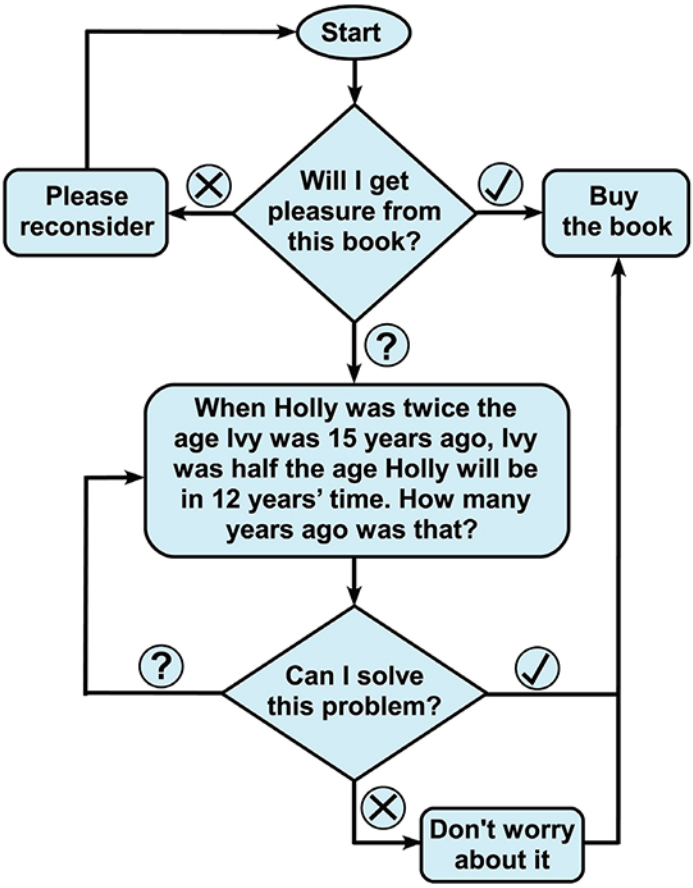

Mathematical Conundrums

Barry R. Clarke

CRC Press
Taylor & Francis Group
AN A K PETERS BOOK

Mathematical Conundrums

Want to sharpen your mathematical wits? If so, then *Mathematical Conundrums* is for you. *Daily Telegraph* enigmatologist, Barry R. Clarke, presents over 120 fiendish problems that will test both your ingenuity and persistence. Between these covers are puzzles in geometry, arithmetic, and algebra (there is even a section for computer programmers). And, for the smartest readers who wish to stretch their mind to its limits, a selection of engaging logic and visual lateral puzzles is included. Although no puzzle requires a greater knowledge of mathematics than the high school curriculum, this collection will take you to the edge. But are you equal to the challenge?

Features

- High school level of mathematics is the only prerequisite.
- Variety of algebraic, route-drawing, and geometrical conundrums.
- Hints section for the lateral puzzles.
- Warm-up exercises to sharpen the wits.
- Full solutions to every problem.

Barry R. Clarke has published over 1,500 puzzles in *The Daily Telegraph* and has contributed enigmas to *New Scientist*, *The Sunday Times*, *Reader's Digest*, *The Sunday Telegraph*, and *Prospect* magazine. His book *Challenging Logic Puzzles Mensa* has sold over 100,000 copies. As well as a PhD in Shakespeare Studies, Barry has a master's degree and academic publications in quantum physics. He is now working on a revised theory of the hydrogen atom. Other skills include mathematics tutor, filmmaker, comedy-sketch writer, cartoonist, computer programmer, and blues guitarist! For more information please visit http://barryispuzzled.com.

AK Peters/CRC Recreational Mathematics Series

Series Editors
Robert Fathauer
Snezana Lawrence
Jun Mitani
Colm Mulcahy
Peter Winkler
Carolyn Yackel

Mathematics of Casino Carnival Games
Mark Bollman

Mathematical Puzzles
Peter Winkler

X Marks the Spot: The Lost Inheritance of Mathematics
Richard Garfinkle, David Garfinkle

Luck, Logic, and White Lies: The Mathematics of Games, Second Edition
Jörg Bewersdorff

Mathematics of The Big Four Casino Table Games: Blackjack, Baccarat, Craps, & Roulette
Mark Bollman

Star Origami: The Starrygami™ Galaxy of Modular Origami Stars, Rings and Wreaths
Tung Ken Lam

Mathematical Recreations from the Tournament of the Towns
Andy Liu, Peter Taylor

The Baseball Mysteries: Challenging Puzzles for Logical Detectives
Jerry Butters, Jim Henle

Mathematical Conundrums
Barry R. Clarke

Lateral Solutions to Mathematical Problems
Des MacHale

For more information about this series please visit: https://www.routledge.com/AK-PetersCRC-Recreational-Mathematics-Series/book-series/RECMATH?pd=published,forthcoming&pg=2&pp=12&so=pub&view=list

Mathematical Conundrums

Barry R. Clarke

CRC Press
Taylor & Francis Group
Boca Raton New York London

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

Designed cover image: Barry R. Clarke

First edition published 2023
by CRC Press
6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL 33487-2742

and by CRC Press
4 Park Square, Milton Park, Abingdon, Oxon, OX14 4RN

CRC Press is an imprint of Taylor & Francis Group, LLC

ISBN: 978-1-032-41478-2 (hbk)
ISBN: 978-1-032-41458-4 (pbk)
ISBN: 978-1-003-35827-5 (ebk)

DOI: 10.1201/9781003358275

Typeset in Optima
by SPi Technologies India Pvt Ltd (Straive)

Contents

Preface

Welcome to *Mathematical Conundrums*! This collection presents over 120 of my best enigmas that have appeared in national newspapers and magazines. They rely primarily on intelligence and persistence, and need no greater knowledge of mathematics than that taught in high school. However, their difficulty lies in the level of ingenuity and persistence required. So who might derive pleasure from this book? It should be ideal for the intelligent high school student who is seeking a challenge beyond the established curriculum, or the university science student who wants to keep their mind in shape, or in general, anyone with a working knowledge of arithmetic and algebra who enjoys thinking. In addition, many of these problems will be excellent preparation for the UKMT Mathematical Challenge and the MAA American Mathematics Competitions for schools. Full solutions are given at the end of each chapter. Good luck!

Barry R. Clarke
Oxford, UK

1 Introduction

The first book I recall seeing on recreational mathematics was Martin Gardner's *Mathematical Puzzles and Diversions* [1]. I was 14 years old at the time and found it in the school library as a hardback copy, with a picture of an arrangement of pentominoes on its green dust cover. The chapter that fascinated me the most was on Sam Loyd (1841–1911) who Gardner billed as "America's Greatest Puzzlist." Since I was interested in conjuring tricks, Loyd appeared to me at the time as an almost supernatural puzzle magician. One of his earliest puzzles, which he claimed to have invented at the age of 9, ran as follows [2].

It is told that three neighbours, who shared a small park, as shown in the sketch (Figure 1.1) had a falling out. The owner of the large house (A), complaining that his neighbour's chickens annoyed him, built an enclosed pathway from his door to the gate at the bottom of the picture (A). Then the man on the right (B) built a path to the gate on the left (B), and the man on the left (C) built a path to the gate on the right (C), so that none of the paths cross, and each man has an exit opposite his door.

Can you draw the paths?

The solution is given at the end of the chapter. In his article, Gardner states that "Loyd's most interesting creation is the famous '14–15' or 'Boss' puzzle." This consists of a four-by-four array of squares in which there are 15 numbered blocks 1–15 with one empty square. The puzzle is set up with the blocks in serial order except for the 14 and 15 which are juxtaposed, the

DOI: 10.1201/9781003358275-1

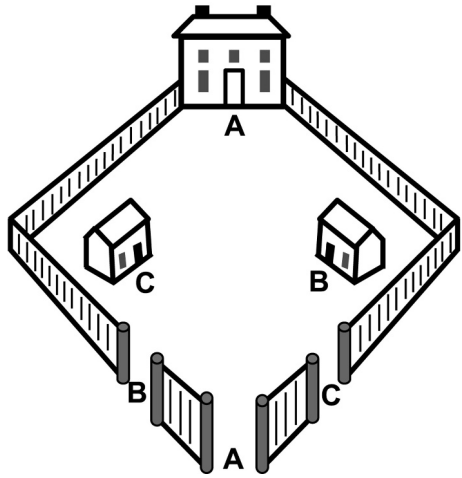

Figure 1.1 Sam Loyd's pathway puzzle.

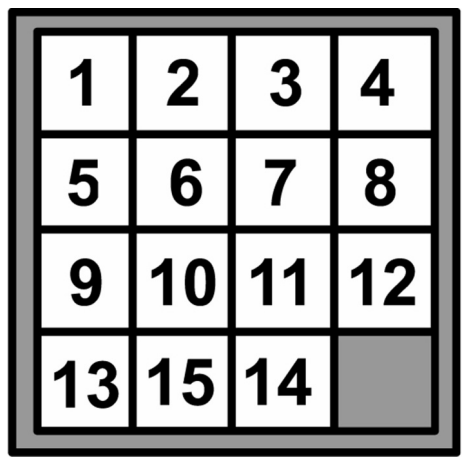

Figure 1.2 Initial configuration of the 14–15 or Boss puzzle.

empty square being left in the bottom right corner (see Figure 1.2). The vacant space is to be used to slide all the blocks into serial order, again leaving the space in the right-hand bottom corner of the grid.

In January 1896, Loyd offered a prize of $1000 in *The Illustrated American* claiming the puzzle as his own invention. He knew he would never have to pay out because a mathematical paper had been published six years earlier demonstrating its impossibility [3]. Not only that, when *The New York Times* reported on the puzzle at the height of its craze on 22 March and 11 June

1880, Sam Loyd received no credit even though he lived in New York. It transpired that the puzzle wasn't his. Noyes Palmer Chapman, a postmaster from Canastota in New York, had filed a patent application dated 21 February 1880 on his "Block Solitaire Puzzle," a 4×4 grid with 15 sliding blocks that could be removed. The application was rejected, likely on the grounds that a patent for a 6×6 grid with 35 sliding pieces had earlier been granted on 20 August 1878 to Ernest U. Kinsey. Apart from the number of sliding blocks, the main difference seemed only to be that Kinsey's "Puzzle Blocks" were held in place by tongues and grooves while Chapman's blocks could be tipped out [4]. Sam Loyd also "borrowed" the ideas for his "P. T. Barnum's Trick Donkeys" [5] and "Get Off the Earth" puzzles from elsewhere. I only learned these facts many years later with great disappointment, and the discovery left me with the conclusion that Sam Loyd excelled more at shameless self-promotion than inspired invention. As he himself said, "people don't care for my puzzles unless they can have them with my name on them" [6].

A contemporary of Sam Loyd also had misgivings about his claims to priority. Henry E. Dudeney (1857–1930) actually accused Loyd of stealing his puzzles and presenting them under his own name. Dudeney was a master of dissection problems and his most celebrated discovery was the rearrangement of the parts of an equilateral triangle into a square (see Figure 1.3). This he called the "Haberdasher's Puzzle" which he published in the *Weekly Dispatch* on 14 June 1903. Dudeney was a prolific compiler, and his main puzzles column appeared in *The Strand Magazine* and ran for 20 years from March 1908.

Although Martin Gardner excelled more at popularisation than compilation, he was the first to present the problem of dissecting a square into eight acute triangles. This he announced in *Scientific American* in February 1960 [7, 8]. As a university physics student, I managed to succeed in producing nine acute triangles using a pentagon in one corner but was disappointed to learn that my solution was not original.

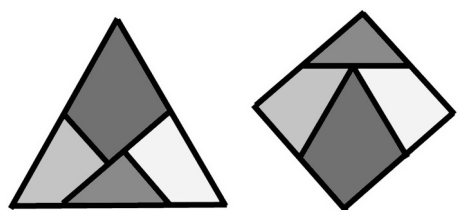

Figure 1.3 Dudeney's equilateral triangle to square dissection.

The inspiration for my own puzzle compilation began with winning the *New Scientist* Enigma prize for 27 March 1986. This was a puzzle entitled "In the bag" set by Christopher Maslanka, the compiler for *The Guardian*, and involved discovering the numbers of red, white, and blue eggs in a bag under various selection conditions. At the time, I was a mathematics PhD student at University College Swansea, a degree I neglected to complete due to my greater interest in quantum physics. I recall that the puzzle, which required the method of exhaustion, took me four hours to crack with just pen and paper. A year later, I bought Chris Maslanka's *The Pyrgic Puzzler* [9] and delighted in the humorous names and drawings of his many quirky characters. For example, here is puzzle No. 9.

> Professor Pembish knows that (with the plug in) his bath fills in three minutes if the cold tap only is turned on full, and in four minutes if the hot tap only is turned on full. With the plug out (and the taps off) the bath empties in two minutes. One day he finds to his horror that Mrs Oldham has mislaid the plug, but he is intelligent enough to realise that he can still fill the bath with both taps turned on full.

How long will it take to fill?

Around 2002, I came across the lateral thinking puzzles of Lloyd King who has invented many ingenious puzzles that demand an alternative interpretation of a visual scene to arrive at the solution [10, 11]. I have met only a handful of people who excel at this type of thinking, and it definitely requires an inventive mind. Something resembling this type of skill is employed for the solution of a geometrical problem, when the imaginative addition of a line leads to new relationships.

Lloyd King's "Snowman" puzzle is shown in Figure 1.4. The problem is as follows.

Which month is indicated in this wintery scene: April, May, June, July, or August?

Of course, it is important to give a reason for your answer! In Chapter 8, I give examples of my own puzzles of this type. Some have criticised this puzzle form as "mind-reading," especially if no hints are given, but my recommendation is that if you see no way forward, don't feel defeated

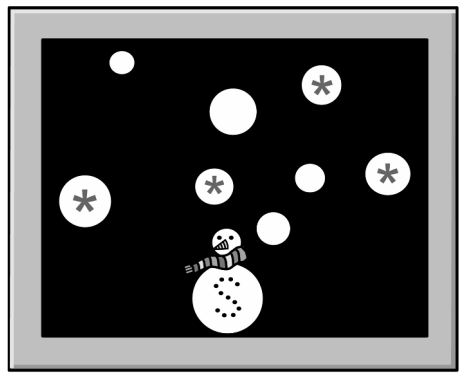

Figure 1.4 The "Snowman" puzzle.

or cheated. Simply look up the solution and try to admire its construction as puzzle art.

My first published puzzle was "One for the road" which appeared in *New Scientist* on 14 July 1988 [12]. It involved five drunks who were trying to push their broken-down car home. They were so inebriated that some pushed at the front of the car, and others opposed them by pushing at the back. The puzzle was to work out the whole-number strength of each man from the given conditions. My second puzzle also appeared in *New Scientist*, this time as "Sum secret" on 26 August 1989. It was an example of the set of digital deletion sums that I had worked out (see Figure 1.5).

In the sum shown, the first row added to the second gives the third, the fourth subtracted from the third gives the fifth, and the fifth added to the

$$\begin{array}{rr} & 6897 \\ + & 2968 \\ \hline & 9865 \\ - & 4968 \\ \hline & 4897 \\ + & 3856 \\ \hline & 8753 \end{array}$$

Figure 1.5 Digital deletion puzzle.

sixth gives the seventh. One digit can be erased from each row (not necessarily the same position in each row) and the gaps can be closed up to leave three columns of digits, then a second digit can be rubbed out in the same way to give two columns, then a third to leave one column, so that a valid sum remains each time. The three sets of seven digits erased (read down the columns) respectively reveal three numbers.

What are the three numbers?

In 1989, I was employed by *The Daily Telegraph* as one of the four members of the Brain Twister team under the editorship of Val Gilbert. Rex Gooch, Angela Newing, David Singmaster, and I provided a weekly puzzle in rotation. Then in the mid-1990s, I became the sole compiler with the assistance of Jacqui Harper's excellent cartoon illustrations. To date the *Telegraph* has published over 1,500 of my enigmas. One of my earliest inventions was the "Mix-and-match" logic puzzle which first appeared in the *Telegraph* on 21 May 1994. An example appears below with four rows and three columns of items. Each item is in the correct column, but only one item in each column is in the correct row. The given facts about the correct order lead to the solution.

(1) Freeman was one place below the boxer from Boston.
(2) The fighter from Seattle was one place above the boxer nicknamed Iron.
(3) McCool was two places below the contestant nicknamed Sugar.

Can you find the correct nickname, surname, and hometown for each position?

My most difficult puzzle of this type has seven rows and four columns of items which has approximately 6×10^{14} possible arrangements [13]. An example appears as puzzle 7.17 in Chapter 7.

Table 1.1 Four by three mix-and-match puzzle

	Nickname	Surname	Hometown
1	Rocky	Tryson	Boston
2	Sugar	Holyhead	Seattle
3	Basher	McCool	Texas
4	Iron	Freeman	New York

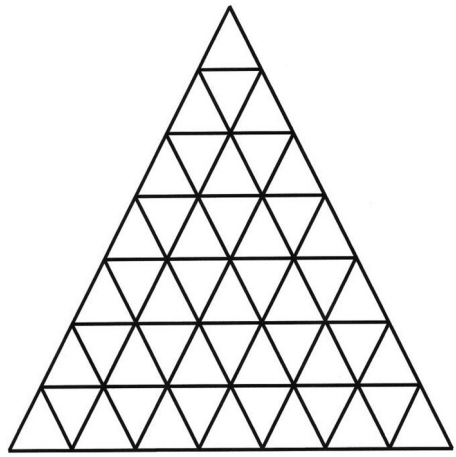

Figure 1.6 A master triangle of side length n = 7.

I have also taken an interest in recreational mathematics problems. One investigation that particularly interested me appeared in my book *Mathematical Puzzles and Curiosities* [14]. It involves the triangle shown in Figure 1.6. For this I managed to deduce formulae that predict both the number of triangles and the number of quadrilaterals of any size in a master triangle of side length n. For the total number of triangles T_n we have the following.

$$T_n = \frac{n(2n+1)(n+2) - \delta_n}{8}, \qquad \delta_n = \begin{cases} 0 \text{ for } n \text{ even} \\ 1 \text{ for } n \text{ odd} \end{cases} \tag{1.1}$$

The total number of quadrilaterals Q_n is given by

$$Q_n = \frac{n(n^2-1)(n+2)(2+\varepsilon_n)}{8}, \quad \varepsilon_n = \begin{cases} \dfrac{n(n-2)}{2(n^2-1)} \text{ for } n \text{ even} \\ \dfrac{n^2-3}{2n(n+2)} \text{ for } n \text{ odd} \end{cases} \tag{1.2}$$

For example, for $n = 3$, we find $\delta_3 = 1$ and $\varepsilon_3 = 1/5$, so $T_3 = \dfrac{3(7)(5)-1}{8} = 13$ and $Q_3 = \dfrac{3(8)(5)(11/5)}{8} = 33$.

These are some of the puzzles that have captured my interest during my enigmatic journey. For the collection of my conundrums that follow, I hope you get as much pleasure solving them as I had compiling them.

Solutions

Sam Loyd's neighbour puzzle

Figure 1.7 appeared in the January 1908 issue of *The Strand Magazine* [15].

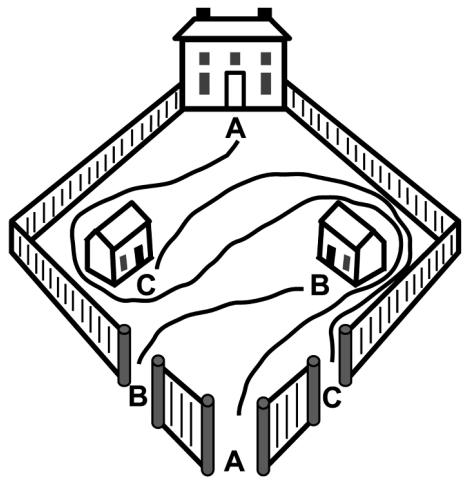

Figure 1.7 Solution to Sam Loyd's pathway puzzle.

Square dissection into eight acute triangles

Points P and Q lie outside the grey semi-circles which shows that the angles are acute (see Figure 1.8).

Professor Pembish's bath

The bath fills in 12 minutes. Let the volume of the bath be V, and the rates for the cold tap, hot tap, and drainage be r_c, r_h, and r_d, respectively. Then $r_c = V/3$, $r_h = V/4$, and $r_d = V/2$. Let the time for the bath to fill with all three rates working together be t. Then $(r_c + r_h - r_d)t = V$. Substituting the rates leads to cancellation of the V and $(1/3 + 1/4 - 1/2)t = 1$. So $t = 12$.

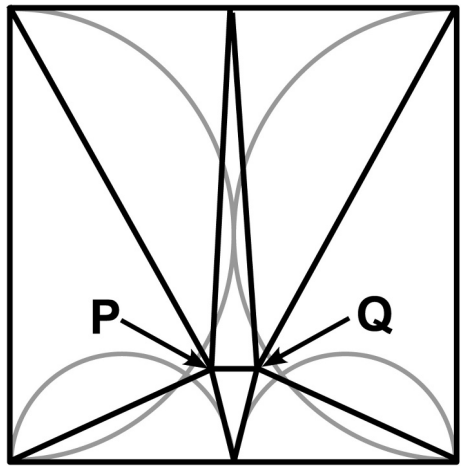

Figure 1.8 Square dissection into eight acute triangles.

Snowman

The month is June. The scene represents our solar system. The body of the snowman is the Sun and the head is Mercury. The planets with asterisks are the Earth, Jupiter, Uranus, and Neptune. We then take their initial letters. The added clever touch about this puzzle is that the title pun "snowman" can be interpreted as "it's no man," that is, what you are looking at is not a man.

Digital deletion sum

The numbers are 7868753, 8656865, and 9999988.

Mix-and-match

Table 1.2 Solution to the four by three mix-and-match puzzle

	Nickname	**Surname**	**Hometown**
1	Sugar	Holyhead	Boston
2	Basher	Freeman	New York
3	Rocky	McCool	Seattle
4	Iron	Tryson	Texas