Control and State Estimation for Dynamical Network Systems with Complex Samplings

Bo Shen
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Control and State Estimation for Dynamical Network Systems with Complex Samplings

This book focuses on the control and state estimation problems for dynamical network systems with complex samplings subject to various network-induced phenomena. It includes a series of control and state estimation problems tackled under the passive sampling fashion. Further, it explains the effects from the active sampling fashion, i.e., event-based sampling is examined on the control/estimation performance, and novel design technologies are proposed for controllers/estimators. Simulation results are provided for better understanding of the proposed control/filtering methods. By drawing on a variety of theories and methodologies such as Lyapunov function, linear matrix inequalities, and Kalman theory, sufficient conditions are derived for guaranteeing the existence of the desired controllers and estimators, which are parameterized according to certain matrix inequalities or recursive matrix equations.

- Covers recent advances of control and state estimation for dynamical network systems with complex samplings from the engineering perspective
- Systematically introduces the complex sampling concept, methods, and application for the control and state estimation
- Presents unified framework for control and state estimation problems of dynamical network systems with complex samplings
- Exploits a set of the latest techniques such as linear matrix inequality approach, Vandermonde matrix approach, and trace derivation approach
- Explains event-triggered multi-rate fusion estimator, resilient distributed sampled-data estimator with predetermined specifications

This book is aimed at researchers, professionals, and graduate students in control engineering and signal processing.

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This book is dedicated to the Dream Dynasty consisting of a group bright people who have been enamored with the challenging research on the control and estimation for dynamical network systems with complex samplings ……

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Preface

In response to the ongoing advances of digital medias, the sampling issue becomes vitally important in the control systems. Over the past decades, periodic sampling has received considerable research attention due to its advantages of easy implementation and analysis. The main feature of periodic sampling is that the state or output signals of a continuous-time system are sampled and then transmitted according to a fixed sampling period. However, in practical applications, it is often the case that the sampling period is time-varying and/or uncertain. This is referred to as complex samplings. Usually, the complex samplings can be classified into two categories, i.e., passive sampling and active sampling. The passive sampling relates to the case that the samplings occur in an uncertain way owing to some undesirable physical constraints such as aperiodic faults in the samplers, fluctuated network loads, intermittent signal quantization/saturation and unwanted changes of some components of the system itself. As for the active sampling, the sampling instants are determined only according to the specific engineering requirements such as energy saving. It is worth mentioning that the utilization of complex sampling mechanisms renders the traditional periodic-sampling-based control and estimation algorithm is no longer effective. Therefore, it makes both theoretical and practical sense to develop new techniques to address the control and estimation issues for various systems with complex samplings such that the required performance can still be guaranteed.

In this book, we focus on the control and state estimation problems for dynamical network systems with complex samplings subject to various network-induced phenomena. The network-induced phenomena under consideration include state/sensor saturations, quantization effects and sensor degradations. The dynamical network systems cover the general networked control systems, sensor networks, neural networks and complex networks. The main content of this book is identified mainly into two parts. In the first part (Chapters 2-3), a series of control and state estimation problems are tackled under the passive sampling mechanism, where sufficient conditions are derived in terms of matrix inequalities to guarantee the existence of the desired controllers and estimators. In the second part (Chapters 4-9), the effects from the active sampling mechanism, i.e., ET sampling, are examined on the control and estimation performance, where novel design technologies are proposed for controllers and estimators with the hope of saving energy while the acceptable control and estimation performances are still guaranteed.

Also, simulation results are provided to have a better understanding of the proposed control/estimation methods.

The compendious frame and description of the book are given as follows. Chapter 1 presents background on complex samplings as well as the recent advances of these topics. Chapter 2 is concerned with the stabilization and control problem for sampled-data systems under noisy sampling intervals. Chapter 3 studies the distributed H_∞ state estimation problem over sensor networks with nonuniform samplings under infinite-distributed delays, where the resilient issue is taken into account in order to accommodate the potential estimator implementation errors. For ET sampling, Chapter 4 addresses the ET control problem for two classes of switched systems with exogenous disturbances. In Chapters 5 and 6, the ET state estimation issues are investigated, respectively, for state-saturated systems and neural networks subject to various network-induced phenomena. In addition, the ET robust fusion estimation problem is investigated in Chapter 7 for multi-rate systems with stochastic nonlinearities, coloured measurement noises and sensor degradations. Chapter 8 is concerned with the dynamic ET synchronization controller design problem for delayed complex networks. In this chapter, a new discrete-time version of the dynamic ET mechanism is put forward to further save energy. Chapter 9 examines the filtering and state estimation problems for several classes of dynamical systems under dynamic ET mechanisms. Chapter 10 presents the conclusion and some possible future research directions.

This book is a research monograph whose intended audiences are graduate and postgraduate students as well as researchers.

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Bo Shen received the B.Sc. degree in mathematics from Northwestern Polytechnical University, Xian, China, in 2003, and the Ph.D. degree in control theory and control engineering from Donghua University, Shanghai, China, in 2011.

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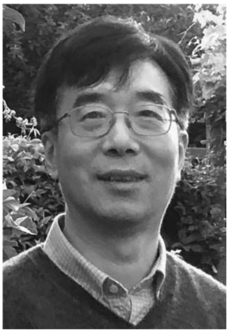
Professor Shen is a program committee member for many international conferences. He serves (or has served) as an Associate Editor or Editorial Board Member for nine international journals, including Systems Science and Control Engineering, Journal of The Franklin Institute, Asian Journal of Control, Circuits, Systems, and Signal Processing, Neurocomputing, Assembly Automation, Neural Processing Letters, IEEE Access, and Mathematical Problems in Engineering.



Zidong Wang was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics from Suzhou University, Suzhou, China, in 1986, and the M.Sc. degree in applied mathematics and the Ph.D. degree in electrical engineering from the Nanjing University of Science and Technology, Nanjing, China, in 1990 and 1994, respectively.

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Symbols

Symbol Description

$\mathbb{R}^n$	The n-dimensional Euclidean space.
$\mathbb{R}^{n\times m}$	The set of all $n \times m$ real matrices.
$\mathbb{Z}_-$	The set of all nonpositive integers.
$\mathbb{Z}_+$	The set of all nonnegative integers.
$\mathbb{N}_+$	The set of positive integers.
$\mathbb{R}_+$	The set of all nonnegative real numbers.
1_n	The n-dimensional vector whose elements are all 1.
$\\|A\\|$	The norm of matrix A defined by $\\|A\\| = \sqrt{\text{trace}(A^T A)}$.
$\\|A\\|_\infty$	The infinite norm of matrix A.
A^T	The transpose of the matrix A.
I	The identity matrix of compatible dimension.
ID	The identity function.
$\otimes$	The Kronecker product.
$\circ$	The Hadamard product.
$\diamond$	The composition of two functions.
$L_2[0,\infty)$	The space of square integrable vector functions.
$l_2[0,\infty)$	The space of square summable vector functions.
$\mathcal{C}([-d,0];\mathbb{R}^n)$	The family of all continuous $\mathbb{R}^n$-value functions ξ on $[-d,0]$ with norm $\\|\xi\\|_d \triangleq \sup\{\\|\xi(s)\\| : -d \leq s \leq 0\}$.
$\mathcal{K}$-function	The function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, strictly increasing and satisfies $\gamma(0) = 0$.
$\mathcal{K}_\infty$-function	The function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is a $\mathcal{K}$-function and also $\gamma(s) \to \infty$ as $s \to \infty$.
$\mathcal{KL}$-function	The function $\gamma(s,k) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is a $\mathcal{K}$-function for each fixed k and also decreasing with $\gamma(s,t) \to 0$ as $t \to \infty$.
$\mathcal{L}_\infty^n$	The set of all locally essentially bounded sequences $\nu : \mathbb{Z}_+ \to \mathbb{R}_+$ with norm $\\|\nu\\|_\infty = ess\sup_{k\geq 0} \\|v(k)\\| < \infty$.
$D^+\varphi(t)$	The upper right-hand Dini derivative of function $\varphi(t)$ with $D^+\varphi(t) = \limsup_{h\to 0^+} \frac{\varphi(t+h)-\varphi(t)}{h}$.
Prob(·)	The occurrence probability of the event "·".
$\mathbb{E}\{x\}$	The expectation of the stochastic variable x.
$\mathbb{E}\{x\|y\}$	The expectation of x conditional on y.

$\mathrm{Var}\{x\}$	The covariance of stochastic variable x.
$(\Omega, \mathcal{F}, \mathrm{Prob})$	A complete probability space.
$\lambda_{\min}(A)$	The smallest eigenvalue of a square matrix A.
$\lambda_{\max}(A)$	The largest eigenvalue of a square matrix A.
$*$	The ellipsis for terms induced by symmetry, in symmetric block matrices.
$\mathrm{tr}(A)$	The trace of a matrix A.
$X > Y$	The $X - Y$ is positive definite, where X and Y are real symmetric matrices.
$X \geq Y$	The $X - Y$ is positive semi-definite, where X and Y are real symmetric matrices.
$\mathrm{diag}\{\cdots\}$	The block-diagonal matrix.
$\mathrm{diag}_N\{A\}$	$= \mathrm{diag}\{\underbrace{A, \cdots, A}_{N}\}$.
$\mathrm{diag}_N^i\{A_i\}$	$= \mathrm{diag}\{A_1, \cdots, A_N\}$.
$\mathrm{vec}_N\{A\}$	$= \underbrace{\begin{bmatrix} A & \cdots & A \end{bmatrix}}_{N}$.
$\mathrm{vec}_N^i\{A_i\}$	$= \begin{bmatrix} A_1 & \cdots & A_N \end{bmatrix}$.
$\mathrm{vec}_N^i\{A_{ij}\}$	$= \begin{bmatrix} A_{i1} & \cdots & A_{iN} \end{bmatrix}$.
$\mathrm{col}_N\{A\}$	$= \underbrace{\begin{bmatrix} A^T & \cdots & A^T \end{bmatrix}^T}_{N}$.
$\mathrm{col}_N^i\{A_i\}$	$= \begin{bmatrix} A_1^T & \cdots & A_N^T \end{bmatrix}^T$.

List of Acronyms

ET	event-triggered
LMIs	linear matrix inequalities
SPs	stochastic parameters
DDSs	delayed differential systems
ZOH	zero-order hold
ROMDs	randomly occurring mixed delays
IMs	incomplete measurements
MJPs	Markovian jumping parameters
TVDs	time-varying delays
GRNs	genetic regulatory networks
mRNAs	messenger ribonucleic acids
PECs	prediction error covariances
FECs	filtering error covariances
CMs	censored measurements
PUs	parameter uncertainties
CI	covariance intersection

1

Introduction

1.1 Background

In recent years, in response to the rapid development of digital technology, sampled-data control systems have attracted recurring research attention both from academic research and industrial application. Typically, a sampled-data control system consists of a continuous-time plant and a discrete-time controller, which are connected together in feedback by the sampler and hold devices. A general architecture of a sampled-data control system is illustrated in Fig. 1.1. In this system, the sensor first converts a physical stimulus of the plant into a readable measurement $y(t)$ and then the sampler takes samples of $y(t)$ to yield the corresponding discrete-time measurement $y(t_k)$, where t_k $(k = 0, 1, \ldots)$ are sampling instants. Subsequently, the controller uses the transmitted measurement $y(t_k)$ to generate a discrete-time control input $u(t_k)$, which is further converted back into a continuous-time control input signal $u(t)$ by using the hold. Finally, the input signal $u(t)$ would be sent to the actuator to control the plant. In reality, the functions of the sampler and the hold are, respectively, implemented with the help of an analogue-to-digital (A/D) converter and a digital-to-analogue (D/A) converter. In fact, due to its practical backgrounds and wide applications, the analysis/synthesis of sampled-data control systems has long been a hot topic of research with a number of excellent results reported in the existing literature, see [16,73,193,209] and the reference therein.

In classic sampled-data control theory, the sampling intervals defined by $T_k \triangleq t_{k+1} - t_k$ are usually expected to be an invariant constant (i.e. $T_k = T > 0$) for its simplicity in analysis, design, and implementation. However, under certain undesirable physical environments such as the unsteady power voltage, the sampler may tremble slightly, and the practically implemented sampling intervals often fluctuate around a nominal sampling period. This is referred to as nonuniform sampling. It is worth mentioning that the nonuniform sampling is essentially performed in a *deterministic* manner. Actually, in many cases such as seismic data extraction, it is quite common that the sampling occurs in a *probabilistic* way, and accordingly, the sampling intervals are stochastic due to unavoidable noises. In such cases, it seems natural to describe the sampling intervals T_k by a random variable obeying certain probability

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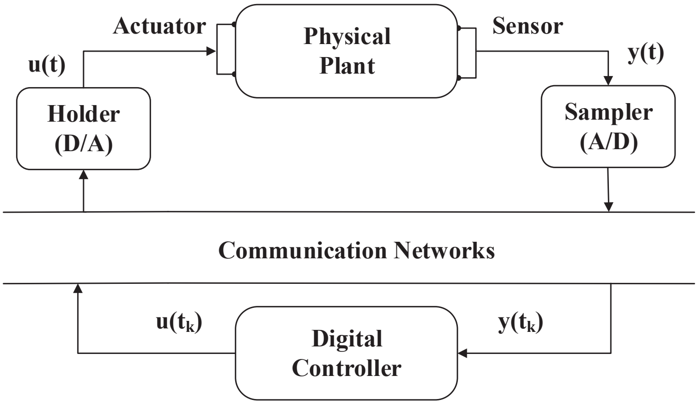

FIGURE 1.1
A general architecture of a sampled-data control system.

distribution. Note that both nonuniform sampling and stochastic sampling can be regarded as passive sampling, which occurs in an uncertain way owing to some undesirable physical constraints such as aperiodic faults in the samplers, fluctuated network loads, and unwanted changes of some components of the system itself.

On the other hand, when resource limit becomes a concern, the so-called ET (or event-based) sampling method has served as an ideal way for energy-saving purposes. The main idea of the ET strategy lies in that the sampling/transmission is executed only if a predefined triggering condition is satisfied, thereby effectively reducing unnecessary transmissions. In view of such an advantage, ET sampling has become more and more popular with wide application potentials [22, 28, 101, 191, 197, 215]. Roughly speaking, the existing triggering conditions can be typically divided into two categories, one is the absolute change triggering condition and the other is the relative change triggering condition. It is noticeable that, in recent years, the ET sampling scheme is under continuous improvement with aim to further reduce the energy consuming, and thus some improved versions of ET strategy have been proposed. For example, a dynamic ET sampling method initiated in [50], which introduces an additional dynamical variable in the typical triggering condition, has recently aroused particular research interests due to its capability of saving resource even further without significantly degrading the system performance. In addition, it also has been shown in [50] that the dynamic ET sampling is more general, which contains the traditional ET one as a special case. So far, a large number of research results have been reported on the investigation of dynamic ET sampling; see e.g. [46, 60, 174] for some latest literature.

Several typical mathematical models for the complex samplings mentioned above, including nonuniform sampling, stochastic sampling, ET sampling, dynamic ET sampling, are shown in Table 1.1.

Types	Mathematical models
Nonuniform sampling	The sampling interval is variable but bounded, i.e., $T_k \in [T_m, T_M]$, where T_m and T_M are known constants.
Stochastic sampling	The sampling interval $T_k = T + v_k$, where T is a constant and v_k is a random variable.
ET sampling	• Absolute change triggering condition: $t_{k+1} = \inf\{t \mid t > t_k \wedge (y(t) - y(t_k))^T(y(t) - y(t_k)) \geq \sigma\}$; • Relative change triggering condition: $t_{k+1} = \inf\{t \mid t > t_k \wedge (y(t) - y(t_k))^T(y(t) - y(t_k)) \geq \sigma y^T(t)y(t)\}$; where σ is a given positive scalar.
Dynamic ET sampling	• Absolute change triggering condition: $t_{k+1} = \inf\{t \mid t > t_k \wedge \big(y(t) - y(t_k)\big)^T\big(y(t) - y(t_k)\big) \geq \frac{1}{\theta}\eta(t) + \sigma\}$ with $\eta(t)$ satisfying $\dot{\eta}(t) = -\lambda\eta(t) + \sigma - \big(y(t) - y(t_k)\big)^T\big(y(t) - y(t_k)\big)$; • Relative change triggering condition: $t_{k+1} = \inf\{t \mid t > t_k \wedge \big(y(t) - y(t_k)\big)^T\big(y(t) - y(t_k)\big) \geq \frac{1}{\theta}\eta(t) + \sigma y^T(t)y(t)\}$ with $\eta(t)$ satisfying $\dot{\eta}(t) = -\lambda\eta(t) + \sigma y^T(t)y(t) - \big(y(t) - y(t_k)\big)^T\big(y(t) - y(t_k)\big)$; where λ, θ and σ are given positive scalars, $\eta(0) \geq 0$ is the initial condition.

TABLE 1.1
Several typical mathematical models of complex samplings.

Nowadays, with the ongoing advances of network technologies, more and more data information of the system components (i.e., sensor, sampler, controller, and actuator) are exchanged through communication networks. Note that the usage of networks has many distinctive advantages such as low installation and maintenance costs, easy manipulation, increased system flexibility, and reduced hardwire. Such advantages have given a great impetus to extensive applications of communication networks in various practical areas such as modern automobiles, wastewater treatment processes, power systems, and transportation systems. Nevertheless, owing primarily to the inherent network constraints such as limited bandwidth, some networked-induced phenomena arise inevitably during the signal transmission. These networked-induced phenomena include, but are not limited to, packet dropouts, saturations, quantizations, and sensor degradations. It is generally acknowledged that these phenomena, if they are not appropriately tackled, could deteriorate the system performance or even result in the instability of the overall dynamical systems. Bearing these in mind, it makes practical sense to look into the effects from the networked-induced phenomena on the sampled-data system (see Fig. 1.1), where the closed-loop sampled-data control is implemented with the help of communication networks to transmit the measured output and the control input signals. Therefore, the aim of this paper is to deal with the control and state estimation problems for complex sampled systems subject to various networked-induced phenomena.

1.2 Recent Advances

In this section, we shall give an overview of the results on complex samplings including nonuniform sampling, stochastic sampling, ET sampling and dynamic ET sampling.

1.2.1 Nonuniform Sampling

In networked and embedded control systems, it is often the case that the sampling intervals are uncertain and/or vary with time due to unpredictable network-induced phenomena. Recently, the analysis/design issues for such nonuniform sampled-data control systems have received considerable research attention. Generally speaking, there have been three different approaches categorized according to the types of the transformed equivalent models of the sampled-data control systems, i.e., 1) continuous-time systems with a delayed control input, 2) discrete-time systems, and 3) hybrid modelling of sampled-data systems.

Up to now, the first approach has been extensively investigated for nonunifrom sampled-data systems. The main idea of this approach is to transform the sampled-data system to a continuous-time counterpart with a bounded TVD. Specifically, if the current sampling instant is t_k and the next sampling instant is t_{k+1}, then the control input between the sequel sampling instants is $u(t) = u(t_k)$, $t \in [t_k, t_{k+1})$. Setting $u(t) = u(t - (t - t_k))$, the underlying sampled-data system can be equivalently transformed to a time-varying input delay system with the delay defined by $\tau(t) = t - t_k$. This approach was proposed in [37], where an LMI-based criterion of the stability for the sampled-data systems has been derived. By using this approach, a number of significant results can be referred to [41, 199, 200], where the stability analysis and control problems have been studied for sampled-data control systems with nonuniform samplings. After that, in [40,116], the scaled small gain theorem has been applied to establish a new simple stability condition which improves the results of [37]. Moreover, the input delay approach has been refined in [36, 100, 131] by introducing a proper time-dependent Lyapunov-Krasovskii functional, where the major contribution lies in that much less conservative stability criteria have been obtained than those [40, 116]. Inspiringly, recent years also have seen a quite frequent use of such an input delay approach for the filtering (or state estimation) issue for nonuniformly sampled systems, see e.g. [4, 8, 44, 45, 57, 71, 83, 109] and the reference therein. More recently, the sampled-data state estimation problem with nonuniform samplings has been examined in [71,109] for a class of Takagi-Sugeno fuzzy systems.

In the past decade, the second approach with discrete-time transformation has also received particular research interests because of the exact integration over a sampling interval leading to less conservative stability conditions. In this approach, a discrete-time controller design would be performed based on a discretized model of the continuous-time system. For example, in [39,150], the stabilization problem of nonuniform sampling systems has been investigated by modelling the considered plant as a linear discrete-time system, where the sampling interval variations are treated as norm bounded matrix uncertainty which is handled by using robust control techniques. Following these works, in [124], an aperiodic sampling has been modeled as parametric uncertainty, rather than matrix uncertainty for less conservative stability analysis, and an LMI-based sufficient condition has been presented accordingly for all possible parameter values. Further improvement can also be found in [69], where the major contribution is to refine the stability conditions proposed in [39,150] by taking into account the positive real property of the time-varying uncertainty in the system. Moreover, the studies in [159] have extended the results in [39,150] to the realm of nonlinear systems with the consideration of large TVDs and packet dropouts. It is worth pointing out that, in [205,210], a novel approach to dealing with the sampled-data filtering problems has been proposed by converting the nonuniform sampling system into a discrete-time switched system with a finite number of subsystems, and then the corresponding distributed H_∞ filtering problem has been studied by restoring to the average dwell time method for switched systems.

A lot of work has been done by following the third approach as well. This approach is based on the representation of the aperiodic sampled-data system in the form of hybrid discrete/continuous model, or more precisely, the impulsive model. In the seminal work [119], by modelling the nonuniformly sampling system with an impulsive system, some sufficient conditions have been derived for asymptotic and exponential stability of such impulsive systems by using a Lyapunov functional with discontinuity at impulsive instants. Later, these results have been extended in [120,158] to the case that the network TVDs are considered; in [10] to the case that the try-once-discard/round-robin protocols are considered; and in [145] to the case that the input saturations are considered. Subsequent work in [19] has provided a less conservative stability condition than the one in [119] by using a piecewise differentiable Lyapunov functional, which is continuous at impulse times but not necessarily positive definite inside the impulse intervals. Besides, in [74,82], by using the conception of average impulse interval, new stability criteria with no restrictions on the bounds of sampling intervals have been obtained for the nonuniform sampling systems. Similarly, the impulsive system approach has also been applied to study the sampled-data filtering issues. For example, the design problem of the impulsive observer has been investigated in [34] for a class of continuous-time dynamical systems with nonuniformly sampled measurements. In [98], the performance index in terms of H_∞ has been proposed, and the corresponding sampled-data filtering issue has been solved. Some other typical results along this approach can be found in [3,12,18,32,113].

1.2.2 Stochastic Sampling

In practice, due to some undesirable physical constraints, the aperiodic samplings often occur in a probabilistic way. In this regard, it seems natural to model the sampling intervals by a stochastic process. For example, [42] has considered two different sampling intervals whose occurrence probabilities are given to be constant and satisfy Bernoulli distribution. In this work, an H_∞ control method has been proposed to ensure both the mean-square exponential stability and H_∞ performance of the sampled-data systems. Following the idea in [42], much effort has been devoted to the system analysis/design problems with such stochastic samplings and many results have been obtained, see e.g. [5, 27, 29, 52, 80, 90, 103, 111, 139, 140, 144, 148, 180, 181, 211]. In [181], the H_∞ filtering issue has been investigated for continuous-time systems under sampled measurements with stochastic sampling, where the input delay approach and Lyapunov theory have been utilized to obtain the stability conditions in the form of LMI. In [139], the distributed H_∞ filtering problem for sensor networks has been investigated by using a stochastic sampled-data approach. The sampled-data synchronization control for complex networks with stochastic sampling has been discussed in [140]. The authors in [52] have considered the consensus under stochastic sampling for multiagent systems and have shown that the results are independent of the number of agents thus facilitating its application to large-scale networked agents.

In recent years, many researchers have attempted to further extend the two different sampling intervals to a more general case of multiple stochastic sampling intervals, see e.g. [56, 75, 76, 127, 129, 202]. The so-called multiple stochastic sampling intervals mainly refer to that the sampling intervals take values in a finite set with m different values (rather than two different values) and switch among these values in a random way with given probability. Based on such a stochastic sampling model, the stochastic sampled-data synchronization issue has been studied in [128] for complex dynamical networks with control packet loss and additive TVDs. For consensus issue of multi-agent systems with stochastic sampling, in [162], a sufficient condition for the mean square node-to-node consensus has been presented in terms of LMIs. A study of exponential synchronization of chaotic Lur'e systems with stochastic sampling has been presented in [208], where a unified probability framework has been proposed to design the synchronization controllers. In [220], a consensus protocol has been proposed in which each Euler-Lagrange system employs the stochastic sampled information to communicate with its neighboring nodes, and it has been proved that the consensus can be reached without utilizing relative coordinate derivatives.

Another way to describe the sampling intervals is treating them as random variables obeying certain probability distributions. For example, in [2], the sampling interval has been modeled by a stochastic variables following Erlang distribution, and a stochastic dynamic programming framework has been used for the solution of the optimal control problems. Following a similar line,