as green arrows:

$\vec{\mathbf{v}}_{BS}$ = velocity of **B**oat with respect to the **S**hore,

$\vec{\mathbf{v}}_{BW}$ = velocity of **B**oat with respect to the **W**ater,

$\vec{\mathbf{v}}_{WS}$ = velocity of the **W**ater with respect to the **S**hore (river current).

When determining relative velocity, it is easy to make a mistake by adding or subtracting the wrong velocities. It is important, therefore, to draw a diagram and use a careful labeling process. Each velocity is labeled by *two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity*. For example, suppose a boat is to cross a river to the opposite side, as shown in Fig. 31. We let $\vec{\mathbf{v}}_{BW}$ be the velocity of the **B**oat with respect to the **W**ater. (This is also what the boat's velocity would be relative to the shore if the water were still.) Similarly, $\vec{\mathbf{v}}_{BS}$ is the velocity of the **B**oat with respect to the **S**hore, and $\vec{\mathbf{v}}_{WS}$ is the velocity of the **W**ater with respect to the **S**hore (this is the river current). Note that $\vec{\mathbf{v}}_{BW}$ is what the boat's motor produces (against the water), whereas $\vec{\mathbf{v}}_{BS}$ is equal to $\vec{\mathbf{v}}_{BW}$ plus the effect of the current, $\vec{\mathbf{v}}_{WS}$. Therefore, the velocity of the boat relative to the shore is (see vector diagram, Fig. 31)

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}. \tag{15}$$

By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 15 are the same, whereas the outer subscripts on the right of Eq. 15 (the B and the S) are the same as the two subscripts for the sum vector on the left, $\vec{\mathbf{v}}_{BS}$. By following this convention (first subscript for the object, second for the reference frame), you can write down the correct equation relating velocities in different reference frames.† Figure 32 gives a derivation of Eq. 15.

Equation 15 is valid in general and can be extended to three or more velocities. For example, if a fisherman on the boat walks with a velocity $\vec{\mathbf{v}}_{FB}$ relative to the boat, his velocity relative to the shore is $\vec{\mathbf{v}}_{FS} = \vec{\mathbf{v}}_{FB} + \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$. The equations involving relative velocity will be correct when adjacent inner subscripts are identical and when the outermost ones correspond exactly to the two on the velocity on the left of the equation. But this works only with plus signs (on the right), not minus signs.

It is often useful to remember that for any two objects or reference frames, A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\vec{\mathbf{v}}_{BA} = -\vec{\mathbf{v}}_{AB}. \tag{16}$$

For example, if a train is traveling 100 km/h relative to the Earth in a certain direction, objects on the Earth (such as trees) appear to an observer on the train to be traveling 100 km/h in the opposite direction.

†We thus would know by inspection that (for example) the equation $\vec{\mathbf{V}}_{BW} = \vec{\mathbf{V}}_{BS} + \vec{\mathbf{V}}_{WS}$ is wrong.

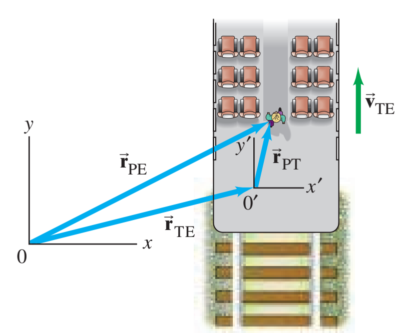


FIGURE 32 Derivation of relative velocity equation (Eq. 15), in this case for a person walking along the corridor in a train. We are looking down on the train and two reference frames are shown: xy on the Earth and $x'y'$ fixed on the train. We have:

$\vec{\mathbf{r}}_{PT}$ = position vector of person (P) relative to train (T),

$\vec{\mathbf{r}}_{PE}$ = position vector of person (P) relative to Earth (E),

$\vec{\mathbf{r}}_{TE}$ = position vector of train's coordinate system (T) relative to Earth (E).

From the diagram we see that

$$\vec{\mathbf{r}}_{PE} = \vec{\mathbf{r}}_{PT} + \vec{\mathbf{r}}_{TE}.$$

We take the derivative with respect to time to obtain

$$\frac{d}{dt}(\vec{\mathbf{r}}_{PE}) = \frac{d}{dt}(\vec{\mathbf{r}}_{PT}) + \frac{d}{dt}(\vec{\mathbf{r}}_{TE}).$$

or, since $d\vec{\mathbf{r}}/dt = \vec{\mathbf{v}}$,

$$\vec{\mathbf{v}}_{PE} = \vec{\mathbf{v}}_{PT} + \vec{\mathbf{v}}_{TE}.$$

This is the equivalent of Eq. 15 for the present situation (check the subscripts!).

CONCEPTUAL EXAMPLE 13 **Crossing a river.** A woman in a small motor boat is trying to cross a river that flows due west with a strong current. The woman starts on the south bank and is trying to reach the north bank directly north from her starting point. Should she (*a*) head due north, (*b*) head due west, (*c*) head in a northwesterly direction, (*d*) head in a northeasterly direction?

RESPONSE If the woman heads straight across the river, the current will drag the boat downstream (westward). To overcome the river's westward current, the boat must acquire an eastward component of velocity as well as a northward component. Thus the boat must (*d*) head in a northeasterly direction (see Fig. 33). The actual angle depends on the strength of the current and how fast the boat moves relative to the water. If the current is weak and the motor is strong, then the boat can head almost, but not quite, due north.

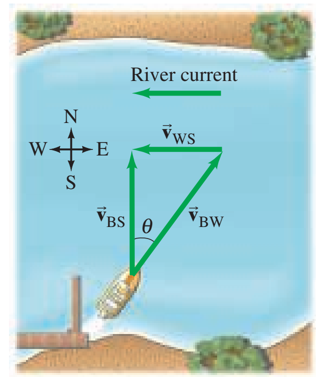


FIGURE 33 Examples 13 and 14.

EXAMPLE 14 **Heading upstream.** A boat's speed in still water is $v_{BW} = 1.85\ \text{m/s}$. If the boat is to travel directly across a river whose current has speed $v_{WS} = 1.20\ \text{m/s}$, at what upstream angle must the boat head? (See Fig. 33.)

APPROACH We reason as in Example 13, and use subscripts as in Eq. 15. Figure 33 has been drawn with $\vec{\mathbf{v}}_{BS}$, the velocity of the **B**oat relative to the **S**hore, pointing directly across the river because this is how the boat is supposed to move. (Note that $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$.) To accomplish this, the boat needs to head upstream to offset the current pulling it downstream.

SOLUTION Vector $\vec{\mathbf{v}}_{BW}$ points upstream at an angle θ as shown. From the diagram,

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\ \text{m/s}}{1.85\ \text{m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a 40.4° angle.

EXAMPLE 15 **Heading across the river.** The same boat $(v_{BW} = 1.85\ \text{m/s})$ now heads directly across the river whose current is still 1.20 m/s. (*a*) What is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) If the river is 110 m wide, how long will it take to cross and how far downstream will the boat be then?

APPROACH The boat now heads directly across the river and is pulled downstream by the current, as shown in Fig. 34. The boat's velocity with respect to the shore, $\vec{\mathbf{v}}_{BS}$, is the sum of its velocity with respect to the water, $\vec{\mathbf{v}}_{BW}$, plus the velocity of the water with respect to the shore, $\vec{\mathbf{v}}_{WS}$:

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS},$$

just as before.

SOLUTION (*a*) Since $\vec{\mathbf{v}}_{BW}$ is perpendicular to $\vec{\mathbf{v}}_{WS}$, we can get v_{BS} using the theorem of Pythagoras:

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(1.85\ \text{m/s})^2 + (1.20\ \text{m/s})^2} = 2.21\ \text{m/s}.$$

We can obtain the angle (note how θ is defined in the diagram) from:

$$\tan\theta = v_{WS}/v_{BW} = (1.20\ \text{m/s})/(1.85\ \text{m/s}) = 0.6486.$$

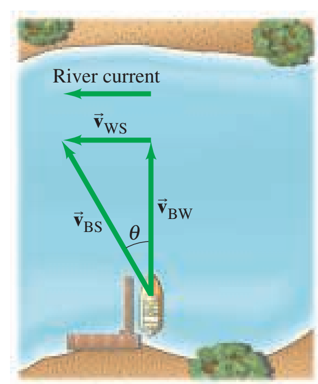
Thus $\theta = \tan^{-1}(0.6486) = 33.0°$. Note that this angle is not equal to the angle calculated in Example 14.

(*b*) The travel time for the boat is determined by the time it takes to cross the river. Given the river's width $D = 110\ \text{m}$, we can use the velocity component in the direction of D, $v_{BW} = D/t$. Solving for t, we get $t = 110\ \text{m}/1.85\ \text{m/s} = 59.5\ \text{s}$. The boat will have been carried downstream, in this time, a distance

$$d = v_{WS}t = (1.20\ \text{m/s})(59.5\ \text{s}) = 71.4\ \text{m} \approx 71\ \text{m}.$$

NOTE There is no acceleration in this Example, so the motion involves only constant velocities (of the boat or of the river).

FIGURE 34 Example 15. A boat heading directly across a river whose current moves at 1.20 m/s.



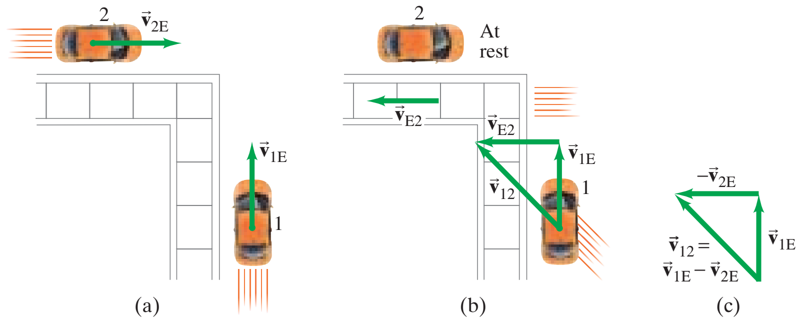


FIGURE 35 Example 16.

EXAMPLE 16 **Car velocities at 90°.** Two automobiles approach a street corner at right angles to each other with the same speed of 40.0 km/h (= 11.11 m/s), as shown in Fig. 35a. What is the relative velocity of one car with respect to the other? That is, determine the velocity of car 1 as seen by car 2.

APPROACH Figure 35a shows the situation in a reference frame fixed to the Earth. But we want to view the situation from a reference frame in which car 2 is at rest, and this is shown in Fig. 35b. In this reference frame (the world as seen by the driver of car 2), the Earth moves toward car 2 with velocity $\vec{\mathbf{v}}_{E2}$ (speed of 40.0 km/h), which is of course equal and opposite to $\vec{\mathbf{v}}_{2E}$, the velocity of car 2 with respect to the Earth (Eq. 16):

$$\vec{\mathbf{v}}_{2E} = -\vec{\mathbf{v}}_{E2}.$$

Then the velocity of car 1 as seen by car 2 is (see Eq. 15)

$$\vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_{1E} + \vec{\mathbf{v}}_{E2}$$

SOLUTION Because $\vec{\mathbf{v}}_{E2} = -\vec{\mathbf{v}}_{2E}$, then

$$\vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_{1E} - \vec{\mathbf{v}}_{2E}.$$

That is, the velocity of car 1 as seen by car 2 is the difference of their velocities, $\vec{\mathbf{v}}_{1E} - \vec{\mathbf{v}}_{2E}$, both measured relative to the Earth (see Fig. 35c). Since the magnitudes of $\vec{\mathbf{v}}_{1E}$, $\vec{\mathbf{v}}_{2E}$, and $\vec{\mathbf{v}}_{E2}$ are equal (40.0 km/h = 11.11 m/s), we see (Fig. 35b) that $\vec{\mathbf{v}}_{12}$ points at a 45° angle toward car 2; the speed is

$$v_{12} = \sqrt{(11.11\text{ m/s})^2 + (11.11\text{ m/s})^2} = 15.7\text{ m/s} \;(= 56.6\text{ km/h}).$$

Summary

A quantity that has both a magnitude and a direction is called a **vector**. A quantity that has only a magnitude is called a **scalar**.

Addition of vectors can be done graphically by placing the tail of each successive arrow (representing each vector) at the tip of the previous one. The sum, or **resultant vector**, is the arrow drawn from the tail of the first to the tip of the last. Two vectors can also be added using the parallelogram method.

Vectors can be added more accurately using the analytical method of adding their **components** along chosen axes with the aid of trigonometric functions. A vector of magnitude V making an angle θ with the x axis has components

$$V_x = V\cos\theta \qquad V_y = V\sin\theta. \tag{2}$$

Given the components, we can find the magnitude and direction from

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}. \tag{3}$$

It is often helpful to express a vector in terms of its components along chosen axes using **unit vectors**, which are vectors of unit length along the chosen coordinate axes; for Cartesian coordinates the unit vectors along the x, y, and z axes are called $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

The general definitions for the **instantaneous velocity**, $\vec{\mathbf{v}}$, and **acceleration**, $\vec{\mathbf{a}}$, of a particle (in one, two, or three dimensions) are

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} \tag{8}$$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt}, \tag{11}$$

where $\vec{\mathbf{r}}$ is the position vector of the particle. The kinematic equations for motion with constant acceleration can be written for each of the x, y, and z components of the motion and have the same form as for one-dimensional motion. Or they can be written in the more general vector form:

$$\begin{aligned} \vec{\mathbf{v}} &= \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t \\ \vec{\mathbf{r}} &= \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 \end{aligned} \tag{13}$$

Projectile motion of an object moving in the air near the Earth's surface can be analyzed as two separate motions if air resistance can be ignored. The horizontal component of the motion is at constant velocity, whereas the vertical component is at constant acceleration, g, just as for an object falling vertically under the action of gravity.

The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the **relative velocity** of the two reference frames, are known.

Answers to Exercises

A: When the two vectors D_1 and D_2 point in the same direction.

B: $3\sqrt{2} = 4.24$.

C: (*a*).

D: (*d*).

E: Both balls reach the same height, so are in the air for the same length of time.

F: (*c*).

Kinematics in Two or Three Dimensions; Vectors Problem Set

Questions

1. One car travels due east at 40 km/h, and a second car travels north at 40 km/h. Are their velocities equal? Explain.
2. Can you conclude that a car is not accelerating if its speedometer indicates a steady 60 km/h?
3. Can you give several examples of an object's motion in which a great distance is traveled but the displacement is zero?
4. Can the displacement vector for a particle moving in two dimensions ever be longer than the length of path traveled by the particle over the same time interval? Can it ever be less? Discuss.
5. During baseball practice, a batter hits a very high fly ball and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball?
6. If $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, is V necessarily greater than V_1 and/or V_2? Discuss.
7. Two vectors have length $V_1 = 3.5$ km and $V_2 = 4.0$ km. What are the maximum and minimum magnitudes of their vector sum?
8. Can two vectors, of unequal magnitude, add up to give the zero vector? Can *three* unequal vectors? Under what conditions?
9. Can the magnitude of a vector ever (*a*) equal, or (*b*) be less than, one of its components?
10. Can a particle with constant speed be accelerating? What if it has constant velocity?
11. Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?
12. A child wishes to determine the speed a slingshot imparts to a rock. How can this be done using only a meter stick, a rock, and the slingshot?
13. In archery, should the arrow be aimed directly at the target? How should your angle of aim depend on the distance to the target?
14. A projectile is launched at an upward angle of 30° to the horizontal with a speed of 30 m/s. How does the horizontal component of its velocity 1.0 s after launch compare with its horizontal component of velocity 2.0 s after launch, ignoring air resistance?
15. A projectile has the least speed at what point in its path?
16. It was reported in World War I that a pilot flying at an altitude of 2 km caught in his bare hands a bullet fired at the plane! Using the fact that a bullet slows down considerably due to air resistance, explain how this incident occurred.
17. Two cannonballs, A and B, are fired from the ground with identical initial speeds, but with θ_A larger than θ_B. (*a*) Which cannonball reaches a higher elevation? (*b*) Which stays longer in the air? (*c*) Which travels farther?
18. A person sitting in an enclosed train car, moving at constant velocity, throws a ball straight up into the air in her reference frame. (*a*) Where does the ball land? What is your answer if the car (*b*) accelerates, (*c*) decelerates, (*d*) rounds a curve, (*e*) moves with constant velocity but is open to the air?
19. If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?
20. Two rowers, who can row at the same speed in still water, set off across a river at the same time. One heads straight across and is pulled downstream somewhat by the current. The other one heads upstream at an angle so as to arrive at a point opposite the starting point. Which rower reaches the opposite side first?
21. If you stand motionless under an umbrella in a rainstorm where the drops fall vertically you remain relatively dry. However, if you start running, the rain begins to hit your legs even if they remain under the umbrella. Why?

Problems

[The Problems in this Section are ranked I, II, or III according to estimated difficulty, with (I) Problems being easiest. Level (III) Problems are meant mainly as a challenge for the best students, for "extra credit." The Problems are arranged by Sections, meaning that the reader should have read up to and including that Section, but this Chapter also has a group of General Problems that are not arranged by Section and not ranked.]

2 to 5 Vector Addition; Unit Vectors

1. (I) A car is driven 225 km west and then 78 km southwest (45°). What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.
2. (I) A delivery truck travels 28 blocks north, 16 blocks east, and 26 blocks south. What is its final displacement from the origin? Assume the blocks are equal length.
3. (I) If $V_x = 7.80$ units and $V_y = -6.40$ units, determine the magnitude and direction of $\vec{\mathbf{V}}$.
4. (II) Graphically determine the resultant of the following three vector displacements: (1) 24 m, 36° north of east; (2) 18 m, 37° east of north; and (3) 26 m, 33° west of south.
5. (II) $\vec{\mathbf{V}}$ is a vector 24.8 units in magnitude and points at an angle of 23.4° above the negative x axis. (*a*) Sketch this vector. (*b*) Calculate V_x and V_y. (*c*) Use V_x and V_y to obtain (again) the magnitude and direction of $\vec{\mathbf{V}}$. [*Note*: Part (*c*) is a good way to check if you've resolved your vector correctly.]
6. (II) Figure 36 shows two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, whose magnitudes are $A = 6.8$ units and $B = 5.5$ units. Determine $\vec{\mathbf{C}}$ if (*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, (*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, (*c*) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}}$. Give the magnitude and direction for each.

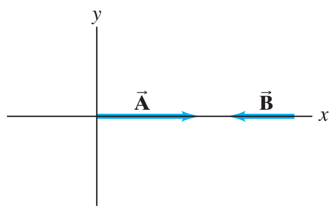


FIGURE 36 Problem 6.

7. (II) An airplane is traveling 835 km/h in a direction 41.5° west of north (Fig. 37). (a) Find the components of the velocity vector in the northerly and westerly directions. (b) How far north and how far west has the plane traveled after 2.50 h?

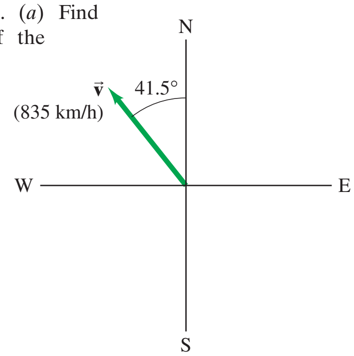


FIGURE 37 Problem 7.

8. (II) Let $\vec{\mathbf{V}}_1 = -6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}$ and $\vec{\mathbf{V}}_2 = 4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}$. Determine the magnitude and direction of (a) $\vec{\mathbf{V}}_1$, (b) $\vec{\mathbf{V}}_2$, (c) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ and (d) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$.

9. (II) (a) Determine the magnitude and direction of the sum of the three vectors $\vec{\mathbf{V}}_1 = 4.0\hat{\mathbf{i}} - 8.0\hat{\mathbf{j}}$, $\vec{\mathbf{V}}_2 = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, and $\vec{\mathbf{V}}_3 = -2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}$. ($b$) Determine $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3$.

10. (II) Three vectors are shown in Fig. 38. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (a) components, (b) magnitude and angle with x axis.

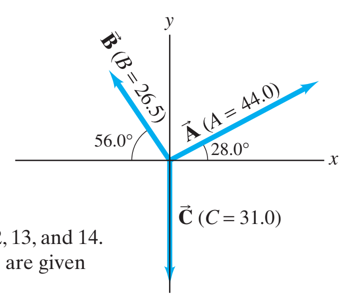


FIGURE 38 Problems 10, 11, 12, 13, and 14. Vector magnitudes are given in arbitrary units.

11. (II) (a) Given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Fig. 38, determine $\vec{\mathbf{B}} - \vec{\mathbf{A}}$. ($b$) Determine $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ without using your answer in (a). Then compare your results and see if they are opposite.

12. (II) Determine the vector $\vec{\mathbf{A}} - \vec{\mathbf{C}}$, given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{C}}$ in Fig. 38.

13. (II) For the vectors shown in Fig. 38, determine (a) $\vec{\mathbf{B}} - 2\vec{\mathbf{A}}$, ($b$) $2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}}$.

14. (II) For the vectors given in Fig. 38, determine (a) $\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}}$, ($b$) $\vec{\mathbf{A}} + \vec{\mathbf{B}} - \vec{\mathbf{C}}$, and ($c$) $\vec{\mathbf{C}} - \vec{\mathbf{A}} - \vec{\mathbf{B}}$.

15. (II) The summit of a mountain, 2450 m above base camp, is measured on a map to be 4580 m horizontally from the camp in a direction 32.4° west of north. What are the components of the displacement vector from camp to summit? What is its magnitude? Choose the x axis east, y axis north, and z axis up.

16. (III) You are given a vector in the xy plane that has a magnitude of 90.0 units and a y component of -55.0 units. (a) What are the two possibilities for its x component? (b) Assuming the x component is known to be positive, specify the vector which, if you add it to the original one, would give a resultant vector that is 80.0 units long and points entirely in the $-x$ direction.

6 Vector Kinematics

17. (I) The position of a particular particle as a function of time is given by $\vec{\mathbf{r}} = \left(9.60t\,\hat{\mathbf{i}} + 8.85\hat{\mathbf{j}} - 1.00t^2\hat{\mathbf{k}}\right)$ m. Determine the particle's velocity and acceleration as a function of time.

18. (I) What was the average velocity of the particle in Problem 17 between $t = 1.00$ s and $t = 3.00$ s? What is the magnitude of the instantaneous velocity at $t = 2.00$ s?

19. (II) What is the shape of the path of the particle of Problem 17?

20. (II) A car is moving with speed 18.0 m/s due south at one moment and 27.5 m/s due east 8.00 s later. Over this time interval, determine the magnitude and direction of (a) its average velocity, (b) its average acceleration. (c) What is its average speed. [*Hint:* Can you determine all these from the information given?]

21. (II) At $t = 0$, a particle starts from rest at $x = 0$, $y = 0$, and moves in the xy plane with an acceleration $\vec{\mathbf{a}} = \left(4.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}\right)\,\text{m/s}^2$. Determine ($a$) the x and y components of velocity, (b) the speed of the particle, and (c) the position of the particle, all as a function of time. (d) Evaluate all the above at $t = 2.0$ s.

22. (II) (a) A skier is accelerating down a 30.0° hill at 1.80 m/s^2 (Fig. 39). What is the vertical component of her acceleration? (b) How long will it take her to reach the bottom of the hill, assuming she starts from rest and accelerates uniformly, if the elevation change is 325 m?

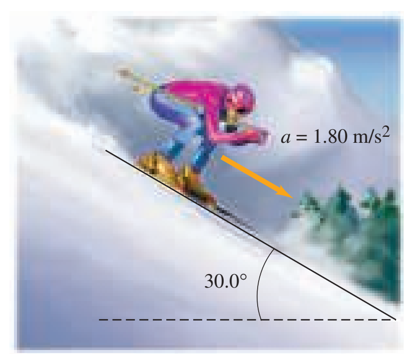


FIGURE 39 Problem 22.

23. (II) An ant walks on a piece of graph paper straight along the x axis a distance of 10.0 cm in 2.00 s. It then turns left 30.0° and walks in a straight line another 10.0 cm in 1.80 s. Finally, it turns another 70.0° to the left and walks another 10.0 cm in 1.55 s. Determine (a) the x and y components of the ant's average velocity, and (b) its magnitude and direction.

24. (II) A particle starts from the origin at $t = 0$ with an initial velocity of 5.0 m/s along the positive x axis. If the acceleration is $(-3.0\hat{\mathbf{i}} + 4.5\hat{\mathbf{j}})\,\mathrm{m/s^2}$, determine the velocity and position of the particle at the moment it reaches its maximum x coordinate.

25. (II) Suppose the position of an object is given by $\vec{\mathbf{r}} = (3.0t^2\hat{\mathbf{i}} - 6.0t^3\hat{\mathbf{j}})\,\mathrm{m}$. (*a*) Determine its velocity $\vec{\mathbf{v}}$ and acceleration $\vec{\mathbf{a}}$, as a function of time. (*b*) Determine $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$ at time $t = 2.5\,\mathrm{s}$.

26. (II) An object, which is at the origin at time $t = 0$, has initial velocity $\vec{\mathbf{v}}_0 = (-14.0\hat{\mathbf{i}} - 7.0\hat{\mathbf{j}})\,\mathrm{m/s}$ and constant acceleration $\vec{\mathbf{a}} = (6.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\,\mathrm{m/s^2}$. Find the position $\vec{\mathbf{r}}$ where the object comes to rest (momentarily).

27. (II) A particle's position as a function of time t is given by $\vec{\mathbf{r}} = (5.0t + 6.0t^2)\,\mathrm{m}\,\hat{\mathbf{i}} + (7.0 - 3.0t^3)\,\mathrm{m}\,\hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$, find the magnitude and direction of the particle's displacement vector $\Delta\vec{\mathbf{r}}$ relative to the point $\vec{\mathbf{r}}_0 = (0.0\hat{\mathbf{i}} + 7.0\hat{\mathbf{j}})\,\mathrm{m}$.

7 and 8 Projectile Motion (neglect air resistance)

28. (I) A tiger leaps horizontally from a 7.5-m-high rock with a speed of 3.2 m/s. How far from the base of the rock will she land?

29. (I) A diver running 2.3 m/s dives out horizontally from the edge of a vertical cliff and 3.0 s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?

30. (II) Estimate how much farther a person can jump on the Moon as compared to the Earth if the takeoff speed and angle are the same. The acceleration due to gravity on the Moon is one-sixth what it is on Earth.

31. (II) A fire hose held near the ground shoots water at a speed of 6.5 m/s. At what angle(s) should the nozzle point in order that the water land 2.5 m away (Fig. 40)? Why are there two different angles? Sketch the two trajectories.

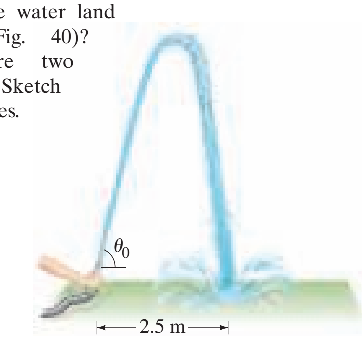


FIGURE 40 Problem 31.

32. (II) A ball is thrown horizontally from the roof of a building 9.0 m tall and lands 9.5 m from the base. What was the ball's initial speed?

33. (II) A football is kicked at ground level with a speed of 18.0 m/s at an angle of 38.0° to the horizontal. How much later does it hit the ground?

34. (II) A ball thrown horizontally at 23.7 m/s from the roof of a building lands 31.0 m from the base of the building. How high is the building?

35. (II) A shot-putter throws the shot (mass = 7.3 kg) with an initial speed of 14.4 m/s at a 34.0° angle to the horizontal. Calculate the horizontal distance traveled by the shot if it leaves the athlete's hand at a height of 2.10 m above the ground.

36. (II) Show that the time required for a projectile to reach its highest point is equal to the time for it to return to its original height if air resistance is neglible.

37. (II) You buy a plastic dart gun, and being a clever physics student you decide to do a quick calculation to find its maximum horizontal range. You shoot the gun straight up, and it takes 4.0 s for the dart to land back at the barrel. What is the maximum horizontal range of your gun?

38. (II) A baseball is hit with a speed of 27.0 m/s at an angle of 45.0°. It lands on the flat roof of a 13.0-m-tall nearby building. If the ball was hit when it was 1.0 m above the ground, what horizontal distance does it travel before it lands on the building?

39. (II) In Example 11 of "Kinematics in Two or Three Dimensions; Vectors" we chose the x axis to the right and y axis up. Redo this problem by defining the x axis to the left and y axis down, and show that the conclusion remains the same—the football lands on the ground 40.5 m to the right of where it departed the punter's foot.

40. (II) A grasshopper hops down a level road. On each hop, the grasshopper launches itself at angle $\theta_0 = 45°$ and achieves a range $R = 1.0\,\mathrm{m}$. What is the average horizontal speed of the grasshopper as it progresses down the road? Assume that the time spent on the ground between hops is negligible.

41. (II) Extreme-sports enthusiasts have been known to jump off the top of El Capitan, a sheer granite cliff of height 910 m in Yosemite National Park. Assume a jumper runs horizontally off the top of El Capitan with speed 5.0 m/s and enjoys a freefall until she is 150 m above the valley floor, at which time she opens her parachute (Fig. 41). (*a*) How long is the jumper in freefall? Ignore air resistance. (*b*) It is important to be as far away from the cliff as possible before opening the parachute. How far from the cliff is this jumper when she opens her chute?

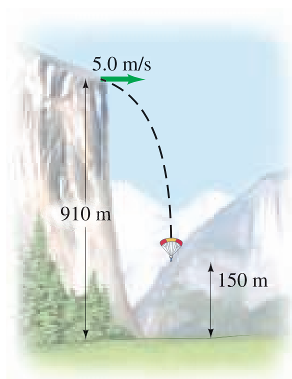


FIGURE 41 Problem 41.

42. (II) Here is something to try at a sporting event. Show that the maximum height h attained by an object projected into the air, such as a baseball, football, or soccer ball, is approximately given by

$$h \approx 1.2t^2\,\mathrm{m},$$

where t is the total time of flight for the object in seconds. Assume that the object returns to the same level as that from which it was launched, as in Fig. 42. For example, if you count to find that a baseball was in the air for $t = 5.0\,\mathrm{s}$, the maximum height attained was $h = 1.2 \times (5.0)^2 = 30\,\mathrm{m}$. The beauty of this relation is that h can be determined without knowledge of the launch speed v_0 or launch angle θ_0.

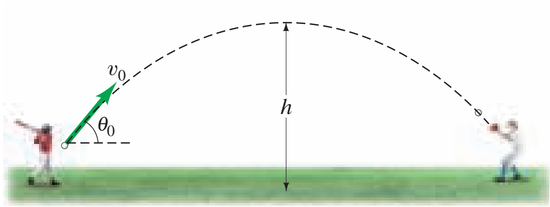


FIGURE 42 Problem 42.

43. (II) The pilot of an airplane traveling 170 km/h wants to drop supplies to flood victims isolated on a patch of land 150 m below. The supplies should be dropped how many seconds before the plane is directly overhead?

44. (II) (*a*) A long jumper leaves the ground at 45° above the horizontal and lands 8.0 m away. What is her "takeoff" speed v_0? (*b*) Now she is out on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5 m, vertically below. If she long jumps from the edge of the left bank at 45° with the speed calculated in (*a*), how long, or short, of the opposite bank will she land (Fig. 43)?

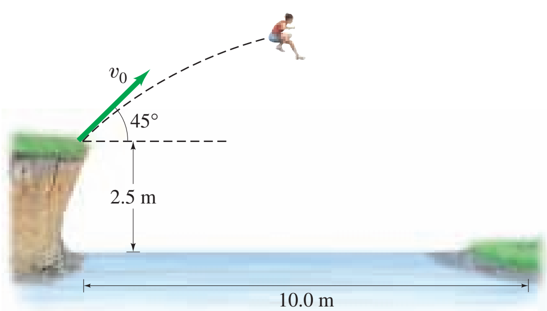


FIGURE 43 Problem 44.

45. (II) A high diver leaves the end of a 5.0-m-high diving board and strikes the water 1.3 s later, 3.0 m beyond the end of the board. Considering the diver as a particle, determine (*a*) her initial velocity, $\vec{\mathbf{v}}_0$; (*b*) the maximum height reached; and (*c*) the velocity $\vec{\mathbf{v}}_f$ with which she enters the water.

46. (II) A projectile is shot from the edge of a cliff 115 m above ground level with an initial speed of 65.0 m/s at an angle of 35.0° with the horizontal, as shown in Fig. 44. (*a*) Determine the time taken by the projectile to hit point P at ground level. (*b*) Determine the distance X of point P from the base of the vertical cliff. At the instant just before the projectile hits point P, find (*c*) the horizontal and the vertical components of its velocity, (*d*) the magnitude of the velocity, and (*e*) the angle made by the velocity vector with the horizontal. (*f*) Find the maximum height above the cliff top reached by the projectile.

47. (II) Suppose the kick in Example 7 of "Kinematics in Two or Three Dimensions; Vectors" is attempted 36.0 m from the goalposts, whose crossbar is 3.00 m above the ground. If the football is directed perfectly between the goalposts, will it pass over the bar and be a field goal? Show why or why not. If not, from what horizontal distance must this kick be made if it is to score?

48. (II) Exactly 3.0 s after a projectile is fired into the air from the ground, it is observed to have a velocity $\vec{\mathbf{v}} = (8.6\hat{\mathbf{i}} + 4.8\hat{\mathbf{j}})$ m/s, where the x axis is horizontal and the y axis is positive

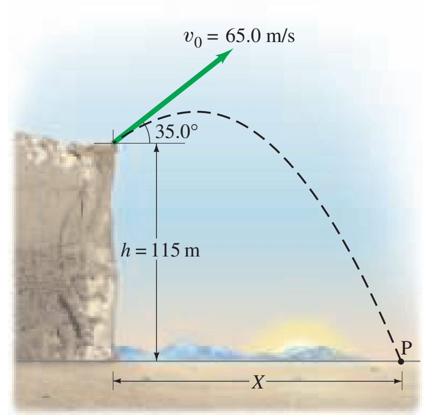


FIGURE 44 Problem 46.

upward. Determine (*a*) the horizontal range of the projectile, (*b*) its maximum height above the ground, and (*c*) its speed and angle of motion just before it strikes the ground.

49. (II) Revisit Example 9 of "Kinematics in Two or Three Dimensions; Vectors," and assume that the boy with the slingshot is *below* the boy in the tree (Fig. 45) and so aims *upward*, directly at the boy in the tree. Show that again the boy in the tree makes the wrong move by letting go at the moment the water balloon is shot.

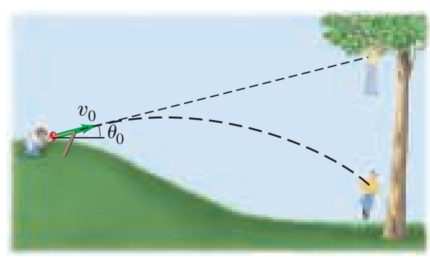


FIGURE 45 Problem 49.

50. (II) A stunt driver wants to make his car jump over 8 cars parked side by side below a horizontal ramp (Fig. 46). (*a*) With what minimum speed must he drive off the horizontal ramp? The vertical height of the ramp is 1.5 m above the cars and the horizontal distance he must clear is 22 m. (*b*) If the ramp is now tilted upward, so that "takeoff angle" is 7.0° above the horizontal, what is the new minimum speed?

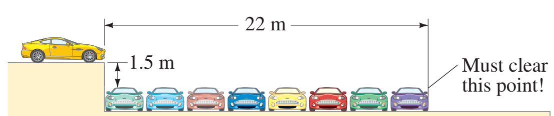


FIGURE 46 Problem 50.

51. (II) A ball is thrown horizontally from the top of a cliff with initial speed v_0 (at $t = 0$). At any moment, its direction of motion makes an angle θ to the horizontal (Fig. 47). Derive a formula for θ as a function of time, t, as the ball follows a projectile's path.

52. (II) At what projection angle will the range of a projectile equal its maximum height?

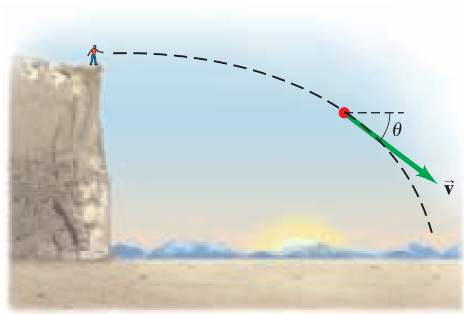


FIGURE 47 Problem 51.

53. (II) A projectile is fired with an initial speed of 46.6 m/s at an angle of 42.2° above the horizontal on a long flat firing range. Determine (*a*) the maximum height reached by the projectile, (*b*) the total time in the air, (*c*) the total horizontal distance covered (that is, the range), and (*d*) the velocity of the projectile 1.50 s after firing.

54. (II) An athlete executing a long jump leaves the ground at a 27.0° angle and lands 7.80 m away. (*a*) What was the takeoff speed? (*b*) If this speed were increased by just 5.0%, how much longer would the jump be?

55. (III) A person stands at the base of a hill that is a straight incline making an angle ϕ with the horizontal (Fig. 48). For a given initial speed v_0, at what angle θ (to the horizontal) should objects be thrown so that the distance d they land up the hill is as large as possible?

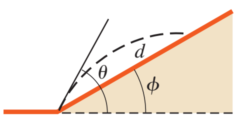


FIGURE 48 Problem 55. Given ϕ and v_0, determine θ to make d maximum.

56. (III) Derive a formula for the horizontal range R, of a projectile when it lands at a height h above its initial point. (For $h < 0$, it lands a distance $-h$ below the starting point.) Assume it is projected at an angle θ_0 with initial speed v_0.

9 Relative Velocity

57. (I) A person going for a morning jog on the deck of a cruise ship is running toward the bow (front) of the ship at 2.0 m/s while the ship is moving ahead at 8.5 m/s. What is the velocity of the jogger relative to the water? Later, the jogger is moving toward the stern (rear) of the ship. What is the jogger's velocity relative to the water now?

58. (I) Huck Finn walks at a speed of 0.70 m/s across his raft (that is, he walks perpendicular to the raft's motion relative to the shore). The raft is traveling down the Mississippi River at a speed of 1.50 m/s relative to the river bank (Fig. 49). What is Huck's velocity (speed and direction) relative to the river bank?

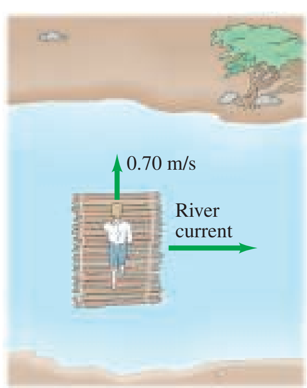


FIGURE 49 Problem 58.

59. (II) Determine the speed of the boat with respect to the shore in Example 14 of "Kinematics in Two or Three Dimensions; Vectors."

60. (II) Two planes approach each other head-on. Each has a speed of 780 km/h, and they spot each other when they are initially 12.0 km apart. How much time do the pilots have to take evasive action?

61. (II) A child, who is 45 m from the bank of a river, is being carried helplessly downstream by the river's swift current of 1.0 m/s. As the child passes a lifeguard on the river's bank, the lifeguard starts swimming in a straight line until she reaches the child at a point downstream (Fig. 50). If the lifeguard can swim at a speed of 2.0 m/s relative to the water, how long does it take her to reach the child? How far downstream does the lifeguard intercept the child?

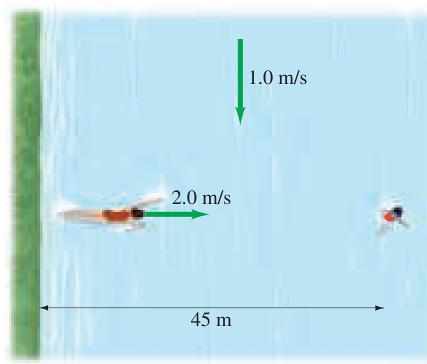


FIGURE 50 Problem 61.

62. (II) A passenger on a boat moving at 1.70 m/s on a still lake walks up a flight of stairs at a speed of 0.60 m/s, Fig. 51. The stairs are angled at 45° pointing in the direction of motion as shown. Write the vector velocity of the passenger relative to the water.

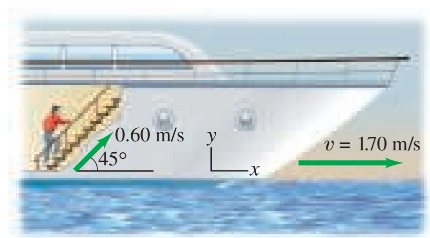


FIGURE 51 Problem 62.

63. (II) A person in the passenger basket of a hot-air balloon throws a ball horizontally outward from the basket with speed 10.0 m/s (Fig. 52). What initial velocity (magnitude and direction) does the ball have relative to a person standing on the ground (*a*) if the hot-air balloon is rising at 5.0 m/s relative to the ground during this throw, (*b*) if the hot-air balloon is descending at 5.0 m/s relative to the ground.

64. (II) An airplane is heading due south at a speed of 580 km/h. If a wind begins blowing from the southwest at a speed of 90.0 km/h (average), calculate (*a*) the velocity (magnitude and direction) of the plane, relative to the ground, and (*b*) how far from its intended position it will be after 11.0 min if the pilot takes no corrective action. [*Hint*: First draw a diagram.]

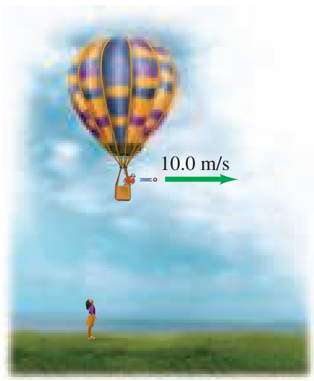


FIGURE 52 Problem 63.

65. (II) In what direction should the pilot aim the plane in Problem 64 so that it will fly due south?

66. (II) Two cars approach a street corner at right angles to each other (see Fig. 35). Car 1 travels at 35 km/h and car 2 at 45 km/h. What is the relative velocity of car 1 as seen by car 2? What is the velocity of car 2 relative to car 1?

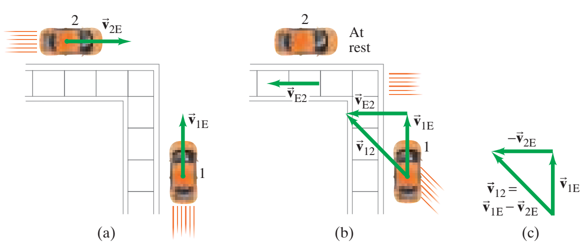


FIGURE 35 Example 16.

67. (II) A swimmer is capable of swimming 0.60 m/s in still water. (*a*) If she aims her body directly across a 55-m-wide river whose current is 0.50 m/s, how far downstream (from a point opposite her starting point) will she land? (*b*) How long will it take her to reach the other side?

68. (II) (*a*) At what upstream angle must the swimmer in Problem 67 aim, if she is to arrive at a point directly across the stream? (*b*) How long will it take her?

69. (II) A motorboat whose speed in still water is 3.40 m/s must aim upstream at an angle of 19.5° (with respect to a line perpendicular to the shore) in order to travel directly across the stream. (*a*) What is the speed of the current? (*b*) What is the resultant speed of the boat with respect to the shore? (See Fig. 31.)

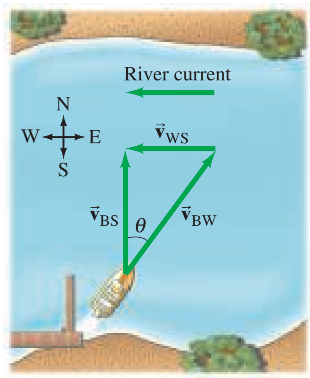


FIGURE 31 To move directly across the river, the boat must head upstream at an angle θ. Velocity vectors are shown as green arrows:
$\vec{v}_{BS}$ = velocity of **B**oat with respect to the **S**hore,
$\vec{v}_{BW}$ = velocity of **B**oat with respect to the **W**ater,
$\vec{v}_{WS}$ = velocity of the **W**ater with respect to the **S**hore (river current).

70. (II) A boat, whose speed in still water is 2.70 m/s, must cross a 280-m-wide river and arrive at a point 120 m upstream from where it starts (Fig. 53). To do so, the pilot must head the boat at a 45.0° upstream angle. What is the speed of the river's current?

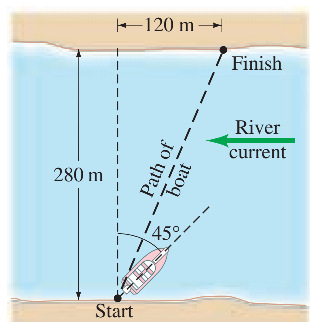


FIGURE 53 Problem 70.

71. (III) An airplane, whose air speed is 580 km/h, is supposed to fly in a straight path 38.0° N of E. But a steady 72 km/h wind is blowing from the north. In what direction should the plane head?

General Problems

72. Two vectors, $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$, add to a resultant $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$. Describe $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ if (*a*) $V = V_1 + V_2$, (*b*) $V^2 = V_1^2 + V_2^2$, (*c*) $V_1 + V_2 = V_1 - V_2$.

73. A plumber steps out of his truck, walks 66 m east and 35 m south, and then takes an elevator 12 m into the subbasement of a building where a bad leak is occurring. What is the displacement of the plumber relative to his truck? Give your answer in components; also give the magnitude and angles, with respect to the *x* axis, in the vertical and horizontal plane. Assume *x* is east, *y* is north, and *z* is up.

74. On mountainous downhill roads, escape routes are sometimes placed to the side of the road for trucks whose brakes might fail. Assuming a constant upward slope of 26°, calculate the horizontal and vertical components of the acceleration of a truck that slowed from 110 km/h to rest in 7.0 s. See Fig. 54.

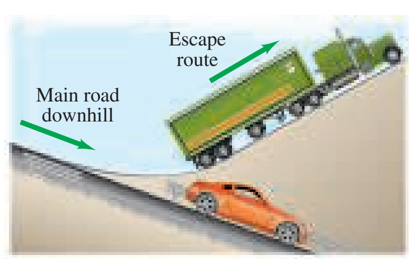


FIGURE 54 Problem 74.

75. A light plane is headed due south with a speed relative to still air of 185 km/h. After 1.00 h, the pilot notices that they have covered only 135 km and their direction is not south but southeast (45.0°). What is the wind velocity?

76. An Olympic long jumper is capable of jumping 8.0 m. Assuming his horizontal speed is 9.1 m/s as he leaves the ground, how long is he in the air and how high does he go? Assume that he lands standing upright—that is, the same way he left the ground.

77. Romeo is chucking pebbles gently up to Juliet's window, and he wants the pebbles to hit the window with only a horizontal component of velocity. He is standing at the edge of a rose garden 8.0 m below her window and 9.0 m from the base of the wall (Fig. 55). How fast are the pebbles going when they hit her window?

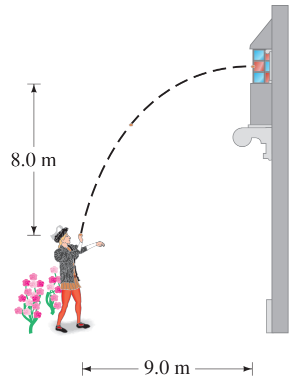


FIGURE 55 Problem 77.

78. Raindrops make an angle θ with the vertical when viewed through a moving train window (Fig. 56). If the speed of the train is v_T, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?

FIGURE 56 Problem 78.

79. Apollo astronauts took a "nine iron" to the Moon and hit a golf ball about 180 m. Assuming that the swing, launch angle, and so on, were the same as on Earth where the same astronaut could hit it only 32 m, estimate the acceleration due to gravity on the surface of the Moon. (We neglect air resistance in both cases, but on the Moon there is none.)

80. A hunter aims directly at a target (on the same level) 68.0 m away. (*a*) If the bullet leaves the gun at a speed of 175 m/s, by how much will it miss the target? (*b*) At what angle should the gun be aimed so the target will be hit?

81. The cliff divers of Acapulco push off horizontally from rock platforms about 35 m above the water, but they must clear rocky outcrops at water level that extend out into the water 5.0 m from the base of the cliff directly under their launch point. See Fig. 57. What minimum pushoff speed is necessary to clear the rocks? How long are they in the air?

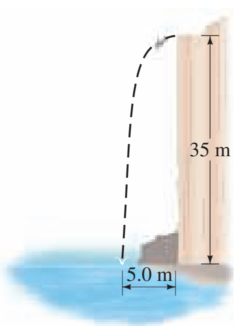


FIGURE 57 Problem 81.

82. When Babe Ruth hit a homer over the 8.0-m-high right-field fence 98 m from home plate, roughly what was the minimum speed of the ball when it left the bat? Assume the ball was hit 1.0 m above the ground and its path initially made a 36° angle with the ground.

83. The speed of a boat in still water is v. The boat is to make a round trip in a river whose current travels at speed u. Derive a formula for the time needed to make a round trip of total distance D if the boat makes the round trip by moving (*a*) upstream and back downstream, and (*b*) directly across the river and back. We must assume $u < v$; why?

84. At serve, a tennis player aims to hit the ball horizontally. What minimum speed is required for the ball to clear the 0.90-m-high net about 15.0 m from the server if the ball is "launched" from a height of 2.50 m? Where will the ball land if it just clears the net (and will it be "good" in the sense that it lands within 7.0 m of the net)? How long will it be in the air? See Fig. 58.

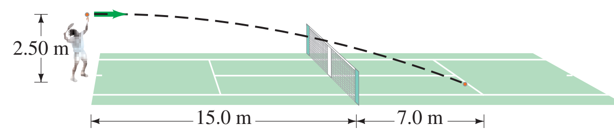


FIGURE 58 Problem 84.

85. Spymaster Chris, flying a constant 208 km/h horizontally in a low-flying helicopter, wants to drop secret documents into her contact's open car which is traveling 156 km/h on a level highway 78.0 m below. At what angle (with the horizontal) should the car be in her sights when the packet is released (Fig. 59)?

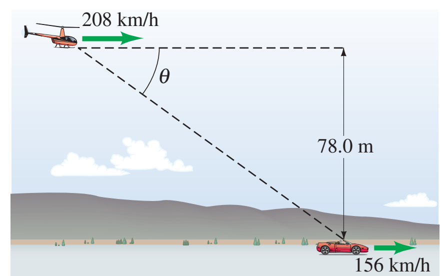


FIGURE 59 Problem 85.

86. A basketball leaves a player's hands at a height of 2.10 m above the floor. The basket is 3.05 m above the floor. The player likes to shoot the ball at a 38.0° angle. If the shot is made from a horizontal distance of 11.00 m and must be accurate to ± 0.22 m (horizontally), what is the range of initial speeds allowed to make the basket?

87. A particle has a velocity of $\vec{\mathbf{v}} = \left(-2.0\hat{\mathbf{i}} + 3.5t\hat{\mathbf{j}}\right)\mathrm{m/s}$. The particle starts at $\vec{\mathbf{r}} = \left(1.5\hat{\mathbf{i}} - 3.1\hat{\mathbf{j}}\right)\mathrm{m}$ at $t = 0$. Give the position and acceleration as a function of time. What is the shape of the resulting path?

88. A projectile is launched from ground level to the top of a cliff which is 195 m away and 135 m high (see Fig. 60). If the projectile lands on top of the

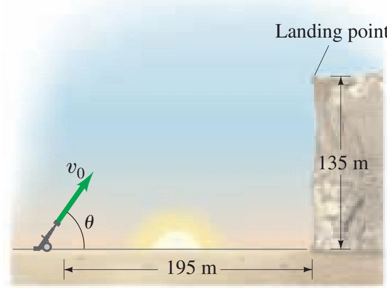


FIGURE 60 Problem 88.

cliff 6.6 s after it is fired, find the initial velocity of the projectile (magnitude and direction). Neglect air resistance.

89. In hot pursuit, Agent Logan of the FBI must get directly across a 1200-m-wide river in minimum time. The river's current is 0.80 m/s, he can row a boat at 1.60 m/s, and he can run 3.00 m/s. Describe the path he should take (rowing plus running along the shore) for the minimum crossing time, and determine the minimum time.

90. A boat can travel 2.20 m/s in still water. (*a*) If the boat points its prow directly across a stream whose current is 1.30 m/s, what is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) What will be the position of the boat, relative to its point of origin, after 3.00 s?

91. A boat is traveling where there is a current of 0.20 m/s east (Fig. 61). To avoid some offshore rocks, the boat must clear a buoy that is NNE (22.5°) and 3.0 km away. The boat's speed through still water is 2.1 m/s. If the boat wants to pass the buoy 0.15 km on its right, at what angle should the boat head?

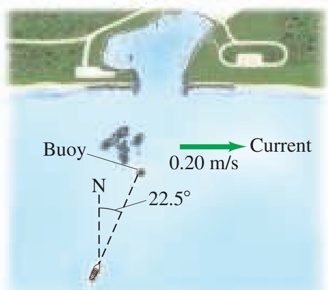


FIGURE 61
Problem 91.

92. A child runs down a 12° hill and then suddenly jumps upward at a 15° angle above horizontal and lands 1.4 m down the hill as measured along the hill. What was the child's initial speed?

93. A basketball is shot from an initial height of 2.4 m (Fig. 62) with an initial speed $v_0 = 12$ m/s directed at an angle $\theta_0 = 35°$ above the horizontal. (*a*) How far from the basket was the player if he made a basket? (*b*) At what angle to the horizontal did the ball enter the basket?

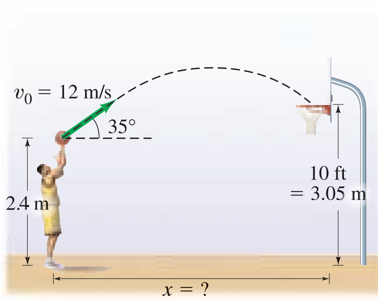


FIGURE 62
Problem 93.

94. You are driving south on a highway at 25 m/s (approximately 55 mi/h) in a snowstorm. When you last stopped, you noticed that the snow was coming down vertically, but it is passing the windows of the moving car at an angle of 37° to the horizontal. Estimate the speed of the snowflakes relative to the car and relative to the ground.

95. A rock is kicked horizontally at 15 m/s from a hill with a 45° slope (Fig. 63). How long does it take for the rock to hit the ground?

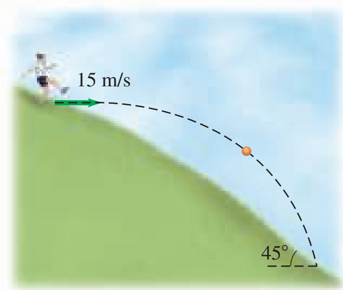


FIGURE 63
Problem 95.

96. A batter hits a fly ball which leaves the bat 0.90 m above the ground at an angle of 61° with an initial speed of 28 m/s heading toward centerfield. Ignore air resistance. (*a*) How far from home plate would the ball land if not caught? (*b*) The ball is caught by the centerfielder who, starting at a distance of 105 m from home plate, runs straight toward home plate at a constant speed and makes the catch at ground level. Find his speed.

97. A ball is shot from the top of a building with an initial velocity of 18 m/s at an angle $\theta = 42°$ above the horizontal. (*a*) What are the horizontal and vertical components of the initial velocity? (*b*) If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?

98. At $t = 0$ a batter hits a baseball with an initial speed of 28 m/s at a 55° angle to the horizontal. An outfielder is 85 m from the batter at $t = 0$ and, as seen from home plate, the line of sight to the outfielder makes a horizontal angle of 22° with the plane in which the ball moves (see Fig. 64). What speed and direction must the fielder take to catch the ball at the same height from which it was struck? Give the angle with respect to the outfielder's line of sight to home plate.

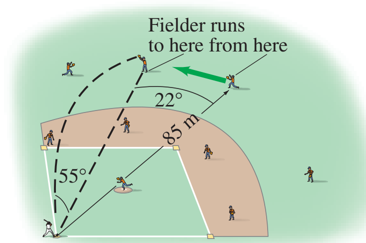


FIGURE 64
Problem 98.

*Numerical/Computer

***99.** (II) Students shoot a plastic ball horizontally from a projectile launcher. They measure the distance x the ball travels horizontally, the distance y the ball falls vertically, and the total time t the ball is in the air for six different heights of the projectile launcher. Here is their data.

Time, t (s)	Horizontal distance, x (m)	Vertical distance, y (m)
0.217	0.642	0.260
0.376	1.115	0.685
0.398	1.140	0.800
0.431	1.300	0.915
0.478	1.420	1.150
0.491	1.480	1.200

(*a*) Determine the best-fit straight line that represents x as a function of t. What is the initial speed of the ball obtained from the best-fit straight line? (*b*) Determine the best-fit quadratic equation that represents y as a function of t. What is the acceleration of the ball in the vertical direction?

*__100.__ (III) A shot-putter throws from a height $h = 2.1\text{ m}$ above the ground as shown in Fig. 65, with an initial speed of $v_0 = 13.5\text{ m/s}$. (*a*) Derive a relation that describes how the distance traveled d depends on the release angle θ_0. (*b*) Using the given values for v_0 and h, use a graphing calculator or computer to plot d vs. θ_0. According to your plot, what value for θ_0 maximizes d?

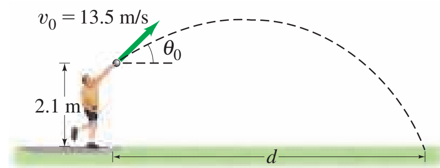
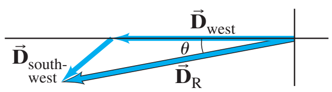


FIGURE 65 Problem 100.

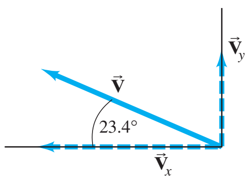
Answers to Odd-Numbered Problems

1. 286 km, 11° south of west.



3. 10.1, −39.4°.

5. (*a*)



(*b*) −22.8, 9.85;

(*c*) 24.8, 23.4° above the $-x$ axis.

7. (*a*) 625 km/h, 553 km/h;
(*b*) 1560 km, 1380 km.

9. (*a*) 4.2 at 315°;
(*b*) $1.0\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}$ or 5.1 at 280°.

11. (*a*) $-53.7\hat{\mathbf{i}} + 1.31\hat{\mathbf{j}}$ or 53.7 at 1.4° above $-x$ axis;
(*b*) $53.7\hat{\mathbf{i}} - 1.31\hat{\mathbf{j}}$ or 53.7 at 1.4° below $+x$ axis, they are opposite.

13. (*a*) $-92.5\hat{\mathbf{i}} - 19.4\hat{\mathbf{j}}$ or 94.5 at 11.8° below $-x$ axis;
(*b*) $122\hat{\mathbf{i}} - 86.6\hat{\mathbf{j}}$ or 150 at 35.3° below $+x$ axis.

15. $(-2450\text{ m})\hat{\mathbf{i}} + (3870\text{ m})\hat{\mathbf{j}} + (2450\text{ m})\hat{\mathbf{k}}$, 5190 m.

17. $(9.60\hat{\mathbf{i}} - 2.00t\hat{\mathbf{k}})\text{ m/s}$, $(-2.00\hat{\mathbf{k}})\text{ m/s}^2$.

19. Parabola.

21. (*a*) $4.0t$ m/s, $3.0t$ m/s;
(*b*) $5.0t$ m/s;
(*c*) $(2.0t^2\hat{\mathbf{i}} + 1.5t^2\hat{\mathbf{j}})\text{ m}$;
(*d*) $v_x = 8.0\text{ m/s}$, $v_y = 6.0\text{ m/s}$, $v = 10.0\text{ m/s}$, $\vec{\mathbf{r}} = (8.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}})\text{ m}$.

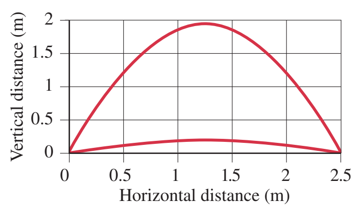
23. (*a*) $(3.16\hat{\mathbf{i}} + 2.78\hat{\mathbf{j}})\text{ cm/s}$;
(*b*) 4.21 cm/s at 41.3°.

25. (*a*) $(6.0t\hat{\mathbf{i}} - 18.0t^2\hat{\mathbf{j}})\text{ m/s}$, $(6.0\hat{\mathbf{i}} - 36.0t\hat{\mathbf{j}})\text{ m/s}^2$;
(*b*) $(19\hat{\mathbf{i}} - 94\hat{\mathbf{j}})\text{ m}$, $(15\hat{\mathbf{i}} - 110\hat{\mathbf{j}})\text{ m/s}$.

27. 414 m at −65.0°.

29. 44 m, 6.9 m.

31. 18°, 72°.



33. 2.26 s.

35. 22.3 m.

37. 39 m.

41. (*a*) 12 s;
(*b*) 62 m.

43. 5.5 s.

45. (*a*) $(2.3\hat{\mathbf{i}} + 2.5\hat{\mathbf{j}})\text{ m/s}$;
(*b*) 5.3 m;
(*c*) $(2.3\hat{\mathbf{i}} - 10.2\hat{\mathbf{j}})\text{ m/s}$.

47. No, 0.76 m too low; 4.5 m to 34.7 m.

51. $\tan^{-1} gt/v_0$.

53. (*a*) 50.0 m;
(*b*) 6.39 s;
(*c*) 221 m;
(*d*) 38.3 m/s at 25.7°.

55. $\dfrac{1}{2}\tan^{-1}\left(-\dfrac{1}{\tan\phi}\right) = \dfrac{\phi}{2} + \dfrac{\pi}{4}$.

57. $(10.5\text{ m/s})\hat{\mathbf{i}}$, $(6.5\text{ m/s})\hat{\mathbf{i}}$.

59. 1.41 m/s.

61. 23 s, 23 m.

63. (*a*) 11.2 m/s, 27° above the horizontal;
(*b*) 11.2 m/s, 27° below the horizontal.

65. 6.3°, west of south.

67. (*a*) 46 m;
(*b*) 92 s.

69. (*a*) 1.13 m/s;
(*b*) 3.20 m/s.

71. 43.6° north of east.

73. $(66\text{ m})\hat{\mathbf{i}} - (35\text{ m})\hat{\mathbf{j}} - (12\text{ m})\hat{\mathbf{k}}$, 76 m, 28° south of east, 9° below the horizontal.

75. 131 km/h, 43.1° north of east.

77. 7.0 m/s.

79. 1.8 m/s^2.

81. 1.9 m/s, 2.7 s.

83. (*a*) $\dfrac{Dv}{(v^2 - u^2)}$;
(*b*) $\dfrac{D}{\sqrt{v^2 - u^2}}$.

85. 54°.

87. $[(1.5\text{ m})\hat{\mathbf{i}} - (2.0t\text{ m})\hat{\mathbf{i}}] + [(-3.1\text{ m})\hat{\mathbf{j}} + (1.75t^2\text{ m})\hat{\mathbf{j}}]$, $(3.5\text{ m/s}^2)\hat{\mathbf{j}}$, parabolic.

89. Row at an angle of 24.9° upstream and run 104 m along the bank in a total time of 862 seconds.

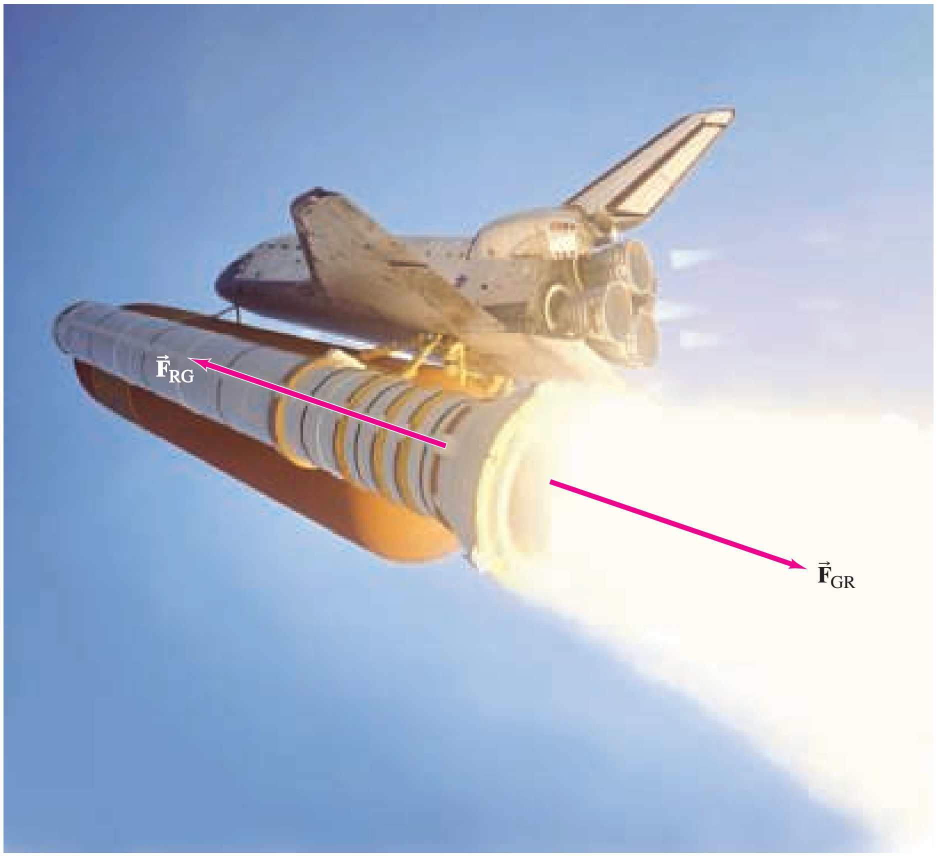
91. 69.9° north of east.

93. (*a*) 13 m;
(*b*) 31° below the horizontal.

95. 5.1 s.

97. (*a*) 13 m/s, 12 m/s;
(*b*) 33 m.

99. (*a*) $x = (3.03t - 0.0265)\text{ m}$, 3.03 m/s;
(*b*) $y = (0.158 - 0.855t + 6.09t^2)\text{ m}$, 12.2 m/s^2.



NASA/John F. Kennedy Space Center

The space shuttle Discovery is carried out into space by powerful rockets. They are accelerating, increasing in speed rapidly. To do so, a force must be exerted on them according to Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. What exerts this force? The rocket engines exert a force on the gases they push out (expel) from the rear of the rockets (labeled $\vec{\mathbf{F}}_{GR}$). According to Newton's third law, these ejected gases exert an equal and opposite force on the rockets in the forward direction. It is this "reaction" force exerted on the **r**ockets by the **g**ases, labeled $\vec{\mathbf{F}}_{RG}$, that accelerates the rockets forward.

Dynamics: Newton's Laws of Motion

CHAPTER-OPENING QUESTIONS—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

A 150-kg football player collides head-on with a 75-kg running back. During the collision, the heavier player exerts a force of magnitude F_A on the smaller player. If the smaller player exerts a force F_B back on the heavier player, which response is most accurate?

(a) $F_B = F_A$.
(b) $F_B < F_A$.
(c) $F_B > F_A$.
(d) $F_B = 0$.
(e) We need more information.

Second Question:

A line by the poet T. S. Eliot (from *Murder in the Cathedral*) has the women of Canterbury say "the earth presses up against our feet." What force is this?

(a) Gravity.
(b) The normal force.
(c) A friction force.
(d) Centrifugal force.
(e) No force—they are being poetic.

CONTENTS

Recall how motion is described in terms of velocity and acceleration. Now we deal with the question of *why* objects move as they do: What makes an object at rest begin to move? What causes an object to accelerate or decelerate? What is involved when an object moves in a curved path? We can answer in each case that a force is required. In this Chapter†, we will investigate the connection between force and motion, which is the subject called **dynamics**.

1 Force

Daly & Newton/Getty Images

FIGURE 1 A force exerted on a grocery cart—in this case exerted by a person.

Intuitively, we experience **force** as any kind of a push or a pull on an object. When you push a stalled car or a grocery cart (Fig. 1), you are exerting a force on it. When a motor lifts an elevator, or a hammer hits a nail, or the wind blows the leaves of a tree, a force is being exerted. We often call these *contact forces* because the force is exerted when one object comes in contact with another object. On the other hand, we say that an object falls because of the *force of gravity*.

If an object is at rest, to start it moving requires force—that is, a force is needed to accelerate an object from zero velocity to a nonzero velocity. For an object already moving, if you want to change its velocity—either in direction or in magnitude—a force is required. In other words, to accelerate an object, a force is always required. In Section 4 we discuss the precise relation between acceleration and net force, which is Newton's second law.

One way to measure the magnitude (or strength) of a force is to use a spring scale (Fig. 2). Normally, such a spring scale is used to find the weight of an object; by weight we mean the force of gravity acting on the object (Section 6). The spring scale, once calibrated, can be used to measure other kinds of forces as well, such as the pulling force shown in Fig. 2.

A force exerted in a different direction has a different effect. Force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition. We can represent any force on a diagram by an arrow, just as we do with velocity. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force.

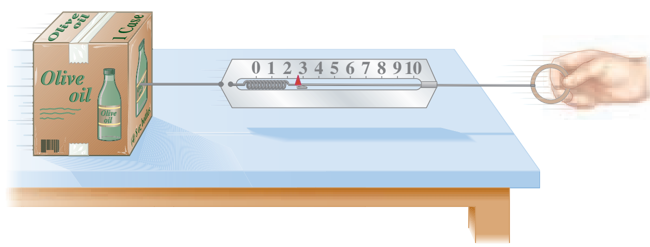


FIGURE 2 A spring scale used to measure a force.

2 Newton's First Law of Motion

What is the relationship between force and motion? Aristotle (384–322 B.C.) believed that a force was required to keep an object moving along a horizontal plane. To Aristotle, the natural state of an object was at rest, and a force was believed necessary to keep an object in motion. Furthermore, Aristotle argued, the greater the force on the object, the greater its speed.

Some 2000 years later, Galileo disagreed: he maintained that it is just as natural for an object to be in motion with a constant velocity as it is for it to be at rest.

To understand Galileo's idea, consider the following observations involving motion along a horizontal plane. To push an object with a rough surface along a

†We treat everyday objects in motion here; the treatment of the submicroscopic world of atoms and molecules, and when velocities are extremely high, close to the speed of light (3.0×10^8 m/s), are treated using quantum theory and the theory of relativity.

tabletop at constant speed requires a certain amount of force. To push an equally heavy object with a very smooth surface across the table at the same speed will require less force. If a layer of oil or other lubricant is placed between the surface of the object and the table, then almost no force is required to keep the object moving. Notice that in each successive step, less force is required. As the next step, we imagine that the object does not rub against the table at all—or there is a perfect lubricant between the object and the table—and theorize that once started, the object would move across the table at constant speed with *no* force applied. A steel ball bearing rolling on a hard horizontal surface approaches this situation. So does a puck on an air table, in which a thin layer of air reduces friction almost to zero.

It was Galileo's genius to imagine such an idealized world—in this case, one where there is no friction—and to see that it could lead to a more accurate and richer understanding of the real world. This idealization led him to his remarkable conclusion that if no force is applied to a moving object, it will continue to move with constant speed in a straight line. An object slows down only if a force is exerted on it. Galileo thus interpreted friction as a force akin to ordinary pushes and pulls.

To push an object across a table at constant speed requires a force from your hand that can balance out the force of friction (Fig. 3). When the object moves at constant speed, your pushing force is equal in magnitude to the friction force, but these two forces are in opposite directions, so the *net* force on the object (the vector sum of the two forces) is zero. This is consistent with Galileo's viewpoint, for the object moves with constant speed when no net force is exerted on it.

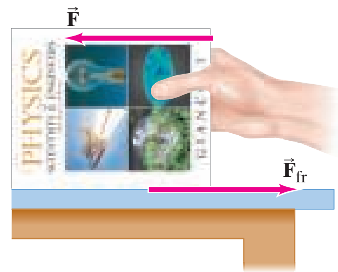


FIGURE 3 $\vec{\mathbf{F}}$ represents the force applied by the person and $\vec{\mathbf{F}}_{fr}$ represents the force of friction.

Upon this foundation laid by Galileo, Isaac Newton (Fig. 4) built his great theory of motion. Newton's analysis of motion is summarized in his famous "three laws of motion." In his great work, the *Principia* (published in 1687), Newton readily acknowledged his debt to Galileo. In fact, **Newton's first law of motion** is close to Galileo's conclusions. It states that

NEWTON'S FIRST LAW OF MOTION

> **Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it.**

The tendency of an object to maintain its state of rest or of uniform velocity in a straight line is called **inertia**. As a result, Newton's first law is often called the **law of inertia**.

CONCEPTUAL EXAMPLE 1 **Newton's first law.** A school bus comes to a sudden stop, and all of the backpacks on the floor start to slide forward. What force causes them to do that?

RESPONSE It isn't "force" that does it. By Newton's first law, the backpacks continue their state of motion, maintaining their velocity. The backpacks slow down if a force is applied, such as friction with the floor.

FIGURE 4
Isaac Newton (1642–1727).

Bettmann/Corbis

Inertial Reference Frames

Newton's first law does not hold in every reference frame. For example, if your reference frame is fixed in an accelerating car, an object such as a cup resting on the dashboard may begin to move toward you (it stayed at rest as long as the car's velocity remained constant). The cup accelerated toward you, but neither you nor anything else exerted a force on it in that direction. Similarly, in the reference frame of the decelerating bus in Example 1, there was no force pushing the backpacks forward. In accelerating reference frames, Newton's first law does not hold. Reference frames in which Newton's first law does hold are called **inertial reference frames** (the law of inertia is valid in them). For most purposes, we usually make the approximation that a reference frame fixed on the Earth is an inertial frame. This is not precisely true, due to the Earth's rotation, but usually it is close enough.

Any reference frame that moves with constant velocity (say, a car or an airplane) relative to an inertial frame is also an inertial reference frame. Reference frames where the law of inertia does *not* hold, such as the accelerating reference frames discussed above, are called **noninertial** reference frames. How can we be sure a reference frame is inertial or not? By checking to see if Newton's first law holds. Thus Newton's first law serves as the definition of inertial reference frames.

3 Mass

Newton's second law, which we come to in the next Section, makes use of the concept of mass. Newton used the term *mass* as a synonym for *quantity of matter*. This intuitive notion of the mass of an object is not very precise because the concept "quantity of matter" is not very well defined. More precisely, we can say that **mass** is a *measure of the inertia* of an object. The more mass an object has, the greater the force needed to give it a particular acceleration. It is harder to start it moving from rest, or to stop it when it is moving, or to change its velocity sideways out of a straight-line path. A truck has much more inertia than a baseball moving at the same speed, and a much greater force is needed to change the truck's velocity at the same rate as the ball's. The truck therefore has much more mass.

To quantify the concept of mass, we must define a standard. In SI units, the unit of mass is the **kilogram** (kg).

CAUTION
Distinguish mass from weight

The terms *mass* and *weight* are often confused with one another, but it is important to distinguish between them. Mass is a property of an object itself (a measure of an object's inertia, or its "quantity of matter"). Weight, on the other hand, is a force, the pull of gravity acting on an object. To see the difference, suppose we take an object to the Moon. The object will weigh only about one-sixth as much as it did on Earth, since the force of gravity is weaker. But its mass will be the same. It will have the same amount of matter as on Earth, and will have just as much inertia—for in the absence of friction, it will be just as hard to start it moving on the Moon as on Earth, or to stop it once it is moving. (More on weight in Section 6.)

4 Newton's Second Law of Motion

Gerard Vandystadt/Agence Vandystadt/Photo Researchers, Inc.

FIGURE 5 The bobsled accelerates because the team exerts a force.

Newton's first law states that if no net force is acting on an object at rest, the object remains at rest; or if the object is moving, it continues moving with constant speed in a straight line. But what happens if a net force *is* exerted on an object? Newton perceived that the object's velocity will change (Fig. 5). A net force exerted on an object may make its velocity increase. Or, if the net force is in a direction opposite to the motion, the force will reduce the object's velocity. If the net force acts sideways on a moving object, the *direction* of the object's velocity changes (and the magnitude may as well). Since a change in velocity is an acceleration, we can say that *a net force causes acceleration*.

What precisely is the relationship between acceleration and force? Everyday experience can suggest an answer. Consider the force required to push a cart when friction is small enough to ignore. (If there is friction, consider the *net* force, which is the force you exert minus the force of friction.) If you push the cart with a gentle but constant force for a certain period of time, you will make the cart accelerate from rest up to some speed, say 3 km/h. If you push with twice the force, the cart will reach 3 km/h in half the time. The acceleration will be twice as great. If you triple the force, the acceleration is tripled, and so on. Thus, the acceleration of an object is directly proportional to the net applied force. But the acceleration depends on the mass of the object as well. If you push an empty grocery cart with the same force as you push one that is filled with groceries, you will find that the full cart accelerates more slowly. The greater the mass, the less the acceleration for the same net force. The mathematical relation, as Newton argued, is that the acceleration of an object is inversely proportional to its mass. These relationships are found to hold in general and can be summarized as follows:

NEWTON'S SECOND LAW OF MOTION

The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

This is **Newton's second law of motion**.

Newton's second law can be written as an equation:

$$\vec{\mathbf{a}} = \frac{\Sigma \vec{\mathbf{F}}}{m},$$

where $\vec{\mathbf{a}}$ stands for acceleration, m for the mass, and $\Sigma\vec{\mathbf{F}}$ for the *net force* on the object. The symbol Σ (Greek "sigma") stands for "sum of"; $\vec{\mathbf{F}}$ stands for force, so $\Sigma\vec{\mathbf{F}}$ means the *vector sum of all forces* acting on the object, which we define as the **net force**.

We rearrange this equation to obtain the familiar statement of Newton's second law:

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \quad \textbf{(1a)}$$

NEWTON'S SECOND LAW OF MOTION

Newton's second law relates the description of motion (acceleration) to the cause of motion (force). It is one of the most fundamental relationships in physics. From Newton's second law we can make a more precise definition of **force** as *an action capable of accelerating an object*.

Every force $\vec{\mathbf{F}}$ is a vector, with magnitude and direction. Equation 1a is a vector equation valid in any inertial reference frame. It can be written in component form in rectangular coordinates as

$$\Sigma F_x = ma_x, \qquad \Sigma F_y = ma_y, \qquad \Sigma F_z = ma_z, \quad \textbf{(1b)}$$

where

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} + F_z\hat{\mathbf{k}}.$$

The component of acceleration in each direction is affected only by the component of the net force in that direction.

In SI units, with the mass in kilograms, the unit of force is called the **newton** (N). One newton, then, is the force required to impart an acceleration of $1\ \mathrm{m/s^2}$ to a mass of 1 kg. Thus $1\ \mathrm{N} = 1\ \mathrm{kg \cdot m/s^2}$.

In cgs units, the unit of mass is the gram (g) as mentioned earlier.† The unit of force is the *dyne*, which is defined as the net force needed to impart an acceleration of $1\ \mathrm{cm/s^2}$ to a mass of 1 g. Thus $1\ \mathrm{dyne} = 1\ \mathrm{g \cdot cm/s^2}$. It is easy to show that $1\ \mathrm{dyne} = 10^{-5}\ \mathrm{N}$.

In the British system, the unit of force is the *pound* (abbreviated lb), where $1\ \mathrm{lb} = 4.448222\ \mathrm{N} \approx 4.45\ \mathrm{N}$. The unit of mass is the *slug*, which is defined as that mass which will undergo an acceleration of $1\ \mathrm{ft/s^2}$ when a force of 1 lb is applied to it. Thus $1\ \mathrm{lb} = 1\ \mathrm{slug \cdot ft/s^2}$. Table 1 summarizes the units in the different systems.

TABLE 1 Units for Mass and Force

System	Mass	Force
SI	kilogram (kg)	newton (N) ($= \mathrm{kg \cdot m/s^2}$)
cgs	gram (g)	dyne ($= \mathrm{g \cdot cm/s^2}$)
British	slug	pound (lb)

Conversion factors: $1\ \mathrm{dyne} = 10^{-5}\ \mathrm{N}$; $1\ \mathrm{lb} \approx 4.45\ \mathrm{N}$.

PROBLEM SOLVING
Use a consistent set of units

It is very important that only one set of units be used in a given calculation or problem, with the SI being preferred. If the force is given in, say, newtons, and the mass in grams, then before attempting to solve for the acceleration in SI units, we must change the mass to kilograms. For example, if the force is given as 2.0 N along the x axis and the mass is 500 g, we change the latter to 0.50 kg, and the acceleration will then automatically come out in $\mathrm{m/s^2}$ when Newton's second law is used:

$$a_x = \frac{\Sigma F_x}{m} = \frac{2.0\ \mathrm{N}}{0.50\ \mathrm{kg}} = \frac{2.0\ \mathrm{kg \cdot m/s^2}}{0.50\ \mathrm{kg}} = 4.0\ \mathrm{m/s^2}.$$

EXAMPLE 2 ESTIMATE Force to accelerate a fast car. Estimate the net force needed to accelerate (*a*) a 1000-kg car at $\frac{1}{2}g$; (*b*) a 200-g apple at the same rate.

APPROACH We use Newton's second law to find the net force needed for each object. This is an estimate (the $\frac{1}{2}$ is not said to be precise) so we round off to one significant figure.

SOLUTION (*a*) The car's acceleration is $a = \frac{1}{2}g = \frac{1}{2}(9.8\ \mathrm{m/s^2}) \approx 5\ \mathrm{m/s^2}$. We use Newton's second law to get the net force needed to achieve this acceleration:

$$\Sigma F = ma \approx (1000\ \mathrm{kg})(5\ \mathrm{m/s^2}) = 5000\ \mathrm{N}.$$

(If you are used to British units, to get an idea of what a 5000-N force is, you can divide by 4.45 N/lb and get a force of about 1000 lb.)

(*b*) For the apple, $m = 200\ \mathrm{g} = 0.2\ \mathrm{kg}$, so

$$\Sigma F = ma \approx (0.2\ \mathrm{kg})(5\ \mathrm{m/s^2}) = 1\ \mathrm{N}.$$

†Be careful not to confuse g for gram with g for the acceleration due to gravity. The latter is always italicized (or boldface when a vector).

EXAMPLE 3 **Force to stop a car.** What average net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

APPROACH We use Newton's second law, $\Sigma F = ma$, to determine the force, but first we need to calculate the acceleration a. We assume the acceleration is constant, so we can use the kinematic equation, to calculate it.

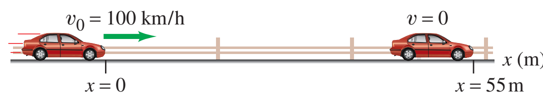


FIGURE 6 Example 3.

SOLUTION We assume the motion is along the $+x$ axis (Fig. 6). We are given the initial velocity $v_0 = 100\,\text{km/h} = 27.8\,\text{m/s}$, the final velocity $v = 0$, and the distance traveled $x - x_0 = 55\,\text{m}$. We have

$$v^2 = v_0^2 + 2a(x - x_0),$$

so

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (27.8\,\text{m/s})^2}{2(55\,\text{m})} = -7.0\,\text{m/s}^2.$$

The net force required is then

$$\Sigma F = ma = (1500\,\text{kg})(-7.0\,\text{m/s}^2) = -1.1 \times 10^4\,\text{N}.$$

The force must be exerted in the direction *opposite* to the initial velocity, which is what the negative sign means.

NOTE If the acceleration is not precisely constant, then we are determining an "average" acceleration and we obtain an "average" net force.

Newton's second law, like the first law, is valid only in inertial reference frames (Section 2). In the noninertial reference frame of an accelerating car, for example, a cup on the dashboard starts sliding—it accelerates—even though the net force on it is zero; thus $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ doesn't work in such an accelerating reference frame ($\Sigma\vec{\mathbf{F}} = 0$, but $\vec{\mathbf{a}} \neq 0$ in this noninertial frame).

EXERCISE A Suppose you watch a cup slide on the (smooth) dashboard of an accelerating car as we just discussed, but this time from an inertial reference frame outside the car, on the street. From your inertial frame, Newton's laws are valid. What force pushes the cup off the dashboard?

Precise Definition of Mass

As mentioned in Section 3, we can quantify the concept of mass using its definition as a measure of inertia. How to do this is evident from Eq. 1a, where we see that the acceleration of an object is inversely proportional to its mass. If the same net force ΣF acts to accelerate each of two masses, m_1 and m_2, then the ratio of their masses can be defined as the inverse ratio of their accelerations:

$$\frac{m_2}{m_1} = \frac{a_1}{a_2}.$$

If one of the masses is known (it could be the standard kilogram) and the two accelerations are precisely measured, then the unknown mass is obtained from this definition. For example, if $m_1 = 1.00\,\text{kg}$, and for a particular force $a_1 = 3.00\,\text{m/s}^2$ and $a_2 = 2.00\,\text{m/s}^2$, then $m_2 = 1.50\,\text{kg}$.

5 Newton's Third Law of Motion

Newton's second law of motion describes quantitatively how forces affect motion. But where, we may ask, do forces come from? Observations suggest that a force exerted on any object is always exerted *by another object*. A horse pulls a wagon, a person pushes a grocery cart, a hammer pushes on a nail, a magnet attracts a paper clip. In each of these examples, a force is exerted *on* one object, and that force is exerted *by* another object. For example, the force exerted *on* the nail is exerted *by* the hammer.

But Newton realized that things are not so one-sided. True, the hammer exerts a force on the nail (Fig. 7). But the nail evidently exerts a force back on the hammer as well, for the hammer's speed is rapidly reduced to zero upon contact. Only a strong force could cause such a rapid deceleration of the hammer. Thus, said Newton, the two objects must be treated on an equal basis. The hammer exerts a force on the nail, and the nail exerts a force back on the hammer. This is the essence of **Newton's third law of motion**:

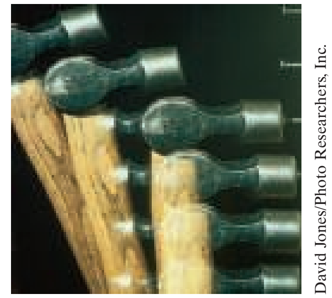


FIGURE 7 A hammer striking a nail. The hammer exerts a force on the nail and the nail exerts a force back on the hammer. The latter force decelerates the hammer and brings it to rest.

NEWTON'S THIRD LAW OF MOTION

> **Whenever one object exerts a force on a second object, the second exerts an equal force in the opposite direction on the first.**

This law is sometimes paraphrased as "to every action there is an equal and opposite reaction." This is perfectly valid. But to avoid confusion, it is very important to remember that the "action" force and the "reaction" force are acting on *different* objects.

CAUTION

Action and reaction forces act on different objects

As evidence for the validity of Newton's third law, look at your hand when you push against the edge of a desk, Fig. 8. Your hand's shape is distorted, clear evidence that a force is being exerted on it. You can *see* the edge of the desk pressing into your hand. You can even *feel* the desk exerting a force on your hand; it hurts! The harder you push against the desk, the harder the desk pushes back on your hand. (You only feel forces exerted *on* you; when you exert a force on another object, what you feel is that object pushing back on you.)

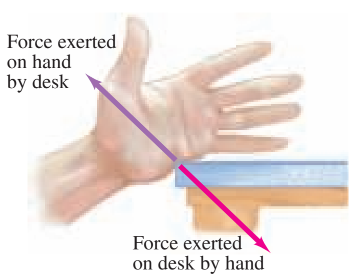
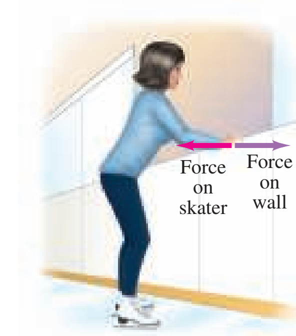


FIGURE 8 If your hand pushes against the edge of a desk (the force vector is shown in red), the desk pushes back against your hand (this force vector is shown in a different color, violet, to remind us that this force acts on a different object).

The force the desk exerts on your hand has the same magnitude as the force your hand exerts on the desk. This is true not only if the desk is at rest but is true even if the desk is accelerating due to the force your hand exerts.

As another demonstration of Newton's third law, consider the ice skater in Fig. 9. There is very little friction between her skates and the ice, so she will move freely if a force is exerted on her. She pushes against the wall; and then *she* starts moving backward. The force she exerts on the wall cannot make *her* start moving, for that force acts on the wall. Something had to exert a force *on her* to start her moving, and that force could only have been exerted by the wall. The force with which the wall pushes on her is, by Newton's third law, equal and opposite to the force she exerts on the wall.

FIGURE 9 An example of Newton's third law: when an ice skater pushes against the wall, the wall pushes back and this force causes her to accelerate away.

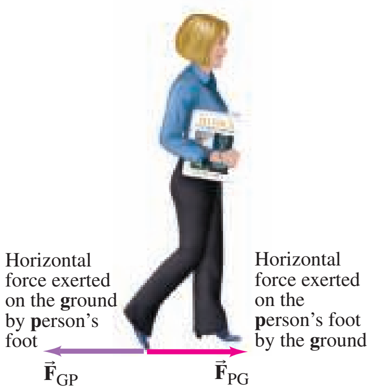


When a person throws a package out of a small boat (initially at rest), the boat starts moving in the opposite direction. The person exerts a force on the package. The package exerts an equal and opposite force back on the person, and this force propels the person (and the boat) backward slightly.

NASA/John F. Kennedy Space Center

FIGURE 10 Another example of Newton's third law: the launch of a rocket. The rocket engine pushes the gases downward, and the gases exert an equal and opposite force upward on the rocket, accelerating it upward. (A rocket does *not* accelerate as a result of its propelling gases pushing against the ground.)

FIGURE 11 We can walk forward because, when one foot pushes backward against the ground, the ground pushes forward on that foot (Newton's third law). The two forces shown *act on different objects*.



Rocket propulsion also is explained using Newton's third law (Fig. 10). A common misconception is that rockets accelerate because the gases rushing out the back of the engine push against the ground or the atmosphere. Not true. What happens, instead, is that a rocket exerts a strong force on the gases, expelling them; and the gases exert an equal and opposite force *on the rocket*. It is this latter force that propels the rocket forward—the force exerted *on* the rocket *by* the gases (see Chapter-Opening photo). Thus, a space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate. When the rocket pushes on the gases in one direction, the gases push back on the rocket in the opposite direction. Jet aircraft too accelerate because the gases they thrust out backwards exert a forward force on the engines (Newton's third law).

Consider how we walk. A person begins walking by pushing with the foot backward against the ground. The ground then exerts an equal and opposite force forward on the person (Fig. 11), and it is this force, *on* the person, that moves the person forward. (If you doubt this, try walking normally where there is no friction, such as on very smooth slippery ice.) In a similar way, a bird flies forward by exerting a backward force on the air, but it is the air pushing forward (Newton's third law) on the bird's wings that propels the bird forward.

CONCEPTUAL EXAMPLE 4 **What exerts the force to move a car?** What makes a car go forward?

RESPONSE A common answer is that the engine makes the car move forward. But it is not so simple. The engine makes the wheels go around. But if the tires are on slick ice or deep mud, they just spin. Friction is needed. On firm ground, the tires push backward against the ground because of friction. By Newton's third law, the ground pushes on the tires in the opposite direction, accelerating the car forward.

We tend to associate forces with active objects such as humans, animals, engines, or a moving object like a hammer. It is often difficult to see how an inanimate object at rest, such as a wall or a desk, or the wall of an ice rink (Fig. 9), can exert a force. The explanation is that every material, no matter how hard, is elastic (springy) at least to some degree. A stretched rubber band can exert a force on a wad of paper and accelerate it to fly across the room. Other materials may not stretch as readily as rubber, but they do stretch or compress when a force is applied to them. And just as a stretched rubber band exerts a force, so does a stretched (or compressed) wall, desk, or car fender.

From the examples discussed above, we can see how important it is to remember *on* what object a given force is exerted and *by* what object that force is exerted. A force influences the motion of an object only when it is applied *on* that object. A force exerted *by* an object does not influence that same object; it only influences the other object *on* which it is exerted. Thus, to avoid confusion, the two prepositions *on* and *by* must always be used—and used with care.

One way to keep clear which force acts on which object is to use double subscripts. For example, the force exerted on the **P**erson by the **G**round as the person walks in Fig. 11 can be labeled $\vec{\mathbf{F}}_{PG}$. And the force exerted on the ground by the person is $\vec{\mathbf{F}}_{GP}$. By Newton's third law

NEWTON'S THIRD LAW OF MOTION

$$\vec{\mathbf{F}}_{GP} = -\vec{\mathbf{F}}_{PG}. \tag{2}$$

$\vec{\mathbf{F}}_{GP}$ and $\vec{\mathbf{F}}_{PG}$ have the same magnitude (Newton's third law), and the minus sign reminds us that these two forces are in opposite directions.

Note carefully that the two forces shown in Fig. 11 act on different objects—hence we used slightly different colors for the vector arrows representing these forces. These two forces would never appear together in a sum of forces in Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Why not? Because they act on different objects: $\vec{\mathbf{a}}$ is the acceleration of one particular object, and $\Sigma\vec{\mathbf{F}}$ must include *only* the forces on that *one* object.

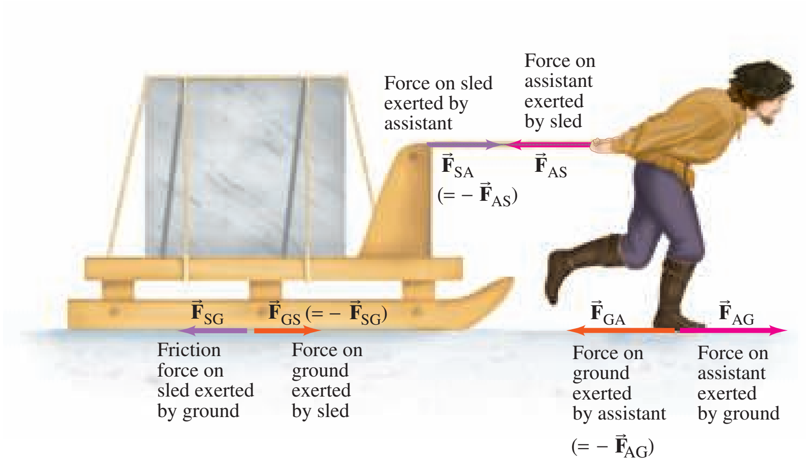


FIGURE 12 Example 5, showing only horizontal forces. Michelangelo has selected a fine block of marble for his next sculpture. Shown here is his assistant pulling it on a sled away from the quarry. Forces on the assistant are shown as red (magenta) arrows. Forces on the sled are purple arrows. Forces acting on the ground are orange arrows. Action–reaction forces that are equal and opposite are labeled by the same subscripts but reversed (such as $\vec{\mathbf{F}}_{GA}$ and $\vec{\mathbf{F}}_{AG}$) and are of different colors because they act on different objects.

CONCEPTUAL EXAMPLE 5 **Third law clarification.** Michelangelo's assistant has been assigned the task of moving a block of marble using a sled (Fig. 12). He says to his boss, "When I exert a forward force on the sled, the sled exerts an equal and opposite force backward. So how can I ever start it moving? No matter how hard I pull, the backward reaction force always equals my forward force, so the net force must be zero. I'll never be able to move this load." Is he correct?

RESPONSE No. Although it is true that the action and reaction forces are equal in magnitude, the assistant has forgotten that they are exerted on different objects. The forward ("action") force is exerted by the assistant on the sled (Fig. 12), whereas the backward "reaction" force is exerted by the sled on the assistant. To determine if the *assistant* moves or not, we must consider only the forces *on the assistant* and then apply $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, where $\Sigma\vec{\mathbf{F}}$ is the net force *on the assistant*, $\vec{\mathbf{a}}$ is the acceleration of the assistant, and m is the assistant's mass. There are two forces on the assistant that affect his forward motion; they are shown as bright red (magenta) arrows in Figs. 12 and 13: they are (1) the horizontal force $\vec{\mathbf{F}}_{AG}$ exerted on the assistant by the ground (the harder he pushes backward against the ground, the harder the ground pushes forward on him—Newton's third law), and (2) the force $\vec{\mathbf{F}}_{AS}$ exerted on the assistant by the sled, pulling backward on him; see Fig. 13. If he pushes hard enough on the ground, the force on him exerted by the ground, $\vec{\mathbf{F}}_{AG}$, will be larger than the sled pulling back, $\vec{\mathbf{F}}_{AS}$, and the assistant accelerates forward (Newton's second law). The sled, on the other hand, accelerates forward when the force on *it* exerted by the assistant is greater than the frictional force exerted backward on it by the ground (that is, when $\vec{\mathbf{F}}_{SA}$ has greater magnitude than $\vec{\mathbf{F}}_{SG}$ in Fig. 12).

PROBLEM SOLVING
A study of Newton's second and third laws

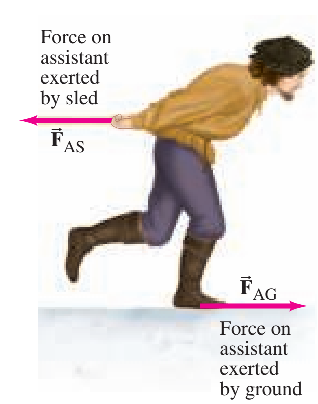


FIGURE 13 Example 5. The horizontal forces on the assistant.

Using double subscripts to clarify Newton's third law can become cumbersome, and we won't usually use them in this way. We will usually use a single subscript referring to what exerts the force on the object being discussed. Nevertheless, if there is any confusion in your mind about a given force, go ahead and use two subscripts to identify *on* what object and *by* what object the force is exerted.

EXERCISE B Return to the first Chapter-Opening Question, and answer it again now. Try to explain why you may have answered differently the first time.

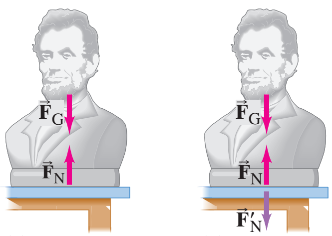
EXERCISE C A massive truck collides head-on with a small sports car. (*a*) Which vehicle experiences the greater force of impact? (*b*) Which experiences the greater acceleration during the impact? (*c*) Which of Newton's laws are useful to obtain the correct answers?

EXERCISE D If you push on a heavy desk, does it always push back on you? (*a*) Not unless someone else also pushes on it. (*b*) Yes, if it is out in space. (*c*) A desk never pushes to start with. (*d*) No. (*e*) Yes.

6 Weight—the Force of Gravity; and the Normal Force

Galileo claimed that all objects dropped near the surface of the Earth would fall with the same acceleration, $\vec{\mathbf{g}}$, if air resistance was negligible. The force that causes this acceleration is called the *force of gravity* or *gravitational force*. What exerts the gravitational force on an object? It is the Earth, and the force acts vertically[†] downward, toward the center of the Earth. Let us apply Newton's second law to an object of mass m falling freely due to gravity. For the acceleration, $\vec{\mathbf{a}}$, we use the downward acceleration due to gravity, $\vec{\mathbf{g}}$. Thus, the **gravitational force** on an object, $\vec{\mathbf{F}}_G$, can be written as

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \tag{3}$$



(a) (b)

FIGURE 14 (a) The net force on an object at rest is zero according to Newton's second law. Therefore the downward force of gravity ($\vec{\mathbf{F}}_G$) on an object at rest must be balanced by an upward force (the normal force $\vec{\mathbf{F}}_N$) exerted by the table in this case. (b) $\vec{\mathbf{F}}'_N$ is the force exerted on the table by the statue and is the reaction force to $\vec{\mathbf{F}}_N$ by Newton's third law. ($\vec{\mathbf{F}}'_N$ is shown in a different color to remind us it acts on a different object.) The reaction force to $\vec{\mathbf{F}}_G$ is not shown.

CAUTION
Weight and normal force are ***not*** *action–reaction pairs*

The direction of this force is down toward the center of the Earth. The magnitude of the force of gravity on an object, mg, is commonly called the object's **weight**.

In SI units, $g = 9.80\ \text{m/s}^2 = 9.80\ \text{N/kg}$,[‡] so the weight of a 1.00-kg mass on Earth is $1.00\ \text{kg} \times 9.80\ \text{m/s}^2 = 9.80\ \text{N}$. We will mainly be concerned with the weight of objects on Earth, but we note that on the Moon, on other planets, or in space, the weight of a given mass will be different than it is on Earth. For example, on the Moon the acceleration due to gravity is about one-sixth what it is on Earth, and a 1.0-kg mass weighs only 1.6 N. Although we will not use British units, we note that for practical purposes on the Earth, a mass of 1 kg weighs about 2.2 lb. (On the Moon, 1 kg weighs only about 0.4 lb.)

The force of gravity acts on an object when it is falling. When an object is at rest on the Earth, the gravitational force on it does not disappear, as we know if we weigh it on a spring scale. The same force, given by Eq. 3, continues to act. Why, then, doesn't the object move? From Newton's second law, the net force on an object that remains at rest is zero. There must be another force on the object to balance the gravitational force. For an object resting on a table, the table exerts this upward force; see Fig. 14a. The table is compressed slightly beneath the object, and due to its elasticity, it pushes up on the object as shown. The force exerted by the table is often called a **contact force**, since it occurs when two objects are in contact. (The force of your hand pushing on a cart is also a contact force.) When a contact force acts *perpendicular* to the common surface of contact, it is referred to as the **normal force** ("normal" means perpendicular); hence it is labeled $\vec{\mathbf{F}}_N$ in Fig. 14a.

The two forces shown in Fig. 14a are both acting on the statue, which remains at rest, so the vector sum of these two forces must be zero (Newton's second law). Hence $\vec{\mathbf{F}}_G$ and $\vec{\mathbf{F}}_N$ must be of equal magnitude and in opposite directions. But they are *not* the equal and opposite forces spoken of in Newton's third law. The action and reaction forces of Newton's third law act on *different objects*, whereas the two forces shown in Fig. 14a act on the *same* object. For each of the forces shown in Fig. 14a, we can ask, "What is the reaction force?" The upward force, $\vec{\mathbf{F}}_N$, on the statue is exerted by the table. The reaction to this force is a force exerted by the statue downward on the table. It is shown in Fig. 14b, where it is labeled $\vec{\mathbf{F}}'_N$. This force, $\vec{\mathbf{F}}'_N$, exerted on the table by the statue, is the reaction force to $\vec{\mathbf{F}}_N$ in accord with Newton's third law. What about the other force on the statue, the force of gravity $\vec{\mathbf{F}}_G$ exerted by the Earth? Can you guess what the reaction is to this force? The reaction force is also a gravitational force, exerted on the Earth by the statue.

EXERCISE E Return to the second Chapter-Opening Question, and answer it again now. Try to explain why you may have answered differently the first time.

[†] The concept of "vertical" is tied to gravity. The best definition of *vertical* is that it is the direction in which objects fall. A surface that is "horizontal," on the other hand, is a surface on which a round object won't start rolling: gravity has no effect. Horizontal is perpendicular to vertical.

[‡] Since $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$ (Section 4), then $1\ \text{m/s}^2 = 1\ \text{N/kg}$.

EXAMPLE 6 **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 15a). (*a*) Determine the weight of the box and the normal force exerted on it by the table. (*b*) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 15b. Again determine the normal force exerted on the box by the table. (*c*) If your friend pulls upward on the box with a force of 40.0 N (Fig. 15c), what now is the normal force exerted on the box by the table?

APPROACH The box is at rest on the table, so the net force on the box in each case is zero (Newton's second law). The weight of the box has magnitude mg in all three cases.

SOLUTION (*a*) The weight of the box is $mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}$, and this force acts downward. The only other force on the box is the normal force exerted upward on it by the table, as shown in Fig. 15a. We chose the upward direction as the positive y direction; then the net force ΣF_y on the box is $\Sigma F_y = F_\mathrm{N} - mg$; the minus sign means mg acts in the negative y direction (m and g are magnitudes). The box is at rest, so the net force on it must be zero (Newton's second law, $\Sigma F_y = ma_y$, and $a_y = 0$). Thus

$$\begin{aligned}\Sigma F_y &= ma_y\\ F_\mathrm{N} - mg &= 0,\end{aligned}$$

so we have

$$F_\mathrm{N} = mg.$$

The normal force on the box, exerted by the table, is 98.0 N upward, and has magnitude equal to the box's weight.

(*b*) Your friend is pushing down on the box with a force of 40.0 N. So instead of only two forces acting on the box, now there are three forces acting on the box, as shown in Fig. 15b. The weight of the box is still $mg = 98.0\,\mathrm{N}$. The net force is $\Sigma F_y = F_\mathrm{N} - mg - 40.0\,\mathrm{N}$, and is equal to zero because the box remains at rest $(a = 0)$. Newton's second law gives

$$\Sigma F_y = F_\mathrm{N} - mg - 40.0\,\mathrm{N} = 0.$$

We solve this equation for the normal force:

$$F_\mathrm{N} = mg + 40.0\,\mathrm{N} = 98.0\,\mathrm{N} + 40.0\,\mathrm{N} = 138.0\,\mathrm{N},$$

which is greater than in (*a*). The table pushes back with more force when a person pushes down on the box. The normal force is not always equal to the weight!

(*c*) The box's weight is still 98.0 N and acts downward. The force exerted by your friend and the normal force both act upward (positive direction), as shown in Fig. 15c. The box doesn't move since your friend's upward force is less than the weight. The net force, again set to zero in Newton's second law because $a = 0$, is

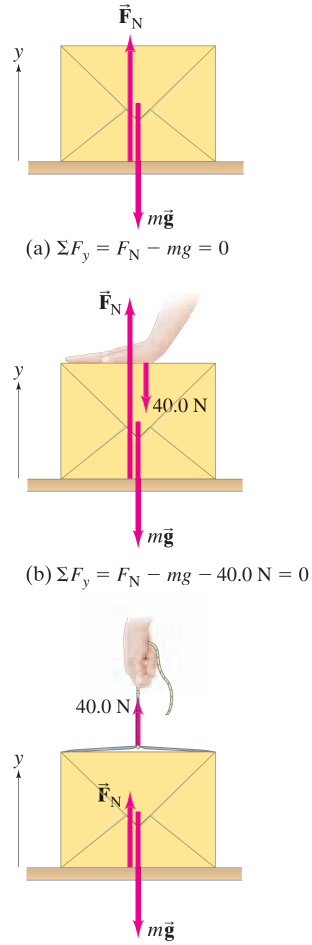
$$\Sigma F_y = F_\mathrm{N} - mg + 40.0\,\mathrm{N} = 0,$$

so

$$F_\mathrm{N} = mg - 40.0\,\mathrm{N} = 98.0\,\mathrm{N} - 40.0\,\mathrm{N} = 58.0\,\mathrm{N}.$$

The table does not push against the full weight of the box because of the upward pull exerted by your friend.

NOTE The weight of the box ($= mg$) does not change as a result of your friend's push or pull. Only the normal force is affected.



(a) $\Sigma F_y = F_\mathrm{N} - mg = 0$

(b) $\Sigma F_y = F_\mathrm{N} - mg - 40.0\,\mathrm{N} = 0$

(c) $\Sigma F_y = F_\mathrm{N} - mg + 40.0\,\mathrm{N} = 0$

FIGURE 15 Example 6. (a) A 10-kg gift box is at rest on a table. (b) A person pushes down on the box with a force of 40.0 N. (c) A person pulls upward on the box with a force of 40.0 N. The forces are all assumed to act along a line; they are shown slightly displaced in order to be distinguishable. Only forces acting on the box are shown.

CAUTION
The normal force is not always equal to the weight

Recall that the normal force is elastic in origin (the table in Fig. 15 sags slightly under the weight of the box). The normal force in Example 6 is vertical, perpendicular to the horizontal table. The normal force is not always vertical, however. When you push against a wall, for example, the normal force with which the wall pushes back on you is horizontal (Fig. 9). For an object on a plane inclined at an angle to the horizontal, such as a skier or car on a hill, the normal force acts perpendicular to the plane and so is not vertical.

CAUTION
The normal force, $\vec{\mathbf{F}}_\mathrm{N}$, is not necessarily vertical

EXAMPLE 7 **Accelerating the box.** What happens when a person pulls upward on the box in Example 6c with a force equal to, or greater than, the box's weight? For example, let $F_P = 100.0\,\text{N}$ (Fig. 16) rather than the 40.0 N shown in Fig. 15c.

APPROACH We can start just as in Example 6, but be ready for a surprise.

SOLUTION The net force on the box is

$$\begin{aligned} \Sigma F_y &= F_N - mg + F_P \\ &= F_N - 98.0\,\text{N} + 100.0\,\text{N}, \end{aligned}$$

and if we set this equal to zero (thinking the acceleration might be zero), we would get $F_N = -2.0\,\text{N}$. This is nonsense, since the negative sign implies F_N points downward, and the table surely cannot *pull* down on the box (unless there's glue on the table). The least F_N can be is zero, which it will be in this case. What really happens here is that the box accelerates upward because the net force is not zero. The net force (setting the normal force $F_N = 0$) is

$$\begin{aligned} \Sigma F_y &= F_P - mg = 100.0\,\text{N} - 98.0\,\text{N} \\ &= 2.0\,\text{N} \end{aligned}$$

upward. See Fig. 16. We apply Newton's second law and see that the box moves upward with an acceleration

$$\begin{aligned} a_y &= \frac{\Sigma F_y}{m} = \frac{2.0\,\text{N}}{10.0\,\text{kg}} \\ &= 0.20\,\text{m/s}^2. \end{aligned}$$

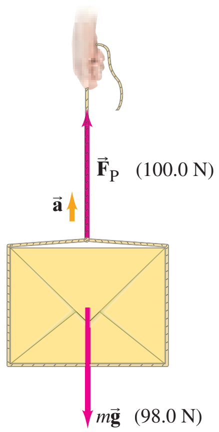
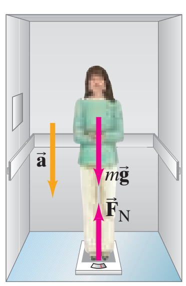


FIGURE 16 Example 7. The box accelerates upward because $F_P > mg$.

EXAMPLE 8 **Apparent weight loss.** A 65-kg woman descends in an elevator that briefly accelerates at $0.20g$ downward. She stands on a scale that reads in kg. (*a*) During this acceleration, what is her weight and what does the scale read? (*b*) What does the scale read when the elevator descends at a constant speed of $2.0\,\text{m/s}$?

FIGURE 17 Example 8. The acceleration vector is shown in gold to distinguish it from the red force vectors.



APPROACH Figure 17 shows all the forces that act on the woman (and *only* those that act on her). The direction of the acceleration is downward, so we choose the positive direction as down (this is the opposite choice from Examples 6 and 7).

SOLUTION (*a*) From Newton's second law,

$$\begin{aligned} \Sigma F &= ma \\ mg - F_N &= m(0.20g). \end{aligned}$$

We solve for F_N:

$$F_N = mg - 0.20mg = 0.80mg,$$

and it acts upward. The normal force $\vec{\mathbf{F}}_N$ is the force the scale exerts on the person, and is equal and opposite to the force she exerts on the scale: $F'_N = 0.80mg$ downward. Her weight (force of gravity on her) is still $mg = (65\,\text{kg})(9.8\,\text{m/s}^2) = 640\,\text{N}$. But the scale, needing to exert a force of only $0.80mg$, will give a reading of $0.80m = 52\,\text{kg}$.

(*b*) Now there is no acceleration, $a = 0$, so by Newton's second law, $mg - F_N = 0$ and $F_N = mg$. The scale reads her true mass of 65 kg.

NOTE The scale in (*a*) may give a reading of 52 kg (as an "apparent mass"), but her mass doesn't change as a result of the acceleration: it stays at 65 kg.

7 Solving Problems with Newton's Laws: Free-Body Diagrams

Newton's second law tells us that the acceleration of an object is proportional to the *net force* acting on the object. The **net force**, as mentioned earlier, is the *vector sum* of all forces acting on the object. Indeed, extensive experiments have shown that forces do add together as vectors. For example, in Fig. 18, two forces of equal magnitude (100 N each) are shown acting on an object at right angles to each other. Intuitively, we can see that the object will start moving at a 45° angle and thus the net force acts at a 45° angle. This is just what the rules of vector addition give. From the theorem of Pythagoras, the magnitude of the resultant force is $F_R = \sqrt{(100\text{ N})^2 + (100\text{ N})^2} = 141\text{ N}$.

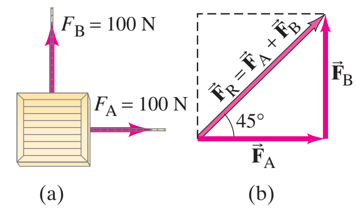


FIGURE 18 (a) Two forces, $\vec{F}_A$ and $\vec{F}_B$, exerted by workers A and B, act on a crate. (b) The sum, or resultant, of $\vec{F}_A$ and $\vec{F}_B$ is $\vec{F}_R$.

EXAMPLE 9 **Adding force vectors.** Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 19a.

APPROACH We add force vectors like any other vectors. The first step is to choose an xy coordinate system (see Fig. 19a), and then resolve vectors into their components.

SOLUTION The two force vectors are shown resolved into components in Fig. 19b. We add the forces using the method of components. The components of $\vec{F}_A$ are

$$F_{Ax} = F_A \cos 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N},$$
$$F_{Ay} = F_A \sin 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N}.$$

The components of $\vec{F}_B$ are

$$F_{Bx} = +F_B \cos 37.0° = +(30.0\text{ N})(0.799) = +24.0\text{ N},$$
$$F_{By} = -F_B \sin 37.0° = -(30.0\text{ N})(0.602) = -18.1\text{ N}.$$

F_{By} is negative because it points along the negative y axis. The components of the resultant force are (see Fig. 19c)

$$F_{Rx} = F_{Ax} + F_{Bx} = 28.3\text{ N} + 24.0\text{ N} = 52.3\text{ N},$$
$$F_{Ry} = F_{Ay} + F_{By} = 28.3\text{ N} - 18.1\text{ N} = 10.2\text{ N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem

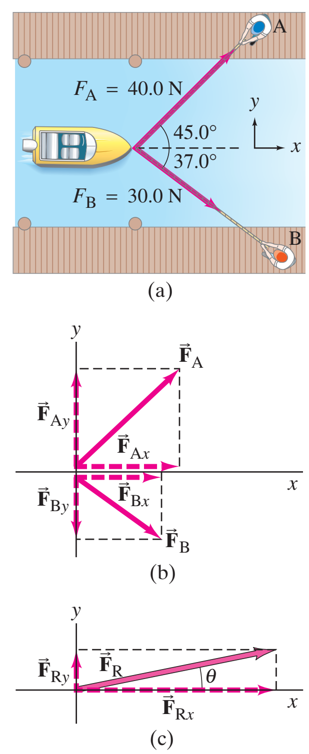
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2}\text{ N} = 53.3\text{ N}.$$

The only remaining question is the angle θ that the net force $\vec{F}_R$ makes with the x axis. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\text{ N}}{52.3\text{ N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0°$. The net force on the boat has magnitude 53.3 N and acts at an 11.0° angle to the x axis.

FIGURE 19 Example 9: Two force vectors act on a boat.



When solving problems involving Newton's laws and force, it is very important to draw a diagram showing all the forces acting *on* each object involved. Such a diagram is called a **free-body diagram**, or **force diagram**: choose one object, and draw an arrow to represent each force acting on it. Include *every* force acting on that object. Do not show forces that the chosen object exerts on *other* objects. To help you identify each and every force that is exerted on your chosen object, ask yourself what other objects could exert a force on it. If your problem involves more than one object, a separate free-body diagram is needed for each object. For now, the likely forces that could be acting are *gravity* and *contact forces* (one object pushing or pulling another, normal force, friction). Later we will consider air resistance, drag, buoyancy, pressure, as well as electric and magnetic forces.

PROBLEM SOLVING
Free-body diagram

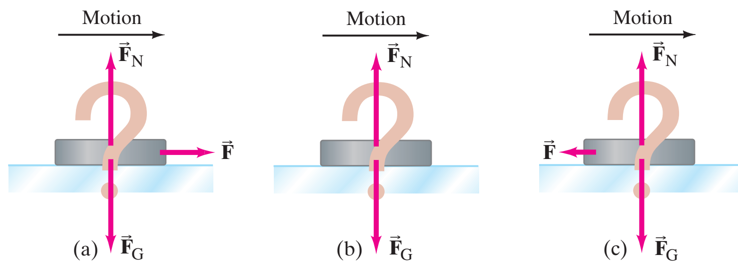


FIGURE 20 Example 10. Which is the correct free-body diagram for a hockey puck sliding across frictionless ice?

CONCEPTUAL EXAMPLE 10 **The hockey puck.** A hockey puck is sliding at constant velocity across a flat horizontal ice surface that is assumed to be frictionless. Which of the sketches in Fig. 20 is the correct free-body diagram for this puck? What would your answer be if the puck slowed down?

RESPONSE Did you choose (*a*)? If so, can you answer the question: what exerts the horizontal force labeled $\vec{\mathbf{F}}$ on the puck? If you say that it is the force needed to maintain the motion, ask yourself: what exerts this force? Remember that another object must exert any force—and there simply isn't any possibility here. Therefore, (*a*) is wrong. Besides, the force $\vec{\mathbf{F}}$ in Fig. 20a would give rise to an acceleration by Newton's second law. It is (*b*) that is correct. No net force acts on the puck, and the puck slides at constant velocity across the ice.

In the real world, where even smooth ice exerts at least a tiny friction force, then (*c*) is the correct answer. The tiny friction force is in the direction opposite to the motion, and the puck's velocity decreases, even if very slowly.

Here now is a brief summary of how to approach solving problems involving Newton's laws.

PROBLEM SOLVING

Newton's Laws; Free-Body Diagrams

1. **Draw a sketch** of the situation.
2. Consider only one object (at a time), and draw a **free-body diagram** for that object, showing *all* the forces acting *on* that object. Include any unknown forces that you have to solve for. Do not show any forces that the chosen object exerts on other objects.

   Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force acting on the object, including forces you must solve for, as to its source (gravity, person, friction, and so on).

   If several objects are involved, draw a free-body diagram for each object *separately*, showing all the forces acting *on that object* (and *only* forces acting on that object). For each (and every) force, you must be clear about: *on* what object that force acts, and *by* what object that force is exerted. Only forces acting *on* a given object can be included in $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for that object.
3. Newton's second law involves vectors, and it is us important to **resolve vectors** into components. **Choose** x and y **axes** in a way that simplifies the calculation. For example, it often saves work if you choose one coordinate axis to be in the direction of the acceleration.
4. For each object, **apply Newton's second law** to the x and y components separately. That is, the x component of the net force on that object is related to the x component of that object's acceleration: $\Sigma F_x = ma_x$, and similarly for the y direction.
5. **Solve** the equation or equations for the unknown(s).

This Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do that will start you thinking and getting involved in the problem at hand.

CAUTION
Treating an object as a particle

When we are concerned only about translational motion, all the forces on a given object can be drawn as acting at the center of the object, thus treating the object as a *point particle*. However, for problems involving rotation or statics, the place *where* each force acts is also important.

In the Examples that follow, we assume that all surfaces are very smooth so that friction can be ignored.

EXAMPLE 11 **Pulling the mystery box.** Suppose a friend asks to examine the 10.0-kg box you were given (Example 6, Fig. 15), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached cord, as shown in Fig. 21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0\,\mathrm{N}$, and it is exerted at a 30.0° angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.

APPROACH We follow the Problem Solving Strategy on the previous page.

SOLUTION

1. **Draw a sketch**: The situation is shown in Fig. 21a; it shows the box and the force applied by the person, F_P.
2. **Free-body diagram**: Figure 21b shows the free-body diagram of the box. To draw it correctly, we show *all* the forces acting on the box and *only* the forces acting on the box. They are: the force of gravity $m\vec{\mathbf{g}}$; the normal force exerted by the table $\vec{\mathbf{F}}_N$; and the force exerted by the person $\vec{\mathbf{F}}_P$. We are interested only in translational motion, so we can show the three forces acting at a point, Fig. 21c.
3. **Choose axes and resolve vectors**: We expect the motion to be horizontal, so we choose the x axis horizontal and the y axis vertical. The pull of 40.0 N has components

$$F_{Px} = (40.0\,\mathrm{N})(\cos 30.0^\circ) = (40.0\,\mathrm{N})(0.866) = 34.6\,\mathrm{N},$$
$$F_{Py} = (40.0\,\mathrm{N})(\sin 30.0^\circ) = (40.0\,\mathrm{N})(0.500) = 20.0\,\mathrm{N}.$$

In the horizontal (x) direction, $\vec{\mathbf{F}}_N$ and $m\vec{\mathbf{g}}$ have zero components. Thus the horizontal component of the net force is F_{Px}.

4. (*a*) **Apply Newton's second law** to determine the x component of the acceleration:

$$F_{Px} = ma_x.$$

5. (*a*) **Solve**:

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\,\mathrm{N})}{(10.0\,\mathrm{kg})} = 3.46\,\mathrm{m/s^2}.$$

The acceleration of the box is $3.46\,\mathrm{m/s^2}$ to the right.

(*b*) Next we want to find F_N.

4′. (*b*) **Apply Newton's second law** to the vertical (y) direction, with upward as positive:

$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

5′. (*b*) **Solve**: We have $mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}$ and, from point 3 above, $F_{Py} = 20.0\,\mathrm{N}$. Furthermore, since $F_{Py} < mg$, the box does not move vertically, so $a_y = 0$. Thus

$$F_N - 98.0\,\mathrm{N} + 20.0\,\mathrm{N} = 0,$$

so

$$F_N = 78.0\,\mathrm{N}.$$

NOTE F_N is less than mg: the table does not push against the full weight of the box because part of the pull exerted by the person is in the upward direction.

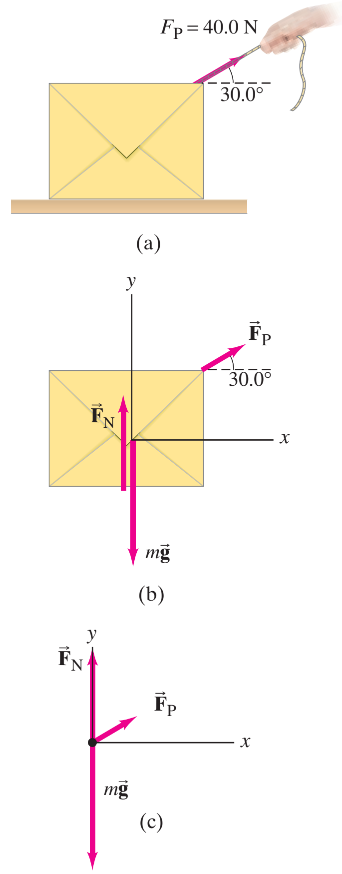


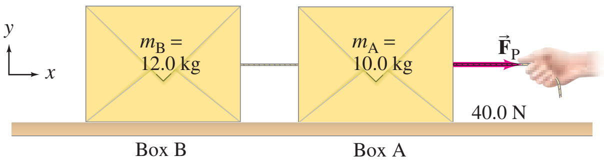
FIGURE 21 (a) Pulling the box, Example 11; (b) is the free-body diagram for the box, and (c) is the free-body diagram considering all the forces to act at a point (translational motion only, which is what we have here).

EXERCISE F A 10.0-kg box is dragged on a horizontal frictionless surface by a horizontal force of 10.0 N. If the applied force is doubled, the normal force on the box will (a) increase; (b) remain the same; (c) decrease.

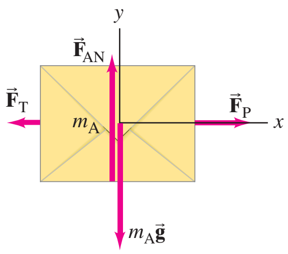
Tension in a Flexible Cord

When a flexible cord pulls on an object, the cord is said to be under **tension**, and the force it exerts on the object is the tension F_T. If the cord has negligible mass, the force exerted at one end is transmitted undiminished to each adjacent piece of cord along the entire length to the other end. Why? Because $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = 0$ for the cord if the cord's mass m is zero (or negligible) no matter what $\vec{\mathbf{a}}$ is. Hence the forces pulling on the cord at its two ends must add up to zero (F_T and $-F_T$). Note that flexible cords and strings can only pull. They can't push because they bend.

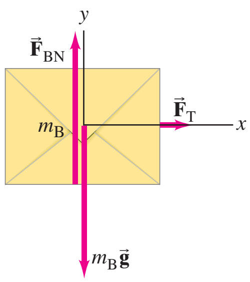
PROBLEM SOLVING
Cords can pull but can't push; tension exists throughout a cord



(a)



(b)



(c)

FIGURE 22 Example 12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0$ N. (b) Free-body diagram for box A. (c) Free-body diagram for box B.

Our next Example involves two boxes connected by a cord. We can refer to this group of objects as a system. A *system* is any group of one or more objects we choose to consider and study.

EXAMPLE 12 **Two boxes connected by a cord.** Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied to the 10.0-kg box, as shown in Fig. 22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord connecting the boxes.

APPROACH We streamline our approach by not listing each step. We have two boxes so we draw a free-body diagram for each. To draw them correctly, we must consider the forces on *each* box by itself, so that Newton's second law can be applied to each. The person exerts a force F_P on box A. Box A exerts a force F_T on the connecting cord, and the cord exerts an opposite but equal magnitude force F_T back on box A (Newton's third law). These two horizontal forces on box A are shown in Fig. 22b, along with the force of gravity $m_A\vec{g}$ downward and the normal force $\vec{F}_{AN}$ exerted upward by the table. The cord is light, so we neglect its mass. The tension at each end of the cord is thus the same. Hence the cord exerts a force F_T on the second box. Figure 22c shows the forces on box B, which are $\vec{F}_T$, $m_B\vec{g}$, and the normal force $\vec{F}_{BN}$. There will be only horizontal motion. We take the positive x axis to the right.

SOLUTION (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_A = a_B = a$. We are given $m_A = 10.0$ kg and $m_B = 12.0$ kg. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\text{ N}}{22.0\text{ kg}} = 1.82\text{ m/s}^2.$$

This is what we sought.

Alternate Solution We would have obtained the same result had we considered a single system, of mass $m_A + m_B$, acted on by a net horizontal force equal to F_P. (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

(*b*) From the equation above for box B $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\text{ kg})(1.82\text{ m/s}^2) = 21.8\text{ N}.$$

Thus, F_T is less than F_P ($= 40.0$ N), as we expect, since F_T acts to accelerate only m_B.

NOTE It might be tempting to say that the force the person exerts, F_P, acts not only on box A but also on box B. It doesn't. F_P acts only on box A. It affects box B via the tension in the cord, F_T, which acts on box B and accelerates it.

CAUTION

For any object, use only the forces on that object in calculating $\Sigma F = ma$

EXAMPLE 13 **Elevator and counterweight (Atwood's machine).** A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 23a, is sometimes referred to as an *Atwood's machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are made similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\,\text{kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\,\text{kg}$. For the latter case $(m_E = 1150\,\text{kg})$, calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

PHYSICS APPLIED
Elevator (as Atwood's machine)

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{\mathbf{F}}_T$. Figures 23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable doesn't stretch). For the counterweight, $m_C g = (1000\,\text{kg})(9.80\,\text{m/s}^2) = 9800\,\text{N}$, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150\,\text{kg})(9.80\,\text{m/s}^2) = 11{,}300\,\text{N}$, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}\,g = \frac{1150\,\text{kg} - 1000\,\text{kg}}{1150\,\text{kg} + 1000\,\text{kg}}\,g = 0.070g = 0.68\,\text{m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\,\text{m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations, setting $a = 0.070g = 0.68\,\text{m/s}^2$:

$$\begin{aligned} F_T &= m_E g - m_E a = m_E(g - a) \\ &= 1150\,\text{kg}\,(9.80\,\text{m/s}^2 - 0.68\,\text{m/s}^2) = 10{,}500\,\text{N}, \end{aligned}$$

or

$$\begin{aligned} F_T &= m_C g + m_C a = m_C(g + a) \\ &= 1000\,\text{kg}\,(9.80\,\text{m/s}^2 + 0.68\,\text{m/s}^2) = 10{,}500\,\text{N}, \end{aligned}$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal $(m_E = m_C)$, then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass $(m_E \neq 0)$ would be predicted by our equation to accelerate at $a = g$, again as expected.

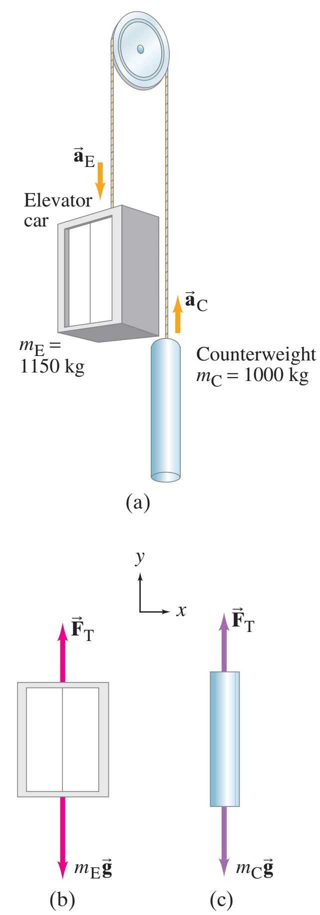


FIGURE 23 Example 13. (a) Atwood's machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

PROBLEM SOLVING
Check your result by seeing if it works in situations where the answer is easily guessed

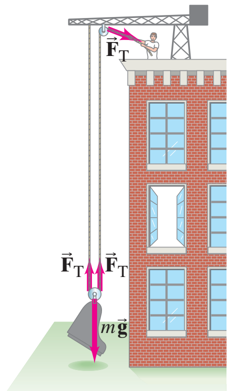


FIGURE 24 Example 14.

CONCEPTUAL EXAMPLE 14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 2000-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore its mass. First notice the forces acting on the lower pulley at the piano. The weight of the piano pulls down on the pulley via a short cable. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m), choosing the upward direction as positive:

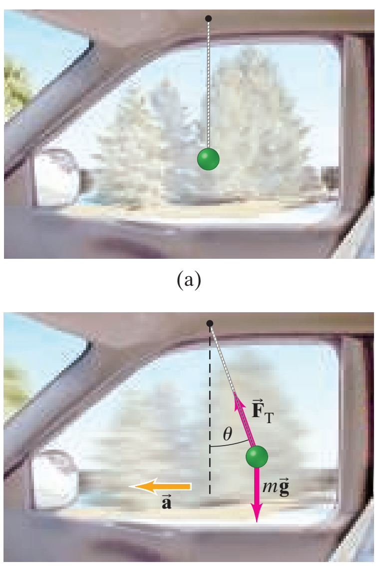
$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The mover can exert a force equal to half the piano's weight. We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

PHYSICS APPLIED
Accelerometer

EXAMPLE 15 **Accelerometer.** A small mass m hangs from a thin string and can swing like a pendulum. You attach it above the window of your car as shown in Fig. 25a. When the car is at rest, the string hangs vertically. What angle θ does the string make (*a*) when the car accelerates at a constant $a = 1.20\,\mathrm{m/s^2}$, and (*b*) when the car moves at constant velocity, $v = 90\,\mathrm{km/h}$?

FIGURE 25 Example 15.



APPROACH The free-body diagram of Fig. 25b shows the pendulum at some angle θ and the forces on it: $m\vec{g}$ downward, and the tension $\vec{F}_T$ in the cord. These forces do not add up to zero if $\theta \neq 0$, and since we have an acceleration a, we therefore expect $\theta \neq 0$. Note that θ is the angle relative to the vertical.

SOLUTION (*a*) The acceleration $a = 1.20\,\mathrm{m/s^2}$ is horizontal, so from Newton's second law,

$$ma = F_T \sin\theta$$

for the horizontal component, whereas the vertical component gives

$$0 = F_T \cos\theta - mg.$$

Dividing these two equations, we obtain

$$\tan\theta = \frac{F_T \sin\theta}{F_T \cos\theta} = \frac{ma}{mg} = \frac{a}{g}$$

or

$$\tan\theta = \frac{1.20\,\mathrm{m/s^2}}{9.80\,\mathrm{m/s^2}} = 0.122,$$

so

$$\theta = 7.0°.$$

(*b*) The velocity is constant, so $a = 0$ and $\tan\theta = 0$. Hence the pendulum hangs vertically $(\theta = 0°)$.

NOTE This simple device is an **accelerometer**—it can be used to measure acceleration.

Inclines

Now we consider what happens when an object slides down an incline, such as a hill or ramp. Such problems are interesting because gravity is the accelerating force, yet the acceleration is not vertical. Solving such problems is usually easier if we choose the xy coordinate system so that one axis points in the direction of the acceleration. Thus we often take the x axis to point along the incline and the y axis perpendicular to the incline, as shown in Fig. 26a. Note also that the normal force is not vertical, but is perpendicular to the plane, Fig. 26b.

PROBLEM SOLVING

Good choice of coordinate system simplifies the calculation

EXAMPLE 16 **Box slides down an incline.** A box of mass m is placed on a smooth (frictionless) incline that makes an angle θ with the horizontal, as shown in Fig. 26a. (a) Determine the normal force on the box. (b) Determine the box's acceleration. (c) Evaluate for a mass $m = 10\,\mathrm{kg}$ and an incline of $\theta = 30°$.

APPROACH We expect the motion to be along the incline, so we choose the x axis along the slope, positive down the slope (the direction of motion). The y axis is perpendicular to the incline, upward. The free-body diagram is shown in Fig. 26b. The forces on the box are its weight mg vertically downward, which is shown resolved into its components parallel and perpendicular to the incline, and the normal force F_N. The incline acts as a constraint, allowing motion along its surface. The "constraining" force is the normal force.

SOLUTION (a) There is no motion in the y direction, so $a_y = 0$. Applying Newton's second law we have

$$\begin{aligned} F_y &= ma_y \\ F_N - mg\cos\theta &= 0, \end{aligned}$$

where F_N and the y component of gravity ($mg\cos\theta$) are all the forces acting on the box in the y direction. Thus the normal force is given by

$$F_N = mg\cos\theta.$$

Note carefully that unless $\theta = 0°$, F_N has magnitude less than the weight mg.

(b) In the x direction the only force acting is the x component of $m\vec{\mathbf{g}}$, which we see from the diagram is $mg\sin\theta$. The acceleration a is in the x direction so

$$\begin{aligned} F_x &= ma_x \\ mg\sin\theta &= ma, \end{aligned}$$

and we see that the acceleration down the plane is

$$a = g\sin\theta.$$

Thus the acceleration along an incline is always less than g, except at $\theta = 90°$, for which $\sin\theta = 1$ and $a = g$. This makes sense since $\theta = 90°$ is pure vertical fall. For $\theta = 0°$, $a = 0$, which makes sense because $\theta = 0°$ means the plane is horizontal so gravity causes no acceleration. Note too that the acceleration does not depend on the mass m.

(c) For $\theta = 30°$, $\cos\theta = 0.866$ and $\sin\theta = 0.500$, so

$$F_N = 0.866mg = 85\,\mathrm{N},$$

and

$$a = 0.500g = 4.9\,\mathrm{m/s^2}.$$

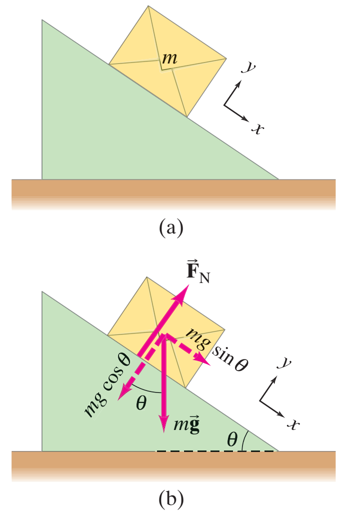


FIGURE 26 Example 16.
(a) Box sliding on inclined plane.
(b) Free-body diagram of box.

8 Problem Solving—A General Approach

A basic part of a physics course is solving problems effectively. The approach discussed here, though emphasizing Newton's laws, can be applied generally for other topics in physics study.

PROBLEM SOLVING

In General

1. **Read** and reread written problems carefully. A common error is to skip a word or two when reading, which can completely change the meaning of a problem.
2. **Draw** an accurate picture or diagram of the situation. (This is probably the most overlooked, yet most crucial, part of solving a problem.) Use arrows to represent vectors such as velocity or force, and label the vectors with appropriate symbols. When dealing with forces and applying Newton's laws, make sure to include all forces on a given object, including unknown ones, and make clear what forces act on what object (otherwise you may make an error in determining the *net force* on a particular object).
3. A separate **free-body diagram** needs to be drawn for each object involved, and it must show *all* the forces acting on a given object (and only on that object). Do not show forces that act on other objects.
4. Choose a convenient xy **coordinate system** (one that makes your calculations easier, such as one axis in the direction of the acceleration). Vectors are to be resolved into components along the coordinate axes. When using Newton's second law, apply $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ separately to x and y components, remembering that x direction forces are related to a_x, and similarly for y. If more than one object is involved, you can choose different (convenient) coordinate systems for each.
5. List the knowns and the unknowns (what you are trying to determine), and decide what you need in order to find the unknowns. For problems in the present Chapter, we use Newton's laws. More generally, it may help to see if one or more **relationships** (or **equations**) relate the unknowns to the knowns. But be sure each relationship is applicable in the given case. It is very important to know the limitations of each formula or relationship—when it is valid and when not. In this text, the more general equations have been given numbers, but even these can have a limited range of validity (often stated in brackets to the right of the equation).
6. Try to solve the problem approximately, to see if it is doable (to check if enough information has been given) and reasonable. Use your intuition, and make **rough calculations**. A rough calculation, or a reasonable guess about what the range of final answers might be, is very useful. And a rough calculation can be checked against the final answer to catch errors in calculation, such as in a decimal point or the powers of 10.
7. **Solve** the problem, which may include algebraic manipulation of equations and/or numerical calculations. Recall the mathematical rule that you need as many independent equations as you have unknowns; if you have three unknowns, for example, then you need three independent equations. It is usually best to work out the algebra symbolically before putting in the numbers. Why? Because (*a*) you can then solve a whole class of similar problems with different numerical values; (*b*) you can check your result for cases already understood (say, $\theta = 0°$ or $90°$); (*c*) there may be cancellations or other simplifications; (*d*) there is usually less chance for numerical error; and (*e*) you may gain better insight into the problem.
8. Be sure to keep track of **units**, for they can serve as a check (they must balance on both sides of any equation).
9. Again consider if your answer is **reasonable**. The use of dimensional analysis can also serve as a check for many problems.

Summary

Newton's three laws of motion are the basic classical laws describing motion.

Newton's first law (the **law of inertia**) states that if the net force on an object is zero, an object originally at rest remains at rest, and an object in motion remains in motion in a straight line with constant velocity.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad \textbf{(1a)}$$

Newton's second law is one of the most important and fundamental laws in classical physics.

Newton's third law states that whenever one object exerts a force on a second object, the second object always exerts a force on the first object which is equal in magnitude but opposite in direction:

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}, \tag{2}$$

where $\vec{\mathbf{F}}_{BA}$ is the force on object B exerted by object A. This is true even if objects are moving and accelerating, and/or have different masses.

The tendency of an object to resist a change in its motion is called **inertia**. **Mass** is a measure of the inertia of an object.

Weight refers to the **gravitational force** on an object, and is equal to the product of the object's mass m and the acceleration of gravity $\vec{\mathbf{g}}$:

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \tag{3}$$

Force, which is a vector, can be considered as a push or pull; or, from Newton's second law, force can be defined as an action capable of giving rise to acceleration. The **net force** on an object is the vector sum of all forces acting on that object.

For solving problems involving the forces on one or more objects, it is essential to draw a **free-body diagram** for each object, showing all the forces acting on only that object. Newton's second law can be applied to the vector components for each object.

Answers to Exercises

A: No force is needed. The car accelerates out from under the cup. Think of Newton's first law (see Example 1).

B: (*a*).

C: (*a*) The same; (*b*) the sports car; (*c*) third law for part (*a*), second law for part (*b*).

D: (*e*).

E: (*b*).

F: (*b*).

Dynamics: Newton's Laws of Motion Problem Set

Questions

1. Why does a child in a wagon seem to fall backward when you give the wagon a sharp pull forward?

2. A box rests on the (frictionless) bed of a truck. The truck driver starts the truck and accelerates forward. The box immediately starts to slide toward the rear of the truck bed. Discuss the motion of the box, in terms of Newton's laws, as seen (*a*) by Andrea standing on the ground beside the truck, and (*b*) by Jim who is riding on the truck (Fig. 27).

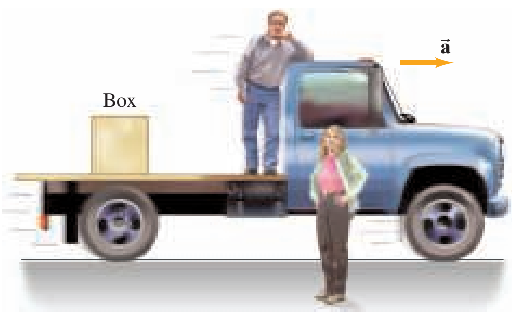


FIGURE 27 Question 2.

3. If the acceleration of an object is zero, are no forces acting on it? Explain.

4. If an object is moving, is it possible for the net force acting on it to be zero?

5. Only one force acts on an object. Can the object have zero acceleration? Can it have zero velocity? Explain.

6. When a golf ball is dropped to the pavement, it bounces back up. (*a*) Is a force needed to make it bounce back up? (*b*) If so, what exerts the force?

7. If you walk along a log floating on a lake, why does the log move in the opposite direction?

8. Why might your foot hurt if you kick a heavy desk or a wall?

9. When you are running and want to stop quickly, you must decelerate quickly. (*a*) What is the origin of the force that causes you to stop? (*b*) Estimate (using your own experience) the maximum rate of deceleration of a person running at top speed to come to rest.

10. (*a*) Why do you push down harder on the pedals of a bicycle when first starting out than when moving at constant speed? (*b*) Why do you need to pedal at all when cycling at constant speed?

11. A father and his young daughter are ice skating. They face each other at rest and push each other, moving in opposite directions. Which one has the greater final speed?

12. Suppose that you are standing on a cardboard carton that just barely supports you. What would happen to it if you jumped up into the air? It would (*a*) collapse; (*b*) be unaffected; (*c*) spring upward a bit; (*d*) move sideways.

13. A stone hangs by a fine thread from the ceiling, and a section of the same thread dangles from the bottom of the stone (Fig. 28). If a person gives a sharp pull on the dangling thread, where is the thread likely to break: below the stone or above it? What if the person gives a slow and steady pull? Explain your answers.

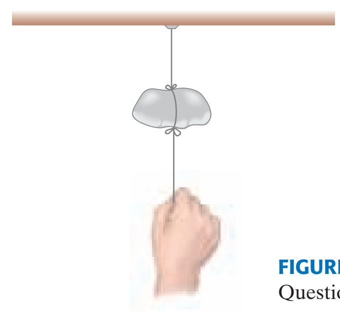


FIGURE 28 Question 13.

14. The force of gravity on a 2-kg rock is twice as great as that on a 1-kg rock. Why then doesn't the heavier rock fall faster?

15. Would a spring scale carried to the Moon give accurate results if the scale had been calibrated on Earth, (*a*) in pounds, or (*b*) in kilograms?

16. You pull a box with a constant force across a frictionless table using an attached rope held horizontally. If you now pull the rope with the same force at an angle to the horizontal (with the box remaining flat on the table), does the acceleration of the box (*a*) remain the same, (*b*) increase, or (*c*) decrease? Explain.

17. When an object falls freely under the influence of gravity there is a net force mg exerted on it by the Earth. Yet by Newton's third law the object exerts an equal and opposite force on the Earth. Does the Earth move?

18. Compare the effort (or force) needed to lift a 10-kg object when you are on the Moon with the force needed to lift it on Earth. Compare the force needed to throw a 2-kg object horizontally with a given speed on the Moon and on Earth.

19. Which of the following objects weighs about 1 N: (*a*) an apple, (*b*) a mosquito, (*c*) a 1200-page textbook, (*d*) you?

20. According to Newton's third law, each team in a tug of war (Fig. 29) pulls with equal force on the other team. What, then, determines which team will win?

FIGURE 29 Question 20. A tug of war. Describe the forces on each of the teams and on the rope.

21. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?

22. Whiplash sometimes results from an automobile accident when the victim's car is struck violently from the rear. Explain why the head of the victim seems to be thrown backward in this situation. Is it really?

23. Mary exerts an upward force of 40 N to hold a bag of groceries. Describe the "reaction" force (Newton's third law) by stating (*a*) its magnitude, (*b*) its direction, (*c*) *on* what object it is exerted, and (*d*) *by* what object it is exerted.

24. A bear sling, Fig. 30, is used in some national parks for placing backpackers' food out of the reach of bears. Explain why the force needed to pull the backpack up increases as the backpack gets higher and higher. Is it possible to pull the rope hard enough so that it doesn't sag at all?

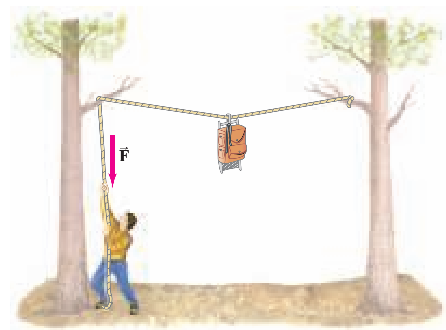


FIGURE 30 Question 24.

Problems

[The Problems in this Section are ranked I, II, or III according to estimated difficulty, with (I) Problems being easiest. Level (III) Problems are meant mainly as a challenge for the best students, for "extra credit." The Problems are arranged by Sections, meaning that the reader should have read up to and including that Section, but this Chapter also has a group of General Problems that are not arranged by Section and not ranked.]

4 to 6 Newton's Laws, Gravitational Force, Normal Force

1. (I) What force is needed to accelerate a child on a sled (total mass = 55 kg) at $1.4\ \mathrm{m/s^2}$?

2. (I) A net force of 265 N accelerates a bike and rider at $2.30\ \mathrm{m/s^2}$. What is the mass of the bike and rider together?

3. (I) What is the weight of a 68-kg astronaut (*a*) on Earth, (*b*) on the Moon $(g = 1.7\ \mathrm{m/s^2})$, (*c*) on Mars $(g = 3.7\ \mathrm{m/s^2})$, (*d*) in outer space traveling with constant velocity?

4. (I) How much tension must a rope withstand if it is used to accelerate a 1210-kg car horizontally along a frictionless surface at $1.20\ \mathrm{m/s^2}$?

5. (II) Superman must stop a 120-km/h train in 150 m to keep it from hitting a stalled car on the tracks. If the train's mass is 3.6×10^5 kg, how much force must he exert? Compare to the weight of the train (give as %). How much force does the train exert on Superman?

6. (II) What average force is required to stop a 950-kg car in 8.0 s if the car is traveling at 95 km/h?

7. (II) Estimate the average force exerted by a shot-putter on a 7.0-kg shot if the shot is moved through a distance of 2.8 m and is released with a speed of 13 m/s.

8. (II) A 0.140-kg baseball traveling 35.0 m/s strikes the catcher's mitt, which, in bringing the ball to rest, recoils backward 11.0 cm. What was the average force applied by the ball on the glove?

9. (II) A fisherman yanks a fish vertically out of the water with an acceleration of $2.5\ \mathrm{m/s^2}$ using very light fishing line that has a breaking strength of 18 N ($\approx$ 4 lb). The fisherman unfortunately loses the fish as the line snaps. What can you say about the mass of the fish?

10. (II) A 20.0-kg box rests on a table. (*a*) What is the weight of the box and the normal force acting on it? (*b*) A 10.0-kg box is placed on top of the 20.0-kg box, as shown in Fig. 31. Determine the normal force that the table exerts on the 20.0-kg box and the normal force that the 20.0-kg box exerts on the 10.0-kg box.

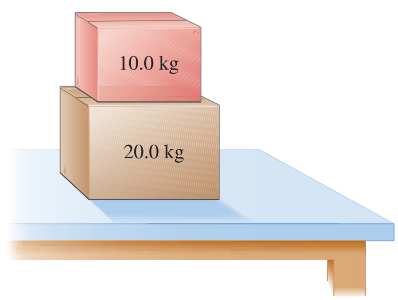


FIGURE 31 Problem 10.

11. (II) What average force is needed to accelerate a 9.20-gram pellet from rest to 125 m/s over a distance of 0.800 m along the barrel of a rifle?

12. (II) How much tension must a cable withstand if it is used to accelerate a 1200-kg car vertically upward at $0.70\ \mathrm{m/s^2}$?

13. (II) A 14.0-kg bucket is lowered vertically by a rope in which there is 163 N of tension at a given instant. What is the acceleration of the bucket? Is it up or down?

14. (II) A particular race car can cover a quarter-mile track (402 m) in 6.40 s starting from a standstill. Assuming the acceleration is constant, how many "g's" does the driver experience? If the combined mass of the driver and race car is 535 kg, what horizontal force must the road exert on the tires?

15. (II) A 75-kg petty thief wants to escape from a third-story jail window. Unfortunately, a makeshift rope made of sheets tied together can support a mass of only 58 kg. How might the thief use this "rope" to escape? Give a quantitative answer.

16. (II) An elevator (mass 4850 kg) is to be designed so that the maximum acceleration is 0.0680 g. What are the maximum and minimum forces the motor should exert on the supporting cable?

17. (II) Can cars "stop on a dime"? Calculate the acceleration of a 1400-kg car if it can stop from 35 km/h on a dime (diameter = 1.7 cm.) How many g's is this? What is the force felt by the 68-kg occupant of the car?

18. (II) A person stands on a bathroom scale in a motionless elevator. When the elevator begins to move, the scale briefly reads only 0.75 of the person's regular weight. Calculate the acceleration of the elevator, and find the direction of acceleration.

19. (II) High-speed elevators function under two limitations: (1) the maximum magnitude of vertical acceleration that a typical human body can experience without discomfort is about $1.2\ \mathrm{m/s^2}$, and (2) the typical maximum speed attainable is about 9.0 m/s. You board an elevator on a skyscraper's ground floor and are transported 180 m above the ground level in three steps: acceleration of magnitude $1.2\ \mathrm{m/s^2}$ from rest to 9.0 m/s, followed by constant upward velocity of 9.0 m/s, then deceleration of magnitude $1.2\ \mathrm{m/s^2}$ from 9.0 m/s to rest. (a) Determine the elapsed time for each of these 3 stages. (b) Determine the change in the magnitude of the normal force, expressed as a % of your normal weight during each stage. (c) What fraction of the total transport time does the normal force not equal the person's weight?

20. (II) Using focused laser light, *optical tweezers* can apply a force of about 10 pN to a 1.0-μm diameter polystyrene bead, which has a density about equal to that of water: a volume of $1.0\ \mathrm{cm^3}$ has a mass of about 1.0 g. Estimate the bead's acceleration in g's.

21. (II) A rocket with a mass of 2.75×10^6 kg exerts a vertical force of 3.55×10^7 N on the gases it expels. Determine (a) the acceleration of the rocket, (b) its velocity after 8.0 s, and (c) how long it takes to reach an altitude of 9500 m. Assume g remains constant, and ignore the mass of gas expelled (not realistic).

22. (II) (a) What is the acceleration of two falling sky divers (mass = 132 kg including parachute) when the upward force of air resistance is equal to one-fourth of their weight? (b) After popping open the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute? See Fig. 32.

Kathleen Schiaparelli

FIGURE 32 Problem 22.

23. (II) An exceptional standing jump would raise a person 0.80 m off the ground. To do this, what force must a 68-kg person exert against the ground? Assume the person crouches a distance of 0.20 m prior to jumping, and thus the upward force has this distance to act over before he leaves the ground.

24. (II) The cable supporting a 2125-kg elevator has a maximum strength of 21,750 N. What maximum upward acceleration can it give the elevator without breaking?

25. (III) The 100-m dash can be run by the best sprinters in 10.0 s. A 66-kg sprinter accelerates uniformly for the first 45 m to reach top speed, which he maintains for the remaining 55 m. (a) What is the average horizontal component of force exerted on his feet by the ground during acceleration? (b) What is the speed of the sprinter over the last 55 m of the race (i.e., his top speed)?

26. (III) A person jumps from the roof of a house 3.9-m high. When he strikes the ground below, he bends his knees so that his torso decelerates over an approximate distance of 0.70 m. If the mass of his torso (excluding legs) is 42 kg, find (a) his velocity just before his feet strike the ground, and (b) the average force exerted on his torso by his legs during deceleration.

7 Using Newton's Laws

27. (I) A box weighing 77.0 N rests on a table. A rope tied to the box runs vertically upward over a pulley and a weight is hung from the other end (Fig. 33). Determine the force that the table exerts on the box if the weight hanging on the other side of the pulley weighs (a) 30.0 N, (b) 60.0 N, and (c) 90.0 N.

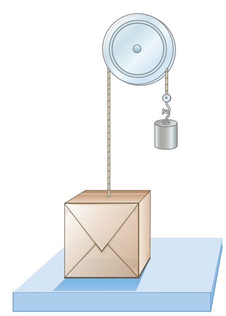


FIGURE 33 Problem 27.

28. (I) Draw the free-body diagram for a basketball player (a) just before leaving the ground on a jump, and (b) while in the air. See Fig. 34.

Brian Bahr/Allsport Concepts/Getty Images

FIGURE 34 Problem 28.

29. (I) Sketch the free-body diagram of a baseball (a) at the moment it is hit by the bat, and again (b) after it has left the bat and is flying toward the outfield.

30. (I) A 650-N force acts in a northwesterly direction. A second 650-N force must be exerted in what direction so that the resultant of the two forces points westward? Illustrate your answer with a vector diagram.

31. (II) Christian is making a Tyrolean traverse as shown in Fig. 35. That is, he traverses a chasm by stringing a rope between a tree on one side of the chasm and a tree on the opposite side, 25 m away. The rope must sag sufficiently so it won't break. Assume the rope can provide a tension force of up to 29 kN before breaking, and use a "safety factor" of 10 (that is, the rope should only be required to undergo a tension force of 2.9 kN) at the center of the Tyrolean traverse. (*a*) Determine the distance x that the rope must sag if it is to be within its recommended safety range and Christian's mass is 72.0 kg. (*b*) If the Tyrolean traverse is incorrectly set up so that the rope sags by only one-fourth the distance found in (*a*), determine the tension force in the rope. Will the rope break?

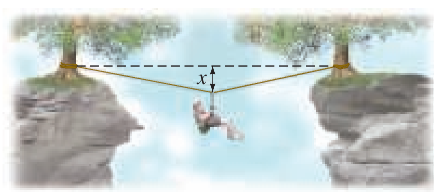


FIGURE 35 Problem 31.

32. (II) A window washer pulls herself upward using the bucket–pulley apparatus shown in Fig. 36. (*a*) How hard must she pull downward to raise herself slowly at constant speed? (*b*) If she increases this force by 15%, what will her acceleration be? The mass of the person plus the bucket is 72 kg.

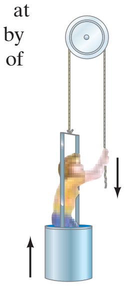


FIGURE 36 Problem 32.

33. (II) One 3.2-kg paint bucket is hanging by a massless cord from another 3.2-kg paint bucket, also hanging by a massless cord, as shown in Fig. 37. (*a*) If the buckets are at rest, what is the tension in each cord? (*b*) If the two buckets are pulled upward with an acceleration of $1.25\ \mathrm{m/s^2}$ by the upper cord, calculate the tension in each cord.

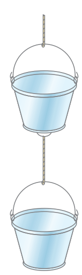


FIGURE 37 Problems 33 and 34.

34. (II) The cords accelerating the buckets in Problem 33b, Fig. 37, each has a weight of 2.0 N. Determine the tension in each cord at the three points of attachment.

35. (II) Two snowcats in Antarctica are towing a housing unit to a new location, as shown in Fig. 38. The sum of the forces $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ exerted on the unit by the horizontal cables is parallel to the line L, and $F_A = 4500\ \mathrm{N}$. Determine F_B and the magnitude of $\vec{\mathbf{F}}_A + \vec{\mathbf{F}}_B$.

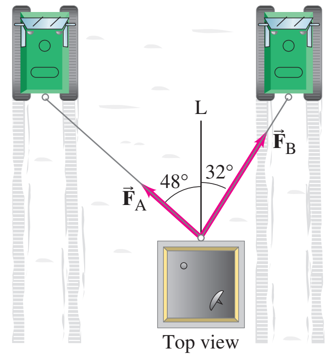


FIGURE 38 Problem 35.

36. (II) A train locomotive is pulling two cars of the same mass behind it, Fig. 39. Determine the ratio of the tension in the coupling (think of it as a cord) between the locomotive and the first car (F_{T1}), to that between the first car and the second car (F_{T2}), for any nonzero acceleration of the train.

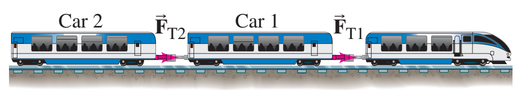


FIGURE 39 Problem 36.

37. (II) The two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ shown in Fig. 40a and b (looking down) act on a 18.5-kg object on a frictionless tabletop. If $F_1 = 10.2\ \mathrm{N}$ and $F_2 = 16.0\ \mathrm{N}$, find the net force on the object and its acceleration for (a) and (b).

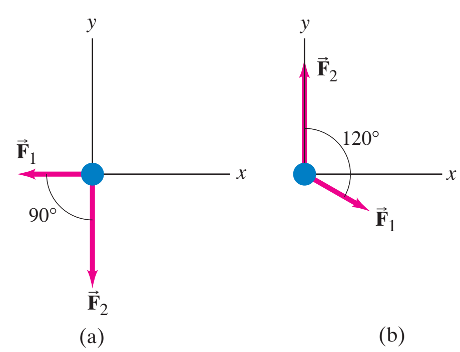


FIGURE 40 Problem 37.

38. (II) At the instant a race began, a 65-kg sprinter exerted a force of 720 N on the starting block at a 22° angle with respect to the ground. (*a*) What was the horizontal acceleration of the sprinter? (*b*) If the force was exerted for 0.32 s, with what speed did the sprinter leave the starting block?

39. (II) A mass m is at rest on a horizontal frictionless surface at $t = 0$. Then a constant force F_0 acts on it for a time t_0. Suddenly the force doubles to $2F_0$ and remains constant until $t = 2t_0$. Determine the total distance traveled from $t = 0$ to $t = 2t_0$.

40. (II) A 3.0-kg object has the following two forces acting on it:

$$\vec{\mathbf{F}}_1 = (16\hat{\mathbf{i}} + 12\hat{\mathbf{j}})\ \mathrm{N}$$
$$\vec{\mathbf{F}}_2 = (-10\hat{\mathbf{i}} + 22\hat{\mathbf{j}})\ \mathrm{N}$$

If the object is initially at rest, determine its velocity $\vec{\mathbf{v}}$ at $t = 3.0\ \mathrm{s}$.

41. (II) Uphill escape ramps are sometimes provided to the side of steep downhill highways for trucks with overheated brakes. For a simple 11° upward ramp, what length would be needed for a runaway truck traveling 140 km/h? Note the large size of your calculated length. (If sand is used for the bed of the ramp, its length can be reduced by a factor of about 2.)

42. (II) A child on a sled reaches the bottom of a hill with a velocity of 10.0 m/s and travels 25.0 m along a horizontal straightaway to a stop. If the child and sled together have a mass of 60.0 kg, what is the average retarding force on the sled on the horizontal straightaway?

43. (II) A skateboarder, with an initial speed of 2.0 m/s, rolls virtually friction free down a straight incline of length 18 m in 3.3 s. At what angle θ is the incline oriented above the horizontal?

44. (II) As shown in Fig. 41, five balls (masses 2.00, 2.05, 2.10, 2.15, 2.20 kg) hang from a crossbar. Each mass is supported by "5-lb test" fishing line which will break when its tension force exceeds 22.2 N (= 5 lb). When this device is placed in an elevator, which accelerates upward, only the lines attached to the 2.05 and 2.00 kg masses do not break. Within what range is the elevator's acceleration?

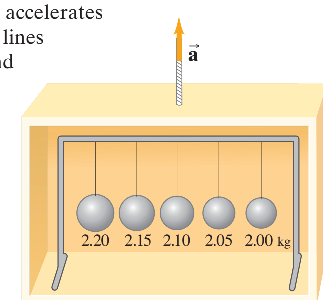


FIGURE 41 Problem 44.

45. (II) A 27-kg chandelier hangs from a ceiling on a vertical 4.0-m-long wire. (*a*) What horizontal force would be necessary to displace its position 0.15 m to one side? (*b*) What will be the tension in the wire?

46. (II) Three blocks on a frictionless horizontal surface are in contact with each other as shown in Fig. 42. A force $\vec{\mathbf{F}}$ is applied to block A (mass m_A). (*a*) Draw a free-body diagram for each block. Determine (*b*) the acceleration of the system (in terms of m_A, m_B, and m_C), (*c*) the net force on each block, and (*d*) the force of contact that each block exerts on its neighbor. (*e*) If $m_A = m_B = m_C = 10.0\ \mathrm{kg}$ and $F = 96.0\ \mathrm{N}$, give numerical answers to (*b*), (*c*), and (*d*). Explain how your answers make sense intuitively.

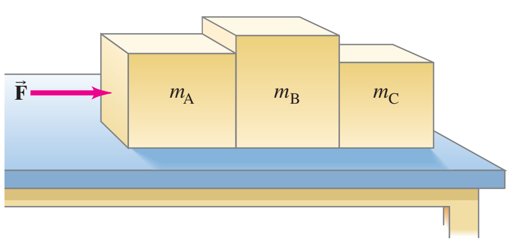


FIGURE 42 Problem 46.

47. (II) Redo Example 13 of "Dynamics: Newton's Laws of Motion" but (*a*) set up the equations so that the direction of the acceleration $\vec{\mathbf{a}}$ of each object is in the direction of motion of that object. (In Example 13, we took $\vec{\mathbf{a}}$ as positive upward for both masses.) (*b*) Solve the equations to obtain the same answers as in Example 13.

48. (II) The block shown in Fig. 43 has mass $m = 7.0\ \mathrm{kg}$ and lies on a fixed smooth frictionless plane tilted at an angle $\theta = 22.0°$ to the horizontal. (*a*) Determine the acceleration of the block as it slides down the plane. (*b*) If the block starts from rest 12.0 m up the plane from its base, what will be the block's speed when it reaches the bottom of the incline?

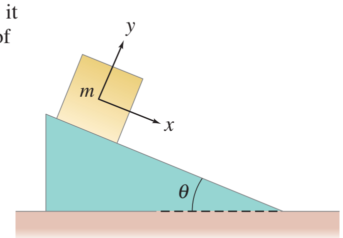


FIGURE 43 Block on inclined plane. Problems 48 and 49.

49. (II) A block is given an initial speed of 4.5 m/s up the 22° plane shown in Fig. 43. (*a*) How far up the plane will it go? (*b*) How much time elapses before it returns to its starting point? Ignore friction.

50. (II) An object is hanging by a string from your rearview mirror. While you are accelerating at a constant rate from rest to 28 m/s in 6.0 s, what angle θ does the string make with the vertical? See Fig. 44.

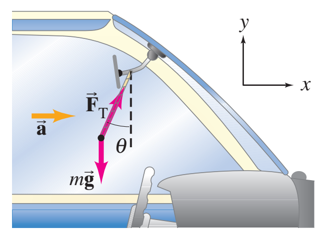


FIGURE 44 Problem 50.

51. (II) Figure 45 shows a block (mass m_A) on a smooth horizontal surface, connected by a thin cord that passes over a pulley to a second block (m_B), which hangs vertically. (*a*) Draw a free-body diagram for each block, showing the force of gravity on each, the force (tension) exerted by the cord, and any normal force. (*b*) Apply Newton's second law to find formulas for the acceleration of the system and for the tension in the cord. Ignore friction and the masses of the pulley and cord.

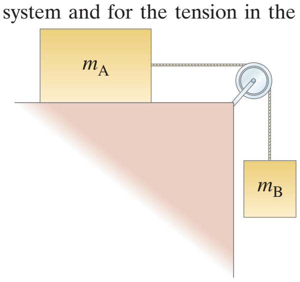


FIGURE 45 Problems 51, 52, and 53. Mass m_A rests on a smooth horizontal surface, m_B hangs vertically.

52. (II) (*a*) If $m_A = 13.0\ \mathrm{kg}$ and $m_B = 5.0\ \mathrm{kg}$ in Fig. 45, determine the acceleration of each block. (*b*) If initially m_A is at rest 1.250 m from the edge of the table, how long does it take to reach the edge of the table if the system is allowed to move freely? (*c*) If $m_B = 1.0\ \mathrm{kg}$, how large must m_A be if the acceleration of the system is to be kept at $\frac{1}{100}g$?

53. (III) Determine a formula for the acceleration of the system shown in Fig. 45 (see Problem 51) if the cord has a non-negligible mass m_C. Specify in terms of ℓ_A and ℓ_B, the lengths of cord from the respective masses to the pulley. (The total cord length is $\ell = \ell_A + \ell_B$.)

54. (III) Suppose the pulley in Fig. 46 is suspended by a cord C. Determine the tension in this cord after the masses are released and before one hits the ground. Ignore the mass of the pulley and cords.

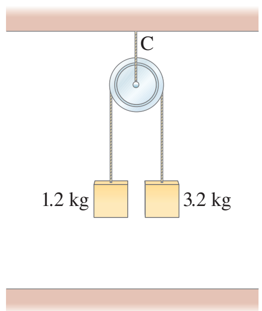


FIGURE 46 Problem 54.

55. (III) A small block of mass m rests on the sloping side of a triangular block of mass M which itself rests on a horizontal table as shown in Fig. 47. Assuming all surfaces are frictionless, determine the magnitude of the force $\vec{\mathbf{F}}$ that must be applied to M so that m remains in a fixed position relative to M (that is, m doesn't move on the incline). [*Hint*: Take x and y axes horizontal and vertical.]

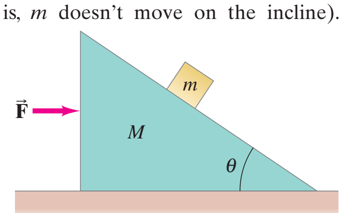


FIGURE 47 Problem 55.

56. (III) The double Atwood machine shown in Fig. 48 has frictionless, massless pulleys and cords. Determine (*a*) the acceleration of masses m_A, m_B, and m_C, and (*b*) the tensions F_{TA} and F_{TC} in the cords.

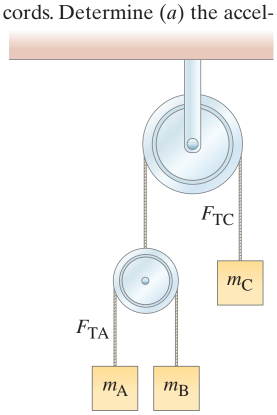


FIGURE 48 Problem 56.

57. (III) Suppose two boxes on a frictionless table are connected by a heavy cord of mass 1.0 kg. Calculate the acceleration of each box and the tension at each end of the cord, using the free-body diagrams shown in Fig. 49. Assume $F_P = 35.0$ N, and ignore sagging of the cord. Compare your results to Example 12 of "Dynamics: Newton's Laws of Motion" and Fig. 22.

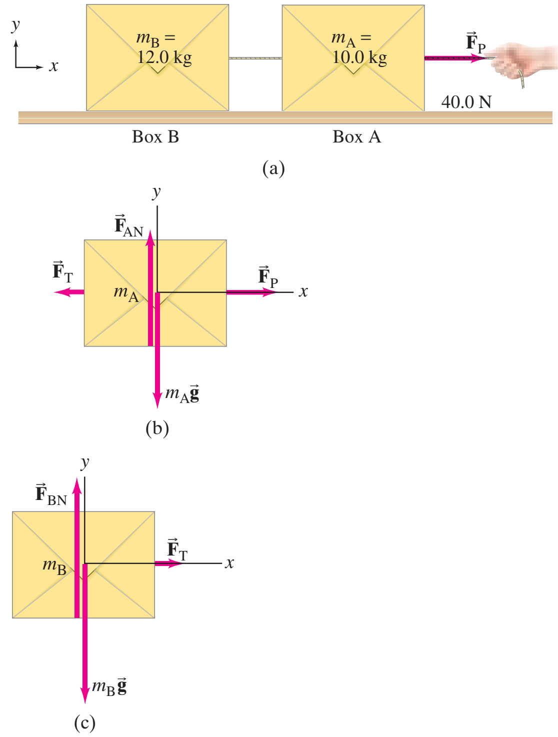


FIGURE 22 Example 12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0$ N. (b) Free-body diagram for box A. (c) Free-body diagram for box B.

58. (III) The two masses shown in Fig. 50 are each initially 1.8 m above the ground, and the massless frictionless pulley is 4.8 m above the ground. What maximum height does the lighter object reach after the system is released? [*Hint*: First determine the acceleration of the lighter mass and then its velocity at the moment the heavier one hits the ground. This is its "launch" speed. Assume the mass doesn't hit the pulley. Ignore the mass of the cord.]

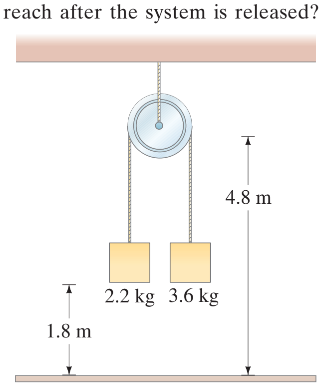


FIGURE 50 Problem 58.

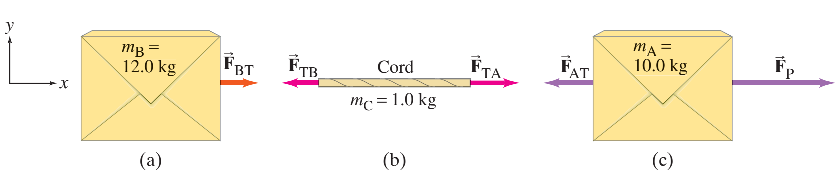


FIGURE 4–49 Problem 57. Free-body diagrams for each of the objects of the system shown in Fig. 4–22a. Vertical forces, $\vec{\mathbf{F}}_N$ and $\vec{\mathbf{F}}_G$, are not shown.

59. (III) Determine a formula for the magnitude of the force $\vec{\mathbf{F}}$ exerted on the large block (m_C) in Fig. 51 so that the mass m_A does not move relative to m_C. Ignore all friction. Assume m_B does not make contact with m_C.

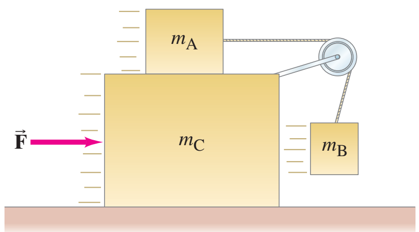


FIGURE 51 Problem 59.

60. (III) A particle of mass m, initially at rest at $x = 0$, is accelerated by a force that increases in time as $F = Ct^2$. Determine its velocity v and position x as a function of time.

61. (III) A heavy steel cable of length ℓ and mass M passes over a small massless, frictionless pulley. (*a*) If a length y hangs on one side of the pulley (so $\ell - y$ hangs on the other side), calculate the acceleration of the cable as a function of y. (*b*) Assuming the cable starts from rest with length y_0 on one side of the pulley, determine the velocity v_f at the moment the whole cable has fallen from the pulley. (*c*) Evaluate v_f for $y_0 = \frac{2}{3}\ell$. [*Hint*: Use the chain rule, $dv/dt = (dv/dy)(dy/dt)$, and integrate.]

General Problems

62. A person has a reasonable chance of surviving an automobile crash if the deceleration is no more than 30 g's. Calculate the force on a 65-kg person accelerating at this rate. What distance is traveled if brought to rest at this rate from 95 km/h?

63. A 2.0-kg purse is dropped 58 m from the top of the Leaning Tower of Pisa and falls 55 m before reaching the ground with a speed of 27 m/s. What was the average force of air resistance?

64. Tom's hang glider supports his weight using the six ropes shown in Fig. 52. Each rope is designed to support an equal fraction of Tom's weight. Tom's mass is 74.0 kg. What is the tension in each of the support ropes?

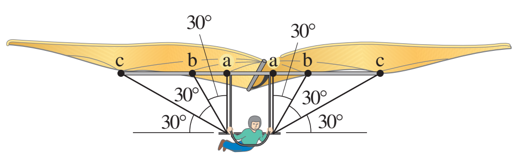


FIGURE 52 Problem 64.

65. A wet bar of soap ($m = 150$ g) slides freely down a ramp 3.0 m long inclined at 8.5°. How long does it take to reach the bottom? How would this change if the soap's mass were 300 g?

66. A crane's trolley at point P in Fig. 53 moves for a few seconds to the right with constant acceleration, and the 870-kg load hangs at a 5.0° angle to the vertical as shown. What is the acceleration of the trolley and load?

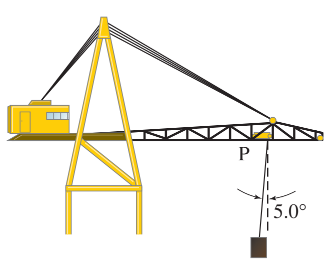


FIGURE 53 Problem 66.

67. A block (mass m_A) lying on a fixed frictionless inclined plane is connected to a mass m_B by a cord passing over a pulley, as shown in Fig. 54. (*a*) Determine a formula for the acceleration of the system in terms of m_A, m_B, θ, and g. (*b*) What conditions apply to masses m_A and m_B for the acceleration to be in one direction (say, m_A down the plane), or in the opposite direction? Ignore the mass of the cord and pulley.

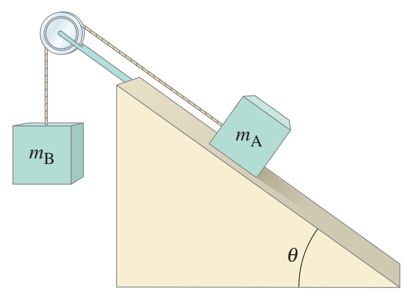


FIGURE 54 Problems 67 and 68.

68. (*a*) In Fig. 54, if $m_A = m_B = 1.00$ kg and $\theta = 33.0°$, what will be the acceleration of the system? (*b*) If $m_A = 1.00$ kg and the system remains at rest, what must the mass m_B be? (*c*) Calculate the tension in the cord for (*a*) and (*b*).

69. The masses m_A and m_B slide on the smooth (frictionless) inclines fixed as shown in Fig. 55. (*a*) Determine a formula for the acceleration of the system in terms of m_A, m_B, θ_A, θ_B, and g. (*b*) If $\theta_A = 32°$, $\theta_B = 23°$, and $m_A = 5.0$ kg, what value of m_B would keep the system at rest? What would be the tension in the cord (negligible mass) in this case? (*c*) What ratio, m_A/m_B, would allow the masses to move at constant speed along their ramps in either direction?

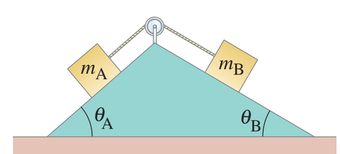


FIGURE 55 Problem 69.

70. A 75.0-kg person stands on a scale in an elevator. What does the scale read (in N and in kg) when (*a*) the elevator is at rest, (*b*) the elevator is climbing at a constant speed of 3.0 m/s, (*c*) the elevator is descending at 3.0 m/s, (*d*) the elevator is accelerating upward at 3.0 m/s^2, (*e*) the elevator is accelerating downward at 3.0 m/s^2?

71. A city planner is working on the redesign of a hilly portion of a city. An important consideration is how steep the roads can be so that even low-powered cars can get up the hills without slowing down. A particular small car, with a mass of 920 kg, can accelerate on a level road from rest to 21 m/s (75 km/h) in 12.5 s. Using these data, calculate the maximum steepness of a hill.

72. If a bicyclist of mass 65 kg (including the bicycle) can coast down a 6.5° hill at a steady speed of 6.0 km/h because of air resistance, how much force must be applied to climb the hill at the same speed (and the same air resistance)?

73. A bicyclist can coast down a 5.0° hill at a constant speed of 6.0 km/h. If the force of air resistance is proportional to the speed v so that $F_{air} = cv$, calculate (*a*) the value of the constant c, and (*b*) the average force that must be applied in order to descend the hill at 18.0 km/h. The mass of the cyclist plus bicycle is 80.0 kg.

74. Francesca dangles her watch from a thin piece of string while the jetliner she is in accelerates for takeoff, which takes about 16 s. Estimate the takeoff speed of the aircraft if the string makes an angle of 25° with respect to the vertical, Fig. 56.

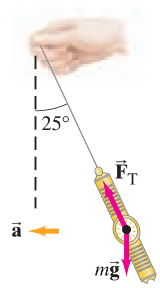


FIGURE 56
Problem 74.

75. (*a*) What minimum force F is needed to lift the piano (mass M) using the pulley apparatus shown in Fig. 57? (*b*) Determine the tension in each section of rope: F_{T1}, F_{T2}, F_{T3}, and F_{T4}.

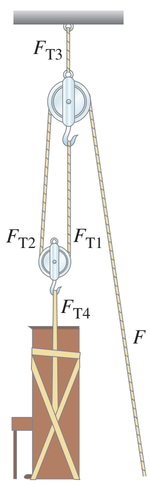
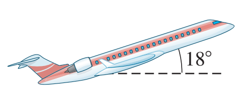


FIGURE 57
Problem 75.

76. In the design of a supermarket, there are to be several ramps connecting different parts of the store. Customers will have to push grocery carts up the ramps and it is obviously desirable that this not be too difficult. The engineer has done a survey and found that almost no one complains if the force required is no more than 18 N. Ignoring friction, at what maximum angle θ should the ramps be built, assuming a full 25-kg grocery cart?

77. A jet aircraft is accelerating at 3.8 m/s^2 as it climbs at an angle of 18° above the horizontal (Fig. 58). What is the total force that the cockpit seat exerts on the 75-kg pilot?

FIGURE 58
Problem 77.



78. A 7650-kg helicopter accelerates upward at 0.80 m/s^2 while lifting a 1250-kg frame at a construction site, Fig. 59. (*a*) What is the lift force exerted by the air on the helicopter rotors? (*b*) What is the tension in the cable (ignore its mass) that connects the frame to the helicopter? (*c*) What force does the cable exert on the helicopter?

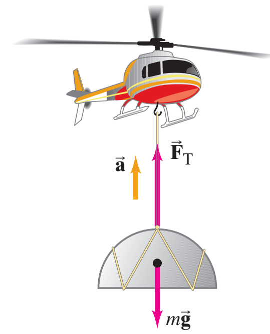


FIGURE 59
Problem 78.

79. A super high-speed 14-car Italian train has a mass of 640 metric tons (640,000 kg). It can exert a maximum force of 400 kN horizontally against the tracks, whereas at maximum constant velocity (300 km/h), it exerts a force of about 150 kN. Calculate (*a*) its maximum acceleration, and (*b*) estimate the force of friction and air resistance at top speed.

80. A fisherman in a boat is using a "10-lb test" fishing line. This means that the line can exert a force of 45 N without breaking (1 lb = 4.45 N). (*a*) How heavy a fish can the fisherman land if he pulls the fish up vertically at constant speed? (*b*) If he accelerates the fish upward at 2.0 m/s^2, what maximum weight fish can he land? (*c*) Is it possible to land a 15-lb trout on 10-lb test line? Why or why not?

81. An elevator in a tall building is allowed to reach a maximum speed of 3.5 m/s going down. What must the tension be in the cable to stop this elevator over a distance of 2.6 m if the elevator has a mass of 1450 kg including occupants?

82. Two rock climbers, Bill and Karen, use safety ropes of similar length. Karen's rope is more elastic, called a *dynamic rope* by climbers. Bill has a *static rope*, not recommended for safety purposes in pro climbing. (*a*) Karen falls freely about 2.0 m and then the rope stops her over a distance of 1.0 m (Fig. 60). Estimate how large a force (assume constant) she will feel from the rope. (Express the result in multiples of her weight.) (*b*) In a similar fall, Bill's rope stretches by only 30 cm. How many times his weight will the rope pull on him? Which climber is more likely to be hurt?

Tyler Stableford/The Image Bank/
Getty Images

FIGURE 60
Problem 82.

83. Three mountain climbers who are roped together in a line are ascending an icefield inclined at 31.0° to the horizontal (Fig. 61). The last climber slips, pulling the second climber off his feet. The first climber is able to hold them both. If each climber has a mass of 75 kg, calculate the tension in each of the two sections of rope between the three climbers. Ignore friction between the ice and the fallen climbers.

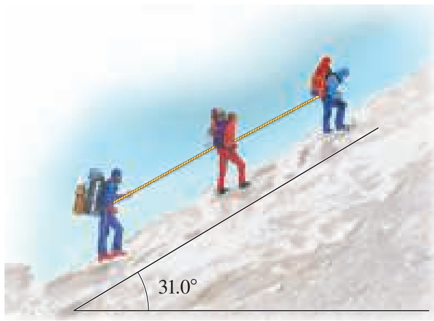


FIGURE 61 Problem 83.

84. A "doomsday" asteroid with a mass of 1.0×10^{10} kg is hurtling through space. Unless the asteroid's speed is changed by about 0.20 cm/s, it will collide with Earth and cause tremendous damage. Researchers suggest that a small "space tug" sent to the asteroid's surface could exert a gentle constant force of 2.5 N. For how long must this force act?

85. A 450-kg piano is being unloaded from a truck by rolling it down a ramp inclined at 22°. There is negligible friction and the ramp is 11.5 m long. Two workers slow the rate at which the piano moves by pushing with a combined force of 1420 N parallel to the ramp. If the piano starts from rest, how fast is it moving at the bottom?

86. Consider the system shown in Fig. 62 with $m_A = 9.5$ kg and $m_B = 11.5$ kg. The angles $\theta_A = 59°$ and $\theta_B = 32°$. (*a*) In the absence of friction, what force $\vec{\mathbf{F}}$ would be required to pull the masses at a constant velocity up the fixed inclines? (*b*) The force $\vec{\mathbf{F}}$ is now removed. What is the magnitude and direction of the acceleration of the two blocks? (*c*) In the absence of $\vec{\mathbf{F}}$, what is the tension in the string?

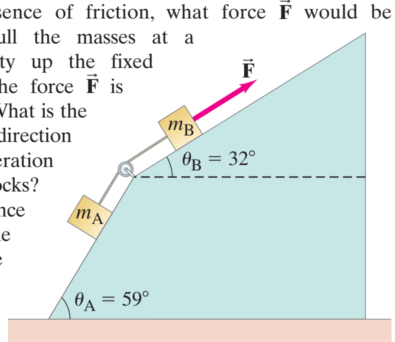


FIGURE 62 Problem 86.

87. A 1.5-kg block rests on top of a 7.5-kg block (Fig. 63). The cord and pulley have negligible mass, and there is no significant friction anywhere. (*a*) What force F must be applied to the bottom block so the top block accelerates to the right at 2.5 m/s^2? (*b*) What is the tension in the connecting cord?

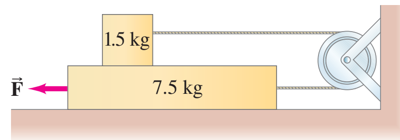


FIGURE 63 Problem 87.

88. You are driving home in your 750-kg car at 15 m/s. At a point 45 m from the beginning of an intersection, you see a green traffic light change to yellow, which you expect will last 4.0 s, and the distance to the far side of the intersection is 65 m (Fig. 64). (*a*) If you choose to accelerate, your car's engine will furnish a forward force of 1200 N. Will you make it completely through the intersection before the light turns red? (*b*) If you decide to panic stop, your brakes will provide a force of 1800 N. Will you stop before entering the intersection?

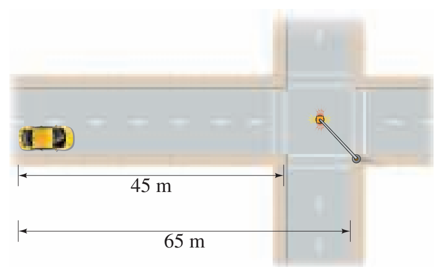


FIGURE 64 Problem 88.

* Numerical/Computer

***89.** (II) A large crate of mass 1500 kg starts sliding from rest along a frictionless ramp, whose length is ℓ and whose inclination with the horizontal is θ. (*a*) Determine as a function of θ: (i) the acceleration a of the crate as it goes downhill, (ii) the time t to reach the bottom of the incline, (iii) the final velocity v of the crate when it reaches the bottom of the ramp, and (iv) the normal force F_N on the crate. (*b*) Now assume $\ell = 100$ m. Use a spreadsheet to calculate and graph a, t, v, and F_N as functions of θ from $\theta = 0°$ to 90° in 1° steps. Are your results consistent with the known result for the limiting cases $\theta = 0°$ and $\theta = 90°$?

Answers to Odd-Numbered Problems

1. 77 N.

3. (*a*) 6.7×10^2 N;
(*b*) 1.2×10^2 N;
(*c*) 2.5×10^2 N;
(*d*) 0.

5. 1.3×10^6 N, 39%, 1.3×10^6 N.

7. 2.1×10^2 N.

9. $m > 1.5$ kg.

11. 89.8 N.

13. 1.8 m/s^2, up.

15. Descend with $a \geq 2.2 \text{ m/s}^2$.

17. -2800 m/s^2, 280 g's, 1.9×10^5 N.

19. (*a*) 7.5 s, 13 s, 7.5 s;
(*b*) 12%, 0%, −12%;
(*c*) 55%.

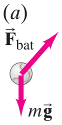
21. (*a*) 3.1 m/s^2;
(*b*) 25 m/s;
(*c*) 78 s.

23. 3.3×10^3 N.

25. (*a*) 150 N;
(*b*) 14.5 m/s.

27. (*a*) 47.0 N;
(*b*) 17.0 N;
(*c*) 0.

29. (*a*)



(*b*)

31. (*a*) 1.5 m;
(*b*) 11.5 kN, no.

33. (*a*) 31 N, 63 N;
(*b*) 35 N, 71 N.

35. 6.3×10^3 N, 8.4×10^3 N.

37. (*a*) 19.0 N at 237.5°, 1.03 m/s^2 at 237.5°;
(*b*) 14.0 N at 51.0°, 0.758 m/s^2 at 51.0°.

39. $\frac{5}{2}\frac{F_0}{m}t_0^2$.

41. 4.0×10^2 m.

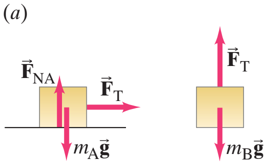
43. 12°.

45. (*a*) 9.9 N;
(*b*) 260 N.

47. (*a*) $m_E g - F_T = m_E a$;
$F_T - m_C g = m_C a$;
(*b*) 0.68 m/s^2, 10, 500 N.

49. (*a*) 2.8 m;
(*b*) 2.5 s.

51. (*a*)



(*b*) $g\frac{m_B}{m_A + m_B}$, $g\frac{m_A m_B}{m_A + m_B}$.

53. $g\dfrac{m_B + \dfrac{\ell_B}{\ell_A + \ell_B}m_C}{m_A + m_B + m_C}$.

55. $(m + M)g\tan\theta$.

57. 1.52 m/s^2, 18.3 N, 19.8 N.

59. $\dfrac{(m_A + m_B + m_C)m_B}{\sqrt{(m_A^2 - m_B^2)}}g$.

61. (*a*) $\left(\frac{2y}{\ell} - 1\right)g$;
(*b*) $\sqrt{2gy_0\left(1 - \frac{y_0}{\ell}\right)}$;
(*c*) $\frac{2}{3}\sqrt{g\ell}$.

63. 6.3 N.

65. 2.0 s, no change.

67. (*a*) $g\dfrac{(m_A\sin\theta - m_B)}{(m_A + m_B)}$;
(*b*) $m_A\sin\theta > m_B$
(m_A down the plane),
$m_A\sin\theta < m_B$
(m_A up the plane).

69. (*a*) $\dfrac{m_B\sin\theta_B - m_A\sin\theta_A}{m_A + m_B}g$;
(*b*) 6.8 kg, 26 N;
(*c*) 0.74.

71. 9.9°.

73. (*a*) $41\frac{\text{N}}{\text{m/s}}$;
(*b*) 1.4×10^2 N.

75. (*a*) $Mg/2$;
(*b*) $Mg/2$, $Mg/2$, $3Mg/2$, Mg.

77. 8.7×10^2 N,
72° above the horizontal.

79. (*a*) 0.6 m/s^2;
(*b*) 1.5×10^5 N.

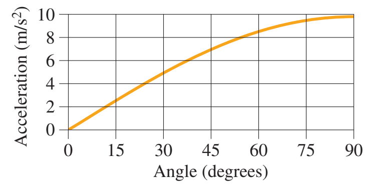
81. 1.76×10^4 N.

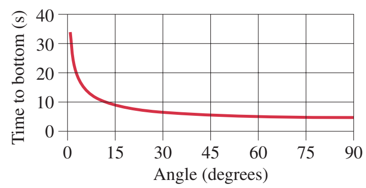
83. 3.8×10^2 N, 7.6×10^2 N.

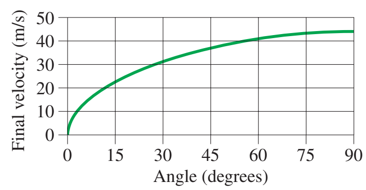
85. 3.4 m/s.

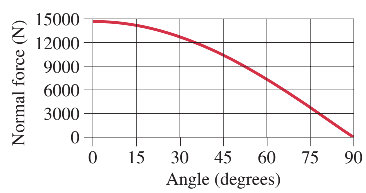
87. (*a*) 23 N;
(*b*) 3.8 N.

89. (*a*) $g\sin\theta$, $\sqrt{\dfrac{2\ell}{g\sin\theta}}$,
$\sqrt{2\ell g\sin\theta}$, $mg\cos\theta$;
(*b*)









The graphs are all consistent with the results of the limiting cases.

Using Newton's Laws: Friction, Circular Motion, Drag Forces

Agence Zoom/Getty Images

Newton's laws are fundamental in physics. These photos show two situations of using Newton's laws. The downhill skier illustrates *friction* on an incline, although at this moment she is not touching the snow, and so is retarded only by air resistance which is a velocity-dependent force. The people on the rotating amusement park ride below illustrate the dynamics of circular motion.

Grant Faint/Getty Images

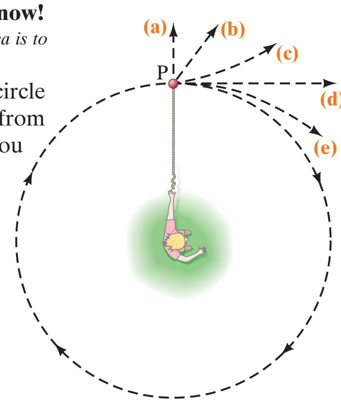
Using Newton's Laws: Friction, Circular Motion, Drag Forces

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

You revolve a ball around you in a horizontal circle at constant speed on a string, as shown here from above. Which path will the ball follow if you let go of the string at point P?



This chapter considers aspects of Newton's laws and emphasizes their fundamental importance in physics. We cover some important applications of Newton's laws, including friction and circular motion.

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

1 Applications of Newton's Laws Involving Friction

Friction must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 1. When we try to slide an object across another surface, these microscopic bumps impede the motion. Exactly what is happening at the microscopic level is not yet fully understood. It is thought that the atoms on a bump of one surface may come so close to the atoms of the other surface that attractive electric forces between the atoms could "bond" as a tiny weld between the two surfaces. Sliding an object across a surface is often jerky, perhaps due to the making and breaking of these bonds. Even when a round object rolls across a surface, there is still some friction, called *rolling friction*, although it is generally much less than when objects slide across a surface. We focus our attention now on sliding friction, which is usually called **kinetic friction** (*kinetic* is from the Greek for "moving").

When an object slides along a rough surface, the force of kinetic friction acts opposite to the direction of the object's velocity. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, experiment shows that the friction force is approximately proportional to the *normal force* between the two surfaces, which is the force that either object exerts on the other and is perpendicular to their common surface of contact (see Fig. 2). The force of friction between hard surfaces in many cases depends very little on the total surface area of contact; that is, the friction force on this book is roughly the same whether it is being slid on its wide face or on its spine, assuming the surfaces have the same smoothness. We consider a simple model of friction in which we make this assumption that the friction force is independent of area. Then we write the proportionality between the magnitudes of the friction force F_{fr} and the normal force F_{N} as an equation by inserting a constant of proportionality, μ_{k}:

$$F_{\mathrm{fr}} = \mu_{\mathrm{k}} F_{\mathrm{N}}. \qquad \text{[kinetic friction]}$$

This relation is not a fundamental law; it is an experimental relation between the magnitude of the friction force F_{fr}, which acts parallel to the two surfaces, and the magnitude of the normal force F_{N}, which acts perpendicular to the surfaces. It is *not* a vector equation since the two forces have directions perpendicular to one another. The term μ_{k} is called the *coefficient of kinetic friction*, and its value depends on the nature of the two surfaces. Measured values for a variety of surfaces are given in Table 1. These are only approximate, however, since μ depends on whether the surfaces are wet or dry, on how much they have been sanded or rubbed, if any burrs remain, and other such factors. But μ_{k} is roughly independent of the sliding speed, as well as the area in contact.

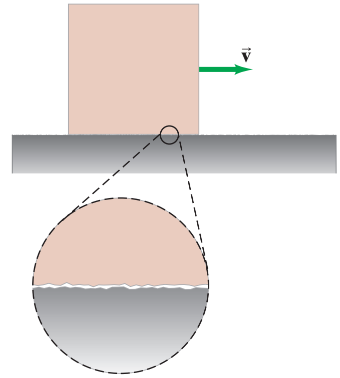


FIGURE 1 An object moving to the right on a table or floor. The two surfaces in contact are rough, at least on a microscopic scale.

FIGURE 2 When an object is pulled along a surface by an applied force $(\vec{\mathbf{F}}_{\mathrm{A}})$, the force of friction $\vec{\mathbf{F}}_{\mathrm{fr}}$ opposes the motion. The magnitude of $\vec{\mathbf{F}}_{\mathrm{fr}}$ is proportional to the magnitude of the normal force (F_{N}).

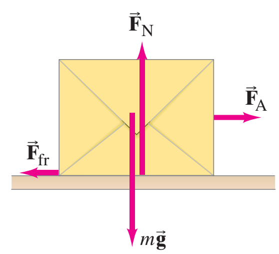


TABLE 1 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_{s}	Coefficient of Kinetic Friction, μ_{k}
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

†Values are approximate and intended only as a guide.