Hardy

A Mathematician's Apology

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BY

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WITH A FOREWORD BY

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To

JOHN LOMAS

who asked me to write it

FOREWORD

IT was a perfectly ordinary night at Christ's high table, except that Hardy was dining as a guest. He had just returned to Cambridge as Sadleirian professor, and I had heard something of him from young Cambridge mathematicians. They were delighted to have him back: he was a *real* mathematician, they said, not like those Diracs and Bohrs the physicists were always talking about: he was the purest of the pure. He was also unorthodox, eccentric, radical, ready to talk about anything. This was 1931, and the phrase was not yet in English use, but in later days they would have said that in some indefinable way he had star quality.

So, from lower down the table, I kept studying him. He was then in his early fifties: his hair was already grey, above skin so deeply sunburnt that it stayed a kind of Red Indian bronze. His face was beautiful—high cheek bones, thin nose, spiritual and austere but capable of dissolving into convulsions of internal gamin-like amusement. He had opaque brown eyes, bright as a bird's—a kind of eye not uncommon among those with a gift for conceptual thought. Cambridge at that time was full of unusual and distinguished faces—

but even then, I thought that night, Hardy's stood out.

I do not remember what he was wearing. It may easily have been a sports coat and grey flannels under his gown. Like Einstein, he dressed to please himself: though, unlike Einstein, he diversified his casual clothing by a taste for expensive silk shirts.

As we sat round the combination-room table, drinking wine after dinner, someone said that Hardy wanted to talk to me about cricket. I had been elected only a year before, but Christ's was then a small college, and the pastimes of even the junior fellows were soon identified. I was taken to sit by him. I was not introduced. He was, as I later discovered, shy and self-conscious in all formal actions, and had a dread of introductions. He just put his head down as it were in a butt of acknowledgment, and without any preamble whatever began:

'You're supposed to know something about cricket, aren't you?' Yes, I said, I knew a bit.

Immediately he began to put me through a moderately stiff viva. Did I play? What sort of performer was I? I half-guessed that he had a horror of persons, then prevalent in academic society, who devotedly studied the literature but had never played the game. I trotted out my credentials, such as they were. He appeared to

find the reply partially reassuring, and went on to more tactical questions. Whom should I have chosen as captain for the last test match a year before (in 1930)? If the selectors had decided that Snow was the man to save England, what would have been my strategy and tactics? ('You are allowed to act, if you are sufficiently modest, as non-playing captain.') And so on, oblivious to the rest of the table. He was quite absorbed.

As I had plenty of opportunities to realize in the future, Hardy had no faith in intuitions or impressions, his own or anyone else's. The only way to assess someone's knowledge, in Hardy's view, was to examine him. That went for mathematics, literature, philosophy, politics, anything you like. If the man had bluffed and then wilted under the questions, that was his lookout. First things came first, in that brilliant and concentrated mind.

That night in the combination-room, it was necessary to discover whether I should be tolerable as a cricket companion. Nothing else mattered. In the end he smiled with immense charm, with child-like openness, and said that Fenner's (the university cricket ground) next season might be bearable after all, with the prospect of some reasonable conversation.

Thus, just as I owed my acquaintanceship with Lloyd George to his passion for phrenology, I

owed my friendship with Hardy to having wasted a disproportionate amount of my youth on cricket. I don't know what the moral is. But it was a major piece of luck for me. This was intellectually the most valuable friendship of my life. His mind, as I have just mentioned, was brilliant and concentrated: so much so that by his side anyone else's seemed a little muddy, a little pedestrian and confused. He wasn't a great genius, as Einstein and Rutherford were. He said, with his usual clarity, that if the word meant anything he was not a genius at all. At his best, he said, he was for a short time the fifth best pure mathematician in the world. Since his character was as beautiful and candid as his mind, he always made the point that his friend and collaborator Littlewood was an appreciably more powerful mathematician than he was, and that his protégé Ramanujan really had natural genius in the sense (though not to the extent, and nothing like so effectively) that the greatest mathematicians had it.

People sometimes thought he was under-rating himself, when he spoke of these friends. It is true that he was magnanimous, as far from envy as a man can be: but I think one mistakes his quality if one doesn't accept his judgment. I prefer to believe in his own statement in *A Mathematician's Apology*, at the same time so proud and so humble:

'I still say to myself when I am depressed and

find myself forced to listen to pompous and tiresome people, "Well, I have done one thing you could never have done, and that is to have collaborated with Littlewood and Ramanujan on something like equal terms."'

In any case, his precise ranking must be left to the historians of mathematics (though it will be an almost impossible job, since so much of his best work was done in collaboration). There is something else, though, at which he was clearly superior to Einstein or Rutherford or any other great genius: and that is at turning any work of the intellect, major or minor or sheer play, into a work of art. It was that gift above all, I think, which made him, almost without realizing it, purvey such intellectual delight. When *A Mathematician's Apology* was first published, Graham Greene in a review wrote that along with Henry James's notebooks, this was the best account of what it was like to be a *creative artist.* Thinking about the effect Hardy had on all those round him, I believe that is the clue.

He was born, in 1877, into a modest professional family. His father was Bursar and Art Master at Cranleigh, then a minor public (English for private) school. His mother had been senior mistress at the Lincoln Training College for teachers. Both were gifted and mathematically inclined. In his case, as in that of most mathema-

ticians, the gene pool doesn't need searching for. Much of his childhood, unlike Einstein's, was typical of a future mathematician's. He was demonstrating a formidably high I.Q. as soon as, or before, he learned to talk. At the age of two he was writing down numbers up to millions (a common sign of mathematical ability). When he was taken to church he amused himself by factorizing the numbers of the hymns: he played with numbers from that time on, a habit which led to the touching scene at Ramanujan's sick-bed: the scene is well known, but later on I shall not be able to resist repeating it.

It was an enlightened, cultivated, highly literate Victorian childhood. His parents were probably a little obsessive, but also very kind. Childhood in such a Victorian family was as gentle a time as anything we could provide, though probably intellectually somewhat more exacting. His was unusual in just two respects. In the first place, he suffered from an acute self-consciousness at an unusually early age, long before he was twelve. His parents knew he was prodigiously clever, and so did he. He came top of his class in all subjects. But, as the result of coming top of his class, he had to go in front of the school to receive prizes: and that he could not bear. Dining with me one night, he said that he deliberately used to try to get his answers wrong so as to

be spared this intolerable ordeal. His capacity for dissimulation, though, was always minimal: he got the prizes all the same.

Some of this self-consciousness wore off. He became competitive. As he says in the *Apology*: 'I do not remember having felt, as a boy, any *passion* for mathematics, and such notions as I may have had of the career of a mathematician were far from noble. I thought of mathematics in terms of examinations and scholarships: I wanted to beat other boys, and this seemed to be the way in which I could do so most decisively.' Nevertheless, he had to live with an over-delicate nature. He seems to have been born with three skins too few. Unlike Einstein, who had to subjugate his powerful ego in the study of the external world before he could attain his moral stature, Hardy had to strengthen an ego which wasn't much protected. This at times in later life made him self-assertive (as Einstein never was) when he had to take a moral stand. On the other hand, it gave him his introspective insight and beautiful candour, so that he could speak of himself with absolute simplicity (as Einstein never could).

I believe this contradiction, or tension, in his temperament was linked with a curious tic in his behaviour. He was the classical anti-narcissist. He could not endure having his photograph taken: so far as I know, there are only five snapshots in

existence. He would not have any looking glass in his rooms, not even a shaving mirror. When he went to a hotel, his first action was to cover all the looking-glasses with towels. This would have been odd enough, if his face had been like a gargoyle: superficially it might seem odder, since all his life he was good-looking quite out of the ordinary. But, of course, narcissism and anti-narcissism have nothing to do with looks as outside observers see them.

This behaviour seems eccentric, and indeed it was. Between him and Einstein, though, there was a difference in kind. Those who spent much time with Einstein—such as Infeld—found him grow stranger, less like themselves, the longer they knew him. I am certain that I should have felt the same. With Hardy the opposite was true. His behaviour was often different, bizarrely so, from ours: but it came to seem a kind of superstructure set upon a nature which wasn't all that different from our own, except that it was more delicate, less padded, finer-nerved.

The other unusual feature of his childhood was more mundane: but it meant the removal of all practical obstacles throughout his entire career. Hardy, with his limpid honesty, would have been the last man to be finicky on this matter. He knew what privilege meant, and he knew that he had possessed it. His family had no money, only a