Encyclopedia of the History of Arabic Science

3 Volume Set

Edited by
Roshdi Rashed

ROUTLEDGE

Encyclopedia of
the History of Arabic Science

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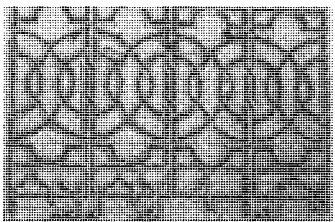
Edited by
ROSHDI RASHED
in collaboration with
RÉGIS MORELON



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Contents

Preface ix

VOLUME 1

1 General survey of Arabic astronomy *Régis Morelon* 1

2 Eastern Arabic astronomy between the eighth and the eleventh centuries *Régis Morelon* 20

3 Arabic planetary theories after the eleventh century AD *George Saliba* 58

4 Astronomy and Islamic society: Qibla, gnomonics and timekeeping *David A. King* 128

5 Mathematical geography *Edward S. Kennedy* 185

6 Arabic nautical science *Henri Grosset-Grange* (*in collaboration with Henri Rouquette*) 202

7 The development of Arabic science in Andalusia *Juan Vernet and Julio Samsó* 243

8 The heritage of Arabic science in Hebrew *Bernard R. Goldstein* 276

9 The influence of Arabic astronomy in the medieval West *Henri Hugonnard-Roche* 284

VOLUME 2

10 Numeration and arithmetic *Ahmad S. Saidan* 306

11 Algebra *Roshdi Rashed* 324

12 Combinatorial analysis, numerical analysis, Diophantine analysis and number theory *Roshdi Rashed* 351

13 Infinitesimal determinations, quadrature of lunules and isoperimetric problems *Roshdi Rashed* 393

14 Geometry *Boris A. Rosenfeld and Adolf P. Youschkevitch* 422

15 Trigonometry *Marie-Thérèse Debarnot* 470

16 The influence of Arabic mathematics in the medieval West *André Allard* 514

17 Musical science *Jean-Claude Chabrier* 556

18 Statics *Mariam Rozhanskaya (in collaboration with I. S. Levinova)* 589

19 Geometrical optics *Roshdi Rashed* 618

20 The emergence of physiological optics *Gül A. Russell* 647

21 The Western reception of Arabic optics *David C. Lindberg* 691

VOLUME 3

22 Engineering *Donald R. Hill* 705

23 Geography *André Miquel* 750

24 Botany and agriculture *Toufic Fahd* 767

25 Arabic alchemy *Georges C. Anawati* 807

26 The reception of Arabic alchemy in the West *Robert Halleux* 840

27 Medicine *Emilie Savage-Smith* 857

28 The influence of Arabic medicine in the medieval West
Danielle Jacquart 917

29 The scientific institutions in the medieval Near East
Françoise Micheau 939

30 Classifications of the sciences *Jean Jolivet* 962

Postface: approaches to the history of Arabic science
Muhsin Mahdi 980

Bibliography (Vol. 1) 999

Bibliography (Vol. 2) 1024

Bibliography (Vol. 3) 1045

Index 1067

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Preface

Ever since the history of science emerged as a discipline at the heart of the Age of Enlightenment in the eighteenth century, Arabic science[1] – or at least certain sectors of it – have constantly been cited by the philosophers and historians of science. For the former, such as Condorcet, it was a guarantee of the continued progress of enlightenment during a period dominated by 'superstitions and darkness'; for the latter, notably Montucla, Arabic science was necessary not for the sketching of a historical picture, but in order to establish the facts of the history of the mathematical disciplines. But philosophers and historians alike had received only the echoes of Arabic science, which had reached them through ancient Latin translations. We must, of course, beware of over-generalization or errors of perspective, and bear in mind that the sciences do not all maintain the same connection with their history; thus, of the mathematical sciences at least, astronomy is the one most firmly linked with its history, if only on account of the values of the observations that were recorded in books over the course of time and consulted by successors. Consequently Arabic astronomy assumed a privileged position, fairly rapidly attracting the attention of historians such as Caussin de Perceval, Delambre and, above all, J.-J. Sédillot – to name but French scholars – at the beginning of the nineteenth century.

Later in the course of the same century, the image of Arabic science began to change and to become shrouded with nuance. German Romantic philosophy, and the German school of philology which it engendered, had given considerable impetus to the philological and historical disciplines. The history of Arabic science gained from this rapid expansion, before becoming its victim: the study of Greek or Latin scientific texts could no longer eschew the Arabic works;[2] but the snare of history through languages – which we have stressed elsewhere[3] – enmeshed the history of Arabic science and bore it into retreat. *De jure*, therefore, it lost its right to exist, while *de facto* it was indispensable to historians, who referred to it increasingly.

This paradox, which is apparent not only in second-order studies but permeates a major work like *Le Système du Monde* by Pierre Duhem, is in fact merely the expression of a profound necessity: the historian of classical science, whatever his doctrinal views, cannot avoid Arabic science when he reviews the facts of the discipline whose history he is retracing. Following in the wake of the Western doctrine of classical science, he can view Arabic science as a repository of Hellenic science, a belated Hellenic science as it were: science as theory is Greek and as experimental method it was born in the seventeenth century. According to this doctrine Arabic science constitutes an excavation site, in which the historian is the archaeologist on the track of Hellenism. This approach has frequently ended up misrepresenting the results of Greek science as well as those of the seventeenth century, a necessary distortion if one wishes to link the two ends of the chain in a continuous history; on the other hand, and not without coincidence, it has led to some famous blunders affecting not only interpretation but comprehension too. These doctrinal views prevented Carra de Vaux (who translated the astronomical treatise of Naṣīr al-Dīn al-Ṭūsī) and the eminent historian P. Tannery (who quotes it) from grasping the innovation that it entailed and which Neugebauer was to emphasize much later. But the historian of classical science has also managed to break away from this doctrine: the other historical practice, contemporaneous with the former, came into being with the work of Alexander von Humboldt, under whose influence certain scholars became involved in the direct and innovative study of the history of Arabic sciences: F. Woepcke and L. A. Sédillot, for example, whose work was later followed up by Nallino, Wiedemann, Suter, Ruska, Karpanski, Hirschberg, Kraus, Luckey, Nazif, etc., resulting in an unprecedented acceleration of this line of research from the 1950s onwards.

Built up over the decades, this work opened the way to a better knowledge of Arabic science and of its contribution to classical science; it also enabled the understanding of one of its essential features, which had hitherto been obscured. In Arabic science a potentiality of Hellenic science was realized: the tendency beginning to germinate in the Greek scholars, to go beyond the frontiers of an area, to break the bounds of a culture and its traditions, to take on world-wide dimensions, was fulfilled in 'a science developed around the Mediterranean not as such but as a forum of exchange of all the civilizations at the centre and at the periphery of the ancient world' (Rashed 1984).

Arabic science was 'international', one can say today, as much on account of its sources as through its developments and extensions. Even if the majority of those sources were Hellenistic, they also comprised writings in Syriac, Sanskrit and Persian. The weights of these different contributions were, of course, unequal, but this does not detract from the fact that their multiplicity was essential to the evolution of Arabic science; and even in the case of mathematics, which no one would deny to be the 'heir' of Greek science, it is essential to go back to other sources for a true understanding. We can see, for example, in the chapter devoted to astronomy the importance of Indian and Persian roots, not only in the development of an astronomy of observation and of astronomical calculation, but also for the new configuration of Ptolemaic astronomy.

Within this new framework, the transmission of findings mattered less than the opportunity which occurred to bring together different scientific traditions, henceforth united within the scope of Islamic civilization. The novelty of this phenomenon was that it was not the fruit of chance meetings, of the regular or unexpected passage of caravans or seafarers; it was the deliberate result of a massive movement of scientific and philosophical translation, undertaken by professionals – sometimes rivals – supported by power and stimulated by the research itself. From this movement was born a library on the scale of the world of its time. Thus traditions from different origins and languages became elements of one civilization whose scientific language was Arabic, and found ways of reacting together to bring about new methods, and sometimes even initially unforeseen new disciplines – see, for example, the chapter on algebra (volume II, chapter 11). The social study of Arabic science will one day enlighten us about the role of Islamic society and of Islamic cities in this historic movement; we may then understand how previously independent scientific currents were able to meet and combine.

This characteristic of Arabic science, which was already marked in its earliest phase, became even more pronounced later. The scholars of the eleventh and twelfth centuries continued to discuss results obtained elsewhere, extending them and integrating them into theoretical structures often foreign to their area of origin. Seen in medicine, in pharmacology or in alchemy, this phenomenon also affected the mathematical sciences, as shown later in the works of al-Bīrūnī or of al-Samaw'al on the Indian methods of quadratic interpolation, or in the formulation by Ibn al-Haytham of the theorem of the Chinese remainder.

With Arabic science it became possible to read in one language the translations and the scientific work of the ancients, as well as the advanced research of the moderns. The latter was produced in Arabic at Samarkand as in Granada, by way of Baghdad, Damascus, Cairo or Palermo. Even when a scholar wrote in his mother tongue, notably Persian – like al-Nasawī or Naṣīr al-Dīn al-Ṭūsī – he undertook to translate his own work into Arabic. In short, from the ninth century onwards, the language of science was Arabic, and that language had in turn acquired a universal dimension: it was no longer the language of one people but of several; it was no longer the language of a single culture but of all learning. Thus previously inexistent channels opened up to facilitate immediate communication between scientific centres from central Asia to Andalusia and exchanges between scholars. Two practices then underwent unprecedented expansion. First, scientific journeys as a means of learning and teaching – ample evidence of which can be found in the biographies of scholars bequeathed to us by the ancient bio-bibliographers – such as those of Ibn al-Haytham between Basra and Cairo; of Maimonides from Córdoba to Cairo; and of Sharaf al-Dīn al-Ṭūsī going from Ṭūs to Damascus, through Ḥamadhān, Mosul and Aleppo. Second, scientific correspondence, a new literary genre, with its usages and its standards, became an instrument for collaboration and the diffusion of research. Arabic science, then, commensurate with the world of its time was, as we see, accompanied by a succession of changes: relations between the old traditions were modified, the composition of the scientific library altered, and the mobility of scholars and ideas was on a different scale.

It is surprising that such a fundamental and obvious feature of Arabic science should have remained obscured and escaped the attention of historians. One can, of course, relate this to the oblique viewpoint of an historical ideology which views classical science as the achievement of European humanity alone. But two considerations need to be added to that: one pertaining to the history, and the other to the historiography, of science. It is a question, first, of the privileged links that unite Arabic science with its Latin extensions and, more generally, with the science developed in western Europe up to the seventeenth century. In fact from the twelfth century onward, Latin science could not be understood without Latin translations from Arabic; nor could the most advanced research in Latin – such as that of Fibonacci and of Jordan of Nemours in mathematics, that of Witelo or of Theodoric of Freiberg in optics – be appreciated without reference to al-Khwārizmī, Abū Kāmil and Ibn al-Haytham. These close links captured the attention of historians and overshadowed the connections which unite the Arabic sciences with other parts of the world, notably India and China. The historiographical fact is the pre-eminence of the science of the seventeenth century. The latter, which is considered – wrongly moreover – to be all of a piece and revolutionary throughout, was invested in the writings of historians with an a-historic transcendence, becoming an absolute reference for situating all previous science. Presented as a postulate, and in the absence of authentic knowledge of the works of the school of Marāgha and of its predecessors in astronomy – of al-Khayyām and of Sharaf al-Dīn al-Ṭūsī in algebra and algebraic geometry, of the Arabic infinitesimalists from Ibn Qurra to Ibn al-Haytham – this absolute pre-eminence has naturally created a vacuum prior to the works of the seventeenth century, and has resulted in a model of Arabic science that flattens its most remarkable peaks of achievement.

It is not that a good knowledge of Arabic science will detract from the innovations of Kepler in astronomy, of Galileo in kinematics or of Fermat in number theory; on the contrary it will enable us to situate them more exactly, by seeking them where they are and not, as is often the case, where they are not. The progress of this knowledge will lead us to a more profound and more rigorous perception of the scientific activities of this great century and of the preceding century. It will encourage us to revise certain representations and certain historiographical methods, and will guard us against ideas of doubtful validity, notably that of the scientific Renaissance, whilst engaging us in the examination of others, like that of the scientific Revolution. But Arabic science must, in turn, recover its cosmopolitan character; this means following its Latin and Italian extensions, also those in Hebrew, Sanskrit and Chinese, not to mention achievements in the languages of Islamic civilization, notably Persian. Finally, for a satisfactory knowledge of Arabic science, it is necessary to restore it to its context, to the society which witnessed its birth, with its hospitals, its observatories, its mosques, its schools ... . How indeed can one understand certain of its developments if one forgets the Islamic city and its institutions, the function that science fulfilled there and the importance of the role that it could play? This necessary reflection will not be slow to dispel the erroneous but still flourishing views engendered by ignorance which confine science to an alleged marginality at the outermost limits of the city, or detect an illusory scientific decadence from the twelfth century onward as the effect of an imaginary theological counter-revolution.

Only at this price will the history of Arabic science accomplish its two principal tasks: to open the way to a genuine understanding of the history of classical science from the ninth to the seventeenth century; and to contribute to the knowledge of Islamic culture itself by according it a dimension which has never ceased to be its own: that of scientific culture.

This book has been conceived and realized to make its contribution towards a history of Arabic science that meets the demands outlined above. It is in fact the *first synthesis* ever carried out in this area and in this spirit, and if such a synthesis is possible today, it is thanks to the research accumulated since the last century and stimulated from the 1950s onwards. The specialists whom we have invited to contribute the different chapters of this edition are writing for the knowledgeable layperson and not merely an inner circle of colleagues, without however over-popularizing their subject; our aim has been to produce a genuine work of reference. We have tried to restore to Arabic science its true aspect and place by emphasizing the analysis of ancient sources and by devoting some chapters to its extensions in Latin and Hebrew. Because of a lack of specialists, other areas of extension have been less favoured. The book as a whole covers the history of Arabic science over about seven centuries.

But a synthesis, and particularly an initial synthesis, cannot precede effective research. This is far from having achieved the same level in the different sectors of science – whence the absence of certain areas of Arabic science, notably the earth and life sciences. Faced with the constraints imposed by the number of pages at our disposal, we have opted for work in depth at the expense of some gaps, rather than producing a so-called comprehensive, but necessarily superficial and insubstantial text. Throughout the work, we have assured ourselves of every humanly possible guarantee: each chapter has been submitted to two other specialists, members or not of the group of co-authors. Among these I should like to thank, in addition to the co-authors themselves, J. Vuillemin, G. Simon, H. Rouquette, E. Poulle, S. Matton, C. Houzel and K. Chemla. My thanks go also to A. Auger.

Roshdi Rashed
Bourg-la-Reine, February 1993

NOTES

Between the time the manuscript was ready and the printing of this work, five authors died: G. C. Anawati, H. Grosset-Grange, D. Hill, A. S. Saidan and A. Youschkevitch. I would like to pay homage to these highly talented authors.

1 By this expression we mean science written in Arabic, in the sense that one speaks of Greek science or Latin science.
2 See, for example, the work of G. Libri, B. Boncompagni, M. Curtze and J. L. Heiberg later.
3 Rashed (1984).

The Editor expresses his gratitude to the Publishers, who generously ordered and supervised the translation from French for a number of chapters.

1

General survey of Arabic astronomy

RÉGIS MORELON

Interest in astronomy has been a constant feature of Arabic culture since the end of the second century AH (eighth century AD), and it is the quantity of study which strikes us first when we begin exploring this subject: the number of scientists who have worked on theoretical astronomy, the number of treatises which have been written in this field, the number of private or public observatories which have been successively active and the number of precise observations recorded there between the ninth and the fifteenth centuries.

This chapter is exclusively concerned with astronomy as an exact science, without considering the question of astrology. In fact, although the same authors sometimes wrote treatises in both disciplines, they never mixed purely astronomical reasoning and purely astrological reasoning in the same book and in most cases the titles of the works indicate unambiguously whether their contents relate to one discipline or the other.

The science of astronomy is chiefly defined by two terms: *ʿilm al-falak*, or 'science of the celestial orb', and *ʿilm al-hayʾa*, or 'science of the structure (of the universe)'; the second term can be translated in many cases as 'cosmography'. In addition, many astronomical works are identified by the word *zīj*, a term of Persian origin corresponding to the Greek *kanôn*; in its proper sense it denotes collections of tables of motion for the stars, introduced by explanatory diagrams which enable their compilation; but it is also often used as a generic term for major astronomical treatises which include tables.[1]

The astronomical term which is generally used to refer to the stars is *kawkab, kawākib*, while a word of similar meaning, *najm, nujūm*, has a more astrological connotation, and astrology is described with the aid of expressions based on the latter term: *ʿilm aḥkām al-nujūm, ṣināʿat al-nujūm, tanjīm* . . . ;[2] however, *ʿilm al-nujūm*, 'the science of the stars', can include both astronomy and astrology, as two different approaches to the same reality.[3]

In the Arabian peninsula, as in all of the ancient Near East, traditions of observing the heavens went back a very long way; one of these traditions is of particular note, having become well-known through its revival in what Arab astronomers called the *Treatises on the Anwā'*.

The term *anwā'* is the plural of *naw'*; it describes a system of computation associated with observation of the heliacal risings and acronycal settings of certain groups of stars, permitting the division of the solar year into precise periods. The appearance of stars on the horizon at a given time of year was considered to be a sign of meteorological phenomena signalling a change of weather, so much so that the term *naw'* acquired the meaning of rain or storm. A brief reminder of the heliacal risings and acronycal settings of the fixed stars is contained in Figure 1.1, which shows a rough projection on the prime vertical of the apparent trajectory of the sun.

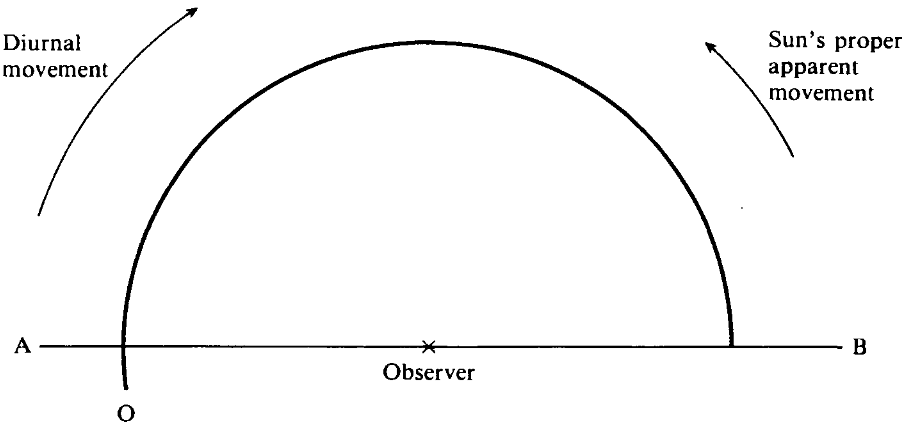


Figure 1.1

AB is the line of the horizon and O is the position of the sun under the horizon before sunrise, so that a star at A, next to the ecliptic, is at the limit of visibility when it rises, and a star at B is at the limit of visibility when it sets, according to the luminosity of the sky on the horizon just before sunrise. This situation shows the heliacal rising of star A and the acronycal setting of star B. The next day, because of the 'apparent movement of the sun' (approximately one degree per day), the sun will be further away from the horizon when A and B are in the same situation, and these two stars will be more visible since the horizon will be less luminous. About six months later, A and B will have exchanged their positions and B will be rising with A setting.

Originally the observation of these phenomena for definite groups of stars allowed the solar year to be divided into fixed periods, probably twenty-eight in number. After the eighth century, under the influence of Indian tradition, this system of calculation became combined with that of the twenty-eight 'lunar mansions' (*manāzil al-qamar*), groups of fixed stars close to the ecliptic, delineating the zones of the sky in which the moon is found night by night during the lunar month. The *Treatises on the Anwā'* which have been handed down – in written form from the ninth century – are like a series of almanacs giving the solar calendar dates for the heliacal risings and acronycal settings of stars which correspond to the lunar mansions, together with the meteorological phenomena that are traditionally associated with them. Under this system the year was divided into twenty-eight periods of thirteen or fourteen days.[4]

This ancient tradition, empirical in origin, was revived as a scientific procedure by Arab astronomers within the framework of their studies concerning the appearance and disappearance of stars on the horizon at the moment of the rising or setting of the sun, which were based in part on the *Phaseis* by Ptolemy, discussed below.[5]

SOURCES OF ARABIC ASTRONOMY

The first scientific astronomical texts translated into Arabic in the eighth century were of Indian and Persian origin, and in the ninth century, Greek sources took precedence. We shall discuss them in chronological order, starting with texts in Greek.

Greek sources

Greek texts were of two types: 'physical' astronomy, in the old sense of the word, and 'mathematical' astronomy.

The aim of 'physical' astronomy was to arrive at a global physical representation of the universe by means of purely qualitative thought; this astronomy was dominated by the influence of Aristotle, with his coherent organization of the world into concentric moving spheres, ranging from a common centre, the earth, and stable at that point. The first celestial sphere was that of the moon – the sub-lunar world being one of generation and corruption, the supra-lunar world one of permanence and uniform circular motion, the only motion that could befit the perfection of the celestial bodies – while each star had its own sphere to move it, and so on out to the sphere of the fixed stars which enclosed the universe.

'Mathematical' astronomy sought a purely theoretical, geometrical representation of the universe, based on precise numerical observations, disregarding if necessary its compatibility with a coherent world of the 'physical' type: to find the geometrical parametric models capable of accounting for measured celestial phenomena, enabling the calculation of the position of the stars at a given moment and the compilation of tables of their movements.

The history of ancient scientific astronomy is built in part on the tension between these two approaches to the same science.

'Mathematical' astronomy developed within the framework of Hellenistic astronomy – especially from the time of Hipparchus (*fl.* 160–126 BC), adapting the work of Apollonius from the previous century – but it was the work of Ptolemy in the second century AD which represented its crowning achievement in the Greek language.

Ptolemy is the scientist whose works have been the most studied, revised, commented on and criticized by later astronomers, until the seventeenth century. His four works on astronomy, in the order of their composition, are the *Almagest*, the *Planetary Hypotheses*, the *Phaseis* and the *Handy Tables*. The first two are the most important.

The *Almagest*, or *Great Mathematical Compendium*, handed down in the original Greek and in several Arabic translations, is regarded as the standard manual, which has served astronomy in the same way as Euclid's *Elements* served mathematics. Suffice it to say that within this monumental work of thirteen volumes Ptolemy synthesized the research of his predecessors, modifying it according to his own observations, and refining the old geometrical models or creating others. It was no accident that the word 'mathematical' was included in the title of the work, because Ptolemy made little reference therein to the 'physical' situation of the universe, even though he took this implicitly into account; he established and detailed the geometrical procedures capable of accounting for observed phenomena, on the basis of two postulates of ancient astronomy: the earth is stable at the centre of the world, and all celestial motion must be explained by a combination of uniform circular movements. He defined his method thus:

1 To collect the greatest possible number of precise observations
2 To identify anomalies in the movements thus observed in relation to uniform circular motion
3 To determine experimentally the laws governing the periods and the magnitudes of the anomalies
4 To combine uniform circular motions with the aid of concentric or eccentric circles and epicycles to account for the observed phenomena

5 To calculate the parameters of these movements in order to compose tables for calculating the positions of stars.

Ptolemy's method was therefore defined very precisely, but his desire to 'save the phenomena' led him in practice to infringe certain of his basic principles and to allow empiricism to intrude on some of his demonstrations, as he states himself in the last volume of his work: 'Each of us must endeavour to make the simplest hypotheses agree with the celestial movements as best he can, but if this is not possible he must adopt the hypotheses which fit the facts'.

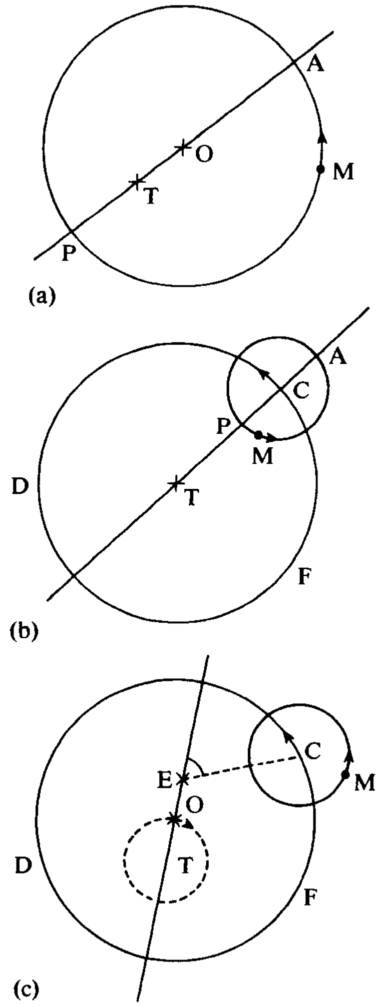


Figure 1.2

Ptolemy based the research for his geometrical models on work carried out by Hipparchus – drawing in turn from Apollonius – when he had developed the system of epicycles and eccentrics. Let the earth be stationary at T, the position of the observer. In the simple eccentric system (Figure 1.2(a)) a star at M travels on the circle MAP in uniform circular motion about the centre O, but the observer notices a different apparent speed when the star is at the apogee A or the perigee P. This geometric model can be applied to account for the apparent movement of the sun. In the simple epicycle system (Figure 1.2(b)), we imagine the observer at T, the centre of a circle CDF (the deferent), on which there travels a small circle with centre C (the epicycle), on the circumference of which moves a star M, the two circular motions being uniform and the angular speed of the centre C corresponding to the mean motion of the planet. This epicycle system, like that of the eccentric, can explain the difference in distance to the earth, but, above all, it can account for the apparent retrograde motion of the planets in a much more convincing way than a pure system of concentric physical spheres: when the planet is at P and its apparent angular speed on the epicycle is greater than that of C, it has an apparent retrograde motion; on the other hand, when it is at A, the two speeds sum and, to the observer at T, it appears to move faster than C.

This system of epicycles is very versatile and lends itself to a more complex combination of the elements concerned: the deferent CDF can be considered as eccentric with respect to the earth (Figure 1.2(c)), and makes in its turn a circular movement around T. One can thus arrive at highly complicated models, such as that of the moon or Mercury. For the larger planets (Mars, Jupiter, Saturn), Ptolemy takes an eccentric deferent CDF, with centre O, with the observer still situated at T, but he asserts that the uniform motion of the centre C of the epicycle is not around O but around the point E such that O is in the middle of TE; the point E is called the 'equant point'. This expedient leads to a better agreement between the theoretical model and the observations but contradicts the basic principle of uniform circular motion.[6]

It is thus possible to find the position of different planets in the heavens; it only requires calculation, based on observations, of the different parameters in each case: eccentricities, relative size of the radii, and angular velocities on the different circles.

The *Planetary Hypotheses* has been preserved partly in Greek (a little less than a quarter of the work) but there is a complete Arabic version.[7] It is much shorter than the *Almagest*, and its general tone is very different. First, Ptolemy calculates the maximum and minimum distances of the stars in terms of the data in the *Almagest* and thus divides the universe into concentric zones, each corresponding to the area in which a given star could move, placing the spheres of fire, air, water and earth under the sphere of the moon, in accordance with Aristotle. Thereafter, his point of view becomes 'physical' in the Aristotelian sense of the term rather than 'mathematical'. He seeks to describe the form of the physical bodies within which the circles which account for the various movements can be conceived, as an expression of the constitution of the real physical universe. He divides the 'ether' into thick globes tangential to one another, recalling the Aristotelian system of homocentric spheres; but Ptolemy also uses eccentric spheres and adds a further arrangement of tori and discs. The result is a kind of highly complex compromise between a purely geometrical system and a coherent physical system such as that defined by Aristotle. Ptolemy had thus attempted to embody his theory in a concrete 'physical' system, but the *Planetary Hypotheses* was to have less influence than the *Almagest*, apart from his calculations of the distances and sizes of stars which would be largely accepted by later astronomers.

The *Phaseis* treats of the appearance and disappearance of fixed stars just before sunrise or just after sunset (heliacal rising and acronycal setting). This work is in two parts, only the second of which is preserved in Greek and which contains a calendar of appearances and disappearances of stars on the horizon in the course of the year. The contents of the first part, a purely theoretical analysis of this particular phenomenon, is only known through an Arabic text.[8]

The *Handy Tables* has been handed down in Greek in Theon of Alexandria's fourth-century *Commentary on the Handy Tables*. It represents a rethinking in practical form of the theoretical results of the *Almagest* through the creation of detailed tables, with modification of certain parameters in accordance with the results in the *Planetary Hypotheses* and in the *Phaseis*.

All these works are cited by Arab astronomers as far back as the ninth century, together with the commentaries on the *Almagest* composed by Pappus and by Theon of Alexandria, and also a series of Greek treatises known as the 'Small astronomy collection' because it was regarded as an introduction to the reading of the *Almagest*: the *Data*, the *Optics*, the *Catoptrica* and the *Phenomena* of Euclid;[9] the *Spherics*, *On Habitations* and *On Days and Nights* of Theodosius;[10] *On the Moving Sphere* and *On Risings and Settings* by Autolycus;[11] *On the Sizes and Distances of the Sun and Moon* by Aristarchus of Samos;[12] *On the Ascensions of Stars* of Hypsicles;[13] and the *Spherica* by Menelaus.[14]

Indian and Persian sources

Three Indian astronomical texts are cited by the first generation of Arab scientists: *Aryabhatiya*, written by Aryabhata in 499 and referred to by Arab authors under the title *al-arjabhar*; *Khandakhadyaka* by Brahmagupta (d. after 665), known in Arabic under the title *zīj al-arkand*; and *Mahassidhanta*, written towards the end of the seventh or at the beginning of the eighth century, which passed into Arabic under the title *Zīj al-Sindhind.*[15]

These texts are based on the yearly cycles corresponding to Indian cosmology, and their scientific tradition is linked with an earlier period of Hellenistic astronomy than that of Ptolemy; they thus preserve a certain number of elements that can be traced back to the time of Hipparchus. They contain few theoretical developments but methods of calculation for creating tables and numerous parameters of the movement of stars. The major scientific innovation of the Indian scientists in this field is the introduction of the *sine* (half-chord of the double arc) in trigonometric calculations, which makes these much less cumbersome than the chords of arcs used in Greek astronomy since Hipparchus (see vol. II, chapter 15).

In Persia, under the Sasanids (AD 226–651), some activity in scientific astronomy developed in the Pahlavi language, under both Indian and Greek influence (Ptolemy's *Almagest* was translated into Pahlavi in the third century). This work seems to have been primarily oriented toward astrology, and the only traces which remain are found in Arabic texts from the end of the eighth century onward; these refer in particular to the 'Royal tables' (*zīj al-Shāh*), several successive versions of which are reported: from 450, 556 and 630 or 640 (under Yazdegerd III). These tables depended principally on Indian parameters.[16]

The chapters which follow detail how the Arab astronomers worked with these different sources.

OBSERVATIONS AND OBSERVATORIES

Small portable instruments and sundials are described in Chapters 4 and 5. Here we shall confine ourselves to a brief presentation of observatories and their large-scale instruments.[17]

Ibn Yūnus reports that astronomical observations were carried out at Gundīshāpūr at the end of the eighth century by al-Nihāwandī (d. AH 174 (AD 790)), whose work has been lost.[18] But the earliest precise observational results to have come down to us were recorded first in the al-Shammāsiyya quarter in Baghdad, and then on Mount Qāsiyūn at Damascus, in the final years of the reign of Caliph al-Ma'mūn (813–33) and through his impetus. They involved a precise programme dealing particularly with the sun and the moon, and at Damascus there was a complete year of continuous observation of the sun in AH 216–17 (AD 831–2). The work does not appear to have continued at these two sites after the death of al-Ma'mūn.

Apart from the numerical results found in later texts, we know little about these two observatories – their functioning, their size, etc. – except that Yaḥyā b. Abī Manṣūr, who was in charge of the observation work at Baghdad, belonged to the famous 'house of wisdom' (*bayt al-ḥikma*), and that the caliph himself had demanded that the instruments used should be the most precise possible. There is no explicit mention of the type of instruments used, but the form of the results and the kind of observations carried out are the same as Ptolemy's, which indicates that the instruments were similar to those described in the *Almagest*, i.e. the equatorial or equinoctial armilla, the meridian armilla, the equatorial quadrant (the plinth), the parallactic rods, the large gnomons, the dioptra of Hipparchus for measuring apparent diameters, and the armillary sphere (Singer *et al.* 1957: III, 586–601); these were the classic instruments of ancient astronomy and were gradually improved by Arab scientists, who sought in particular to construct larger and larger circles to achieve greater precision.[19]

In the wake of the first series at Baghdad and Damascus, a number of other observations were recorded during the course of the ninth century by Ḥabash al-Ḥāsib, the Banū Mūsā, al-Māhānī, Sinān b. Thābit, etc. In the majority of cases only the place is mentioned (Baghdad, Damascus, Sāmarrā or Nishāpūr, for example) with no indication of the setting in which these observations were made, which indicates that they were carried out from private observatories, outside any collective structure.

All these accumulated observations had not yet been organized systematically, but, by way of comparison, it should be noted that Ptolemy based all the work in his *Almagest* on ninety-four observations made between 720 BC and AD 141, the oldest having been recorded in Babylon and the latest (thirty-five in all) being due to Ptolemy himself (Pedersen 1974: 408–22). It is therefore evident that, from the ninth century, the Arabic astronomers had at their disposal the results of a far greater number of recent observations than those available to Ptolemy when creating his work.

At the turn of the ninth and tenth centuries, al-Battānī emerged as one of the major observers of the first period of the history of Arabic astronomy. For a period of about thirty years he followed a systematic programme of observations at Raqqa in the north of present-day Syria, and in the context of locating the first crescent moon on the horizon, he made what appears to be the first reference to 'observation tubes' in an astronomical treatise in the Greco–Arabic tradition.[20] These tubes, without lenses, enabled the observer to focus on a part of the sky by eliminating light interference.[21] Al-Battānī only mentions them, but the work of al-Bīrūnī includes an exact description of this type of apparatus, in a section that is also dedicated to verifying the presence of the new crescent on the horizon:[22]

> This tube is fixed on a column and is capable of two movements: the first is the movement of the column itself, enabling one to turn the tube in all directions; the other is around an axis so that the tube moves in the plane of the circle of elevation in which it lies. The tube must be not less than five cubits in length and one cubit in section. The view is concentrated and strengthened because of the shadow of the tube and its darkness, augmented by its internal blackness. When the column is placed at the centre of the Indian circle, it can be turned round until the plumbline fixed at the end of the tube is in line with the azimuth of the crescent; then the other movement is used until the tube makes an angle with the surface of the earth equal to the height of the crescent; this is simple with a quadrant divided into 90 degrees attached to the column and turning with it parallel to the tube.

This observation tube, whose use is thus attested in the Arabic world from at least the end of the ninth or the beginning of the tenth century, passed into the medieval Latin West where it became a standard astronomical instrument.[23]

Numerous other observations were recorded in the East in the course of the tenth century. Let us briefly mention in particular the work carried out at the end of that century by al-Qūhī and Abū al-Wafā' al-Būzjānī from the large observatory built in the gardens of the royal palace at Baghdad under Sharaf al-Dawla (AH 372–9 (AD 982–9)); that of ʿAbd al-Raḥmān al-Ṣūfī (d. AH 376 (AD 986)), who systematically observed the fixed stars at Isfahan, measured their position, and published as a result his famous catalogue of stars, which was a complete revision of Ptolemy's;[24] and that of Ibn Yūnus at Cairo, at the turn of the tenth and eleventh centuries.[25] But let us look more closely at the observatory of Rayy.

It was at Rayy (12 km south of Teheran), in the reign of Fakhr al-Dawla (AH 366–87 (AD 977–97)) who subsidized it, that al-Khujandī (d. *c*. AH 390 (AD 1000)) devised and built a very large sextant for solar observations, based on the principle of the black box: a dark room with a small opening in the roof (Bruin 1969).

The building was oriented north–south along the meridian; it was composed of two parallel walls, 3.5 m apart, about 20 m in length and 10 m high (see Figure 1.3); it was devoid of light, but a small opening was made in the southern corner of the roof of the building. The ground was partially excavated between the two walls so that a sextant of 20 m radius could be drawn with the opening in the roof as its centre. The interior of the arc of the sextant was covered in copper plate where the image of the sun formed when it was at the meridian, and the markings permitted measurement of its height above the horizon or its distance at the zenith. Each degree measured approximately 35 cm; it was divided into 360 parts of 10 seconds each, and the image of the sun passing at the meridian formed a circle about 18 cm in diameter; by finding the centre of the circle, a precise angle could be read off the copper surface. In 994, al-Khujandī measured the obliquity of the ecliptic as 23; 32, 19 and the latitude of Rayy as 35; 34, 39, but we have no other point of reference to indicate for how long a period this sextant was used.

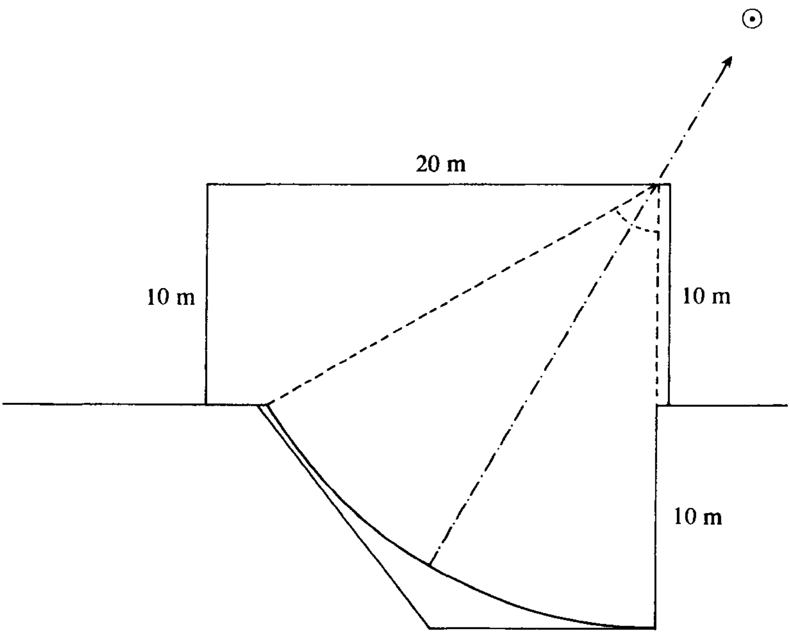


Figure 1.3

There are several allusions to large-scale instruments in various earlier observatories – for example, a construction of spherical shape, 12.5 m in diameter, in the observatory of Sharaf al-Dawla at Baghdad, for following the path of the sun – but the description of the great sextant at Rayy is the first to be given in such precise detail about a large-scale structure within the environment of a permanent observatory; most instruments of Hellenistic design were portable or could be made in one place and transported to another for ongoing use there, including large-sized copper circles or tubes like those of al-Battānī.

One other instrument of great size, cut into a permanent base of masonry, is described by Ibn Sīnā (AH 370–428 (AD 980–1037)) in his treatise *Maqāla fī al-ālāt al-raṣadiyya.*[26] On the top of a circular wall about 7 m in diameter lay a completely horizontal graduated circle. At the centre of the circle was a pillar bearing a double, vertically jointed rule, which could pivot horizontally around the centre. The lower rule lay on the graduated circle and allowed measurement of the azimuth; the upper rule carried a sighting system, and the angle between the two rules gave the height of the object observed. This construction was therefore based on a similar principle to that of the 'observation tube' described by al-Bīrūnī. About two centuries later, at Marāgha, Ibn Sīnā's instrument was further developed by the addition of a second set of jointed rules – or by an analogous arrangement of two vertical sighting devices pivoting independently around the centre of the large stone circle – enabling simultaneous measurement of the height and azimuth of two celestial objects.

The instrument described by Ibn Sīnā – and probably invented by him – is of particular interest because its new sighting system was much more precise than that of earlier instruments, giving independent readings of degrees and minutes. The upper rule was equipped with two identical

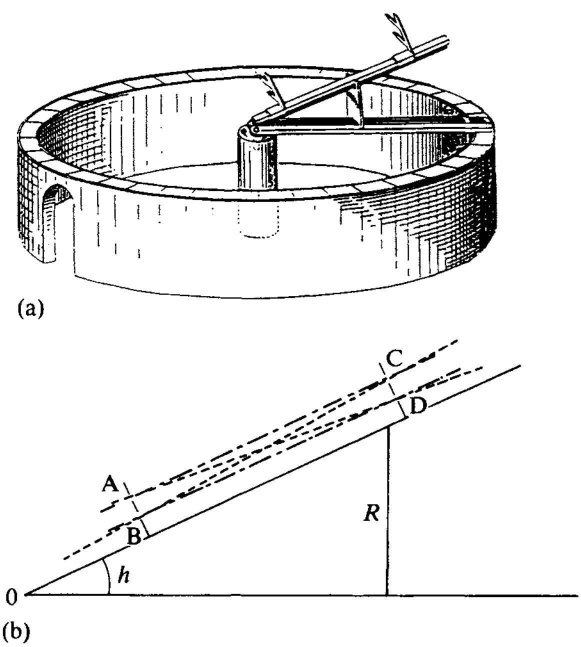


Figure 1.4

movable sights, each comprising two superposed aligning grooves (Figure 1.4(b)), A and B on the first sight, and C and D on the second, so that AB = CD. Calling the angle CAD a and the angle CBD b, we know these two angles by the respective positions of the two sights, read from the upper rule. If we focus on a star through the two grooves A and C – or B and D – the required height of the celestial object being observed will be the angle h, determined by the position of the smaller rule R on the lower rule. If we observe the same object through the two grooves A and D, the position of R will need to be altered to give an angle at O of value h_1 such that $h = h_1 - a$; if we sight through grooves B and C, we must again modify the angle at O to a value h_2 such that $h = h_2 + b$. It is therefore possible in this way to bring the small rule R to a position corresponding to the whole number of degrees that is closest – h_1 greater or h_2 less – to the true height of the observed object, and then to manipulate the position of the two sights to observe the star through A and D or B and C, so that one only has to subtract an angle a or add an angle b, according to the particular case, these angles being less than a degree and being accurately determined on the upper rule. The position of the scale small rule R thus gives the number of degrees, and the position of the sights AB and CD the number of minutes. This procedure represented a major advance in the precision of recorded measurements.

Around 1074, probably in the region of Isfahan, a large and highly organized observatory was founded by Malikshāh (AH 465–85 (AD 1072–92)), counting al-Khayyām in particular among its scientists. Observations there were planned to take place over thirty years, the period of one complete revolution of Saturn, the planet then considered to be the most distant from the earth (Sayili 1960: 160–6). In fact it only operated for eighteen years, until the death of its founder, but it was the first official observatory to have had such long continuous activity backed by such a precisely planned structure, and it was specifically in this tradition that the well-documented Marāgha observatory was constructed in the second half of the thirteenth century, marking an important turning point in the history of Arabic astronomy (Sayili 1960: 188–223; Vardjavand 1980).

The observatory at Marāgha (in northwest modern Iran) enabled the creation of a new set of astronomical tables, known as the 'Ilkhanian tables' but above all it gave the scientists who worked there the opportunity of producing better geometrical models than those of Ptolemy to account for the movements of celestial bodies, thanks to the high quality of its instruments, the rigorous organization of the work and the number of extremely high-calibre researchers who were able to work there simultaneously. Naṣīr al-Dīn al-Ṭūsī (AH 597–672 (AD 1201–74)) had chief responsibility for the work, and al-ʿUrḍī (d. AH 664 (AD 1266)) undertook the design of the instruments. The building was financed by Hūlāgū Khān (d. AH 663 (AD 1265)), who assigned the observatory large sums of revenue from a protected legacy (*waqf*) for its maintenance. This is the first time, to our knowledge, that an observatory was accorded this privilege, and it explains how work was able to continue there following the death of its founder Hūlāgū, finances not having been abruptly terminated by the disappearance of the princely patron, as had happened with the observatory of Malikshāh, for example.

Building began at Marāgha in AH 657 (AD 1259) and seems to have been completed in AH 661 (AD 1263). The group of buildings was situated over an area of 280 m × 220 m; in addition to the various instruments, it included a very important scientific library and a foundry for the construction of the copper apparatus. The instruments designed by al-ʿUrḍī were those that were already known, improved in size and precision, except for one which seems to have been created for Marāgha: the azimuthal circle equipped with two quadrants, permitting the simultaneous measurement of the height of two stars above the horizon.

A programme of continuous observations was intended by Naṣīr al-Dīn al-Ṭūsī to last for thirty years, as at the observatory of Malikshāh and for the same reason, but was reduced to twelve years, the period of rotation of Jupiter, and the 'Ilkhanian tables' were in fact published after this period. A great many scientists worked at Marāgha – the most famous being Naṣīr al-Dīn al-Ṭūsī and Mu'ayyid al-Dīn al-ʿUrḍī themselves, and Muḥyī al-Dīn al-Maghribī and Quṭb al-Dīn al-Shīrāzī, who will be covered in the following chapters – all of whom participated in the task of extending the astronomy of Ptolemy. Thus a veritable 'school' grew up around Marāgha which would have an important influence on all later developments in astronomy in the East.

Traces of activity at the observatory last until AH 715 (AD 1316), the date of the death of its last known director, Aṣīl al-Dīn, who was in charge from AH 704 (AD 1304), but the buildings were in ruins by about 1350. We are therefore sure that Marāgha functioned for more than fifty years, although it is not possible at present to date the ending of work at the site precisely.

This observatory had a marked influence, not only due to the importance of the scientific work that it nurtured, which will be explained below, but also because it acted as a model for the large observatories built later, of which the most celebrated, because of the quality of their instruments, were those at Samarkand and Istanbul. The observatory at Samarkand was founded in AH 823 (AD 1420) by the sovereign Ulugh Beg, who was also a scientist of high standing, and it remained active until nearly 1500 (Sédillot 1853). The one at Istanbul was built by the astronomer Taqī al-Dīn from AH 982 (AD 1575) and only functioned for a few years (Sayili 1960:

259–305). The last great observatories in the Marāgha tradition were founded in India in the eighteenth century by Jaï Singh, notably the one at Jaipur (1740), most of whose instruments are still in place.

This brief survey has offered us some idea of the evolution of observatories in the East. In the Muslin West, Andalusia and the Maghreb, astronomical observation was far less developed; it did not form part of an ongoing tradition and there is no trace of organized public observatories. The only precise observations that have survived were carried out from private observatories, at the end of the fourth century AH (tenth century AD) by Maslama al-Majrīṭī and in the fifth century AH (eleventh century AD) by al-Zarqāllu, whose 'Toledan tables' had a marked influence in the medieval Latin West.[27]

PROBLEMS OF PRACTICAL ASTRONOMY

From the end of the eighth century, with the development of the exact sciences in the particular context of an organized Muslim society, scientists from various disciplines were called upon to resolve a number of practical questions relating to social or religious matters. It therefore fell to astronomers, for example, to respond technically to the demands of the astrologers, whose official social role was important; the astronomical tables for calculating the position of the heavenly bodies were set up in part for this purpose. But above all the astronomers were required to help solve practical problems of calendars, time, or bearings on land or sea. This is illustrated by Ibn Yūnus at the start of his 'Hakemite tables' written at the beginning of the eleventh century:

> The observation of heavenly bodies is connected with religious law, since it permits knowledge of the time of prayer, of the time of sunrise which marks the prohibition of drinking and eating for him who fasts, of the moment when daybreak finishes, of the time of sunset whose ending marks the start of the evening meal and cessation of religious obligations, and moreover knowledge of the moment of eclipses so that the corresponding prayers can be made, and also knowledge of the direction of the Kaʿba (towards Mecca) for all those who pray, and equally knowledge of the beginning of the months and of days involving doubt, and knowledge of the time of sowing, of the pollination of trees and the harvesting of fruit, and knowledge of the direction of one place from another, and of how to find one's way without going astray.[28]

All these subjects gave rise to important theoretical developments which went far beyond the bounds of the practical problems involved. They will be discussed in detail in the following chapters on gnomonics and the science of time, the question of the 'qibla' for determining the direction of Mecca from a given place, calculation of the visibility of the crescent, mathematical geography and the computation of the latitude and longitude of a place, nautical science for navigating at sea, etc. Let us give some attention here to the question of calendars.

In the Arab world, the official calendar is lunar. Year one of the Muslim era began on Friday 16 July AD 622, date of the Hijra (hence the European custom of referring to Muslim years as AH), and the lunar year is made up of twelve months of twenty-nine or thirty days; the change in date takes place at sunset, and the passage to the following month occurs when the first crescent moon is sighted on the horizon just after sunset. Ptolemy had passed on a very accurate value for the average length of the lunar month at a little over twenty-nine and a half days (by about forty-four minutes); a lunar year of twelve months is therefore equal on average to 354.367 days. This value was verified and re-adopted from the ninth century by Arab astronomers who then introduced a cycle of thirty years to create an official calendar with alternating months of twenty-nine and thirty days, eleven of the years in this cycle having an additional day in the last month (which normally consisted of twenty-nine days); these were the years 2, 5, 7, 10, 13, 16, 18, 21, 24, 26 and 29 of the cycle. The astronomic correspondence is thus closely respected in the long term, but the visibility of the first crescent on the horizon on the evening of the twenty-ninth day always brought in a change of month for the place where this observation was made, so that there could be a difference of one unit in the day of the month from one end of the Muslim world to the other. Although actual visibility of the crescent was required in principle by religious law, the question facing astronomers was how to calculate the visibility of the lunar crescent in advance at a given place on the evening of the twenty-ninth day of the month, whatever the reading on the official calendar (which is what Ibn Yūnus meant by 'days involving doubt' in the earlier quotation). This is a difficult problem in view of the number of parameters involved – celestial co-ordinates of the sun and the moon, apparent relative speed of these 'two luminaries', latitude of the place, brightness of the sky on the horizon, etc. – and numerous astronomers studied the question, thereby producing important theoretical developments concerning the visibility of heavenly bodies on the horizon just after sunset.

In Persia the solar calendar was always used in parallel with the lunar calendar and corresponded at first to 'the era of Yazdegerd' which began on 16 June AD 632. As in the 'Egyptian calendar' used by Ptolemy in the *Almagest*, the year was divided into twelve equal months of thirty days, and five extra days – six every four years for leap years – were added at the end of the year; these were called the 'epagomenes days' and allowed the legal year to coincide with the astronomical solar year. This is the calendar which was adopted from the beginning by the astronomers of Baghdad, because the solar cycle is at the basis of astronomical measurements, and it was easier to create tables of the movements of heavenly bodies for months that always equalled thirty days. But the length of the solar year is a little less than 365.25 days, and at the end of the eleventh century Jalāl al-Dawla Malikshāh – founder of the great observatory described above – asked the astronomers whom he had appointed to review the composition of this calendar and make the necessary corrections to avoid accumulating the slight discrepancy with the apparent movement of the sun. Thus began 'the era of Jalālī', instituted in AH 467 (AD 1075) and comprising eight leap years in thirty-three years – instead of the thirty-two years in the earlier computation – which corresponded well with the astronomical calculations. This correction was of the same order as the one which waited until 1582 in the West, when the Julian calendar changed to the Gregorian calendar.[29]

But, apart from what we have called practical astronomy, the most important contribution of Arab astronomers is found in the arena of pure theoretical astronomy, which is not unrelated to the above.

GREAT PERIODS IN THE HISTORY OF ARABIC ASTRONOMY

The history of Arabic astronomy can be broadly divided into two great periods, the eleventh century being at the turning point between the two.

From the ninth to the eleventh century, the work was almost exclusively in the area of geometrical models inherited from Ptolemy, reworked and criticized on the basis of new observations, and in the eleventh century Ibn al-Haytham (AH *c.* 354–430 (AD *c.* 965–1039)) made an evaluation of the scientific papers accumulated for two centuries in his work *al-Shukūk 'alā Baṭlamyūs* ('Doubts concerning Ptolemy').[30] He drew up a catalogue of all the still unresolved inconsistencies to be found in three of Ptolemy's works, the *Almagest*, the *Planetary Hypotheses* and the *Optics* – but without proposing solutions.

This critical assessment led to a temporary impasse, since solutions could only be found outside the framework in which astronomy had confined itself. Solutions of two very different kinds were therefore sought, one in the Muslim West and the other in the East.

In Andalusia there was a proposal to re-adopt Aristotelian principles by abandoning epicycles and eccentrics and returning to homocentric spheres, which would be much more consistent from a 'physical' astronomy point of view. The most characteristic representative of this school was al-Biṭrūjī (end of the twelfth century), but his bases were almost entirely philosophical, and it was impossible to make any calculations from his conclusions or to verify them by numerical observations. This approach was therefore unproductive, even though the underlying philosophical processes remain interesting.

In the East the response was scientific and gave rise to what we have called the second great period of Arabic astronomy when the search took place to account for the movement of heavenly bodies by means of new geometrical models of epicycles and eccentrics that were geocentric but non-Ptolemaic. The essential part of that work was carried out by the team connected with the Marāgha observatory, described above.

The history of the development of theoretical astronomy in the Arab world is therefore divided by the two following chapters in accordance with the two great Eastern periods, and the work of the astronomers in the Muslim West is described in the chapter on Arab science in Andalusia (chapter 7).

NOTES

1 For example, al-Battānī's important work, *al-Zīj al-Ṣābī*, or al-Bīrūnī's *Al-Qānūn al-Mas*ʿ*ūdī* – where a transcription of the Greek term is retained – cited in the bibliography; see also the following chapter.

2 See Rashed's note on the term *munajjim* in Diophante (1984: vol. III, pp. 99–102).

3 See, for example, Abū ʿAbd Allāh al-Khwārizmī, pp. 210ff.

4 For the *Anwā*', cf. C. A. Nallino (1911: 117–40, conferences 18 and 19) and *The Encyclopaedia of Islam*, I, pp. 523–5. For the lunar mansions, cf. 'Manāzil' in *The Encyclopaedia of Islam*, VI, pp. 374–6.

5 In particular Sinān b. Thabit b. Qurra (d. 331 AH (943 AD)) reproduced part of the second book of *Phaseis* in his *Kitāb al-Anwā*'; see Neugebauer (1971).

6 For a short and precise description of the geometrical planetary models proposed by Ptolemy in the *Almagest*, see Neugebauer (1957: appendix I, French translation, pp. 239–55).

7 See Ptolemy, *Planetary Hypotheses*. I have personally undertaken the edition of the Arabic version of this text (Morelon 1993).

8 The contents of this book were found described in a passage of the work by al-Bīrūnī, *al-Qānūn al-Mas*ʿ*ūdī*; see Morelon (1981).

9 Euclid lived around 300 BC; his *Data* contains diverse definitions of the elements involved in geometry; his *Optics* develops a theory of vision and of perspective; his *Catoptrica* is a study on mirrors; his *Phenomena* contains a geometrical study of the celestial sphere.

10 Theodosius lived in the second century BC; his *Spherics* concerns the geometry of the spheres; in *On Habitations* he shows which portions of the celestial sphere are visible according to the regions of the earth; in *On Days and Nights* he determines the portions of the ecliptic traversed by the sun each day over the whole year.

11 Autolycus lived in the third century BC; in *On the Moving Sphere* he describes the different circles of the celestial sphere and the modification of their respective

positions caused by the movement of the sphere; in *On Risings and Settings* he describes the phenomena of the visibility of the stars on the horizon at their rising or setting.

12 Aristarchus lived in the third century BC and is famous for having proposed a short-lived heliocentric hypothesis; in his treatise *On the Sizes and Distances of the Sun and Moon* he calculates their distance from the earth and their respective size based on their position in quadrature and on eclipses.

13 Hypsicles lived around 150 BC; in his *Ascensions* he determines the rising of the different signs of the zodiac for a given place in terms of the relation between the longest and shortest day at that place.

14 Menelaus lived in the first century AD; his book on the *Spherica* contains the fundamental formulae of spherical trigonometry used by Ptolemy in the *Almagest*, introducing equal proportions between the chords of arcs on a complete spherical quadrilateral (see the chapter on trigonometry in vol. II).

15 See al-Hāshimī, *Book of the Reasons*, pp. 201–11.

16 See 'Astrology and Astronomy in Iran' in *Encyclopedia Iranica* (1987: vol. II, pp. 858–71) and Kennedy (1958).

17 On the question of observatories, see Sayili (1960).

18 See Ibn Yūnus, *Le Livre*, pp. 140–1.

19 In particular at Baghdad and Damascus, from the time of the first observations.

20 See al-Battānī, *Al-Battānī*, vol. 3, pp. 137–8; vol. 1, pp. 91 and 272.

21 See Eisler (1949), 'The polar sighting tube'. These 'observation tubes' are not mentioned explicitly in any of the texts of Hellenistic astronomy that have come down to us, but they have been known in China since the sixth century; see Needham and Wang Ling (1959: 332–4).

22 Al-Bīrūnī, *Al-Qānūn*, p. 964, treatise 8, chapter 14, 2nd section.

23 See Eisler (1949), 'The polar sighting tube'.

24 See al-Ṣūfī, *Kitāb ṣuwar al-kawākib*.

25 See Ibn Yūnus, *Le Livre*.

26 Arabic text edited and translated into German with notes by Wiedemann-Juynboll. The following two figures are taken from this publication; the drawing of the instrument was made by J. Frank from data in the text and from the author's knowledge of the instruments of the observatory of Marāgha.

27 See the entries for these two scientists in the *Dictionary of Scientific Biography*.

28 Ibn Yūnus, *Le Livre*, pp. 60–1.

29 See 'Djalālī' in *The Encyclopaedia of Islam*, II, pp. 397–9.

30 Ibn al-Haytham, *Shukūk*.

2

Eastern Arabic astronomy between the eighth and the eleventh centuries

RÉGIS MORELON

Al-Qifṭī notes that the first Arab scientist to be interested in astronomy was Muḥammad b. Ibrāhīm al-Fazārī in the second half of the eighth century, at the beginning of the reign of the Abbasids.[1] His name is connected with a famous tradition according to which an Indian delegation with an astronomer in its ranks was received in Baghdad by Caliph al-Manṣūr around the year 770; the name of this astronomer is not known but the tradition reports that he had with him at least one astronomy text, written in Sanskrit, which was translated into Arabic under the title *Indian Astronomical Table* (*Zīj al-Sindhind*)[2] by al-Fazārī and Ya'qūb b. Ṭāriq[3] under the supervision and direction of this Indian astronomer. Whatever the historic value of this tradition as far as its details are concerned, the two mentioned authors have been presented by all their successors as the men who introduced scientific astronomy into the Arab world from its origins in India.

The works of al-Fazārī and Ya'qūb b. Ṭāriq are lost but a certain number of fragments survive in the work of later authors.[4] It is known that al-Fazārī wrote *The Great Indian Table* (*Zīj al-Sindhind al-kabīr*), and later quotations from this text show that he mixed Indian parameters with elements of Persian origin from *The Royal Table* (*Zīj al-Shāh*). We have traces of three works by Ya'qūb b. Ṭāriq: *Table Solved in India Degree by Degree* (*Zīj maḥlūl fī-l-Sindhind li-daraja daraja*), *The Composition of Orbs* (*Tarkīb al-aflāk*) and *The Book of Causes* (*Kitāb al-'ilal*); the basis of his reasoning in these is clearly the same as that of his contemporary. These two authors had the great merit of introducing scientific astronomy into the Arab world but their works, to judge from what remains of them, appear to be a compilation of elements which they had at their disposal, unverified by observation and without any attempt at proper internal coherence.

The first work of Arabic astronomy to have reached us in its entirety is that of Muḥammad b. Mūsā al-Khwārizmī (*c.* 800–*c.* 850) and is also called *Indian Astronomical Table* (*Zīj al-Sindhind*); it is in keeping with the preceding tradition but with the addition of elements from Ptolemaic astronomy. The Arabic text is lost and the work has been transmitted through a Latin translation made in the twelfth century by Adelard of Bath from a revision made in Andalusia by al-Majrīṭī (d. AH 398 (AD 1007–8)).[5]

Al-Khwārizmī is equally renowned as a mathematician for his work in algebra. His treatise on astronomy was written under al-Ma'mūn (813–33) and does not include any theoretical elements: it is a set of tables concerning the movements of the sun, the moon and the five known planets, introduced by an explanation of its practical use. Most of the parameters adopted are of Indian origin, and so are the methods of calculation described, including in particular the use of the sine. But some elements are taken from Ptolemy's *Handy Tables* (Neugebauer 1962a: 101–8), without any attempt by the author to achieve coherence between the differing results drawn from the Indians and from Ptolemy. Here we have the same problem as with al-Fazārī and Ya'qūb b. Ṭāriq in their simultaneous use of Indian and Persian sources.

During the ninth century, the Arab astronomers of Baghdad fairly rapidly assigned the Indian tradition, which comprised only methods of calculation and sets of parameters for the composition of tables, to second place in favour of Ptolemaic astronomy, which was well endowed with theoretical reasoning and therefore enabled the development of astronomy as an exact science. But the Indian tradition continued to have an influence of some significance in the compilation of astronomy tables in the Muslim West (Andalusia and Maghreb) (Kennedy and King 1982).

The introduction of Greek astronomy

The eleven short treatises in Greek listed in the preceding chapter, considered to be a preparation for the reading of Ptolemy and grouped under the title 'Small astronomy collection', were all translated during the ninth century by confirmed scientists with a sound knowledge of both Greek and Arabic: Ḥunayn b. Isḥāq (d. 877), his son Isḥāq b. Ḥunayn (d. 911), Thābit b. Qurra (d. 901) and Qusṭā b. Lūqā (d. *c.* 900).[6]

The four works of astronomy by Ptolemy mentioned in Chapter 1 were also translated into Arabic in the ninth century. The *Almagest* is the most important because of the influence it exerted.[7] Several successive translations were made, as noted by twelfth-century author Ibn al-Ṣalāḥ:

> There were five versions of *Almagest* in different languages and translations: a Syriac version which had been translated from the Greek, a second version translated from Greek to Arabic by al-Ḥasan b. Quraysh, for al-Ma'mūn, a third version translated from Greek to Arabic by al-Ḥajjāj b. Yūsuf b. Maṭar and Halyā b. Sarjūn, also for al-Ma'mūn, a fourth version translated from Greek to Arabic by Isḥāq b. Ḥunayn for Abū al-Ṣaqr b. Bulbul – we have Isḥāq's original in his own hand – and a fifth version revised by Thābit b. Qurra from the translation of Isḥāq b. Ḥunayn.[8]

Three of these versions have been lost: the first, an anonymous Syriac version; the second, the Arabic version of al-Ḥasan b. Quraysh, of which traces remain, particularly in the work of al-Battānī in the tenth century (Kunitzsch 1974: 60–4); and the fourth, the version of Isḥāq b. Ḥunayn before its revision by Thābit. At present we do have the third and the fifth versions, in manuscript form:[9] the translation made by al-Ḥajjāj at the behest of al-Ma'mūn around 827–8 and that of Isḥāq b. Ḥunayn revised by Thābit b. Qurra around 892, both translated from Greek to Arabic. Another revision, or in fact a re-writing, of the *Almagest* should be added to the list drawn up by Ibn al Ṣalāḥ: it was produced after that author's time, in the middle of the thirteenth century, using the Isḥāq–Thābit version, by Naṣīr al-Dīn al-Ṭūsī; it became widely known among Arabic-speaking astronomers from this period onwards.

Let us compare the two versions that we have from the ninth century. That of al-Ḥajjāj is very close to the Greek text, and the sentence structure of the original Greek is largely preserved; the scientific vocabulary used in the Arabic is sometimes vague, making it necessary in certain cases to return to the original text in order to understand the reasoning properly even though it is expressed in Arabic. These deficiencies in the translation of such a fundamental text led to the Isḥāq–Thābit version, made towards the end of the ninth century, after more than fifty years of work in scientific astronomy in the Hellenistic tradition. This version removes all need to refer to the Greek text: its language and vocabulary are perfectly clear and devoid of ambiguity. We thus have two precise points of reference from which to conclude that an Arabic scientific language was developed for astronomy between 827 and 892.

We do not have such precise information about the translation of the three other works of Ptolemy. The *Planetary Hypotheses* is cited in Arabic from at least the middle of the ninth century, under the title *Kitāb al-iqtiṣāṣ*, or *Kitāb al-manshūrāt* (especially by al-Bīrūnī). A complete but hitherto unpublished translation preserves the last three-quarters of the work which have been lost in the original Greek (see pp. 6–7). The name of the translator has not been passed on but one of the two complete manuscripts containing the work states that the text was corrected by Thābit b. Qurra.[10]

The *Book concerning the Appearance of the Fixed Stars*, or *Phaseis*, is quoted by Thābit b. Qurra under the title *Kitāb fī ẓuhūr al-kawākib al-thābita*. Thābit knew Greek and this reference is not enough to confirm that the text had been translated. But the Arabic translation is quoted by al-Mas'ūdī (d. *c*. AH 345 (AD 956),[11] and was used by Sinān b. Thābit (d. AH 332) (AD 943)) in his *Kitāb al-Anwā'* (see chapter 1). The Arabic translation of the *Phaseis* must therefore have been made at the beginning of the tenth century at the latest, and although we do not have the original translation, we have numerous references to it by Arab astronomers.

Ptolemy's *Handy Tables* was used by al-Khwārizmī, as we have seen, and then by Qusṭā b. Lūqā (in the mid-ninth century),[12] and we find traces of it subsequently in the work of many other authors, but we do not possess the Arabic translation and we have no knowledge of the circumstances in which it was produced.

We should add in the context of Ptolemaic astronomy that the comments on the *Almagest* by Theon of Alexandria were available in Arabic during the ninth century, since we find lengthy literal quotations from them in the work of astronomy *Kitāb fī-l-ṣinā'at al-'uẓmā* by Ya'qūb b. Isḥāq al-Kindī (d. *c*. 873).[13] The Arabic translation of Theon's work has not survived.

As we have said, the evolution of astronomy as an exact science beginning in Baghdad in the third century AH (ninth century AD) was primarily based on the works of Ptolemy. Only very few of the earliest extant Arabic studies of astronomy have yet been edited or undergone detailed scientific comment, and in most cases reference must be to the manuscript sources. All attempts at synthesis must therefore be provisional at this stage and will be continually subject to review in the light of any seriously edited and commented texts that may emerge. Here we shall take some examples of significant works and demonstrations in order to outline the first phase of the evolution of scientific astronomy in Arabic, concentrating more on the progressive transformation of methods of reasoning than on calculations of the different parameters for the movement of the stars despite their specific interest.

ARABIC ASTRONOMY IN THE EAST DURING THE NINTH CENTURY

As an introduction to the early development of Arabic astronomy in the East, we shall group the contributions of the different scientists who began work in this area into subjects of study, from the simplest to the most complex: the dissemination of Ptolemy's astronomy, the critical analysis of his results, and finally the rigorous mathematization of astronomical reasoning; an additional section is devoted to the celebrated astronomer al-Battānī, who worked at the turn of the ninth to tenth century in Raqqa.

The dissemination of Ptolemy's astronomy

From the first half of the ninth century, several treatises were written to present the findings of the *Almagest* in a simple manner, or to summarize it so that this fundamental work could reach the largest possible audience beyond the restricted circle of specialized astronomers. The best-known example of this type of literature is the book by Aḥmad b. Muḥammad b. Kathīr al-Farghānī. His was also the most widely distributed work – first in Arabic (as attested by the large number of manuscripts listed from all periods and all regions), then in Latin (there were two successive Latin translations in the twelfth century) – and it was passed on under several titles, the most common being *Compendium of the Science of the Stars* (*Kitāb fī jawāmiʿ ʿilm al-nujūm*).[14] We know little of the author, except that he was a member of the team of scientists formed by al-Ma'mūn (813–33) and that he died sometime after 861. His book was probably written after 833 and before 857. It is a sort of manual of cosmography, comprising about a hundred pages in its published version and containing thirty chapters in which al-Farghānī explains the state of the universe according to the findings of Ptolemy. It is purely descriptive without any mathematical demonstration. It contains, in the following order: a description of the computations for the months and years of the different eras (Arab, Syrian, Byzantine, Persian and Egyptian); the justification of the sphericity of the sky and of the earth, the latter being stationary at the centre of the universe whereas the sky has two circular movements; the inclination of the ecliptic over the equator; a description of the inhabited part of the earth with seven climates and the different regions and towns; the size of the earth; the movement of the seven 'wandering stars' both in longitude and latitude, explained by the model of eccentrics and epicycles; the precessional movement of the fixed stars; the distances of all the heavenly bodies from the earth and their sizes; the heliacal risings and acronycal settings; the phases of the moon, its parallax and the eclipses of the moon and sun.

This work thus sets out the principal problems of ancient scientific astronomy, which is why it became a subject of repeated commentary by scientists of the highest calibre, including in particular al-Bīrūnī.[15] Al-Farghānī uses Ptolemy as his virtually exclusive source, but corrects him on several points according to the results obtained by the astronomers of al-Ma'mūn, such as the rectification of the obliquity of the ecliptic from 23;51 to 23;33, the assertion that the apogees of both the sun and the moon follow the precessional movement of the fixed stars, and the use of the measurement of the earth's circumference determined under al-Ma'mūn. In addition, after claiming that Ptolemy had only calculated the distance and the size of the sun and the moon, which demonstrates that al-Farghānī knew the *Almagest* but not the *Planetary Hypotheses*, he gives numerical values identical to those in the latter book, without indicating the source from which he drew his data.

Several other works of a similar nature have survived, including in particular a treatise by Qusṭā b. Lūqā, as yet unpublished, and two more scientific treatments by Thābit b. Qurra, focusing especially on the movements of the heavenly bodies and taking their reasoning from the first part of the *Planetary Hypotheses.*[16]

These texts made scientific astronomy accessible, bringing together its findings in an understandable form that would nowadays be dubbed a 'popularization of the highest standard', produced by professional astronomers and widely diffused among the educated circles of the time. This tradition continued in all the accounts of the *Almagest* written by the authors of encyclopedias such as Ibn Sīnā, who included a summary in his great survey of philosophy, the *Shifā'*.

Critical analysis of Ptolemy's results

Once the *Almagest* became available in Arabic under al-Ma'mūn, the work of verifying its findings began, and this was the reason for the setting up of the first programme of astronomical observations in Baghdad and Damascus referred to in chapter 1. About 700 years lay between Ptolemy and the al-Ma'mūn astronomers, who found in the *Almagest* schemes of computation and tables permitting the theoretical calculation of the position of the celestial bodies for a given date. The results of these calculations, made for a period of 700 years, were set against the data from the observations recorded at Baghdad and Damascus, and a discrepancy was noted between the two sets of figures obtained.

This discrepancy, inevitable over such a long period of time, led the Baghdad astronomers not just to 're-set their clocks' – i.e. simply to add a correction to all the lines of a table so that it could be used again – but to re-examine the theoretical base of Ptolemy's results, in order to revise the mechanisms he had proposed and recalculate the parameters of the different movements. Three examples will serve to illustrate this work, which was undertaken from the beginning of the ninth century: the 'Verified table' (*al-Zīj al-mumtaḥan*), the 'Book on the solar year' (*Kitāb fī sanat al-shams*) and the work of Ḥabash al-Ḥāsib.

The 'Verified table'

The Arabic term *al-zīj al-mumtaḥan* is in itself a generic one, denoting a set of tables compiled on the basis of observations and thereby offering all possible guarantees of scientific accuracy. But the term 'Verified table', used alone, refers specifically to the first set of astronomical tables in Arabic based on the results of observations carried out at the observatories of Baghdad and Damascus; it was Yaḥyā b. Abī Manṣūr (d. AH 217 (AD 832)) who was nominated by al-Ma'mūn to co-ordinate this research. The tables had a great influence in so far as they continued the first series of precise scientific observations recorded since Ptolemy, within the same tradition of Hellenistic astronomy, and they were widely quoted by later Arab astronomers, such as Ibn Yūnus and al-Bīrūnī.

The complete original text has not come down to us.[17] However, the recorded results from it cited by later authors show that the different parameters of the motion of the heavenly bodies had been recalculated.[18] But the most important conclusion from the observations in these tables concerned the movement of the sun: they showed that the apogee of the solar orb was connected with the precession of the fixed stars, contrary to the view of Ptolemy, who considered this apogee to be subject to diurnal movement only.[19]

Although we cannot establish a clear relationship between this conclusion from the 'Verified table' and the 'Book on the solar year', it is the latter text that contains the demonstration of the link between the movement of the sun and that of the fixed stars.

The 'Book on the solar year'[20]

Manuscript tradition attributes this text to Thābit b. Qurra, but close critical analysis reveals that it pre-dates this author and that it probably came from within the team of researchers that formed around the Banū Mūsā prior to the arrival of Thābit, i.e. before the middle of the ninth century.

The author of this treatise defines his position in relation to Ptolemy with regard to the movement of the sun and the calculation of the length of the solar year. Figure 2.1 is a reminder of what the *Almagest* has to say on this subject.

E is the position of the observer on a stationary earth at the centre of the world. The sun moves in a uniform circular orb on an eccentric circle in relation to the earth: circle I, with centre D, of which the most important points are the apogee A and the perigee P. E is also the centre of the ecliptic circle II, the apparent trajectory of the sun in the sky during the year; the reference points of the ecliptic are the equinoxes B and C, and the solstices

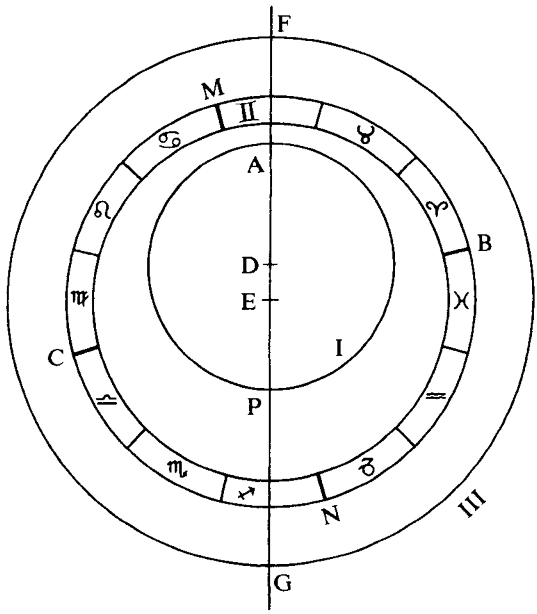


Figure 2.1

M and N. The common plane of these two circles intersects the sphere of the fixed stars according to circle III, also centred on E.

In one year, the sun completes one revolution on its eccentric orb I, with a uniform regular motion. The time of this complete revolution is constant, whatever its starting point. This is the value of the *anomalous year*, the time taken for the sun to return to the same point in its orb. This value is the only one that can be considered as a constant reference; but it is not directly measurable from E, as the eccentric in itself does not contain any sufficiently precise element of reference. The observer must first position circle I clearly in relation to circle II or circle III.

When we observe from E the motion of the sun on circle II, and we measure the time interval between two successive passages of the sun at the same point, for instance B, the spring equinox, we obtain the value of the *tropical year*.

When we observe from E the sun's movement on circle III, and measure the time interval between two successive conjunctions of the sun with the same star, we obtain the value of the *sidereal year*.

If the circles I, II and III were fixed relative to each other, the three values of the solar year defined above would be absolutely identical – but this is not the case. The problem for the ancient astronomers was to try to find, from observations of the irregular motion of the sun on circles II or III, the value of the anomalous year over orb I, the only absolute constant.

The study of the sun's movement is detailed in the third book of the *Almagest*. Here Ptolemy begins by noting, in line with Hipparchus, that the sidereal year is slightly longer than the tropical year, but he concentrates on the latter, in order to prove that this is the sought-for absolute constant. He thus makes the tropical year and the anomalous year coincide, causing circles I and II to combine while circle III moves in relation to them by the motion of the precession of the equinoxes, evaluated by Ptolemy at 1° per century.

To calculate the parameters of the eccentric solar orb, Ptolemy uses the model illustrated in Figure 2.2. ABCD is the circle of the ecliptic, with centre E, the position of the observer; the circle MNPO, with centre G, is the eccentric orb around which the sun moves, A and C are the two equinoxes and B is the summer solstice. The figure is completed by the straight lines MQGP and NGXO parallel to DEB and AEC respectively, and the straight line EGH which intersects the eccentric orb at H, its apogee. By observing the moment at which the sun passes over A, B and C, one can obtain, by a simple calculation based on the mean movement of the sun, the value of the arcs IL, IK, KL, IN, PK, LO on the eccentric,

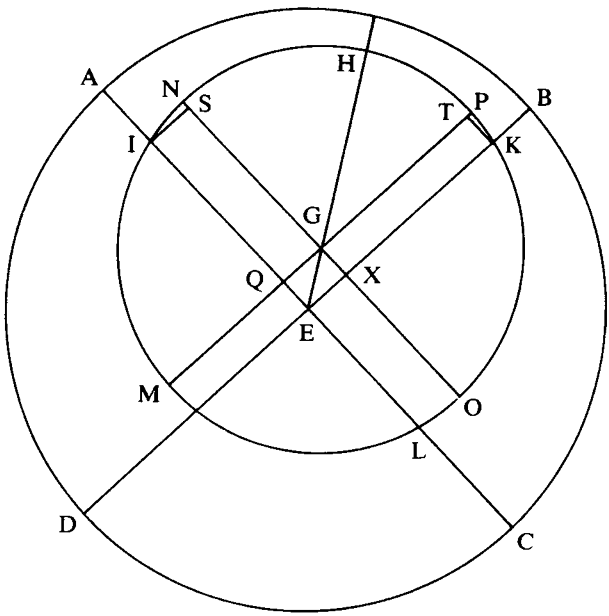


Figure 2.2

and consequently calculate all the parameters. Assuming by convention that the radius of the eccentric orb is 60 'parts', Ptolemy then finds that the eccentricity EG is equal to 2; 30 of these parts, that the apogee H is situated at 5; 30° of Gemini, that this apogee is fixed on the ecliptic, and that the length of the tropical solar year (passage of the sun over the same point of the ecliptic) is constant and equal to 365; 14, 48 days.

Following the observations made in Baghdad between 830 and 832 – some 700 years after the *Almagest* and 950 years after Hipparchus – the author of the 'Book on the solar year' notes that the sun's apogee in his era is at 20; 45° of Gemini and that this shift of 15; 15° since Hipparchus' observations is analogous to that due to the precession of the fixed stars, which had been measured on Regulus as 13; 10°, excepting observational errors of which the author was well aware. This therefore lends him to link circles I and III of Figure 2.1 and to conclude that the apogee of the eccentric orb of the sun is subject to precessional motion. The anomalous year is thus found not in the tropical year but in the sidereal year, the only absolute constant. But since the sidereal year is only a theoretical reference, it is necessary to derive the value of the tropical year from it, as the only practical reference which will permit the marking of terrestrial time throughout the year.

Since the eccentric orb shifts in relation to the circle of the ecliptic, we cannot measure the length of the tropical year directly by observing the time interval separating two successive passages of the sun over the same point of the ecliptic; the length of the tropical year can only be derived from a calculation based on the values of the sidereal year and the constant of precession. In fact, if one takes the mean motion of the sun along the eccentric from the apogee, it shifts slightly because of precessional movement and, to relate the mean motion to the ecliptic, one must add these two shifts which have a constant value.

In this way, the author of the 'Book on the solar year' radically challenges Ptolemy's conclusions, his computations and even the quality of his observations: he compares these last with his own and with those of Hipparchus, and deduces that all of Ptolemy's observations should be rejected in favour of a return to Hipparchus. He concludes his vigorous critique as follows:

> As well as the error of calculating the duration of the solar year from a point on the ecliptic, Ptolemy has created further error as a result of his observations themselves: he did not conduct them as they should have been conducted and it is this part of the error that has most seriously damaged the method of computation that he has proposed.[21]

Despite his criticisms, the author considers that Ptolemy's is still the best geometrical method for calculating the parameters of the sun's orb. He reworks the third book of the *Almagest*, quoting it at length by adopting its geometrical method, partly re-organizing the plan of the book while retaining its content, and relying solely on the observations of Hipparchus and himself. His calculation of the parameters of the solar orb is based on Ptolemy's model as shown in Figure 2.2, but he alters the orientation of the observations: A, B and C no longer correspond to the two observed equinoxes and one solstice for, as the author says:

> Given that observations of solstices are difficult, we shall not include any solstice observation results in our three measurements. In the three measurements from which he identified the solar anomaly, Ptolemy included one measure of the summer solstice. We do not agree with this; on the contrary, we consider that this gave him little safety from error.[22]

During a solstice, the variation of the sun's declination is effectively very slight, and it was difficult to determine the exact moment of the sun's passage at this point. The author of the treatise therefore changes the three observations by 45° and measures the passage of the sun on the ecliptic in the middle of Aquarius, Taurus and Leo. He reformulates the method of computation in the *Almagest* by 'modernizing' it, i.e. by reasoning with sines of arcs instead of with their chords (see vol. II, chapter 15), and he obtains the following results:[23]

Position of the apogee of the sun 20; 54° of Gemini (22; 53°)
Constant of precession 0; 0, 49, 39° per year (0; 0, 50, 1)
Sidereal year 365; 15, 23, 34, 33 days (365; 15, 22, 53, 59)
Tropical year 365; 14, 33, 12 days (365; 14, 32, 9, 20)
Eccentricity of the solar orb 2; 6, 40

In addition to the good level of accuracy of the preceding results, bearing in mind the means of observation of the period, the 'Book on the solar year' is extremely important for the understanding of how Arabic astronomy first developed from the heritage of Ptolemy. This treatise was written in the first half of the ninth century, thus shortly after the Arabic translation of the *Almagest* by al-Ḥajjāj, which is liberally quoted in over a third of the text. It enables us to see first how some Arab astronomers of the first generation used this fundamental text, and second to identify a certain number of scientific innovations which became established as a result of their work.

To summarize the foregoing, we see that the author of the 'Book on the solar year' concludes, on the one hand, that Ptolemy has made some errors of computation, particularly over the precession constant, and on the other hand, that his observations are less reliable than those of Hipparchus. He therefore dismisses the observations and the results. After having established the displacement of the solar apogee and its relationship with the precession of the fixed stars, he works out a method which will permit him to determine the time it will take the sun to return to the same star, in order to calculate the length of the sidereal year. He retains Ptolemy's geometrical reasoning, and all the subjects treated in *Almagest* III, slightly modifying the order by moving two chapters, and redrafts all these elements. The result suggests that the 'Book on the solar year' was not composed as an isolated treatise, but was part of a vast project aimed at rewriting the *Almagest*, keeping its structure and its theoretical reasoning but eliminating Ptolemy's observations and calculations; its author keeps Hipparchus's observations in order to compare them with recent observations made in Baghdad and Damascus, and he creates new methods of computation based on the theories proposed by Ptolemy.[24] We do not know how far this project of a 'new *Almagest*' may have been pursued, but the content and structure of the book we have been discussing clearly demonstrate that this important work was started in Baghdad in the first half of the ninth century in the school formed around the Banū Mūsā.

The 'Book on the solar year' also contains a number of innovations which were adopted by later astronomers. First, following the composition of this treatise it was accepted that the apogee of the solar orb moved in relation to the ecliptic, and that a relationship needed to be established between the sidereal year, the constant of precession and the tropical year (although it would be the end of the eleventh century before the Andalusian astronomer al-Zarqāllu calculated the real supplementary movement of the solar apogee at 19 minutes per century). Next, contrary to Ptolemy, the author of the treatise connects the movement of the apogee of the sun's orb and that of the moon's orb with the precessional motion of the sphere of fixed stars, in the same manner as the apogee of the orbs of all the other planets; the motion of the sphere of the fixed stars therefore carries with it all the celestial spheres; the sun and the moon are not special cases in the universe. In this way, the circle of the ecliptic becomes a purely theoretical circle beyond the sphere of the fixed stars, located by the passage of terrestrial time and the rhythm of the seasons. Finally, the displacement by 45° of the three solar observations, introduced in order to avoid errors in solstice observation, were used by later astronomers in their calculations of the parameters of solar motion.[25]

The work of Ḥabash al-Ḥāsib

We know little about the life of Ḥabash. He was one of the astronomers of Caliph al-Ma'mūn, and he was alive in AH 254 (AD 859), as a calculation is attributed to his name in that year, but we do not know the date of his death. Only one of his original, but incomplete, works has been published:

his short treatise on the sizes and distances of the heavenly bodies, partially preserved in a single manuscript (Langermann 1985). A lengthy work of his, *al-Zīj al-dimashqī* ('Damascus tables'), has come down in two different versions, one to be found in Istanbul and the other in Berlin. The text of the Berlin manuscript has obviously been much revised by later hands; the Istanbul version appears to be quite close to Ḥabash's original work but has not yet been published.[26]

This work is in the Ptolemy tradition, but is not a reworking of the *Almagest* as the 'Book on the solar year' is in part. Ḥabash merely selects those areas which he considers susceptible to modification in the light of his own studies and of the data acquired from the early work of theoretical astronomy at Baghdad and Damascus. His text is therefore for use alongside the *Almagest* and is not intended to replace it. An important part of the 'Damascus tables' concerns trigonometry: Ḥabash 'modernizes' the theories of the *Almagest*, introducing sines, cosines and tangents in place of chords, and proposes complete formulae to be applied in the different astronomical computations. This will be examined in detail in volume II, chapter 15; here we shall consider certain points raised by Ḥabash concerning pure astronomy.

The first section of the 'Damascus tables' deals with chronology and the passage between the different eras for the calculation of the equivalences of dates – under Persian, Egyptian, Greek, Hegirian, etc. calendars – focusing on tables of concordance. But in addition Ḥabash sets out to draw up tables for the motion of the heavenly bodies based on the lunar year, recalculated with great care because this was the legal year in his society; however, this attempt was not pursued by Arab astronomers because the lunar year was much less well adapted to the computations and theories of astronomy than the solar year, which was used both in the Hellenistic world of Ptolemy and by the Persians with their regular months of thirty days.

Throughout his work, Ḥabash compares the parameters calculated by Ptolemy for the motion of the different heavenly bodies with his own calculations and systematically modifies the composition of his tables accordingly for each one, without returning to the theoretical aspect of the geometrical models. But Ḥabash's most important theoretical innovation occurs in his study of the visibility of the crescent moon.

This problem of the visibility of the crescent is not treated in the Greek tradition of astronomy, but some methods of calculation were elaborated in the Indian tradition. Before looking at Ḥabash's solution, let us consider two previous solutions based on various elements of reference on the celestial sphere.

In the situation of a stationary earth at the centre of the world, the sun and the moon each have their 'proper motion' daily, in the opposite direction to the diurnal motion, being slightly less than 1° with respect to the ecliptic for the sun and about 13° for the moon on either side of the ecliptic (its maximum latitude is 5°). Thus each month, the moon 'catches up' with the sun and overtakes it, the crescent then becoming visible again on the western horizon just after sunset, signalling the beginning of a new lunar month. Figure 2.3 shows the moon setting at D; its latitude is DG, and the sun is at O under the horizon. HDA is the horizon from the point of observation, E is the closest equinoctial point (here the autumn equinox), OGE is the ecliptic and MAE is the celestial equator. OM is the position of the horizon at the moment of sunset, OH represents the distance from the sun to the horizon when the moon is setting, OG is the longitudinal distance between the sun and the moon, and the angle A between the horizon and the equator is equal to the complement of the latitude of the place.

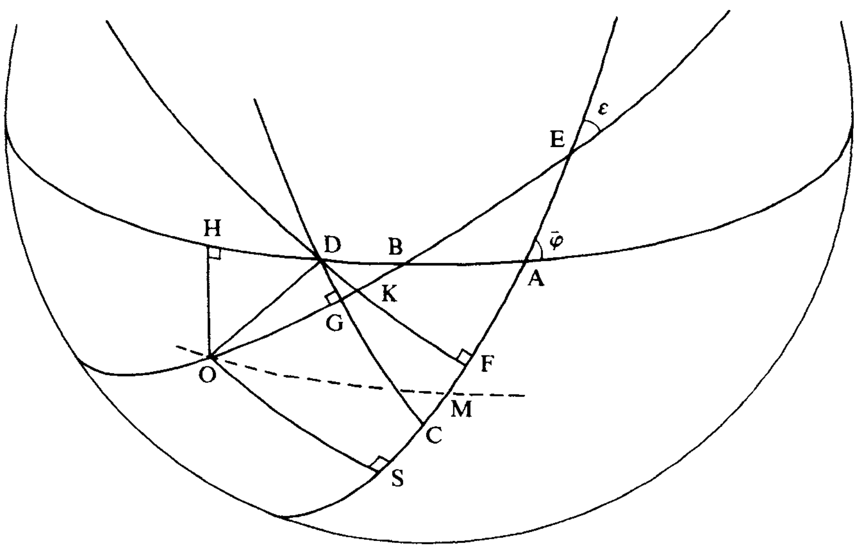


Figure 2.3

Yaʿqūb b. Ṭāriq and al-Khwārizmī, whom we mentioned earlier, both adopted an Indian solution based on the time that elapses between sunset and the setting of the moon, i.e. on the arc AM in Figure 2.3.[27] According to them, the crescent will be visible if the calculation for the chosen day shows this arc to equal at least 12°, which corresponds to an interval of 48 minutes between the setting of one and the other body.

Ḥabash follows the tradition created by Ptolemy in his study of the visibility of the fixed stars and of the planets on the horizon.[28] Ptolemy had never been concerned with the visibility of the crescent moon, but he had based his entire study of the visibility of the other heavenly bodies at their rising or setting on the luminosity of the atmosphere on the horizon, and thus on 'the arc of depression of the sun under the horizon' before its rise or after its setting – i.e. OH in the case of Figure 2.3. He had determined the value of this arc necessary for a given body to be visible on the horizon; the arc was later known in the Latin tradition as *arcus visionis*, or 'arc of visibility'. Ḥabash took this idea and applied it to the case of the moon; he determined as a result of observations and calculations that 'the arc of depression of the sun under the horizon', or 'arc of visibility of the crescent', OH, should have a value of at least 10° for the lunar crescent to be visible after sunset, on the twenty-ninth night of the lunar month.

This method of Ḥabash became famous; it was taken up unchanged some two centuries later by al-Bīrūnī, and cited by many subsequent authors as one of the typical means of approaching the difficult problem of the visibility of the crescent.

Ḥabash therefore emerges as an observer who studied the *Almagest* in order to verify its data, pursuing the work begun under al-Ma'mūn by the group which had compiled *al-Ẓīj al-mumtaḥan*; however, his work goes further than that of his immediate predecessors, as he also adapted and developed certain of Ptolemy's ideas after having completely assimilated them, but without tackling his theoretical demonstrations as such. Another author was to undertake that task, as we shall see in the following section.

The mathematization of astronomical reasoning

A single author is our subject here: Thābit b. Qurra. He was born probably in AH 209 (AD 824) and died in AH 288 (AD 901); he came from Ḥarrān in upper Mesopotamia and his mother tongue was Syriac but he knew Greek perfectly and his working language was Arabic. Joining the team of the Banū Mūsā in Baghdad, Thābit produced original works in all the known sciences of his time. Especially famous as a mathematician, he wrote more than thirty treatises on astronomy, nine of which have been handed down under his name, including the 'Book on the solar year' discussed earlier, which is incorrectly attributed to him; we therefore have only eight of his books from which to judge the work in astronomy of this author.[29] We shall look at just three of these eight treatises: the first concerning the theoretical study of the motion of a heavenly body on an eccentric; the second about the choice of time intervals for determining the different motions of the moon; and the third concerning a method of calculating the visibility of the crescent.

Theoretical study of the motion of a heavenly body on an eccentric[30]

When Ptolemy discusses the motion of the sun on its eccentric orb, he notes the inequality of its apparent motion:

> The greatest difference between the mean motion and the motion which appears irregular, the difference by which we know the passage of the heavenly bodies in their mean distances, occurs when the apparent distance from the apogee is a quarter of a circle and the heavenly body takes longer to go from the apogee to this mean position than from the latter to the perigee.[31]

Ptolemy thus establishes only that the slowest apparent motion occurs on the side of the apogee, the fastest on the side of the perigee, and that between the two there is a point of 'mean passage' at a quarter of a circle from the apogee.

Thābit studies this question and proves Ptolemy right. When any heavenly body, or the centre of an epicycle, moves along an eccentric ABC with centre D with uniform circular motion, this movement is observed from the earth at E on the circle of the ecliptic A′B′C′, and it is an irregular apparent movement. Thābit takes equal arcs on the eccentric, which are thus covered by the moving body in question in equal times: GF on either side of the apogee A, HI on either side of the perigee C, BK on the side of A and LM on the side of C (see Figure 2.4).

Based on the reasoning in Euclid's *Elements* he proves that the arcs of apparent motion observed on the ecliptic are such that $G'F' < B'K' < L'M' < H'I'$, so that he can conclude precisely:

> When the motion of a heavenly body or of the centre of any orb is uniform on an eccentric, its slowest apparent motion, on the ecliptic, will be produced when the moving body is at the apogee of its eccentric, and its fastest apparent movement will occur when it is at its perigee. For the rest, apparent motion is slower when it occurs close to the apogee than when it occurs far away from it.

Note that Thābit refers to the speed of a moving object at the apogee or the perigee. As far as we know, this is the first reference in history to the notion of speed at a point.

This was the first theorem of Thābit's treatise. The second theorem is no less important. Thābit takes an eccentric ABC with centre D, apogee A and perigee C, and he positions points B and F as 'those for which the distance to the apogee, in apparent motion on the ecliptic, is a quarter of a circle' (see Figure 2.5).

He then demonstrates, again with the help of reasoning from the *Elements* of Euclid, that the arc of mean motion IH, the sum of HB

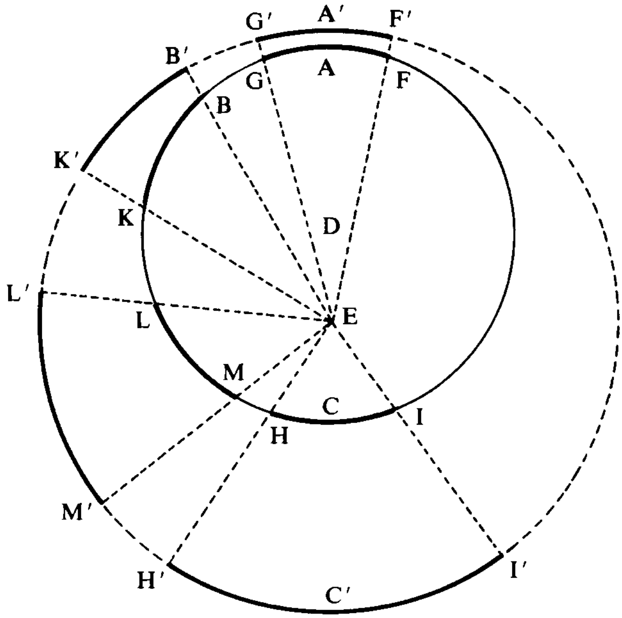


Figure 2.4

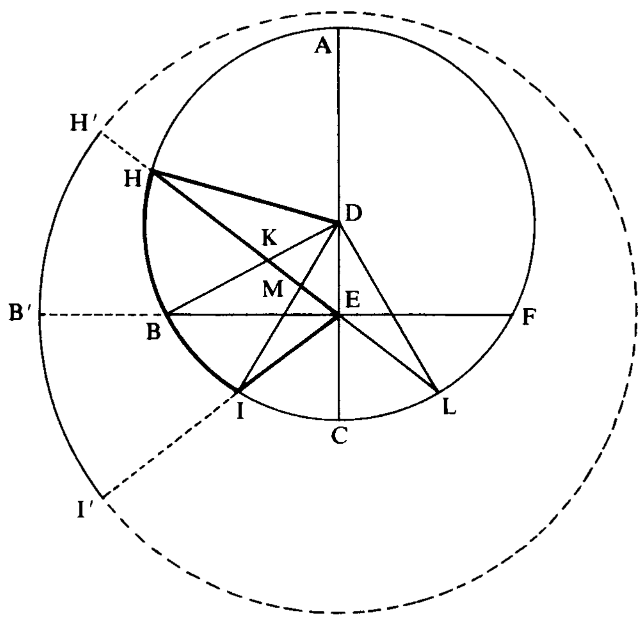


Figure 2.5

and BI, is equal to the arc I′H′, which is the sum of the arcs of apparent movement H′B′ and B′I′, that there is an 'approach to equality between the mean motion and the apparent motion when point B is approached ... and that the same is true when the motion occurs at point F'. He concludes, in conjunction with the preceding theorem:

> The closer the apparent motion to one of these two points, B or F, the closer it is to equality with the mean motion; and each time that two equal arcs of apparent motion on the ecliptic are taken on either side of each of these two points, their sum is in fact equal to the mean motion. These are the two points which resemble two points of mean motion.

This purely mathematical proof permits him to analyse precisely the apparent motion and the mean uniform motion relative to one another and to situate two axes: AC, an axis of symmetry for the mean uniform motion as observed from point E; and BF, an axis of symmetry for the apparent motion on the ecliptic. Thus for Thābit it became possible to analyse theoretically a geometric model as such that had been postulated to account for the movement of a heavenly body, using all the resources offered by the development of mathematics, leading him here to carry out the first mathematical analysis of a movement.

The choice of time intervals for determining the motions of the moon[32]

Here again Thābit takes a problem posed by Ptolemy, this time at the beginning of *Almagest* IV. Ptolemy based his whole study of lunar movements on the observations of lunar eclipses, as these could give the relative positions of the sun and moon without any error of parallax entering in to distort the results. The movement of the sun had been studied in *Almagest* III, and the next problem was to choose the time intervals at the limits of which lunar eclipses are found periodically, so as to ensure that the moon would have accomplished complete revolutions of each of its different orbs; once the number of these revolutions was known, the periodicity of the different movements of the moon could be determined. Before considering how Ptolemy solved this problem, we shall look at the way in which Thābit poses it.

He concentrates first on the sun, taking the two axes of symmetry determined in his preceding treatise for the movement of a body on an eccentric, shown in Figure 2.6 as AP and BC, O being the position of the observer at the centre of the ecliptic, and E the centre of the eccentric. In the first time interval t_1 the sun goes from M_1 to M_2, in a second interval $t_2 = t_1$ it goes from N_1 to N_2, and these two arcs of mean motion on the eccentric are then equal, $M_1M_2 = N_1N_2$, whereas the question remains for the corresponding arcs of apparent motion observed on the ecliptic, $M_1'M_2'$ and $N_1'N_2'$, the relationship between these two arcs being dependent on the respective position of M_1 and M_2 on the eccentric according to the results of the treatise described earlier (see note 30).

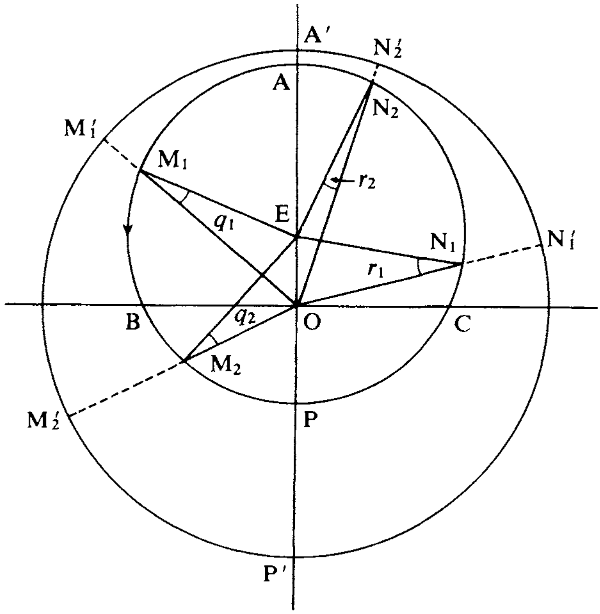


Figure 2.6

Let us call q_1 and q_2, r_1 and r_2 respectively the difference between mean motion and apparent motion for M_1 and M_2, N_1 and N_2; we obtain

$$M_1M_2 - M_1'M_2' = q_2 - q_1$$

and

$$N_1N_2 - N_1'N_2' = r_2 - r_1$$

Keeping $t_1 = t_2$, i.e. for two intervals of equal time, Thābit obtains seven types of combination between the two movements, which can be expressed in a purely theoretical way in terms of relations between $q_2 - q_1$ and $r_2 - r_1$, immediately applicable in the case of the sun.

1 In t_1 the sun leaves M_1 and returns to the same point after a number of complete revolutions; in t_2 it leaves N_1 and returns to it. Obviously we then have $q_1 = q_2$ and $r_1 = r_2$.

2 $q_2 - q_1 = r_2 - r_1 = 0$
3 $q_2 - q_1 = r_2 - r_1 > 0$
4 $q_2 - q_1 = r_2 - r_1 < 0$
5 $|q_2 - q_1| = |r_2 - r_1|$
but each is of opposite sign.
6 $q_2 - q_1 \neq r_2 - r_1$
7 $q_2 - q_1 = 0$ and $r_2 - r_1 \neq 0$

In these two equal time intervals, there is equality between the apparent movements for the cases 1 to 4, and inequality for cases 5 to 7; in cases 1 and 2 there is equality between the mean motion and the apparent motion (case 2 corresponds to the preceding second theorem). Figure 2.6 shows the general case of mode 6.

With the help of the two theorems from the preceding treatise, and with reference to the two axes of symmetry, we can easily situate the points M_1 and M_2, and N_1 and N_2, corresponding to the points of departure and arrival of the sun in the two equal time intervals for each of these seven cases.

The case of the moon is more complex in that it moves on an epicycle, itself mobile on an eccentric. But we are in the situation where eclipses of the moon are at the limits of the two time intervals cited, which allows us to relate the movement of the moon to that of the sun, since they are then in opposition, as shown in Figure 2.7.

The sun being at O and the earth at T, the moon on its epicycle can be found at L or L′ at the moment of opposition. In this situation, Thābit finds seven forms of combination for lunar motion, analogous to those of the sun. If the sun, in each of the two time intervals, has covered equal angular distances in apparent motion, so has the moon. But, in order for all its motions to be restored to its different orbs, it is necessary to eliminate the cases where the moon would pass from L to L′ on its epicycle between the two limits of the time intervals considered. Looking again at the seven cases, cases 5 to 7 are set aside because of the situation of the sun which presents unequal apparent movements at the limits of the time intervals, and cases 2 to 4 are eliminated since the moon would then pass from L to L′ on its epicycle. Only the first case is retained, when the moon and sun leave from the same point of the ecliptic to return there, because only in this situation will they have completed a number of revolutions on their various respective orbs.

Ptolemy had also discussed two analogous time intervals, choosing four cases for the sun.[33]

(a) In t_1 and t_2, it travels complete circles – equivalent to Thābit's case 1.

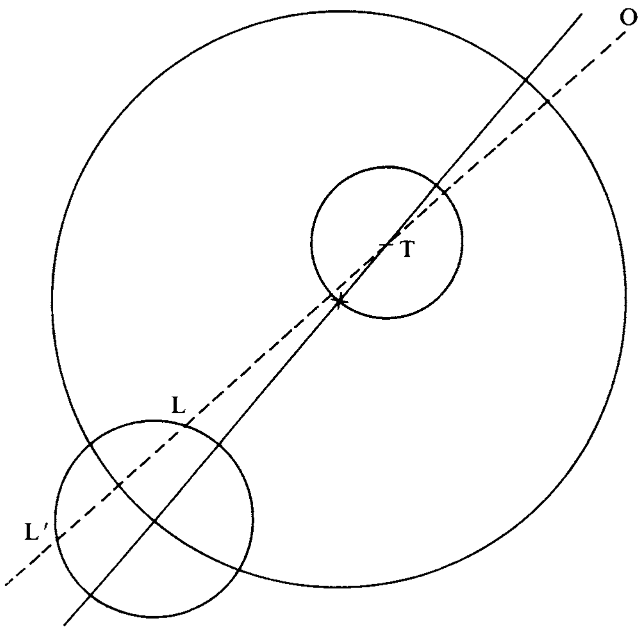


Figure 2.7

(b) In t_1 it goes from perigee to apogee and in t_2 it goes from apogee to perigee – a special case of Thābit's mode 2.
(c) In t_1 and t_2, the sun departs from the same point on the ecliptic – a special case of Thābit's modes 3 or 4.
(d) The point of departure for t_1 is symmetrical, with respect to the apogee or the perigee, to the point of arrival for t_2, and vice versa – equivalent to Thābit's cases 3 or 4.

Ptolemy then considers the question of the moon and eliminates cases (b), (c) and (d), keeping only the first – Thābit's mode 1. The conclusions of both thinkers are similar, but Ptolemy reasons only from particular cases, while Thābit regards the problem in its entirety, his analysis is exhaustive and he reaches a conclusion which becomes irrefutable (in the frame of the geometrical models concerned) due to the impeccable precision of his theoretical analysis.

The visibility of the crescent

Like all the Arab astronomers, Thābit studied the problem of the visibility of the crescent moon, and two of his treatises on this subject have been preserved: 'The visibility of the crescent by calculation' and 'The visibility of the crescent by tables'. The first is purely theoretical, and the second is a simplification of the first for practical application with the aid of tables.[34]

In a general sense, Thābit looks for a quantifiable relationship between the luminosity of the first lunar crescent and that of the horizon just after sunset. As we have seen above, in his study of the visibility of the fixed stars and the planets, Ḥabash took from Ptolemy the notion of the 'arc of visibility' of the crescent, giving it a fixed value of 10°. Thābit followed the same tradition, but his solution is much more complex because, for him, this 'arc of visibility' is no longer a constant and he needs to modify its values by successive calculations based on the four variables that he defines.

The first three variables are the three sides of the fundamental spherical triangle OHD in Figure 2.8, O being the position of the sun under the horizon, H the 'brightest point on the horizon' in a vertical line from the sun, and D the position of the moon at its setting; let us call these three arcs α_1, α_2, α_3 (Figure 2.8).

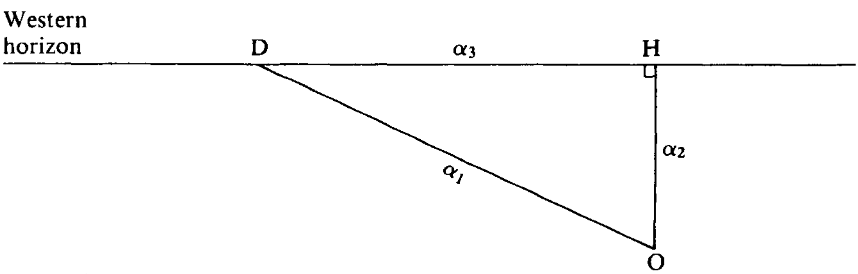


Figure 2.8

The first, α_1, is the angular distance between the moon and the sun; it is this arc which will determine the portion of the crescent, as seen from the earth, that is illuminated by the sun. The luminosity of the sky at point H on the horizon after sunset depends on the second arc, α_2; this is 'the arc of depression of the sun under the horizon'. On the third arc, α_3, depends the luminosity of the sky at the point where the moon sets; it is the distance from D to H, the 'brightest point on the horizon'. This fundamental triangle allows two limit situations (Figures 2.9 and 2.10).

When the moon sets on the vertical line from the sun, at the 'brightest point on the horizon' (Figure 2.9), $\alpha_3 = 0$, the crescent can be visible if α_1 and α_2 have a value at least equal to what we shall call α_0, the precise limit value for these two arcs together; α_0 is the absolute value for the 'arc of visibility' of the crescent, to be determined as a function of the earth–moon

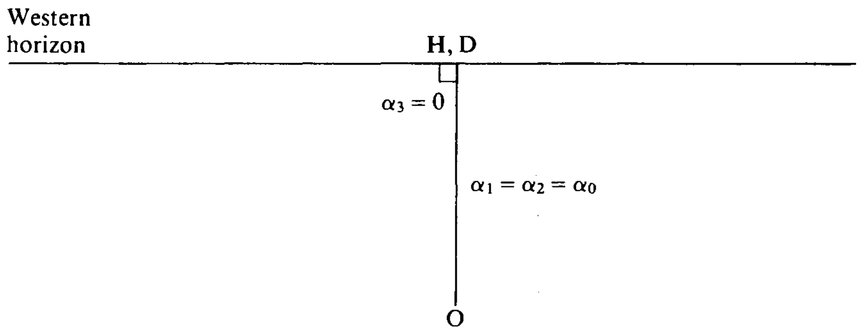


Figure 2.9

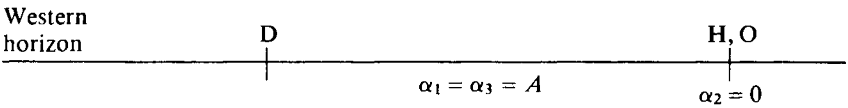


Figure 2.10

distance. Thābit states, without justifying it, that its minimum value in degrees is 10; 52; thus if $\alpha_0 < 10;52$ the crescent will be invisible.

When the crescent is at the limit of visibility while the moon and the sun are setting together, the angular distance between the two must be such that the crescent can be visible by day (Figure 2.10). We then obtain $\alpha_2 = 0$ and $\alpha_1 = \alpha_3 = A$, the limit value beyond which the crescent will be visible in all possible conditions. Thābit states that for $A > 25^\circ$ the crescent will be visible by day whatever the value of the other variables. This upper limit of 25° seems to correspond to an observation; in fact, recent observations show that in the middle of the day the moon is at the limit of visibility when its angular distance to the sun is close to 25°.

The fourth variable refers to the distance between the moon and the earth, on which depends the apparent diameter of the moon and therefore its luminosity for that portion of crescent illuminated. At the moment when the crescent is first visible, the position of the centre of the moon's epicycle could be mistaken for the apogee of its eccentric; the true angle of anomaly a is the only variable which intervenes in the distance from the moon to the earth (Figure 2.11).

The moon is furthest from the earth when $a = 0$ and closest when $a = 180$. If R is the radius of the eccentric orb, e its eccentricity and r the radius of the epicycle, the distance from earth to moon will go from $R + e + r$ to $R + e - r$ as a goes from 0 to 180.

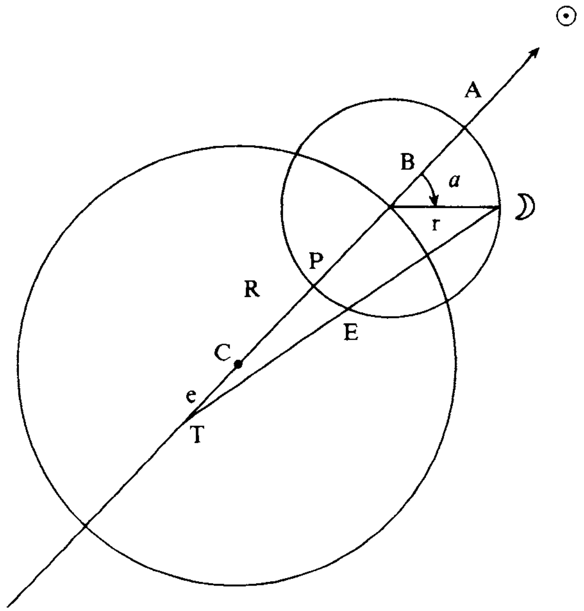


Figure 2.11

First stage: relation between α_1 and α_2

The main point of the discussion concerns arcs α_1 and α_2 of Figure 2.8, which are the two most important variables. If α_1 increases, the crescent becomes brighter, and if α_2 decreases the sky's luminosity will be stronger on the horizon. A balance needs to be found between the variation of these two arcs and will need to be modified in accordance with the other two variables. Let $V(\alpha_1, \alpha_2)$ stand for the relation between a specific pair of these variables when the crescent is at the limit of visibility. Thābit looks for the relation that must exist between the 'increasing' $\Delta\alpha_1$ and the 'decreasing' $\Delta\alpha_2$ so that we can express the following identity:

$$V(\alpha_1, \alpha_2) \Leftrightarrow V(\alpha_1 + \Delta\alpha_1, \alpha_2 - \Delta\alpha_2)$$

The second term of this expression means that the crescent is once again at the limit of visibility for the pair in question. Thābit then states that the relation between $\Delta\alpha_1$ and $\Delta\alpha_2$ is a constant $k = \Delta\alpha_1/\Delta\alpha_2 = (A - \alpha_0)/\alpha_0$, A and α_0 being the values defined above. This constant is found when the crescent passes from one limit to the other (see Figures 2.9 and 2.10), i.e. from $\alpha_1 = \alpha_2 = \alpha_0$ to $\alpha_1 = A$ and $\alpha_2 = 0$. Using the previous expression we can then say that $V(\alpha_0, \alpha_0) \Leftrightarrow V(A, 0)$, with $\Delta\alpha_1 = A - \alpha_0$ and $\Delta\alpha_2 = \alpha_0$ which gives the proposed constant relation k between $\Delta\alpha_1$ and $\Delta\alpha_2$. We then obtain $A = (k+1)\alpha_0$, and Thābit states that this relation k is known. The numerical data in the text give us $k = 1;11,46$, a figure possibly derived from the author's work on the values for the 'arc of visibility' of different planets given by Ptolemy in the *Planetary Hypotheses.*[35]

Second stage: intervention of α_3

α_0 is the absolute value of the 'arc of visibility' of the crescent for, in the limit situation of Figure 2.9, the moon sets in a vertical line from the sun and $\alpha_3 = 0$. When the moon moves away from H, the 'brightest point on the horizon', the value of α_0 must be modified in order to find a new slightly fainter 'arc of visibility', for at this point the horizon will be slightly less luminous than at H. Thābit then applies the formula put forward by Ptolemy in the *Phaseis* for the visibility of the fixed stars at any point on the horizon (Morelon 1981: 3–14), and he gives the first formula for modification of the arc of visibility: $\alpha_0' = \alpha_0(360 - \alpha_3)/360$.

Third stage: intervention of the distance earth–moon (as a function of a)

As we have seen, Thābit gives $\alpha_0 = 10;52$ as the absolute minimum of the arc of visibility, and $A = 25$ as the maximum above which the moon is visible by day whatever other conditions might be. Thus for him, $\alpha_0 = 10;52$ corresponds to the best conditions of visibility, and therefore the moon's closest position to the earth ($a = 180$ in Figure 2.11), and $A = 25$ corresponds to the worst conditions of visibility, i.e. when the moon is furthest from the earth ($a = 0$). When the moon moves from one distance to another its apparent diameter changes, and consequently α_0 and A must be calculated again.

To solve this problem, Thābit draws an analogy with the visibility of Venus as explained in the *Planetary Hypotheses*: there Ptolemy determines that the arc of visibility of Venus is 5° when the planet is at its minimum distance from the earth (166 times the earth's radius according to the figures accepted at the time) and 7° at its maximum distance (1,079 times the earth's radius), whilst the figures given for the moon in the same work correspond to $R + e - r = 53$ and $R + e + r = 64$. Without explicitly justifying his calculation, Thābit then declares that the corresponding differences in the arc of visibility in the case of the moon are 0;31 for α_0 and 1;8 for A. He then deduces that $10;52 \leqslant \alpha_0 \leqslant 11;23$ and $23;52 \leqslant A \leqslant 25$ when $0 \leqslant a \leqslant 180$.

The only method of calculation that will enable a close approximation of these figures to be found is to establish, term by term, a corresponding geometrical progression for the distances and an arithmetical progression for the values of the arc of visibility. The result is as follows: for Venus the arithmetical progression is of ratio 1, and the two arcs of visibility are obviously of rank 5 and 7; the geometrical progression is of ratio 2.712 and we find 147 and 1,079 of ranks 5 and 7. For the moon the arithmetical progression is of ratio 0; 31, and we find 10; 51 and 11; 22 of ranks 21 and 22; the geometrical progression is of ratio 64/53, and we find 53 and 64 of ranks 21 and 22. This correspondence is sufficiently close for us to conclude that this was the procedure used. If the relation k is known, as Thābit states, the single value of $A = 25$, corresponding to one observation, is sufficient to find the limit values of α_0 and A.

This term-by-term correspondence of two progressions only yields the extreme values of α_0 and A, those which correspond to $a = 0$ and $a = 180$. To go from one to the other, Thābit uses a simple interpolation formula which Ptolemy calculated by making a table[36] for $I(a)$ such that $0 \leqslant I(a) \leqslant 1$ when $0 \leqslant a \leqslant 180$. He then sets out as a function of a:

$$\alpha_0 = 11;23 - 0;31 I(a)$$

and

$$A = 25 - 1;8 I(a)$$

The discussion then turns to the arc α_2 (arc of depression of the sun under the horizon), in order to compare it with the 'arc of visibility' calculated in stages by giving a fixed value to one or another of the variables.

1 Thābit writes $\alpha_3 = 0$ and $\alpha_1 = 10;52$ (its absolute minimum); he calculates as a function of a the value of $\alpha_0 = 11;23 - 0;31 I(a)$ and concludes that for $\alpha_2 \geqslant \alpha_0$ the crescent will be visible.
2 He takes the real value of α_3 and keeps $\alpha_1 = 10;52$; he calculates the corresponding decrease of the arc of visibility $\alpha_0' = \alpha_0 - \Delta\alpha_0$ using the formula of Ptolemy in the *Phaseis*: $\alpha_0' = \alpha_0(360 - \alpha_3)/360$. He then concludes that for $\alpha_2 \geqslant \alpha_0'$ the crescent will be visible.
3 He substitutes all the variables by their real values and calculates $\alpha_0'' = \alpha_0' - \Delta\alpha_0'$. The corresponding decrease will depend on α_1, the real angular distance between the moon and the sun, giving the true width of the visible crescent, and another factor is added, acting to increase α_1 from its absolute minimum of 10; 52 and introducing A' as the value of A modified like α_0 by the formula from the *Phaseis*. The final expression is as follows:

$$\alpha_0'' = [11;23 - 0;31 I(a)]\ \frac{360 - \alpha_3}{360}\ \frac{A' - \alpha_1}{A' - 10;52}$$

He concludes that for $\alpha_2 \geqslant \alpha_0''$ the crescent will be visible.

This theory of the visibility of the crescent thus involves six elements: an observation, $A = 25$; a constant relation k between the 'increase' of α_1 and the 'decrease' of α_2; a term-by-term correspondence of two progressions, one arithmetical and the other geometrical; the situation of the three main variables with respect to their limit values, $\alpha_0 \leqslant \alpha_1 \leqslant A$, $0 \leqslant \alpha_2 \leqslant \alpha_0$, $0 \leqslant \alpha_3 \leqslant A$; a simple interpolation formula taken from Ptolemy; the formula of the *Phaseis* to modify the result according to the position of the moon over the horizon.

Thābit bases his study on an analogy between the case of the crescent and that of the fixed stars, using the formula from the *Phaseis*, and another analogy with the case of the planets, using the example of Venus. This means that, for him, there is just one problem of visibility for every luminous celestial body on the horizon after sunset or before sunrise: the lunar crescent, fixed stars and planets all take part in this unique phenomenon which Thābit tries to analyse in mathematical terms using a relation between the magnitudes linked to the luminosity of the body in question and to that of the horizon at that moment. He seems, then, to have looked for a general law which he has tried to apply numerically to the case of the crescent.

This author thus attempts to deal with the problems of astronomy in a rigorously mathematical way, looking at them in general terms and studying the models proposed by Ptolemy from a purely geometrical point of view without questioning them as such. He recognizes that the degree of accuracy of pure reasoning cannot always match that of observation, for as he says, 'what is perceived by sense does not lend itself to such precision' (Thābit, p. 108, l. 6). Verification by observation will always be necessary, and the conclusion of his purely theoretical treatment of the visibility of the crescent is entirely devoted to this matter, including conditions of observation and personal factors associated with the quality of the observer.

Al-Battānī

Al-Battānī, an astronomer of great reputation, lived at the turn of the ninth and tenth century: he was born about the middle of the ninth century and died in AH 317 (AD 929). Originally from Ḥarrān, like Thābit, he lived for most of his life in Raqqa, on the Euphrates in the north of present-day Syria, where for more than thirty years he made many high-quality observations from a private observatory. He wrote a survey of his work in a monumental book called 'The Sabian tables' (*al-Zīj al-Ṣābī*).[37] This had a great influence on the astronomy of the Latin West in the Middle Ages and at the beginning of the Renaissance, because it is the only complete treatise on Arabic scientific astronomy of this era to have been translated in its entirety into Latin in the twelfth century (and then directly into Spanish in the thirteenth century), naming the author Albategni or Albatenius. His was therefore the only majorly important work of eastern astronomy in the Arabic tradition that was known and studied until relatively recent times, which is why al-Battānī is so renowned and has been hailed as 'the greatest Arab astronomer' by successive authors of most of the books on the history of astronomy.

He was indeed a great observer, but his work in theoretical astronomy is not of major importance; it depends almost entirely on his immediate Arab predecessors, who are never explicitly cited although al-Battānī frequently refers to Ptolemy. He recalculates certain parameters and compares the results of his own observations with some preceding theories without criticizing them or making any notable additions.

His important contribution, then, lies in the area of pure observation. He measures the obliquity of the ecliptic with a high degree of accuracy (23; 35); he finds that the apogee of the sun's orb on the ecliptic is at 22; 50; 22 of Gemini, which is much closer to the true position in his day than the value recorded in the 'Book on the solar year', and he thus confirms the mobility of this apogee. He calculates the length of the tropical year and finds it equal to 365; 14, 26, a value slightly less accurate than that in the 'Book on the solar year'. Having checked it but without naming his source, he accepts the value of the precession constant given in the 'Verified table' (*al-Zīj al-mumtaḥan*) (1° every sixty-six years), and from this he recalculates the figures in the catalogue of fixed stars in the *Almagest*, reducing their number by slightly more than half (489 stars instead of 1,022).

His most famous observation, and deservedly so, is that of the variation of the apparent diameter of the sun and the moon, on the basis of which he comes to the conclusion, for the first time in the history of astronomy, that annular eclipses of the sun are possible as long as the apparent diameter of the moon at its minimum is slightly less than that of the sun. He finds that the apparent diameter of the moon, when in conjunction with the sun, can vary from 0; 29, 30 to 0; 35, 20 (real variation from 0; 29, 20 to 0; 33, 30), and that the apparent diameter of the sun can vary from 0; 31, 20 to 0; 33, 40 (real variation from 0; 31, 28 to 0; 32, 32), whereas Ptolemy had considered that the apparent diameter of the sun would remain equal to 0; 31, 20 – curiously disregarding the difference in its distance from the earth during its movement on the eccentric – and that this value was the same as the minimum apparent diameter of the moon, thereby eliminating the possibility of annular eclipse.[38]

In conclusion, to attempt a brief summary of the study of astronomy under the Abbasids in the ninth century, we can say first that original research took place in this domain as soon as the basic resources became available to scholars, whether those resources were Indian, Persian, Syriac or above all, Greek. Translation into Arabic of earlier sources and pure scientific research went hand in hand, for astronomy as for all the exact sciences, right from the beginning and during the whole of this century (Rashed 1989).

The work of astronomical research really got underway with the establishment of a collective programme of continuous observations under al-Ma'mūn a little before 830, and the caliph strongly encouraged this fundamental research, many of his successors doing the same. It is clear that right from this period astronomers stressed the precision of the instruments and the necessity for continuous and repeated observations – for the sun and the moon at first in Baghdad and Damascus, and then for the rest of the heavenly bodies – whereas the sources transmitted only observations that were isolated in space and time; this programme was continued and developed throughout succeeding history.

The collective aspect of the work should also be emphasized even outside a purely observational framework for in addition to the existence of communal structures such as the observatories at Baghdad and Damascus financed by central power, numerous traces of scientific correspondence between astronomers are to be found cited in ancient Arabic bio-bibliographic works concerning this era. We can therefore speak of the founding of a real 'school of Baghdad' in astronomy of the ninth century.

The constant movement back and forth between theory and observation, markedly more systematic than in the astronomy of Hellenistic tradition, allowed an early and sometimes vigorous critique of parts of Ptolemy's theories or results, but still exclusively within the framework of the system and the geometrical models that he had proposed.

During this century the progress of spherical trigonometry, considered at the time to be just an 'auxiliary science' to astronomy, enabled much more rigorous and elaborate geometrical reasoning concerning the arcs of the celestial sphere, due to the systematic utilization of sine and cosine and the introduction of tangents and cotangents (see vol. II, chapter 15); finally, the research begun by Thābit to apply to astronomy the results achieved by mathematicians, who were often also astronomers, was continued by most of his great successors, consequently making astronomy increasingly 'scientific'.

Thus the subsequent developments in Arabic astronomy had already taken root in the ninth century, especially in Baghdad, for the programme and methods of work which reached a high state of organization there would be followed without notable change, at least in basic principles, for several centuries.[39]

ASTRONOMY IN THE TENTH AND ELEVENTH CENTURIES UNTIL AL-BĪRŪNĪ

We saw in Chapter 1 that it was between the tenth and eleventh centuries that decisive progress was made with regard to the conception and organization of permanent large-scale observatories in Baghdad or in Persia, and the chapter on trigonometry (vol. II, chapter 15) shows the importance of the results achieved during the tenth century for the development of this science, on which the accuracy of astronomical calculations partly depends.

However, few but fragmentary or incomplete texts of theoretical astronomy have come down from this era, and it is paradoxically more difficult to sketch the evolution of eastern Arabic astronomy in the tenth century than in the preceding century. We shall therefore simply take three examples of scientists from this period, who seem to have worked in greater isolation than their predecessors of the ninth century, and then turn our attention to the line that, from master to student, leads to al-Bīrūnī, who lived partly in the tenth and partly in the eleventh century and who stands at the summit of this first period of eastern astronomy.

Abū Jaʿfar al-Khāzin, ʿAbd al-Raḥmān al-Ṣūfī and Ibn Yūnus

Abū Jaʿfar al-Khāzin was a brilliant mathematician; originally from Khurāsān, he spent part of his life in Rayy and he died between AH 350 and 360 (AD 961 and 971). He composed several treatises of theoretical astronomy, but only some fragments of his *Commentary on the Almagest* – mainly on trigonometry – remain; however, the allusions to this work made by certain later authors, notably al-Bīrūnī, demonstrate its importance for his successors. He had studied the motion of the sun and, unlike al-Battānī, he used Ptolemy's observation of the constant value of its apparent diameter, therefore necessitating a fixed distance from the earth. He thus proposed a new model for the motion of the sun: not on an eccentric but on a circle concentric with the earth, the uniform motion occurring around an eccentric point in an analogous way to the movement of the epicycle around the 'equant point' in the Ptolemaic model of the upper planets.[40] This is currently the only point which reveals that he had made a critical evaluation of Ptolemy's models.

He also wrote the *Kitāb fī sirr al-ʿālamīn* ('Book on the secret of the worlds'), which is lost in its entirety and in which he proposed a new global conception of the universe based on Ptolemy's results in the *Planetary Hypotheses*.[41] Although we cannot yet determine its precise measure, the work of Abū Jaʿfar al-Khāzin had an undoubted influence, a century later, on the work of Ibn al-Haytham, *al-Shukūk ʿalā Baṭlamyūs*, relating to his criticism of the Ptolemaic system, which is frequently based on arguments of a cosmological nature (see the following chapter).

ʿAbd al-Raḥmān al-Ṣūfī (AH 291–376 (AD 903–86)) was born in Rayy and worked in Shiraz and Isfahan. Several of his observations, on the obliquity of the ecliptic and on the motion of the sun or the length of the solar year, have been reported, but he is most famous for his *Kitāb ṣuwar al-kawākib al-thābita* ('Book concerning the constellations of the fixed stars');[42] this is a reworking of the catalogue of fixed stars from the *Almagest*, written around 965. In his introduction, al-Ṣūfī defines his position in relation to the Arab astronomers of the preceding generation who dealt with the fixed stars or to the makers of celestial spheres, criticizing the way in which one or another constellation was handled, and he chooses the value of the precession constant calculated by the authors of the 'Verified table' under al-Ma'mūn – 1° in sixty-six years – instead of the 1° per century stated by Ptolemy. This work is not just an adaptation of the catalogue of the *Almagest* achieved by modifying the longitude of each star with the aid of the correction corresponding to the precessional movement between the second and tenth centuries, because al-Ṣūfī also made many verifications by observation, for the magnitude of the stars as well as for their ecliptic longitude – he states that he preserved the value for the latitudes given by Ptolemy – and introduced notes on the apparent colours of the principal stars. Al-Ṣūfī's book was widely read in Arabic, and from the twelfth century was translated and disseminated in Latin – the name of its author being transcribed as 'Azophi' – resulting in many stars being given names of Arabic origin in the West.

The work describes each of the forty-eight constellations according to a unique format: first a presentation of the constellation concerned, listing all its stars and the different Arabic names under which they could be known; then a table giving their ecliptic co-ordinates and their magnitude. All copies of the book, from the earliest, contain miniatures of the mythical figures representing each of the constellations with the positions of its different stars, always sketched twice in symmetrical fashion – 'as seen in the sky' and 'as seen on the sphere' (i.e. on a representation, in wood or metal, of the celestial sphere) – thus enabling easy location of the constellations even by a beginner. The author intended a double purpose for his work, at once theoretical and practical, for example in orientation on land or sea, and this was part of the reason for its success. A number of illustrations printed here show the quality and diversity of the representations of constellations in the manuscripts of this famous work.

Ibn Yūnus (d. AH 399 (AD 1009)) was a great Egyptian astronomer, and above all an observer, who worked in Cairo during the first period of the Fatimids and probably had his observatory on Mount Muqattam, east of

Cairo. His most important work is *al-Zīj al-Ḥākimī al-Kabīr* ('The great Hakemite table') – from the name of the Fatimid sultan al-Ḥākim who reigned in Cairo from AH 386 (AD 996) to AH 411 (AD 1021) – a monumental book in eighty-one chapters of which only a little more than half is preserved.[43] Ibn Yūnus set out to produce a complete treatise of astronomy including the greatest possible number of previous observations, critically reviewed and analysed and enriched with the results of his own numerous observations. His work is therefore a means of gaining access to much scientific material of the ninth and tenth centuries which is known only through his quotation of it.

There is very little theoretical reasoning in this work of Ibn Yūnus; it is a 'zīj' in the strict sense of the term, i.e. a work that concentrates exclusively on the compilation of tables of the movements of heavenly bodies, with calculations of their various parameters and details of how to use them. Since his results became available in translation at the beginning of the nineteenth century, the accuracy of his observations has been exploited by modern scientists to gain, for example, a better knowledge of the secular acceleration of the moon.

Al-Bīrūnī

Al-Bīrūnī was born in AH 362 (AD 973) in the Khwārizm and died around AH 442 (AD 1050), probably in Ghazna (modern Afghanistan). He was the pupil of Abū Naṣr Manṣūr b. ʿIrāq, himself the pupil of Abū al-Wafā' al-Būzjānī; he explicitly recognized these two scholars as his masters, and he worked in Rayy with al-Khujandī. These three contributed toward al-Bīrūnī's becoming simultaneously a mathematician, an astronomical theorist and an observer.

Abū al-Wafā' al-Būzjānī, mathematician and astronomer, who was born in AH 328 (AD 940) at Būzjān in Persia, and died at Baghdad in AH 388 (AD 998), represents a return to the tradition of astronomical research of the 'Baghdad school', which had been so strong in the preceding century, as we have seen, for he gained his scientific training in this environment and worked in Baghdad thereafter. For his astronomical research Abū al-Wafā' used the large observatory built under the patronage of Sharaf al-Dawla in the gardens of the royal palace in Baghdad. He entitled his principal astronomical treatise *Almagest*, but only part of this text has been preserved and this mainly concerned with questions of trigonometry, a science considerably developed by this author.[44] We thus know little about his contributions to theoretical astronomy, but al-Bīrūnī makes numerous allusions to his studies concerning the motion of the sun and the value of the precession constant.[45]

We have less information about al-Bīrūnī's immediate 'master', Abū Naṣr Manṣūr b. 'Irāq, who died around AH 427 (AD 1036) at Ghazna. We know that he was a student of Abū al-Wafā' al-Būzjānī, and his surviving works consist mainly of important texts on trigonometry, written partly at al-Bīrūnī's request when the latter was puzzling over specific problems (Samsó 1969). Al-Khujandī, who died *c.* AH 390 (AD 1000), devoted a great deal of study to the question of observational instruments, about which he wrote several books, and he was responsible in particular for the building of the great sextant at Rayy described in the preceding chapter.

Al-Bīrūnī is a scholar of exceptional stature, who wrote about 150 works on all the known sciences of his time. These included thirty-five treatises of pure astronomy of which only six have survived; his other works – on India, for example, or about chronology – contain numerous references to astronomical matters. His major survey in this area is *al-Qānūn al-Mas'ūdī* ('Tables Dedicated to Mas'ūd'), a work consisting of eleven treatises written around AH 426 (AD 1035) which comprises 1,482 pages in the original edition (Boilot 1955).

His mother tongue was Persian, but his chief working language was Arabic and he also knew Sanscrit fluently because he used it and he made several translations of scientific texts from Sanscrit to Arabic. He thus had direct access to all the sources of Indian scientific astronomy, to which he constantly refers alongside the Greek sources or the works in Arabic of his predecessors, whereas since the transmission of Sanscrit texts at the end of the eighth century, those predecessors seem only to have had access to a few Indian astronomical documents or to secondary sources, while the Greek scientific tradition was much more widespread. Al-Bīrūnī could therefore bring together and study directly the entire astronomical heritage of his day, from the Greek world, the Indian world and the Arab world, and all his work does in fact tend toward the goal of a rigorously conducted synthesis. Rather than attempting to present all the astronomical work of al-Bīrūnī, which would be particularly difficult, we shall consider some aspects of his method.

In the first treatise of *al-Qānūn al-Mas'ūdī*, al-Bīrūnī states some general principles of astronomy and sets out the bases of chronology in different cultures, including that of China. In Chapter 2 he deals with the position of the heavens in relation to the earth, and considers the hypothesis of the rotation of the earth about itself to explain diurnal movement.[46] He states that this hypothesis was supported in India by Aryabhata and his disciples but that it is not compatible with one of Ptolemy's arguments according to which a body in free fall will not fall vertically if the earth has a rotational movement; al-Bīrūnī then asserts that a 'great scholar' (whom he does not name) contends that Ptolemy's argument is not valid in so far as every terrestrial body is carried by this rotation along the vertical line through which the body falls. After setting out this argument, which he appears to find consistent, al-Bīrūnī returns to the question, considers the problem of horizontal motion, calculates the speed of a point on the earth in the hypothesis of its rotation about itself, and concludes that this great speed could only be added to or subtracted from the other movements of terrestrial bodies from East to West, which cannot be verified, and therefore, according to him, it is not possible for the earth to rotate about itself.

In general, al-Bīrūnī deals with a specific astronomical problem according to the following scheme: first, he outlines some general principles about the problem in question; then he gives the various solutions proposed by the Indians, by Ptolemy and by the Arab astronomers, all presented and critically analysed on the basis of the general principles stated at the beginning; then, where relevant, he details the earlier observations that are most important or most noteworthy for the phenomenon in question, and describes his own observations; finally he selects one of the preceding solutions, or proposes his own solution based on all the foregoing material. As an example, we shall take the question of the visibility of the crescent as set forth in *al-Qānūn al-Masʿūdī*.[47]

Treatise VI of this work concerns the motion of the sun, treatise VII concerns the motion of the moon and treatise VIII concerns the observable phenomena on the connection between the motions of the sun and of the moon, i.e. the question of eclipses of one or other of these 'two luminaries' and that of the visibility of the crescent. Chapter 13 of treatise VIII is devoted to the morning and evening twilight, with a description of this phenomenon as the approach of the horizon to the limit of the cone of shadow created on the earth by the sun, and al-Bīrūnī states that 'the astronomers' (without citing them) have determined that the beginning of the morning twilight in the East, or the end of the evening twilight in the West, occurs when 'the arc of depression of the sun below the horizon' is 17° or 18° – without choosing between the two values. Chapter 14 then deals with the visibility of the crescent.

General principles

The ability of the eye to see the crescent depends on several factors: first, the distance between the moon and the sun, which determines the portion of the moon's surface which is illuminated; then the earth–moon distance on which its apparent luminosity for the same amount of illumination depends; then the luminosity of the atmosphere on the horizon depending on the inclination of the ecliptic on the horizon, and therefore on both the position of the sun on the ecliptic and the latitude of the place; finally the position of the setting moon on the horizon, more or less close to the 'brightest point on the horizon', and thus to the vertical of the sun's position below the horizon.[48] Al-Bīrūnī concludes that all these parameters must be carefully taken into account.

Earlier solutions

Ptolemy did not study this question because the problem did not arise in his culture. Four Arab astronomers, al-Fazārī, Ya'qūb b. Ṭāriq, al-Khwārizmī and al-Nayrīzī, used an Indian method, taking the difference in time between the setting of the sun and of the moon, but this criterion did not allow for the inclination of the ecliptic on the horizon, and it was therefore not valid; al-Nayrīzī, however, did a little better than the other three, because unlike them, he took account of the correction of the lunar parallax. Following several corrections, al-Battānī took into consideration the distance between the sun and the moon both on the equator and the ecliptic but did not pay sufficient attention to the inclination of the ecliptic on the horizon. Finally, Ḥabash used as his principal criterion 'the arc of depression of the sun below the horizon', which can only be calculated from all the other parameters.

Conclusion

Ḥabash's method must be chosen. Al-Bīrūnī does not offer a personal solution, and concludes his chapter by describing the means of finding the lunar crescent on the horizon with the aid of the observation tube described in the preceding chapter.

The problem of the motion of the sun according to al-Bīrūnī has been studied by Hartner and Schramm (1963). As well as all the stages of the preceding scheme, al-Bīrūnī includes large numbers of observations of the sun in addition to his own, and a mathematical study of the apparent motion on the eccentric which recalls that of Thābit b. Qurra described earlier. Following a critical analysis of the findings of previous authors, al-Bīrūnī establishes in definitive manner the motion of the apogee of the sun, recalculates all the parameters and draws up tables of its movement.

This type of astronomical work did not disrupt the overall system of astronomy as perceived by al-Bīrūnī, because he remained faithful to the model of epicycles and eccentrics defined by Ptolemy. However, al-Bīrūnī reviewed everything in detail, continuing, for example, the movement toward the mathematization of astronomy begun by Thābit a century and a half before him,[49] and rigorously taking stock of the current state of the science in all its aspects. In so far as such a comparison is possible, this work is analogous to that carried out by Ptolemy in the *Almagest* eight centuries earlier: establishing a rigorous scientific tradition, but without major global innovation, using all the preceding research and all the mathematical tools available to the astronomer at the given time.

Al-Bīrūnī accomplished this synthesis brilliantly; it was the crowning achievement of the first period of Arabic astronomy, remaining within the general framework erected by Ptolemy. It was his contemporary Ibn al-Haytham who began to break free of that framework, a development that might not have been possible without the precise contribution of al-Bīrūnī.

NOTES

1 See al-Qifṭī.
2 See the Indian sources referred to in Chapter 1.
3 Cf. al-Bīrūnī, *Kitāb fī Taḥqīq*, pp. 351–2. Al-Bīrūnī is usually very reliable when reporting traditions of a scientific nature, particularly with regard to India, and the tradition recounted here is probably based on historical fact, but certain elements are lacking, without which we cannot be absolutely convinced of the authenticity of everything that has been reported concerning the episode: the various Arabic sources disagree on an exact date; who was the Indian astronomer and in which language did the exchanges between him and his questioners take place; was this the translation of a text in the proper sense of the term – and if so, which text, since the expression *Zīj al-Sindhind* may be purely generic – or was it simply a transmission of results in the form of tables? And so on .... .
4 For al-Fazārī, see Pingree (1970). For Ya'qūb b. Ṭāriq, see Pingree (1968: 97–125).
5 Latin text edited by Suter (1914); translation with commentary by Neugebauer (1962a).
6 The four texts by Euclid were translated by Ḥunayn and Thābit; the three texts by Theodosius were translated by Qusṭā; the two texts by Autolycus were translated by Isḥāq and by Qusṭā respectively; the text by Aristarchus and that of Hypsicles were translated by Qusṭā; and the book by Menelaus was translated by Ḥunayn or his son Isḥāq.
7 For the transmission of the *Almagest* in Arabic, see Kunitzsch (1974).
8 Ibn al-Ṣalāḥ, Arabic text, p. 155, ll. 12–18.
9 Only one part of these two versions has been published: the star catalogue of the *Almagest*. See Ptolemy, *Almagest: Der Sternkatalog*, edited and translated into German by Kunitzsch.
10 See Leiden, or. ms. 180, fol. 1a.
11 al-Mas'ūdī, *Kitāb al-Tanbīh*, pp. 15–16.
12 In his book *Hay'at al-aflāk*, ms. Oxford, Bodl., Seld. 3144.
13 See al-Kindī for the edition of the text and Rosenthal for his analysis.
14 See al-Farghānī.

15 We no longer possess his commentary, which numbered 200 folios.
16 Thābit Ibn Qurra, treatises 1 and 2. For Quṣṭā's text see note 12.
17 The Arabic manuscript Escurial 927 carries the explicit title 'Table verified according to the observations of al-Ma'mūn', but the text contains many elements dating from after the ninth century; see the analysis by Vernet (1956) and Kennedy (1956: 145–7).
18 They are regrouped in tabular form in al-Hāshimī, pp. 225–6.
19 Cited in Thābit, treatise 2, p. 22, lines 4–5, and in al-Farghānī, pp. 50–3.
20 The Arabic text of this treatise can be found in Thābit, pp. 26–67; see the introduction, pp. XLVI–LXXV, and the complementary notes, pp. 189–215, where the arguments condensed here are detailed.
21 Thābit, treatise 3, p. 61.
22 Thābit, treatise 3, p. 49.
23 Recalculated results for the period (year 830) are given in parentheses.
24 For details of the reasoning see Thābit, pp. LX–LXIII.
25 See the commentary on this point in Neugebauer (1962b: 274–5).
26 The contents of this manuscript have been analysed in detail by Debarnot (1987).
27 See Kennedy (1965, 1968), reprinted in Kennedy (1983: 151–63).
28 See the detailed exposition in Thābit, pp. XXVI–XXX.
29 His works on astronomy preserved in Arabic have been edited and annotated (see Thābit); the account that follows is summarized from that study.
30 Treatise entitled 'Ralentissement et accélération du mouvement apparent sur l'écliptique selon l'endroit où ce mouvement se produit sur l'excentrique'; cf. Thābit, pp. LXXVI–LXXIX, 69–82, 216–21.
31 Ptolemy, *Almagest*, Heiberg, vol. I, p. 220; Toomer (1984), p. 246.
32 Treatise entitled 'Clarification d'une méthode rapportée par Ptolémée, à l'aide de laquelle ceux qui l'avaient précédé avaient déterminé les divers mouvements circulaires de la lune, qui sont des mouvements uniformes' or 'Le mouvement des deux luminaires'; cf. Thābit, pp. LXXX–XCII, 84–92, 222–9.
33 Ptolemy, *Almagest*, Heiberg, vol. I, pp. 272–5, and Toomer (1984), pp. 176–8.
34 See Thābit, pp. XCIII–CXVII, 94–116, 230–59, for details of the description summarized here in an attempt to reconstruct the reasoning of the author.
35 For the explanation of this hypothesis see Thābit, pp. CXIII–CXV.
36 Ptolemy, *Almagest*, Heiberg, Vol. I, p. 524; Toomer (1984), p. 308.
37 The full name of this author is Abū 'Abd Allāh Muḥammad b. Jarīr b Sinān al-Battānī al-Ṣābī al-Ḥarrānī; see the bibliography under al-Battānī.
38 For these various observations see al-Battānī (translation and commentary, vol. 1; Arabic text, vol. 3; tables, vol. 2): the obliquity of the ecliptic, translation p. 12, commentary pp. 157–62, Arabic p. 18, l. 14; apogee of the solar orb, translation p. 72, Arabic p. 107, l. 23 to p. 108, l. 7; tropical year, translation p. 42, commentary pp. 210–11, Arabic p. 63, l. 22 to p. 64, l. 1; precession, translation p. 128, Arabic p. 192, ll. 1–5; catalogue of fixed stars, translation pp. 144–86, Arabic pp. 245–79; apparent diameters of the sun and the moon, translation p. 58, commentary pp. 236–7, Arabic p. 88, ll. 3–15.
39 This topic is discussed further in Morelon (1994), under the following areas: the link between theory and observation, the 'mathematization' of astronomy, and the link between 'mathematical' astronomy and 'physical' astronomy.

40 See al-Bīrūnī, *al-Qānūn al-Masʿūdī*, pp. 630–2. Al-Bīrūnī (ibid., p. 1312) also refers to a *Book on the Sizes and the Distances of Heavenly Bodies* by the same author.
41 Al-Kharaqī al-Thābitī, an author from the twelfth century, refers to al-Khāzin while mentioning similar work by Ibn al-Haytham, in the introduction to his book of cosmology: *Muntahā-l-idrāk fī taqāsīm al-aflāk* ('The farthest point of knowledge for the divisions of celestial spheres') – manuscript: Paris, B.N., Ar. 2499.
42 See al-Ṣūfī, ʿAbd al-Raḥmān.
43 See Ibn Yūnus for the edition in French translation of the first chapters of this work.
44 The Paris manuscript, B.N., Ar. 2494, is very incomplete; it was studied by Carra de Vaux (1892). The use by this author of a particular term sparked a controversy, begun by L. A. M. Sédillot, concerning the discovery by Abū al-Wafāʾ of the motion of lunar variation, by showing that this was not what was referred to in the text.
45 See al-Bīrūnī *al-Qānūn al-Masʿūdī*, pp. 640–77.
46 al-Bīrūnī *al-Qānūn al-Masʿūdī*, pp. 42–53; see Pines (1956).
47 al-Bīrūnī *al-Qānūn al-Masʿūdī*, pp. 950–65.
48 See Figure 2.3 and the discussion of it, including a description of the different methods. Note that al-Bīrūnī was obviously unaware of Thābit b. Qurra's method, described above, which uses all the parameters cited in a more complete way than that of Ḥabash.
49 For the complexity of the interpolation methods employed by al-Bīrūnī for the use of the tables, see Rashed (1991).

3

Arabic planetary theories after the eleventh century AD

GEORGE SALIBA

This chapter takes its starting date as the eleventh century AD for several reasons. First, one can argue that it was in the eleventh century that Arabic astronomy was finally 'acclimatized' within the Islamic environment and from then on it began to be coloured with whatever prerequisites that environment demanded. From this perspective several works began to be characterized by original production, and were no longer mere repetitions of problems that were discussed in the Greek tradition. Figures such as Abū Sahl al-Kūhī, Abū al-Wafā' al-Būzjānī, Bīrūnī, Manṣūr ibn Naṣr ibn 'Irāq, etc., who lived just around the turn of the previous century, were setting the grounds for this new production in astronomical research. This work could still be considered as a continuation of that of Ḥabash al-Ḥāsib, Thābit Ibn Qurra, Khwārizmī and others of the previous ninth century.

Second, the eleventh century witnessed a series of works all characterized by a genuine interest in the philosophical basis of Greek astronomy. As a result, these works led to a new school of writers on astronomical subjects whose main concern was to point out the problems that were inherent in the Greek astronomical system. One should recall the works of Ibn al-Haytham in his *Shukūk*, Abū 'Ubayd al-Jūzjānī in his *Tarkīb al-Aflāk* and the anonymous Spanish astronomer in his *Istidrāk*. The problems raised by these astronomers were later taken up by 'Urḍī, Ṭūsī, Quṭb al-Dīn al-Shīrāzī and Ibn al-Shāṭir, among others. The last four astronomers have been referred to in the literature as the 'School of Marāgha', mainly because of the association of the first three with the observatory built by the Ilkhānid monarch Hūlāgū in the city of Marāgha in northwest modern Iran in AD 1259. If one were to take their works alone, one could show that within the thirteenth century, when the first three lived, there occurred a real revolution in astronomical research and a definite change in attitude towards astronomical presuppositions. This tradition, started in the eleventh century, reached a sophisticated maturity during the thirteenth century, and climaxed with the works of Ibn al-Shāṭir in the fourteenth, but lingered on well into the fifteenth and sixteenth centuries if one takes into consideration the works of a student of Ulugh Beg, ʿAlāʾ al-Dīn al-Qushjī (d. 1474), and *al-Hayʿa al-Manṣūrīya* of Manṣūr ibn Muḥammad al-Dashtāghī (1542).

By defining this genre of writing as the main motivating direction of astronomical research after the eleventh century, one has to accept the fact that from this perspective the works of someone like Jamshīd b. Giyāth al-Dīn al-Kāshī in the fifteenth century, especially in his *zīj-i Khāqānī*, represent a return to the older tradition as represented by such works as those of Khwārizmī and Bīrūnī, where the main concern is mathematical and computational and not at all concerned with theory of science and philosophy.

Other important figures of the fifteenth and the sixteenth centuries such as Abū ʿAlī al-Birjandī seem to have taken it upon themselves to write commentaries on earlier works, mainly the works of Ṭūsī, and have produced very little original material that could be classified either way. Works of people like Jaghmīnī and Qushjī were elementary indeed, and of the two authors only Qushjī seems to have understood the originality of the Marāgha School, as we shall see later.

In what follows, it will be shown that the Marāgha School astronomers not only produced original mathematical astronomy, but also left their imprint on later astronomical research, mainly in the Latin West, and may perhaps have laid the foundation for Copernican astronomy itself.

This chapter will introduce the problems that were the main concern of the New School, discuss their various solutions by various authors and conclude with an evaluation of their possible relationship to Copernican astronomy.

THE CONTROVERSIAL PROBLEMS

The most outstanding problems in the Ptolemaic astronomy as expounded in the *Almagest* and the *Planetary Hypotheses* were seen to be (1) the problem of prosneusis, (2) the problem of inclination and deviation of the spheres of Mercury and Venus, (3) the problem of the equant in the model of the superior planets, and (4) the problem of the consistency of the planetary distances as they were perceived to be within nested shells.[1] But this list could be enlarged depending on the seriousness with which one took the various lists compiled during the later centuries such as the list compiled by al-Akhawayn sometime during the latter years of the fifteenth century and the early years of the sixteenth. The list of problems – called *ishkālāt* – that was the subject of the treatise of al-Akhawayn will be reproduced here as an example of the kind of comprehensive coverage that these problems received.

According to al-Akhawayn, the general *ishkālāt* in astronomy were classified as follows:

Ishkāl 1 Speeding up, slowing down and average motion are inappropriate motions in the heavens and need a special solution. Such an *ishkāl* is easily solved, in the case of the sun for example, by the adoption of either the eccentric model or the epicyclic one.

Ishkāl 2 Planetary bodies appear sometimes to be larger than at other times. One such problem involves the explanation of the reason why there is a total solar eclipse when the sun is in the middle of its slow motion sector, while there is only an annular eclipse when it is on the opposite side in its fast moving sector, knowing that in both cases the sun is covered by the same-size body, i.e. the moon. The solution would obviously follow from the kind of model one would adopt for the solution of the first *ishkāl*, i.e. if one were to take the eccentric model then it is easy to see why the sun would look smaller when it is on the far side of the eccentric than when it is on the near side.

Ishkāl 3 Stationary, retrograde and forward motion of the planets are phenomena which seem to deny the assumed regular motions of the planets. Here again, the adoption of the epicyclic model could explain the three phenomena without any inconsistencies with respect to the general principles of uniform circular motions as being the only appropriate motions for the heavenly bodies.

All of the three problems mentioned above could have been solved by relying on principles already enunciated by Ptolemy in the *Almagest* and without recourse to any conditions contrary to the general principles.

Ishkāl 4 Uniform motion is measured around a point which is not the centre of the body that generates this motion. This is the general problem of the *equant* directly applicable to the model of all the planets, but in a special application includes the motion of the moon which is uniform with respect to the earth rather than the centre of its deferent.

It is this problem which generated a great amount of research, for it implied a contradiction in the Ptolemaic theory between the physical assumptions and mathematical ones. In what follows, we shall discuss in great detail the various solutions that were applied to this problem.

Ishkāl 5 The uniform motion is around a point, in spite of the fact that the moving body draws sometimes nearer to that point than at other times. The solution of this problem required the development of a mathematical theorem – now called the 'Ṭūsī Couple' – that became an integral part of most astronomical research after its discovery.

Ishkāl 6 A problem which results from requiring in the motion of a sphere that its diameter be slanted away from the centre of the carrying sphere that moves it. This *ishkāl* will be made clear in the discussion of the prosneusis problem mentioned above. The most notable illustration of this problem is in the model of the moon as it was perceived by Ptolemy.

Ishkāl 7 The problem of incomplete circular motions being part of the motion of the heavenly bodies. The best illustration of this *ishkāl* is the assumed motion of the epicyclic diameter in the Ptolemaic latitude theory of the lower planets. This is the same problem referred to above as the problem of the inclination and the deviation.

PTOLEMAIC THEORY OF THE MOTION OF THE PLANETS IN LONGITUDE

In order to begin to appreciate the seriousness of these problems, and the nature of the criticism as well as the solutions applied to them, it is best that we review very quickly the Ptolemaic theory for the motion of the planets.

The motion of the sun

The motion of the sun is described in *Almagest* III by Ptolemy in either one of two models, namely the eccentric model or the epicyclic model. The equality of these models had already been shown by Apollonius (Neugebauer 1959; re-edited in 1983), and was incorporated by Ptolemy as an integral part of the *Almagest* terminology. In Figure 3.1, the observer is supposed to be at point O, the centre of the ecliptic. The sun could either move along the eccentric ABCD with uniform speed so that it will appear to the observer on the earth to be moving fast when in the lower half of the eccentric BCD and to slow down when in the upper half DAB – of course it would seem to be at its slowest point when it is at the apogee A – or the whole motion could equally be described if one assumed the sun to be moving on an epicycle with centre E in the direction contrary to the order of the signs (i.e. in the direction indicated by the arrow, which is also called 'forward', while the opposite motion is called 'to the rear' or 'backward'),[2] while the centre E itself is moved on a concentric circle (the broken one in the figure) in an equal and contrary motion to that of the epicycle. The resulting motion will obviously be the same as the one anticipated by the eccentric model. The equivalence of the two models, and thus the resulting motions, is best described in *Almagest* III, 3.

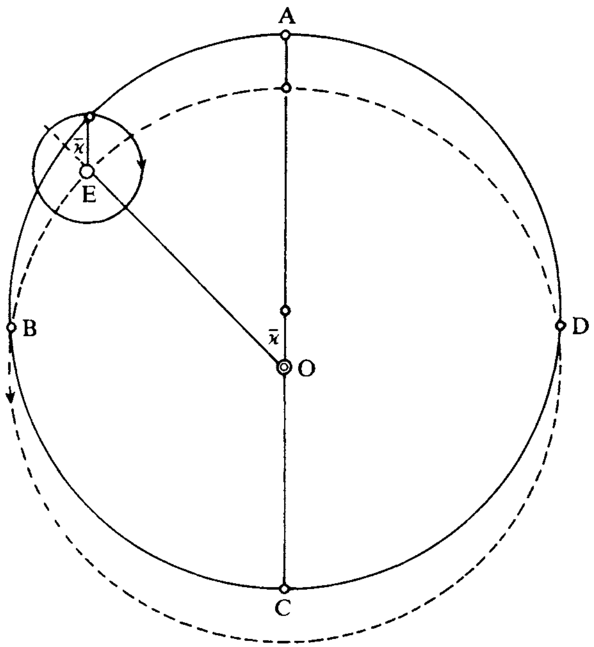


Figure 3.1

Although the motion of the sun could appear to the novice as implying a contradiction with the original principles of uniform motion, the Ptolemaic double-model explanation was found to be perfectly satisfactory, in that all the motions were indeed taking place around the centre of a sphere which was different from the position of the observer, in the first case, but could also be explained as identical with it in the epicyclic model and still be a combination of motions all uniform and all around centres of spheres – hence satisfying the primary principles.

The motion of the moon

In the case of the moon, however, the situation is quite different, for the motion is much more complex. In *Almagest* IV, Ptolemy first tried the Hipparchian model, which was essentially an extension of that of the sun, but soon realized that it did not describe all the motions properly. After a lengthy description that seems like a change of mind, he finally adopted a rather complicated model, in *Almagest* V, to account for the lunar movements.[3] In Figure 3.2 the observer is supposed to be at point O, the centre of the ecliptic. An encompassing sphere, called the sphere of the nodes, would move uniformly forward around the centre of the universe carrying with it the apogee of the deferent A. The deferent itself moves in the opposite direction around its own centre F, such that angles $\bar{S}OA$ and $\bar{S}OC$ are equal and opposite. This is obviously *ishkāl* 4 of al-Akhawayn's list mentioned above, for we have a deferent moving in a non-uniform motion around its centre F but moving uniformly around another point O. The epicycle carrying the moon is supposed in this model to be at point C, moving backward. The moon itself moves with its epicycle in the forward direction but measuring the forward angle from the extension of the line that connects point N – a point diametrically opposite to F from the centre of the universe called the prosneusis point – to the centre of the epicycle C, and extended to the mean apogee H at the epicycle's circumference. Since point N is always in motion, to remain opposite to the moving point F, it was thought to be a non-stable point for one to begin the description of motion from it, thus giving rise to the *ishkāl* of prosneusis mentioned above.

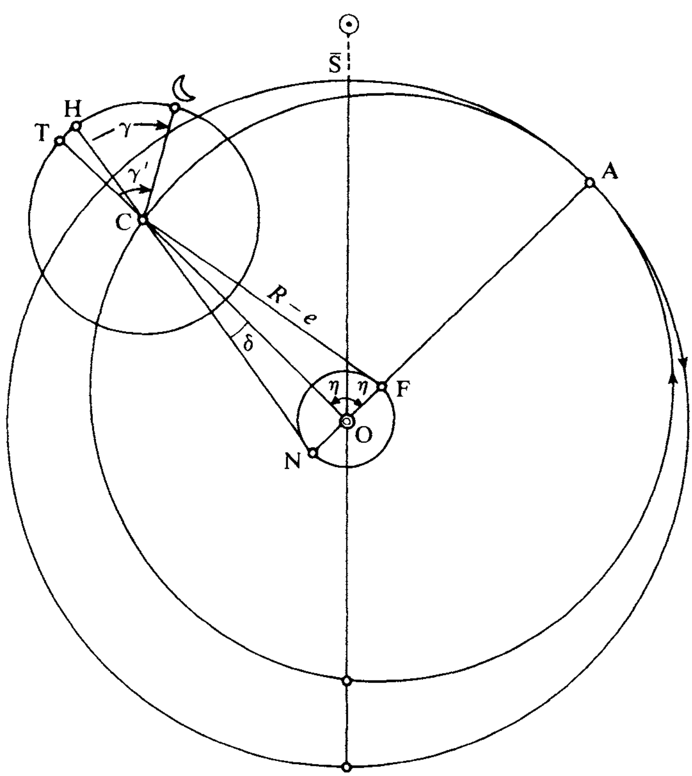


Figure 3.2

To summarize therefore, in the Ptolemaic lunar model one had to accept inconsistencies that gave rise to serious problems when one thought of the heavenly spheres as actual solid spheres. For it was impossible to move these spheres uniformly with respect to a centre other than their own, or with respect to a moving point that is not a fixed reference point for equal motion. All the criticism and the reformulations of Ptolemaic astronomy were really centred around these two points.

The motion of the upper planets (Saturn, Jupiter, Mars) and Venus

The motion of the upper planets as described by Ptolemy was relatively simpler than that of the moon, and involved the following elements. In Figure 3.3 the observer is taken to be at point O. The centre of the sphere that carries the epicycle of the planet – i.e. the deferent – in a backward motion is at point T. The epicycle of the planet itself moves backward around its centre, point C. The planet, P, is carried by the epicycle in its backward motion at a uniform speed measured by the angle of the anomaly. The reference point for the measurement of the anomaly, however, is located along the extension of the line joining the centre of the epicycle C to a point

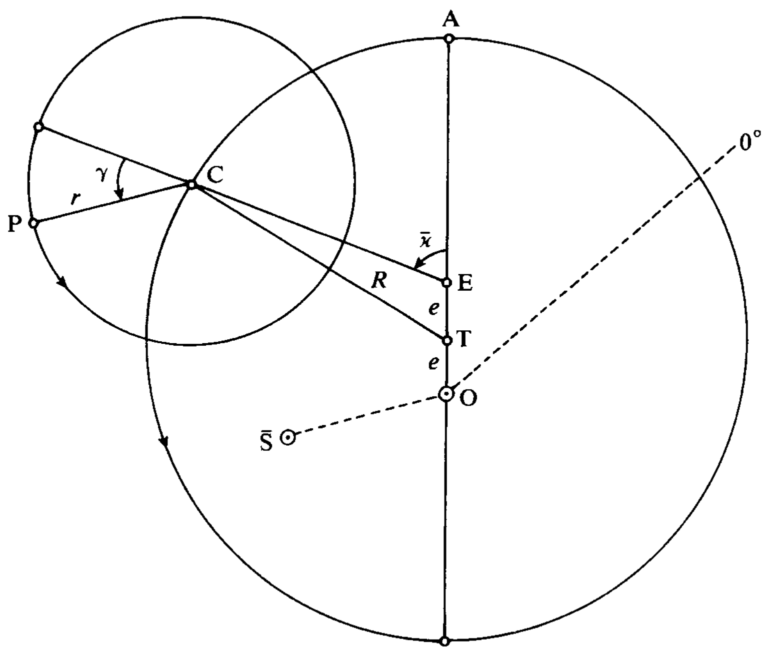


Figure 3.3

E, which is located along the line of centres OTA and is at such a distance from O that T bisects the line OE.

The difficulty in this model is in the motion of the deferent. For, as it was described by Ptolemy, the deferent carried the epicycle in a backward motion but did not do so in such a way that point C, the centre of the epicycle, would describe equal arcs in equal times around the centre of the deferent T, but rather around point E, the so-called *equant*. In essence then, the deferent, assumed by Ptolemy to be a physical sphere in the *Planetary Hypotheses*, is here being forced to move uniformly around a point different from its own centre. Put differently, the condition seems to require a sphere to move uniformly around an axis that did not pass through the centre of that sphere, which is an impossibility (*muḥāl*).

The motion of Mercury

Because Mercury is difficult to observe on account of its proximity to the sun and on account of its relatively fast motion, the Ptolemaic model for this planet was conceived to involve very complicated motions, which could not be included in the models described above. Moreover, this planet was unlike any other planet, in that Mercury was thought to have two perigees instead of one like all the others. These perigees were supposed to be symmetrically located with respect to the line of centres at angles equal to 120° from the apogee.

For an observer at point O, the centre of the universe, the motion of Mercury could be described as resulting from the following motions.[4] Let there be an encompassing sphere, analogous to the one that carried the nodes of the moon (Figure 3.4), and let it move forward around centre B in such a way that it carries the deferent's apogee with it. Let that apogee fall along the extension of line BG. Now let the deferent itself move backward around its own centre G, thereby carrying the epicycle's centre to point C, and making angle AEC equal to angle ABG. The epicycle itself moves backward around centre C, thus moving the planet M with the anomaly motion, which is measured from the extension of line EC. This model will allow the centre of the epicycle C to come close to the earth – i.e. to reach the perigee – twice per revolution, namely when ABG = 120° and 240° approximately. In both of these situations line GC will pass through E. Moreover, since the mean anomaly is always measured from the extension of the line EC, point E therefore will always act as the *equant* for the planet Mercury in an analogous fashion to the *equant* of the upper planets.

It is obvious that this model of Mercury suffers from difficulties similar to the ones encountered so far in connection with the models of the moon

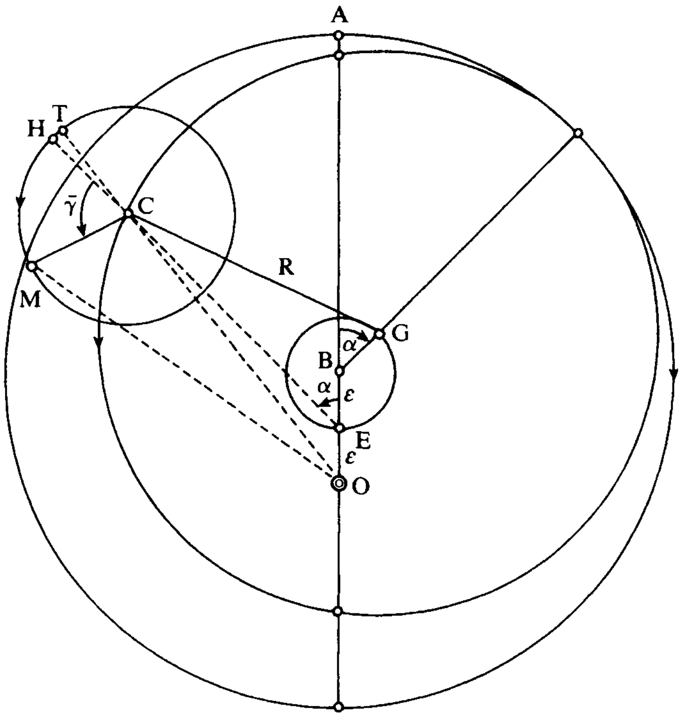


Figure 3.4

and the upper planets. In the first instance, the crank-like mechanism suggested here, with which the deferent is moved in one direction around a centre different from its own, while it moves on its own centre in the opposite direction, is very similar to the type of mechanism used in connection with the moon. The main difference between the two models is that in the case of the moon the mean anomaly was measured from a line that connected the centre of the epicycle with a moving point called the prosneusis point N, while here, in the case of Mercury, the analogous point is taken to be the midpoint of line OB, and thus a fixed *equant*, like that of the upper planets. In both models though, the deferent was supposed to move around its own centre at a non-uniform speed, while its uniform motion is supposed to occur around another centre, the centre of the universe O, in the case of the moon, and around the *equant* E in the case of Mercury.

It is not surprising therefore to find that the objections that were raised against Ptolemy's lunar model – especially in regard to the prosneusis point – and the model of the upper planets, in connection with the *equant* problem in particular, were also raised against the Ptolemaic model for

Mercury, for it seemed to have had the disadvantages of both of the other earlier models.

THE MOTION OF THE PLANETS IN LATITUDE

The previous description of the Ptolemaic models assumed that their motion in latitude is either negligible or that if it existed it did not affect the motion in longitude, which is not true. The facts are such that the planets are rarely seen to coincide with the plane of the ecliptic where the longitude motion is really measured, and that the latitude component is sometimes considerable and has to be taken into consideration. But in a typical Ptolemaic approach, this latitude component is thought of as an adjustment to the longitude and was left to be described in a separate section by itself.

The models accounting for the latitude theory were described in the *Almagest* in three separate models, i.e. one for the moon, one for the upper planets Saturn, Jupiter and Mars, and one for the inferior planets Venus and Mercury. This order also represents their order of complexity.

The latitude of the moon

For the moon, the model is rather simple, on account of the fact that the lunar orbit does pass through the earth, and thus for a geocentric observer the calculation of the lunar latitude is not too difficult. In fact, since the lunar orbit is at a fixed angle with respect to the ecliptic, and since the observer is at the centre of the ecliptic, the computation of the lunar latitude is very similar to the computation of the solar declination with respect to the celestial equator.

But because the lunar orbit is inclined at a fixed angle with respect to the ecliptic, the angle being about 5°, this means that the maximum latitude of the moon will also reach around 5° which it does according to observations. On the other hand, the observations have also shown that the moon's maximum latitude is not reached at any specific position on the ecliptic, but that it rather 'moves' around. This, coupled with the fact that solar eclipses also happen at various points on the ecliptic, meant that the line of intersection between the lunar orbit and the ecliptic, the nodal line, was also moving. That could only happen if one assumes the existence of a sphere that carries the whole configuration around so that the sphere whose cincture (*minṭaqa*) is the lunar orbit itself, or in Ptolemaic terms the deferent, is also moved by this assumed sphere. This last sphere is called the sphere of the parecliptic (*mumaththal*), or that of the nodes (*jawzahar*), and was supposed to move at about three minutes per day in the direction opposite to the order of the signs (i.e. forward in the sense used above).

To summarize, the lunar model, in its complete form, included the following spheres: (1) a sphere called the sphere of the pareclíptic (*mumaththal*) which moves the nodes and everything else in a forward direction; (2) an inclined sphere (*al-mā'il*), which also moves in the same forward direction and also accounts for the lunar latitude, and whose *minṭaqa* has the same plane as the deferent; (3) the sphere of the deferent, which moves in the backward direction in its own motion; and finally (4) the sphere of the epicycle which carries the body of the moon itself and is itself carried by the deferent sphere.

We have seen above the two objections raised against this model in terms of its performance when describing the lunar motion in longitude. These objections do not apply to the motion in latitude, for in this case all the motions that are needed to account for the motion in latitude are performed by spheres moving around their own centre, in this case the centre of the universe as well.

Latitude of the upper planets

For the upper planets, the situation is more complex, just because the actual orbit of these planets does not pass through the earth, the centre of the universe, but rather through the sun. For an observer on the earth, the transformation of the motion in latitude to geocentric co-ordinates involves more complicated procedures than the ones used in the lunar latitude which was just described.

To use the analogy of the lunar model, the upper planets' deferents (Figure 3.5) were also thought to be inclined, at a fixed inclination i_1 with respect to the ecliptic. The line along which the deferent intersects the ecliptic plane is also called the line of nodes, where the point at which the

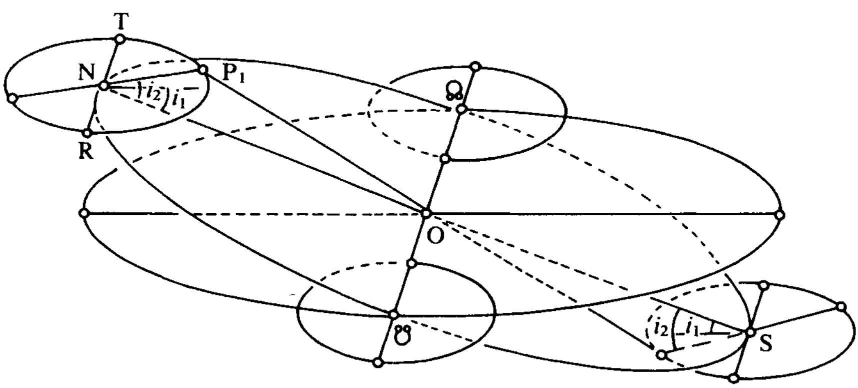


Figure 3.5

epicycle ascends from the south to the north is called the ascending node (or the head) and the diametrically opposite one is called the descending node (or the tail). The line issuing from the observer's point, perpendicular to the line of nodes, defines the top of the deferent N when it intersects the northern circumference of the deferent, and the bottom of the deferent S when it intersects the southerly one. This line is not in general the same as the apsidal line for it only passes through the centre of the ecliptic O, not through the centre of the deferent and the *equant* as the apsidal line does.

But unlike the lunar model, the epicycles of the upper planets do not lie in the plane of the deferent, as they were assumed to do when the component of the longitude was being considered on its own. Instead the plane of the epicycles themselves becomes inclined with respect to the plane of the ecliptic by an angle i_2 when the epicycle moves from the nodes. This last angle of inclination (called a deviation) reaches a maximum northerly inclination when the epicyclic centre is at the top of the deferent. The same deviation reaches a maximum southerly value, a value larger than the northerly one in absolute terms, when the epicycle is at the bottom of the deferent. This situation occurs because the portion of the deferent that is to the north of the ecliptic is larger than the southerly portion, which implies that the bottom part of the deferent is closer to the observer, and thus subtends a larger angle.

But when the centre of the epicycle is along the line of nodes, the plane of the epicycle is supposed to go back to lie in the plane of the ecliptic. Both angles of latitude, that of the inclined deferent and that of the deviation of the epicycle, will be equal to zero.

In effect, therefore, the plane of the epicycle seems to undergo a see-saw motion about an axis, RNT, which is perpendicular to the line joining the real apogee to the real perigee of the epicycle, and is always parallel to the plane of the ecliptic. This result is in itself an awkward one, for it involves a see-saw kind of motion in a portion of the heavens that was supposed to allow only complete circular motions. To account for it, Ptolemy suggested in *Almagest* XIII, 2, that small circles be attached to the tip of the see-sawing diameter P_1 of the epicycle such that the radius of the small circle would be equal to the maximum angle of deviation and the plane of the small circle would be perpendicular to the plane of the deferent from which the deviation is measured. With the insertion of such small circles, the line connecting the real apogee of the epicycle to the real perigee will no longer see-saw, but would have its tip moving along these small circles. But here again, since the time the epicycle takes to move along the larger northerly portion of the deferent is in general greater than the time it takes to cover the southern portion of the same deferent, and since the period along the small circle for the tip of the epicyclic diameter is equal to the period of the epicyclic motion along the deferent, the motion of the tip of the epicyclic diameter along the small circle is not uniform, and like the epicyclic centre has to be moving uniformly along its own *equant*.

This must have proved embarrassing to Ptolemy, since he begs the reader not to consider this arrangement as over-complicated, 'for it is not appropriate to compare human [constructions] with divine, nor to form one's beliefs about such great things on the basis of very dissimilar analogies' (*Almagest* XIII, 2). He goes on to say that he accepted that only because it yields a simple representation of the motions of the heavens.

It was this specific point that was objected to above (*ishkāl* 7) and was thought to be a violation of the accepted premises of the discipline of astronomy. As we shall see, the invention of what later came to be called the 'Ṭūsī Couple' would allow for the solution of this problem. In fact there is enough evidence to support the claim that the 'Couple' was specifically invented by Ṭūsī to solve this inconsistency, and was only applied later to produce a linear motion as a combination of two circular motions. Moreover, the 'Couple', being composed of two circular motions, allows the tip of the epicyclic diameter to oscillate back and forth in the same plane, without violating the circular motion principle, and thus allows the longitude component not to be disturbed.

Latitude of the lower planets

The Ptolemaic model for the lower planets is still more complicated, and assumes, in the case of Venus for example, that the inclination of the deferent plane is not fixed, but that it oscillates back and forth; that the deviation of the epicycle, like that of the upper planets, also undergoes a see-sawing motion about an axis that passes through the centre of the ecliptic; and finally, that the epicyclic plane also see-saws about another axis perpendicular to the first, and thus undergoes two see-sawing motions of its own. In the case of Mercury, all these motions are also required except that they are taken to be in the opposite direction.

To illustrate the case of the model for Venus, we take (Figure 3.6) the eccentric deferent to be inclined with respect to the plane of the ecliptic at an angle i_0, and let it intersect the plane of the ecliptic along the nodal line that passes through the point of the observer at the centre of the ecliptic. Unlike the case of the upper planets, in this model the apsidal line itself is perpendicular to the nodal line. But the inclination of the deferent is no longer fixed as it used to be in the case of the upper planets and the moon. In this case, the inclination of the deferent i_0 is coupled with the motion of the epicycle so that the plane of the deferent would coincide with the plane

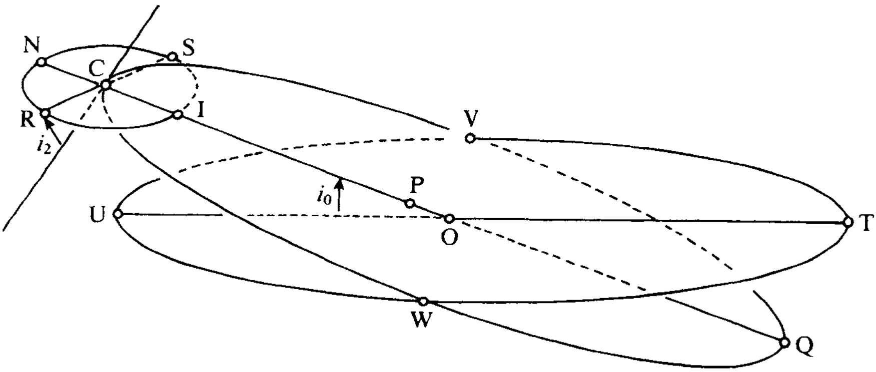


Figure 3.6

of the ecliptic when the epicycle is along the ascending node. As the epicycle begins to move towards the north, the inclination of the deferent begins to increase also in the northerly direction to reach a maximum inclination i_0 when the epicycle reaches the apogee. The inclination will then decrease as the epicycle moves from the apogee to the descending node, to get back to the plane of the ecliptic as shown in Figure 3.7. But as the epicycle moves from the descending node towards the perigee, the inclination of the deferent will increase again in the northerly direction, as shown in Figure 3.8, to reach another maximum value i_0 when the epicycle reaches the perigee. As the epicycle goes back to the ascending node, the plane of the deferent goes back to its original position in the plane of the ecliptic as in Figure 3.7. This is the first see-sawing motion in the model for Venus.

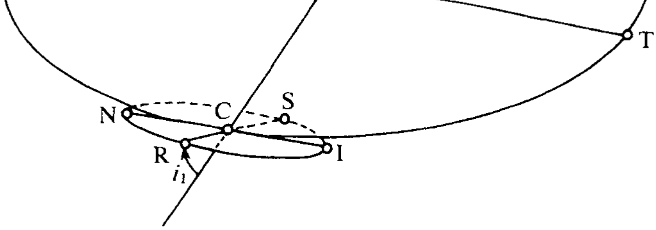


Figure 3.7

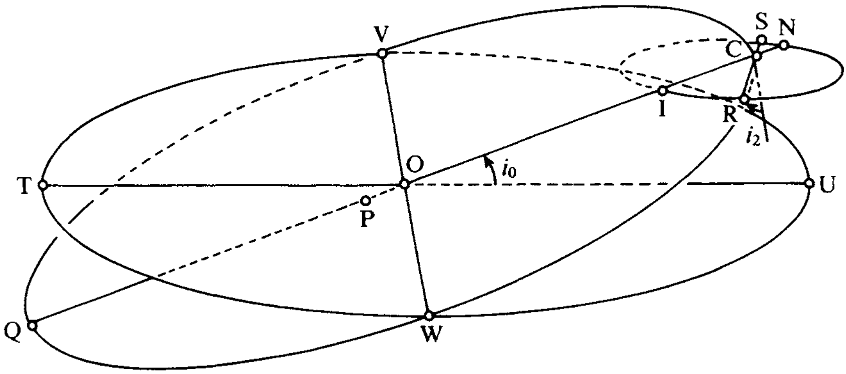


Figure 3.8

The second see-sawing motion is called a 'slant'. To explain it, Ptolemy assumed the plane of the deferent to coincide with the plane of the ecliptic when the epicycle is at the ascending node as in Figure 3.7. The line COD defined by the intersection of the plane of the ecliptic with the perpendicular plane produced by the line joining the apparent epicyclic apogee R, H and perigee S, E and the centre of the ecliptic is taken to be the first axis about which the slanting motion will take place. The line perpendicular to this last axis KDL, NCI, and which passes through the centre of the epicycle D, C (the mean diameter), is then taken to be the second axis about which the deviation motion will be described.

When the epicycle is at the ascending node, the epicyclic mean diameter KDL lies in the plane of the ecliptic, and thus has no latitude component for that slant. But at that position the plane of the epicycle is 'deviated' in such a way that it reaches its maximum deviation i_1 just at that position. As the epicycle begins to move towards the apogee, the plane of the deferent moves towards the north as in Figure 3.6, and the plane of the epicycle begins to 'deviate' back from its maximum position to reach zero deviation when the epicycle reaches the apogee, while the slant which was equal to zero at the node begins to increase to reach a maximum slant i_2 when the epicycle is at the apogee.

When the epicycle reaches the apogee, the plane of the deferent will be at its maximum inclination i_0, the plane of the epicycle will be slanted such that the eastern side of the epicycle will be northerly at a maximum slant i_2, while the line connecting the centre of the ecliptic to the apparent perigee and apogee of the epicycle will lie in the plane of the deferent having a zero deviation.

As the epicycle moves to the descending node as in Figure 3.7, the plane of the deferent goes back to coincide with the plane of the ecliptic, while the epicyclic plane reaches its maximum deviation i_1 with the apogee of the epicycle being in the northerly direction, and at that position the epicycle will have a zero slant.

But when the epicycle moves to the perigee of the deferent as in Figure 3.8, the inclination of the deferent increases to bring the perigee of the deferent to a maximum northerly direction i_0. The plane of the epicycle is slanted in that position to bring the easterly side of the epicycle to its maximum northerly slant i_2, as it did when the epicycle was in the apogee of the deferent. Here again, the deviation of the epicycle will be zero.

In the case of Mercury, the inclination of the deferent, the slant and the deviation of the epicycle are all in the direction opposite to that of Venus. When the epicycle is at the nodes, the deviation is northerly for Mercury when it was southerly for Venus and vice versa. At the apogee the inclination of Mercury's deferent moves to its maximum southerly inclination when that of Venus moves to its maximum northerly direction. Similarly at the apogee the slant is southerly for Mercury where it was northerly for Venus.

If the deviation in the case of the upper planets had been an embarrassment for Ptolemy in terms of his having to use small circles to account for the motion of the deviation of the epicycles of the upper planets, the inclination, the deviation and the slant of the inferior planets, all of them requiring such small circles to allow them to see-saw about their various axes, are a triple embarrassment, and it should not be surprising that such models were considered to be inconsistent with the original premises of the discipline of astronomy. Here again, the 'Ṭūsī Couple' could be efficiently used to account for all those linear motions of the tips of the various axes as resulting from circular motions.

This then is a brief description of the main features of the Ptolemaic theory of latitude. And as we have seen, it was easy to find many faults with it in spite of its observational base and its ability to predict positions of specific planets at specific times. The main problem that permeates the whole theory at all levels is the one referred to above as *ishkāl* 7, and could be simply described as the problem of admitting oscillating linear motion among the heavenly motions which are supposed to be circular. Once those oscillating motions could be produced by circular motions, as in the case of the 'Ṭūsī Couple', the problem would then be reduced to adjusting the periods of motion so that the circles of the 'Couple' will themselves move at a uniform speed, which is not an easy matter.

THE REFORM OF THE PTOLEMAIC PLANETARY MODELS

It was said above that serious criticisms of the Ptolemaic models started, as far as we know, sometime during the eleventh century. In that century, two main lines of research seem to have been developed simultaneously; namely, the line of criticism that was limited to identifying the main defects of the Ptolemaic models, and that of finding alternative models to replace the defective Ptolemaic ones.

The first line of research, that devoted to criticism, was represented by Ibn al-Haytham (*c.* AD 965–1039) in his work *al-Shukūk ʿalā Baṭlamyūs*, and an anonymous astronomer whose work *al-Istidrāk ʿalā Baṭlamyūs* has not yet been located.[5] From Ibn al-Haytham's work, we know that the criticism was not only limited to Ptolemy's planetary models, but that it also included other works of Ptolemy such as the *Optics*. This means that the whole work was probably motivated by considerations that were much more general than the astronomical ones. One could argue that this genre of writing was in all likelihood in the same tradition as the work of the tenth-century physician Abū Bakr al-Rāzī (d. *c.* AD 925) who wrote a similar work against Galen (second century AD), which he called *al-Shukūk ʿalā Jālīnūs*. The astronomical contents of Ibn al-Haytham's work will be summarized in the following section. The work of the anonymous astronomer, on the other hand, seems to have been devoted to astronomical issues. For in his surviving treatise, whenever he reaches one of the difficult points in the Ptolemaic models that were noted above, he says that the point is rather difficult to accept and is explained in his work *al-Istidrāk*.

Contents of Ibn al-Haytham's *Shukūk*[6]

The book begins with an introduction in which Ibn al-Haytham sets down the principles that he intends to follow in his work. After admitting the excellence of Ptolemy's works, he goes on to say that, in this book, he will only mention those problems (*shukūk*) that cannot be explained away, and are in direct contradiction with the accepted original principles.

Apparent size of the sun

The rest of the book is divided into three main parts; each is devoted to the contradictory positions in one of the three works of Ptolemy, i.e. the *Almagest*, the *Planetary Hypotheses* and the *Optics*. In the first part, following the order of the *Almagest*, Ibn al-Haytham begins with the problem in Book I,3, which is the problem of the apparent size of the sun, as it appears bigger when the sun is closer to the horizon than when it is in the middle of the heaven. Here Ibn al-Haytham uses Ptolemy's own results from the *Optics* against his statement in the *Almagest.*

Directions from the centre of the world

In regard to chapter 5 of Book I of the *Almagest,* Ibn al-Haytham requires of Ptolemy more precision in his use of his own concepts, and objects to Ptolemy's use of the earth being 'higher' or 'lower' with respect to the centre of the world when there are no such directions with respect to the centre of the universe, since all directions from the centre are in the 'higher' direction. This kind of mistake (*ghalaṭ*), Ibn al-Haytham identifies as a mistake in conceptualization (*taṣawwur*) rather than a contradiction. Similarly, when Ptolemy uses the terms east and west to describe the position of the earth, he would be committing the same conceptual error.

The value of the chord of 1°

In the same vein, Ibn al-Haytham objects to Ptolemy's use of a value being bigger and smaller than another quantity, as a proof of its being equal to that quantity. He would have forgiven Ptolemy had he said at that point that the value of the chord of 1° was approximately equal to that quantity and that it differed from it by some small number rather than being at the same time greater than and smaller than that quantity.

The inclination of the ecliptic

Ibn al-Haytham also objects to Ptolemy's method for determining the inclination of the ecliptic, for he said that he observed the sun along the meridian circle and found the difference between the highest position of the sun at the summer solstice and its position at the winter solstice to be 47° and a value greater than $\frac{2}{3}^\circ$ but less than $\frac{1}{2}^\circ$ and $\frac{1}{4}^\circ$.

The reason he objects to that is that the solstices need not occur when the sun is crossing the meridian circle of that specific locality, and Ptolemy knew that. But he agreed to take the approximate value, when he ought to have explained how such a value could be determined with precision. Moreover, he also knew that the sun will never return to the same point on the meridian circle in an integral number of days in the years that follow. But in spite of that he still said that he observed the sun cross that point of the solstice year after year, which could not be true. Because several parameters are connected to this measurement, Ibn al-Haytham concludes that neither the solar year, nor the solstice point, nor the declination, nor the equinoctial points are known from Ptolemy's statements.

The proof that Ptolemy did not really determine these parameters is that modern astronomers have found them to be different. They found the declination to be different and the solar apogee to be moving when Ptolemy had determined that it was fixed.

The prosneusis point

The objection about the prosneusis point is the same as the one mentioned by al-Akhawayn under *ishkāl* 6. It derives from the Ptolemaic lunar model in which it was required that the mean epicyclic apogee be determined by the extension of a line joining the centre of the epicycle and the prosneusis point, which was itself defined as being diametrically opposite to the centre of the deferent with respect to the centre of the universe. Such an apogee is not only imaginary to Ibn al-Haytham, but it could not be a reference point from which one could measure motion. Ibn al-Haytham's real concern, however, is expressed in the following terms:

> The epicyclic diameter is an imaginary line, and imaginary lines have no perceptible motion that produces an existing entity in the world. Similarly, the plane of the ecliptic is also an imaginary plane, and imaginary planes do not exhibit an observable motion. And nothing moves in an observable motion, which produces an existing entity in this world, except the bodies that do [indeed] exist in this world.
>
> (*Shukūk*, p. 16)

Moreover, even if one were to accept the existence of such an imaginary line, and thus the existence of a mean apogee defined by it, one still could not explain according to the accepted principles the motion of this line, for it seems to oscillate back and forth producing positive and negative angles within a period of half a lunar month. None of these motions seemed to have been produced by full revolutions of spheres moving at uniform speed as they ought to be. Ibn al-Haytham concludes this section with a tirade of criticisms, exhausting all possible excuses for Ptolemy, and finally rejecting the existence of such lines or bodies that could move these lines in this manner. 'If such bodies were then found to be impossible [to exist] [*muḥāl*] then it is impossible for the diameter of the epicycle to move in such a way that it would be in line with the assumed prosneusis point' (p. 19).

The later attempts by other astronomers to modify the Ptolemaic lunar model included in one way or another a statement about the problem of the prosneusis point and an effort to avoid using it.

Limits of eclipses

In this section Ibn al-Haytham objects to Ptolemy's apparent use of an approximate method when he determined the limits of eclipses. The brunt of the objection centres around Ptolemy's use of an arc – equal to the sum of the solar and lunar radii – perpendicular to the orbit of the moon rather than the ecliptic, as Ibn al-Haytham would have preferred. Ibn al-Haytham then argues that Ptolemy's choice of that procedure does not allow him to compute the beginning of the eclipse, nor its middle nor its end, and 'his assumption that this arc could determine the limits of an eclipse in longitude and in latitude is an obvious error without any doubt'.[7]

The equant problem

This section is by far the most important of Ibn al-Haytham's criticisms of Ptolemaic astronomy. It deals with the problem referred to above as *ishkāl* 4, which simply states that a sphere could not possibly move uniformly around an axis which does not pass through its centre as Ptolemy assumed. To construct his argument, however, Ibn al-Haytham started by showing that Ptolemy was quite aware that he was violating his own premises in regard to the *equant* problem.

Ibn al-Haytham begins by pointing to *Almagest* IX, 2, where it is clearly stated that the upper planets are supposed to move uniformly[8] just like the other planets that he had already described. This section is then contrasted with *Almagest* IX, 5, where Ptolemy clearly states that in the model for the upper planets 'We find, too, that the epicycle centre is carried on an eccentre which, though equal in size to the eccentre which produces the anomaly, is not described about the same centre as the latter'. (Toomer 1984: 443) Later on, in *Almagest* IX, 6, Ptolemy describes his model for the upper planets in more detail. It is there that Ptolemy defines the *equant* (to use the later medieval term) as simply the point around which the centre of the epicycle moves uniformly. Without any proof, Ptolemy also states in this chapter that the centre of the deferent lies midway between the centre of the universe and that of the *equant*.

To this Ibn al-Haytham says: 'What we have reported is the truth of what Ptolemy had established for the motion of the upper planets; and that is a notion [*maʿn^{an}*] that necessitates a contradiction' (p. 26). The proof of the contradiction is then constructed as follows. Since Ptolemy accepted the principle of uniform motion, and since he had shown in the case of the sun that if any body moves uniformly around one point it must necessarily move non-uniformly around any other point, therefore, Ptolemy must have contradicted himself by stating that the centre of the epicycle moves uniformly around the *equant* for then it means that it does not move uniformly around the centre of its own deferent, which is impossible.

In the details of the response, Ibn al-Haytham states quite clearly that his objection is actually based on the fact that these motions are supposed to be motions of real bodies, not imaginary ones, 'since imaginary circles do not move by themselves in any perceptible motion' (p. 28). Moreover, Ibn al-Haytham makes the obvious remark that if a body is supposed to move uniformly around a point, it also means that the body must be always equidistant from that point. In effect, if the bodies that were described by Ptolemy were supposed to be real physical bodies, then a sphere could move uniformly only around an axis that passes through its centre.

Ibn al-Haytham then extends his criticism to the Ptolemaic model of Mercury, *Almagest* IX, 9, for it suffers from the same contradiction. He goes on to conclude this section by casting doubt about the method in which Ptolemy determined the eccentricities of the planets.

In order to clinch his argument, Ibn al-Haytham quotes Ptolemy, *Almagest* IX, 2, which proves that even Ptolemy himself had already admitted that he was using hypotheses that were contrary to the accepted principles (*khārija ʿan al-qiyās*). Since Ptolemy

> had already admitted that his assumption of motions along imaginary circles was contrary to [the accepted] principles, then it would be more so for imaginary lines to move around assumed points. And if the motion of the epicyclic diameter around the distant center [i.e. the equant] is also contrary to [the accepted] principles, and if the assumption of a body that moves this diameter around this center is also contrary to [the accepted] principles for it contradicts the premises [*al-uṣūl*], then the arrangement, which Ptolemy had organized for the motions of the five planets, is also contrary to [the accepted] principles. And it is impossible for the motion of the planets, which is perpetual, uniform, and unchanging to be contrary to [the accepted] principles. Nor should it be permissible to attribute a uniform, perpetual, and unchanging motion to anything other than correct principles, which are necessarily due to accepted assumptions that allow no doubt. Then it becomes clear, from all that we have shown so far, that the configuration, which Ptolemy had established for the motion of the five planets, is a false [*bāṭila*] configuration, and that the motions of these planets must have a correct configuration, which includes bodies moving in a uniform, perpetual, and continuous motion, without having to suffer any contradiction, or be blemished by any doubt. That configuration must be other than the one established by Ptolemy.
>
> (*Shukūk*, pp. 33–4)

Motion in latitude

Ibn al-Haytham's objections to the theory of latitude begin with a long quotation from *Almagest* XIII, 1, which treats the motion of the inferior planets in latitude. He then follows that with his own summary of Ptolemy's statement, and concludes by saying that

> This is an absurd impossibility [*muḥāl fāḥish*], in direct contradiction with his earlier statement about the heavenly motions – being continuous, uniform and perpetual – because this motion has to belong to a body that moves in this manner, since there is no perceptible motion except that which belongs to an existing body.
>
> (*Shukūk*, p. 36)

Moreover, since the motions of the inclined plane in which the deferent lies are contrary in direction, Ibn al-Haytham concludes that Ptolemy did indeed commit a great error in allowing the same body to have two different natures, thus signalling a possibility of change in the heavens which is contrary to the accepted principles.

Conclusion

The concluding section of the critique of the *Almagest* is a long reflective statement by Ibn al-Haytham on the reasons why Ptolemy said what he said. He admits that there are places where these contradictions might have occurred as a result of negligence from which no human is free. At these places, Ptolemy could be excused. But at the places where he intentionally falls into contradiction, he has no excuse whatsoever. To prove that Ptolemy did indeed intend to accept the contradictions, Ibn al-Haytham quotes the famous passage of the *Almagest* IX, 2, in which Ptolemy says that he was obliged to employ devices that were contrary to the accepted principles (*khārija ʿan al-qiyās*) and that he demonstrated his proof by using imaginary circles. Ibn al-Haytham then isolates the main problem with Ptolemy's configuration for the upper planets as being exactly that; i.e. that he had demonstrated the motion of these planets in reference to imaginary circles and lines. But once the existence of real bodies was assumed, the contradiction then became clear.

Nor would Ibn al-Haytham accept the statement of an apologist who would say that these configurations are all imaginary, and that they would not affect the behaviour of the real planets, because one need not assume a contradictory configuration to describe the motions of existent bodies. Nor could Ptolemy be excused for saying as he did (*Almagest* IX, 2) that he reached a correct description of the motion of the planets without being able to describe the method by which he reached those conclusions. He should rather have admitted first that the configuration that he was describing was not the real one, and that he had not yet come to understand the correct one. Only then would he be excused.

This section is followed by Ibn al-Haytham's summary of Ptolemy's models for the planets, a straightforward rendering of the models described in the *Almagest* (*Shukūk*, pp. 39–41). He concludes by saying that Ptolemy

> had gathered together all the motions that he could verify from his own observations and from the observations of those who have preceded him. Then he sought a configuration of real existing bodies that could exhibit such motions, but could not realize it. He then resorted to an imaginary configuration based on imaginary circles and lines, although some of these motions could possibly exist in real bodies. He resorted to this method simply because he could not devise another one. But if one imagines a line to be moving in a certain fashion according to his own imagination, it does not follow that there would be a line in the heavens similar to the one he had imagined moving in a similar motion. Nor is it true that if one imagined a circle in the heaven, and then imagined the planet to move along that circle, that the [real] planet would indeed move along that circle. Once that is accepted, then the configuration assumed by Ptolemy for the five planets is a false configuration [*hay'a bāṭila*], and he established it knowing that it was false for he could not devise anything else. But the motions of the [real] planets have a correct configuration in [real] existing bodies that Ptolemy did not comprehend, nor could he achieve. For it is not true that there should be a uniform, perceptible, and perpetual motion which does not have a correct configuration in existing bodies.
> (*Shukūk*, pp. 41–2)

Doubts concerning the Planetary Hypotheses

In his doubts engendered by the text of the *Planetary Hypotheses*, Ibn al-Haytham starts by enumerating the points of variation between that text and the text of the *Almagest*. He enumerates, for example, the number of motions attributed to the planets in the *Almagest*, which were found to be thirty-six, with the number of motions of those in the *Planetary Hypotheses*, which were found to be only twenty-six.

Then, in the description of the movements of the epicyclic spheres in the first book of the *Planetary Hypotheses*, Ibn al-Haytham finds Ptolemy's text wanting in that it did not include the 'small circles', referred to in the *Almagest*, that carried the epicycles in latitude, nor did he find any account of how the planet is supposed to be moved in latitude (*Shukūk*, pp. 43–4).

He then concludes that Ptolemy's statements in the first book of the *Planetary Hypotheses* not only describe an erroneous configuration (*hay'a fāsida*), but are in fact contrary to what is found by observation – in terms of the latitudinal motion of the planets – and to what is found in the *Almagest* itself.

In analysing the causes (*'ilal*) for the planetary movements, Ptolemy proposes in the first book of the *Planetary Hypotheses* that each of these planets has two movements: 'One that is voluntary [*irādīya*], and the other that is by compulsion [*yuḍṭarru ilayha*]' (Goldstein 1967b: 26, ll. 16–18). In the second book of the *Planetary Hypotheses*, he goes on to say that 'each of these various movements that vary in quantity and kind must have a body that produces it by moving about some poles ... in such a way that it undergoes no forcing or compelling from outside' (*Shukūk*, pp. 45–6).

Ibn al-Haytham finds these two statements to be contradictory, for how could a body be compelled to move in one case and in the other accept no compulsion from an outside agent?

Then he attacks Ptolemy for using the idea of spherical shells (*manshūrāt*) instead of spheres, saying that instead of solving the problems under discussion, the planetary shells suffer from the same disadvantages and introduce some of their own in addition.[9]

This brings Ibn al-Haytham back to the theory of latitude for the inferior planets, and the 'small circles' which were assumed, in the *Almagest*, to move the epicycles of the inferior planets along two perpendicular axes. These 'circles' are not mentioned in the *Planetary Hypotheses*. To this Ibn al-Haytham says:

> If one were to explain them in the same manner as before [i.e. in the case of the *Almagest*] then they would produce the same impossibilities, but if not, then one has to assume that Ptolemy had made a mistake in their regard [by not mentioning them here] or that he had made a mistake by mentioning them in the *Almagest*.
>
> (*Shukūk*, p. 58)

Similarly, Ptolemy did not mention in the *Planetary Hypotheses* the oscillating motion of the inclined planes of the inferior planets, as he did in the *Almagest*.

Moreover, Ptolemy dropped from consideration the motion of the prosneusis point while describing the spheres of the moon, which he had included among the movements of the moon in the *Almagest*.

At the end of the second book of the *Planetary Hypotheses*, Ptolemy seemed to have accepted the fact that planets could move by themselves, without the need for any bodies to move them. But to this Ibn al-Haytham says that it necessitates the existence of void in the heavens, by allowing the planet to 'empty' one place and 'fill' another. Then he goes on to object to the motion as being one of rolling (*tadaḥruj*), and he concludes by saying:

> If Ptolemy could find it permissible that a planet could move by itself, without the need for any body to move it, then all the spherical shells, which he had proposed for the planets, as well as the spheres themselves would be invalid.
> (*Shukūk*, p. 62)

This section is concluded like the one concerning the criticism of the *Almagest* by saying that Ptolemy

> either knew of the impossibilities that would result from the conditions that he assumed and established, or he did not know. If he had accepted them without knowing of the resulting impossibilities, then he would be incompetent in his craft, mislead in his attempt to imagine it and to devise configurations for it. And he would never be accused of that. But if he had established what he established while he knew the necessary results – which may be the case befitting him – with the reason being that he was obliged to do so for he could not devise a better solution, and [on top of that] he went ahead and knowingly fell into these contradictions, then he would have erred twice: once by establishing these notions that produce these impossibilities, and the second time by committing an error when he knew that it was an error.
>
> When all is considered, and to be fair, Ptolemy would have established a configuration for the planets that would have been free from all these impossibilities, and he would not have resorted to what he had established – with all the resulting grave impossibilities – nor would he have accepted that if he could produce something better.
>
> The truth that leaves no room for doubt is that there are correct configurations for the movements of the planets, which exist, are systematic, and entail none of these impossibilities and contradictions, but they are different from the ones established by Ptolemy. And Ptolemy could not comprehend them, nor could his imagination come to grips with them.
> (*Shukūk*, pp. 63–4)

As if that condemnation was not enough, Ibn al-Haytham then reminds the reader once more that Ptolemy neglected to mention in the *Planetary Hypotheses* the 'small circles' that he had used in the *Almagest* to account for the latitudinal motion. Ibn al-Haytham then guesses that Ptolemy did not do so either because he knew of the contradictions it would lead to if he adopted the model of the spherical shells, or because he wanted to avoid the cumbersome additional spheres if he adopted the model of the spheres. 'He then saw that it was better to remain silent about this (latitudinal) motion than to fall into these contradictions that it entailed' (*Shukūk*, p. 64).

Contents of *al-Istidrāk ʿalā Baṭlamyūs*

We know very little about the author of this work, or about the work itself, which has not yet been located. But whatever we can glean from the existing treatise by the same author, called *Kitāb al-Hay'a*, now kept at the Osmania University Library in Hyderabad (Deccan, India), seems to indicate that the author of *Kitāb al-Hay'a* had lived in Spain sometime during the eleventh century; he spoke of the famous Spanish astronomer al-Zarqāel (or al-Zarqāllu) (d. 1099) as his personal friend. The author also mentioned that in one of his works he described the instrument used for the observations that were conducted at Toledo, without specifying the year.

This author of *Kitāb al-Hay'a* tells us that he had found some of the statements of Ptolemy to have been objectionable, and states quite explicitly that he did not want to interject his objections in this elementary text which he was writing, for he had already devoted a special book to such objections which he called *al-Istidrāk ʿalā Baṭlamyūs* ('Recapitulation Regarding Ptolemy').

The manner in which he refers to this work is quite revealing of the subject matter that the book must have included. While speaking of the inaccuracy of the instrument which 'was found in Toledo, in al-Andalus', he says that 'the instrument was set in accordance with (the position) designated by the man who actually used it for observations, Abū Isḥāq Ibrāhīm b. Yaḥyā known as al-Zarqael (*sic*) as he himself had told me' (fol. 15^{v}). On folio 16^{r}, the author says that he had composed a book that he called *al-Istidrāk ʿalā Baṭlamyūs*. And while discussing the solar apogee, the author says: 'It was during the time of al-Ma'mūn at twenty degrees of Gemini and about two thirds of a degree. These matters ought to be better mentioned in the book *al-Istidrāk*' (fol. 41^{v}).

When the author discussed the motions of the moon, he had the following to say: 'I may object to Ptolemy in these matters in several ways, but I ought to mention that in what is simpler (?) than this book, and I will mention it in *al-Istidrāk* if God wills it' (fol. 48^{r}).

Finally, while discussing the planetary apogees, the author had the following to say: 'And Ptolemy found that the motion of the (origin of the) longitudes of these five planets takes place at the rate of one degree every one hundred years, while the more recent (astronomers) found it to be one degree in about sixty six years. We will mention the reasons for this variation in *Kitāb al-Istidrāk*' (fol. 68^{r}).

ALTERNATIVES TO PTOLEMY'S PLANETARY MODELS

The two critical works that were cited above represent what is now known of this type of literature. But that does not mean that the scope of the critical activity was limited to those two, nor does it mean that other criticisms were not as influential as those. From the recovered works that were written in the later centuries we can assert that the criticism of Ibn al-Haytham was taken very seriously by astronomers, and more than one astronomer took it upon himself to find an alternative set of models that was free of the contradictions that had bedevilled Plotemaic astronomy.

At this point, and in the interest of space and time, it is useful to divide the response to such criticisms – which found expression in the attempts to construct planetary models which were construed as alternatives to the Ptolemaic ones – into two schools; namely, the Andalusian School and the Eastern School.

The Andalusian school

The anonymous Andalusian astronomer who wrote *al-Istidrāk* may have been the forerunner of a later school of astronomers who continued his work and added some of their own criticisms; they all attempted to reformulate the Ptolemaic models. Names such as those of Jābir ibn Aflaḥ (d. *c.* middle of the twelfth century), Al-Biṭrūjī (fl. *c.* 1190) and Averroes (d. 1198) are but a few of those whose works have been critical of Ptolemy's models and have been subjected to some study.[10]

Considering Jābir ibn Aflaḥ's work, *Iṣlāḥ al-Majisṭī* ('Correction of the *Almagest*'), the main contribution of that work is that it lists some ten to fifteen problems – called 'errors' by Jābir – through which the reader is led step by step to realize the difficulties and the problems in the Ptolemaic text. One such major difficulty, for example, is the treatment of the planetary distances in the *Almagest*, for, according to Ptolemy's values, at least Venus would have to be placed above the sun.[11] Noting this difficulty, Jābir argues[12] that his computations required that both Venus and Mercury ought to be above the sun.

The main arguments of Jābir for placing both Venus and Mercury above the sun are as follows. (1) Ptolemy admits that the sun exhibits a parallax of about three minutes of arc, and that Venus and Mercury do not exhibit any observable parallax. This, according to Jābir, could only mean that they were further away than the sun, and hence above the sun in the arrangement of the heavenly spheres. (2) Jābir uses Ptolemy's values for the ratio between the epicyclic radii of Venus and Mercury and their respective deferents, and proves that if these values were to be taken seriously then Venus and Mercury would have to exhibit a parallax of about six to seven minutes, almost twice as much as that of the Sun. Since none of that takes place then they must be above the sun.

After citing the full text of Ptolemy in regard to the relative distances of the spheres of the planets, Jābir remarks in the following manner: 'I am extremely amazed at this man, and quite perplexed by him, because he appears to contradict himself without even knowing it' (fol. 78^{v}).

Since the absolute distances of the planets could not be determined with any certainty, this problem of the relative order of the planetary spheres remained a challenging one throughout the Middle Ages, and was taken up, as we shall see below, by al-Biṭrūjī and by Mu'ayyad al-Dīn al-ʿUrḍī (d. 1266) among others.

For al-Biṭrūjī, as for Ibn al-Haytham, the main problem with Ptolemaic astronomy was that it was not sufficiently Aristotelian. But unlike Ibn al-Haytham, who understood motion along an eccentric as being acceptable in the Aristotelian sense, al-Biṭrūjī did not even tolerate eccentrics and epicycles if these were to be understood in the traditional Ptolemaic sense. His main concern was that the universe must have only one point around which all other points must revolve, and that point had to be fixed and must coincide with the centre of the earth. This purist Aristotelian attitude is supposed to have been first championed by al-Biṭrūjī's teacher Ibn Ṭufayl (d. 1185), who promised that he would produce a book in which such an astronomy would be described, but does not seem to have done so.

These attempts were then followed by al-Biṭrūjī's book *Kitāb al-Hay'a* ('Book on Astronomy') which was written especially to develop such an astronomy, and later by the work of Ibn Rushd (mainly his commentary on the Aristotelian *Metaphysics*), who only recorded his objections in a qualitative manner.

All this activity remained limited in its applicability and scope, simply because the new proposed configurations – such as the one proposed by al-Biṭrūjī – were not successful enough in their ability to duplicate the Ptolemaic observational and analytical results. There was a real need therefore for a set of new models that would avoid the Ptolemaic shortcomings, with the provision that these models had also to conform to Ptolemy's valid observations, and save the same phenomena that were saved by Ptolemy's models.

The real progress in that regard was achieved in the eastern part of the Islamic world, where generations of astronomers, beginning in the eleventh century and continuing well beyond the fourteenth, had achieved several results first by isolating the Ptolemaic problems and then by applying new techniques to them which rendered them consistent with the original principles of the Aristotelian cosmos.

The Eastern School

The Eastern School discussed here has been referred to in the literature as the 'Marāgha School', [13] simply because all but one of the astronomers who were then known to have discussed non-Ptolemaic astronomical models had worked at one time or other during the latter part of the thirteenth century at the Marāgha Observatory in northwest modern Iran. But since we now know more about this activity, and we know that it was not restricted to the environs of the Marāgha Observatory, nor to the thirteenth century, we chose a term that contrasts the activities that were carried out in this region of the Islamic Empire with those that have been described as belonging to the Andalusian Revolt.

Luckily, the activities in the Eastern School have some cohesion, and could therefore be characterized as belonging to the same tradition. The general attitude of the astronomers belonging to this School towards Aristotle and Aristotelian cosmology was distinctly different from the attitude of their Western colleagues in Andalus. As the Andalusians were concerned with the inadmissibility of eccentric and epicyclic motion, for it violated the principle of the Aristotelian centre of the universe which marked the centroid around which all circular motion had to take place, the Eastern astronomers did indeed realize that this problem was only a pseudo-problem. For as Ibn al-Shāṭir put it:

> The existence of small spheres such as the epicyclic spheres, that do not surround the earth, is not impossible, except in the ninth sphere. The proof of that is that [it is similar] to the existence of a planet in each sphere and a multitude of stars in the eighth sphere where each of them [i.e. the planets or the stars] is greater than some of the epicycles of some planets, and the planet [or star] is different from the body of the [heavenly] sphere. Thus it is not inadmissible that there be epicycles and such things. From this, it is understood that the [heavenly] spheres have some sort of composition [*fīhā tarkīb^{un} mā*] and that the only one which is absolutely simple [*basīṭ muṭlaq*] is the ninth [sphere] for it is not possible to imagine in it the existence of a planet [or star] or such similar thing. [14]

Ibn al-Shāṭir expresses the same opinion later on when he says:

> They [i.e. the astronomers] have disagreed about the motion of the small spheres which do not encompass the center of the universe, such as the sphere of the epicycle and the like. They have all accepted that it could move towards any assumed direction, for they indicated that the epicyclic sphere has an upper and a lower half. So if it moves in the direction of the signs in the upper

half, it would move in the direction opposite to the signs in the lower half, and vice versa. Its motion would not be by compulsion [*qasrīya*] nor by accident [*ʿaraḍīya*], but would rather be natural [*ṭabīʿīya*]. They have also agreed that it was permissible to have epicycles in other than the ninth [sphere], because we see planets in the spheres. And the existence of a planet in a sphere signifies some sort of composition [*tarkīb*^{un} *mā*]. And whoever says that the spheres are all simple, and thus no epicycles could exist in them, and every motion that is not a motion around the center could not be a simple motion, I would say that the motion of the epicycles has been proven to exist. And if that could ever be proven with a conclusive proof [*burhān qaṭʿīy*], then the composition and the lack of simplicity in the spheres would be proven. My opinion is that it is composed of simple [essences] and not of elements [*ʿanāṣir*], except for the ninth [sphere]. And only God knows best.[15]

The problem for the Eastern School astronomers was therefore a problem of devising models that would preserve Ptolemy's observations, save the phenomena and be consistent mathematically as well as physically. In other words, their main concern was to find a set of models that would describe the motion of the physical spheres that carry the various planets by using geometrical mathematical terms, without having the mathematical statements contradict the physical assumptions.

The general thrust of the research that was initiated at the Eastern School is usually described in the literature as being of a philosophical nature, for it had accepted in general all the observational results of Ptolemy and only expressed some philosophical objections to the Ptolemaic models.

In a separate article, I have argued that Ibn al-Shāṭir's model for the sun was, as far as we can tell, the only model that seems to have been motivated by philosophical as well as observational considerations (Saliba 1987a). In that article, I discussed in great detail Ibn al-Shāṭir's attitude to observations, and the manner in which his solar model was definitely conceived to accommodate his observational results and that it was not proposed only as a philosophical objection to Ptolemy's model which, as we have seen, was in no way philosophically objectionable. In fact I know of no other astronomer who considered Ptolemy's solar model as objectionable, or who had offered any alternative to that model.

In order to trace the general activities of the astronomers of the Eastern School, however, I shall single out Ibn al-Shāṭir's solar model, since it is unique in its conception and the only model for the sun, lay down the main outlines of the argument against the Ptolemaic solar model and then proceed to give a short description of the model itself. For the sake of economy, and to avoid duplication of material, the models for the other planets will be treated in a thematic fashion, by giving, in a chronological progression, all the known alternative models that were proposed for each planet.

Ibn al-Shāṭir's solar model

The Ptolemaic model for the sun (Figure 3.1) was conceived as being either an eccentric or an epicyclic model, both alternatives being philosophically acceptable for they could actually describe the behaviour of physical bodies. But from other considerations of the solar theory, Ptolemy had assumed, for example, that the apparent disk of the sun always had the same size, $0;31,20^\circ$, at all distances of the sun, and was therefore equal to the size of the lunar disk when the moon is at its maximum distance from the earth. Naturally, this assumption implies, on the one hand, that the solar eccentricity has at best a negligible effect on the size of the apparent diameter of the sun, and denies, on the other hand, the possibility of annular eclipses. The first statement is obviously an approximate one, and the second is simply contradicted by observations.

We unfortunately do not have an explicit statement by Ibn al-Shāṭir in which he objects to this Ptolemaic assumption. But from his remarks throughout his *Nihāyat al-Sūl* ('The Final Quest'),[16] we know, for example, that, in contradistinction to Ptolemaic theory, Ibn al-Shāṭir did admit the possibility of annular eclipses (fol. 38^r). From his other observational results, which are only quoted in the *Nihāya*, we also know that he considered the apparent size of the solar disk to be variable, again contrary to Ptolemaic theory. Ibn al-Shāṭir referred the reader to another one of his texts, namely *Taʿlīq al-Arṣād* ('Discourse on Observations'), in which these observations were supposed to have been analysed in detail. Unfortunately, the *Taʿlīq* itself has not yet been identified, and is presumed to be lost.

In the *Nihāya*, in at least two places (fols 12^v, 41^r), Ibn al-Shāṭir gives the apparent size of the solar diameter to be

at apogee	$0;29,5^\circ$
at mean distance	$0;32,32^\circ$
at perigee	$0;36,55^\circ$

which proves beyond doubt that he must have been reporting only observational results as he himself says in many different expressions such as '*taḥarrara bi-l-raṣd*' and '*ḥaqqaqtu dhālika bi-l-raṣd*', both meaning 'to verify by observation'.

In a different context (fol. 3^r), Ibn al-Shāṭir also says that he observed the sun in the middle of the seasons and found that the maximum equation of the sun, a function of the eccentricity, was different from the one given by Ptolemy; Ibn al-Shāṭir's maximum equation was $2;2,6^\circ$, which implies an eccentricity of about 2; 7 parts instead of 2; 30 parts as given by Ptolemy.

As long as we do not know the details of Ibn al-Shāṭir's observational methods, we refrain from making any comments on the plausibility or the accuracy of these reports. We simply state that Ibn al-Shāṭir must have convinced himself that his results were indeed more accurate than those of Ptolemy, and thus require a new model that accommodates them for they could not be accommodated by the Ptolemaic model. What he had to do, therefore, was to devise a model that has the effect of a smaller eccentricity than that of Ptolemy, to accommodate the smaller maximum equation, but at the same time allow the sun to move much closer to the earth to appear at an angle of $0;36,55^\circ$, and farther from the earth to appear at an angle of $0;29,5^\circ$. The ratio of the maximum size to the minimum one should therefore be approximately the same as $0;36,55/0;29,5 = 1.26934$.

To do that, he assumed (Figure 3.9) the following orbs for the solar model. (1) An orb of radius 60 parts, which he called the parecliptic, concentric with the observer at point O, the centre of the world, and moving in the direction of the signs at the same speed as the daily mean motion of the sun, namely $0;59,8,9,51,46,57,32,3^\circ$ per day. This parecliptic orb carries another smaller one (2), called the deferent, of radius 4; 37 parts in

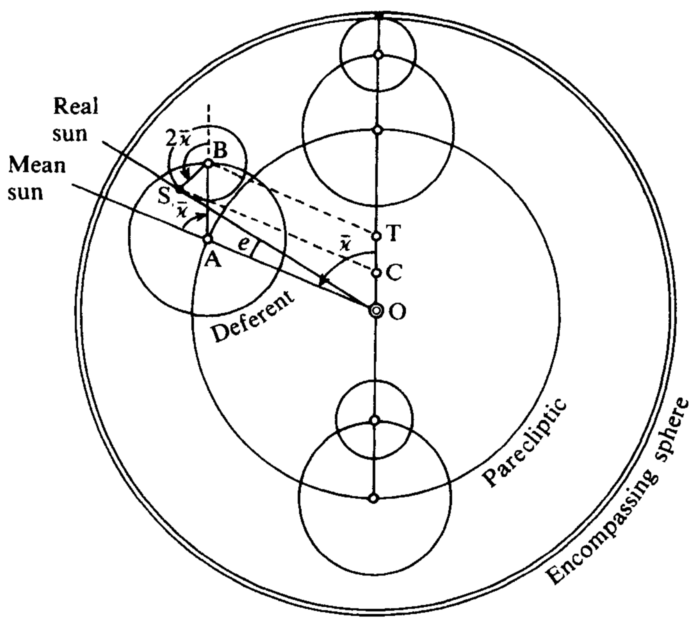


Figure 3.9

the same units that make the radius of the first orb 60 parts. The second orb moves on its own centre at the same speed as the first, but in the opposite direction, thus keeping line AB always parallel to OCT and having the same effect as transferring the eccentricity OT to an epicycle with centre A (as in Figure 3.9). (3) The third orb, called the director, of radius 2; 30, is carried by the deferent in a direction opposite to that of the signs, but moves on its own centre in the opposite direction at twice the speed of the first orb. This third orb carries the body of the sun S, which now seems, according to 'Urḍī's lemma (discussed later), to be moving at uniform speed around point C. Finally, all of these orbs are embedded within a final orb (4) called the encompassing one (*al-shāmil*), that moves at the same speed as the solar apogee, in the direction of the signs, which was found to be 1° per 60 Persian years.

The effect of this model is to allow the sun S to move uniformly around point C, i.e. eccentricity OC = 4; 37 − 2; 30 = 2; 7 which is smaller than the Ptolemaic eccentricity of 2; 30, and thus achieve longitudes close to those of Ptolemy, to be later corrected for the maximum equation. But unlike the Ptolemaic model, that of Ibn al-Shāṭir allows for a variation in the apparent size of the solar disk in the magnitude of

$$\frac{67;7}{52;53} = 1.26914$$

which is very close to the value predicted by the observations of the apparent size of the solar diameter. As an additional benefit, Ibn al-Shāṭir adds that his model yields a further advantage in that all the angles for mean motion are measured around point O, the observer, rather than around the centre of the eccentric, as would have been required by the Ptolemaic model.

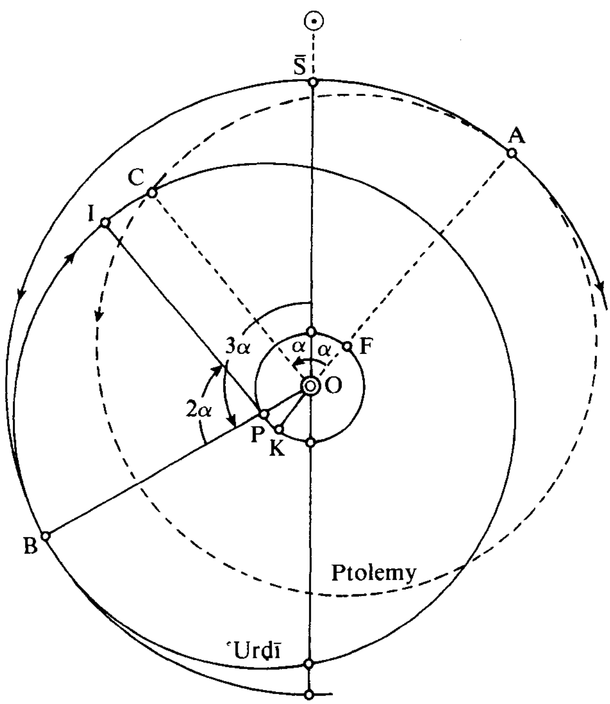
The lunar models

As we have seen above (Figure 3.2), the Ptolemaic model for the moon suffered from two main contradictions: (1) the impossibility (*muḥāl*) of the motion of the deferent sphere, as it seemed to describe, according to Ptolemy, equal arcs in equal times around the centre of the universe rather than around its own centre as it should; and (2) the unaccountability for a mechanism that could allow the diameter of the epicycle that connects the mean apogee and the epicyclic centre to be always directed towards the prosneusis point rather than towards the centre of the deferent.

The astronomical reforms of the thirteenth century included several suggestions for alternative lunar models. One such alternative was proposed by the Damascene astronomer Mu'ayyad al-Dīn al-ʿUrḍī (d. 1266) sometime before 1259.[17]

ʿUrḍī's lunar model

In order to avoid the first impossibility, ʿUrḍī required that the direction of the motion of the Ptolemaic inclined sphere be reversed so that it now moves, according to ʿUrḍī, in the same direction as that of the signs instead of the reverse. In this new arrangement (Figure 3.10) the apogee of the deferent will be carried in the same direction as that of the order of the signs, say to point B. ʿUrḍī further required that the motion of the inclined sphere be in absolute value three times as much as the motion required by the Ptolemaic model. Since the inclined sphere is concentric with the centre of the universe, it meant that angle $\bar{S}OB$ would be three times as large as angle $\bar{S}OA$.



Lunar model
Figure 3.10

Once the whole deferent is carried with the inclined sphere in the same direction as that of the order of the signs, then the apogee that would have reached point A in the Ptolemaic model would now be carried to point B. Now 'Urḍī requires that the deferent itself move around its own centre P in the direction opposite to that of the order of the signs, by an absolute amount equal to twice the motion required by Ptolemy. This would require that point B would be carried back to point I, thus making line PI, parallel to OC, the original direction of the Ptolemaic epicyclic centre from the observer's position O. All of these motions described so far are performed by spheres that move around their own centres, and thus do not contradict the principles of uniform motion. 'Urḍī notes at this point that his model is only describing mean motions, just like the Ptolemaic model, and thus the direction PI should be taken as equivalent to the direction OC since it is parallel to it. With this configuration the epicyclic centre could then be carried to the same position required by the Ptolemaic model without having to fall into the same contradiction as the first one mentioned above.

The new model also solves the second contradiction, i.e. that of the prosneusis point, since now one should notice that line PI passes, in general, very close to point N, K in Figure 3.10, thus making it appear at point I as if it is coming from point N, the Ptolemaic prosneusis point. The mean apogee will therefore be, in this model, a fixed point defined as the common point of tangency between the deferent and the epicycle, falling naturally at the extremity of the line connecting the centre of the deferent to that of the epicycle.

By reversing the order of the motions, and by changing their magnitudes, 'Urḍī managed, therefore, to retain the Ptolemaic observations and to reproduce the predictable motions of the moon without any compromise on the physical principles that were accepted by Ptolemy himself. He was quite aware of this major step that he had taken, and of the differences between his model and that of Ptolemy. But that did not bother him greatly, for he tells his reader that Ptolemy could only demand that his observations be taken for facts, not the mathematical methods – such as the directions and amount of motions – that he used to account for these facts. These, 'Urḍī claims, are only guesses (*ḥads*) on the part of Ptolemy, and no one should be held responsible for them, because anyone's guesses could be as good as those of Ptolemy.

'Urḍī then takes up the problem of the variation between his model and that of Ptolemy, and computes the variations between the equation due to the prosneusis point as predicted by his model and the one predicted by Ptolemy's model. After a lengthy argument he concludes that the variation between the two models is less than two and a half minutes of arc, which he considers as quite permissible since Ptolemy himself had accepted variations from the facts of up to four minutes of arc and said that such variations could escape even the best of observers. For that reason he felt quite satisfied with his model, and urged the reader to accept it and to reject that of Ptolemy, since the latter had been shown to have been riddled with contradictions.

In 'Urḍī's words, the alternative would be to accept that there are spheres that move irregularly on their own centres, and

> if we were to accept that there is a sphere that moves around its own center, sometimes speeding up and other times slowing down, then there would be no need for all the efforts expended in regard to this astronomy, and the final quest would then be the knowledge of the equations to be applied to the motions, even if those were based on false notions.
>
> ('Urḍī, *Kitāb al-Hay'a*, p. 135)

Ṭūsī's lunar model

Ṭūsī discusses in Chapter 7 of his most famous astronomical work *al-Tadhkira fī 'ilm al-hay'a* the lunar model according to Ptolemy. At the difficult points, however, he mentions that there were problems with that model and that he intended to treat them later on. In fact, after he finishes surveying the remaining models for the upper planets and for Mercury, he devotes a special chapter to the solution of all those difficulties that had been encountered so far. The strategy of that approach becomes obvious when one considers that the model which Ṭūsī finally proposed for the lunar motion was also applied to the motion of the upper planets, and hence placing it at the end meant that he could take advantage of combining both models under one solution.

Ṭūsī's understanding of the difficulties in the Ptolemaic model for the moon seem to have been centred around the inability of that model to allow the centre of the epicycle to approach the centre of the universe and to draw away from it without having to incorporate the crank-like mechanism of Ptolemy. If one could, by some method, keep the centre of the deferent sphere at the centre of the universe, and only allow the line joining the centre of the deferent to that of the epicycle to be shortened at quadratures and be elongated at conjunction and opposition, then that could allow the deferent to move uniformly around the centre of the universe, and, at the same time, account for the great observable variation in the equation due to the epicyclic radius.

Once the problem was conceived as such it was reduced to having to devise a mechanism that would allow a vector-like magnitude to be shortened and elongated as a result of circular motion only. Put differently, the problem could be solved once the tip of a vector line could be made to oscillate back and forth as a result of uniform circular motion. The same problem was faced when Ptolemy had to consider the oscillating planes of the latitude model for all the planets with the exception of the moon. It was in that context that Ṭūsī proposed a new mechanism in one of his other books called *Taḥrīr al-Majisṭī* ('A Redaction of the Almagest', composed in 1247), with the help of which he managed to get the tip of the oscillating diameters to be mounted on a pair of circles – discussed elsewhere as the 'Ṭūsī Couple' – and thus made that tip to move in an oscillating linear motion which was produced by circular motion. All that Ṭūsī had to do was to generalize that solution which he had proposed for the latitude model and apply it to the special requirement of the lunar model, and then use the same technique for the model of the upper planets as well.

It is not surprising, therefore, that Ṭūsī would begin the chapter in which he proposed his alternative models with a statement of the lemma that came to be called the 'Ṭūsī Couple' lemma and a formal proof of the same lemma. That was done in Chapter 11 of his *Tadhkira* mentioned above.

The lemma is stated in the specific case first, i.e. the plane case, and afterwards generalized to include the spherical case.[18] In the plane case a paraphrase of that lemma states the following: Let there be two circles (Figure 3.11), such that one of them is tangent to the other from the inside, and whose diameter is half the diameter of the other encompassing circle. If we then assume that the smaller interior circle moves in a direction opposite to the direction of motion of the exterior encompassing circle, and at twice the speed, then the point on the diameter of the larger circle and the circumference of the smaller circle, namely, the initial point of tangency, will oscillate back and forth between the extremities of the diameter of the larger circle.

Once that result was proved, Ṭūsī remarked that instead of these circles one could take two spheres whose diameters and positions have the same relationship to each other as these two circles. If that were true, then those spheres could be taken to be of an appropriate thickness to encompass other spheres such as the epicycle of the moon in the Ptolemaic model. In fact, Ṭūsī replaces the initial point of tangency with the initial position of the centre of the lunar epicycle which is now embedded inside two such spheres. What that did was to allow the centre of the lunar epicycle to oscillate back and forth along the diameter of the larger sphere. Once that was achieved, there was no longer any need for the eccentric deferent of Ptolemy, nor for his crank-like mechanism, both of which were originally required to bring the lunar epicycle closer to the earth at quadrature and farther away at conjunction and opposition.

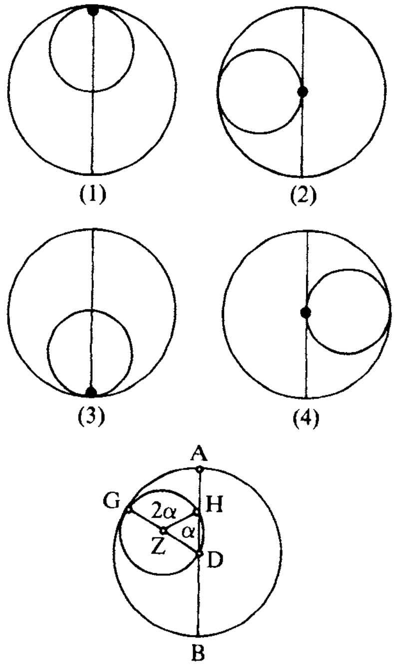


Figure 3.11

If one ascribes the appropriate motions to these spheres in such a way that they would match the Ptolemaic observed motions, then one could devise a model (Figure 3.12) whereby the deferent of the moon could move uniformly around the centre of the universe, to solve the first difficulty of the Ptolemaic model, and the epicycle could be drawn closer to the earth at quadratures and farther at conjunction and opposition in order to approximate the maximum equations observed by Ptolemy. For the prosneusis point, Ṭūsī adopts a spherical 'Couple' which, like the plane one, allows the tip of the epicyclic diameter to oscillate back and forth covering in each direction an angle equal to the maximum equation of Ptolemy.

With that resolved Ṭūsī goes on to show that the resulting path of the centre of the epicycle around the earth is not a circle, although it looks like one.

But once the advantage of the 'Couple' was realized, Ṭūsī goes on to use it in the model for the upper planets, discussed below, and in the latitude theory, as was mentioned above.

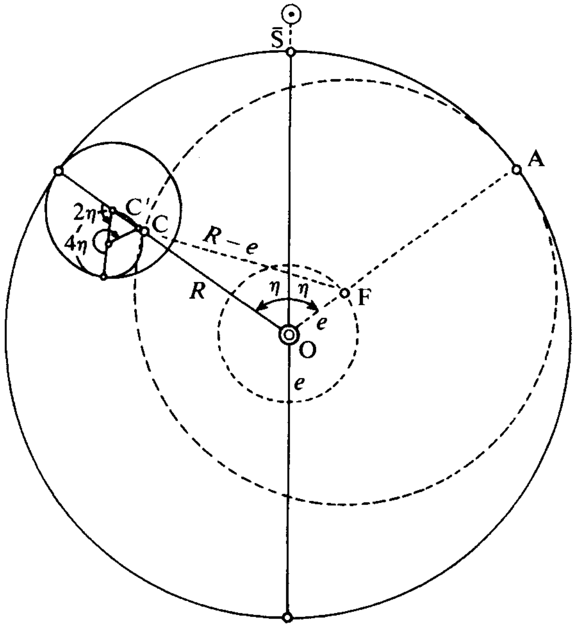


Figure 3.12

The lunar model of Quṭb al-Dīn al-Shīrāzī (d. 1311)

In his *Nihāyat al-Idrāk*,[19] Shīrāzī begins the discussion of the lunar model, on folio 54ʳ, with a general survey of the conditions for the Ptolemaic model, concluding in effect that the Ptolemaic model answers very well for a description of the observational phenomena. After giving the observational facts that would account for the existence of the spheres of the lunar model, he lists in detail the number of spheres that the lunar model must have in order to account for all the observational variations. The following section is then devoted to the various motions of these spheres and to the way in which such motions could be combined to produce the observational variations, giving in each case the mean motion for each of these spheres. That synopsis is immediately followed by a description of the variations that are observed between the mean motions produced by these spheres and the actual motions of the moon. Here, in this section, he gives the maximum equations, which, like the mean motion parameters quoted above, are strictly Ptolemaic.

On folio 60ʳ, he recapitulates and summarizes the objections that have been raised to this Ptolemaic model that he had just finished describing. In effect, he gives the two famous objections quoted so far, namely, the inadmissibility of the motion of the deferent around its centre while it describes equal arcs in equal times around the centre of the universe, and the prosneusis point.

Then he quickly says that such objections could be answered. One of those answers, which had to do with the objection to the uniform motion around the centre of the universe rather than around the deferent centre, had already been given by the principle of 'the large and the small (spheres)' – an obvious reference to the 'Ṭūsī Couple'. Moreover, from the description that he gives for this principle, and the way in which it responds to the objections against the Ptolemaic model, it becomes very clear that he is only summarizing Ṭūsī's solution that was given in Chapter 11 of the *Tadhkira*, and which was described above. Even the terminology used is transparently that of Ṭūsī, and, at best, one could say that the model offered in the *Nihāya*, so far, was nothing other than a paraphrase of Ṭūsī's model.

As for the objection concerning the prosneusis point, 'that is a matter of some subtlety' (*maḥall naẓar*). He asserts that it was difficult to achieve, and without repeating the statements of Ṭūsī in this regard moves on to say that it could be answered by using the ninth principle – referring to a principle that he had described earlier in the text – which he now calls the principle of inclination '*aṣl al-mayl*'. On the other hand, Shīrāzī does not offer a detailed description of how that principle, which was mainly applied to the latitude theory problem, could be applied here for the prosneusis point. Nor is it clear as to how Shīrāzī intended to combine this principle with Ṭūsī's model. He then continues to describe the behaviour of the Ptolemaic model that necessitated the assumption of the prosneusis point.

Then, without any introduction, Shīrāzī juxtaposes a long quotation from 'Urḍī's text *Kitāb al-Hay'a*, beginning it simply with the statement 'one of the learned men of the moderns here, who is versed in this discipline had said' (*qāla baʿḍ afāḍil al-muta'akhkhirīn min ahl al-ṣināʿat hāhunā*).[20] That is followed by a detailed paraphrase of ʿUrḍī's lunar model which apparently had been accepted by Shīrāzī as the preferable solution, for he ends this section with the following statement:

> This is the configuration for the spheres of the moon, the magnitudes of their motions, and the manner in which it could be taken according to the chosen method [*al-wajh al-mukhtār*] that suffers from none of the objections [*al-ishkālāt*] and which conforms to the principles and the observations. The method is only different from that which has been accepted by common opinion, but that should not be detrimental to it once it is the true one. For the truth is beloved, and the teacher is beloved, but the truth is lovelier still.[21]

To summarize, therefore, Shīrāzī, who had promised in the introduction of his *Nihāya* to offer an anthology of solutions that would answer the objections raised against the Ptolemaic models, had offered, in the case of the moon, two models – one by Ṭūsī, which he did not think answered both objections to the lunar model, and one by ʿUrḍī, which he seems to have preferred.

But in the *Tuḥfa*, which Shīrāzī had written later, he proposed a model of his own. The model consists of vector connections that will ultimately allow the centre of the epicycle to move uniformly around the centre of the universe as a result of a combination of two other uniform motions. Instead of the regular eccentric sphere of Ptolemy, Quṭb al-Dīn proposed (Figure 3.13) an eccentric of his own, DHK, which has only half the Ptolemaic eccentricity. He then allowed this new eccentric sphere to move in the direction of the order of the signs at a speed equal to twice the speed of the inclined sphere of Ptolemy, ABG, which carried the apogee D in the direction opposite to that of the order of the signs. Then at the circumference of the cincture of that eccentric, Quṭb al-Dīn required the introduction of another small sphere with centre H whose radius is equal to half the Ptolemaic eccentricity. He required that the smaller sphere should move in the same direction as the new eccentric, and at the same speed. This allowed the epicyclic centre E, which is placed at the cincture of this sphere, to be

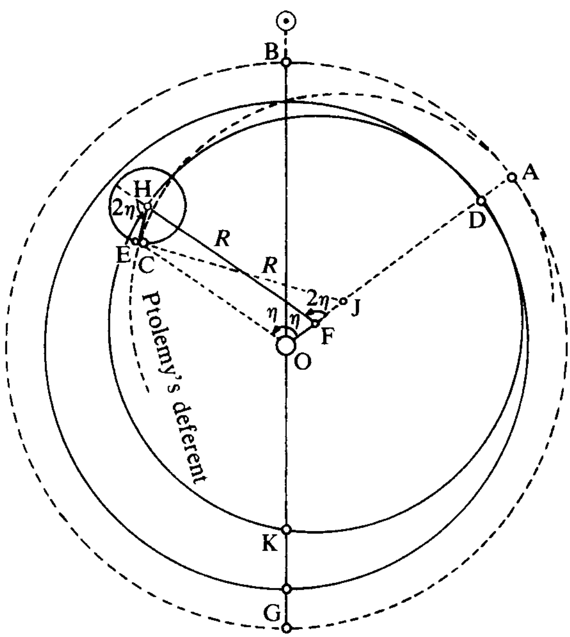


Figure 3.13

brought very close to the old Ptolemaic epicyclic centre C, and to move uniformly around the centre of the universe.

The improvement in the new model is that one could show that the new position of the epicyclic centre will appear as if it is moving uniformly around O, the centre of the universe, when in reality it is moving uniformly around H, the centre of its own carrier – the smaller sphere – which in turn is moving uniformly around F, the centre of its own carrier as well – the centre of the new eccentric proposed by Quṭb al-Dīn. To prove that such a relationship actually exists, Quṭb al-Dīn used a theorem that was first proposed by Mu'ayyad al-Dīn al-ʿUrḍī which will be discussed below as ʿUrḍī's lemma. What the new model does to the Ptolemaic one is to remove the first objection that was raised in connection with the Ptolemaic model, namely that of having a sphere move uniformly around a point which is not its own centre.

But what it does not do is solve the second objection, namely that of the prosneusis point. Quṭb al-Dīn remains silent on this second issue, in Chapter 10 of the *Tuḥfa*, and takes it up again at the end of Chapter 12 of the same book. But, even then, Quṭb al-Dīn did not seem to have succeeded in responding to the second objection. In the words of a later astronomer by the name of ʿUbaydallāh b. Masʿūd b. ʿUmar Ṣadr al-Sharīʿa (d. AH 747 AD 1346/7), who attempted to solve this specific problem of Quṭb al-Dīn's model,[22] he said that the author of the *Tuḥfa* 'had spoken profusely about the prosneusis point, without any apparent success, for the import of his statement was that the motion of the eccentric alone was sufficient to exhibit the difference between the two (epicyclic) apogees. But there is no doubt that that would not do so.'[23] The work of Ṣadr al-Sharīʿa himself, which is encyclopedic in nature, has now been studied by Dallal (1995a). What he seems to have done (Figure 3.14) is to suggest the addition of yet another sphere – of radius r_1 – to be carried at the tip of the epicyclic radius, whose own radius is equal to 0; 52 parts in the same units that make the radius of the inclined sphere 60 units. This additional sphere is supposed to move at the same speed as the deferent and in the same direction, i.e. in the direction opposite to that of the epicycle. The effect of this small additional vector is to increase the anomaly by an amount proportional to the first equation at the intermediate points between syzygies and quadratures, and to leave it as is, i.e. have the effect of a zero equation, at syzygies and quadratures. This small epicyclet could also allow the radius of the epicycle to appear larger at quadrature, and smaller at syzygies as required by the Ptolemaic observations.

A more successful astronomer, and a better known one, a contemporary of Ṣadr al-Sharīʿa, was a Damascene by the name of Ibn al-Shāṭir (d. 1375),