The Principle of Relativity

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by

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PREFACE

THE controversial note which has been characteristic of discussions in respect of the Principle of Relativity has prevented the significance of the principle from being seen in its proper proportions and in its relation to general physical theory. On the one hand, there have been those who have magnified its importance, and assigned to it an unduly revolutionary power, while on the other hand, there are those who have scoffed at it as fantastic and reared on the most slender of physical bases. It has therefore seemed desirable in the first part of this book to outline the way in which the Principle of Relativity grew out of electrical theory, so that it might be made clear that there is a real place for it as a hypothesis supplementary to and independent of electrical theory owing to the limitations to which that theory is subject.

It is hoped that by drawing a clear distinction between the 'mode of measurement,' and the 'nature' of space and time, the author will escape from the charge of venturing unduly upon debatable metaphysical questions.

In the Second Part an attempt has been made to present in a simple form the more attractive of the two mathematical methods devised by Minkowski for the purpose of putting in evidence the relative nature of electrical and other phenomena.

The Third Part seeks to indicate some of the most fundamental points in which mechanical theory needs modification if the principle is accepted as universal. It has not been thought advisable to give an account of the purely formal and rather academic developments of special branches of mechanics such as hydromechanics, and elasticity, as these might tend to divert attention from the bearing of the principle on what are generally classified as the fundamental concepts. Some account of these is given by M. Laue in the second edition of his book, *Das Relativitätsprinzip*, Braunschweig, 1913. No attempt has been made to present the highly speculative attempt of Einstein at a generalization of the principle in connection with a physical theory of gravitation.

Throughout the intention has been as far as possible to consider those aspects of the principle which bear directly on practical physical questions. The mathematical part has been compressed to as small a compass as is consistent with furnishing sufficient apparatus for a systematic consideration of the problems suggested.

In the preparation of the book the author has received great help from Mr H. R. Hassé, who read the whole of the manuscript and made many suggestions for its improvement, besides reading the proofs of nearly the whole work. Mr R. W. James has also given valuable assistance in reading both manuscript and proofs. Especially would the author wish to acknowledge his debt to Sir Joseph Larmor, both personally and through his published works, for much stimulus and encouragement in the study of theoretical physics, and for valuable criticism of the earlier part of this book.

To the staff of the Cambridge University Press for care and courtesy in the work of printing the author is most grateful.

E. C.

CAMBRIDGE,
June 1914.

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A characteristic of the Newtonian conception of space is that an ideal rigid body always has the same volume and shape whatever its motion. There is no ambiguity in the meaning of the measurements of lengths and intervals of time. Also there is no ambiguity in the measurement of *mass* and *force*. Whatever frame of reference is used these have the same value. Not so, however, with *energy* and *momentum*.

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But the experimental failures to determine a unique aether extend into regions where the electron theory is by no means sufficient as an explanation, e.g. the optical properties of solid bodies, the conductivity of metals, the rigidity of the sandstone and pine of Michelson and Morley's experiments. We are therefore tempted to examine the consequences of the general assumption that physical phenomena will never discriminate between the various frames of reference permitted by the electron theory . . . . **31—41**

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The classical experiments of Kaufmann and others on the so-called *electromagnetic mass of an electron* considered from this point of view are experiments on the variation of acceleration with velocity of an electron in an electric field, and, in so far as they are consistent with the results suggested by the theory, afford confirmation of the Einstein kinematics and also of the associated conception of the relative nature of the electric and magnetic intensities . . . . . . **53—70**

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By introducing hypotheses as to the geometrical configuration of an electron in motion, it is possible to find a relation between the total external *force* on an electron, its velocity, and its acceleration, the total external force being supposed to be balanced by aethereal reaction. Experiment is on the whole in good agreement with the calculations based on Lorentz' hypothesis of an electron subject to the FitzGerald contraction . . . . **135—153**

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The automatic contraction of a body, when its velocity relative to an observer is accelerated, can be interpreted as the taking up of the most probable state consistent with the altered velocity.

Certain hypotheses made for attacking the problem of radiation are shewn to be suggested by this criterion of equal probability.

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THE PRINCIPLE OF RELATIVITY IN RELATION TO GENERAL PHYSICAL THEORY

CHAPTER I

INTRODUCTION

1. The Relativity of the Newtonian Dynamics.

It is a commonplace observation in respect of Newtonian dynamics that although the fundamental laws assume at the outset an absolute frame of reference, yet they are not sufficient to determine *uniquely* what that frame of reference must be; or in other words, although to every moving point a velocity is assigned, yet the laws of dynamics as stated by Newton are not sufficient to determine a velocity which, more than any other arbitrary velocity, can be said to be *the* velocity of any particular point. Put more definitely, it is known that if any set of axes in space can be specified relative to which the laws of dynamics are satisfied by a system of bodies, then any other set of axes which moves continually with a constant velocity of translation and with no rotation relatively to the former set, is also a valid framework for the dynamics of the same system of bodies.

It is important to distinguish this *dynamical relativity* from any philosophic dogma as to the *a priori* impossibility of the mind conceiving of an absolute position or motion in space. For such a proposition would be of much wider content than that stated above. It would imply not only that the velocity of a point is an undetermined quantity but also that its acceleration cannot be determined. Further there is nothing to exclude from the scope of such a supposed philosophic truth the impossibility of defining uniquely the velocity of rotation of a body. In fact apart from physical phenomena, the mind can no more and no less conceive of an absolutely fixed direction than of an absolutely fixed position.

In the Newtonian description of the modes of motion of bodies however, absolute standards of position and direction are at the outset assumed, not as a philosophic doctrine, but as a means of co-ordinating the phenomena observed as simply as possible. The justification for the assumption lies in the fact that the co-ordination is possible, and the laws so framed contain within themselves the definition of the terms 'absolute position' and 'direction,' and they also *define*, though not uniquely, 'the frame of reference.'

A *straight line* from the point of view of Newtonian dynamics is nothing more than the path of a particle which moves under the influence of no external agencies*, or again it is the locus of the positions at any instant of a number of particles which were projected in a common direction with different velocities from a given point at some previous moment.

2. The Space and Time of dynamics.

But more than this, the laws of dynamics contain statements as to distances and intervals of time. In the scheme as finally drawn up it appears as if the measurements of distance and of time were almost intuitive processes, the method to be adopted being undefined. It appears almost as if space were marked out permanently, and as if there were an absolute clock to

* The differential equations

$$\frac{d^2x}{dt^2}=0, \quad \frac{d^2y}{dt^2}=0, \quad \frac{d^2z}{dt^2}=0$$

lead at once to the equations

$$x=a_1t+b_1, \quad y=a_2t+b_2, \quad z=a_3t+b_3,$$

out of which all the descriptive geometry of the straight line and plane follow immediately.

which all time is referred. But on examination it is clear that our intuitive conception of distance is not sufficiently definite, and that all possible clocks are too dependent on dynamical mechanism to be of use as primary standards of time.

Thus, as in the case of position and direction, the definitions of the 'measures of space and time' in dynamics are partly contained within the laws of motion. Those laws at once express a uniformity in the phenomena of motion, and define a scheme of quantities in terms of which that uniformity may be simply expressed, but, as has been said, the scheme so defined is by no means unique. In the light of the principles of conservation of linear and angular momentum of a system of particles, all that can be stated is that we have criteria of certain fixed directions, namely the directions of the resultant linear and angular momenta, and also a criterion of uniform motion in a fixed direction, namely the motion of the centre of mass, of the system.

It was said above that the definitions of the measures of space and time are *partly* contained in the laws of dynamics. The two concepts are not in fact completely defined by the dynamics of separate particles. If we had an independent measure of time, then uniform motion would give a means of passing to a measure of space. Conversely if we had an independent measure of space, then the measure of time could be obtained. The clock would be a free particle traversing a graduated straight line.

But if within the term 'dynamics' we include the phenomena of a rigid body, then both measures are defined—subject to a choice of units. For in this branch of dynamics the ideal rigid body is conceived as having a definite size; the distance between two points of it is conceived as always defining the same interval of distance, whether the body is moving or not. It may be looked upon as a means of measuring space. Or again if, avoiding the usual somewhat artificial deductions of the equations of motion of rigid bodies from those of separate particles, we take those equations for granted as the appropriate extension of Newton's equations, then we have an ideal clock in a body rotating about an axis of symmetry, and hence, by means of the motion of free particles, a method of graduating space. But it is implied in our scheme of dynamics that if space were so graduated then the distance between two given points of any rigid body would occupy the same interval wherever it was placed, and whatever were the free particle which was defined to be at rest.

3. There is then a **Newtonian principle of relativity** which may be stated thus:

It is impossible by means of any dynamical phenomena to ascertain absolutely the velocity of any material particle, but the relative velocity of any point with respect to any other is a uniquely determinable quantity.

It is worth noticing at once that in considering the same dynamical system from two different frames of reference moving relatively to one another with a certain uniform velocity of translation, there are certain quantities that have the same value in the two cases. The 'mass' of a particle, its 'acceleration,' the 'force' acting on a particle, the 'distance between two marked points of a rigid body,' and the 'measure of an interval of time' are the most fundamental. On the other hand the velocity, momentum, and energy vary with the frame of reference.

It will be important in the ensuing discussion to emphasize the invariant quantities, but the main outlook is towards the *invariant relations* between the quantities which are not themselves invariant.

For example, if the vector $\mathbf{u}$* is the velocity of a moving particle relative to a certain Newtonian frame of reference we have

$$m\frac{d\mathbf{u}}{dt}=\mathbf{P},$$

where $\mathbf{P}$ is the force acting.

* Throughout the book vector quantities are represented by Clarendon type, and $(\mathbf{uv})$ represents the scalar product of $\mathbf{u}$ and $\mathbf{v}$, i.e. $|\mathbf{u}||\mathbf{v}|\cos\theta$, where $|\mathbf{u}|$ is the magnitude of $\mathbf{u}$ and θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

If $\mathbf{u}'$ is the velocity referred to a new frame of reference whose velocity relative to the first is $\mathbf{v}$

$$\mathbf{u}' = \mathbf{u} - \mathbf{v},$$

so that $\mathbf{v}$ being constant

$$m \frac{d\mathbf{u}'}{dt} = \mathbf{P}.$$

Here m and $\mathbf{P}$ are *invariant quantities* and the equation is an *invariant relation.* It is the relation which is all important.

Again the equation of energy in the first system is

$$\frac{d}{dt}(\tfrac{1}{2}m\mathbf{u}^2) = (\mathbf{Pu}),$$

and in the second $$\frac{d}{dt}(\tfrac{1}{2}m\mathbf{u}'^2) = (\mathbf{Pu}');$$

the latter can be obtained from the former by means of the equation of momentum.

But conversely *if we assume the equation of energy to be an invariant relation, the equation of momentum may be deduced.*

Thus $$\frac{d}{dt}(\tfrac{1}{2}m\mathbf{u}'^2) = \frac{d}{dt}(\tfrac{1}{2}m\mathbf{u}^2) - \frac{d}{dt}(m\mathbf{uv}) + \frac{d}{dt}(\tfrac{1}{2}m\mathbf{v}^2),$$

or since $\mathbf{v}$ is constant

$$(\mathbf{Pu}') = (\mathbf{Pu}) - \left(\mathbf{v}\frac{d}{dt}m\mathbf{u}\right),$$

or $$(\mathbf{vP}) = \left(\mathbf{v}\frac{d}{dt}m\mathbf{u}\right).$$

If this is true whatever the value of the relative velocity $\mathbf{v}$ of the two frames of reference, we must have

$$\mathbf{P} = \frac{d}{dt}m\mathbf{u}.$$

4. Remembering that a force is not a primary physical quantity*, it will be as well to refer to the more fundamental physical results of the conservation of energy and momentum for a system of particles under no forces but their own mutual actions.

* As commonly presented in present day teaching—but see § 5 and appendix, pp. 9 and 10.

From the equations

$$\Sigma \tfrac{1}{2} m\mathbf{u}^2 + V = \text{const.},$$
$$\Sigma m\mathbf{u} = \text{const.},$$

in which the potential energy of the configuration is V, it follows that

$$\Sigma \tfrac{1}{2} m (\mathbf{u} - \mathbf{v})^2 + V = \text{const.},$$

where $\mathbf{v}$ is any constant vector. Thus if it be taken that V is a function of the *relative* positions of the particles (which are invariant quantities), so that V is an invariant, it follows that the sum of the kinetic and potential energies is constant whatever be the velocity.

In the same way if it be assumed that V is independent of $\mathbf{v}$ and that the constancy of the sum of the energies is an invariant relation, it follows that $\Sigma m\mathbf{u}$ is constant; or, in other words—*the principle of the conservation of linear momentum follows from the principle of energy combined with the principle of relativity.*

In this statement of course the mass m becomes merely a constant in the equation of energy—its definition is included in the assumption of the existence of such an equation, and the familiar definition in terms of accelerations takes a derived position.

5. It is striking that the invariant quantities *mass* and *force* are precisely those which have, in the point of view represented by Mach (*Science of Mechanics*) and Karl Pearson (*Grammar of Science*) among others, been relegated from the position of primary concepts to that of mathematical numbers characteristic of certain uniformities and relations between the accelerations of bodies. But it is at least capable of being argued* that even if the means of measuring them is lacking, qualitatively they are matters of apprehension as direct as time and space; and, as for measurement, the ordinary means applied to the latter depend upon the properties of mass, force, and rigidity. It is as easy to imagine the reproduction of the physical

* See quotations in the appendix to this chapter.

conditions giving rise to what we call a *force*, as to conceive of an ideally periodic phenomenon which will serve as a clock.

In passing from the 'dynamics of experiment,' in which forces are looked upon as prior to motion, as for example in the projection of a bullet from a rifle, or the throwing of a ball by muscular effort, to the 'dynamics of theory,' we are, as in nearly all logical schemes, extracting from the complex of experience of motion in space ideal conceptions of abstract space and time merely as a foundation or background for an ordered description of the permanent relations between different factors in that experience. When we have obtained these two conceptions by an analytic process from the directly perceived facts of motion, we can by a synthetic process build up from these materials an ideal conception of motion which records the facts of experience in a form amenable to mathematical treatment.

But as the essence of a picture is not in the canvas, but in the painting thereon as a record and interpretation of some part or aspect of human experience; so the value of dynamical theory is not in its reducing 'force' and 'mass' to abstractions by making 'space' and 'time' the prime concepts, but in its giving a fuller and more precise significance to those fundamental elements of experience by making a picture of them which is complete in itself*.

6. Statement of the General Principle of Relativity.

The Principle of Relativity which is the subject of this work holds the same place in the physical thought of to-day, that the foregoing principle of dynamical relativity held in the time when the laws of dynamics were considered as ultimate and all-embracing. It consists in the general hypothesis, based on a certain amount of experimental evidence, that the *problem of determining in a physical sense the absolute velocity of a body is one that can no more be solved uniquely by the help of optical and electrical phenomena, than it could by means of dynamical observations.*

* Cf. Larmor, *Aether and Matter*, Appendix B, especially § 3, pp. 271–3. See also pp. 9, 10.

If we speak of a 'fixed aether' as the background of electrical activity, it is the hypothesis that *the velocity of any piece of matter relative to the aether is unknowable.*

To put it more precisely, it states *that we neither have nor expect to have any experimental evidence of the uniqueness of the framework which we call 'the aether,' but that if there is one, there is an infinite number of such frames of reference, any one of which has, relative to any other, a uniform velocity of translation without rotation, this velocity being of arbitrary magnitude.*

In this statement, it will be seen, the old philosophic difficulty as to *absolute direction* or *angular velocity* remains. The domain of the principle is co-extensive with the relativity which follows from the laws of dynamics. But as in that case, the assertion of the principle is not a metaphysical dogma. It is an *empirical* principle, suggested by an observed group of facts, namely the failure of experimental devices for determining the velocity of the earth relative to the luminiferous aether, and would make it a *criterion* of theories of matter that they should give an account of this failure, and it suggests modifications where the theory is insufficient to do so. But like all physical principles, it is to be probed by further experience.

It will be seen that it involves a reconsideration of many old-established preconceptions. It emphasizes very strongly what has been said as to the derivative nature of *metrical* space and time; though, of course, as a physical principle it has nothing to say against the reality of the perception of spatial extent and temporal duration. It is in fact completely dependent upon such perception.

It will be found that many commonly accepted terms such as 'simultaneity,' 'electric force,' 'aether' are incapable of unique definition or specification, and to that extent lose that reality which they seemed to possess. A criterion of *objective* or *physical* reality is strongly suggested in the requirement of *uniqueness of definition* and *invariance of magnitude* whatever the frame of reference chosen out of an infinite number that are possible.

Thus, for example, in the light of what has been said, the metrical absolute space of Newton is not unique. On the other hand, in *force* and *mass* we are dealing with quantities which the relations perceived between phenomena do not leave ambiguous*, so that, in this sense, we may say that they have a *physical* significance such as we cannot attribute to velocity, or energy.

Thus we are reminded that mechanics was in its origin a calculus of *forces*, that Galileo's great achievement was in the clear statement of the property of *mass* or inertia as a permanent property of matter, and that only at a later date was the attempt made to frame a system of Mechanics in which these quantities took a secondary place, while motion relative to a conceptual and undefined† framework was given the pre-eminence as the basis of the science.

APPENDIX

It may be worth while for the sake of emphasizing the point of view to place side by side quotations representing different schools of thought.

Karl Pearson—*Grammar of Science*, 3rd edition, p. 332.

"The definition of force we have reached is a perfectly intelligible one; it is completely freed from any notion of matter as the moving thing, or from any notion of a metaphysical cause of motion....Force is an arbitrary conceptual measure of motion without any perceptual equivalent."

Contrast with this

Larmor—*Aether and Matter*, p. 272.

"To say, as is sometimes done, that force is a mere figment of the imagination which is useful to describe the motional changes that are going on around us in Nature, is to assume a scientific attitude that is

* Subject of course to the restriction of the phenomena in question to a certain limited field.

† That is, undefined apart from the relations to be subsequently developed.