FLATLAND

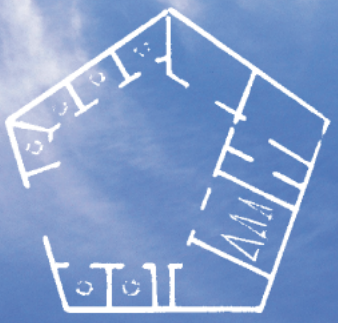
by

Edwin A. Abbott

An Edition with

Notes and Commentary by

William F. Lindgren & Thomas F. Banchoff



Flatland

An Edition with Notes and Commentary

Flatland, Edwin Abbott Abbott's story of a two-dimensional universe, as told by one of its inhabitants who is introduced to the mysteries of three-dimensional space, has enjoyed an enduring popularity from the time of its publication in 1884. This fully annotated edition enables the modern-day reader to understand and appreciate the many "dimensions" of this classic satire with commentary on language and literary style, including numerous definitions of obscure words and an appendix on Abbott's life and work. Historical commentary, writings by Plato and Aristotle, and citations from Abbott's other writings work together to show how this tale relates to Abbott's views of society in late-Victorian England and classical Greece. Approaching the book from a mathematical stance, additional notes and illustrations enhance the usefulness of *Flatland* as an elementary introduction to higher-dimensional geometry.

William F. Lindgren is a professor of mathematics at Slippery Rock University. He is the coauthor of *Quasi-Uniform Spaces* (with Peter Fletcher).

Thomas F. Banchoff is a professor of mathematics at Brown University. He is the author of *Beyond the Third Dimension*, *Linear Algebra through Geometry* (with John Wermer), and an introduction to a new edition of Henry P. Manning's *The Fourth Dimension Simply Explained*.

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The Mathematical Association of America

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Contents

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It is altogether fitting that there be a "Cambridge *Flatland*." Abbott graduated from the University of Cambridge, as did most of his closest friends and many of his students. His son was a fellow of Jesus College, Cambridge, and his daughter attended Girton College. His last great work, the fourteen-volume "Diatessarica" series, was printed in Cambridge at the University Press. We believe that he would be pleased that Cambridge University Press is publishing *Flatland*, and it is to his memory that this edition is dedicated.

Introduction

Edwin Abbott Abbott's *Flatland* is the story of a two-dimensional universe as told by one of its inhabitants, a square who is introduced to the mysteries of three-dimensional space by a sphere. Since the time of its publication in 1884, Victorian customs have become obscure, the meanings of words have changed, and historical allusions, which were obvious to a contemporary, now require explanation. The present edition is intended to enable a modern-day reader to understand and appreciate the "many dimensions" of this classic satire. The extensive annotations to the text include mathematical notes and illustrations, which enhance the usefulness of *Flatland* as an elementary introduction to higher-dimensional geometry; historical notes, which show connections to late-Victorian England and to classical Greece; citations from Abbott's other writings and the works of Plato and Aristotle, which serve to interpret the text; commentary on close parallels between *Flatland* and Plato's "parable of the cave"; notes on the language and literary style of the book, including definitions of obscure words; and an appendix, which gives a comprehensive account of Abbott's life and work.

A Romance of many dimensions.

The word "romance" in *Flatland*'s subtitle means a prose narrative that treats imaginary characters involved in events quite different in time and place from those of ordinary life. The two-dimensional world of *Flatland*, inhabited by geometric figures, is manifestly different from that of ordinary life; nonetheless, a great majority of its early readers were well acquainted with the "physical space" of Flatland. Flatland's space is the familiar Euclidean plane, the principal object of study in Euclid's *Elements*, a fixture in the curriculum of Victorian public schools.

The phrase "of many dimensions" is a play on various meanings of "dimension." In the literal (geometric) sense of the word, *Flatland* is a

primer on the geometry of higher dimensions. But Abbott was not a mathematician and did not intend to write a geometry text; he would be surprised to learn that his "romance of many dimensions" has become a standard introduction to higher-dimensional geometry. He meant "dimension" primarily in a figurative sense and, in this sense, *Flatland* indeed has many dimensions: It is an extended metaphor expressed in the language of mathematics; it is a satirical commentary on Victorian society; it is a geometric version of Plato's parable of the cave; it is an expression of religious principle; and it is an illustration of Abbott's theory that imagination is the basis of all knowledge. Another dimension of *Flatland*, Abbott's use of play, has frequently been misjudged. The book certainly abounds with cleverness and play of wit, and Abbott certainly intended it to be amusing. But those commentators who have characterized *Flatland* as a mere humorous trifle have badly underestimated a book that is written in the same spirit of playful seriousness that pervades the Platonic dialogues.

Flatland *as Victorian England*/Flatland *as classical Greece.*

Although Abbott was not the first person to posit a two-dimensional universe inhabited by geometric figures, he was the first to imagine such a space endowed with a highly developed social and political structure. Abbott's primary model for this structure is not that of late-Victorian England, which is unquestionably the target of his satire, but rather that of classical Greece. Abbott's contemporaries would have found Flatland's society as familiar as its space, for the traditional public school education placed a heavy emphasis on the Greek and Roman classics.

There was a widespread conviction among Victorian writers that the Greeks had been like the English and "that the historical situations of the two civilizations were essentially similar. Although this attitude did not survive much beyond the first quarter of the twentieth century, it was fundamental to Victorian intellectual life and determined the outlook of much Victorian scholarship, criticism, and commentary on the Greeks." To maintain the similarity between the two civilizations, writers like Matthew Arnold had to rationalize away fundamental differences and ignore morally distasteful elements in Greek society. Others (with whom Abbott would have agreed) argued that classical Greece had not been like England, and England should not model itself on classical Greece (Turner 1981, 11, 61, 252).

In writing *Flatland*, Abbott used "historical imagination" not to reconstruct the past but to reconstruct the present in the past. He devised an extended geometric metaphor by projecting late-Victorian England onto a two-dimensional space with a "civilization" in various ways similar to that of classical Greece. Further, he heightened his satirical commentary on the present by making prominent in this imaginary civilization some of

the very aspects of classical Greece that its Victorian apologists had rationalized away – for example, slavery, a rigid class system, misogyny, and ancient forms of social Darwinism.

Flatland *and Plato's parable of the cave.*

Several writers have noted the most significant "Greek connection" in *Flatland* – its parallels with Plato's parable of the cave. In the seventh book of the *Republic*, Socrates describes a cave containing prisoners who have been kept fixed since childhood by bonds on their legs and necks. Their bonds prevent them from turning their heads, and so they cannot see the fire burning above and behind them. Between the fire and the prisoners there is a rampart bordered by a low wall on which certain men place various artifacts; the fire casts shadows of these objects on the cave wall in front of the prisoners. These shadows and the voices of the men on the rampart are the only "reality" that each prisoner knows (*Republic*, 514a–518b).

In *Flatland*, Abbott has amplified Plato's metaphor and rendered it into the language of geometry by substituting a two-dimensional plane populated by geometric figures for the cave and its prisoners. Although Abbott does not explicitly acknowledge the source of his model, and the details of the two stories are by no means identical, the derivation of *Flatland* from Plato's parable seems unmistakable. Most significant, each text contains a metaphorical account of both the nature of the human condition and the journey of an individual soul from ignorance to knowledge.

Abbott's "geometrization" of the cave parable is particularly appropriate because of the importance that Plato attaches to mathematics. Plato included geometry in the educational curriculum for the guardians of the State because it "is the knowledge of the eternally existent" and tends to draw the soul to truth, and is "productive of a philosophical attitude of mind, directing upward the faculties that are now wrongly turned earthward" (*Republic*, 527b).

Precursors of Flatland.

The study of higher-dimensional geometry began with works by Hermann Grassmann (1844), Arthur Cayley (1846), and Bernhard Riemann (1854); by the time *Flatland* appeared, hundreds of articles on the subject had been published. Interest in higher-dimensional spaces was by no means confined to the scientific community, and a number of these articles were directed at a general readership. Several writers of popular essays on higher dimensions illustrated the difficulty of understanding four-dimensional space by portraying two-dimensional beings living on a surface and unable to perceive anything of three-dimensional space. This

dimensional analogy, which is fundamental to *Flatland*, was used by three writers whose works might have influenced Abbott: Gustav T. Fechner, Hermann von Helmholtz, and C. Howard Hinton.

The German psychologist and philosopher Fechner was the first person to use "flatland" as a device for understanding higher dimensions. In two essays, "*Der Schatten ist lebendig*" ("The shadow is alive") and "*Der Raum hat vier Dimensionen*" ("Space has four dimensions"), first published in 1845, he describes a shadow man capable of moving about on surfaces and interacting with other shadows. Abbott, who was fluent in German, might have read them in *Kleine Schriften,* a collection of Fechner's satirical essays (Fechner 1875, 243–276).

Between 1868 and 1879, the German scientist and philosopher Helmholtz published several lectures and papers with similar titles and contents that considered how humans come to understand the nature of space. He used an example of two-dimensional beings whose movements are confined to the surface of a solid to argue that our notion of space is not, as Immanuel Kant supposed, an *a priori* intuition but rather is determined by our experience. Abbott could have seen the essay, "The origin and meaning of geometrical axioms," which appeared in the English philosophical journal, *Mind* (1876).

Hinton has been described as a "hyperspace philosopher." His essay "What is the fourth dimension?" (1880, 1883) is the first of several that he wrote to popularize four-dimensional space. Abbott might have learned of Hinton's essay from his friend Howard Candler, who was the mathematics master at Uppingham School where Hinton was the science master between 1880 and 1886.

Another precursor of *Flatland*, an essay entitled "A new philosophy," appeared without attribution in the November 1877 issue of the *City of London School Magazine*. The thesis of this satirical essay is that mathematics is the only "science" that can provide an unshakable foundation for a philosophy or religion. According to the proposed "Geometrical Philosophy," the universe is composed of an ascending chain of spaces, each having its dimension one greater than that of its predecessor, as well as a descending chain of spaces, each having its dimension one less than that of its predecessor. Although the *City of London School Magazine* was a student publication, it is possible that Abbott himself was the author of this noteworthy essay (New Philosophy 1877; Valente 2004).

Abbott could have found the idea of using a mathematical setting for his story in Lewis Carroll's *Dynamics of a Parti-cle*. Carroll prefaces this small pamphlet of political satire with a brief account of a love affair between a pair of linear creatures moving across a plane surface. This preface, he says, illustrates "the advantage of introducing the human element into the hitherto barren region of mathematics" (Carroll 1874).

Edwin Abbott Abbott.

The author of *Flatland*, the eminent biblical and English scholar and Victorian headmaster Edwin Abbott Abbott, was born in 1838 in Marylebone, where his father was headmaster of the Philological School. Abbott was educated at the City of London School (CLS) under George F. W. Mortimer before entering St. John's College, Cambridge, where he was senior classic and senior Chancellor's Medallist in 1861. He was ordained deacon in 1862 and priest the following year. After teaching briefly at King Edward's School, Birmingham, and Clifton College, Bristol, he returned to CLS as headmaster in 1865. The school that he inherited from Mortimer was highly regarded, and under Abbott it became one of the best day schools in England. He reformed the traditional curriculum, brought new methods of instruction, and improved the quality of the assistant masters. Abbott was a gifted teacher who sent a large number of students to Oxford and Cambridge. At the same time, as an administrator he ensured that the greater number, those not meant for the universities, received a sound general education.

His student and biographer, Lewis Farnell, asserted that Abbott's "claim to be remembered must chiefly rest upon what can only be called his genius for teaching." Nonetheless, the enduring interest of his life was the problem of presenting Christianity to his contemporaries in a way that would ensure the permanence of traditional beliefs without requiring the acceptance of miracles (Obituary 1926b). He "retired" at the age of 51 and devoted himself to biblical scholarship; between 1900 and 1917, he published an immensely detailed, fourteen-volume study of the four Gospels.

Various commentators have remarked that *Flatland* seems out of place with the rest of Abbott's literary output; nevertheless, it is quite similar to two other pseudonymous, first-person accounts that Abbott wrote: *Philochristus* (1878), the story of a Pharisee in the early first century, and *Onesimus* (1882), the story of the Greek slave in St. Paul's Epistle to Philemon. In these stories, as in *Flatland*, a protagonist is transformed by the revelation of a being of a higher order, but when he attempts to spread this good news, he meets with frustration and even persecution.

The first and second editions of Flatland.

The first edition of *Flatland* probably appeared in late October 1884. *Flatland* is on Seeley and Company's "List of New Books" in *The Literary Churchman* (24 October 1884, 452). Abbott autographed copies to several friends in October 1884, and the earliest review appeared in *The Oxford Magazine* (5 November 1884) (see Appendix A4).

We do not know how many copies of the first edition of *Flatland* were sold. In the nineteenth century, publishers typically issued 500 to 1,000 copies, and the great majority of books never went into a second edition. In any case, the first edition sold out quickly, and a second edition was published. The title page of this edition is dated 1884, but the following evidence suggests that it did not appear until 1885: The significant changes to the text are based on a letter, which the Square sent to *The Athenaeum* in response to a review of *Flatland*, and the reviewer's response to that letter. Both the Square's letter and the reviewer's response appeared in *The Athenaeum* on 6 December 1884 (Appendix A2). An advertisement for Seeley and Co. in the *Times* of 21 January 1885, describes *Flatland* as "just published" with no mention that it is a second edition. Abbott inscribed a second edition to Howard Candler in February 1885. The first advertisement for *Flatland* in the *Times* that mentions a second edition appeared in the 18 March 1885 issue.

Subsequent editions.

Flatland had been long out of print in England when it was reissued in June 1926 by Sir Basil Blackwell. The text of Blackwell's edition was only slightly different from the second edition, and it had much the same appearance as the original. This edition included an introduction written by the physicist, William Garnett, who was among Abbott's first pupils at the City of London School. In his introduction, Garnett cites an essay written six years earlier in which he characterized Abbott as a prophet who had foreseen the relevance of the dimensional analogy for understanding the passage of time in relation to space.

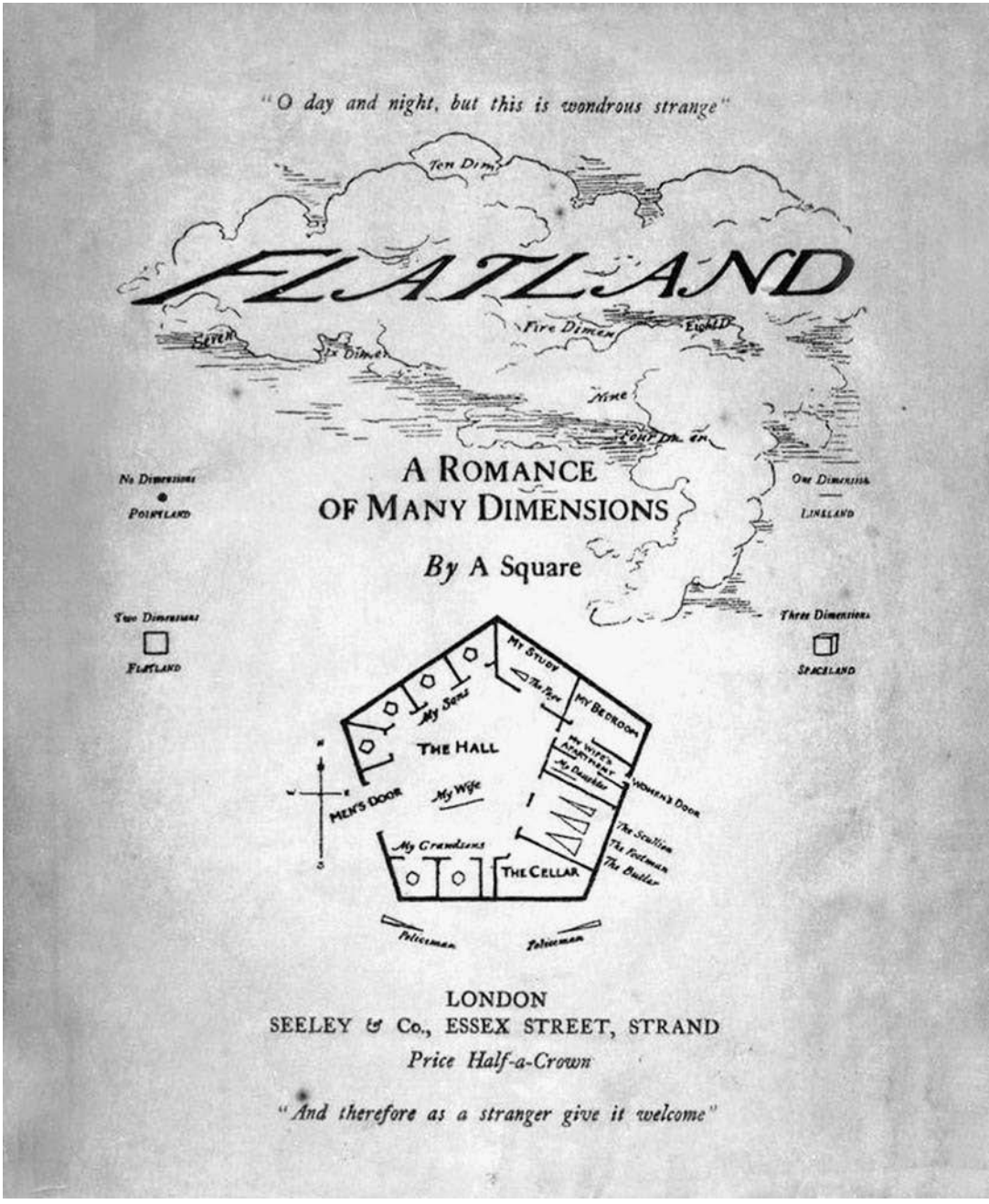
In 1885, Roberts Brothers of Boston issued the first American edition of *Flatland*, essentially the uncorrected first edition with Americanized spelling. In 1898, Roberts Brothers was acquired by Little, Brown and Co., which continued publishing *Flatland* until the middle of the twentieth century. Much of the popularity of *Flatland* in the United States is attributable to a co-founder of Dover Publications, Hayward Cirker, who chose *Flatland* as one of his firm's first titles in mathematics in 1952. The publication of the Dover *Flatland* made the second edition readily available to American readers for the first time. In the past thirty years, publishers have issued dozens of "editions" of the book, which differ from one another only in their introductions. The first translation of *Flatland*, the Dutch *Platland*, appeared in 1886; since then, it has been translated into sixteen other languages.

***This edition of* Flatland.**

The text of *Flatland* that follows is the second edition with one change: The Preface to that edition is the Epilogue to this one. We have made this

change because this Preface/Epilogue can be properly understood only by a person who has read the rest of the text. Furthermore, it really is an epilogue – the concluding section of a work in which a character, at a somewhat later time, reflects on the preceding events and gives additional details, which serve to interpret the story. Finally, reading this section before reading the text would spoil the effect of the narrative in which the Square describes his passage from the unenlightened "Square he once was" through his "initiation into the mysteries of space" and his subsequent "miserable Fall."

In his essay "Good readers and good writers," Vladimir Nabokov maintains: "In reading, one should notice and fondle details... Curiously enough, one cannot *read* a book: one can only reread it. A good reader, a major reader, an active and creative reader is a rereader" (Nabokov 1980, 3). It is to the rereaders that we address this annotated edition of *Flatland*.



LONDON
SEELEY & Co., ESSEX STREET, STRAND
Price Half-a-Crown

"And therefore as a stranger give it welcome"

Notes and Commentary

Cover. As he did with *Philochristus* and *Onesimus*, Abbott wrote *Flatland* in an archaic style to create an impression of antiquity. In the case of the first edition of *Flatland*, the impression begins with its cover, which is made of vellum (a fine kind of parchment prepared from the skins of calves) wrapped on cardboard. Abbott may have chosen this cover as a reminiscence of the principal form of the "book" in the ancient world – a roll made of glued sheets of papyrus wound about a wooden stick and often kept in a parchment cover.

Cover epigraph. The epigraph, which is taken from *Hamlet* 1.5, points toward and provides commentary on *Flatland*'s central event, the nocturnal "appearance" in Flatland of a sphere, who has come to initiate the Square into "the mysteries of space." The first line, "O day and night, but this is wondrous strange," is spoken by Horatio, who has just seen the ghost of Hamlet's slain father appear and disappear. In his reply, "And therefore as a stranger give it welcome," Hamlet alludes to the ancient practice of hospitality to strangers. Shakespeare is urging us not to be limited by Horatio's rationalism but, together with Hamlet, to welcome "things not dreamt of in (our) philosophy."

It was natural for Abbott to select epigraphs from the works of Shakespeare. He established his scholarly reputation with the publication of *A Shakespearian Grammar*, which he wrote to furnish students of Shakespeare with an account of the differences between Elizabethan and Victorian English.

A Square. Edwin Abbott Abbott's name contains the surname of both of his parents, Edwin Abbott and Jane Abbott, who were first cousins. The pseudonym Abbott has chosen is a pun – it refers to his own initials, EAA = EA^2, as well as the modest social status of the "author" who is an ordinary square. *Flatland*'s everyman narrator does not tell us the names of any of his contemporary Flatlanders; nor does he tell us his own name – he is "a square" ("A Square" as it is typeset on the title page), not A. Square.

Abbott made the Square the author of *Flatland* not to avoid responsibility for the book but rather to present it as the memoirs of a two-dimensional being. The anonymous publication of books was common in the nineteenth century, but books that received any degree of celebrity were typically attributed within a few months. The first public indication that Abbott was the author of *Flatland* appeared in the Literary Gossip column of *The Athenaeum* (see Appendix A2, footnote 3). Abbott himself may well have been the source of this item. Certainly, he was the one who revealed that he was the author of *Philochristus*: "I shall publish it anonymously: but shall carefully let it be known that I am the author, for there are reasons why (though I may not like to be abused by *name* in the religious papers) I have no right to shirk the odium of heterodoxy, for the book is heterodox" (Abbott 1874).

Seeley & Co. The firm that published *Flatland* was owned by Richmond Seeley, the second son of Robert Benton Seeley and an elder brother of Abbott's friend and mentor, John R. Seeley. In 1857, Richmond Seeley took control of his father's share in the family business, which had been founded about 1784. Seeley continued the

FLATLAND

A Romance of Many Dimensions

With Illustrations

by the Author, A SQUARE

"*Fie, fie, how frantically I square my talk!*'

LONDON

SEELEY & Co., 46, 47 & 48, ESSEX STREET, STRAND

(*Late of* 54 FLEET STREET)

1884

Cover notes continued

traditions of the firm, which under his father's influence had acquired a strong tone of evangelical churchmanship.

In addition to *Flatland*, Seeley published ten of Abbott's books – *Bacon and Essex: A Sketch of Bacon's Earlier Life* and nine school books, including *English Lessons for English People* (with John R. Seeley) and *How to Write Clearly*. Abbott published these books at his own risk. He employed Richard Clay & Sons to produce the printed pages. Then Seeley & Co. bound, distributed, and advertised the books for a percentage of the revenue from sales. The income from the sales of his school books enabled him to retire in 1889 at the age of 51 (Abbott 1877e).

Price Half-a-crown. A half crown was 2½ shillings, or ⅛ of a pound. Although changes in relative values of goods make it impossible to translate this price into a modern value, a rough indication is given by Leone Levi's estimate that in 1884 the average daily income of an English working man was somewhat less than 3 shillings (Levi 1885, 2–4).

Title Page.

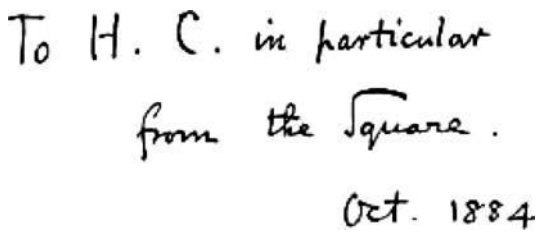
Title page epigraph. "Fie, fie, how franticly I square my talk" means "how madly I adjust my language." It is taken from Shakespeare's *Titus Andronicus* 3.2, where Titus responds angrily to his brother's urging him to moderate his language of grief and despair. This epigraph is a play on the verb "to square" and the "name" of *Flatland*'s narrator. It refers to the Square's struggle to make his narrative description consistent with the "reality of Flatland," and it alludes to Abbott's own efforts to write *Flatland* in "the language of the Square."

Abbott was a philologist in the literal sense, a lover of words, and he took great care in constructing the Square's language, which is not merely the ordinary English of the late nineteenth century. It includes archaisms from Elizabethan English, biblical diction, mathematical and geometrical vocabulary, and a number of words peculiar to the "idiom of Flatland." The prose occasionally borders on the poetic; alliteration and other rhetorical forms are common. There is a large element of wordplay, including several clever puns, which, like this epigraph, often call attention to a noteworthy aspect of the text.

To
The Inhabitants of SPACE IN GENERAL
And H. C. IN PARTICULAR
This Work is Dedicated
By a Humble Native of Flatland
In the Hope that
Even as he was Initiated into the Mysteries
Of THREE Dimensions
Having been previously conversant
With ONLY TWO
So the Citizens of that Celestial Region
May aspire yet higher and higher
To the Secrets of FOUR FIVE OR EVEN SIX Dimensions
Thereby contributing
To the Enlargement of THE IMAGINATION
And the possible Development
Of that most rare and excellent Gift of MODESTY
Among the Superior Races
Of SOLID HUMANITY

Dedication.

H. C. in particular. In *Apologia* (1907), Abbott explicitly identified his closest friend, Howard Candler, as "the 'H. C.' to whom *Flatland* was dedicated many years ago." When he inscribed the title page of a copy of *Flatland* for Candler, Abbott wrote,



That volume was given in 1969 to the library of Trinity College, Cambridge University, by Christopher Candler, a grandson of Howard Candler.

Initiated into the mysteries. Abbott uses the initiation rituals of the ancient Greek mystery cults as a figure for the Square's passage from intellectual darkness into light and his subsequent inability to describe his experience to others.

Imagination. For Abbott, imagination is the basis of all knowledge. In *The Kernel and the Husk*, he maintains that our knowledge of the external world and ourselves comes not from sensations as interpreted by reason but, at least to a large extent, from sensations as interpreted by imagination.

Modesty. In *The Spirit on the Waters*, Abbott says that an illustration set in geometric space may lead us to wider views of possible circumstances and existences, and thereby "develop in us modesty, respect for facts, a deeper reverence for order and harmony, and a mind more open to new observations and to fresh inferences from old truths" (Abbott 1897, 32–33).

PART 1

THIS WORLD

"Be patient, for the world is broad and wide."

Part I: THIS WORLD

"Be patient, for the world is broad and wide." (*Romeo and Juliet* 3.3) These words are spoken by Friar Laurence in an effort to comfort Romeo, who has been banished from Verona, by assuring him that the world outside Verona is spacious. Romeo responds:

> "There is no world without Verona walls,
> But purgatory, torture, hell itself.
> Hence – banished is banish'd from the world."

Romeo's insistence that nothing exists outside the world of his experience is a theme that is repeated in the text by inhabitants of Pointland, Lineland, Flatland, and Spaceland.

§1
Of the Nature of Flatland

I CALL OUR WORLD FLATLAND, not because we call it so, but to make its nature clearer to you, my happy readers, who are privileged to live in Space.

Imagine a vast sheet of paper on which straight Lines, Triangles, Squares, Pentagons, Hexagons, and other figures, instead of remaining fixed in their places, move freely about, on or in the surface, but without the power of rising above or sinking below it, very much like shadows – only hard and with luminous edges – and you will then have a pretty correct notion of my country and countrymen. Alas, a few years ago, I should have said "my universe": but now my mind has been opened to higher views of things.

In such a country, you will perceive at once that it is impossible that there should be anything of what you call a "solid" kind; but I dare say you will suppose that we could at least distinguish by sight the Triangles, Squares, and other figures, moving about as I have described them. On the contrary, we could see nothing of the kind, not at least so as to distinguish one figure from another. Nothing was visible, nor could be visible, to us, except straight Lines; and the necessity of this I will speedily demonstrate.

Place a penny on the middle of one of your tables in Space; and leaning over it, look down upon it. It will appear a circle.

But now, drawing back to the edge of the table, gradually lower your eye (thus bringing yourself more and more into the condition of the inhabitants of Flatland), and you will find

Notes on Section 1.

1.1. I call our world Flatland. The Square never reveals what his countrymen call their land; he has chosen the adjective "flat" for the sake of his readers from three-dimensional space. By "flat" he means without curvature, but he may also intend it to mean dull or monotonous. "Flatland" deviates from the naming convention for the spaces of other dimensions; the term analogous to Pointland, Lineland, and Spaceland is "Planeland." Several authors have illustrated the problem of determining the nature of space with a story of beings confined to a two-dimensional surface. The most fully developed of these stories is found in *The Shape of Space*, Jeffrey Weeks's beautifully written introduction to the basic geometry of two- and three-dimensional space. See also Dionys Burger's *Sphereland: A Fantasy about Curved Spaces and an Expanding Universe.*

1.2. happy. Lucky.

1.3. Space. Three-dimensional space. Abbott's peculiar use of capitals is irregular, but a few guiding principles are apparent. The names of geometric figures are capitalized whenever they represent Flatlanders. Dimension is always capitalized; "truth" is capitalized whenever it means the truth of the existence of higher-dimensional space. Occasionally, capitalization is used to express emphasis or personification, or to call attention to metaphor, but there are manifest inconsistencies. "Space" is capitalized in 51 of the 54 instances where it means one-, two-, or three-dimensional space; there is no obvious reason for the exceptions.

1.4. straight Lines. The Square consistently uses "straight line" for what is now called "line segment" and "line" for what is now called "curve," the trace of a moving point. His usage is consistent with Euclid's *Elements*, where a "straight line" is infinite only in the sense that it may be extended indefinitely.

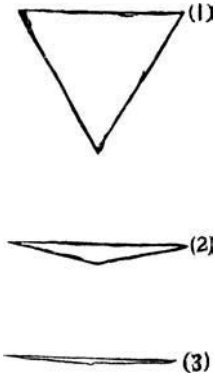
1.7. on or in the surface. For an illustration of the important distinction between "on a surface" and "in a surface," see Weeks's account of the "Universal Survey of all of Flatland" in which A Square and his fellow surveyors (who live in their two-dimensional universe) all return to Flatsburgh as their own mirror images. Such a reversal of orientation could not be achieved by a figure that remains on a surface (Weeks 2002, 3–9; 45–49; 65–69).

1.8. like shadows. Perhaps an allusion to Sophocles' *Ajax* ("Phantoms, all we that live, mere fleeting shadows.") or 1 Chronicles 29:15 ("Our days on the earth are as a shadow"). In any case, the representation of humans as two-dimensional figures symbolizes the insubstantiality of human existence.

1.9. luminous. The *Oxford English Dictionary* credits Charles Darwin with being the first person to use "luminous" to describe animals or plants that emit light. In Flatland, the identification of both animate and inanimate objects from their appearance depends upon the luminosity of their perimeters.

1.25. gradually lower your eye. A similar thought experiment appeared in an essay, "On space of four dimensions," which appeared in the 1 May 1873 issue of *Nature* (Rodwell 1873). This essay is a revision of a lecture that George F. Rodwell delivered at a meeting of the Natural History Society at Marlborough

the penny becoming more and more oval to your view; and at last when you have placed your eye exactly on the edge of the table (so that you are, as it were, actually a Flatlander) the penny will then have ceased to appear oval at all, and will have become, so far as you can see, a straight line.



The same thing would happen if you were to treat in the same way a Triangle, or Square, or any other figure cut out of pasteboard. As soon as you look at it with your eye on the edge on the table, you will find that it ceases to appear to you a figure, and that it becomes in appearance a straight line. Take for example an equilateral Triangle – who represents with us a Tradesman of the respectable class. Fig. 1 represents the Tradesman as you would see him while you were bending over him from above; figs. 2 and 3 represent the Tradesman, as you would see him if your eye were close to the level, or all but on the level of the table; and if your eye were quite on the level of the table (and that is how we see him in Flatland) you would see nothing but a straight line.

When I was in Spaceland I heard that your sailors have very similar experiences while they traverse your seas and discern some distant island or coast lying on the horizon. The far-off land may have bays, forelands, angles in and out to any number and extent; yet at a distance you see none of these (unless indeed your sun shines bright upon them revealing the projections and retirements by means of light and shade), nothing but a grey unbroken line upon the water.

Well, that is just what we see when one of our triangular or other acquaintances comes towards us in Flatland. As there is neither sun with us, nor any light of such a kind as to make shadows, we have none of the helps to the sight that you have in Spaceland. If our friend comes close to us we see

College, where he was the science master. That a prestigious journal like *Nature* published this essay, which is addressed to a general audience, indicates the extensive popular interest in the fourth dimension in the late nineteenth century.

1.27. oval. A penny viewed obliquely appears to be an elliptical disc, in popular language, an oval. An ellipse is a closed curve formed by the intersection of a right circular cone and a plane.

Figure 1.1. An old English penny "becoming more and more oval."

1.29. Flatlander. The first edition reads "Flatland citizen."

1.44. Figures. The crudely drawn illustrations in *Flatland*, described on the title page as "by the Author, A Square," are one of several means that Abbott employs to give his text an ancient character.

1.49. When I was in Spaceland. As we shall see in Part II, the Square has visited Spaceland, our three-dimensional space.

We noted in the introduction that there are a number of parallels between *Flatland* and Plato's parable of the cave. One significant difference between these stories is their points of view. Plato's story is told in the third person by Socrates; *Flatland* is Plato's parable told as the first-person account of a prisoner who has escaped the cave and returned.

1.49. your sailors. For Abbott's brother Edward, a sailor who died at sea, see Appendix B1, 1859.

1.60. nor any light of such a kind as to make shadows. In Flatland, there is a diffuse light that provides uniform illumination and so does not produce highlights or shadows, which would give visual clues.

1.61. If our friend. Many sentences in *Flatland* contain alliteration, but none contains more than the following specimen, which ends with an "else" to flag the parade of "l's":

> If our friend comes **c**loser to us we see his **l**ine becomes **l**arger; if he **l**eaves us it becomes sma**ll**er: but sti**ll** he **l**ooks **l**ike a straight **l**ine; be he a Triang**l**e, Square, Pentagon, Hexagon, Circ**l**e, what you wi**ll** – a straight **l**ine he **l**ooks and nothing **else**.

The repeated "l's" do not constitute alliteration in its current, most restrictive sense: the repetition of an initial consonant sound in two or more neighboring words or syllables. However, Abbott and Seeley's definition of alliteration includes what they call "concealed alliteration" – cases where the alliteration depends not upon the initial but upon the middle syllables of words or where the alliterative words are separated from one another. There is a good deal of this type of "alliteration" throughout *Flatland*, even though Abbott and Seeley