Gödel's Theorem in focus

Edited by S.G. SHANKER

GÖDEL'S THEOREM IN FOCUS

PHILOSOPHERS IN FOCUS SERIES

GÖDEL'S THEOREM IN FOCUS
Edited by S.G. Shanker

Gödel's Theorem in focus

Edited by S.G. Shanker

ROUTLEDGE

London and New York

First published in 1988 by
Routledge

Reprinted 1989, 1990, 1991 and 1997 by
Routledge
2 Park Square, Milton Park, Abingdon, Oxon, OX14 4RN
270 Madison Ave, New York NY 10016

Transferred to Digital Printing 2007

British Library Cataloguing in Publication Data

Gödel's theorem in focus.
1. Gödel's theorem
I. Shanker, S.G.
511.3 QA9.65

ISBN 0-7099-3357-6
ISBN 0-415-04575-4 (Pbk)

Library of Congress Cataloguing-in-Publication Data

ISBN 0-7099-3357-6
ISBN 0-415-04575-4 (Pbk)

Typeset in 10pt Times Roman by Leaper & Gard Ltd, Bristol

Publisher's Note
The publisher has gone to great lengths to ensure the quality of this reprint but points out that some imperfections in the original may be apparent

Contents

Preface

An ever-growing number of sub-disciplines in philosophy — ranging from the philosophies of science and language to the philosophy of mind and aesthetics — now demand a working acquaintance with Kurt Gödel's incompleteness theorems from their students. For Gödel's theorems raise issues which lie at the very heart of modern attempts to revitalise metaphysics and/or the Mechanist Thesis. Unfortunately, given the highly technical nature of Gödel's proof these debates have remained relatively inaccessible to those not trained in mathematical logic. The present book has been designed to meet these needs by providing a lucid introduction to the mechanics and mathematical import of Gödel's proof. We begin with a short biographical sketch of Kurt Gödel by John W. Dawson, Jr., followed by Stephen Kleene's overview of Gödel's work in mathematical logic. With this background in place we then address the mounting controversy in the philosophy of mathematics surrounding the philosophical significance of Gödel's theorems.

Some will no doubt regard the latter phenomenon as a reflection of the inevitable time-lag between scientific and mathematical discoveries versus philosophical comprehension. Perhaps it will even be seen to corroborate the increasingly popular thesis in the sociology of knowledge that it is only once a science has digested the full implications of a breakthrough that it becomes the property of philosophers to exaggerate and distort. But unlike standard mathematical results Gödel's theorems are inextricably linked to the epistemological disputes which they have sparked off; indeed, nowhere could this be more evident than in the writings of Gödel himself. It is not surprising, therefore, that Gödel's theorems should present us with a catalogue of philosophical problems, many of which we are only just beginning to recognise, let alone resolve.

To begin with there is the anomalous reception of Gödel's first incompleteness theorem; in the words of John W. Dawson, Jr., how was it that 'one of the most profound discoveries in the history of logic and mathematics was assimilated promptly and almost without objection by Gödel's contemporaries'? As Solomon Feferman shows us, this issue is intimately connected with the larger question of how Gödel's subsequent work in mathematical logic and the growing confidence with which he expressed his platonist convictions relate to his earlier interpretation and presentation of the second incompleteness theorem. This brings us to the point where we can address the main concerns of the final three papers. First Michael Resnik outlines the seriousness of the sceptical problem created by Gödel's theorems. Michael Detlefsen then takes up the challenge of attempting to rescue Hilbert's Programme from the impasse in which Gödel apparently left it. In the final paper I examine the nature of this dilemma in the light of Wittgenstein's attempt to resolve by dissolving the crisis created by Gödel's theorems.

Like all philosophical sceptical problems the issues raised by Gödel's theorems are pregnant with possibilities and fraught with dangers. Chief amongst the latter is an inevitable tendency to become distanced from the *fons et origo* of these developments. For it becomes ever more tempting and acceptable to rely on the findings of commentators who might themselves have based their readings on earlier summaries. To be sure, it is common practice to accept the verdict obtained by the experts in a field without inspecting their findings. But such custom presupposes concord. The ultimate aim of this book has been to outline the basis of the consensus which has hitherto obtained in order then to question it. It is our hope that this will stimulate renewed interest in the ongoing interpretation of the significance of Gödel's incompleteness theorems.

S.G. Shanker
Atkinson College, York University

Acknowledgements

I am deeply indebted to John W. Dawson, Jr. and Michael Detlefsen for their generous assistance in the design of this book. I have also benefited greatly from many fruitful discussions with Rainer Born. Lastly I would like to thank the Canada Council for the Postdoctoral Fellowship which enabled me to work on this project.

We are grateful to the authors and their publishers for permission to reprint the following articles:

John W. Dawson, Jr., 'Kurt Gödel in Sharper Focus', *The Mathematical Intelligencer*, vol. 6, no. 4 (1984), by permission of Springer-Verlag.

John W. Dawson, Jr., 'The Reception of Gödel's Incompleteness Theorems', *Philosophy of Science Association*, vol. 2 (1984). Copyright © 1985 by the Philosophy of Science Association.

Michael Detlefsen, 'On Interpreting Gödel's Second Theorem', *Journal of Philosophical Logic*, vol. 8, no. 3 (June 1979), pp. 297-313. Copyright © 1979 by D. Reidel Publishing Company, Dordrecht, Holland.

Solomon Feferman, 'Kurt Gödel: Conviction and Caution', a revised and expanded version of the author's contribution to Section 13 of the 7th International Congress of Logic, Methodology and Philosophy of Science (held in Salzburg, 11-16 July 1983), and published in a separate issue of *Philosophia Naturalis*, Paul Weingartner and Christine Pühringer (eds.), *Philosophy of Science — History of Science. A Selection of Contributed Papers of the 7th International Congress of Logic, Methodology and Philosophy of Science, Salzburg, 1983*, Meisenheim/Glan, Verlag Anton Hain (1984).

Kurt Gödel, 'On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I', Jean van Heijenoort (trans.), in Jean van Heijenoort (ed.), *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931* (Cambridge, Mass., Harvard University Press, 1967). Copyright © 1967 by the President and Fellows of Harvard College.

Stephen C. Kleene, 'The Work of Kurt Gödel', *The Journal of Symbolic Logic*, vol. 41, no. 4 (1976).

Stephen C. Kleene, 'An Addendum to "The Work of Kurt Gödel"', *The Journal of Symbolic Logic*, vol. 41, no. 4 (1978).

Michael D. Resnik, 'On the Philosophical Significance of Consistency Proofs', *Journal of Philosophical Logic*, vol. 3/1-2 (1974), pp. 133-47. Copyright © 1974 by D. Reidel Publishing Company, Dordrecht, Holland.

I

Kurt Gödel in Sharper Focus

John W. Dawson, Jr.

1. INTRODUCTION

The lives of great thinkers are sometimes overshadowed by their achievements — a phenomenon perhaps no better exemplified than by the life and work of Kurt Gödel, a reclusive genius whose incompleteness theorems and set-theoretic consistency proofs are among the most celebrated results of twentieth-century mathematics, yet whose life history has until recently remained almost unknown.

Several tributes to Gödel have appeared since his death in 1978, most notably the obituary memoirs of Curt Christian, Georg Kreisel, and Hao Wang (Christian, 1980; Kreisel, 1980; Wang, 1978).[1] But none of those authors developed a close personal acquaintance with Gödel before the 1950s, and there are discrepancies among their accounts. To resolve them, to substantiate or refute various rumors that have circulated, and to learn further details about Gödel's life and work, scholars must therefore turn to primary documentary sources. In the remainder of this article I shall highlight some aspects of Gödel's life that have been thrown into sharper focus as the result of my own explorations among such sources; I have drawn primarily on my experiences over the past two years in cataloguing Gödel's *Nachlass* at the Institute for Advanced Study in Princeton, supplemented by personal interviews, visits, and correspondence with various individuals here and abroad. Of necessity, I have repeated some biographical information already available in the memoirs cited. My aim, however, has been to amplify or correct details of those accounts, insofar as primary documentation is available.

I am grateful to the Institute for Advanced Study for permission to quote from, and to reproduce photographs of, unpublished items in the *Nachlass*; to Rudolf Gödel, Kurt's brother and only surviving close relative, for his gracious responses to my inquiries; and to H. Landshoff for assistance in preparing illustrations for this article.

2. THE GÖDEL *NACHLASS*: PROVENANCE, ARRANGEMENT, AND DISPOSITION

The scientific *Nachlass* of Kurt Gödel, including correspondence, drafts, notebooks, unpublished manuscripts, books from his library, and all manner of loose notes and memoranda was donated to the Institute for Advanced Study after Gödel's death by his widow. Gödel himself made no provision for the disposition of his papers, although correspondence in the *Nachlass* indicates that the Library of Congress solicited them from him. Indeed, his attitude toward posthumous preservation or publication of his papers seems to have been ambivalent. Thus he spoke to Dana Scott of his desire to have certain papers published posthumously and even asked Scott to prepare typescripts of some of them. Yet on the other hand, he declined several invitations to consider publication of his collected works, maintaining that the most important of them were readily available and that the rest were only of historical and biographical interest. Ultimately, his papers were gathered into boxes and placed in a cage in the basement of the Institute's historical studies library (which has no archival facilities) to await further disposition.

My own involvement with Gödel's papers began in the fall of 1980. In an effort to track down Gödel's lesser-known publications, I consulted the bibliography in Bulloff, Holyoke and Hahn (eds, 1969), prepared on the occasion of Gödel's sixtieth birthday. Though I presumed that that listing would be complete, a colleague soon called my attention to an item not cited there, and I later found two more. Subsequently, I wrote to the IAS to inquire whether Gödel himself had ever prepared a bibliography of his own work; in response, I received a typewritten list identical to that in Bulloff, Holyoke and Hahn. Concluding that no comprehensive bibliography had ever been attempted, I undertook to compile one myself, at the same time embarking on the more ambitious program of translating all of Gödel's previously untranslated works into English.[2] Eventually I was offered the opportunity of cataloguing the *Nachlass*, and in June 1982 I arrived in Princeton to begin work.

At the outset, there were three major problems to be faced: the sheer bulk of the materials comprising the *Nachlass*; my lack of training in archival technique; and the necessity of penetrating Gödel's Gabelsberger shorthand, an obsolete German script he used extensively.[3] It was at first somewhat daunting to find that the *Nachlass* occupied two large filing cabinets plus some *sixty*-odd cardboard packing boxes. The majority of the boxes, however, contained books from Gödel's library, back issues of journals he received, and reprints and offprints sent to him by others. The remaining, much more manageable, body of primary documentary material required about three months to survey in order to devise an appropriate arrangement scheme. (See Dawson (1983b) for further details.)

My ignorance of archival principles was remedied by consultations with archivists and by reference to Gracy's helpful manual (1977). Gödel's eminently logical mind, extremely methodical habits, and clear handwriting were also a boon to my efforts. In particular, one fundamental archival conflict — that of preserving the original order of a manuscript collection while facilitating scholarly access to it — has seldom arisen. My task has rather been that of restoring Gödel's order to papers gathered somewhat haphazardly after his death.

The shorthand problem proved the most difficult. Early on, my wife volunteered to learn the script, provided suitable instruction manuals could be found. At the same time we began a search for 'native' stenographers. Eventually, both approaches proved successful. In the *Nachlass* itself, Gödel's own shorthand textbook turned up, along with several 'Rosetta stones' — vocabulary notebooks in several languages with foreign words in longhand paired with their German synonyms in Gabelsberger — that allowed comparison of Gödel's individual 'hand' with the textbook examples. Much later, we also located a German émigré, Hermann Landshoff, who had learned the Gabelsberger script as a youth and was willing to assist us.

Cataloguing of the *Nachlass* should be completed by the summer of 1984. After that, it is expected that the papers will be donated to the library of Princeton University, where they will be made available to scholars.

3. GÖDEL'S CHILDHOOD AND YOUTH

Kurt Friedrich Gödel was born 28 April 1906 in Brünn, Moravia (then part of the Austro-Hungarian empire; now Brno, Czechoslovakia), the second of the two children of Rudolf and Marianne Gödel. Then as now, Brünn was a major textile centre, and Kurt's father worked as director of the Friedrich Redlich textile factory. (Redlich himself was Kurt's godfather, later killed by the Nazis. From him the infant's middle name was presumably taken. Gödel officially dropped his middle name when he became a U.S. citizen, but the initial 'F.' survives on his tombstone.) Patent correspondence preserved among Gödel's papers attests to his father's inventiveness in the textile field, and even Marianne Gödel's maiden name, Handschuh ('glove'), suggests the garment trade. (As noted in Christian (1980) and Kreisel (1980), her father was in fact a weaver.)

The bare facts about Gödel's birth are recorded on a copy of his *Taufschein* (baptismal certificate), preserved in the *Nachlass* along with his naturalization papers. That document shows that he was born at 5 Bäckergasse (now Pekařska) and baptized in the German Lutheran congregation of Brünn. Later the family moved to a villa at 8A Spilberggasse (now Pellikova), a residence more befitting their moderately wealthy circumstances.

Gödel's ethnic heritage was thus neither Czech nor Jewish, as has sometimes been asserted. Though his parents were both born in Brünn, they were part of the German community there, and the children attended German-language schools. Kurt did not enrol in optional courses in the Czech language, and both he and his brother gave up their postwar Czech citizenship after they became students at the University of Vienna.

Other items in the *Nachlass* pertaining to Gödel's youth include report cards from both elementary and secondary schools and several of his school notebooks. Particularly quaint is his first arithmetic workbook, which contains but a single error in computation. The report cards are indicative of curricula of the time, which laid heavy stress on science and languages, in addition to such required courses as religion, drawing, and penmanship. Latin and French were required, and Gödel chose English as the second of his two elective subjects (after shorthand). In general, he seems to have been quite interested in languages. His library contains many foreign-language dictionaries, and there are vocabulary and exercise notebooks in Italian and Dutch in addition to the languages already mentioned.

All of Gödel's school records attest to his diligence and outstanding performance as a student. Indeed, only once did he receive less than the highest mark — in mathematics. But the report cards also record a rather large number of excused absences, including exemptions during the years 1915-16 and 1917-18 from participation in physical education. The earlier of those exemptions probably corresponds to a childhood bout with rheumatic fever, an episode Rudolf Gödel believes to have been the source of his brother's later hypochondria.

Some later material in the *Nachlass* also relates to Gödel's youth. One especially valuable item is a questionnaire sent to Gödel in 1974 by the sociologist Burke D. Grandjean — a document Gödel dutifully filled out but never returned. In response to some of its queries, Gödel noted that his interest in mathematics began at about age 14, stimulated by an introductory calculus text in the well-known Göschen collection; that his family was little affected by World War I and the subsequent inflation; that he was never a member of any religious congregation, although he was a believer (describing himself as a theist rather than pantheist, 'following Leibniz rather than Spinoza'); and that prior to his enrolment at the University of Vienna he had little contact with Vienna's intellectual or cultural life except through the newspaper *Neue Freie Presse.*

4. VIENNA YEARS, AND VISITS TO PRINCETON

Aside from his doctoral diploma and some of his course notebooks, the *Nachlass* contains few records of Gödel's university career. According to his own account (again in response to the Grandjean questionnaire), he entered the University of Vienna in 1924 intending to major in physics. Once there, however, he was influenced by the mathematical lectures of Philipp Furtwängler and the lectures of Heinrich Gomperz on the history of philosophy. He switched into mathematics in 1926. At about the same time, under the guidance of Hans Hahn, he began to attend meetings of the Vienna Circle, with whose views, however, he disagreed, and from which he later took pains to dissociate himself (as revealed, for example, in several letters found in the *Nachlass*). Gödel submitted his dissertation in the autumn of 1929, a year marked not only by worldwide financial collapse, but by the premature death of Gödel's father on 23 February, five days before his fifty-fifth birthday. On 6 February 1930, Gödel's doctorate in mathematics was conferred by the University of Vienna (not, as E.T. Bell asserts in his *Mathematics, Queen and Servant of Science*, by 'the University of Brno in engineering').

Shortly afterward, in an attempt to pursue the aims of Hilbert's Programme, Gödel sought to find an interpretation of analysis within arithmetic. In so doing, he came to realize that the concept of provability could be defined arithmetically. This led to his incompleteness proof, which, ironically, overturned Hilbert's Programme (at least as originally envisioned). Yet Gödel announced his momentous discovery almost casually, toward the end of a dicussion on foundations at a conference in Königsberg where only the day before he had lectured on his dissertation results (the completeness of the first-order predicate calculus).[4] Reaction (not always accompanied by understanding) was immediate, ranging from profound appreciation by von Neumann (who two months later nearly anticipated Gödel's discovery of the *second* incompleteness theorem) to vigorous criticism by Zermelo (see Dawson (reprint) and Grattan-Guinness (1979)) and even to a claim to priority by Finsler (see van Heijenoort's note, pp. 438-440 in van Heijenoort (ed., 1967)), dismissed by Gödel with uncharacteristic disdain. The incompleteness paper appeared early in 1931. Later Gödel submitted it to the University of Vienna as his *Habilitationsschrift*, thereby earning the right to teach as *Privatdozent*. In the meantime he participated actively in Karl Menger's colloquium, where he presented nearly a dozen papers and collaborated in editing volumes 2-5 and 7-8 of the colloquium proceedings (*Ergebnisse eines mathematischen Kolloquiums*).

Officially, Gödel's tenure as *Privatdozent* extended from 1933 to 1938. In fact, however, his lecturing at the University of Vienna was repeatedly interrupted, both by visits to America and by episodes of ill health. Indeed, on the basis of enrolment slips saved by Gödel and records of the University of Vienna, it appears that Gödel actually taught only three courses there: foundations of arithmetic, in the summer of 1933; selected chapters in mathematical logic, in the summer of 1935; and axiomatic set theory, in the spring of 1937.

From the published memoirs it is difficult to piece together a coherent chronology of Gödel's visits to America; there were actually three such prior to his emigration in 1940. He first came in 1933-34 to lecture on his incompleteness theorems at the Institute for Advanced Study, where he spent the academic year. It was the Institute's first year of operation, without a building of its own and with titles for the visiting scholars yet to be decided upon. The official IAS *Bulletin* for that year lists Gödel simply as a 'worker'. In April, Gödel also travelled to New York and Washington, where he lectured before the Philosophical Society of New York and the Washington Academy of Sciences.

After his return to Europe Gödel suffered a nervous breakdown. He entered a sanatorium and was forced to postpone an invitation to return to the IAS for the second term of 1934-35. In the meantime he began his investigations in set theory; and when he did return to the IAS in October 1935, he told von Neumann of his consistency proof for the axiom of choice. A month later Gödel suddenly resigned, suffering from depression and overwork. Veblen saw him aboard ship in New York and telegraphed ahead to Gödel's family. More time in sanatoria followed, and, as noted above, Gödel only resumed his teaching in Vienna in the spring of 1937.

Later that summer,[5] Gödel saw how to extend his consistency proof to the generalized continuum hypothesis. In the fall of 1938 he returned once more to America, spending the first term at the IAS and the second, at Menger's invitation, at Notre Dame. (Not 'Rotterdam', as stated in Wang (1981)). At both institutions he lectured on his consistency results, and at Notre Dame he and Menger offered a joint course on elementary logic. The *Nachlass* contains manuscripts of all those lectures — carefully written in English, except for a single page of examination questions, effectively concealed in Gabelsberger.

5. EMIGRATION AND AMERICAN CAREER

Gödel intended to return again to Princeton in the fall of 1939, but personal and political events intervened. The previous September, only about two weeks before his departure for America, he had married Adele Nimbursky (nee Porkert) in

Vienna. Though he had known Adele for over a decade, their marriage had been delayed by opposition from Gödel's family; for she was not only a divorcee, older than Kurt, but she had worked as a dancer and was somewhat disfigured by a facial birthmark. It proved nevertheless to be an enduring union. Their first year of marriage, however, was spent apart, as Adele remained behind in Vienna during the academic year 1938-39.

After his return to Vienna to rejoin his bride in the summer of 1939, Gödel was called up for a military physical by the Nazi government. Writing to Veblen in November, Gödel reported that contrary to his expectation, 'I was mustered and found fit for garrison duty.'[6] At the same time, to retain his right to teach at the University of Vienna, he was obliged to apply to Nazi authorities for appointment as a *Dozent neuer Ordnung*, thereby subjecting himself to political and racial scrutiny; and though his mother and brother lived unmolested in Brno and Vienna throughout the Nazi occupation, Gödel was suspect because of his association with Jewish intellectuals such as Hahn. Eventually his application was approved, but only after his emigration to America.

Even there, however, he was thought by many to be Jewish. Thus Bertrand Russell declared in the second volume of his *Autobiography*:

> I used to go to [Einstein's] house, once a week to discuss with him and Gödel and Pauli. These discussions were in some ways disappointing, for, although all three of them were Jews and exiles and, in intention, cosmopolitans, I found that they all had a German bias toward metaphysics ... Gödel turned out to be an unadulterated Platonist, and apparently believed that an eternal 'not' was laid up in heaven, where virtuous logicians might hope to meet it hereafter.

In 1971, Gödel's attention was called to this passage by Kenneth Blackwell, curator of the Russian archives at McMaster University. Gödel drafted a reply (never actually sent) that is preserved in the *Nachlass*:

> As far as the passage about me [in Russell's autobiography] is concerned, I have to say *first* (for the sake of truth) that I

am not a Jew (even though I don't think this question is of any importance), 2.) that the passage gives the wrong impression that I had many discussions with Russell, which was by no means the case (I remember only one). 3.) Concerning my 'unadulterated' Platonism, it is no more 'unadulterated' than Russell's own in 1921 when in the *Introduction [to Mathematical Philosophy]* he said '[Logic is concerned with the real world just as truly as zoology, though with its more abstract and general features].' At that time evidently Russell had met the 'not' even in this world, but later on under the influence of Wittgenstein he chose to overlook it.

(In Gödel's draft, only an ellipsis (...) appears between the quotation marks. The bracketed passage inserted here was quoted by Gödel in his 1944 essay 'Russell's mathematical logic'.)

Somehow, in the midst of the political turmoil, Gödel succeeded in obtaining exit visas: his passport, preserved in the *Nachlass*, testifies to his frantic efforts to obtain transit documents from consulates in Vienna and Berlin. By then it was too dangerous to risk crossing the Atlantic, so, in January 1940, he and Adele travelled through Lithuania and Latvia to board the trans-Siberian railway at Bigosovo. After crossing Russia and Manchuria they made their way to Yokohama and thence by ship to San Francisco, where they arrived 4 March 1940.

At the IAS Gödel found an intellectual haven from which he was rarely to venture again. Reclusive by nature, he seems not to have minded (and perhaps even to have sought) his growing isolation. Several of Gödel's acquaintances, however, have remarked that Princeton society proved unreceptive (not to say hostile) toward Adele, and she appears to have led a very lonely life there.

Professionally, the IAS offered Gödel relative job security. Yet he was appointed on a yearly basis until 1946, when he was finally made a permanent member. Only in 1953, five years after becoming a U.S. citizen and two years after sharing the first Einstein award,[7] was he promoted to professor. Gödel himself seems never to have expressed dissatisfaction about this long delay, but others have called for an explanation. In particular, Stanislaw Ulam (1976, p. 80) and Freeman Dyson (1983, p. 48) have brought the matter to public attention. Ulam, indeed, quoting remarks made to him by von Neumann, has suggested that Gödel's treatment was occasioned by the personal opposition of some unnamed IAS colleague. The *Nachlass* itself sheds no light on the matter, but some 'old-timers' at the Institute have suggested that a division of opinion prevailed among Gödel's colleagues: some felt that Gödel would not welcome the administrative responsibilities entailed by faculty status, while others feared that if he were promoted, his sense of duty and legalistic habit of mind might impel him to undertake such responsibilities all too seriously, perhaps hindering efficient decision-making by the faculty. In the event, such fears seem to have been justified; but one should also note Gödel's own statement in 1946 in a letter to C.A. Baylis, in which he noted that 'apprehension that cooperation of this kind [service in offices or on committees] would be expected' was 'the very reason' he had so belatedly joined the Association for Symbolic Logic.

In any case, the Institute allowed Gödel freedom to pursue a broad range of intellectual interests. At first he labored to prove the independence of the axiom of choice and the continuum hypothesis, but the latter especially proved unyielding, and eventually he gave up the attempt.[8] Instead, he turned to philosophy. The transition is marked by his 1944 essay 'Russell's mathematical logic', solicited by P.A. Schilpp for his *Library of Living Philosophers* series. Subsequently, Schilpp was to solicit essays for the Einstein, Carnap, and Popper volumes of the series as well. Gödel accepted all but the last, and he devoted great care to each — so much so that in every case his essay was among the last to be received. Schilpp displayed great patience and diplomacy, but Gödel could not be hurried; and when his Russell essay was received too late for Russell to reply, Gödel considered withdrawing it altogether. Eventually he yielded to Schilpp's entreaties, but when the situation recurred a few years later Schilpp was forced to send the Carnap volume to press without Gödel's contribution. It remains unpublished in the *Nachlass.*

In contrast, Gödel's contribution to the Einstein volume was not only commented upon, but praised by Einstein as marking a significant advance in the physical, as well as philosophical, understanding of relativity; for Gödel had in fact discovered an unexpected solution to Einstein's field equations of gravitation, one permitting 'time travel' into the past.[9] The contribution to Schilpp's anthology is quite brief, but Gödel prepared a much longer essay that has remained unpublished. It too is preserved in the *Nachlass* — in six different versions. It is worth noting that Gödel's interest in relativity went beyond the purely theoretical. In his essay he argues in favour of the possible relevance of his models to our own world, and in the *Nachlass* there are two notebooks devoted to tabulations of the angular orientations of galaxies (which Gödel hoped might exhibit a preferred direction). Freeman Dyson has remarked that even much later, Gödel maintained a keen interest in such observational data.

Yet another unpublished item in the *Nachlass* is the text of Gödel's Gibbs Lecture to the American Mathematical Society, delivered at Brown University on 26 December 1951. Titled 'Some basic theorems on the foundations of mathematics and their philosophical implications', it is Gödel's contribution to the debate on mechanism in the philosophy of mind.

In addition to such relatively finished papers, there are a great many pages of notes by Gödel, including 16 mathematical workbooks, 14 philosophical notebooks, and voluminous shorthand notes on Leibniz. The latter appear to be partly bibliographic, but a recently discovered memorandum suggests that there are about 1000 pages of Gödel's own philosophical assertions as well.

Gödel's long-standing interest in Leibniz is also indicated by an extensive correspondence with archivists, conducted jointly with Oskar Morgenstern during the years 1949-53. The object of the correspondence was the microfilming of some of Leibniz's unpublished manuscripts in Hannover, with the aim not only of preserving them, but of making them available to American scholars. Ultimately the attempt failed, but copies of the manuscripts were later deposited at the University of Pennsylvania through the independent efforts of Prof. Paul Schrecker.

6. LATER YEARS

Gödel's last published paper appeared in 1958. Based on results obtained nearly eighteen years earlier (on which Gödel lectured at Yale, 15 April 1941), it presented a consistency proof for arithmetic by means of 'a [t]hitherto unused extension' of principles formulated in intuitionistic mathematics. As such, it represented a return to earlier mathematical interests (and to the German language — it was the only paper Gödel published in German after his emigration); nevertheless, it is decidedly philosophical in character and was published in the philosophical journal *Dialectica*. Unfortunately, the paper is notoriously difficult to translate. In the early 1970s Gödel himself prepared a revised and expanded English version that reached the stage of galley proofs but never appeared.

After 1958, Gödel devoted himself to revisions of earlier papers, to a search for new axioms to settle the continuum hypothesis (in the wake of Cohen's independence proofs), and to a study of the philosophy of Edmund Husserl. Honorary degrees, academy memberships, and awards (including the National Medal of Science in 1975) were bestowed upon him from many quarters, but he became increasingly withdrawn and preoccupied with his health. He consulted doctors but distrusted their advice, which he often failed to heed. Thus, in the late 1960s, he refused recommended surgery for a prostate condition, despite the urgings of concerned colleagues; and earlier, in the 1940s, he delayed treatment of a bleeding duodenal ulcer until life-saving blood transfusions had to be administered. Afterwards he observed a strict dietary regimen, and as the years wore on his figure became even more gaunt.

During the final decade of Gödel's life, his wife underwent surgery and suffered two strokes that led to her placement in a convalescent home. By all accounts, Gödel attended her devotedly, but he soon began to display signs of depression and paranoia. Correspondence, even with is brother, virtually ceased during the last two years of Gödel's life. In the end, his paranoia conformed to a classic syndrome: fear of poisoning leading to self-starvation. After a relatively brief hospitalization, Gödel died 14 January 1978, of 'malnutrition and inanition' caused by 'personality disturbance'. (His death certificate, from which these phrases are quoted, is on file in the Mercer County courthouse, Trenton, N.J.). He is buried beside his wife and mother-in-law in the historic Princeton Cemetery.

7. PROSPECTS

To what extent can study of Gödel's *Nachlass* be expected to reveal hitherto unknown discoveries? It seems unlikely that major new mathematical results will be found in Gödel's notebooks, although the reputed 'general' consistency proof for the axiom of choice mentioned in Wang (1978) and Wang (1981) may prove to be an exception — if, indeed, it can be reconstructed. Gödel was certainly cautious and overly fastidious in submitting his work for publication, but there is no evidence that he actively withheld important mathematical discoveries; and though details of his researches remain largely concealed behind his shorthand, the topics of his investigations can nonetheless be determined to a large extent from (longhand) headings in the notebooks. On that basis, it seems safe to predict that some lesser results of mathematical interest will be found there, along with some anticipations of or alternative approaches to results of others (such as Gödel's early recognition of errors in Herbrand's work, cited in Kreisel (1980), or his partial independence results in set theory). Of course, details of Gödel's explorations should be of great interest to mathematical historians. Beyond that, I would venture to predict that of all the unpublished material in the *Nachlass*, Gödel's philosophical investigations will turn out to be of greatest interest. Certainly they figure most prominently among the items he left in relatively finished form and that he himself considered to be potentially publishable.

Plans for publication of Gödel's collected works are now well under way.[10] Two volumes are presently envisioned, under the editorship of Solomon Feferman (editor-in-chief), myself, Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort. The first volume, now in preparation, will contain all of Gödel's published articles and reviews, together with his doctoral dissertation (in its original unpublished form) and his revised English version of the *Dialectica* paper, as well as three short notes appended to galley proofs of the latter. Papers in German will be accompanied by English translations on facing pages, and each article will be preceded by introductory commentary. Textual notes, a short biographical essay, and an extensive bibliography will round out the volume.

Detailed contents of Volume 2 remain to be determined, subject both to the success of our decipherment efforts and to our ability to obtain necessary funding and copyright permissions. We hope, however, to include all the relatively finished papers mentioned previously, plus other lecture texts, excerpts from the mathematical notebooks, and selected correspondence, including not only extensive exchanges with other mathematicians, but individual letters of interest. Should there be enough material to warrant it, further volumes may also be considered. The editors will welcome contributions of correspondence with Gödel or recollections of him.

NOTES

1. Wang's later memoir (Wang, 1981) contains some interesting additional information, but on the whole it is less reliable, even though it alone was written before Gödel's death and was submitted to Gödel for his approval and correction. In particular, it is marred by several errors in references to dates and places.

2. The bibliography has now appeared (Dawson, 1983a); note also Dawson (1984a). The translations will be included in the first volume of our edition of Gödel's collected works.

3. Devised by Franz Xaver Gabelsberger, for whom it is named, the script was one of two competing German shorthand systems in widespread use during the early decades of this century. Eventually the two systems merged to form the modern *Einheitskurzschrift* ('unified shorthand'); alas, however, those trained only in the modern system can read *neither* the Gabelsberger nor Stolze–Schrey scripts from which it was derived. Yet there is a need for some younger scholars to learn to read these scripts, since many prominent intellectuals used them in their daily lives — not, as is often supposed, for reasons of secrecy, but as efficient means for rapid and concise recording of events and ideas.

4. An abridged transcript of the discussion appeared in *Erkenntnis 2* (1931), pp. 135-151; for an English translation and commentary, see Dawson (1984b).

5. This date is based on Gödel's correspondence with von Neumann. On 13 July 1937, von Neumann wrote Gödel from Budapest, saying that he expected to visit Vienna in a few weeks, and that while there he hoped to speak with Gödel and learn more about his plans. In the same letter he urged Gödel to consider publishing his work on the axiom of choice in the *Annals of Mathematics*. In his next letter, however, written 14 September from New York, von Neumann advised Gödel that the editors of the *Annals* were prepared to expedite publication of his work on the generalized continuum hypothesis. In the end, both consistency results were announced late the following year in the *Proceedings of the National Academy of Sciences*.

6. By 'mustered' Gödel apparently meant only that he had to report for the physical examination. It seems very unlikely that he was actually sworn in.

7. With Julian Schwinger; not, as stated in Quine (1978), with von Neumann. Von Neumann was actually a member of the awards committee, and he may have introduced Gödel's name for consideration. In any case, there is evidence in von Neumann's papers that Schwinger alone was originally proposed for the award.

8. Rumours have persisted that Gödel actually obtained the independence of the axiom of choice in the early 1940s but refused to

publish his results. In particular, after Cohen's proofs in 1964, Mostowski asserted, 'Es ist seit 1938 bekannt, dass Gödel einen Unabhängigkeitsbeweis dieser Hypothesen besitzt; trotz vielen Anfragen verriet aber nie sein Geheimnis' (*Elemente der Mathematik* 19, p. 124). But Gödel himself denied this. In a letter to Wolfgang Rautenberg (published in *Mathematik in der Schule 6*, p. 20) he stated explicitly:

> Die Mostowskische Behauptung ist insofern unrichtig, als ich bloss in Besitzte gewissen Teilresultate war, nämlich von Beweisen für die Unabhängigkeit der Konstruktibilitäts- und Auswahlaxioms in der Typentheorie. Auf Grund memer höchst unvollständigen Aufzeichnungen von damals (d.h. 1942) könnte ich ohne Schwierigkeiten nur den ersten dieser beiden Beweise rekonstruieren. Meine Methode hat eine sehr nahe Verwandtschaft mit der neuerdings von Dana Scott entwickelten, weniger mit den Cohenschen.

It is clear from Gödel's correspondence that he had great respect for Cohen's work; indeed, he described Cohen's achievements as 'the greatest advance in abstract set theory since its foundation by Georg Cantor.' Nevertheless, the method by which Gödel obtained his partial results may still prove to be of interest.

9. Technical details were published that same year (1949) in *Reviews of Modern Physics*, and a year later Gödel spoke on his results at the International Congress of Mathematicians in Cambridge, Massachusetts.

10. A nearly complete edition of Gödel's published works, excluding reviews, has already appeared in Spanish translation; see Jesús Mosterin, ed., *Kurt Gödel, Obras Completas*, Alianza Editorial, Madrid, 1981.

REFERENCES

Bulloff, Jack, T.C. Holyoke and S.W. Hahn (eds.) (1969) *Foundations of Mathematics: Symposium Papers Commemorating the Sixtieth Birthday of Kurt Gödel*, New York: Springer-Verlag.

Christian, Curt (1980) Leben und Wirken Kurt Gödels. *Monatshefte für Mathematik* 89, 261-73.

Dawson, J.W., Jr. (1983a) The published work of Kurt Gödel: an annotated bibliography. *Notre Dame Journal of Formal Logic* 24, 255-84.

Dawson, J.W., Jr. (1983b) Cataloguing the Gödel *Nachlass* at the Institute for Advanced Study. *Abstracts of the 7th International Congress of Logic, Methodology, and Philosophy of Science* 6, 59-61.

Dawson, J.W., Jr. (1984a) Addenda and corrigenda to 'The published work of Kurt Gödel'. *Notre Dame Journal of Formal Logic* 25, to appear.

Dawson, J.W., Jr. (1984b) Discussion on the foundation of mathematics. *History and Philosophy of Logic* 5, 111-29.
Dawson, J.W., Jr. (reprint) Completing the Gödel-Zermelo correspondence. *Historia Mathematica*, 12, 66-70.
Dyson, Freeman (1983) Unfashionable pursuits. *The Mathematical Intelligencer* 5:3, 47-54
Feferman, Soloman, John W. Dawson, Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay and Jean van Heijenoort (eds.) (1986) *Kurt Gödel: Collected Works. Volume I: Publications, 1929-1936*, Oxford: Oxford University Press.
Gracy, David B., II(1977) *Archives & Manuscripts: Arrangement & Description* (Basic Manual Series). Chicago: Society of American Archivists.
Grattan-Guinness, Ivor (1979) In memoriam Kurt Gödel: his 1931 correspondence with Zermelo on his incompletability theorem. *Historia Mathematica* 6, 294-304.
Kreisel, Georg (1980) Kurt Gödel, 1906-1978, elected For. Mem. R.S. 1968. *Biographical Memoirs of Fellows of the Royal Society* 26, 148-224; (1981) corrigenda, 27, 697; (1982) further corrigenda, 28, 718.
Quine, Willard V. (1978) Kurt Gödel (1906-1978). *Year Book of the American Philosphical Society*, 81-84.
Ulam, Stanislaw (1976) *Adventures of a Mathematician.* New York: Scribner's.
van Heijenoort, Jean (ed.) (1967) *From Frege to Gödel, A Source Book in Mathematical Logic, 1879-1931.* Cambridge: Harvard.
Wang, Hao (1978) Kurt Gödel's intellectual development. *The Mathematical Intelligencer* 1, 182-4.
Wang, Hao (1981) Some facts about Kurt Gödel. *The Journal of Symbolic Logic* 46, 653-9.

II

On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I[1] (1931)

Kurt Gödel

1.

The development of mathematics toward greater precision has led, as is well known, to the formalization of large tracts of it, so that one can prove any theorem using nothing but a few mechanical rules. The most comprehensive formal systems that have been set up hitherto are the system of *Principia mathematica* (*PM*)[2] on the one hand and the Zermelo–Fraenkel axiom system of set theory (further developed by J. von Neumann)[3] on the other. These two systems are so comprehensive that in them all methods of proof today used in mathematics are formalized, that is, reduced to a few axioms and rules of inference. One might therefore conjecture that these axioms and rules of inference are sufficient to decide *any* mathematical question that can at all be formally expressed in these systems. It will be shown below that this is not the case, that on the contrary there are in the two systems mentioned relatively simple problems in the theory of integers[4] that cannot be decided on the basis of the axioms. This situation is not in any way due to the special nature of the systems that have been set up but holds for a wide class of formal systems; among these, in particular, are all systems that result from the two just mentioned through the addition of a finite number of axioms,[5] provided no false propositions of the kind specified in note 4 become provable owing to the added axioms.

Before going into details, we shall first sketch the main idea of the proof, of course without any claim to complete precision. The formulas of a formal system (we restrict ourselves here to the system *PM*) in outward appearance are finite sequences of primitive signs (variables, logical constants, and parentheses or punctuation dots), and it is easy to state with complete precision *which* sequences of primitive signs are meaningful formulas and which are not.[6] Similarly, proofs, from a formal point of view, are nothing but finite sequences of formulas (with certain specifiable properties). Of course, for metamathematical considerations it does not matter what objects are chosen as primitive signs, and we shall assign natural numbers to this use.[7] Consequently, a formula will be a finite sequence of natural numbers,[8] and a proof array a finite sequence of finite sequences of natural numbers. The metamathematical notions (propositions) thus become notions (propositions) about natural numbers or sequences of them;[9] therefore they can (at least in part) be expressed by the symbols of the system *PM* itself. In particular, it can be shown that the notions 'formula', 'proof array', and 'provable formula' can be defined in the system *PM*; that is, we can, for example, find a formula $F(v)$ of *PM* with one free variable v (of the type of a number sequence)[10] such that $F(v)$, interpreted according to the meaning of the terms of *PM*, says: v is a provable formula. We now construct an undecidable proposition of the system *PM*, that is, a proposition A for which neither A nor *not-A* is provable, in the following manner.

A formula of *PM* with exactly one free variable, that variable being of the type of the natural numbers (class of classes), will be called a *class sign*. We assume that the class signs have been arranged in a sequence in some way,[11] we denote the nth one by $R(n)$, and we observe that the notion 'class sign', as well as the ordering relation R, can be defined in the system *PM*. Let α be any class sign; by $[\alpha; n]$ we denote the formula that results from the class sign α when the free variable is replaced by the sign denoting the natural number n. The ternary relation $x = [y; z]$, too, is seen to be definable in *PM*. We now define a class K of natural numbers in the following way:

$$n \,\varepsilon\, K \equiv \overline{Bew}\,[R(n); n] \qquad (1)$$

(where $Bew\ x$ means: x is a provable formula.[11a] Since the notions that occur in the definiens can all be defined in *PM*, so can the notion K formed from them; that is, there is a class sign S such that the formula $[S; n]$, interpreted according to the meaning of the terms of *PM*, states that the natural number n