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2nd Edition

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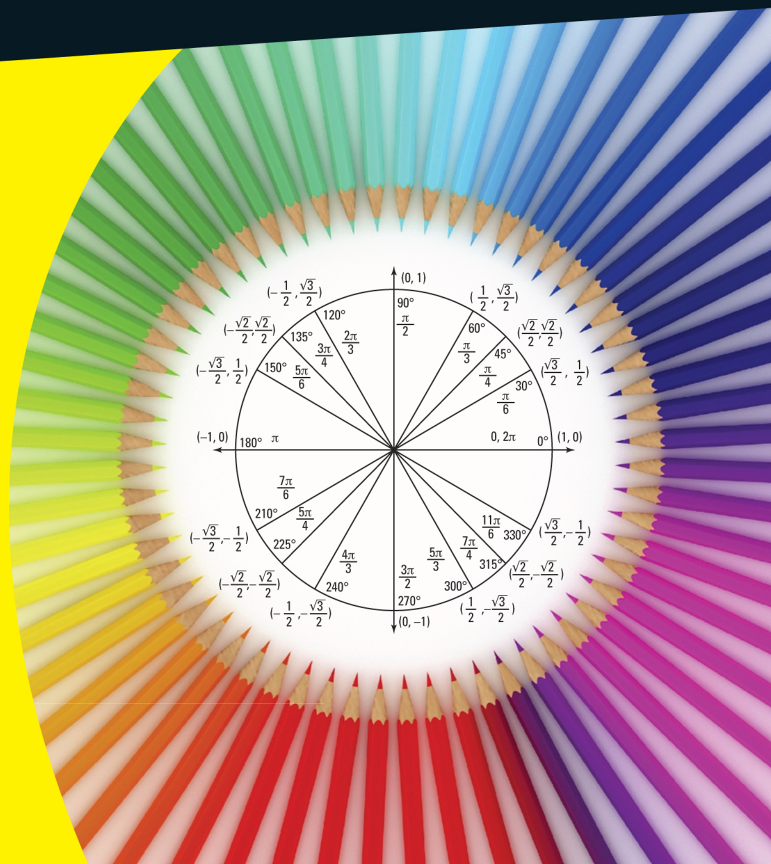

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Pre-Calculus FOR DUMMIES®

2ND EDITION

by Yang Kuang and Elleyne Kase

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About the Authors

Yang Kuang is a proud father of three, Youny, Foris, and Belany. He has been a professor of mathematics at Arizona State University since 1988. He entered the University of Science and Technology of China in 1980, at age 14, went to the University of Oxford in 1984 for M.Phil study, completed a PhD in mathematics in 1988, and became a mathematics professor at ASU at age 22.

Yang Kuang has published over 130 refereed publications and authored and edited many books. He has directed a dozen PhD dissertations and many funded research projects in mathematical biology and medicine. He has founded and continues to edit the *Journal of Mathematical Biosciences and Engineering.* He has given mathematical research talks all over the world and organized many international mathematical biology and medicine meetings. He is well known for his pioneering work in the application of delay differential equation in models of biology and medicine as well as his ongoing work in establishing a solid foundation and framework to build accurate and dynamic rich population models that explicitly include resource quality dynamics.

His more recent interest is to formulate scientifically well-grounded and computationally tractable mathematical models to describe the rich and intriguing dynamics of various within-host diseases and their treatments, including various types of cancer, diabetic diseases, and virus-induced diseases such influenza and HBV. He hopes that these models will help speed up the much-needed personalized medicine development.

Yang Kuang teaches and uses calculus all the time. He is keenly aware of the many challenges facing students and the instructors. He hopes this book will be helpful to many of those who want to confront those challenges.

Elleyne Kase is a multi-disciplined artist and graphics professional whose creative expertise has been applied commercially in many different avenues, such as fine art designs for the commercial interior design industry, Wayfinding programs for large hospital systems, 3D displays, website design, and national brand identity. A bachelor's degree in graphic design from San Jose State University began her professional career in the San Francisco Bay Area, which continued in San Diego design agencies as her family grew.

Book design, a favorite of hers, began early on in her career and has now been applied to two of her own exam prep publications, *Visual Quick Notes for Life Insurance* and *Visual Quick Notes for Life and Health.* Both use her mind mapping system, an evolution of her study on how the mind absorbs data, to explain simple terms and concepts in a visual matrix of graphic elements that improve memory retention.

Authors' Acknowledgments

I would like to thank Elleyne Kase again for inviting me to take part in this interesting and timely book project and all the people involved in the process. It was a joy and rewarding experience working with all of you. Of course, I am forever grateful to my wife, Aijun, for her beautiful creation of and the great attention devoted to our family so that I can have time to do my work, including carrying out this time-consuming project. —Yang Kuang

Throughout my varied career I have had the great pleasure of working with professionals of many disciplines. The *For Dummies* series combines the skills of many very talented people. I want to express my gratitude to Bill Gladstone, who gave me this rare opportunity to participate; to the editors at Wiley, Tracy Boggier, Tim Gallan, and Caitie Copple, for their professionalism, understanding, and skill at bringing such a complex product to market; and to Yang Kuang, who juggled his demanding teaching schedule to accommodate this project. Above all, I want to thank my dear mother who has always supported me in all my varied interests, and to my three children, Seth, Shane, and Mimi, who have grown to become wonderful, nurturing, caring adults and my best friends. —Elleyne Kase

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We're proud of this book; please send us your comments at `http://dummies.custhelp.com`. For other comments, please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002.

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Contents at a Glance

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Introduction

Welcome to *Pre-Calculus For Dummies,* a nondiscriminatory, equal-opportunity book. You're welcome to participate whether you are a genius or (like us) you need a recipe to make ice. Don't let the title throw you. If you've gotten this far in math, in no way are you a dummy! You may be reading this book for a few perfectly great reasons. Maybe you need a reference book that you can actually *understand* (we've never met a pre-calc text that we liked). Perhaps your guidance counselor told you that taking pre-calc would look good on your college application, but you couldn't care less about the subject and just want to get a good grade. Or, maybe you're contemplating buying this book and you want to check us out to see if we're a good match (not unlike looking at your blind date through the window before you walk into the restaurant). Regardless of why you opened up this book, it will help you navigate the tricky path that is pre-calc.

You may also be wondering, "When will I ever really use pre-calculus?" You're not alone. Some of our students have referred to it as pre-calc-uselessness. Well, they quickly found out how wrong they were. The concepts throughout this book are used in many real-world applications.

This book has one goal and one goal only — to teach you pre-calculus in as painless a way as possible. If you thought that you could never tackle this subject and you end up with a decent grade in this class, would you mind sending us a letter? E-mail's good too. We love to hear our students' success stories!

About This Book

This book is not necessarily meant to be read from beginning to end. It's structured in a way that you can flip to a particular chapter and get your needs met (those pesky needs we all have). Sometimes we may tell you to look in another chapter to get a more in-depth explanation, but we have tried to allow each chapter to stand on its own.

All vocabulary is mathematically correct and clear. We have taken liberties at some points throughout this book to make the language more approachable and likable. It's just more fun that way.

Pre-calc is its own special math topic. You see, some states, like California, don't have any set standards that students need to learn to officially master pre-calculus. As a result, the subject of pre-calc varies between districts, schools, and individual teachers. Because we don't know what your teacher is going to want you to take away from this course, we've covered pretty much every concept in pre-calc. We may have covered areas that you'll never be required to tackle. That's okay. Just use this book according to your individual needs.

If you use this book only to prop open a door or as a bug smoosher, you won't get what you need from it. We suggest two alternatives:

- Look up only what you need to know when you need to know it. This book is handy for this. Use the index, the table of contents, or better yet, the quickie contents at a glance found in the very front of this book to find what you need.
- Start at the beginning and read through the book, chapter by chapter. This approach is a good way to tackle this subject because the topics sometimes build on previous ones. Even if you're a math god and you want to skim through a section that you feel you know, you may be reminded of something that you forgot. We recommend starting at the beginning and slowly working your way through the material. The more practice you have, the better.

Conventions Used in This Book

For consistency and ability to navigate easily, this book uses the following conventions:

- Math terms are *italicized* to indicate their introduction and to help you find their definition.
- Variables are also *italicized* to distinguish them from common letters.
- The step-by-step problems are always **bold** to help you identify them more easily.
- The symbol for imaginary numbers is a lowercase *i*.

Foolish Assumptions

We can't assume that just because we absolutely love math that you share the same enthusiasm for the subject. We can assume, however, that you opened this book for a reason: You need a refresher on the subject, need to learn it for the first time, are trying to relearn it for college, or have to help your kid understand it at home. We can also assume that you have been exposed, at least in part, to many of the concepts found in this subject because pre-calc really takes geometry and Algebra II concepts to the next level.

We also assume that you're willing to do some work. Although pre-calculus isn't the end-all to math courses out there, it's still a higher level math course. You're going to have to work a bit, but you knew that, didn't you?

We also are pretty sure that you're an adventurous soul and have chosen to take this class, because pre-calculus is not necessarily a required high school course (in most U.S. high schools, anyway). Maybe it's because you love math like we do, or you have nothing better to do with your life, again like us, or because the course will enhance your college application. Obviously, you managed to get through some pretty complex concepts in geometry and Algebra II. We can assume that if you made it this far, you'll make it even farther. We're going to help!

How This Book Is Organized

This book is broken down into four sections dealing with the most frequently taught and studied concepts in pre-calc.

Part I: Set It Up, Solve It, Graph It

The chapters in Part I begin with a review of material you should already know from Algebra II. Then we review real numbers and how to operate with them. From there we cover functions, including polynomial, rational, exponential, and logarithmic functions and graphing them, solving them, and performing operations on them.

Part II: The Essentials of Trigonometry

The chapters in Part II begin with a review of angles, right triangles, and trig ratios. Then we build the glorious unit circle. Graphing trig functions may or may not be a review, depending on the Algebra II course you've taken, so we show you how to graph the parent graph of the six basic trig functions and then explain how to transform those graphs to get to the more complicated ones.

This part also covers the harder formulas and identities for trig functions, breaking them down methodically so you can internalize each identity and truly understand them. We then move right along into simplifying trig expressions and solving for an unknown variable using those formulas and identities. Finally, this part covers how to solve triangles that are not right triangles using the Law of Sines and Law of Cosines.

Part III: Analytic Geometry and System Solving

Part III covers a multitude of pre-calc topics. It begins with understanding complex numbers and how to perform operations with them. Next comes polar-coordinate graphing and finally conics. Systems of equations live in this part, as do sequences and series and binomial expansion. Finally, this part concludes with calculus and the study of limits and continuity of functions.

Part IV: The Part of Tens

After you've covered everything up to this point in the book, you may be eyeing the next big math challenge: calculus. (And if you decide to stop with pre-calc, that's okay, too.) But before you head off for even more complex concepts, you need to do two things: Pick up some good math habits to take into calculus and break any bad habits you've developed along the way. This part helps you with those tasks. Both ends of this spectrum are critical for success because the problems get longer and teachers' patience for algebra errors gets shorter.

Icons Used in This Book

Throughout the book you'll find little drawings (called *icons*) that are meant to draw your attention to something important or interesting to know.

PRE-CALC RULES

Pre-calc rules are the basic rules of pre-calc. The rules marked with this icon must be observed every time to make problems come out correctly.

MATH MUMBO JUMBO

Math Mumbo Jumbo alerts you to information that is helpful but not required to gain full knowledge of the concept in that section.

TIP

We love Tips! When you see this icon, you know that it points out a way to make your life a lot easier. Easier is good.

You'll see this icon when we mention an old idea that you should never forget. We use it when we want you to recall a previously learned concept or a concept from a lower math course.

Think of Warnings as big stop signs. The presence of this icon alerts you to common errors or points out something that can be a bit tricky.

Where to Go from Here

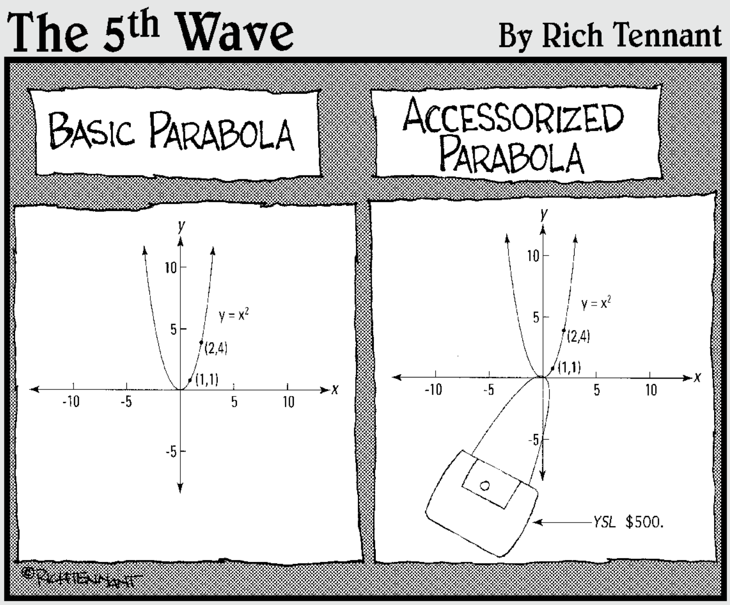
If you have a really firm background in basic algebra skills, feel free to skip Chapter 1 and head right over to Chapter 2. If you want a brush-up, we suggest reading Chapter 1. In fact, everything in Chapter 2 is also a review except interval notation. So if you're really impatient or are a math genius, ignore everything until interval notation in Chapter 2. As you work through this book, keep in mind that many concepts in pre-calc are take-offs from Algebra II, so don't make the mistake of completely skipping chapters because they sound familiar. They may sound familiar but are likely to include some brand-spanking-new material. We also didn't sit next to you when you took Algebra II, so we can't be sure what your teacher covered. So here's a brief list of sections that may sound familiar but include new concepts that you should pay attention to:

- Translating common functions
- Solving polynomials
- All the trig information
- Complex numbers
- Matrices

So where do you go from here? You get going straight into pre-calc! Good luck.

Part I

Set It Up, Solve It, Graph It



In this part . . .

A major goal of pre-calculus is to bring the big ideas of algebra to the surface and sharpen the skills most needed for calculus. This part pulls together and expands on those algebra concepts. Perhaps most importantly, it identifies the most common mistakes students make in algebra so you can conquer those before moving on to higher-level concepts.

The chapters in Part I move through a review of working with real numbers, including the ever-elusive radicals. From there, we review functions — from how to graph them by transforming their parent graphs to how to perform operations on them. Then we move on to polynomials and review how to solve polynomials using common techniques, including factoring, completing the square, and the quadratic formula. We also explain how to graph complex polynomial and rational functions. Lastly, we show you how to deal with exponential and logarithmic functions.

Chapter 1

Pre-Pre-Calculus

In This Chapter

- Refreshing your memory on numbers and variables
- Accepting the importance of graphing
- Preparing for pre-calculus by grabbing a graphing calculator

Pre-calculus is the bridge (or purgatory?) between Algebra II and calculus. In its scope, you review concepts you've seen before in math, but then you quickly build on them. You see some brand-new ideas, but even those build on the material you've seen before; the main difference is that the problems get much harder (for example, going from systems to nonlinear systems). You keep on building until the end of the course, which doubles as the beginning of calculus. Have no fear! We're here to help you cross the bridge (toll free).

Because you've probably already taken Algebra I, Algebra II, and geometry, we assume throughout this book that you already know how to do certain things. Just to make sure, though, we address each of them in this chapter in a little more detail before we move on to the pre-calculus that is pre-calculus.

If we cover any topic in this chapter that you're not familiar with, don't remember how to do, or don't feel comfortable doing, we suggest that you pick up another *For Dummies* math book and start there. Don't feel like a failure in math if you need to do that. Even the pros have to look up things from time to time. These books can be like encyclopedias or the Internet — if you don't know the material, you look it up and get going from there.

Pre-Calculus: An Overview

Don't you just love movie previews and trailers? Some people show up early to movies just to see what's coming out in the future. Well, consider this section a trailer that you see a couple months before the *Pre-Calculus For*

Dummies movie comes out! (We wonder who will play us in the movie.) In the following list, we present some material you've learned before in math, and then we give you some examples of where pre-calculus will take you next:

- **Algebra I and II:** Dealing with real numbers and solving equations and inequalities.

  Pre-calculus: Expressing inequalities in a new way called *interval notation.*

  Before, your solutions to inequalities were given as set notation. For example, one solution might look like $x > 4$. In pre-calc, you express this solution as an interval: $(4,\infty)$. (For more, see Chapter 2.)

- **Geometry:** Solving right triangles, where all sides are positive.

  Pre-calculus: Solving non-right triangles, where the sides aren't necessarily always positive.

  You've learned that a length can never be negative. Well, in pre-calc you use negative numbers for sides of triangles to show where these triangles lie in the coordinate plane (they can be in any of the four quadrants).

- **Geometry/trigonometry:** Using the Pythagorean theorem to find the length of a triangle's sides.

  Pre-calculus: Organizing some frequently used angles and their trig function values into one nice, neat package known as the *unit circle* (see Part II).

  In this book, we give you a handy shortcut to finding the sides of triangles, which is an even handier shortcut to finding the trig values for the angles in those triangles.

- **Algebra I and II:** Graphing equations on a coordinate plane.

  Pre-calculus: Graphing in a brand-new way, with the polar coordinate system (see Chapter 11).

  Say goodbye to the good old days of graphing on the Cartesian coordinate plane. You have a new way to graph, and it involves goin' round in circles. We're not trying to make you dizzy; actually, polar coordinates can make you some pretty pictures.

- **Algebra II:** Dealing with imaginary numbers.

  Pre-calculus: Adding, subtracting, multiplying, and dividing complex numbers gets boring when the complex numbers are in rectangular form ($A + Bi$). In pre-calc, you become familiar with something new called *polar form* and use that to find solutions to equations that you didn't even know existed.

All the Number Basics (No, Not How to Count Them!)

When entering pre-calculus, you should be comfy with sets of numbers (natural, integer, rational, and so on). By this point in your math career, you should also know how to perform operations with numbers. We quickly review these concepts in this section. Also, certain properties hold true for all sets of numbers; some math teachers may want you to know them by name, so we review these in this section, too.

The multitude of number types: Terms to know

Dorky mathematicians love to name things simply because they can; it makes them feel special. In this spirit, mathematicians attached names to many sets of numbers to set them apart and cement their places in math students' heads for all time:

- **The set of natural or counting numbers: {1, 2, 3 . . .}**. Notice that the set of natural numbers doesn't include 0.
- **The set of whole numbers: {0, 1, 2, 3 . . .}**. The set of whole numbers does include the number 0, however.
- **The set of integers: {. . . –3, –2, –1, 0, 1, 2, 3 . . .}**. The set of integers includes positives, negatives, and 0.

  Dealing with integers is like dealing with money: Think of positives as having it and negatives as owing it. This becomes important when operating on numbers (see the next section).
- **The set of rational numbers: the numbers that can be expressed as a fraction where the numerator and the denominator are both integers.** The word *rational* comes from the idea of a ratio (fraction or division) of two integers.

  Examples of rational numbers include (but in no way are limited to) 1/5, –7/2, and 0.23. If you look at any rational number in decimal form, you notice that the decimal either stops or repeats.

  Adding and subtracting fractions is all about finding a common denominator. And roots must be like terms in order to add and subtract them.

- **The set of irrational numbers: all numbers that can't be expressed as fractions.** Examples of irrational numbers include $\sqrt{2}, \sqrt{21}$, and π.
- **The set of all real numbers: all the sets of numbers previously discussed.** For examples of a real number, think of a number . . . any number. Whatever it is, it's real. Any number from the previous bullets works as an example. The numbers that aren't real numbers are imaginary.

  Like telemarketers and pop-up ads on the Net, real numbers are everywhere; you can't get away from them — not even in pre-calculus. Why? Because they include all numbers except the following:

  - **A fraction with a zero as the denominator:** Such numbers don't exist and are called *undefined.*
  - **The square root of a negative number:** These numbers are called *complex numbers* (see Chapter 11).
  - **Infinity:** Infinity is a concept, not an actual number.
- **The set of imaginary numbers: square roots of negative numbers.** Imaginary numbers have an imaginary unit, like i, $4i$, and $-2i$. Imaginary numbers used to be made-up numbers, but mathematicians soon realized that these numbers pop up in the real world. We still call them imaginary because they're square roots of negative numbers, but they do exist. The imaginary unit is defined as $i = \sqrt{-1}$. (For more on these numbers, head to Chapter 11.)
- **The set of complex numbers: the sum and difference of a real number and an imaginary number.** Complex numbers appear like these examples: $3 + 2i$, $2 - \sqrt{2i}$, and $4 - \frac{2}{3}i$. However, they also cover all the previous lists, including the real numbers (3 is the same thing as $3 + 0i$) and imaginary numbers ($2i$ is the same thing as $0 + 2i$).

  The set of complex numbers is the most complete set of numbers in the math vocabulary, because it includes real numbers (any number you can possibly think of), imaginary numbers (i), and any combination of the two.

The fundamental operations you can perform on numbers

From positives and negatives to fractions, decimals, and square roots, you should know how to perform all the basic operations on all real numbers. These operations include adding, subtracting, multiplying, dividing, taking powers of, and taking roots of numbers. The *order of operations* is the way in which you perform these operations.

TIP

The mnemonic device used most frequently to remember the order is PEMDAS, which stands for

1. **P**arentheses (and other grouping devices)
2. **E**xponents
3. **M**ultiplication and **D**ivision, whichever is first from left to right
4. **A**ddition and **S**ubtraction, whichever is first from left to right

REMEMBER

One type of operation most of our students overlook or forget to include on the previous list: the absolute value. *Absolute value* is the distance from 0 on the number line. Absolute value should be included with the parentheses step, because you have to consider what's inside the absolute-value bars first (because the bars are a grouping device). Don't forget that absolute value is always positive. Hey, even if you're walking backward, you're still walking!

The properties of numbers: Truths to remember

Remembering the properties of numbers is important because you use them consistently in pre-calc. The properties aren't often used by name in pre-calc, but you're supposed know when you need to utilize them. The following list presents the properties of numbers:

- **Reflexive property: $a = a$.** For example, $10 = 10$.
- **Symmetric property: If $a = b$, then $b = a$.** For example, if $5 + 3 = 8$, then $8 = 5 + 3$.
- **Transitive property: If $a = b$ and $b = c$, then $a = c$.** For example, if $5 + 3 = 8$ and $8 = 4 \cdot 2$, then $5 + 3 = 4 \cdot 2$.
- **Commutative property of addition: $a + b = b + a$.** For example, $2 + 3 = 3 + 2$.
- **Commutative property of multiplication: $a \cdot b = b \cdot a$.** For example, $2 \cdot 3 = 3 \cdot 2$.
- **Associative property of addition: $(a + b) + c = a + (b + c)$.** For example, $(2 + 3) + 4 = 2 + (3 + 4)$.
- **Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.** For example, $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$.
- **Additive identity: $a + 0 = a$.** For example, $-3 + 0 = -3$.

- **Multiplicative identity: $a \cdot 1 = a$.** For example, $4 \cdot 1 = 4$.
- **Additive inverse property: $a + (-a) = 0$.** For example, $2 + -2 = 0$.
- **Multiplicative inverse property: $a \cdot \frac{1}{a} = 1$.** For example, $2 \cdot \frac{1}{2} = 1$.
- **Distributive property: $a(b + c) = a \cdot b + a \cdot c$.** For example, $10(2+3) = 10 \cdot 2 + 10 \cdot 3 = 50$.
- **Multiplicative property of zero: $a \cdot 0 = 0$.** For example, $5 \cdot 0 = 0$.
- **Zero-product property: If $a \cdot b = 0$, $a = 0$ or $b = 0$.** For example, if $x(x + 2) = 0$, then $x = 0$ or $x + 2 = 0$.

If you're trying to perform an operation that isn't on the previous list, then the operation probably isn't correct. After all, algebra has been around since 1600 BC, and if a property exists, someone has probably already discovered it. For example, it may look inviting to say that $10(2+3) = 10 \cdot 2 + 3 = 23$, but that's incorrect. The correct answer is $10(2+3) = 10 \cdot 2 + 10 \cdot 3 = 20 + 30 = 50$. Knowing what you *can't* do is just as important as knowing what you *can* do.

Visual Statements: When Math Follows Form with Function

Graphs are great visual tools. They're used to display what's going on in math problems, in companies, and in scientific experiments. For instance, graphs can be used to show how something (like real estate prices) changes over time. Surveys can be taken to get facts or opinions, the results of which can be displayed in a graph. Open up the newspaper on any given day and you can find a graph in there somewhere.

Hopefully the preceding paragraph answers the question of why you need to understand how to construct graphs. Even though in real life you don't walk around with graph paper and a pencil to make the decisions you face, graphing is vital in math and in other walks of life. Regardless of the absence of graph paper, graphs indeed are everywhere.

For example, when scientists go out and collect data or measure things, they arrange the data as x and y values. Typically, scientists are looking for some kind of general relationship between these two values to support their hypotheses. These values can then be graphed on a coordinate plane to show trends in data. A good scientist may show that the more you read this book, the more you understand pre-calculus! (Another scientist may show that people with longer arms have bigger feet. Boring!)

Basic terms and concepts

Graphing equations is a huge part of pre-calc, and eventually calc, so we want to review the basics of graphing before we get into the more complicated and unfamiliar graphs you see later in the book.

Although some of the graphs in pre-calc will look very familiar, some will be new — and possibly intimidating. We're here to get you familiar with these graphs so that you can study them in detail in calculus. However, the information in this chapter is mostly information that your pre-calc teacher or book assumes that you remember from Algebra II. You did pay attention then, right?

Each point on the coordinate plane on which you construct graphs — made up of the horizontal (x-) axis and the vertical (y-) axis, creating a plane of four quadrants — is called a coordinate pair (x, y), which is often referred to as a *Cartesian coordinate pair.*

MATH MUMBO JUMBO: The name *Cartesian coordinates* comes from the French mathematician and philosopher who invented all this graphing stuff, René Descartes. Descartes worked to merge algebra and Euclidean geometry (flat geometry), and his work was influential in the development of analytic geometry, calculus, and cartography.

A *relation* is a set (which can be empty, but in this book we consider only nonempty sets) of ordered pairs that can be graphed on a coordinate plane. Each relation is kind of like a computer that expresses x as input and y as output. You know you're dealing with a relation when it's set in those curly brackets (like these: { }) and has one or more points inside. For example, $R = \{(2, -1), (3, 0), (-4, 5)\}$ is a relation with three ordered pairs. Think of each point as (input, output) just like from a computer.

The *domain* of a relation is the set of all the input values, usually listed from least to greatest. The domain of set R is $\{-4, 2, 3\}$.The *range* is the set of all the output values, also often listed from least to greatest. The range of R is $\{-1, 0, 5\}$. If any value in the domain or range is repeated, you don't have to list it twice. Usually, the domain is the x-variable and the range is y.

REMEMBER: If different variables appear, such as m and n, input (domain) and output (range) usually go alphabetically, unless you're told otherwise. In this case, m would be your input/domain and n would be your output/range. But when written as a point, a relation is always (input, output).

Graphing linear equalities and inequalities

When you first figured out how to graph a line on the coordinate plane, you learned to pick domain values (x) and plug them into the equation to solve for the range (y). Then you went through the process multiple times, expressed each pair as a coordinate point, and connected the dots to make a line. Some mathematicians call this the ol' *plug-and-chug method.*

After a while of that tedious work, somebody said to you, "Hold on! You can use a shortcut." That shortcut is called *slope-intercept form,* and it's expressed as $y = mx + b$. The variable m stands for the slope of the line (see the next section), and b stands for the y-intercept (or where the line crosses the y-axis). You can change equations that aren't written in slope-intercept form by solving for y. For example, graphing $2x - 3y = 12$ requires you to subtract $2x$ from both sides first to get $-3y = -2x + 12$. Then you divide every term by –3 to get

$$y = \frac{2x}{3} - 4$$

This graph starts at –4 on the y-axis; to find the next point, you move up two and right three (using the slope). Slope is often expressed as a fraction because it's rise over run — in this case 2/3.

Inequalities are used for comparisons, which are a big part of pre-calc. They show a relationship between two expressions (we're talking greater than, less than, or equal to). Graphing inequalities starts exactly the same as graphing equalities, but at the end of the graphing process (you still put the equation in slope-intercept form and graph), you have two decisions to make:

- Is the line *dashed,* meaning $y <$ or $y >$, or is the line *solid,* meaning $y \leq$ or $y \geq$?
- Do you shade under the line — $y <$ or $y \leq$ — or do you shade above the line — $y >$ or $y \geq$? Simple inequalities (like $x < 3$) express all possible answers. For inequalities, you show all possible answers by shading the side of line that works in the original equation.

  For example, when graphing $y < 2x - 5$, you follow these steps:

  1. Start off at –5 on the y-axis and mark a point.
  2. Move up two and right one to find a second point.
  3. When connecting the dots, you produce a straight dashed line.
  4. Shade on the bottom half of the graph to show all possible points in the solution.

Gathering information from graphs

After getting you used to coordinate points and graphing equations of lines on the coordinate plane, typical math books and teachers begin to ask you questions about the points and lines that you've been graphing. The three main things you'll be asked to find are the distance between two points, the midpoint of the segment connecting two points, and the exact slope of a line that passes through two points. Away we go in the following sections!

Calculating distance

Knowing how to calculate distance by using the information from a graph comes in handy in pre-calc in a big way, so allow us to review a few things first. *Distance* is how far apart two objects, or two points, are. To find the distance, *d*, between the two points (x_1, y_1) and (x_2, y_2) on a coordinate plane, for example, use the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

You can use this equation to find the length of the segment between two points on a coordinate plane whenever the need arises. For example, to find the distance between A(–6, 4) and B(2, 1), first identify the parts: $x_1 = -6$ and $y_1 = 4$; $x_2 = 2$ and $y_2 = 1$. Plug these values into the distance formula: $d = \sqrt{(2-(-6))^2 + (1-4)^2}$. This problem simplifies to $\sqrt{73}$.

Finding the midpoint

Finding the midpoint of a segment pops up in pre-calc topics like conics (see Chapter 12).To find the midpoint of the segment connecting two points, you just average their *x* values and *y* values and express the answer as an ordered pair:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

You can use this formula to find the center of various graphs on a coordinate plane, but for now you're just finding the midpoint. You find the midpoint of the segment connecting the two points (see the previous section) by using the previous formula. This would give you

$\left(\frac{-6+2}{2}, \frac{4+1}{2} \right)$, or (–2, 5/2).

Figuring a line's slope

When you graph a linear equation, slope plays a role. The slope of a line tells how steep the line is on the coordinate plane. When you're given two points (x_1, y_1) and (x_2, y_2) and are asked to find the slope of the line between them, you use the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If you use the same two points A and B from the previous sections and plug the values into the formula, the slope is –3/8.

Positive slopes always move up and to the right or move down and to the left on the plane. Negative slopes either move down and right or up and left. (Note that if you moved the slope down and left, it would be –/–, which is really positive.) Horizontal lines have zero slope, and vertical lines have undefined slope.

If you ever get the different types of slopes confused, remember the skier on the ski-slope:

- When he's going uphill, he's doing a lot of work (+ slope).
- When he's going downhill, the hill is doing the work for him (– slope).
- When he's standing still on flat ground, he's not doing any work at all (0 slope).
- When he hits a wall (the vertical line), he's dead and he can't ski anymore (undefined slope)!

Get Yourself a Graphing Calculator

We highly recommend that you purchase a graphing calculator for pre-calculus work. Since the invention of the graphing calculator, math classes have begun to change their scope. Some teachers feel that the majority of the work should be done using the calculator. More conservative math teachers, however, won't even let you use one. Your instructor should make his views quite apparent on the first day of school. A graphing calculator does so many things for you, and even if a teacher won't allow you to use one on a test, you can almost always use one to check your work on homework problems.

Many different types of graphing calculators are available, and their individual inner workings are all different. To figure out which one to purchase, ask advice from someone who has already taken a pre-calc class, and then look around on the Internet for the best deal.

Just a hint: If you can find one with an exact/approximate mode, you'll thank us later because it will give you exact values (rather than decimal approximations), which is often what teachers are looking for.

If by chance you're allowed to use a graphing calculator, we recommend that you still do the work by hand. Then use your graphing calculator to check your work. This way you won't become dependent on technology to do work for you; someday, you may not be allowed to use one (on a college math-placement test, for example).

Many of the more theoretical concepts in this book, and in pre-calc in general, are lost when you use your graphing calculator. All you're told is, "Plug in the numbers and get the answer." Sure, you get your answer, but do you really know what the calculator did to get that answer? Nope. For this purpose, this book goes back and forth between using the calculator and doing complicated problems longhand. But whether you're allowed to use the graphing calculator or not, be smart with its use. If you plan on moving on to calculus after this course, you need to know the theory and concepts behind each topic.

We can't even begin to teach you how to use your unique graphing calculator, but the good *For Dummies* folks at Wiley supply you with entire books on the use of them, depending on the type you own. We can, however, give you some general advice on their use. Here's a list of hints that should help you use your graphing calculator:

- **Always double check that the mode in your calculator is set according to the problem you're working on.** Look for a button somewhere on the calculator that says *mode.* Depending on the brand of calculator, this button allows you to change things like degrees or radians, or $f(x)$ or $r(i)$, which we discuss in Chapter 11. For example, if you're working in degrees, you must make sure that your calculator knows that before you ask it to solve a problem. The same goes for working in radians. Some calculators have more than ten different modes to choose from. Be careful!
- **Make sure you can solve for y before you try to construct a graph.** You can graph anything in your graphing calculator as long as you can solve for y. The calculators are set up to accept only equations that have been solved for y.

  Equations that you have to solve for x often aren't true functions and aren't studied in pre-calc — except conic sections, and students generally aren't allowed to use graphing calculators for this material because it's entirely based on graphing (see Chapter 12).
- **Be aware of all the shortcut menus available to you and use as many of the calculator's functions as you can.** Typically, under your calculator's graphing menu you can find shortcuts to other mathematical concepts (like changing a decimal to a fraction, finding roots of numbers,

or entering matrices and then performing operations with them). Each brand of graphing calculator is unique, so read the manual. Shortcuts give you great ways to check your answers!

- **Type in an expression exactly the way it looks, and the calc will do the work and simplify the expression.** All graphing calculators do order of operations for you, so you don't even have to worry about the order. Just be aware that some built-in math shortcuts automatically start with grouping parentheses.

  For example, the calculator we use starts a square root off as √(, so all information we type after that is automatically inside the square root sign until we close the parentheses. For instance, $\sqrt{}(4+5)$ and $\sqrt{}(4)+5$ represent two different calculations and, therefore, two different values (3 and 7, respectively). Some smart calculators even solve the equation for you. In the near future, you probably won't even have to take a pre-calc class; the calculator will take it for you!

Okay, after working through this chapter, you're ready to take flight into pre-calculus. Good luck to you and enjoy the ride!

Chapter 2

Playing with Real Numbers

In This Chapter

- Working with equations and inequalities
- Mastering radicals and exponents

If you're taking a pre-calculus class, you've already taken Algebra I and II and survived (whew!). You may also be thinking, "I'm sure glad that's over; now I can move on to some new stuff." Although pre-calculus presents many new and wonderful ideas and techniques, these new ideas build on the solid-rock foundation of algebra. Alas, we must refresh your memory a bit and test just how sturdy your foundation is.

We assume that you have certain algebra skills down cold, but we begin this book by reviewing some of the tougher ones that become the fundamentals of pre-calculus. In this chapter, we review solving inequalities, absolute-value equations and inequalities, and radicals and rational exponents. We also introduce a new way to express solution sets: interval notation.

Solving Inequalities

By now you're familiar with equations and how to solve them. Pre-calculus teachers generally assume that you know how to solve equations, so most courses begin with inequalities. An *inequality* is a mathematical sentence indicating that two expressions aren't equal. The following symbols express inequalities:

Less than: $<$

Less than or equal to: $\leq$

Greater than: $>$

Greater than or equal to: $\geq$

Recapping inequality how-tos

Inequalities are set up and solved the same way as equations; the inequality sign doesn't change the method of solving. In fact, to solve an inequality, you treat it exactly like an equation — with one exception.

If you multiply or divide an inequality by a negative number, you must change the inequality sign to face the opposite way.

For example, if you must solve $-4x + 1 < 13$, your work follows these steps:

$$-4x < 12$$

$$x > -3$$

You first subtract 1 from both sides and then divide both sides by –4, at which point the less-than sign changes to the greater-than sign. You can check this solution by picking a number that's greater than –3 and plugging it into the original equation to make sure you get a true statement. If you check 0, for instance, you get $-4(0) + 1 < 13$, which is a true statement.

Switching the inequality is a step that many students forget. Look at an inequality with numbers in it, like $-2 < 10$. This statement is true. If you multiply 3 on both sides, you get $-6 < 30$, which is still true. But if you multiply –3 on both sides — and don't fix the sign — you get $6 < -30$. This statement is false, and you always want to keep the statements true. The only way for the equation to work is to switch the inequality sign to read $6 > -30$. The same rule applies if you divide $-2 < 10$ by –2 on both sides. The only way for the problem to make sense is to read $1 > -5$.

Solving equations and inequalities when absolute value is involved

If you think back to Algebra I, you'll likely remember that an absolute-value equation usually has two possible solutions. Absolute value is a bit trickier to handle when you're solving inequalities. Similarly, though, inequalities have two possible solutions:

- One where the quantity inside the absolute-value bars is greater than a number
- One where the quantity inside the absolute-value bars is less than a number

In mathematical terminology, the inequality $|ax \pm b| < c$ — where *a, b,* and *c* are real numbers — always becomes two inequalities:

$$ax \pm b < c \text{ AND } ax \pm b > -c$$

The "AND" comes from the graph of the solution set on a number line, as seen in Figure 2-1a.

The inequality $|ax \pm b| > c$ becomes

$$ax \pm b > c \text{ OR } ax \pm b < -c$$

The "OR" also comes from the graph of the solution set, which you can see in Figure 2-1b.

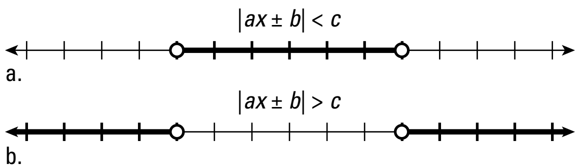


Figure 2-1: The solution to $|ax \pm b| < c$ and $|ax \pm b| > c$.

Here are two caveats to remember when dealing with absolute values:

- **If the absolute value is less than (<) or less than or equal to (≤) a negative number, it has no solution.** An absolute value must always be zero or positive (the only thing less than negative numbers is other negative numbers). For instance, the absolute-value inequality $|2x - 1| < -3$ doesn't have a solution, because the inequality is less than a negative number.

  Getting 0 as a possible solution is perfectly fine. It's important to note, though, that having no solutions is a different thing entirely. No solutions means that no number works at all, ever.

- **If the result is greater than or equal to a negative number, the solution is all real numbers**. For example, given the equation $|x - 1| > -5$, x is all real numbers. The left-hand side of this equation is an absolute value, and an absolute value always represents a positive number. Because positive numbers are always greater than negative numbers, these types of inequalities always have a solution. Any real number that you put into this equation works.

To solve and graph an inequality with an absolute value — for instance, $2|3x - 6| < 12$ — follow these steps:

1. **Isolate the absolute-value expression.**

   In this case, divide by both sides by 2 to get $|3x - 6| < 6$.

2. **Break the inequality in two.**

   This process gives you $3x - 6 < 6$ and $3x - 6 > -6$. Did you notice how the inequality sign for the second part changed? When you switch from positives to negatives in an inequality, you must change the inequality sign.

   Don't fall prey to the trap of changing the equation inside the absolute-value bars. For example, $|3x - 6| < 6$ doesn't change to $3x + 6 < 6$ or $3x + 6 > -6$.

3. **Solve both inequalities.**

   The solutions to this problem are $x < 4$ and $x > 0$.

4. **Graph the solutions.**

   Create a number line and show the answers to the inequality. Figure 2-2 shows this solution.

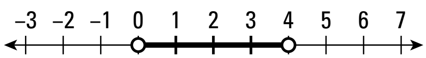


Figure 2-2: The solution to $2|3x - 6| < 12$ on a number line.

Expressing solutions for inequalities with interval notation

Now comes the time to venture into interval notation to express where a set of solutions begins and where it ends. *Interval notation* is another way to express the solution set to an inequality, and it's important because it's how you express solution sets in calculus. Most pre-calculus books and some pre-calculus teachers now require all sets to be written in interval notation.

TIP: The easiest way to find interval notation is to first draw a graph on a number line as a visual representation of what's going on in the interval.

If the coordinate point of the number isn't included in the problem (for < or >), the interval is called an *open interval.* You show it on the graph with an open circle at the point and by using parentheses in notation. If the point is included in the solution (≤ or ≥), the interval is called a *closed interval,* which you show on the graph with a filled-in circle at the point and by using square brackets in notation.

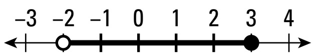
For example, the solution set $-2 < x \leq 3$ is shown in Figure 2-3. ***Note:*** You can rewrite this solution set as an *and* statement:

$-2 < x$ AND $x \leq 3$

In interval notation, you write this solution as (–2, 3].

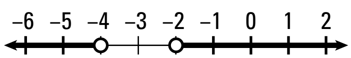
The bottom line: Both of these inequalities *have* to be true at the same time.

Figure 2-3: The graph of $-2 < x \leq 3$ on a number line.



You can also graph *or* statements (also known as *disjoint sets* because the solutions don't overlap). *Or* statements are two different inequalities where one or the other is true. For example, Figure 2-4 shows the graph of $x < -4$ OR $x > -2$.

Figure 2-4: The graph of the *or* statement $x < -4$ OR $x > -2$.



Writing the set for Figure 2-4 in interval notation can be confusing. x can belong to two different intervals, but because the intervals don't overlap, you have to write them separately:

- The first interval is $x < -4$. This interval includes all numbers between negative infinity and –4. Because –∞ isn't a real number, you use an open interval to represent it. So in interval notation, you write this part of the set as (–∞, –4).

- The second interval is $x > -2$. This set is all numbers between –2 and positive infinity, so you write it as $(-2, \infty)$.

  You describe the whole set as $(-\infty, -4) \cup (-2, \infty)$. The symbol in between the two sets is the *union symbol* and means that the solution can belong to either interval.

REMEMBER

When you're solving an absolute-value inequality that's greater than a number, you write your solutions as *or* statements. Take a look at the following example: $|3x - 2| > 7$. You can rewrite this inequality as $3x - 2 > 7$ OR $3x - 2 < -7$. You have two solutions: $x > 3$ or $x < -5/3$.

In interval notation, this solution is $\left(-\infty, \frac{-5}{3}\right) \cup (3, \infty)$.

Variations on Dividing and Multiplying: Working with Radicals and Exponents

Radicals and exponents (also known as *roots* and *powers*) are two common — and oftentimes frustrating — elements of basic algebra. And of course they follow you wherever you go in math, just like a cloud of mosquitoes follows a novice camper. The best thing you can do to prepare for calculus is to be ultra-solid on what can and can't be done when simplifying with exponents and radicals. You'll want to have this knowledge so that when more challenging math problems come along, the correct answers come along also. This section gives you the solid background you need for those challenging moments.

Defining and relating radicals and exponents

Before you dig deeper into your work with radicals and exponents, make sure you remember the facts in the following list about what they are and how they relate to each other:

- **A *radical* is a root of a number.** Radicals are represented by the root sign, $\sqrt{\ }$. For example, if you take the 2nd root of the number 9 (or the *square root*), you get 3 because $3 \cdot 3 = 9$. If you take the 3rd root (or the *cube root*) of 27, you get 3 because $3 \cdot 3 \cdot 3 = 27$. (In equation form, you write $\sqrt[3]{27}$).