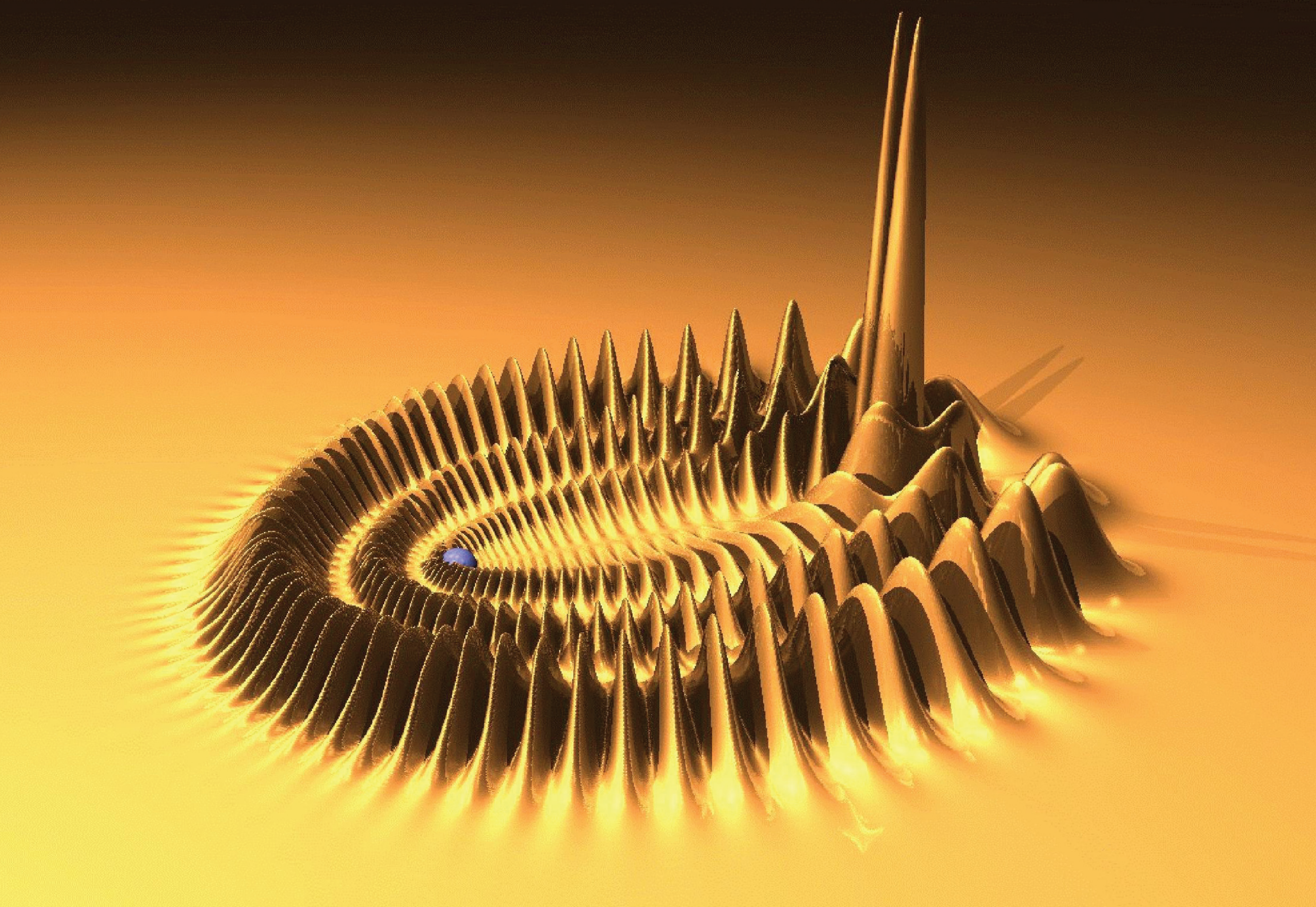
The New

Quantum Universe

Tony Hey and Patrick Walters



A fully revised and updated edition of the bestselling **The Quantum Universe**

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The New Quantum Universe

Quantum mechanics gives an understanding of not only atoms and nuclei, but also all the elements and even the stars. It makes possible the silicon chip and the myriad of laser applications, it explains the structure of Jupiter and provides an understanding of the mechanism of energy generation in our Sun and other stars. Quantum mechanics has given us great insights into the nature of the universe, promising an unlimited supply of energy from nuclear power and unlocking the awesome capability for self-destruction through nuclear weapons.

Following the success of *The Quantum Universe*, first published in 1987, a host of exciting new discoveries has been made in the field of quantum mechanics. *The New Quantum Universe* provides an accessible introduction to the essential ideas of quantum physics, and demonstrates how it affects our everyday life. Quantum paradoxes and the eventful life of Schrödinger's cat are explained, along with the Einstein–Podolsky–Rosen paradox and the Bell inequality. The book looks ahead to the coming nanotechnology revolution, describing quantum cryptography, quantum computing and quantum teleportation, and ends with an account of quantum mechanics and science fiction.

Explaining quantum mechanics in a simple non-mathematical way, this book is a fascinating and accessible introduction to one of the most important scientific disciplines of the twenty-first century. It is suitable for final-year school students, science undergraduates, and anyone wishing to appreciate how physics has made possible the new technologies that are changing our lives.

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Preface

The popularization of science is now an established business for booksellers and publishers. The traditional formula for a 'trade' book on popular science is a text of about 100 000 words or so (around 200 pages) with relatively few diagrams or pictures. The target audience is the educated reader with a general interest in science. At the other end of the scale we have popular science reference books such as encyclopedias and atlases. Our target audience lies in between these two extremes. We wish to write a book that will not only interest the 'educated reader' as above but, more importantly, capture the interest and imagination of young people. We believe that it is vitally important to give young people a glimpse of the excitement of physics so that they may be motivated to take up the challenge themselves. Nowadays, there are many more alternatives for young people and there is a general perception that science and mathematics are 'hard' subjects. It is certainly true that understanding in these subjects does not come without effort and real mastery may indeed take years. So we cannot promise instant gratification. What we can promise is that the study of science and mathematics will provide a gateway to a deeper understanding of a fascinating universe – our universe, a quantum universe. And paradoxically, as our world becomes more and more dependent on science and technology, it has also become increasingly technologically fragile in that fewer people understand the technology on which we all depend. Civilization requires us to inspire and motivate young people to take up the challenge of science. This is the true target audience for this book. But we hope that our text and our extensive use of diagrams, colour photographs and biographies of great scientists will also be interesting and entertaining to the 'educated reader'!

Our first book on quantum mechanics, *The Quantum Universe*, was published in 1987. At the time, it seemed to us that there was a clear need to communicate the strange ideas of quantum mechanics to a wider audience, since this is the theory that underpins the operation of many 'high tech' objects in daily use. So, after a look at the fundamentals, we concentrated on explaining how quantum mechanics gives us an understanding of not only atoms and nuclei, but also all the elements and even the stars. Quantum mechanics makes possible the incredible silicon chip as well as all the myriad of applications of lasers that we see today. It explains not only the structure of Jupiter but also provides us with an understanding of the mechanism of energy generation in our Sun and other stars. Because of the bizarreness of quantum theory at a fundamental level, we deliberately avoided all philosophical issues and followed Richard Feynman's advice in adopting a very pragmatic stance. We therefore concerned ourselves with demonstrating that the theory, no matter how strange it may seem, clearly works in practice. Since quantum theory was developed in the 1920s by Niels Bohr, Erwin Schroedinger, Werner Heisenberg, Paul Dirac and others, it seemed that, apart from more applications, there was little new to be discovered.

To our surprise, the last fifteen years have been years of great advances in quantum technology. Although no new results have arisen to challenge the supremacy of the underlying quantum theory, there have been many exciting new discoveries. In the main, these developments all demonstrate our increasing control of quantum systems. So much so that we believe that we are seeing the emergence of a new field of scientific endeavour – 'quantum engineering'. This term signifies our belief that this new century will see our increasing mastery over manipulating matter at the quantum level leading to new and spectacular applications of such 'nanotechnology'. There will certainly be significant implications for the semiconductor industry. We will see the end of 'Moore's Law' – the prediction that the number of transistors on computer chips, and hence their computational speed and memory capacity, doubles every eighteen months. In ten years or so, the dimensions of features on a silicon chip will have shrunk to such a size that the properties of individual atoms and electrons will play a determining role. Such quantum objects do not behave in a classically describable way. Unless quantum engineers are able to come up with some competitive new technology, Moore's Law will come to an end – along with the necessity to upgrade PCs every 18 months! One possible new technology on the horizon is 'quantum computing'. Instead of bits of information restricted to be either '1' or '0', as in present-day 'classical' computers, a quantum computer would allow the possibility of algorithms using quantum bits – 'qubits' – that are somehow simultaneously '1' and '0'! This observation has led to the development of a whole new area of research – 'quantum information theory' – and there are already possibilities of its practical application in cryptography. Although we retain our original approach to quantum mechanics in this book, the chapters on quantum applications have been extensively re-written and updated. In addition, a new chapter on 'quantum engineering' introduces the ideas and technologies of nanotechnology and quantum information.

As we have said, in our earlier book on quantum mechanics, we followed Feynman and avoided asking the question 'But how can it be like that?'. However, the last fifteen years have seen an upsurge of interest in understanding what quantum mechanics implies about the physical reality of the world in which we live. We have therefore included a chapter on 'quantum paradoxes' in which we introduce the reader to the unfinished debate between Niels Bohr and Albert Einstein. It was Bohr who formulated the orthodox 'Copenhagen' view of quantum mechanics and who was its most robust defender. According to Bohr's interpretation, uncertainty and unpredictability are intrinsic features of quantum theory, and the actual physical reality of quantum objects is debatable. Against such orthodoxy, Albert Einstein, Bohr's long-term friend and colleague, fought for the remainder of his life. He summed up his opposition to the Copenhagen interpretation in the memorable phrase 'God does not play dice!' After a lengthy but ultimately inconclusive debate, Einstein died still a non-believer in quantum theory. Soon after his death, the Irish physicist John Bell came up with a way to distinguish between the orthodox quantum mechanics of Bohr and the deterministic approach favoured by Einstein. Experiments to test 'Bell's Inequality' have now come down in favour of quantum mechanics and Einstein would have to think again! Bell's result is of such importance for quantum mechanics that we include an intuitive explanation of the Bell Inequality. Our presentation closely follows one given by John Bell himself in a meeting in Geneva. The other essential creature in any discussion of the interpretation of quantum mechanics is Schrödinger's Cat. The paradox of the cat graphically illustrates the so-called 'measurement problem' in quantum mechanics. We discuss how this problem is resolved – to a greater or lesser degree – by the ever popular 'Many Worlds' interpretation of quantum mechanics of Hugh Everett or by the 'Decoherence' mechanism favoured by Wojtek Zurek and others.

Finally, as a light-hearted 'afterword', we look at the treatment of quantum mechanics in Science Fiction. H.G. Wells led the way with his account of an atomic-bomb-induced Armageddon in his book *The World Set Free*. In the early years of quantum mechanics, SF writers struggled to incorporate the new understanding of the atom into a fictional context. Modern SF has now moved on to include multiple universes and nanotechnology as part of its standard technology base. Finally, in Michael Crichton's recent book, *Timeline*, quantum computers, teleportation and time travel are woven together to create yet another new dimension for Science Fiction to explore.

The distinguished theoretical physicist and author Paul Davies has made the following prediction:

> The nineteenth century was known as the machine age, the twentieth century will go down in history as the information age. I believe the twenty-first century will be the quantum age.

In the course of the next decades we will see how far this vision will be realized. Certainly, we believe that the influence on our society of this coming nanotechnology revolution, underpinned by quantum mechanics, will be at least as substantial as the fall-out from the present bio-informatics explosion. We hope that this book will assist in stimulating the imagination of a new generation of quantum engineers.

Some acknowledgements are in order. Once again we wish to thank our families for their invaluable support and forbearance – Marie Walters, and Jessie, Nancy, Jonathan and Christopher Hey. We are also grateful to colleagues who have read and commented on draft chapters, especially Phil Charles, Malcolm Coe, Jeff Mandula and Steve King. In Southampton, we

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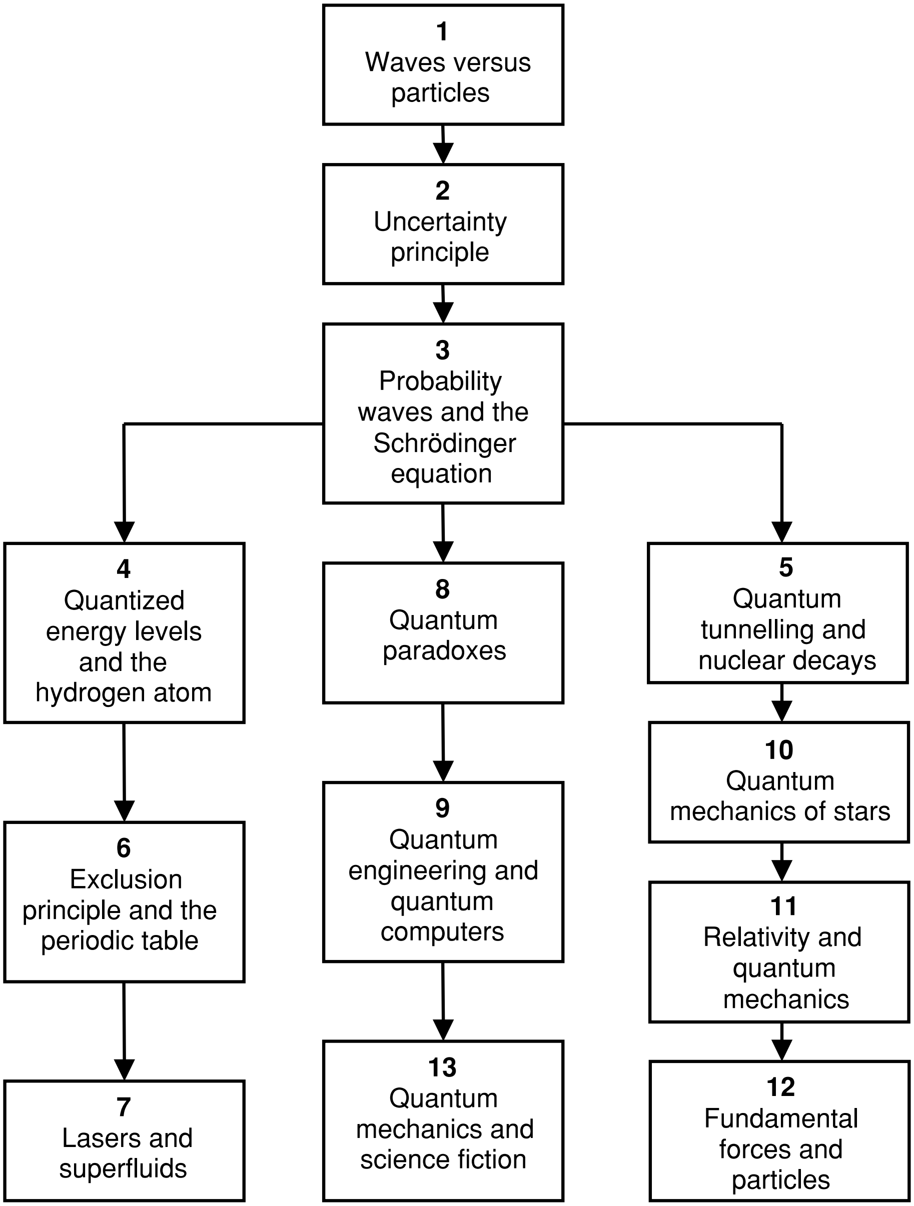
Prologue

Poets say science takes away from the beauty of the stars – mere globs of gas atoms. Nothing is 'mere'. I too can see the stars on a desert night, and feel them. But do I see less or more? The vastness of the heavens stretches my imagination – stuck on this carousel, my little eye can catch one-million-year- old light . . . Or see them [the stars] with the greater eye of Palomar, rushing all apart from some common starting point when they were perhaps all together. What is the pattern, or the meaning, or the why? It does not do harm to the mystery to know a little about it. For far more marvellous is the truth than any artists of the past imagined! Why do the poets of the present not speak of it?

Finally, may I add that the main purpose of my teaching has not been to prepare you for some examination – it was not even to prepare you to serve industry or the military. I wanted most to give you some appreciation of the wonderful world and the physicist's way of looking at it, which, I believe, is a major part of the true culture of modern times. (There are probably professors of other subjects who would object, but I believe they are completely wrong.) Perhaps you will not only have some appreciation of this culture; it is even possible that you may want to join in the greatest adventure that the human mind has ever begun.

Richard Feynman

Route map



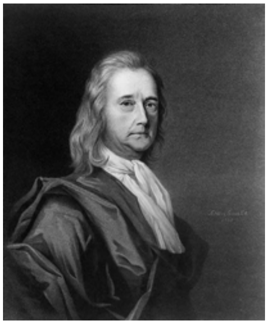
The three major strands of interconnected topics through the book. Broadly speaking, the left-hand strand is concerned with the quantum mechanics of the solid state; the right-hand strand focuses on the quantum mechanics of stars and elementary particles; and the middle strand explores quantum paradoxes and quantum engineering, before looking at the fictional realization of these ideas.

1 Waves versus particles

> . . . I think I can safely say that nobody understands quantum mechanics.
> Richard Feynman

Science and experiment

Science is a special kind of explanation of the things we see around us. It starts with a problem and curiosity. Something strikes the scientist as odd. It doesn't fit in with the usual explanation. Maybe harder thinking or more careful observation will resolve the problem. If it remains a puzzle, it stimulates the scientist's imagination. Perhaps a completely new way of looking at things is needed? Scientists are perpetually trying to find better explanations – better in the sense that any new explanation must not only explain the new puzzle, but also be consistent with all of the previous explanations that still work well. The hallmark of any scientific explanation or 'theory' is that it must be able to make successful predictions. In other words, any decent theory must be able to say what will happen in any given set of circumstances. Thus, any new theory will only become generally accepted by the scientific community if it is able not only to explain the observations that scientists have already made, but also to foretell the results of new, as yet unperformed, experiments. This rigorous testing of new scientific ideas is the key feature that distinguishes science from other fields of intellectual endeavour – such as history or even economics – or from a pseudoscience such as astrology.



Isaac Newton (1642–1727) published his book *Optics* in 1704 that explained the rainbow and put forward the 'corpuscular' theory of light. In his 1687 book *Mathematical Principles of Natural Philosophy* Newton set down the principles of mechanics and gravity that guided science until the mid nineteenth century.

In the seventeenth century Isaac Newton and several other great scientists developed a wonderfully successful explanation of the way things move. This whole theoretical framework is called 'classical mechanics', and its scope encompasses the motion of everything from billiard balls to planets. Newton's explanation of motion in terms of forces, momentum and acceleration is encapsulated in his 'laws of motion'. These principles are incorporated into so many of our machines and toys that classical mechanics is familiar from our everyday experience. We all know what to expect in

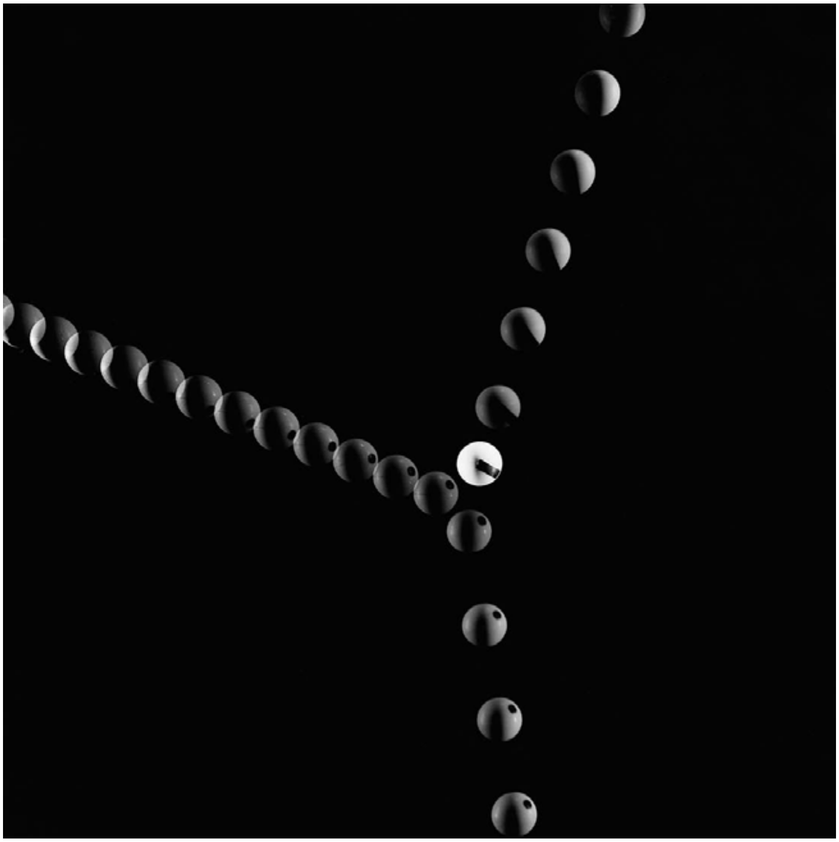


Fig. 1.1 A multi-flash photograph of a billiard ball collision. The motions of the balls can be calculated using Newton's laws but we have a good feel for what will happen from watching snooker on television or playing ourselves.

the collision of two billiard balls. Perhaps the most spectacular application of classical mechanics is in the exploration of space. Nowadays, it surprises no one that the astronaut and the space shuttle float side by side and neither falls dramatically to Earth. A hundred years ago it was not so 'obvious', and in Jules Verne's famous story *A Trip Around The Moon* the passengers of the spacecraft were amazed to find the body of a dog that died on takeoff, and which they had jettisoned outside the craft, floating side by side with them all the way to the Moon. Today, you may not know how Newton's theory works in detail but you can see that it works. It is part of our daily experience.

All this brings us to the problem most of us have in coming to terms with 'quantum mechanics'. It is just this. At the very small distances involved in the study of atoms and molecules, things do *not* behave in a familiar way. Classical mechanics is inadequate and an entirely new explanation is needed. Quantum mechanics is that new explanation, and it is cunningly constructed so that it not only works in the quantum realm of very-short length scales, but also so that, for larger distances, its predictions are identical with those of Newton. An atom is a typical quantum thing – it cannot be understood from the standpoint of classical physics. One popular

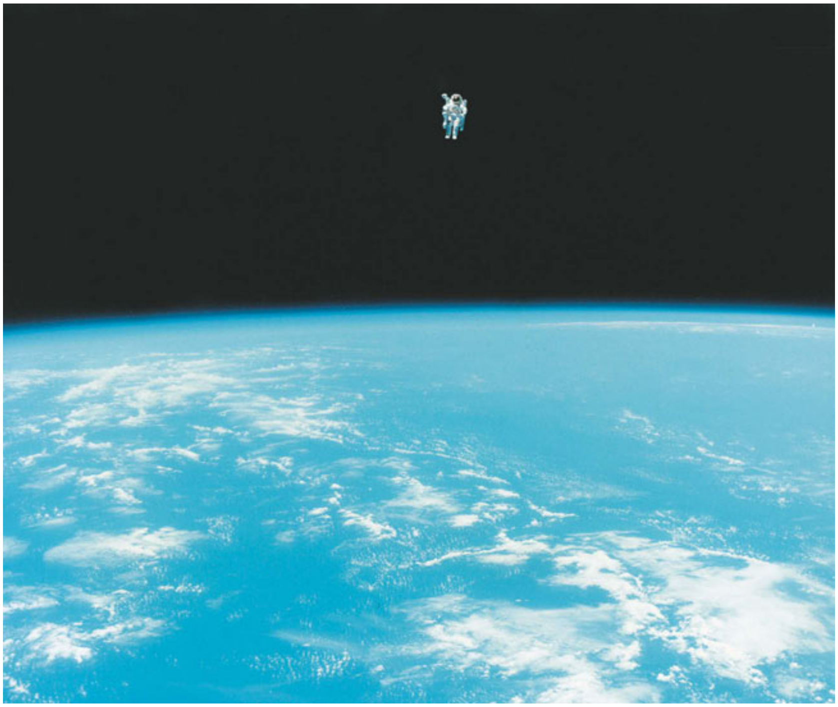


Fig. 1.2 Astronaut Bruce McCandless floats in space during the first untethered space walk on February 7th, 1984. The astronaut is essentially an independent spacecraft in orbit near the shuttle. McCandless commented 'Well that may have been one small step for Neil [Armstrong] but it's a heck of a big leap for me!'

visualization of an atom imagines electrons orbiting the nucleus of the atom much in the way planets orbit the Sun in the solar system. In fact, for negatively charged electrons in orbit round a positively charged nucleus, this simple model is unstable! According to classical physics the electrons would spiral into the centre and the atom would collapse. This nice and comforting model of the atom cannot account for even the existence of real atoms, let alone predict their expected behaviour. It is important to be aware at the outset that there is *no* simple picture that can accurately describe the behaviour of electrons in atoms. This is the first hurdle faced by the newcomer to the quantum domain: the inescapable and unpalatable fact that the behaviour of quantum objects is totally unlike anything you have ever seen.

How can we convince you that quantum mechanics is both necessary and useful? Well, a physicist, just like a good detective, sifts through the evidence and remembers the old maxim of Sherlock Holmes that 'when you have excluded the impossible, whatever remains, however improbable, must be the truth'. Nonetheless, it was only with much reluctance that twentieth-century physicists became convinced that the whole magnificent edifice of classical physics was not 'almost right' for describing the behaviour of atoms, but had, instead, to be radically rebuilt. Nowhere, was the confusion generated by this painful realization more evident than in their attempts to understand the nature of light.

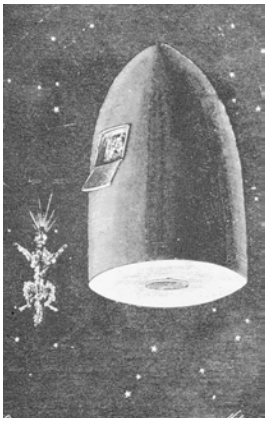


Fig. 1.3 In the story by Jules Verne *A Trip Around The Moon*, published in 1865, the dog 'Satellite' died on take-off and was jettisoned from the space-ship. Much to the surprise of the occupants the dog's body floated along with them all the way to the Moon!

Fig. 1.4 The interference pattern produced by two vibrating sources in water.

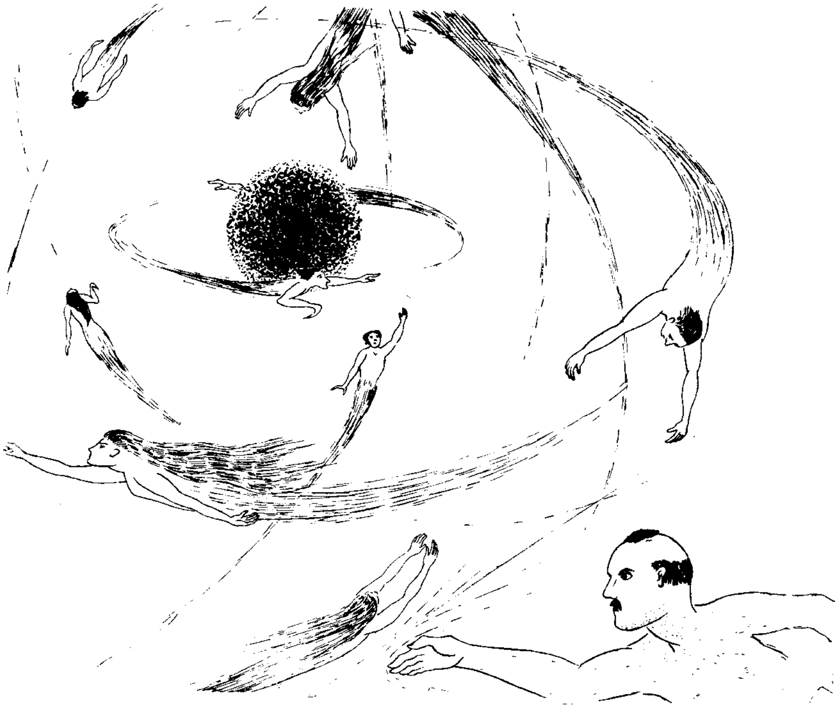


Fig. 1.5 George Gamow's rather whimsical view of the planetary model of the atom in *Mr Tompkins Explores The Atom.*

Thomas Young (1773–1829) was an infant prodigy who could read at the age of two. During his youth he learnt to speak a dozen languages. He is best remembered for his work on vision and for establishing the wave theory of light. However, he was also the first to make progress on deciphering the hieroglyphic script of the ancient Egyptians.

Light and quantum mechanics

In the seventeenth century, Isaac Newton suggested that light should be regarded as a stream of particles, rather like bullets from a machine gun. Such was Newton's reputation that this view persisted, apart from some isolated pockets of opposition, until the nineteenth century. It was then that Thomas Young and others conclusively showed that the particle picture of light must be wrong. Instead, they favoured the idea that light was a kind of wave motion. One property of waves that is familiar to us is that of 'interference', to use the physicists' term for what happens when two waves collide. For example, in Fig. 1.4 we show the 'interference' patterns produced by two sources of water waves on the surface of the water. Using his famous 'double-slit' apparatus to make two sources of light, Thomas Young had observed similar interference patterns using light.

Alas, physicists were not able to congratulate themselves for long. Experiments at the end of the nineteenth century revealed effects that were inexplicable by a wave theory of light. The most famous of such experiments concerns the so-called 'photo-electric' effect. Ultraviolet light shone onto a negatively charged metal caused it to lose its charge, while shining visible light on the metal had no effect. This puzzle was first explained by Albert Einstein in the same year that he invented the 'theory of relativity' for which he later became famous. His explanation of the photo-electric effect resurrected the particle view of light. The discharging of the metal was caused by electrons being knocked out of the metal by light energy concentrated into individual little 'bundles' of energy, which we now call 'photons'. According to Einstein's theory, ultraviolet photons have more energy than visible-light ones, and so no matter how much visible light you shine on the metal, none of the photons has enough energy to kick out an electron.

After several decades of confusion in physics, a way out of this dilemma was found in the 1920s with the emergence of quantum mechanics, pioneered by physicists such as Heisenberg, Schrödinger and Dirac. This theory is able to provide a successful explanation of the paradoxical nature of light, atoms and much else besides. But there is a price to pay for this success. We must abandon all hope of being able to describe the motion of things at atomic scales in terms of everyday concepts like waves or particles. A 'photon' does not behave like anything anyone has ever seen. This does not, however, mean that quantum mechanics is full of vague ideas and lacks predictive power. On the contrary, quantum mechanics is the only theory capable of making definite and successful predictions for systems of atomic sizes or smaller, in much the same way that classical mechanics makes predictions for the behaviour of billiard balls, rockets and planets. The difficulty with quantum things such as the photon is that, unlike billiard balls, their motion cannot be visualized in any accurate pictorial way. All we can do is summarize our lack of a picture by saying that a photon behaves in an essentially quantum mechanical way.

J. J. Thomson (1856–1940) measured the charge-to-mass ratio of the electron thus establishing it as a new elementary particle. He was awarded the Nobel Prize in 1906.

There is one sense in which Nature has been kind to us. Viewed from the perspective of classical physics, photons and electrons are very different kinds of objects. Remarkably, in the quantum domain both photons and electrons, and indeed all quantum objects, behave in the same strange quantum mechanical way. This is at least some compensation for our inability to picture quantum things! There is a curious little irony in the history of our attempts to understand the nature of electrons. In 1897 J. J. Thomson measured the charge-to-mass ratio of the electron and established the electron as a new elementary particle of Nature. Thirty years later, his son, G. P. Thomson, and also Davisson and Germer in the USA, performed a beautiful series of experiments that conclusively revealed that electrons behave like waves. The historian Max Jammer wrote: ‘One may feel inclined to say that Thomson, the father, was awarded the Nobel Prize for having shown that the electron is a particle, and Thomson, the son, for having shown that the electron is a wave’.

Our intention in this book is to impress even the most skeptical reader with the enormous range and diversity of the successful predictions of quantum mechanics. The apparently absurd ideas of de Broglie, Schrödinger and Heisenberg have now led to whole new technologies whose very existence depends on the discoveries of these pioneers of quantum mechanics. The modern electronics industry, with its silicon chip technology, is all based on the quantum theory of materials called semiconductors. Likewise, all the multitude of applications of lasers are possible only because of our understanding, at the fundamental quantum level, of a mechanism for radiation of light from atoms first identified by Einstein in 1916. Moreover, understanding how large numbers of quantum objects behave when packed tightly together leads to an understanding of all the different types of matter ranging from ‘superconductors’ to ‘neutron stars’. In addition, although originally invented to solve fundamental problems concerned with the existence of atoms, quantum mechanics was found to apply with equal success to the tiny nucleus at the heart of the atom, and this has led to an understanding of radioactivity and nuclear reactions. As everyone knows, this has been a mixed blessing. Not only do we now know what makes the stars shine, but we also know how to destroy all of civilization with the awesome power of nuclear weapons.

Before we can explain how quantum mechanics made all these things possible, we must first attempt to describe the strange quantum mechanical behaviour of objects at atomic distance scales. This task is clearly difficult given the absence of any accurate analogy for the mathematical description of quantum behaviour. However, we can make progress if we use a mixture of analogy and contrast. Young’s original ‘double-slit’ experiment used a screen with two slits in it to make two sources of light which could interfere and produce his famous ‘interference fringes’ – alternating light and dark lines (Fig. 1.6). We shall describe the results of similar ‘double-slit’ experiments carried out using bullets, water waves and electrons. By

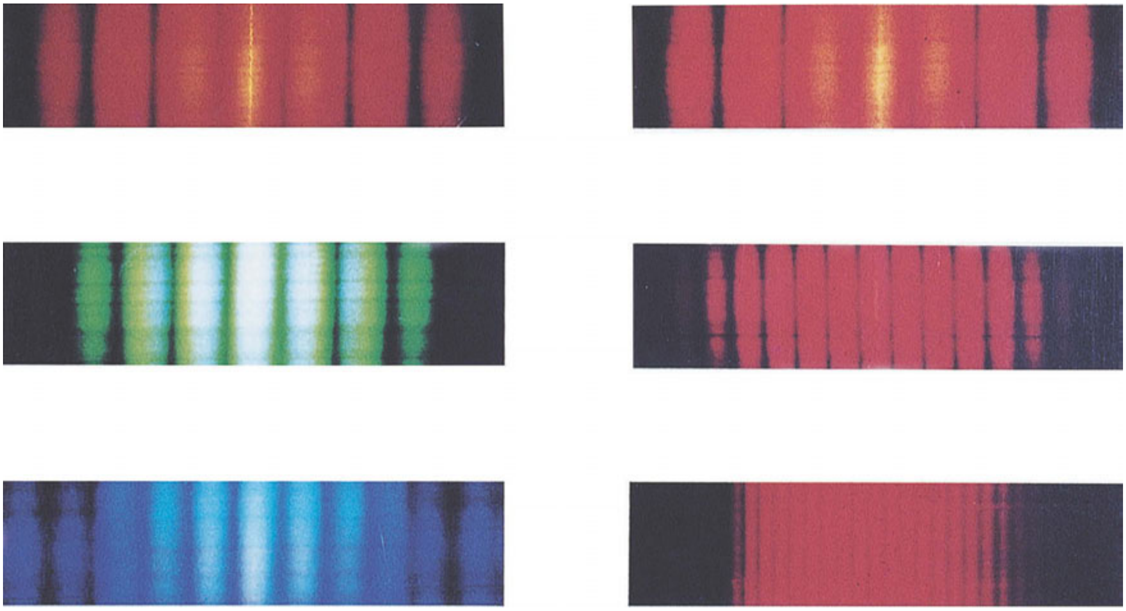


Fig. 1.6 Double-slit interference patterns for light, usually taken as demonstrating that light is a wave motion. In the left-hand pictures, as the wavelength of the light is decreased and the colour changes from red to blue, the interference fringes become closer together. On the right, for red light, the decrease in the fringe separation is caused by increasing the separation of the slits.

comparing and contrasting the results obtained with the three different materials we shall be able to give you some idea of the essential features of quantum mechanical behaviour. Quantum mechanics textbooks contain detailed discussion of many types of experiment, but this double-slit experiment is sufficient to reveal all the mystery of quantum mechanics. All of the problems and paradoxes of quantum physics can be demonstrated in this single experiment.

A word of warning before we begin. To avoid running into a frustrating psychological cul-de-sac, try to be content with mere acceptance of the observed experimental facts. Try not to ask the question ‘but how can it be like that?’ As Richard Feynman says ‘nobody understands quantum mechanics’. All we can give you is an account of the way Nature appears to work. Nobody knows more than that. Only **after** we have convinced you that quantum mechanics really works will we examine what quantum mechanics has to say about the very nature of reality, with a discussion about Schrödinger’s cat, Einstein and dice.

The double-slit experiment

This section may be rather hard going first time through. If so, just glance at the pictures and pass on quickly to the next chapter!

With bullets

Source: a wobbly machine gun that, as it fires, spreads the bullets out into a cone, all with the same speed but with random directions.

Screen: armour plate with two parallel slits in it.

Detector: small boxes of sand to collect the bullets.

Results: the gun fires at a fixed rate and we can count the number of bullets that arrive in any given box in a given period of time. The bullets that go through the slits can either go straight through or else bounce off one of the edges, but must always end up in one of the boxes. The bullets we are using are made of a tough enough metal so that they never break up – we can never have half a bullet in a box. Moreover, no two bullets ever arrive at the same time – we have only one gun, and each bullet is a single identifiable 'lump'.

If we let the experiment run for an hour and then count the bullets in each of the boxes, we can see how the 'probability of arrival' of a bullet varies with the position of the detector box. The total number of bullets arriving at any given position is clearly the sum of the number of bullets going through slit 1 plus the number going through slit 2. How this 'probability of arrival' varies with position of the sand boxes is shown in Fig. 1.7. We shall label this result P_{12} – the probability of arrival of bullets when both slits are open. We also show in Fig. 1.7 the results obtained with slit 2 closed, which we call P_1, and those obtained with slit 1 closed, which we call P_2. Looking at the figures, it is evident that the curve labelled P_{12} is obtained by adding curves P_1 and P_2. We can write this mathematically as the equation:

$$P_{12} = P_1 + P_2$$

For reasons that will become apparent in a moment, we call this result the case of *no interference*.

With water waves

Source: a stone dropped into a large pool of water.

Screen: a jetty with two gaps in it.

Detector: a line of small floating buoys whose jiggling up and down gives a measure of the amount of energy of the wave at that position.

Results: Ripples spread out from the source and reach the jetty. On the far side of the jetty ripples spread out from each of the gaps. At the detector, the resulting disturbance of the water is given by the sum of the disturbances of the ripples coming from both gaps. As we look along the line of buoys, there will be some places where the crest of a wave from slit 1 coincides with the arrival of a crest from slit 2, resulting in

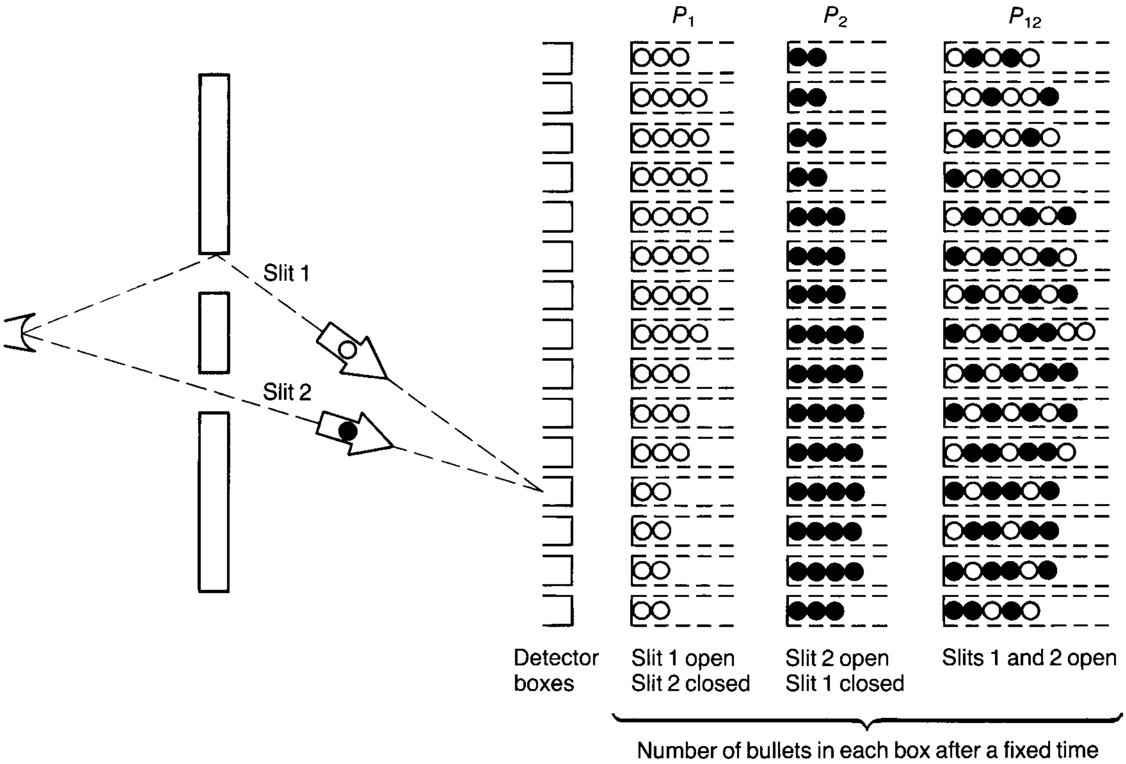


Fig. 1.7 A double-slit experiment with bullets. The experimental set-up is shown on the left of the figure and the results of three different experiments indicated on the right. We have shown bullets that pass through slit 1 as open circles and bullets through slit 2 as black circles. The column labelled P_1 shows the distribution of bullets arriving at the detector boxes when slit 2 is closed and only slit 1 is open. Column P_2 shows a similar distribution obtained with slit 1 closed and slit 2 open. As can be seen, the maximum number of bullets appears in the boxes directly in line with the slit that is left open. The result obtained with both slits open is shown in the column labelled P_{12}. It is now a matter of chance through which slit a bullet will come and this is shown by the scrambled mixture of black and white bullets collected in each box. The important point to notice is that the total obtained in each box when both slits are open is just the sum of the numbers obtained when only one or other of the slits is open. This is obvious in the case of bullets since we know that bullets must pass through one of the slits to reach the detector boxes.

a very large up-and-down motion for the buoy. At other places, a crest from one slit will coincide with a trough from the other so there will be no movement of the buoy at that position. At yet other places, the motion of the buoys will be somewhere between these two extremes. For water waves, it is certainly plausible that the energy of a wave at any given position is related to how big the waves are at that point. In fact, it can be shown that the energy of a wave depends on the square of the maximum height of the wave. Let us call the amount of energy arriving per second the 'intensity' and label this by the symbol I. If we label the

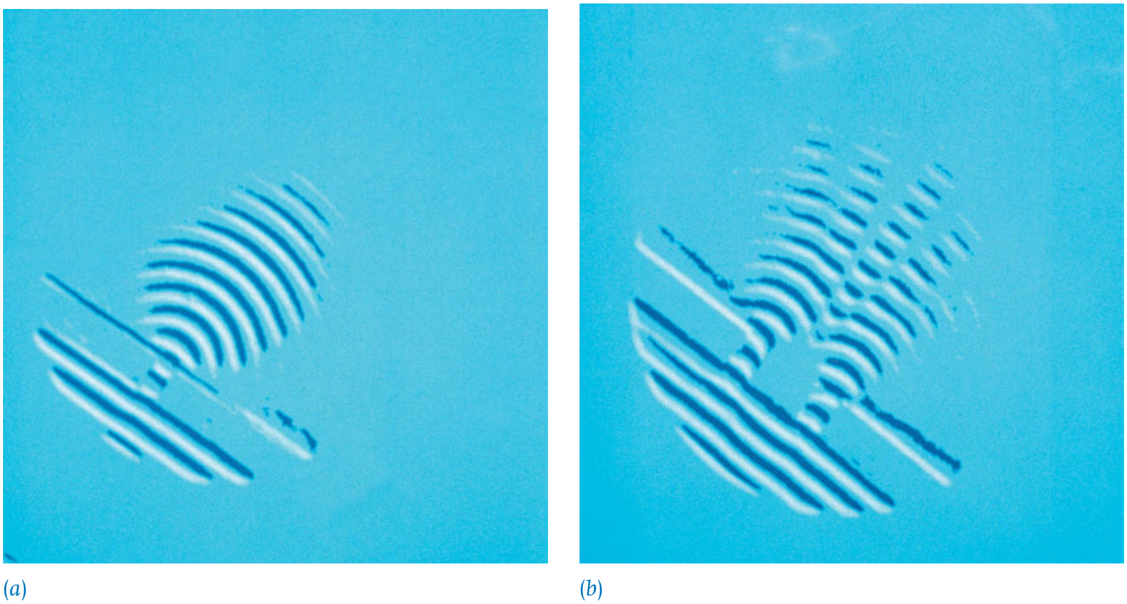


Fig. 1.8 Wave patterns with water waves. (*a*) A wave spreading out from a single slit; (*b*) the interference obtained with two slits.

maximum height of the wave by h, we can write the relation between I and h as the following equation:

$$I = h^2$$

$$\text{intensity} = \text{height squared}$$

In contrast to our experiment with bullets, we see that the energy of the waves does not arrive at the detector in definite-sized lumps. There, bullets only arrived at one particular position at one particular time. Here, since the height of the resulting wave at the detector varies smoothly from zero up to some maximum value as we move along the detector, we see that the energy of the original wave is spread out. The curve showing how the intensity varies with position along the detector is shown in Fig. 1.9. Since this is the intensity obtained with both slits open, we shall call this curve I_{12}. This intensity pattern has a very simple mathematical explanation. The total disturbance of the water at any position along the detector is given by the sum of the disturbances caused by the waves from slit 1 and slit 2. If we label the height of the wave from slit 1 by h_1, the height from slit 2 by h_2, and the total height obtained when both slits are open as h_{12},we can write this result as the equation:

$$h_{12} = h_1 + h_2$$

Remember that each of these heights can be positive or negative depending on whether the corresponding wave disturbance raises or lowers the

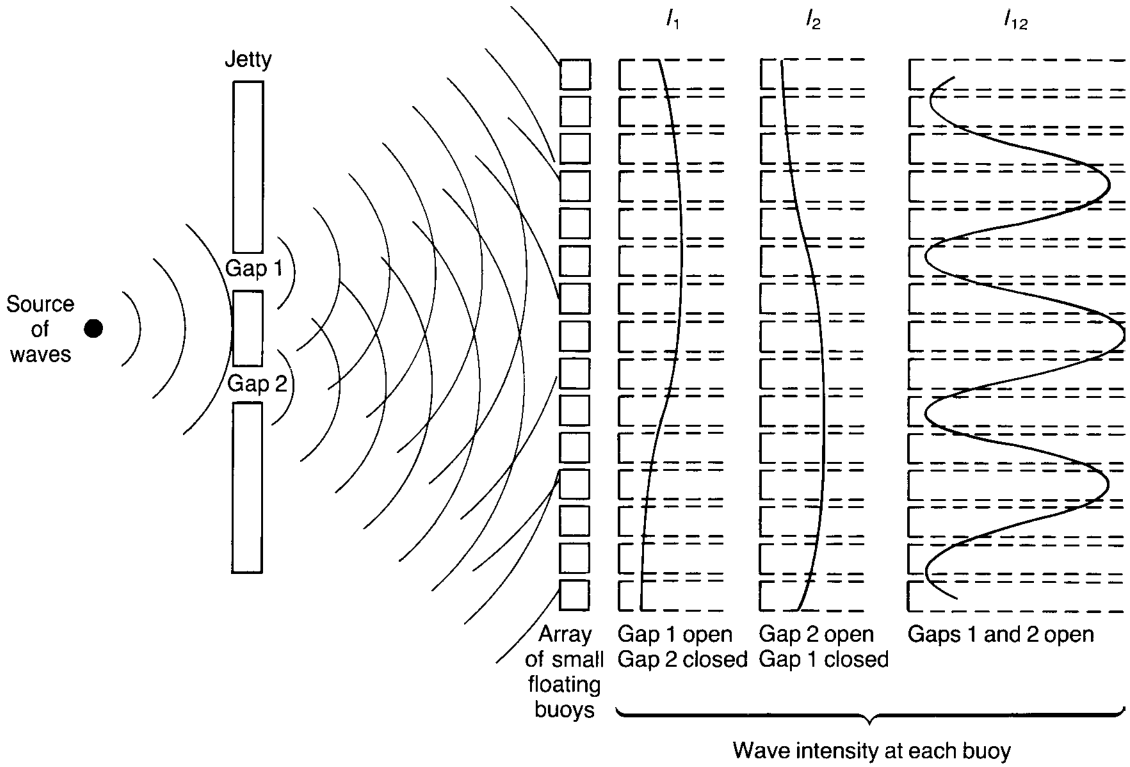


Fig. 1.9 A double-slit experiment with water waves. The detectors are a line of small floating buoys whose jiggling up and down provides a measure of the wave energy. The wave crests spreading out from each slit are shown in the figure and can be compared with Fig. 1.8. The column labelled I_1 shows the smoothly varying wave intensity obtained when only gap 1 is open. Notice that this is very similar to the pattern P_1 obtained with bullets in Fig. 1.7 with only slit 1 open. Again, it is largest at the detector directly in line with gap 1 and the source. The second column shows that a similar pattern, I_2, is obtained when gap 1 is closed and gap 2 is open. The final column, I_{12}, shows the wave intensity pattern obtained with both slits open. It is dramatically different from the pattern obtained for bullets with both slits open. It is not equal to the sum of the patterns I_1 and I_2 obtained with one of the gaps closed. This rapidly varying intensity curve is called an interference pattern.

water level. The resulting intensity is the square of this height or 'wave amplitude'

$$I_{12} = h_{12}^2$$

so that

$$I_{12} = (h_1 + h_2)^2$$

We could now repeat the experiment with one of the gaps closed. In this case we find the results shown in Fig. 1.9. We label the corresponding intensity pattern I_1, since it is the intensity obtained with slit 1 open and slit 2 closed. The curve I_1 is just given by the square of the disturbance caused by the wave from slit 1

$$I_1 = h_1^2$$

Similarly, the curve I_2 is the result obtained with slit 2 open and slit 1 closed, and, in the same way as before, we have the result

$$I_2 = h_2^2$$

It is clear that these two curves are much less wiggly than the pattern I_{12}. Furthermore, the pattern I_{12}, for both slits open, cannot be obtained just by adding up the two intensity patterns, I_1 and I_2, that are each obtained with one of the slits closed. Mathematically, we can see this from our equations as follows:

$$\begin{aligned} I_{12} &= (h_1 + h_2)^2 \\ &= (h_1 + h_2) \times (h_1 + h_2) \end{aligned}$$

This may be expanded to read

$$I_{12} = h_1^2 + 2h_1h_2 + h_2^2$$

which is clearly **not** equal to the sum of I_1 and I_2

$$I_1 + I_2 = h_1^2 + h_2^2$$

For wave motion, we say there is *interference.* Unlike the case with bullets, you do not obtain the pattern for 'both slits open' by adding the patterns for 'one slit closed'. It was the observation of such interference patterns for light that convinced Thomas Young that light must be a wave motion. In fact, life is not so simple! We will now describe the results of the double-slit experiment performed with electrons, but similar results would be obtained if the experiment were repeated with light.

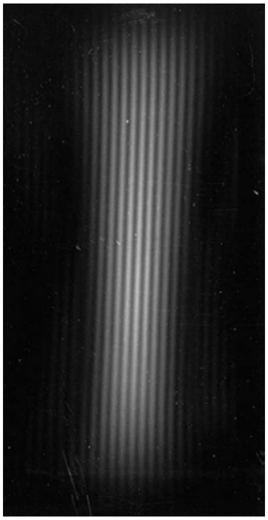


Fig. 1.10 An experimental observation of the two-slit interference pattern obtained with electrons.

With electrons

Source: an electron 'gun', consisting of a heated wire to 'boil off' electrons from the metal, together with an electric potential to accelerate them.

Screen: a thin metal plate with two very narrow slits in it.

Detector: a screen coated with a chemical 'phosphor' that produces a flash of light every time an electron arrives at it.

Results: Flashes of light signal the arrival of electrons at the detector. Electrons arrive singly, in individual 'lumps' of the same size, only at a single place at any one time – just as in the case with bullets. If we turn down the intensity of the electron gun, and thus boil off fewer electrons per minute, we still see the same size flashes at the detector but with fewer electrons arriving per minute. Again, exactly as for bullets, we can count up the number of flashes we see at any given position of the detector during a given interval of time. As for bullets, this allows us to measure how the probability of arrival of electrons varies as we move along the detector. The magic of quantum mechanics will now be revealed! The pattern we see (shown in Fig. 1.11) is the interference pattern characteristic of waves, although, as we have said, the electrons always arrive like bullets!

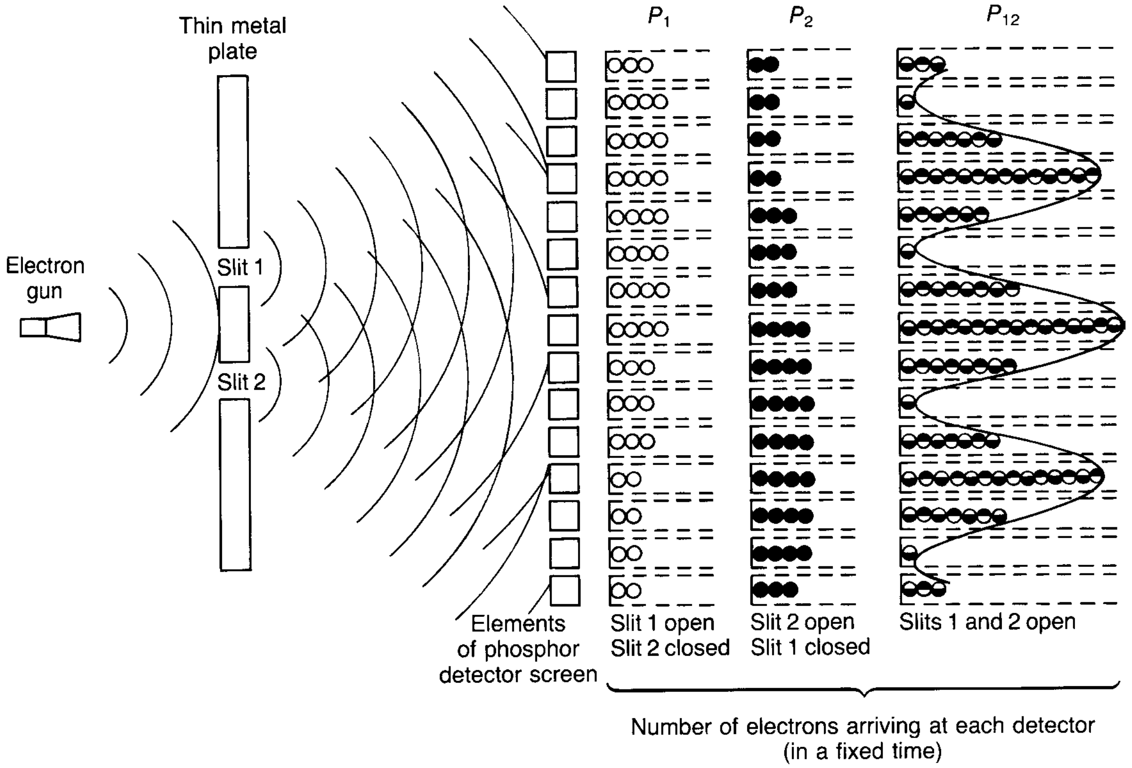


Fig. 1.11 A double-slit experiment with electrons. Electrons always arrive with a flash at the phosphor detector at one point, in the same way that bullets always end up in just one of the detector boxes rather than the energy being spread out, as in a wave. The column marked P_1 shows the pattern obtained with only slit 1 open. Electrons that have gone through slit 1 are represented as open circles, like the bullets of Fig. 1.7. Column P_2 shows the same thing with only slit 2 open and the electrons that have gone through slit 2 indicated by black circles. These two patterns are exactly the same as those obtained with bullets. The difference lies in the column headed P_{12}, which shows the pattern obtained for electrons when both slits are open. This is like the interference pattern obtained with water waves and requires some kind of wave motion arising from each slit as indicated on the figure. It is not the sum of P_1 and P_2 and so we cannot say which slit any electron goes through. We have indicated this lack of knowledge by drawing the electrons, which still arrive like bullets, as half white and half black circles. This fact, that quantum objects such as electrons possess attributes of both wave and particle motion but behave like neither, is the central mystery of quantum mechanics.

This is already very strange – but things become even more mystifying as we look at this result in more detail.

Let us look at a place where the detector observes a dip or 'minimum' of the interference pattern obtained with both slits open. At such positions we find fewer electrons than would be the case if we repeated the experiment with one slit closed! If we do such a 'one slit closed' experiment with electrons, we see the patterns shown in Fig. 1.11, exactly as for waves. But if electrons arrive like bullets, how can this be? Does the electron somehow

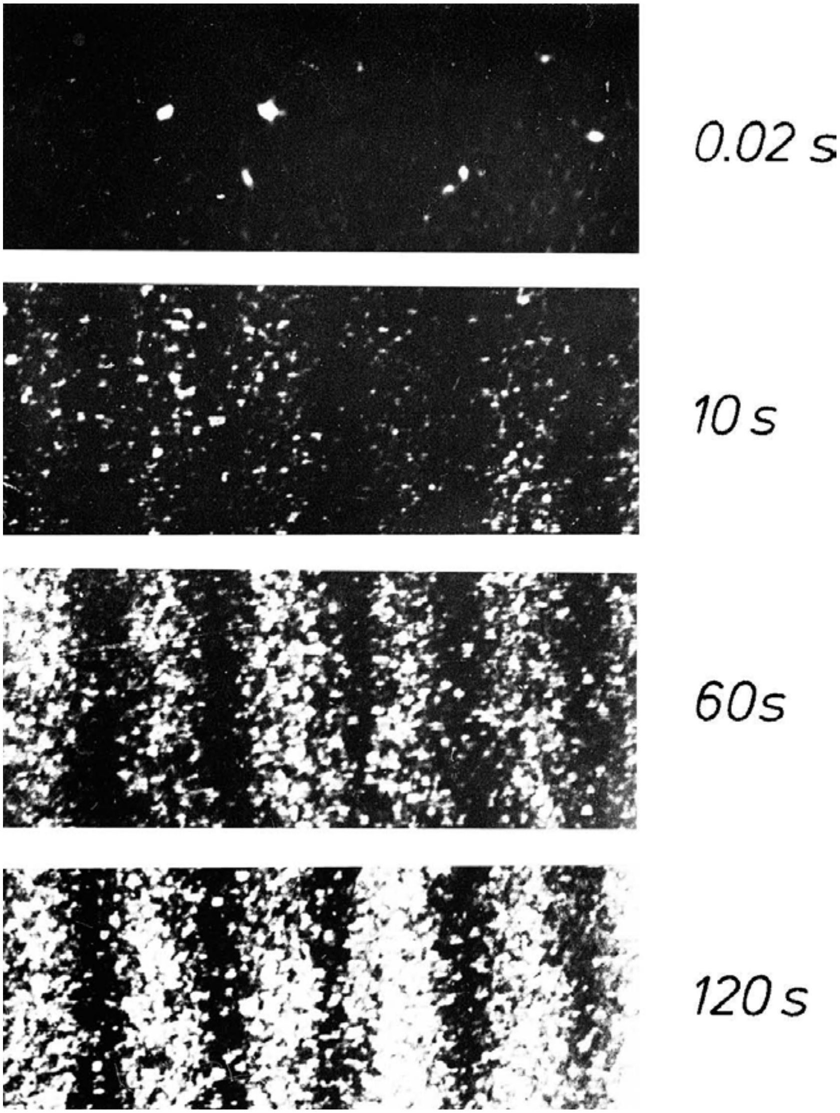


Fig. 1.12 More details of a double-slit experiment performed with electrons. Although interference patterns were once thought of as evidence for wave motion, when looked at in detail it can be seen that the electrons arrive in individual lumps. The top photograph shows a short exposure in which so few electrons have arrived that an almost random pattern of hits is seen. The pictures below show what happens as the exposure becomes longer: more and more electrons arrive until eventually the familiar interference pattern becomes visible.

split up into two and half an electron go through each slit? No! Electrons are never seen in halves – just like bullets they are either all there or not at all. Since the invention of quantum mechanics, many people have struggled to resolve this dilemma. No one has yet succeeded. It is as if the electrons start as particles at the electron gun, and finish as particles when they arrive at the detector, but the arrival pattern of electrons observed at the detector is as if they travelled like waves in between!

We have seen that the mathematics of the interference curve can be summarized in a very simple equation. We also saw, in the case of water waves, that the interference arose from adding the wave heights or ‘amplitudes’ for waves from the source to go via slit 1 and via slit 2. The intensity or energy of the wave was then related to the square of the sum of these amplitudes. The same mathematics must hold for the electron interference pattern. In the case of electrons, however, we are not measuring the intensity of a real wave motion but rather the probability of arrival of the electrons. From the mathematics of the interference curve, we see that there must be something like the height of a wave in the case of electrons. But what is the meaning of the 'height' of an electron wave? Since the square of this 'height' must give the corresponding probability, it is called a 'quantum probability amplitude'. We shall denote such quantum 'heights' or amplitudes by the symbol a. Thus, our equations for the probability of arrival of electrons will all have exactly the same form as for water waves, except that we shall use the symbol P for probability, instead of I for intensity, and a for quantum amplitude, instead of h for height. With these substitutions, the equations for the probability of the arrival of electrons with 'both slits open' and with 'one slit closed' take the form

$$\begin{aligned} P_{12} &= (a_1 + a_2)^2 \\ P_1 &= a_1^2 \\ P_2 &= a_2^2 \end{aligned}$$

and, as before, P_{12} is not equal to the sum of P_1 and P_2:

$$P_{12} \neq P_1 + P_2$$

We conclude that electrons show wave-like interference in their arrival pattern despite the fact that they arrive in lumps, just like bullets. It is in this sense that we can say that quantum objects sometimes behave like a wave and sometimes behave like a particle. You may find this all rather mysterious. It is! We cannot do more to explain the magic of quantum mechanics – all we can do is describe the way quantum things behave. This description **is** quantum mechanics.

2 Heisenberg and uncertainty

> A philosopher once said 'It is necessary for the very existence of science that the same conditions always produce the same results'. Well, they don't!
>
> Richard Feynman

Watching electrons

We have seen that quantum mechanics does not allow us the comfort of being able to visualize the motion of a quantum particle. In a normal game of billiards we can imagine the paths taken by the individual balls (Fig. 1.1). Figure 2.1 shows the physicist George Gamow's attempt to give some impression of how the same game might look if played with quantum particles. Besides illustrating that the notion of a path is no longer valid in quantum mechanics, this cartoon also illustrates another significant difference between the quantum and classical worlds: the exact position of the white ball is not known. Uncertainty has entered physics and replaced the determinism of Newtonian mechanics.

By the nineteenth century, physicists had been able to explain vast amounts of experimental observations on objects as different as planets and billiard balls. If an observation differed from the predictions of classical physics, they looked for something they had overlooked to explain the deviation. In 1864, physicists' confidence in the whole edifice of classical physics seemed to be spectacularly verified by an analysis of some irregularities in the orbit of Uranus. These were attributed to the existence of a then undiscovered planet – the subsequent discovery of Neptune was a triumph for Newtonian physics. By the turn of the twentieth century, it seemed that all of physics followed from Newton's laws. If one was given a box containing a certain number of particles, all one had to do to be able to predict the motions of every particle any time in the future (or in the past for that matter) was to measure the present positions and speeds of all the particles. By measuring the speeds and positions sufficiently accurately, these predictions could be made as precise as required. This was the deterministic view of Nature encouraged by the success of classical physics. The caveat about 'sufficiently accurately' hardly seemed necessary. After all,

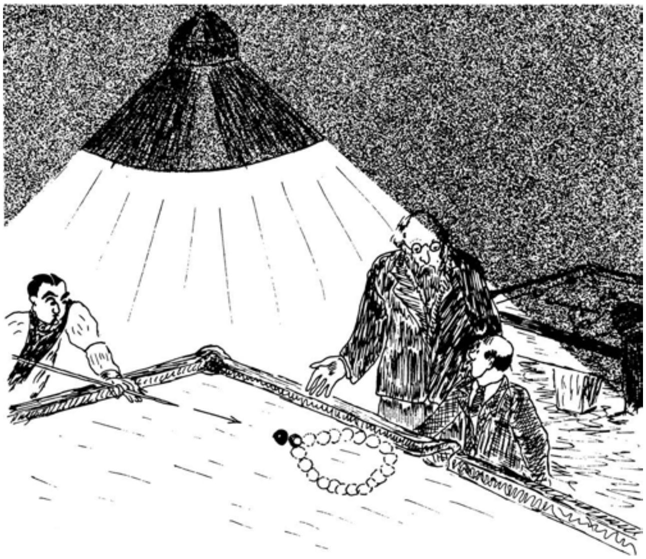


Fig. 2.1 In this illustration George Gamow has Mr Tompkins playing billiards with quantum billiard balls. The original caption is ‘The white ball went in all directions!’ In such a world, quantum uncertainty would be a familiar experience.

it was ‘obvious’ that one could measure anything with essentially no limit to the accuracy of the measurement – all one required was a sufficiently sensitive measuring device.

Quantum mechanics does away with this deterministic view of the future once and for all, and an essential element of uncertainty enters the predictions of physics. How does this come about? It arises because the seemingly innocuous belief of the classical physicists, that they could measure both the position and the velocity of a particle as accurately as they wished, is wrong! In quantum mechanics there is a fundamental limit to the accuracy we can achieve, no matter how ingenious or sensitive we make our measuring devices.

To illustrate this point let us return to the double-slit experiment once more. Remember that we talked in terms of the probability of where an electron would hit the screen. This is because we cannot say with certainty where any particular electron will land. We can only predict the relative chance of it landing at any particular position on the screen.

Now recall the experiment with bullets. This was also described in terms of probabilities. But there is a crucial difference between bullets and electrons. In the case of bullets the probability description was used because of our ignorance of the exact initial direction of the bullet – because of the wobbly gun. However, we could make a video of the firing of an individual bullet and then watch the bullet’s trajectory to the screen on a slow-motion playback. Even if we only saw a part of the bullet’s path, that would be sufficient, according to Newton, to determine the rest of the path. Obviously, the bullet must pass through one of the slits, and we can determine which one by looking at the video replay.

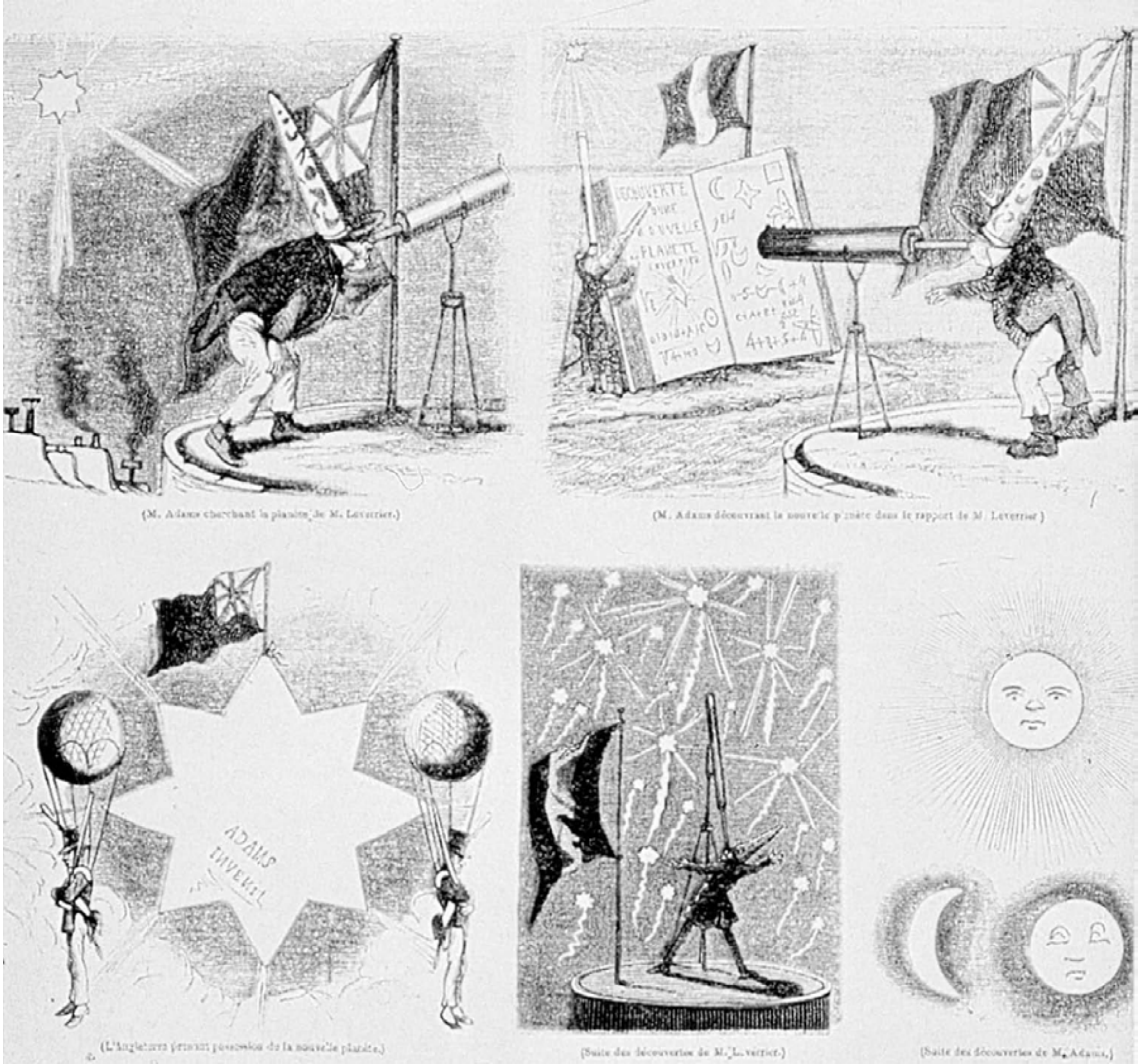


Fig. 2.2 The existence of the planet Neptune was predicted by Adams in England and by Le Verrier in France at about the same time. After Neptune's discovery there was a familiar Anglo-French squabble about priority, and this French cartoon depicts Adams stealing Le Verrier's results. Le Verrier tried to repeat his success with Neptune by using an anomaly in the orbit of Mercury to predict the existence of another planet, Vulcan, closer to the Sun. This was, of course, not the correct interpretation of the anomaly, which is now understood using Einstein's theory of general relativity (see our companion book *Einstein's Mirror*).

Why can't we do the same sort of thing for electrons? Let us imagine how we would go about trying to establish through which slit the electron passes. To see the electron just after it has passed through one of the slits we must shine some light on it and observe the reflected light. Let us therefore modify the experimental apparatus by inserting a light source behind the

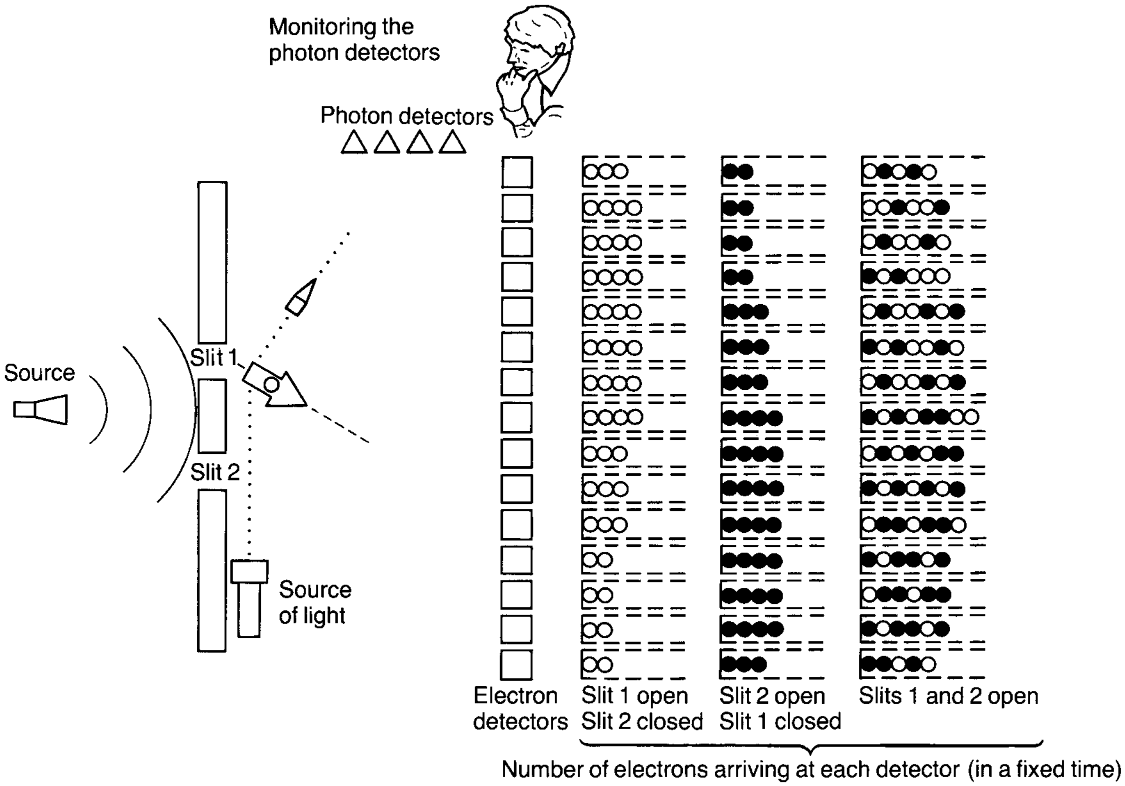


Fig. 2.3 The experimental set-up required to observe through which slit the electron passes in a double-slit experiment. Light, in the form of photons, is directed at the slits. In the figure a photon, represented as a small bullet, has hit an electron behind slit 1. The electron is disturbed slightly in its motion and the scattered photon is observed at the photon detectors. The electron patterns obtained with only one of the slits open are almost the same as before, when we did not observe the electron behind the slits. The surprise occurs with both slits open: there is no interference pattern. The small nudges given to the electrons in their collisions with the photons are always sufficient to wash out the interference pattern completely! We can now say with certainty through which slit the electron went but now the electrons are behaving just like bullets. The observed pattern is just the sum of the patterns for slit 1 and slit 2 separately.

slits (Fig. 2.3). We now arrange things so that if an electron passes through slit 1 we see a flash behind slit 1, and similarly for slit 2. If we now do the experiment what do we see? Well, the first important result is that we never see a half-flash behind both slits simultaneously. There is always a whole flash behind either slit 1 or slit 2. We can now divide up the electrons arriving at the detector into two groups, according to whether they went through slit 1 or slit 2. What's all this quantum nonsense then? The electron obviously either goes through slit 1 or slit 2. So indeed it does, when we watch the electron. But, if we now look at the arrival pattern of electrons at the screen, we see no interference pattern! The result is just the same as we obtained with bullets!