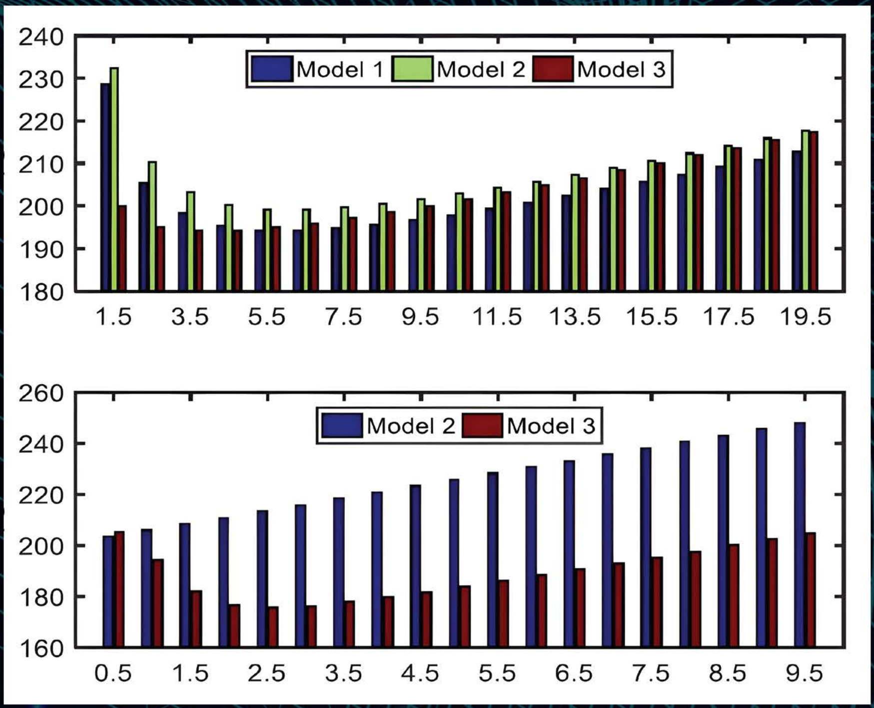
MATHEMATICAL ENGINEERING, MANUFACTURING, AND MANAGEMENT SCIENCES

Statistical Modeling and Applications on Real-Time Problems

Enhancing Understanding and Practical Implementation



Edited by
Chandra Shekhar
Raghaw Raman Sinha

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Statistical Modeling and Applications on Real-Time Problems

In the dynamic landscape of modern data analysis, this curated guide by global experts explores the latest in statistical methodologies, modeling techniques, and optimization strategies. This comprehensive text offers insights into diverse fields such as engineering, economics, medicine, and agriculture, addressing real-world challenges. It delves into the intricacies of the Lomax distribution under a Type II censoring scheme, exploring various loss functions. The compilation uncovers estimators for population proportion, product of two population means, and more, supported by empirical and simulation studies. Additionally, it scrutinizes the prevalence of caesarean section deliveries in India, correlating with socio-economic factors.

This book:

- Traverses diverse fields for insights into real-world challenges.
- Delves into the intricacies of the Lomax distribution under a Type II censoring scheme.
- Uncovers estimators supported by empirical and simulation studies.
- Scrutinizes the prevalence of caesarean section deliveries in India, correlating with socio-economic factors.

This compilation promises a holistic exploration of advanced statistical and optimization methods, offering readers valuable insights into their pragmatic applications across a spectrum of real-world issues.

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Contents

Preface

In our contemporary milieu, mathematical and statistical models emerge as indispensable instruments, adeptly addressing uncertainties and refining numerical, observational, and calculation-based data extensively employed by governmental and private entities. These models serve as pivotal tools in deciphering optimal solutions, with statistical prediction models providing nuanced insights into probabilistic future behaviors across diverse domains such as climate, population, and production.

Under the banner of *Statistical Modeling and Applications on Real-Time Problems: Enhancing Understanding and Practical Implementation,* this tome aspires to serve as an exhaustive guide, shedding light on recent developments in statistical modeling, methodologies, and optimization techniques. It traverses varied fields, encompassing engineering, economics, medicine, and agriculture, presenting a compendium of 11 chapters contributed by authors globally. The central focus lies on the estimation and optimization of real-time problems through the application of sophisticated statistical modeling and methods.

The book meticulously delves into advanced statistical methodologies and modeling techniques. It introduces the Lomax distribution under a Type II censoring scheme, exploring diverse loss functions. Each ensuing chapter contributes a unique perspective, intricately weaving a rich tapestry of statistical applications. Devoted to performance analysis, practical applications in stochastic systems, optimization techniques, and Multiple-criteria decision-making essentials, the book promises an exhaustive journey toward a profound understanding of analytical methodologies with practical insights — transcending from theoretical breakdowns to real-world applications.

Chapter 1 introduces the Lomax distribution under a Type II censoring scheme, considering three different loss function names: Al-Bayyati loss function, Minimum expected loss function, and DeGroot loss function. Chapter 2 proposes two classes of estimators for estimating the population proportion under stratified random sampling, incorporating empirical and simulation comparative studies. Chapter 3 concentrates on the estimation of the proportion and product of two population means, presenting classes of estimators utilizing the mean and rank of auxiliary variables under pps sampling, along with their properties. Chapter 4 introduces a modified regression-type estimator of the population mean on current occasions, based on samples selected over current and previous occasions.

Chapter 5 scrutinizes the prevalence of cesarean section deliveries in India, correlating preferences with socio-economic factors. Chapter 6 addresses the challenges of estimating the ratio and product of two population means using auxiliary information under complete and incomplete information. Chapter 7 conducts a detailed analysis of the agriculture domestic product cost for different zones of India, providing a comparative study spanning 15 years (from 2003–2010 to 2011–2017).

Chapter 8 delves into the performance analysis of three models using the queueing theoretic approach for server breakdown, working breakdown, and recovery policy. Chapter 9 explores practically beneficial aspects through a single-server stochastic queueing system with encouraged arrivals attracted to discounts and offers. Chapter 10 acquaints readers with the current literature on optimization techniques, emphasizing practical utilization without delving into intricate technical details. Chapter 11 addresses the essential characteristics of Multiple-criteria decision-making (MCDM), introducing statistical and mathematical tools to assist decision-makers in qualitative assessments of performance, including classifications and their real-life applications.

Collectively, under the overarching title "*Statistical Modeling and Applications on Real-Time Problems: Enhancing Understanding and Practical Implementation*," these chapters provide a comprehensive exploration of advanced statistical and optimization methods, furnishing readers with valuable insights into their pragmatic applications across a spectrum of real-world issues.

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Jalandhar, India

Acknowledgments

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विद्या नाम नरस्य कीर्तिरतुला भाग्यक्षये चाश्रयो
धेनुः कामदुधा रतिश्च विरहे नेत्रं तृतीयं च सा।
सत्कारायतनं कुलस्य महिमा रत्नैर्विना भूषणम्
तस्मादन्यमुपेक्ष्य सर्वविषयं विद्याधिकारं कुरु।।

The creation of the book, titled "Statistical Modeling and Applications on Real-Time Problems: Enhancing Understanding and Practical Implementation," has been made possible through the generous collaboration with esteemed institutions, namely BITS Pilani and NIT Jalandhar. Our sincere appreciation extends to these institutions for their invaluable administrative and academic support.

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We extend our deepest appreciation to the erudite reviewers, whose constructive feedback has significantly elevated the quality of the chapters. The contributors, representing diverse corners of the globe, have shared a rich array of manuscripts covering topics in statistical modeling, statistical methods, optimization techniques, and challenges within both statistical and optimization fields. Special thanks are due to the contributors for sharing their content-rich insights.

Our sincere thanks go to our friends, guardians, and members of society whose guidance has illuminated our path to progress. To both our known and unknown supporters, we express profound gratitude for their encouragement and valuable contributions.

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Chapter 1

Comparison between Ml, Bayes and E-Bayes estimates of $\gamma(t)$ and $\rho(t)$ for Lomax distribution under type II censoring scheme

Sumit Koul

1.1 INTRODUCTION

There are many life-time distributions which are considered by different authors, viz, commonly used exponential distribution for life time data, Weibull distribution for bathtub analysis, gamma distribution, etc. In this context a mixture of exponential and gamma distribution is introduced, known as Lomax distribution [commonly known as Pareto distribution of second kind] for failure data. Lomax (1954) introduced this distribution for analysing business failure data, after that many authors introduced this distribution in their literature. Table 1.1 shows the literature review of the work by different authors. Table 1.2 represents the nomenclature used in the chapter.

The purpose of the present note is many-fold. One is finding the Bayesian and E-Bayesian estimation of the Lomax distribution under type II censoring scheme. [27,28] has considered the Lomax distribution for estimating the parameter α, the reliability and hazard function based on the balanced squared error loss function under type II censoring scheme. After being influenced by the research article of [27], I have considered the Lomax distribution, estimating the unknown parameter, the reliability function for Bayes and E-Bayes under type II censoring scheme using the three different loss functions namely Al-Bayyati, Minimum expected and DeGroot loss function. The organization of the chapter is as follows. In section 1.2, define the probability density function and some related definitions are given. In section 1.3, likelihood inference, MLE and Bayesian estimation are discussed under type II censoring scheme. E-Bayesian estimation under type II scheme for parameter $\gamma(t)$ which is an unknown quantity and function of reliability is considered, also the characteristics for unknown parameter and the reliability function are described in section 1.4. Monto-Carlo simulation study is obtained in section 1.5. Lastly in section 1.6, concluding remarks are made.

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Table 1.1 Comparative study on different authors on Bayesian and E-Bayesian inferences

Reference	*Year*	*Aspect discussed*	*Description*
[1,2]	1983	Pareto distribution	Considered the Pareto distribution and its application
[3,4]	2011	Pareto distribution with progressive type II censoring	Derived the progressive type II censoring competing risk data for Lomax distribution
[5]	2009	E-Bayesian method	E-Bayesian estimation and hierarchical Bayesian estimation of failure rate is considered
[6]	2011	Burr type XII model with E-Bayesian	Proposed E-Bayesian estimation for the Burr type XII model based on type II censoring
[7,8]	2012	Weibull distribution with E-Bayesian	Developed an E-Bayesian estimation of system reliability with Weibull distribution of components based on type II censoring
[9]	2017	Rayleigh Model with E-Bayesian model	Considered E-Bayesian Estimation for Rayleigh Model Using Progressive type II Censoring Data
[10]	2018	E-Bayesian with Binomial distribution	Considered different loss function to obtained E-posterior risk as well as reliability function for E-Bayesian estimation. Reliability function as well as parameter under E-Bayesian gives better result
[11,12]	2019	E-Bayesian estimation with Exponential distribution	Considered the record values to obtained the E-Bayesian estimation as well as Bayesian estimation for the Exponential distribution. E-Bayesian estimation performs well as compared to Bayesian estimation under squared error loss function
[13–16]	2021	Frechet Distribution with E-Bayesian technique	Considered E-Bayesian estimation to compare it with Bayesian parameter using squared error loss function for parameters of Frechet Distribution. It has been shown that E-Bayesian method gives better result as compared with Bayesian method
[17,18]	2021	Hierarchical Bayesian Estimations with Inverse Weibull distribution	Author proposed new formulation to compare E-Bayesian and Hierarchical Bayesian Estimations. They found that Hierarchical Bayesian Estimations is less efficient that E-Bayesian
[19,20]	2022	Bayesian estimation using Power-Modified Lindley distribution	Using stress-strength reliability to drawn an inference based on Bayesian and non-Bayesian for Power-Modified Lindley Distribution

[21]	2021	E-Bayesian with Poisson distribution	A new approach for estimating parameter of Poisson distribution i.e., Empirical E-Bayesian estimation as compared to E-Bayesian estimation is considered. Various risk functions are computed under both methods. Empirical E-Bayesian estimation comparatively gives better results as compared with E-Bayesian estimation
[22]	2021	E-Bayesian with Lomax distribution	Proposing new loss function which is named as Weighted Composite LINEX Loss Function with other loss function for comparing E-Bayesian and Bayesian estimation for the Lomax distribution. The new loss function provides better result than other loss under E-Bayesian estimator
[23,24]	1978	Burr distribution using Bayesian approach	Developed Bayesian model using failure model: Burr distribution
[25]	2022	E-Bayesian method with Kumaraswamy distribution	Established method such as Bayesian and E-Bayesian to estimate parameters of Kumaraswamy distribution
[26]	2022	E-Bayesian estimator using inverse Rayleigh distribution	Using different loss function to estimate E-Bayesian and Hierarchical Bayesian estimator of scale parameter of inverse Rayleigh distribution.

Table 1.2 Nomenclature

S.No.	*Symbol*	*Description*
1	Pdf	probability density function
2	Cdf	cumulative distribution function
3	γ	Shape parameter
4	ϑ	Scale parameter
5	$\rho(t)$	Reliability function
6	$h(t)$	Hazard rate
7	MLE	Maximum likelihood estimator
8	ABLF	Al-Bayyati Loss Function
9	MELF	Minimum Expected Loss Function
10	DGLF	DeGroot Loss Function

1.2 DEFINITION AND PRELIMINARIES

Let $\mathfrak{X}$ be a life time of an item having the Lomax distribution then its probability density function (pdf) and cumulative distribution function (cdf) are defined as

$$f(\mathfrak{X};\gamma,\vartheta) = \gamma\vartheta(1+\vartheta\mathfrak{X})^{-(\gamma+1)}, \quad \mathfrak{X} > 0, \quad (\gamma,\vartheta) > 0 \tag{1.1}$$

and

$$\mathcal{F}(\mathfrak{X};\gamma,\vartheta) = (1+\vartheta\mathfrak{X})^{-\gamma}, \tag{1.2}$$

where, γ and ϑ are the shape and scale parameter, respectively.

The reliability $\rho(t)$ of an item, is the probability of failure free probability until time t, reliability function at a specified mission time t is given by

$$\begin{aligned} \rho(t) &= P(\mathfrak{X} > t) \\ &= 1 - \mathcal{F}(t) \\ &= (1+\vartheta t)^{-\gamma}, \quad t > 0 \end{aligned} \tag{1.3}$$

and hazard function is given by combining equations (1.1) and (1.2) as,

$$\begin{aligned} h(t) &= \frac{f(t)}{\rho(t)} \\ &= \frac{\gamma\vartheta}{(1+\vartheta t)}. \quad t > 0 \end{aligned} \tag{1.4}$$

1.3 ML AND BAYES UNDER TYPE II CENSORING SCHEME

Suppose n items are put to test and the first r ordered observations are recoded when the test is terminated. Let us denote by $\mathfrak{X}_1, \mathfrak{X}_2, \ldots, \mathfrak{X}_r$, $0 < r < n$, the lifetime of first r failures, obviously, $(n - r)$ items survived until $\mathfrak{X}_r$. From equation (1.1) and integration out $\mathfrak{X}_{(1)} \leq \mathfrak{X}_{(2)} \leq ,\ldots, \mathfrak{X}_{(r)}$ is given as

$$\ell(\underline{\mathfrak{X}}; \gamma, \vartheta) = \frac{n!}{(n - r)!} \gamma^r \varphi(\vartheta, \underline{\mathfrak{X}}) \, e^{-\gamma Q_r}, \tag{1.5}$$

where,

$$Q_r = Q(\vartheta; \underline{\mathfrak{X}}) = \sum_{i=1}^{r} \ln(1 + \vartheta \mathfrak{X}_i) + (n - r)(1 + \vartheta \mathfrak{X}_r), \tag{1.6}$$

$$\varphi(\vartheta, \underline{\mathfrak{X}}) = \frac{\vartheta^r}{\prod_{i=1}^{r} (1 + \vartheta \mathfrak{X}_i)}; \; \underline{\mathfrak{X}} = \mathfrak{X}_{(1)}, \mathfrak{X}_{(2)}, \ldots, \mathfrak{X}_{(r)} \tag{1.7}$$

1.3.1 Maximum likelihood estimator under type II censoring scheme

When ϑ is known and from equation (1.6), the MLE of γ comes out to be

$$\tilde{\gamma} = \frac{r}{Q_r} \tag{1.8}$$

1.3.2 Bayes estimators under type II censoring scheme

Conjugate prior is considered as gamma distribution for known γ and is given as

$$\varnothing(\gamma; \, g, \, h) = \frac{h^g}{\Gamma(g)} \gamma^{g-1} \, e^{-h\gamma}, \quad \gamma > 0, (g, h) > 0 \tag{1.9}$$

where g, h are the parameters of the gamma distribution.

Papadopoulos (1978) was the first to use this prior. On combining (1.5) and (1.9), the posterior density of γ given $\underline{\mathfrak{X}}$ is of form

$$\psi(\underline{\mathfrak{X}}; \gamma) = \tau \gamma^{r+g-1} e^{-(h+Q_r)\gamma}, \quad \gamma > 0 \tag{1.10}$$