issues in mathematics teaching

Edited by Peter Gates

## Issues in Mathematics Teaching

Issues in Mathematics Teaching brings together a number of key, and sometimes controversial, issues in mathematics education, which will be of concern to all those teaching the mathematics. It focuses in particular upon the social context of teaching mathematics and its impact upon teachers and pupils with diverse social and cultural backgrounds. The issues covered include:

- the role played by mathematics textbooks
- the national numeracy strategy
- incorporating ICT
- international comparisons of mathematical attainment
- the values adopted, implicitly and explicitly in mathematics teaching
- how to use-and not to use-classroom resources

All chapters benefit from being informed by current state-of-the-art research in mathematics education and each of the contributors has an international reputation in his or her field. Every chapter offers opportunities to reflect on the issues involved and directs the reader's attention and activity to the critical aspects of each issue. Written with new teachers in mind, Issues in Mathematics Teaching makes complex issues accessible without trivialisation and will be of interest to all those charged with teaching mathematics.

Peter Gates teaches at the University of Nottingham's School of Education and has previously taught at the Open University and the University of Bath. Prior to this he worked as a mathematics teacher for thirteen years, including four years as head of department at a Milton Keynes comprehensive and a spell in Moçambique. He has edited and written numerous books and articles on teaching mathematics and is currently interested in how practices in schools in general, and mathematics education in particular, contribute to social exclusion. Peter is a member of the International Committee of the International Group for the Psychology of Mathematics Education. Peter is a member of the International Committee of the International Group for the Psychology of Mathematics Education.

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## Contributors

Janet Ainley is a senior lecturer in the Mathematics Education Research Centre, University of Warwick, and is also Deputy Director of the Warwick Institute of Education. Her main research interest is in the design of purposeful pedagogic settings that exploit the potential of computers in the learning and teaching of data handling and algebra. She is co-director of the Primary Laptop Project, based in a Warwickshire school, and has a national and international reputation in this field. She publishes regularly through international conferences and journals, and is the author of a popular book on the use of IT in primary mathematics.

Paul Andrews spent thirteen years teaching mathematics in three secondary comprehensive schools, the last five of which were as head of department. Since then he has spent ten years working in teacher education in both Manchester and Cambridge and has extensive experience of both pre-service and in-service award-bearing courses. His principal area of interest is in making mathematics accessible to all children-hence his exploration of the ways in which the subject is taught overseas-and sees equality of opportunity as a much compromised principle in British education, not least because of successive governments' obsessive drive towards differentiation with its systematic denial, for so many pupils, of access to the curriculum. He is also fascinated by teachers' understanding of, and beliefs about, mathematics and how they impact on classroom activity and, importantly, children's learning. He rejects unequivocally utilitarianism as a curricular justification for the subject.

Mike Askew is lecturer in mathematics education at King's College, University of London. A former primary school teacher for six years, he now researches, lectures and writes on teaching primary mathematics. He was a member of the advisory group to the National Numeracy Project and worked on both the content of the Project's Framework for teaching mathematics from Reception to Year 6 and the associated training materials. He was director of the Teacher Training Agency funded 'Effective Teachers of Numeracy in Primary Schools' and Nuffield Foundation funded 'Raising Attainment in Numeracy Project'. Both of these are now being followed up in the five-year Leverhulme Numeracy Research Programme, of which Mike is deputy director.

Alan Bishop is a professor in the Faculty of Education at Monash University in Melbourne, Australia where he has worked since 1992 after leaving Cambridge University and England. He has taught in different school situations and has been involved in teacher initial and in-service training all his working life. He was president of the Mathematical Association in 1991. His research has involved many aspects of classroom teaching and in particular he has researched the area of socio-cultural aspects of mathematics teaching for the last twentyfive years. He is now the director of two large government-funded research projects, one concerned with excellence in mathematics teaching, and the other with values in mathematics education.

Jo Boaler taught mathematics in secondary schools in Inner London and worked at the headquarters of the National Union of Teachers, developing equal opportunities policies. She worked for seven years as a lecturer and researcher at King's College London, and now lives and works in California, where she is an associate professor of mathematics education. Jo is the author of two books. Experiencing School Mathematics, was published by the Open University Press in 1997. It reports upon a study of two schools using different mathematics approaches and won a national book award for education in 1997. Jo's more recent book is Multiple Perspectives on Mathematics Teaching and Learning, published by Ablex in 2000. It is the first of an international series on mathematics education. Jo is the current president of the International Organization for Women and Mathematics Education.

Barry Cooper is currently Professor of Education, and Director of Research, in the School of Education, University of Durham. His research interests include the sociological study of the school curriculum and assessment, maths education, and Indian primary education. He has recently directed two projects investigating national curriculum testing in maths, both funded by the Economic and Social Research Council. His publications include Renegotiating Secondary School Mathematics (Falmer 1985) and Assessing Children's Mathematical Knowledge: Social Class, Sex and Problem-Solving (with Máiréad Dunne, Open University Press 2000). He was previously a Reader in Education at the University of Sussex.

Tony Cotton has taught in Sheffield, Leicester and Nottingham. He has worked as a mathematics teacher, an advisory teacher for anti-racist and multicultural education and an advisor to an educational publisher. He grew up in Liverpool and Manchester so is very much attached to northern industrial towns. His main claim to fame is having taught Jarvis Cocker and helping to put on Pulp's first ever gig in City School Hall, Sheffield. Apart from that he has written books on mathematics for pupils and books on teaching for teachers. He has also written several articles for Mathematics Teaching. He now works at the Nottingham Trent University in the Faculty of Education and would like to think he was asked to write this chapter because it is an area of work he has been committed to since starting teaching in 1981, rather than because of the fact he is a close colleague of the editor.

Kevin Delaney. After graduating with a degree in chemistry in 1969, Kevin Delaney took the obligatory year off to find himself and then found himself on a primary PGCE course at Southampton University. One of his tutors was Bill Brookes whose wild intellectual ways and intriguing ideas changed his life; he also introduced him to many kindred spirits who belonged to the Association of Teachers of Mathematics (ATM). He taught in 'normal' primary schools for five years in Southampton and Harrow before meeting John Dichmont at an ATM conference and joining him at Lynncroft Primary School in Nottingham in 1976. Here, under John's inspired guidance, his life changed for a second time and he discovered what working with primary age children could really be like when a creative and thoughtful staff dared each other to do useful and interesting things with children. Ten years later, after an M.Phil, with Bill Brookes exploring aspects of his own teaching, he joined Nottingham Trent University to teach on primary mathematics courses and is currently a senior lecturer there. In 1988 his life changed for a third time when he started a timeconsuming second career as the father of Jack (11) and Alice (8); this and his first career seem to have taken up most of his waking hours since that time so he is particularly pleased to have found time to write a chapter on a subject close to his heart. From 1990 to 1998 he was consultant editor, with Adrian Pinel, for Strategies. He is excited by Islamic patterns and his own and other people's 'Aha!' moments during the teaching and learning of mathematics. He also remains open to the possibility of having his life changed a fourth time.

Paul Dowling is a sociologist. He taught mathematics in schools in and around London for fifteen years before joining the Institute of Education in 1987. He is currently a member of the Culture Communication and Societies academic group at the Institute. His research activities centrally concern the development of 'semiotic sociology' in and through the analysis of texts and practices associated with, amongst other areas: mathematics education; information and communications technologies; non-school pedagogic sites; the media; and the relationships between pedagogic practices and social and community structure. He has been involved with consultancies in the area of research methods in education around the world, particularly in South Africa. His Falmer Press publications include The Sociology of Mathematics Education: Mathematical Myths, Pedagogic Texts, Doing Research/Reading Research: A Mode of Interrogation for Education (co-authored with Andrew Brown), Mathematics Versus the National Curriculum (co-edited with Richard Noss).

Paul Ernest was born in New York but educated in England, where he became a school mathematics teacher in the 1970s. He studied mathematics, logic and philosophy at Sussex and London Universities where he obtained his B.Sc., M.Sc. and Ph.D. degrees. He has previously held lecturing positions in the Universities of Cambridge and the West Indies. Paul Ernest is currently Professor of the Philosophy of Mathematics Education at the University of Exeter, where he directs the masters and doctoral degree programmes in mathematics education. He edits the Philosophy of Mathematics Education Journal (web location: http:/ /www.ex.ac.uk/~PErnest/). His main research interests concern fundamental
questions about the nature of mathematics and how it relates to teaching, learning and society. His publications include: The Philosophy of Mathematics Education, Falmer 1991; Social Constructivism as a Philosophy of Mathematics, State University of New York Press, 1998; Mathematics for Primary Teachers, Routledge, 2000.

Peter Gates trained as a mathematics teacher in 1975 having successfully remained a student in Manchester for six years. He then taught in the East End of London where mere 11 year olds were so tough they had the capacity to turn his legs to jelly. From there he worked as a 'cooperante' in the People's Republic of Moçambique, in the aftermath of independence. As a provincial mathematics advisor he worked in rural areas supporting teachers and children. It was the commitment and dedication of people who had so little that has helped him to retain his socialist inclinations ever since. After Moçambique, Peter worked as head of mathematics at Stantonbury Campus in Milton Keynes (with Anne Watson-see below). He considers himself immensely lucky in managing to work with other teachers so fully committed both to mathematics and to teaching for diversity, and held a commitment to teaching in all-attainment groups. Unlike Tony Cotton (see above) he has no major claim to fame-though he did once shake hands with the Duke of Edinburgh who is unlikely to remember the experience. He has so far edited two booksConceptualising Reflection in Education (co-edited with James Calderhead and published by Falmer Press), and this one. He is currently a so-called academic at Nottingham University where he works with students on initial teacher education courses and with local mathematics teachers and children. With Tony Cotton, he established the first Mathematics Education and Society Conference, which is now an important international forum for discussion and dissemination of the social implications of teaching and learning mathematics. Peter got his Ph.D. as this book was going to print, with a thesis exploring the ideological orientations of mathematics teachers. He now spends much of his time trying to explain to his elderly relatives that even though he is a doctor he doesn't know how to cure their bunions or ingrowing toenails. Peter has two children-Sophie (born 30th October 1994) and Megan (born 8th July 1998) who, in spite of all the trials and tribulations, make Peter's and his partner Jane's life worth living.

Derek Kassem entered teaching 'late’ after a number of different jobs that includes capstan operator, tube guard/motorman and a welfare rights advisor in a law centre. Though he trained for middle schools he actually started teaching in the only all-boys school in Brixton-Tulse Hill. After a couple of years he switched to primary and has been a maths co-coordinator in schools in Tower Hamlets, Southwark and Lambeth. In the early '90s he became involved in providing Inset for a number of LEAs. Since 1996 he has been senior lecturer in mathematics education at Liverpool John Moores University. He has written teaching materials for the primary classroom and co-authored a number of conference papers on primary pupils' attitudes on a range of issues. He is currently researching for his $\mathrm{Ph} . \mathrm{D}$. on issues relating to ethnic minority attainment and the National Numeracy Strategy.

Candia Morgan taught mathematics for a number of years in secondary schools and at university level. She has also worked as an advisory teacher for mathematics. In her current position as senior lecturer in mathematics education at the Institute of Education of the University of London, she works with students in initial teacher education and with teachers undertaking further postgraduate study. Her current research interests include the role of language within mathematics education and teachers' assessment of pupils' mathematical activity. She is the author of Writing Mathematically: The Discourse of Investigation (Falmer 1998).

Mike Ollerton 'graduated' from the bottom stream (1960-3), leaving school with six O levels, gathered over four years. This experience was to have a profound effect upon his pedagogy for teaching mathematics. He started his teaching career in 1971 at Lowca, a small West Cumbrian primary school. In 1973 he began teaching at Wyndham, an 11-18 school in Egremont. Here he worked with Eric Love, a visionary and inspirational head of department. In 1986 Mike took up a head of department post at an 11-16 comprehensive in Telford, where he guided the department towards constructing mixed-ability teaching methods throughout the age range. He also taught mathematics without using textbooks, instead adopting problem-solving, equipment-based approaches. His work was supported by colleagues and members of the Association of Teachers of Mathematics; from 1986 until 1992 he was involved in the ATM/SEG pilot GCSE project, which at its inception, was based upon a 100 per cent coursework model. From 1990 until 1995 Mike worked part-time (at Keele and Manchester University) as a teacher trainer, continuing as a part-time head of department in Telford. Since 1995 Mike has worked full time at St Martin's College, Lancaster, with pre-service and in-service teachers, teaching undergraduate, postgraduate and Masters students. Mike gained his first degree at the age of 42, an Advanced Diploma at age 43 and completed his M.Phil. in 1997, aged 49. In this project he was strongly supported by Christine Shiu and Anne Watson. All of these qualifications were gained through the Open University, an institution to which he extends his heart-felt admiration. Mike frequently writes articles in Mathematics Teaching (the journal of the Association of Teachers of Mathematics); many of which focus on ways of teaching mathematics without needing to separate children into different supposed 'ability' groups. He has recently completed a book with Anne Watson (see below) about teaching mathematics in inclusive classroomsInclusive Mathematics 11-18 (Cassell 2001).

Carrie Paechter is Director of the Centre for Curriculum and Teaching Studies in the Faculty of Education and Language Studies at the Open University. After a degree in Philosophy at Cambridge followed by a return to a further education college to take a second set of A levels, she trained as a primary school teacher and then went to work in a mathematics department in East London. After two more schools and a period as head of mathematics, she went to work as a researcher at King's College, London, looking at how teachers and students negotiated interdisciplinary coursework projects. This stimulated
her emerging interest in power/knowledge, gender and curriculum negotiation, on which her research has been focused ever since. She lives in London with her partner, two sons and two stepdaughters, and is currently trying to work out how she can get other parents to take her ideas and concerns about gender and schooling seriously. Her publications include Crossing Subject Boundaries (HMSO 1995), Educating the Other: Gender, Power and Schooling (Falmer 1998) and Changing School Subjects: Power, Gender and Curriculum (Open University Press 2000).

Malcolm Swan taught for several years in a secondary school near Derby before being appointed as a lecturer at the University of Nottingham, where he has now been working for the past twenty years (gulp!). He is particularly interested in the design of teaching and assessment and has been involved in many curriculum, assessment and teacher education research and development projects. His particular interests include finding strategies for engaging people in reflection and discussion about mathematical ideas and approaches. At one time he was involved in creating and then being Chief Examiner at the NEAB (now AQA) for a GCSE called 'Mathematics Through Problem Solving' which, among other things, involved learning number through planning day trips and going on them, space and shape through designing 3-D pop-up cards and board games, and handling data through consumer testing. (This was just before the days when maths was cut up into tiny pieces in the National Curriculum; it all sadly came to an end when exam boards were forced to reduce the number of syllabuses.) He is currently working on balanced assessment of problem solving in mathematics for the Mathematics Assessment Resource Service (MARS) at Nottingham and in the US. (Maybe if it is assessed, teachers will be able to spend more time doing it.)

Anne Watson was a secondary mathematics teacher for many years, working in comprehensive schools. Subsequently she moved into teacher education, professional development and mathematics education research. Her doctoral research was into the informal assessment techniques used by teachers of mathematics and she also works on and writes about other aspects of mathematics classrooms, particularly those which improve achievement for all. Currently she is a lecturer in mathematics education at the University of Oxford, and she has also worked in Pakistan, South Africa and Jamaica. With Mike Ollerton (see above) she has written Inclusive Mathematics 11-18 (Cassell 2001) which combines her twin passions for mathematics and social justice.

Dylan Wiliam. Despite pressure from his headteacher to introduce setting, Dylan Wiliam taught mathematics to mixed-ability classes in inner-city schools for seven years before joining King's College, London to work on developing innovative assessment materials for the Graded Assessment in Mathematics Project. Between 1989 and 1991 he co-ordinated the development of Key Stage 3 assessment tasks in English, mathematics, science and technology, where his efforts met with critical acclaim such as 'elaborate nonsense' (Kenneth

Clarke, Secretary of State for Education) and 'getting children to do weird things in corners' (John Major, Prime Minister). He returned to King's where he is currently head of the School of Education and Professor of Educational Assessment and spends as much time as he can amassing information on the negative effects of ability grouping in mathematics.

Jan Winter is a lecturer in education at the University of Bristol. She taught in Bristol schools for eight years before working as an advisory teacher for Avon LEA. This experience provided her with a wealth of experiences of working with other teachers, through which she learned an enormous amount. For the last seven years she has worked at the University of Bristol on pre-service training courses for secondary mathematics teachers as well as masters degrees and other courses of in-service training. She has taken part in many research projects in the areas of assessment and of mathematics learning and continues to be fascinated by the learning process, both her own and that of pupils in school. She was chair of the General Council of the Association of Teachers of Mathematics for three years. She is currently starting work on a major research project on home-school links-improving the use of knowledge about learning between these two settings in which children learn.

Robyn Zevenbergen trained as a teacher at Deakin University in Australia in the 1980s. Robyn is a senior lecturer at Griffith University, Australia, where she works in (primary) mathematics education. Her research centres on issues of equity and social justice within a critical sociological framework. Currently, her work is exploring the role of language and its impact on learning and equity. She is also working on a project examining reforms in mathematics education and the barriers that these pose to learning and participation, with the intention of making explicit aspects of invisible pedagogy that hinder access to mathematics and success. While the main emphasis in her work is with students from working-class backgrounds, she has also been involved in projects with indigenous students and students with hearing difficulties-particularly in exploring the relationships between these groups.

## Glossary

| ALBSU | Adult Literacy and Basic Skills Unit |
| :--- | :--- |
| APU | Assessment and Performance Unit |
| ATM | Association of Teachers of Mathematics |
| BCME | British Congress of Mathematics Education |
| BSRLM | British Society for Research into Learning Mathematics |
| BSU | Basic Skills Unit |
| DFE | Department for Education |
| DfEE | Department for Education and Employment |
| EMTAG | Ethnic Minority and Traveller Achievement Grant |
| ESN | Educationally Sub-Normal |
| ESRC | Economic and Social Research Council |
| GAMMA | Gender and Mathematics Association |
| GCE | General Certificate of Education |
| GCSE | General Certificate of Secondary Education |
| GNVQ | General National Vocational Qualification |
| HMCI | Her Majesty's Chief Inspector of Schools |
| HMI | Her Majesty's Inspector |
| HMSO | Her Majesty's Stationery Office |
| ILEA | Inner London Education Authority |
| ITE | Initial Teacher Education |
| LEA | Local Education Authority |
| LMS | Local Management of Schools |
| NCC | National Curriculum Council |
| NFER | National Foundation for Educational Research |
| NNP | National Numeracy Pilot |
| NNS | National Numeracy Strategy |
| NQT | Newly Qualified Teacher |
| OFSTED | Office for Standards in Education |
| PSE | Personal and Social Education |
| QCA | Qualifications and Curriculum Authority |
| SATs | Standard Assessment Tests |
| SCAA | Schools Curriculum and Assessment Authority |
| SMP | Schools Mathematics Project |
| TES | Times Education Supplement |
| TGAT | Task Group on Assessment and Testing |
| THES | Times Higher Educational Supplement |
| TIMSS | Third International Mathematics and Science Study |
| TTA | Teacher Training Agency |
|  |  |

## Introduction

# How to make the most of this book 

Peter Gates

## Who this book is for

This book has been specifically written with classroom teachers in mind, particularly new teachers. By 'new teachers', we mean student teachers on courses of initial teacher education (PGCE or B.Ed. courses or those on other routes into the profession, such as SCITT schemes, fast track programmes, modular courses etc.) as well as teachers in their first few years of teaching. We hope the book will be read by both teachers in primary schools who need to teach mathematics, and mathematics teachers in secondary schools. Most of the issues we address are issues that affect both primary and secondary phases-and we have avoided focusing this book exclusively on one phase or another because we feel there are issues of which all those who teach mathematics need to be aware. Due to the contextualisation of some of the issues, it might appear that some chapters are more appropriate for one phase rather than another. We would, however, urge all readers, whether working in the primary or secondary phase, to see that each chapter has something to offer once you have seen through that context into the wider issues. Hence, the content of the book and the issues raised in it will be of interest to all those responsible for teaching mathematics and mathematics education in all phases. The issues covered in the book are both contemporary and controversial, and are those currently under debate by the profession more generally. This audience has dictated also the style and approach of the writing used in the book. All contributors have attempted to make the issues as accessible as possible. This is not always easy given the complex nature of educational issues and at times, you might have to work hard. We make no apologies for that.

The book is likely also to be useful to teachers who are working as mentors to new teachers-either on ITE courses or NQT induction programmes. In addition it is likely to be of interest to higher education tutors of students on mathematics education courses, since it not only covers contemporary issues, but also offers reflective activities.

Because of the way in which current state-of-the-art research in mathematics education has been used by the contributors, (most of whom are currently active researchers in the area on which they write) this book is likely also to be of interest to those researching areas of mathematics education in order to foster a broader perspective on some complex issues.

## How to read this book

With the exception of the first introductory chapter, which forms the overview and the backdrop to the book, all chapters have a common structure. The issue that each chapter addresses is briefly introduced and summarised in Introducing the Issue, to give you a feeling for what is about to come. This is not merely in order to follow a literary tradition, but is intended to mentally prepare you. These introductions, along with the Key Questions that follow, are intended to give you the opportunity to think ahead. Therefore, before you read each chapter, you might read the introduction, and then ponder over the key questions before fully engaging with the chapter.

Each chapter contains points where we think you might benefit by Reflecting on the Issue.... These opportunities have been designed to help you interact with and confront some of the essential ideas in each chapter. Naturally, we have no power over how or whether you undertake these activities-we are not going to put you through some assessment process! It is unlikely that you will always be in a position to stop reading and undertake some activity, since a number of the activities suggest you move away from the book to work with children or other teachers. However many of the Reflecting on the Issue...activities can be-and indeed need to be-worked on there and then because they require you to consider some important aspect before reading on. There will possibly be a tendency to just keep on reading because this is often the easiest way. (It's a bit like finding an excuse for not doing your homework isn't it?) Of course, how you want to use this book is up to you.

At the end of each chapter is a further Invitation to Reflect. This invites you to reconsider in some way, some of the arguments in the chapter and to relate what you have read to your own experience.

Finally, each chapter closes with Further Suggested Readings, which offer some ideas where you might usefully look if you want to learn more about the issues discussed in the chapter, and the References to material used or drawn on in the chapter. We have tried to make the suggested readings accessible and available.

My experience in working with teachers and pupils over many years leads me to suggest that you might find it helpful to keep a notebook-or use any blank pages at the back-to write down your responses to the key questions and the activities. You might write a response to the activity-or if this is less appropriate to your context (i.e. you might be a teacher educator, or researcher) then you might benefit from collecting your thoughts on the activities as activities? How might you use them? Why did the mathematics educators who wrote this book construct them as they did?

Naturally, we hope you enjoy engaging with this book, and that in some small way the world might become a slightly better place as a result.

## Acknowledgements

The contributors to this book have agreed to donate all the royalties to the Save the Children Fund to contribute to their work with disadvantaged children
throughout the world who in many cases do not even get the benefit of the largely imperfect education system critiqued throughout this book.

We would like to thank:

- Cambridge University Press, for permission to use the SMP 11-16 extracts in Chapter 12;
- The Metropolitan Police, for permission to use the photograph of the police also in Chapter 12;
- The Association of Teachers of Mathematics, for permission to adapt and enlarge Jan Winter's ATM Conference address, previously published in Mathematics Teaching 172, for Chapter 13.
- The Qualifications and Curriculum Authority, for permission to reproduce the SAT questions in Chapter 16.


## Part I

## Setting the scene; raising the issues

# 1 What is an/at issue in mathematics education? 

Peter Gates

## The introduction to the issues

Those of us with an interest in-and indeed a passion for-teaching mathematics learn pretty quickly not to divulge this information too readily in public places. One has to be careful responding to the apparently innocent 'and what do you do?' question for fear of throwing the hairdresser into a rant about how they never enjoyed maths, could never do it or understand all those letters. You then find that you are being looked at a bit askance in order to examine what other defects or peculiarities you might have. Hating mathematics is as much a national pastime as complaining about the weather and mathophobia is so acceptable that one can readily make light of it-as I just have, but of course, it is a very serious matter, that has serious implications for very many children. Mathematics is not just a complex collection of skills, concepts and ideas that we endeavour to pass on to the next generation. As many chapters in this book will go on to argue, mathematics serves as a 'badge of eligibility for the privileges of society' (Atweh, Bleicher and Cooper 1998:63). Being successful at mathematics brings with it opportunities and riches; one stands a better chance of higher paid careers if one holds a higher qualification in mathematics. In addition, of course the converse is true. In order to keep certain sectors of the population away from such success, they have to be made to fail at mathematics. However, I am running away with myself here and need to develop this argument a little more.

Of course, many children enjoy mathematics-usually more so between the ages of 5 and 11 than 11 and 16 it has to be said, but there's another story! For many pupils, mathematics is a series of challenges and hurdles, which they face with passion and determination. For many others however, mathematics is a daily experience of continued failure and irrelevance. Mathematics education fails too many children; it fails children on the margins of society, it fails children from ethnic minorities, and it fails children from social and cultural backgrounds that are different from the majority of mathematics teachers.

In 1982, one submission to the Cockcroft Report into the teaching of mathematics in schools, Mathematics Counts, said 'Mathematics lessons in secondary schools are very often not about anything. You collect like terms, or learn the laws of indices, with no perception of why anyone needs to do such things' (Department for Education and Science 1982: para 462, p. 141). This seems to me to sum up what must be
many children's experience of the subject. Mathematics is about all manner of things, but about nothing at all. It is about things that seem quite divorced from our everyday lives, interests or needs. Yet while this might reflect what goes on in many classrooms, it is a rather naïve description because it merely takes some of the surface features of the mathematics classroom and ignores the underlying complexity and the unintended outcomes of that complexity. Paradoxically, it is exactly because someone can say 'mathematics lessons are not about anything' that the situation is more worrying-showing how the processes of exclusion, rejection and de-motivation are hidden, obscured and misinterpreted.

Part of that complexity lies in the role that schools play in the construction of one's identity-forging how one compares oneself to others during some particularly difficult times for young people-growing independence, the embarrassment of puberty and the frustrations of adolescence. So, in this gradual process of selfawareness and self-efficacy what are the issues for mathematics teachers? What do we need to think about? In a book titled Issues in Mathematics Teaching, one might naturally expect a bunch of mathematicians to begin with the definitions, and I'll not disappoint. 'Issues' is the easy one, so I will start there. To be an issue means to be important and requiring of a decision; to be at issue implies something is under discussion due to disagreement; to take issue implies to disagree, whereas to issue forth means to expound. This book then is an opportunity for some writers to expound on matters about which there is likely to be some disagreement and which are under debate. As for defining 'mathematics' and 'teaching', little will be gained by opening that can of worms because the terms are so contentious and so slippery. I will 'leave it as an exercise for the reader'—as it used to say in my university mathematics books. Defining such terms is really only useful insofar as it engages you in discussion with others. Coming to understand and define what it means to teach mathematics requires a long-term professional commitment of critical engagement in debate, not a textbook definition.

## The background to the issues

It can hardly be contested that we live in an uneven and unjust society where access to education and to justice depend on the capital one can appropriate and accumulateparticularly through the benefits the education system bestows on some individuals. There is ample evidence in the academic and research literature in education to support this contention such that it is hardly now contentious. Yet, unfairness, injustice and prejudice are not abstract concepts of some macro-social analysis of an internecine class war. They are felt through the disappointment, hopelessness and frustrations of ordinary people as they get though their everyday lives. They exist in the knots in the pit of the stomach and the tears in the eyes. Injustice exists in the disappointments many children face when they are not endowed with financial resources to have what other children have and take for granted. Injustice exists in the frustration, anger and self-depreciation when a pupil is placed in a low set for mathematics based on some assessment procedure over which they have no control and which they feel is unfair. Injustice is a process that goes on all around us, even when-and arguably especially when-we do not look for it or recognise it.

I have spent many years in a variety of classrooms as a mathematics teacher, and continue to do so in my current role as tutor to student teachers, and what I see often upsets me, and I guess it was this which, to some extent, encouraged me to put this book together. I feel uncomfortable when I see children labelled as 'less able' placed into 'bottom sets' and fed diets of at worst, tedium, or at best, irrelevant and uninteresting exercises. I feel uncomfortable when the majority of pupils I see in those bottom sets seem to have had very similar life experiences reflecting varying degrees of deprivation. I feel uncomfortable when I can see they realise that there really is no point in working hard to learn mathematics because the structure of the school means they cannot achieve high GCSE grades whatever they do. I feel frustrated when I see mathematics envisaged by pupils, parents and teachers as little more than a collection of techniques to be captured rather than an approach to understand and to tackle society's ills. Finally, (for now) I feel angry when I hear teachers criticise parents for not being interested enough in their children to come to parents' evenings when, of course, 'these are just the ones you want to see!'. Well that may be so, but they clearly don't want to see you very much-and that ought to be where we begin to ask questions.

However, this book is not about my feelings; this is about those children who give up on mathematics; because many of the children who give up on mathematics (or better, those whom mathematics gives up for sacrifice) give up on society. OK. You might think this is a bit extreme, but it is my contention that mathematics education in schools plays a significant role in organising the segregation of our society, and conversely as a mathematics teacher, you will play your part too.

Sue Willis is an Australian who works in mathematics education, and who in a book titled Real Girls Don't do Maths provocatively argued:

Mathematics is not used as a selection device simply because it is useful, but rather the reverse.
(Willis 1989:35)

Hence she claims mathematics is not useful because it is useful; it is not useful because of what it helps you do. Mathematics is useful because it is organised and conceptualised in such a way that certain people can't do it. Now that is certainly a challenging and controversial claim-one you might take issue with or one you believe is right on. It certainly is an issue whichever way you look at it.

So, what makes something an issue in mathematics teaching? Let me give you one classic example—setting by ability. Why is setting an issue? Well, the answer to this will be obvious if you spend any time looking at the make-up of the different ability groups in any comprehensive school. Furthermore, setting is a mechanism for legitimising the very process of differential privileging of cultural background. We know from a great deal of international research that setting does not actually have an effect on raising overall standards-though the likelihood is that you will not accept my argument here. You might prefer the argument put to me by Alan Brown, a head of mathematics:

I think it's probably only fair to say that I have a fairly high degree of scepticism about a lot of the qualitative and quantitative research. I think it tends to be done by people who, with the best will in the world, have an axe to grind. The people who've argued about mixed ability tend to be people who've moved out of the classroom.
(Gates 2000:299)
Which puts me firmly in my place (and as it happens all the other contributors to this book too). I don't intend here to argue the merits or demerits of this issue, because that type of controversy and disparity of perspective is in the nature of an issue-and because I don't expect any chapter in any book to radically alter deeply held beliefs on its own. What I hope this book does is to raise issues and expose the underlying values that need to be confronted. What is important is that you recognise the controversies, and enter into them with an awareness of what is at stake. When it comes down to it, it is a matter of whose side you are on and that is for you to decide by considering your own values. Of course, this book, like any other similar book, is deeply saturated with values. The difference here, that all contributors have striven to ensure, is that our values are somewhat more explicit and transparent than many others you might read. This does not make the book less useful, on the contrary. Because you will be able to ascertain from whence each writer is coming, you will be better placed to consider, evaluate and position yourself.

Actually, all the contributors to this book are qualified classroom teachers, trained and experienced either at primary or at secondary level, with decades of classroom experience between them, and they have something important to say about that experience. For half of the contributors in this book, the chapters they have written also derive directly from their own doctoral research studies.

## About the issues in this book

Of course, any book represents someone's selection of material to include, and this book is no exception. We are not suggesting the issues we have written about are the only issues that are currently important in mathematics teaching, but they are the issues that engage the twenty-one of us who have contributed to the book. The importance of our selection is not only in its content, but is in the way that the politics of mathematics teaching is being made explicit throughout the book. We discuss a wide sweep of issues, and provide sources and resources for those wanting to know more. In an overview of many of the contemporary issues in mathematics education, Peter Bailey concludes that teachers of mathematics can play a crucial role in making the world a fairer place' (Bailey 1999:84), and that is our starting point-which surely must be of interest to us all.

Our focus in this book is on teaching mathematics in schools, but no one interested in children's learning of mathematics can overlook the significant issue of the difference between doing mathematics in schools and being mathematical outside of school. This is a fast-growing area of study-and is something that teachers ought not ignore (though it will have to remain as the subject of another
book). The number of studies of adults and children doing mathematics outside of school show that lack of competence in one context (usually school) is no indicator of lack of competence in the other. See for example the work of Mary Harris, (Harris 1991), Gelsa Knijnik (Knijnik 1996), Madelena Santos (Santos and Matos 1999), Jean Lave (Lave 1988) and Terezinha Nunes (Nunes, Schliemann and Carraher 1993). It is perhaps interesting that so many of these studies are carried out by female researchers in mathematics education. One small way of marking the contribution that women make to the academic literature is to identify gender through the use of first as well as second names. This is a practice utilised throughout this book.

## The social context of mathematics education

The chapters forming Part II-Issues in the Social Context of Mathematics Education, really cover a broad sweep of contemporary issues at the heart of current debates about the teaching and learning of mathematics; social justice (Chapter 2), language, social class and social inclusion (Chapter 3), gender (Chapter 4) and ethnicity (Chapter 5) all have a central place in these debates, yet have not all had a central place in mainstream literature on mathematics education. However, in addition to these broad, macro-issues, there are chapters here that look more into the personal dimension of mathematics teaching-pupils' perspectives and emotions on their learning (Chapter 6), one's own values as a teacher (Chapter 7) and the ways in which one's values influence one's teaching style in the context of the current teaching of numeracy (Chapter 8).

In Chapter 2, 'Mathematics Teaching in the Real World', Tony Cotton discusses the relationship between mathematics education and social justice, an area in which Tony's work is well known. Tony's work-and his discussion of it in this chapter-is important because it helps us understand the nature of social exclusion and its manifestation in and through mathematics education. Furthermore, Tony offers some blueprints for strategies that we might incorporate into our teaching to try to make a difference. What comes across in Tony's chapter-and this issue is picked up at several other points in this book-is that pupils have a view, they have a right to a view, and a right to be listened to, and that no school, department or teacher can claim to be socially just without listening to and acting on those views. Of course, for this to be successful, effective channels of communication need to be established between the dominant voice of the teachers and the oftsuppressed voices of the pupils.

Robyn Zevenbergen problematises the idea that language is merely a means of communication. In Chapter 3, 'Language, social class and underachievement in school mathematics', she claims that the language forms and strategies we use in mathematics teaching differentially favour some social groups over others. What often passes as a lack of 'ability' in mathematics or a lack of understanding is more likely to be a result of differences in language use between the school and the home context. What makes this doubly complex is how the school acts as if it is the protector of appropriate behaviour and communication rather than just one arbitrary context.

Carrie Paechter's writing is often challenging but inspiring, and in Chapter 4, 'Gender, Reason and Emotion in Secondary Mathematics Classrooms' she looks beyond the issue of differential mathematics attainment between genders. This has been a rather unhelpful diversion in recent years, something that is now becoming clearer as girls begin to race ahead in the achievement stakes. For while girls are showing a tendency to be more successful they show little interest in studying mathematics (or indeed the sciences) beyond school. To understand this issue more fully, we need to adopt concepts and perspectives that get below the surface features. Carrie does this by looking back over the development of rational thought, and suggests that, empirically, girls and boys approach mathematics, and therefore rationality, rather differently; boys seem to have a predilection for decontextualised rationality while girls seem to be more comfortable basing their decisions on emotional morality. Now we need to be clear about what is being claimed here. What is not being argued is that there is something 'essentially' rational about the male, and something 'essentially' emotive about the female. Such a position is usually called essentialism-or 'the belief that individuals have a unique essence that transcends historical and cultural boundaries' (Gale and Densmore 2000:128). For example this would include the belief that women have an inherent capacity as carers and nurturers, thus being subjected to a 'stereotyping upheld by the fact that in practice women do more actual caring and feeding of children than men do' (Gale and Densmore $2000: 128)$. So rather than accepting stereotypical roles as essences of gender, we have instead the social construction of identity, with the result that some groups or sections of society are forced, encouraged or constrained to adopt preferred ways of behaving, interacting and responding to challenges that go on to be undervalued by the education system. Hence, the issue of gender becomes not merely who does best, but how, as teachers of mathematics, we influence and constrain some pupils more than others through our classroom practices and our curriculum.

In Chapter 5, 'Ethnicity and Mathematics Education' Derek Kassem describes how pupils' ethnic background places them at a disadvantage when success at mathematics is being distributed and he pulls no punches. He reminds us that while, in discussions about the mathematics curriculum over recent years, the issue of ethnicity might not have had the same high profile as gender, we are equally culpable here of some subtle discriminatory practices. An issue that Derek raises-which again permeates this book-is the importance of pupils' cultural inheritance to their attainment, especially since mathematics educational practices tend to favour one cultural tradition over others.

Jo Boaler and Dylan Wiliam explore the highly disputed issue of ability grouping from the pupils' point of view in Chapter 6. "We've still got to learn!" Students' perspectives on ability grouping and mathematics achievement'. This is a story that needs to be heard, and heard more widely because it lays a most significant dimension onto the usually polarised debate on setting or target grouping in mathematics teaching. There is currently considerable research underway in the UK into the efficacy of the practice of segregation by 'ability' - a practice that is not common elsewhere in the world (as we see later in Chapter 18). Jo and Dylan
identify the considerable disadvantages that such segregation brings and which beg us all at least to reconsider the practice.

Research evidence suggesting there are disadvantages in grouping pupils by ability for mathematics teaching contradicts the widely held perception that ability discrimination enhances attainment. Indeed, it is this perception that leads to the almost universal application of ability grouping in mathematics at secondary level in the UK. The widely held professional logic is that in 'ability groups' teachers can better match work to the ability levels of pupils since the spread of ability is more narrow than would otherwise be the case in all attainment groups. One argument which may explain findings that setting restricts attainment can be found in the paradoxical claim that when teaching in setted groups, teachers actually respond less to pupils individual needs than they do when teaching all attainment classes. The significance of this draws Jo and Dylan to conclude that 'the traditional British concern with ensuring that some of the ablest students reach the highest possible standards appears to have resulted in a situation in which the vast majority of students achieve well below their potential'.

Grouping by ability or attainment (setting) has not been part of the culture of primary schools in the UK for a long time, however it has recently been reported that 'nearly two-thirds of primary schools appear to be adopting ability grouping in response to government pressure for higher educational standards' (TES 1999b). The assumption here is that educational standards may be raised by utilising some form of grouping of pupils by ability—becoming termed 'target grouping' when used in primary schools. However, Anita Straker, Director of the National Numeracy Strategy, speaking at the annual conference of the Association of Teachers of Mathematics in 1999, advised caution, telling schools not to put pupils in ability groups due to the inconclusive nature of the evidence (TES 1999a).

A review of the literature on grouping by ability across all school subjects, carried out by NFER has suggested that 'it is possible to identify a general trend which suggests that setting, compared with mixed ability teaching, has no significant effect on pupil achievement' (Sukhnanden 1999:6-7). There are some interesting findings from the research. In comparison to pupils in high ability streams, those in low ability streams are provided with a lower quality of instruction and with teachers who are less experienced and less qualified. Furthermore, placement in low ability groups reinforces differences relating to social class, gender and ethnicity by lowering self-concepts and attitudes (Sukhnandan and Lee 1998:11). In terms of pupil achievement, research indicates that grouping by ability has no effect on average achievement. What Jo and Dylan illustrate here, and in their work elsewhere, is that there are some very clear disadvantages for pupils across the school in the adoption of forms of ability segregation. The real question here is, can a school rightly claim to be providing an education that helps each child achieve their best while adopting some form of ability segregation? Jo and Dylan seem to be suggesting the only answer to this question is 'no!'. My argument here is that teachers do not actually know setting 'works', but can actually not conceive of doing it another way. (All is not lost, however, because Mike Ollerton discusses some strategies for inclusive teaching of mathematics in Chapter 16.) However, the practice of ability (or target) grouping in schools rests not on some systematic
analysis of efficacy or effectiveness, but actually on the underlying beliefs and values of teachers-something discussed in the next chapter.

In Chapter 7, 'What values do you teach when you teach mathematics?', Alan Bishop opens up the issue of what constitutes a 'value' and further explores how values are both explicitly and implicitly conveyed in our teaching of mathematics. He offers us some examples of where we have opportunities to make decisions about the way in which our values can be incorporated into our teaching through different types of classroom activity and teacher-pupil relationships.

Arguably, one of the most significant developments in the mathematics curriculum in recent years has been the introduction of the National Numeracy Strategy (NNS) and in Chapter 8, 'Policy, practices and principles in teaching numeracy: What makes a difference?' Mike Askew looks into the history and the chronology of the NNS. He explores the roots of the current pedagogical imperatives and locates some of these in the international comparative studies of mathematical attainment. Mike identifies that such comparisons are not always to be taken at face value and that we need to be vigilant in order not to lose sight of what might lie behind classroom strategies that could be obscured by simplistic comparisons focused on superficial forms of classroom organisations. He draws on a research study which indicates that what seems to make teachers effective in the teaching of numeracy is being able to help pupils construct a rich interconnecting web of mathematical ideas. Nothing really surprising here perhaps, but it does suggest than many of the teaching strategies which result in fragmentation of mathematics might be rather less effective than their widespread use might suggest.

## Teaching and learning mathematics

Part III-Issues in the teaching and learning of mathematics, moves on to consider some of the issues in the practical day-to-day work of teachers. Kevin Delaney looks into the thorny issue of the use of resources in Chapter 9, 'Teaching mathematics resourcefully'. The struggle to find the right language here illustrates perhaps some of the complexity in the issue. The terms: teaching aids, learning aids, apparatus, equipment, practical equipment, manipulatives, etc. all seem to be widely used, yet how, why and when they are used perhaps received less attention. Perhaps the most correct term-'physical embodiment' hardly rolls off the tongue, yet does suggest that some form of representation of some mathematics can rest in or be embodied in some physical entity, albeit temporarily. Kevin opens up this issue and illustrates where the real issues lie by looking first at the processes of learning in the classroom and secondly at the practice of teaching by considering the proposals in the National Numeracy Strategy—which Kevin suggests need to be carefully scrutinised and critiqued.

Teachers of mathematics might learn a great deal about this issue by looking at how adults, children and babies learn outside the classroom. Babies seem to do most of their learning before they even get to school. Dave Hewitt gives a very passionate description of his own baby girl learning mathematics in 'The first
two years' (Hewitt, 2000) which suggests what active learners very young children are and how objects are used to learn then cast aside when they have no further utility. I have to say my own experience with my own children has made me realise how impotent the 'teacher' often is when real learning is taking placeand how they seem to be absent too with learning coming as the result of creative synergy between learner and object.
'Dealing with misconceptions in mathematics' is the topic Malcolm Swan addresses in Chapter 10. He makes a clear distinction between mistakes and errors on the one hand, and misconceptions and alternative conceptions on the other. Malcolm stresses how the widespread nature of misconceptions across both space and time ought to underline for us all the conceptual roots of misconceptions. He locates his discussion of the issue in the process of coming to understand mathematics and draws on Vygotskian ideas. As one comes to expect from Malcolm, the discussion becomes very practical by looking at the implications for classroom strategies and offers some activities that help teachers and pupils to confront and address misconceptions. The use of this language is important. Malcolm cautions against the dangerous over-simplification in 'official' government statements, which require new teachers to be taught how to prevent misconceptions arising. Assuming that misconceptions can be simply remedied or avoided has to be a misconception in itself.

No discussion of issues in mathematics teaching can justifiably overlook the impact of new technology and Janet Ainley looks at how computers need to influence teaching styles in Chapter 11, 'Adjusting to the newcomer: Roles for the computer in mathematics classrooms'. A major consideration for Janet is not just how computers can be used, or what software or hardware there is available, but how one interacts with a computer both as a learner and as a teacher. She offers three different roles a computer can play in the classroom-each of which have their own rationales, objectives and patterns of interaction. A greater clarity in the diversity between these roles can only help the classroom teacher more effectively utilise the potential that computer technology can bring. One aspect that Janet touches on is the way in which pupils' differential access to computer technology can influence their engagement and eventual success in the classroom. There is a clear issue of social exclusion here that needs confronting unless we are to unwittingly privilege those children from families where access to computers is as accepted as access to microwaves, digital camcorders and DVD players at the expense of children from less advantaged backgrounds—large families, small overcrowded properties or even bed and breakfast accommodation.

While for teachers the computer might be the newcomer, the mathematics textbook has been around for decades. More's the pity some would say, and Paul Dowling offers a fresh perspective on them in Chapter 12, 'Reading mathematics texts'. There are some difficult ideas in this chapter-and it might be easy to consider that Paul 'doth protest too much'. However, the real issue here is looking at the everyday, ordinary, taken-for-granted and reading it differently. Looking at things differently has been, of course, one major way in which mathematics itself has developed over the centuries. What Paul's work helps me to do is to see more clearly why many children can't see the point in much of the mathematics we
give them to do in textbooks. While much of it is pretty daft and pointless, this is because the real purpose is hidden, obscured and unrecognised. The purpose, as agued by Paul, seems to be to position pupils differentially within the social hierarchy.

One dimension of children's educational experience that seems to be largely overlooked in teaching mathematics is addressed by Jan Winter in Chapter 13, 'Personal, spiritual, moral, social and cultural issues in teaching mathematics'. Jan discusses each of these five elements in terms of what they might mean in the context of teaching mathematics, and offers some suggestions and strategies for the classroom. This is a very important chapter inasmuch as it places mathematics teaching more closely at the centre of children's spiritual, moral, social, cultural and personal education than it currently lies.

## Assessing mathematics

In Part IV-Issues in the assessment of mathematics, three contributors look into some of the more controversial aspects of assessing pupils' level of attainment. Anne Watson writes about 'Making Judgements about Pupils' Mathematics' in Chapter 14. Anne is particularly interested in the nature of the judgements that teachers make about their pupils' mathematics. These judgements are important because they can greatly influence much of what goes on in the classroom, and furthermore can influence significant key decisions made on behalf of the pupilsuch as examination entry and ability group allocation for example. This chapter then needs to be read in the context of a number of the chapters in Part 2. Anne's concern is over the way there might be a tendency for decisions to be made about pupils based not just on evidence, but also on teachers' expectations and assumptions about what a particular child ought to be able to do. There are some very good reasons for valuing and utilising teacher assessments of pupil capability-and Anne lists six, yet there are a number of reasons to be cautious. Teacher assessment has to be based on evidence, gleaned from what pupils say, write or do, yet none of these are unproblematic since each depends to some extent on communication between participants who may have differing cultural backgrounds.

One of these controversial areas forms the central plank of Candia Morgan's issue in Chapter 15, 'The place of pupil writing in learning, teaching and assessing mathematics'. Candia describes the ways in which writing contributes to and potentially enhances learning. Yet this is not without some disadvantages and conflicts especially in the social class differences in the incorporation of writing into communicative strategies. Unlike speaking, where children become largely fluent around the age of three, children do not just pick up the skill of writing and it takes many years to hone one's writing capability. Writing leaves a permanent record that can be held up for scrutiny and analysis; the interactive dynamic is stripped away, leaving the result heavily dependent on conventions and acceptable styles of grammar, syntax and semantics. The task of the teacher of mathematics then is to balance the positive and negative aspects of writing to learn.

The final issue in assessment of pupils' mathematical ability looks at the official Standard Assessment Tests (SATs) and Barry Cooper looks at the underlying bias
inherent in such tests in Chapter 16, 'Social class and 'real-life' mathematics assessments'. The main issue in Barry's chapter is the role of context in the framing and interpretation of mathematical test questions. For many years it has been assumed that mathematical tasks and problems become more meaningful for the learner, and by implication more easily internalised, if they are placed in some 'real-life' context. Now, many of us know that most of these contexts are contrived and largely meaningless, and assume that this state of affairs is basically harmless-even though it loses opportunities for making mathematics really meaningful. What Barry's work highlights is that this process of fabrication of context is far from harmless-but has the effect of further disadvantaging those pupils from more disadvantaged social groups, and Barry shows us in some detail how this comes about.

Now, there is a game being played here. We claim most strongly that mathematics is a real-world discipline and that it is useful to solve real-world problems; we construct real-world problems for pupils to solve in the comfort of the classroom, away from the real realities of everyday life. Yet in the problems we offer, pupils are only supposed-or better, only allowed-to take the reality so far. This is the goldilocks principle of the reality behind mathematical problems-not too much, not too little-just enough. You have to learn your place and your role, whether you are a teacher or a pupil. Pupils have to learn to be a learner, but more than this, they have to learn the limitation of what they have to learn. In achieving this, children learn the rules of social order-what Basil Bernstein called 'hierarchical rules'. Basil Bernstein argued that the way in which school practices organise the educational experiences for different children separates the local from the lesslocal. The children who tend to fall through this net are often those children from lower working class families who become constrained into local, contextdependent skills.

## The culture of teaching mathematics

Making mathematics more real gets taken up in the next section, Part VIssues in the Culture of Mathematics Teaching. Mike Ollerton opens with Chapter 17 on 'Inclusion, learning and teaching mathematics: Beliefs and values'. Here Mike returns to the issue raised earlier by Jo and Dylan, but offers strategies for teaching in a way that is inclusive rather than exclusive. Mike is one of several contributors to this book who have actually taught mathematics throughout the secondary phase in all attainment groups, and who therefore know how to make it work. Of course it's not easy—but then again it's not hard either. Mike's argument is that the persistence of exclusive teaching (that is grouping pupils by ability) depends on what you believe in and what you value. Conversely, teaching inclusively demands a determination and a commitment to put inclusive strategies into practice.

The importance of one's belief systems gets taken up again by Paul Ernest in Chapter 18, 'Critical mathematics education'. Paul begins by discussing the nature of mathematics-or more accurately the natures of different mathematics-and shows that while these might rest upon different philosophical traditions, they also reflect different ideological positions. A fundamental issue here is whether

