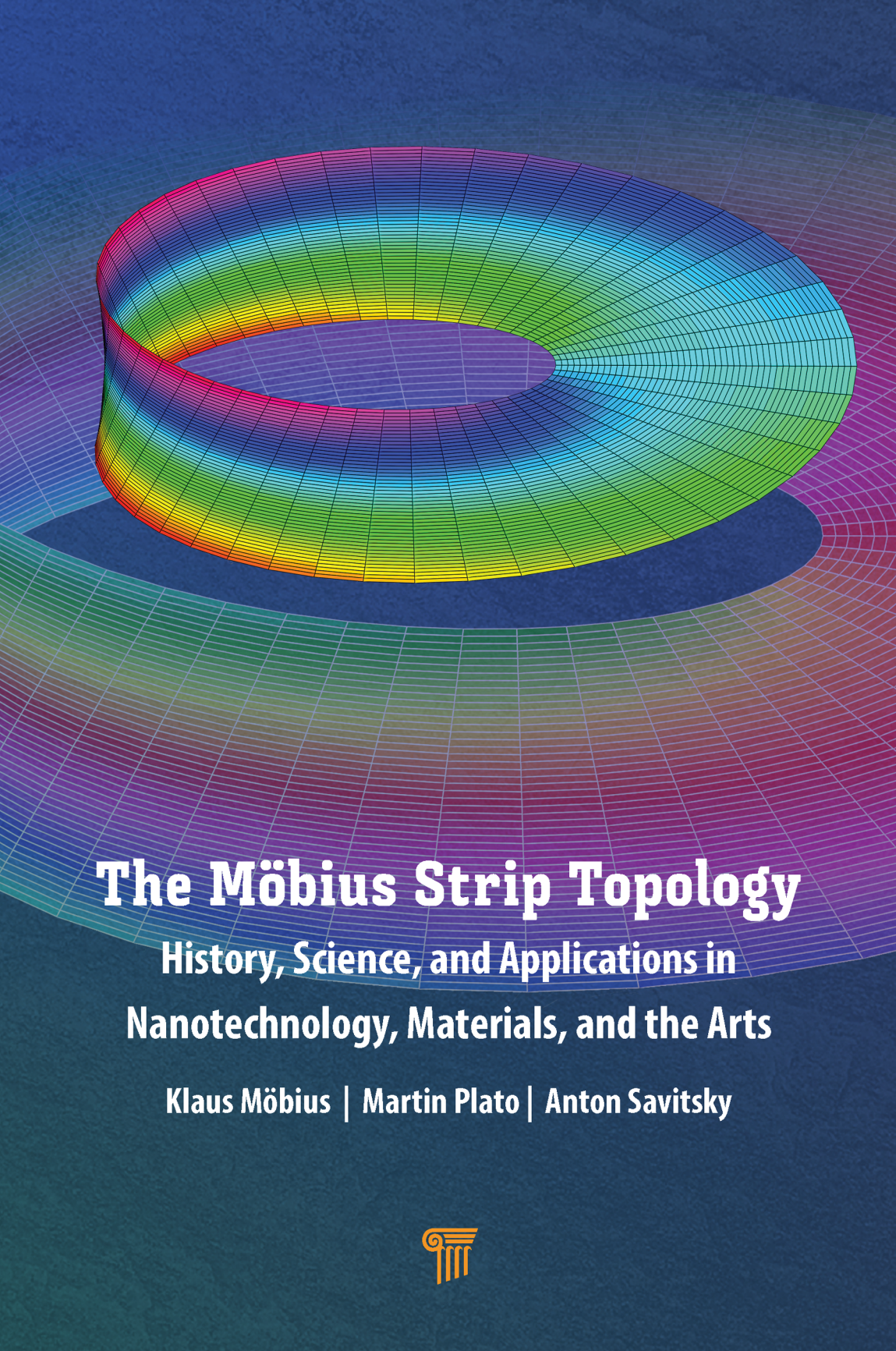
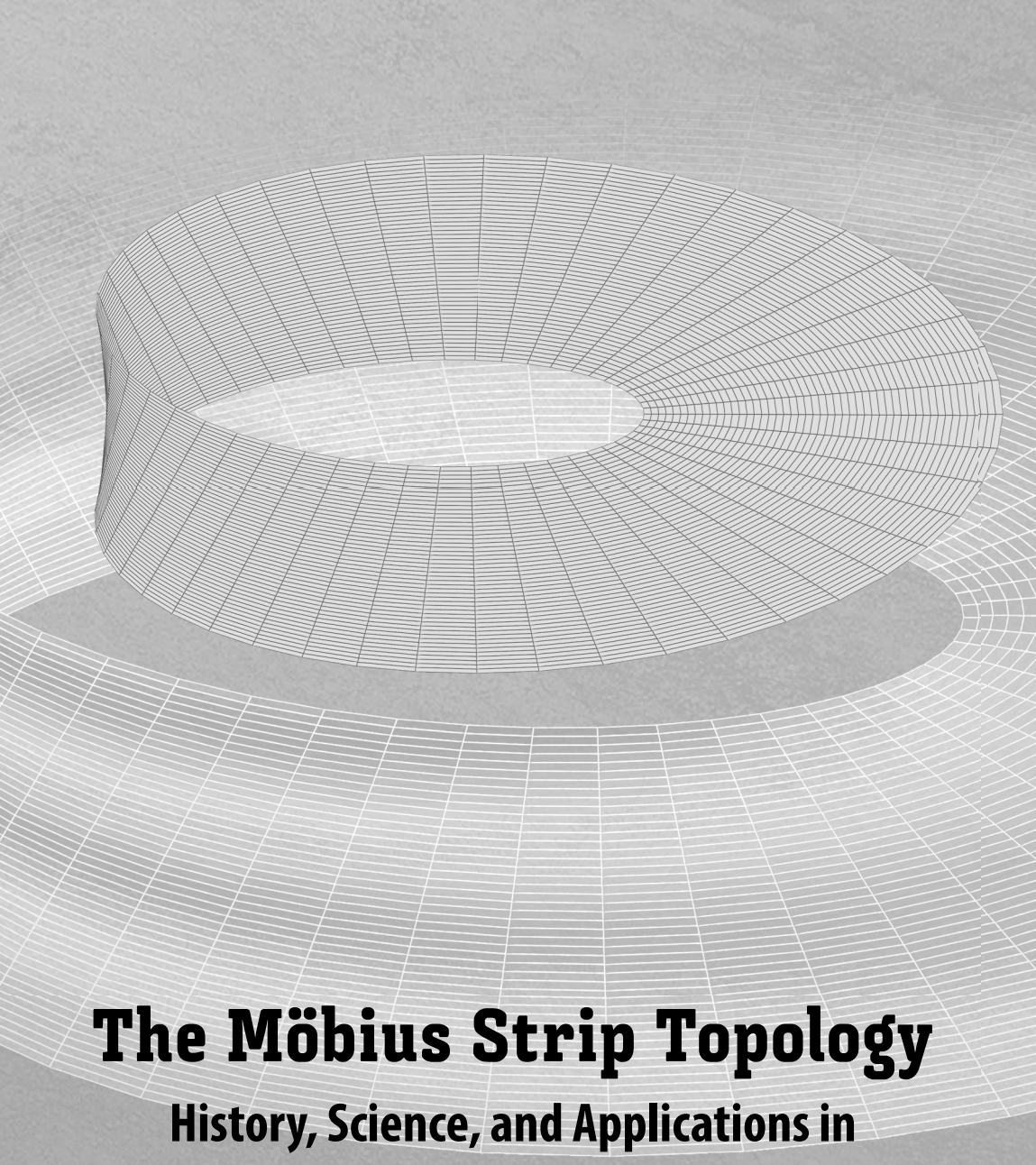


The Möbius Strip Topology

History, Science, and Applications in Nanotechnology, Materials, and the Arts

Klaus Möbius | Martin Plato | Anton Savitsky



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Acknowledgments

Since June 2015, when our correspondence with Jenny Stanford Publishing (formerly Pan Stanford Publishing) started with the invitation to write a book on Möbius Strip Topology based on our magnetic-resonance research on Hückel–Möbius aromaticity switching in chemistry, the content of the book has substantially extended: It should now include Möbius topology applications also in other fields of science, in the arts and architecture as well as give historical perspectives. We are grateful to Ms. Jenny Rompas and Mr. Stanford Chong from Jenny Stanford Publishing for having encouraged us for this extension and their continuing support for more than 5 years, which we finally needed to realize this book project.

The historical parts of the book naturally include biographical observations about our protagonist, August Ferdinand Möbius (1790–1868), a distinguished mathematician and astronomer in Leipzig. We also describe the time in which he lived in. We are particularly thankful to André Loh-Kliesch, Leipzig, for having sent us his PhD dissertation (*August Ferdinand Möbius, Leben und Werk*, Universität Leipzig, 1994) as an authentic and often unique historic source for Möbius's biography. Decisive periods of his life were marked by the Napoleonic Wars in Central Europe with subsequent wars of liberation and social revolutions. After initial successes, the democratic uprisings were put down with military force. What followed was the restauration period of the absolutist monarchies with their wide-ranging censorship and suppression of the request for democratic freedoms. All this had serious consequences also for the development of sciences and societies in Central Europe. For the assessment of these historical consequences in mathematics, André Loh-Kliesch's dissertation was very helpful. We acknowledge with gratitude the scientific cooperation with Lechosław Latos-Grażyński and his coworkers (University of Wroclaw) who were the first to synthesize expanded porphyrin molecules with Möbius–Hückel topology switching capabilities—as we describe in one of the Case

studies of the book. We thank Takeji Takui (Osaka City University) for his help in the evaluation of a popular Japanese novel whose plot follows the structure of a Möbius strip with a twist of the characters as they move through the loop of the story. We are indebted to Lorenz Haarmann (Berlin) for gifting us his magnificent photos of the Phoenix Media Center in Beijing, which was recently built in the form of a giant Möbius–Strip torus.

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Preface

The intention of this *Preface* is to give some latest news headlines from 2021, a *postscriptum* of our book in a way, which are related to August Ferdinand Möbius (1790–1868) and his heritage as mathematician and astronomer.

For more than one and a half thousand years, long before the discovery of the "Möbius strip" as a topological figure with a one-sided surface in the 19th-century mathematics, the mysterious symmetry behavior of half-twisted ribbons has played a role in Western and Eastern cultures as a symbol of unity despite diversity. Recently, the Möbius strip has received an unexpected actualization and visibility as the logo of the European Union (EU) Council Presidency by Germany on July 1, 2020, for a period of 6 months (see Fig. 1). Germany took over the Presidency in the midst of the first global wave of the Corona Covid-19 pandemic. The EU logo designed for this period, a Möbius strip representation, symbolizes an "inclusive and innovative Europe in which the most diverse people and interests come together to form a common entity," according to the official statement by the German government.

August Ferdinand Möbius was not only an outstanding mathematician but also a renowned astronomer. For example, in February 1822, Möbius observed for a month the opposition of Mars and Earth. This was a real astronomical spectacle: During the orbits of planets Earth and Mars around the Sun, their different orbiting velocity leads to the "opposition" situation in which the two planets are on opposite sides of the Sun. During opposition, Mars and Sun are on directly opposite sides of Earth. From our perspective, Mars rises in the East just as the Sun sets in the West. Then, after staying up in the sky the entire night, Mars sets in the West just as the Sun rises in the East.

August Ferdinand Möbius would certainly have liked to receive the following spectacular news of planet Mars from the future, 199 years ahead:

Figure 1 *Left*: Möbius-strip logo of the European Union Council Presidency by Germany from July 1 to December 31, 2020. It is a symbol of unity and connectedness. Because even if you start on the Möbius strip from different sides and in different directions, you will still meet on the same side: there exits only one side. The EU member states have voluntarily associated themselves across all differences, national interests, and disagreements to ultimately move forward together on the democratic European course. In the Presidency Program "*Together for Europe's recovery*", the focus is put on overcoming the Covid-19 pandemic and seeking answers to the challenges of the climate change mitigation. *Right*: Angela Merkel, from November 22, 2005, to December 8, 2021, Chancellor of the Federal Republic of Germany, in times of Corona pandemic.

After a 480-million-kilometer flight to Mars, the lander vehicle carrying the *Perseverance* rover unit together with its small *Ingenuity* helicopter was announced to have successfully landed in *Jezero Crater* of Mars at 20:55 UTC on February 18, 2021; the landing signal from Mars took 11 min to arrive at Earth. The landing of the *Perseverance* rover on Mars is part of the "Mars 2020 Mission" by NASA; it is NASA's fifth Mars rover mission in the frame of the *Mars Exploration Program*. Using an Atlas V launch vehicle, the Mars lander unit was launched from Cape Canaveral on July 30, 2020.

Perseverance was designed to study Mars rocks in detail with respect to bio-signatures, geological processes, and the planet's geological history in order to gain, among other things, insights into possible earlier life on Mars (see Fig. 2).

August Ferdinand Möbius's legacy in mathematics and astronomy has turned into our daily experience of science and technology. Therefore, we focus the present book on typical examples of current activities in science, art, and architecture; they are examined against the background of political events and social and cultural tendencies.

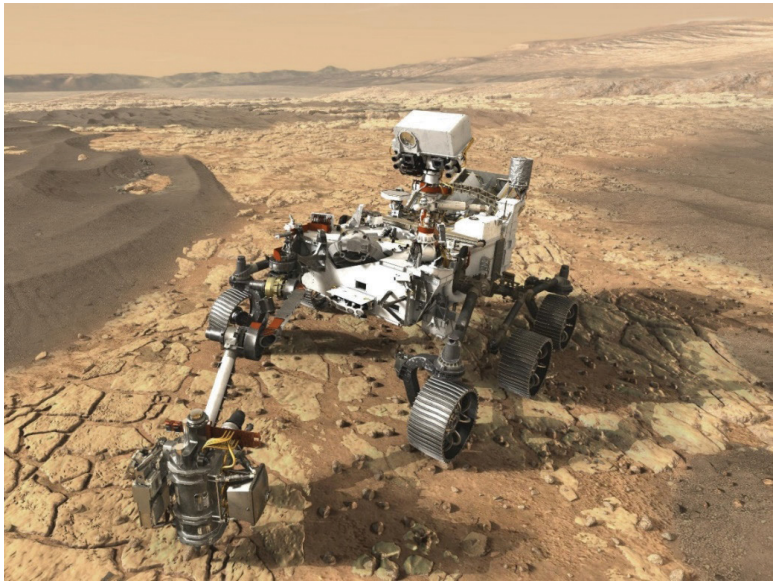

Figure 2 NASA's newest robotic explorer *Perseverance* landed safely on Mars in the *Jezero Crater* on February 18, 2021, ready to begin seeking for signs of ancient life. The approach and landing maneuvers of the *Perseverance* on Mars had to be fully automated in advance because of the long signal propagation time of about 11 min between Earth and Mars. The landing stage was able to detect any unforeseen obstacles that might appear on the surface of Mars and, if necessary, to change the landing site with sideways movements of up to 300 m. *Perseverance* runs on nuclear power batteries because on Mars, global dust storms render solar panels useless. (Image Credit: NASA/JPL-Caltech).

Klaus Möbius
Martin Plato
Anton Savitsky
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Introduction

August Ferdinand Möbius (1790–1868) and the half-twisted Möbius loop: a topological paradigm of one-sided surfaces

> "*What makes a great mathematician? A feel for form, a strong sense of what is important. August Ferdinand Möbius had both in abundance. He knew that topology was important. He knew that symmetry is a fundamental and powerful mathematical principle…. His mathematical taste was imaginative and impeccable. And, while he may have lacked the inspiration of genius, whatever he did he did well, and he seldom entered a field without leaving his mark…. No body of deep theorems, but a style of thinking, a working philosophy for doing mathematics effectively and concentrating on what's important. That is Möbius's modern legacy. We couldn't ask for more.*"

(Ian Stewart in John Fauvel et al., *Möbius and His Band*, Oxford University Press, 1993). Ian Stewart (born 1945) is a British mathematician at the University of Warwick, UK. He received the prestigious Michael Faraday Prize of the Royal Society in 1995 for his exceptional contributions to the popularization of mathematical sciences).

It is *Möbius's modern legacy* that has fascinated us during the years of doing research for our book project "*The Möbius Strip Topology: History, Science, and Applications in Nanotechnology, Materials, and the Arts*." While doing this work, we concluded that we should not try to write a biography about the mathematician August Ferdinand Möbius, a former pupil of Gauss, in the traditional manner, in which a historic account of the protagonist's life from birth to death is presented chronologically, with a grouping of noteworthy events around the main character. Rather, we should report on August Ferdinand Möbius in the mirror of his time with its political and social peculiarities—and in the context of his contemporaries, especially Carl Friedrich Gauss and Alexander von Humboldt, whom he admired and who strongly influenced him. Who played a major role in creating an intellectual environment for the abstract sciences such as mathematics and astronomy in Germany and beyond, in which intellectual achievements like topology were produced. In fact, Möbius's legacy reaches into today's modern sciences and arts and is being rediscovered time and again. Thus, we consider August Ferdinand Möbius not so much as the sole protagonist of our book, but rather as a representative of civil society in the period between the 18th and 19th centuries when a fundamental change took place in the political, scientific, and cultural world of Central Europe.

The one-sided Möbius strip (or band) is a paradigmatic example of the fascination and significance of mathematical topology for understanding unexpected consequences of complex object formation. In 2021, the Möbius strip has its 160th birthday, i.e., it is not really a young object, but still going strong in its vibrancy. The birth of the Möbius strip was accompanied by the birth of topology (from the Greek τόπος, place, and λόγος, study); this branch of mathematics is concerned with the invariants of objects that are preserved under their continuous deformations such as stretching, crumpling, and bending.

Over the years, topology grew and grew and became an important branch of abstract mathematics, with significant impact in almost all

areas of the sciences and humanities. The study of topology does not only introduce new concepts and theorems in abstract mathematics but also creates fascinating practical applications of strange symmetry aspects for weird geometrical objects and underlying principles, for instance in science, art, and architecture.

For the preservation of August Ferdinand Möbius's scientific legacy, the Saxon Society of Sciences had established a Möbius archive in Leipzig, which contained Möbius's rich correspondence with other mathematicians and astronomers. Sadly, this archive was burned in 1943 due to the effects of World War II, actually, by allied area bombing. A detailed appreciation of the scientific work of A. F. Möbius is given (in German) in the doctoral dissertation of André Loh at the University of Leipzig from the year 1994, entitled "*August Ferdinand Moebius (1790–1868) – His life and work*" [1]. In this excellently researched biography of Möbius, the author discusses in detail his major works, books, and essays, as well as Möbius's influence on the development of astronomy and mathematics. Many testimonies on polyhedra, the fundamental geometric object for the development of topology, were first compiled by Johann Benedict Listing, another pupil of Gauss, in his "*Der Census räumlicher Complexe oder Verallgemeinerung des Euler'schen Satzes von den Polyedern*" (A census of spatial complexes, or a generalization of Euler's theorem on polyhedra) (1861), as well as somewhat later but independently, by August Ferdinand Möbius in his "*Über die Bestimmung des Inhalts eines Polyeders*" (On the determination of the volume of a polyhedron) (1861, 1865).

The dissertation of André Loh is the first comprehensive Möbius biography describing the life as well as the scientific oeuvre of the eminent Leipzig astronomer and mathematician; hereby it combines socio-historical and ideas-historical approaches to the subject. In doing so, the strong influence of adverse external life circumstances caused by the Napoleonic Wars and the subsequent political reorganization of Central Europe on the scientific work of a scholar in the first half of the 19th century is presented in a comprehensive and convincing manner. Loh's evaluation of archival documents and publications that have not been addressed so far provides a number of revealing details, which are highly significant for the historical positioning of August Ferdinand Möbius in his time.

The description of Möbius's life and work by André Loh follows his academic *Curriculum Vitae* from a student of a Prince College in Schulpforta (near Naumburg at the river Saale), to a full professor at the University of Leipzig. It also covers his more than 50 years of activity as a researcher and teacher in his various periods of creativity. There is considerable coverage of Möbius's family and collegial environment, which was formative for him over the years in crucial stages of life.

As you would expect, the book we are presenting is not the first book *in English* on the history of August Ferdinand Möbius and his topological masterstroke, what is now called the Möbius strip (or band). It is, in fact, the third book in English to sketch his contributions to mathematics and astronomy, all embedded in 19th-century Central Europe's history, society, and science. Specifically, we refer here to the two earlier books [2, 3].

Actually, there is still another earlier book (2013) on the history of August Ferdinand Möbius; it was written *in French* by the French historian of geometry Dominique Flament, and the title is "*August Ferdinand Möbius: Entre polyèdres et corrélation élémentaire*" (August Ferdinand Möbius: Between polyhedra and elementary correlation) [4]. In his elaborate study, Flament examines the evolution of the idea of the polyhedra from the Platonic solids to the Möbius band. He summarizes that when looking at the transformations the polyhedron has undergone since its first state as a "Platonic solid," it can be seen that the Möbius strip actually belongs to the great conceptions of the theory of polyhedral and leads to the "topological toy" that still fascinates us today.

The previous books by Fauvel et al. and Pickover are still great reading experiences, which have lost nothing of their charm and topicality since the years of their publication (1993 and 2006, respectively). On the other hand, in the meantime, a large number of modern research results and breathtaking applications of Möbius strip topology in chemistry, physics, and even in art and architecture have appeared, either in specialized journals or in public media, and we have tried to include representative examples of them to a certain detail in our book.

Furthermore, we studied in a comprehensive framework of historical facts the political conditions in Central Europe of the 19th century, where and when our protagonists lived. Over the

last 5 years, we have carried out intensive Internet research in international historical encyclopedias, libraries, and archives, made virtual and real local museum visits, where traces and testimonies of August Ferdinand Möbius and his contemporaries in science are still preserved and accessible, to elucidate the details of the historical processes and try to understand the actions (or nonactions) of our protagonists. Unfortunately, since 2020, this historical source search has become much more difficult by the global Corona pandemic with the subsequent lockdowns of public institutions. Generally, their websites in the Internet remained accessible; they often reveal, however, only the standard, mostly rather patriotic, views of the 18th- and 19th-century kings and emperors, army generals, and field marshals. In rare cases, they also contain descriptions of the suffering of the local people under the Napoleonic Wars: With the unimaginable cruelties of the soldiers of the mercenary armies, gigantic death tolls and destructions, particularly in the central German states Saxony and Prussia, where many characters of our book lived.

In view of the impression we got from our historical research, we were wondering why this specific 19th-century period of Napoleonic Wars and forced invasion—and the concomitant renunciation of all humanistic values—was so little reflected in the self-testimonies of the mathematical elites in Germany of that time, first and foremost by Carl Friedrich Gauss and his student August Ferdinand Möbius. Möbius's academic education and subsequent academic career took place in Leipzig, in the immediate vicinity of the battlefields of the Napoleonic Wars, like those of Jena-Auerstedt (1806) and of the Battle of Nations (1813) near Leipzig.

These questions and many facets of the historical events in the period from 1790 to 1868, in which August Ferdinand Möbius's academic education and his academic career took place, are explored in Chapters 2, 3, and 4.

The historical sources are full of appreciations of the resurrection of the German universities and pure sciences, in particular abstract mathematics, after the Napoleonic regime of war and subjugation. The rise largely occurred as a result of the acclaimed Stein–Hardenberg reforms of the civil societies in Prussia and other German states. On the cultural level, they were initiated and assisted by then very popular Wilhelm von Humboldt and his younger brother Alexander

von Humboldt. The Stein–Hardenberg reforms, introduced in the years 1807–15, created the basis for Prussia's transformation from an absolutist and agrarian state to an enlightened national and industrial state.

The 18th and 19th centuries are often viewed as the centuries of revolutions. In America, 1776 marks the end of the American revolution. In France, 1789 marks the end of the French Revolution, and soon after, the period of the Napoleonic regime is heralded. Napoleon Bonaparte (1769–1821) temporarily gained dominion over large parts of continental Europe, enforced by his mercenary *Grande Armée*.

In the German states and Austrian Empire, 1848 marks the revolutionary events of the liberal, bourgeois-democratic and national rebellions for national unity and independence and against the restructuring efforts of the Holy Alliance, to which the ruling houses in large parts of Central Europe had allied themselves from 1815 onward (Congress of Vienna). From Berlin to Vienna, the 1848 Revolution forced the appointment of liberal governments in the individual German states and the conduct of elections to a constituent National Assembly. It convened on May 18, 1848, in the *Paulskirche* in the then Free City of Frankfurt (Main).

By July 1849, the first attempt to create a democratically constituted, unified German nation-state was put down by the Holy Alliance with military force, predominantly by Prussian and Austrian troops. The subsequent persecution of supporters of a liberal, republican-democratic or socialist ideology caused tens of thousands to flee the German states in the years after 1848–49. They initially found asylum mainly in France, England, or Switzerland.

These political and social revolutions in Central Europe are to be seen in the context of the global Industrial Revolution where scientific methods were being applied to increase productivity and life standard. Both France and Germany were caught up in the age of revolution, but the two countries reacted quite differently. The 18–19th century period saw an impressive rise in the sciences and technologies. For example, an increase in breadth and complexity of mathematical concepts unprecedented in the history of mathematics.

After the French Revolution, Napoleon emphasized the practical usefulness of mathematics in times of war and preparations of new ones. And his military ambitions gave French mathematics a big

boost in Applied Mathematics, as exemplified by the giants Lagrange, Laplace, Legendre, and Fourier.

Germany, after the crushing defeat of Prussia and its allied German constituent states by Napoleon's *Grande Armée* at the double battle of Jena and Auerstedt in 1806, took a different approach to implementing pressing scientific and cultural reforms. Under the influence of the great humanist and cosmopolitan scholars Wilhelm and Alexander von Humboldt, Germany was supporting, for example, pure learning and research in pure mathematics for its own sake, basically detached from the demands of the military leaders for extended applications of mathematics in modern armament development. Through Humboldt's initiatives, the Berlin University was founded in 1810. Even today, this university is considered the "mother of all modern universities." Wilhelm von Humboldt advocated a "*Universitas litterarum*" in which the unity of teaching and research would be realized and an in-depth humanistic education for students would be made possible. This idea proved successful, spread worldwide, and gave birth to many universities of the same type in the following century and a half.

Separated from the *Universitas litterarum*, in the States of the German-speaking Europe, technical and military education and research continued to take place in the first decades of the 19th century in a large number of trade schools and polytechnic colleges, some of which are still important and significant today (including Berlin in 1821, Karlsruhe in 1825, Munich in 1827, Stuttgart in 1829, Hanover in 1831, Darmstadt in 1837). The underlying reasons for these foundations were, on the one hand, the requirements of the civil service and, on the other hand, the requirements of the developing industrial society. The tasks of the technical civil service had grown considerably; they were mainly in state mining, the military sector, building approvals, and from 1840 also railroad construction, which was nationalized in the course of the 19th century.

It was in the environment of the *Universitas litterarum* that the young Carl Friedrich Gauss, sometimes called the "Prince of Mathematics," worked at the prestigious University of Göttingen. Some of Gauss's ideas were a hundred years ahead of their time and touched on many different parts of the mathematical world, including geometry, number theory, calculus, algebra, and probability. He is widely regarded as one of the three greatest mathematicians of all

times, along with Archimedes and Newton. Among his students was August Ferdinand Möbius.

August Ferdinand Möbius is best known for his 1858 discovery of the "Möbius strip," which became a paradigmatic subject of the then new mathematical discipline topology. As a side note on modernity, the importance of topology in today's science is still persisting, as was demonstrated, for example, when the Nobel Prize in Physics 2016 was awarded to David Thouless, Duncan Haldane, and Michael Kosterlitz "*for their theoretical discoveries of topological phase transitions and topological phases of matter*."

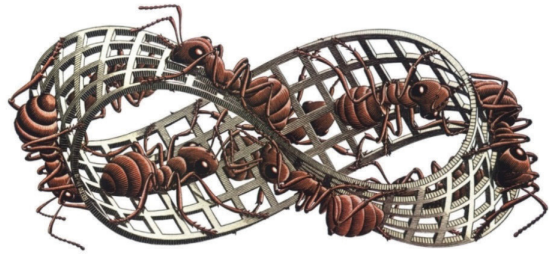

Figure 1 M. C. Escher, *Möbius Strip II (Red Ants)*, woodcut 1963. It shows ants crawling on the Möbius strip with the intention to traverse all parts of its (sweet) surface area. To their surprise, they are enjoying to crawl on all parts of the surface without ever having to cross an edge to the other side of the strip. The essence of one sidedness of the Möbius strip is that there is no "other side"; the irritated ants simply have to crawl far enough to go around twice before coming back to where they started. Mathematically translated, this means that on a Möbius strip a vector normal can be shifted to any point of the surface without ever crossing an edge.

Figure 2 Logos inspired by the Möbius strip with twisted loops (from *left* to *right*): Universal Recycling Symbol, Commerzbank, German Mathematicians Association.

It is fair to state that the concept of Möbius topology is fundamental in almost every branch of science, arts, and commerce. For instance, the topology of the Möbius strip with its one-sided surface (that requires twice as much paint to color as one might have thought) has inspired Maurits Cornelis (M. C.) Escher (1898–1972) to create his famed woodcuts and lithographs, among them is the 1963 woodcut *Möbius Strip II (Red Ants)* (see Fig. 1).

The public's appreciation of a personality of science or a scientific invention can sometimes be inferred from banknotes or stamps issued by the respective national bank or postal services. Figure 3 shows banknotes with the portraits of Leonhard Euler and Carl Friedrich Gauss, the two towering mathematical geniuses of the 18–19th century.

Figure 3 Banknotes (front). *Left*: Leonhard Euler, 10 Swiss Franc; *Right*: Carl Friedrich Gauss, 10 Deutsche Mark.

Figure 4 shows a selection of historical stamps from around the world that have a reference to mathematics and topology and their applications, as well as the scholars who left their deep marks in these fields.

Apparently, the Möbius strip is one of the few icons of mathematics that is been recognized by people from a broad variety of cultures. Its topological beauty is sensed even by laymen and inspired them to startling interpretations—also when designing, for example, furniture, fashion, and jewelry. Beyond such ephemeral creations, they are found in the enduring creative domains of music, literature, painting and sculpture, architecture, engineering, and science, all the way to dining culture.

Figure 4 International postal stamps with mathematicians involved in the early development of topology. Pierre-Simon Laplace (1749–1827): "*Read Euler, read Euler, he is the master of us all.*"

The nine chapters in this book cover a considerable range of subjects and characters. After the initial chapter, in which we give the timeline and historical background of science and cultural breakthroughs in Europe, we describe in Chapter 2 and Chapter 3 the time of August Ferdinand Möbius's early life and academic education (1790–1815) and his academic career (1815–68), respectively. Chapter 4 analyzes the reality of the Napoleonic Wars in Europe,

the reality as it presented itself to the victims and eyewitnesses in the battle areas, for example in Saxony, where August Ferdinand Möbius lived. Chapters 5 to 8 are devoted to applications of Möbius strip topology in mathematics, astronomy, chemistry, as well as physics and nanomaterials. Finally, Chapter 9 highlights quite a few fascinating examples of Möbius strip topology from the arts and architecture.

Figure 5 *I'll never be able to finish this Möbius strip steak.* (Adapted from a cartoon by Dan Piraro, June 23, 2011. Source: Bizarro Comics. com).

If we take astronomy (Chapter 6) as an example, there was a revolution in astronomical instrument development, specifically in large high-resolution telescopes, which took place over the period of Möbius's lifetime and opened new horizons in quantifying stellar observations of the Milky Way galaxy. August Ferdinand Möbius at the Leipzig Observatory, for example, and his contemporary Friedrich Wilhelm Bessel at the Königsberg Observatory, depended on a small number of experienced physicists, optical experts, and precision mechanics, like Joseph von Fraunhofer (1787–1826) and Johann Georg Repsold (1771–1830), who built telescopes for them, which enabled observations of admirable numerical precision. In other words, the manufacturing technology of the instruments redefined our view of the universe. This point will be discussed in detail in Chapter 6.

Figure 6 (*Left*): Möbius strip chair (Takeshi Miyakawa, 2014. Born and educated in Tokyo, Takeshi Miyakawa is an artist based in Brooklyn, New York.) (*Middle*): Hand-knitted Möbius strip roundabout scarf that features a half-twist. (Möbius scarf knitting instructions are available on the Internet). (*Right*): Möbius strip necklace jewelry carved from Jade gemstone (designed and manufactured in China).

In the case of modern astronomy, the evolution of instruments to progressively larger telescopes became a major scientific and engineering project, which, due to the enormous costs involved, can only be coped with through international cooperation. This is especially true for space-based telescopes, for example the Optical and UV Hubble Space Telescope, which NASA and the ESA have been operating since 1990. It is named after the American astronomer Edwin P. Hubble (1889–1953), who confirmed that the universe is expanding, thereby providing the foundation for the Big Bang cosmological model of the origin of the universe. Another example is the Infrared Herschel Space Telescope, named after the German–British astronomer Wilhelm Herschel (1738–1822); it was operated from 2009 until 2013 by the ESA.

The Herschel Space Telescope sees infrared and sub-millimeter light, which can readily penetrate through the dust hovering between the bustling center of our galaxy and us. Herschel's detectors are also suited to see the coldest material in our galaxy. During and after operation, Herschel left a legacy of treasured data that are still being analyzed to produce a multitude of amazing scientific results every year.

For instance in September 2018, the ESA announced that a bizarre, twisted ring of dense gas and dust at the center of our Milky Way galaxy became visible from the analysis of the astronomical data that had been captured by the far-infrared cameras on board the Herschel Space Observatory (Fig. 7). Herschel's view reveals the entire ring for the first time. The nearly continuous strip of dense and cold clumps of material forms a loop figure that resembles either an infinity symbol (?), or a lateral number 8 (?), or a Möbius strip (?) that stretches across more than 600 light years of space. In this image, the strip twists around an invisible axis running roughly from the top left to the bottom right.

Figure 7 Astronomers of the "Herschel infrared Galactic Plane Survey" project using the Herschel Space Observatory have discovered that a suspected ring at the center of our galaxy is warped into an infinity-shaped loop. The image combines observations at three different wavelengths: 70 μm (blue), 160 μm (green), and 250 μm (red). The loop is estimated to have 30 million solar masses, and it is made up of dense gas and dust at a temperature of as low as 15° above absolute zero (such cold regions are coded in blue color). The loop and its surroundings harbor a number of star-forming regions and young stars that stand out in bright blue. This area is part of the Central Molecular Zone, a region at the center of the Milky Way permeated with molecular clouds, which are ideal sites for star formation. (Image credit: ESA/NASA/JPL-Caltech).

Chapter 7 is devoted to topology applications from chemistry. We analyze, for example, the concept of chemical stability versus instability of organic molecules on the basis of "Hückel aromaticity" (introduced in 1931 by Erich Hückel, 1896–1980) and "Möbius aromaticity" (introduced in 1964 by Edgar Heilbronner, 1921–2006). Both approaches are based on theoretical considerations of symmetry and topology of the electronic and geometrical structures of a molecular system with either planar or 180°-twisted skeletons (see Fig. 8).

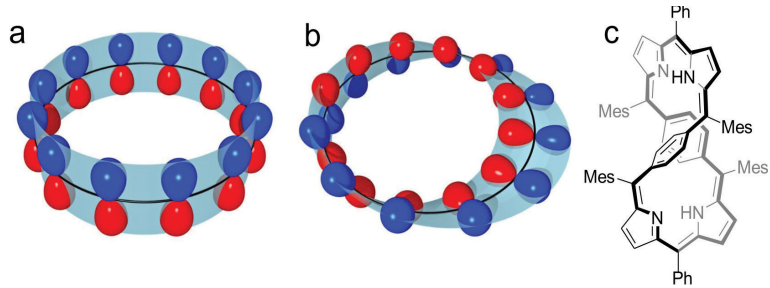

Figure 8 2p orbitals in aromatic molecules. Conventional planar (Hückel) 12-annulene, stable (**a**) and hypothetical 180° twisted (Möbius) 12-annulene, instable (**b**). Note the phase change by 180° between the two leftmost 2p orbitals in Möbius annulene (**b**). (Sources: P. B. Karadakov, Department of Chemistry, University of York, UK; D. L. Cooper, Department of Chemistry, University of Liverpool, UK; (**c**): Expanded porphyrin di-p-benzi[28]hexaphyrin(1.1.1.1.1.1), compound **1**, with Möbius topology (Mes = mesityl, Ph = phenyl). The 28-π-electron conjugation pathway is shown in bold. The expanded porphyrin was synthesized by L. Latos-Grażyński and coworkers at the University of Wroclaw, Poland [5].

In 2007, the group of Lechosław Latos-Grażyński at the University of Wroclaw, Poland, succeeded in synthesizing the compound di-p-benzi[28]hexaphyrin(1.1.1.1.1.1), which can dynamically switch between Hückel and Möbius conjugation depending, in a complex manner, on the polarity and temperature of the surrounding solvent. This discovery of "topology switching" between the two-sided (Hückel) and one-sided (Möbius) molecular state with closed-shell electronic configuration was based primarily on the results of NMR spectroscopy and DFT calculations. The present EPR and ENDOR work on the radical cation state of compound **1** is the first study of

a ground-state open-shell system, which exhibits a Hückel–Möbius topology switch that is controlled by temperature, like in the case of the closed-shell precursor. The unpaired electron interacting with magnetic nuclei in the molecule is used as a sensitive probe for the electronic structure and its symmetry properties [6].

Many more examples of fascinating Möbius strip molecules of unique chemical stability against environmental perturbations are also discussed in Chapter 7. For instance, the interest in topology considerations for synthetic chemists and bioengineers has grown tremendously with the discovery of the "cyclotides," those cyclic disulfide-rich mini-protein ribbons isolated from certain plants. Their extraordinary chemical and biological stability is considered of paramount relevance in ongoing computer-aided medical drug design. The stability can be rationalized in terms of knot theory and topology invariants of the ribbon-shaped molecular fragments and the orientation of their π-orbitals.

The cyclotides fall into two main structural subfamilies: "Möbius cyclotides" contain a cis-proline in a loop that induces a local 180° backbone twist (like in a Möbius strip), whereas "Bracelet cyclotides" do not. Their tertiary structure is typically maintained by disulfide bridges. The cyclotides represent a new class of natural plant compounds considered ideal "template" molecules for drug design.

An extraordinary example is the cyclotide *Kalata B1* that adopts both Möbius strip and trefoil-knot topologies. Here, we give special recognition to David Craik from the University of Queensland, Australia. He was among the first scientists to discover the cyclotide family of stable globular circular proteins and pioneered their biomolecular synthesis. In traditional medicine in Central Africa, boiling water extracts of the local plant *kalata-kalata* were given as a tea to pregnant women to speed up childbirth. In 1995, the X-ray crystal structure of *Kalata B1*, the active constituent of the traditional tea, was solved and its cyclic structure with the cystine knot motif was elucidated [7]. The structural beauty of the new circular mini-proteins has also stimulated artists to create fascinating sculptures; a prominent example is Julian Voss-Andreae, born 1970 in Hamburg, Germany, see Fig. 9).

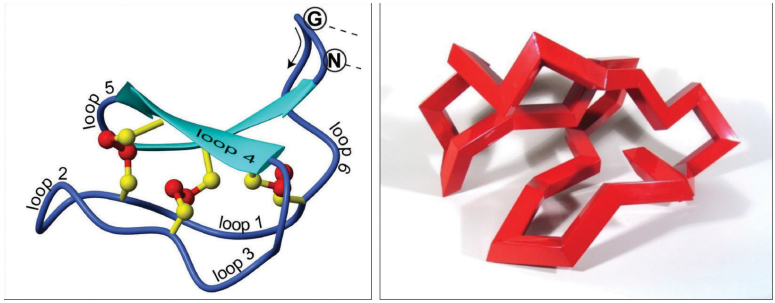

Figure 9 *Left*: Kalata B1 cyclotide, the glycine (G) and asparagine (N) amino acids are the terminal residues that are linked in a peptide bond to cyclize the peptide, i.e., give it a circular backbone [7]. *Right*: Julian Voss-Andreae, *Kalata* (2002). From his early works inspired by proteins, the molecular building blocks of life. (Source: Wikimedia Commons). For Julian Voss-Andreae, the main idea underlying his sculptures is the analogy between the technique of mitered cuts and protein folding. The sculptures offer a sensual experience of a world of science that is usually accessible only through the intellect [8].

Chapter 8 is devoted to topology applications from physics. It focuses on a novel class of solid-state material, the "graphenes," and the "topological insulators." In recent years, the study of band-structure topology has gained popularity after the discovery of materials that, at first glance, appear to be traditional band insulators, but when examined more closely turn out to be of a topologically different character. For these materials, a simple, continuous deformation of the band structure does not influence the topological invariants and, therefore, cannot change the band structure into that of a trivial insulator. The topological insulator phase in graphene was theoretically predicted in 2005 [9]. The behavior of the quantum spin state is topologically distinct from all previously known states of matter [10].

Recently, graphene nanoribbons (GNRs) have emerged as promising quasi-one-dimensional materials for next-generation electronic nanodevices. Because of the effect of edges and quantum confinement, GNRs can exhibit band gaps for transistor operation with exceptional switching speed and high carrier mobility. Particularly, GNRs with zigzag edges are expected to host edge-localized states, which may serve as key elements for graphene-based electronics and spintronics.

An interesting question arises as to what would happen to the quantum properties of the material when a zigzag-edged GNR torus transforms into a zigzag-edged Möbius GNR, causing the two zigzag edges to become one (see Fig. 10 *Right*). Answers to this question are also discussed in Chapter 8.

Some of the highlights with regard to fundamental issues of symmetry and topology in the arts and architecture are touched in Chapter 9. Large fractions of the chapter are devoted to fascinating examples of modern sculptures, houses, office buildings, and bridges inspired by Möbius strip topology.

But before continuing with our introductory remarks, we realized that it is time for a breathing pause for reflections:

When apparently in danger of turning the Introduction into an essay about why it is so important to know the history of topology in general and of the Möbius strip topology in particular, we remembered the famous quote by Antoine de Saint-Exupéry:

> "*If you want to build a ship, don't drum up the men to gather wood, and don't assign them tasks and work, but rather teach them to yearn for the vast and endless sea.*"

And so we stop continuing the Introduction.

Have we caught your interest? Just glance through the book, and we hope you will get stuck and start reading!

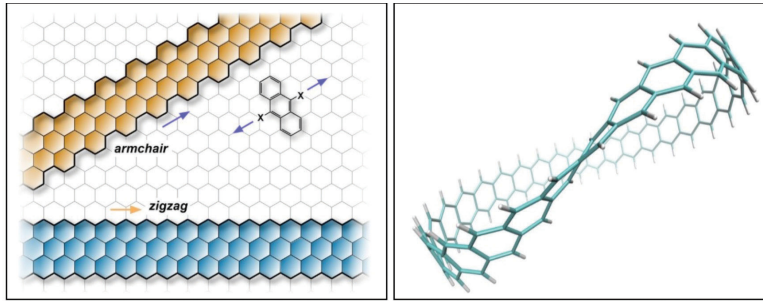

Figure 10 *Left*: Types of edges in graphene nanoribbon (GNR). Armchair GNRs are nonmagnetic semiconductors with the band gap determined by the width of the ribbon; zigzag GNRs are magnetic semiconductors with spin-polarized antiferromagnetic states at the edge (Source: Cheap Tubes Inc. (2019, September 26). A Guide to Graphene. AZoNano. Retrieved on March 10, 2021 from https://www.azonano.com/article.aspx?ArticleID=4841). *Right*: Structure of Möbius 30-cyclacene, which contains 30 fused benzene rings forming a closed loop with a single half-twist [11].

Appendix: The situation of the historic sources for the biographical Chapters 2 and 3

The overall source situation for the biographical part of this book on the life and work of A. F. Möbius is quite unfavorable, as was documented by the historic dissertation of André Loh from Leipzig University, 1994 [1]. Today it is very difficult to find original documents on the life and work of A. F. Möbius at all. The main reason for this is the destruction of the "Möbius Archive" of the Saxon Academy of Sciences in Leipzig. It burned to ashes during Allied bombing in December 1943. For the biographical Chapters 2 and 3 of our book, André Loh's dissertation was the main source of references. In the "Möbius Archive," all available documents by or about Möbius had been collected since 1885 by Leipzig scholars, of whom Curt Reinhardt (1855–1940) and Heinrich Liebmann (1874–1939) were of particular distinction; they had saved manuscripts from the Möbius's legacy, letters, diaries, documents, etc. Unfortunately, in most cases, not only the originals themselves but also their contents were lost because copies of only a few documents were made and stored elsewhere, for example a few letters between A. F. Möbius and C. F. Gauss and Alexander von Humboldt.

Because of this precarious source situation, André Loh tried to locate as many other repositories of documents by or about A. F. Möbius as possible, e.g., at the main stations of his life, such as the former *Fürstenschule Schulpforta*, where Möbius spent his childhood and school years, the universities of Leipzig, Göttingen, and Halle-Wittenberg, where Möbius spent his university student years. The archive searching there was mostly without success. The source situation concerning Möbius's professional life is much better. In the Leipzig University Archives, for example, there are still the personnel files and other holdings of the Faculty of Philosophy and the Leipzig Observatory, respectively, which refer to A. F. Möbius—and which André Loh was able to consult.

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Chapter 1

Nineteenth-Century Science Breakthroughs in Europe: Historical Background

During the course of August Ferdinand Möbius's lifetime (1790–1868), the pursuit of mathematics in the German-speaking part of Central Europe was transformed from a few leading figures considered as "lighthouses" of science, such as Gottfried Wilhelm Leibniz (1646–1716), Johann Bernoulli (1667–1748), Leonhard Euler (1707–1783), or Carl Friedrich Gauss (1777–1855), to distributed "centers of excellence" around eminent mathematicians, among them August Ferdinand Möbius (1790–1868) in Leipzig, Carl Brandan Mollweide (1774–1825) in Leipzig, Carl Gottfried Neumann (1832–1925) in Leipzig, Peter Gustav Lejeune Dirichlet (1805–1859) in Göttingen, Johann Benedict Listing (1808–1882) in Göttingen, Bernhard Riemann (1826–1866) in Göttingen, Felix Klein (1849–1925) in Göttingen, David Hilbert (1862–1943) in Göttingen, Hermann Minkowski (1864–1909) in Göttingen, Rudolf Friedrich Alfred Clebsch (1833–1872) in Göttingen, Carl Gustav Jakob Jacobi (1804–1851) in Berlin, Niels Henrik Abel (1802–1829) in Berlin, Karl Weierstrass (1815–1897) in Berlin, Leopold Kronecker (1823–1891) in Berlin, August Leopold Crelle (1780–1855) in Berlin, Paul Gustave Du Bois-Reymond (1831–1889) in Berlin, Jakob Steiner

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Klaus Möbius, Martin Plato, and Anton Savitsky

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(1796–1863) in Berlin, Martin Ohm (1792–1872) in Berlin, Bernard Bolzano (1781–1848) in Prague, Richard Baltzer (1818–1887) in Giessen, Richard Dedekind (1831–1916) in Braunschweig, Johann Friedrich Pfaff (1765–1825) in Halle (Saale), Georg Cantor (1845–1918) in Halle (Saale), Friedrich Wilhelm Bessel (1784–1846) in Königsberg, Christian Ludwig Gerling (1788–1864) in Marburg, Hermann Günther Grassmann (1809–1877) in Stettin, Hermann Hankel (1839–1873) in Tübingen, Carl Louis Ferdinand von Lindemann (1852–1939) in Munich, Alfred Pringsheim (1850–1941) in Munich, Hans Karl Friedrich von Mangoldt (1854–1925) in Danzig, Julius Plücker (1801–1868) in Bonn, and Arthur Moritz Schoenflies (1853–1928) in Frankfurt (Main).

This (incomplete) list of renowned 19th-century mathematicians in Germany reflects an impressive advancement by all standards—and this within a relatively short period of only a few decades. And the question arises if it were possible to trace reasons for this advancement. For instance, it is often suggested that among them are the large-scale reforms of the political and cultural background after the Napoleon-induced collapse of the "Holy Roman Empire of the German Nation." Reforms that were initiated in the Kingdom of Prussia after 1806, known as the Stein–Hardenberg and Wilhelm von Humboldt reforms, had diffused into many other German territories and states. The reforms were a reaction to the Prussian defeat by Emperor Napoleon I at Jena-Auerstedt in 1806, leading to the second Treaty of Tilsit, in which Prussia lost about half its territory and was forced to make massive tribute payments to France. To comply with this demand, Prussia needed to rationalize its administration, military, and educational organization on a large scale. Prussia's 1806 defeat and subjection also demonstrated the weaknesses, even inferiority of its absolutistic monarchical model of statehood—and excluded it from belonging to the dominating powers of Europe.

To become a great power again, Prussia initiated reforms from 1807 onward, based on Enlightenment ideas. In the context of our discussion on mathematics in Germany, of these reforms the far-reaching educational reforms headed by Wilhelm von Humboldt turned out to be of particular importance. In Humboldt's thinking, universities represented the crowning glory of intellectual education, and his postulation "Freedom and unity of teaching and research" (*Freiheit und Einheit von Lehre und Forschung*) was

a serious cornerstone in it. It would provide the fertile ground for independent thinking, creative research, promotion of talents, humane performance, and idealism. Could this reform ideal also have stimulated mathematical sciences leading to their spectacular flourishing in the 19th-century Germany?

Quite a number of historians favorably discuss such lines of thought (see, for instance, Fauvel et al. [1]). They conclude that the dramatic changes in mathematical science and education in the German-speaking part of Europe were most likely related to a more liberalized political development over this period in the early 19th century. To substantiate this kind of reasoning, or challenge it, a brief historic detour, strolling to specific historic events and landmarks of the 19th century, is in order, as they relate also to August Ferdinand Möbius and his career as mathematician and astronomer in Leipzig. There is no doubt that everybody's life—also that of August Ferdinand Möbius and that of his much-admired teacher Carl Friedrich Gauss—is coined by the time he lives in, i.e., its history of thought in terms of politics, culture, and conception of freedom.

August Ferdinand Möbius was born at the fin de siècle of the 18th century, a century that is generally considered the beginning of the Age of Enlightenment in Central Europe. Generally speaking, the Age of Enlightenment allowed more individualistic approaches to challenge established dogmas in science and humanities. Enlightened monarchs promoted rationality, often fostered education and research, allowed religious tolerance and freedom of speech, at least within certain limits set by the government and police control. Governmental responses to the ideas of Enlightenment had varied widely throughout Europe: The British government generally ignored the Enlightenment's leaders. In France, the government's posture was one of hostility toward Enlightenment, and the government reacted with increasing censorship. In several other states of Europe, notably in Spain, leaders of the Enlightenment were welcomed at court and helped design laws and programs to reform the political system, typically to build stronger national states. In Prussia, the leading figure in enlightened absolutism was Frederick II, called "the Great," who ruled Prussia from 1740 to 1786. In his youth, as Crown Prince, Frederick was more interested in music and philosophy than in strategies of war. These early years showed him as an enthusiastic patron of the arts and sciences, a gifted musician and

a correspondent with the top minds of the Enlightenment like, for instance, Voltaire. Upon ascending to the Prussian throne, however, he changed his priorities. Was it because of *raison d'état*? Or the thirst for power? Anyway, he attacked Austria and claimed Silesia during the Silesian Wars, winning military acclaim for himself and Prussia. Near the end of his life, Frederick conquered Polish territories in the First Partition of Poland. Aside from these early efforts as an idealist, he became an influential military theorist whose analysis emerged from his extensive personal battlefield experience and covered issues of strategy, tactics, mobility, and logistics of warfare.

Our historic detour will focus on the German states, specifically Saxony and Prussia, and their influential neighbors in Central Europe, notably France. The selections and valuations of the landmarks of this historic stroll will be mainly based on published historic essays, as have been compiled in the literature, for example, by Wikipedia and Britannica [2, 3]. And it will be based on elaborations of eyewitnesses, as are collected in various archives in museums of local history as, for instance, those of Naumburg or Leipzig in Saxony. And it will focus on salient events during the 18th and 19th centuries to characterize time and place of the environment our protagonist August Ferdinand Möbius had experienced (see the *Timeline* in Section 1.1, a chronological synopsis until 1871 when Saxony became part of the united German Empire, the *Deutsches Reich*).

1.1 Timeline of Key Historical Events in Central Europe of the 18th/19th Century

In this section, we give a chronological compilation of important historical events in Central Europe of the 18th/19th century with an emphasis on the consequences of the Napoleonic Wars on Saxony and Prussia and their political and cultural systems.

Timeline

800–1806 The Holy Roman Empire (*Sacrum Romanum Imperium, Heiliges Römisches Reich*) (see Figs. 1.1 and 1.2) was a multi-ethnic complex of states and territories in Central Europe that developed

during the Early Middle Ages and continued until its dissolution by Napoleon Bonaparte and his *Grande Armée* in 1806. In the end, about 300 full members belonged to the Holy Roman Empire.

Figure 1.1 Holy Roman Empire and its state symbol, the double-headed eagle, with coats of arms of individual states and territories. Painting from 1510. (Source: Wikipedia, Public domain).

Around 1800, today's territory of Germany was a "patchwork rug" of more than 300 small- and medium-sized states that were connected with each other but did not form a unified state. In contrast to England and France, where centralized states had formed over the centuries, Germany was dominated by miniaturized states (*Kleinstaaterei*). In the long term, this small-scale sectionalism impeded the formation of a nation-state in Germany until 1871.

1618–1648 The Thirty Years' War was a series of wars in Central Europe. It was one of the longest and most destructive conflicts in European history, resulting in millions of casualties, the arson of numerous towns and devastations of large regions. What initially started as a war between various Protestant and Catholic states in the Holy Roman Empire, gradually developed into a more general

conflict between most of the great powers, as a continuation of the France–Habsburg rivalry for European political and military supremacy.

Figure 1.2 Map of the Holy Roman Empire (1789). Due to its pre- and supranational character, the Holy Roman Empire never developed into a nation-state but remained a monarchically led, corporative structure consisting of the Emperor and imperial estates. There were status groups and privileged classes, but only a few common imperial institutions.

1685 Johann Sebastian Bach, eminent German composer of Baroque music, born (March 21, 1685 in Eisenach, Saxony).

1714–1838 The Georgian era in Great Britain. In British history, this is a period spanning the reigns of the first four Hanoverian kings of Great Britain who were all named George: George I, George II, George III, and George IV. The last Hanoverian monarch of Great Britain was Queen Victoria, who is the namesake of the following historical era, the Victorian, which is usually defined as occurring from the start of her reign (1837) and continuing until her death (1901). The Georgian era saw continual warfare in Europe and Overseas, including in continental Europe the Seven Years' War

(1756–63), the French Revolutionary Wars (1792–1802), and the Napoleonic Wars (1803–15). In effect, Britain won all these together with her often changing allies. The Georgian era was also a time of immense social changes in Britain, with the beginnings of the Industrial Revolution intensifying the class divisions and, accordingly, stirring up the activities of rival political parties like the Whigs and Tories.

1724 Immanuel Kant, grand German philosopher and mathematician who is considered a central figure in modern philosophy, born (April 22, 1724 in Königsberg). Kant was one of the foremost thinkers of the Enlightenment and arguably one of the greatest philosophers of all time. He inaugurated a new era in the development of philosophical thought. In his epochal work were subsumed new trends that had begun with the rationalism (stressing reason) of René Descartes and the empiricism (stressing experience) of Francis Bacon. Politically, Kant was one of the earliest exponents of the idea that perpetual peace could be secured through universal democracy and international cooperation. He believed that this will be the eventual outcome of universal history, although it is not rationally planned. In his 1795 essay "Perpetual Peace: A Philosophical Sketch" (*Zum ewigen Frieden. Ein philosophischer Entwurf*), Kant proposed a peace program to be implemented by governments, containing concrete "Articles" describing the indispensable steps toward peace that should be taken immediately. Kant's "Articles" lost none of their relevance; in fact his essay in some ways resembles modern democratic peace theory. Kant was also a student and a teacher of mathematics throughout his career, and his reflections on mathematics and mathematical practice had a profound impact on his philosophical thought. He developed philosophical views on the status of mathematical judgment, the nature of mathematical definitions, axioms, and proof—and the relation between pure mathematics and the natural world.

1737 Luigi Aloisio Galvani, multi-talented Italian physician, physicist, biologist, and philosopher at the University of Bologna, born in Bologna (September 9, 1737), died in Bologna (December 4, 1798). Another discipline Luigi Galvani learned alongside medicine was surgery, both in terms of theory and practice. This helped him in his experiments with animals and helped familiarize Galvani

with the manipulation of a living body. He discovered animal electricity by his famous frog experiments thereby pioneering the field of "bioelectricity" or "bioelectromagnetics" that still studies the electrical patterns and signals from tissues such as the nerves and muscles (now commonly named "electrophysiology"). In 1780, he discovered that the muscles of a dead frog's legs twitched when struck by an electrical spark from touching a bimetallic metal clamp. His frog experiments immediately became famous throughout Europe and were enthusiastically, or controversially, discussed in scientific, literary, and religious circles. Fascinated by Galvani's experiments, literally electrified were, for instance, Alexander von Humboldt (1769–1859), August Ferdinand Möbius (1790–1868), and Mary Shelley, the young eccentric English writer (1797–1851), together with her friend and mentor Lord Byron (1788–1824) (resulting in Shelley's spectacular novel *Frankenstein or The Modern Prometheus*, see Chapter 4).

1744 Johann Gottfried Herder (1744–1803), German poet and philosopher.

1749 Johann Wolfgang von Goethe, ingenious German poet, writer, statesman, born (August 28, 1749 in Frankfurt/Main).

1750 Johann Sebastian Bach, German ingenious composer, died (July 28, 1750 in Leipzig).

1756–1763 The Seven Years' War was the last major conflict before the French Revolution to involve all the great powers of Europe; France, Austria, Saxony, Sweden, and Russia were aligned on one side, fighting against Prussia, Hanover, and Great Britain on the other side. The main players of the war were King Frederick II ("the Great") of Prussia, Empress Maria Theresa of Austrian Habsburgs, Empress Elizabeth of Russia, King Louis XV of France, and King George III of Great Britain. The Seven Years' War also involved overseas colonial struggles between Great Britain and France for control of North America and India. Britain's unusual alliance with Prussia was undertaken partly in order to protect electoral Hanover, the British ruling dynasty's continental possession, from the threat of a French takeover. The Electorate of Saxony was among the losers of the Seven Years' War: Its territory was heavily ravaged as a battlefield and, albeit in the peace treaty of Hubertusburg, Prussia

agreed to withdraw its troops from Saxony within a few weeks and the pre-war state was restored, Saxony had to conform to Prussia's dominating interests and lost its claim of the Polish crown.

1759 Friedrich von Schiller, German poet, philosopher, historian, born (November 10, 1759 in Marbach/Neckar).

1760–1840 The Industrial Revolution began in Great Britain and spread all over continental European countries, albeit with different speed. The Industrial Revolution marks a major turning point in history; almost every aspect of daily life across all social classes was influenced in some way or the other. The Industrial Revolution was the transition to new manufacturing processes that included going from hand production methods to machines with improved efficiency of water and steam power, enabling new chemical manufacturing and iron production processes. Textiles were the dominant industry of the Industrial Revolution in terms of employment, value of output, and capital invested. The British textile industry was the first to use modern mass production methods for large-scale export, thereby creating excessive poverty of the traditional weavers in continental countries succumbing to the cheaper British textile imports. The historic, socioeconomic, and sociocultural impact of the Industrial Revolution was probably higher than that of the French Revolution starting about 30 years later.

1767 Wilhelm von Humboldt, German scholar, writer and statesman, born (June 22, 1767 in Potsdam).

1769 Alexander von Humboldt, German natural scientist and explorer, born (September 14, 1769 in Berlin).

1769 Napoleon Bonaparte, French military and political leader, born (August 15, 1769 in Ajaccio).

1770 Ludwig van Beethoven, German ingenious composer at transition between Classical and Romantic eras, born (December 16, 1770 in Bonn).

1770 Georg Wilhelm Friedrich Hegel (1770–1831), eminent German philosopher.

1776 E. T. A. Hoffmann, in full Ernst Theodor Amadeus Hoffmann, German writer, composer, jurist, and painter, born (January 24, 1776 in Königsberg; died June 25, 1822 in Berlin). He was one of the major

authors of the Romantic movement writing influential tales and novels full of fantasy and horror, masterpieces of subtle psychological analysis of the acting characters. In his stories, often supernatural and sinister characters move in and out of men's lives, ironically revealing tragic or grotesque sides of human nature. His novella "The Sandman" (*Der Sandmann*) is prototypical for Hoffmann's art to successfully blend reality and fantasy in the life of his protagonist, Nathaniel. By doing so, Hoffmann criticizes the "blind" advocate's values of the Enlightenment and addresses the subjectivity of reality, at least as we observe it. Of central importance of the story is the deep-rooted fear of losing one's eyes, which, within the framework of German Romanticism, represents the fear of losing the ability to distinguish between what is real and what is not. Also of central importance is the idea of outside forces effectively controlling what one sees and, therefore, controlling one's reality. This can be easily altered simply by modifying what that person sees—clearly, rather irritating statements for staunch defenders of the widely held new rationalism the Enlightenment had placed on science and scientific advancement. However, Hoffmann does not go to the other extreme with his characters by placing too much emphasis on emotion and feeling up to the total exclusion of reason.

Hoffmann has also a reputation as an acute observer of the immense sufferings of the people, soldiers, and civilians alike, in the course of the Battle of Dresden on August 26–27, 1813. This was an important battle during the War of the Sixth Coalition against Napoleonic troops (see below). During the days, Hoffmann would roam in town, watching the fighting with curiosity and increasing horror. The inner city of Dresden was bombarded; many people were killed by artillery bombs directly in front of him. After the main battle was over, he visited the gory battlefield. His account can be found in his essay *"Die Vision auf dem Schlachtfelde bei Dresden*," published 1814.

1777 Carl Friedrich Gauss, German ingenious mathematician, astronomer, and geophysicist, born (April 30, 1777 in Braunschweig).

1784 Friedrich Wilhelm Bessel, German astronomer, mathematician, physicist, and geodesist, born (July 22, 1784 in Minden, Westphalia). He became widely known already at the age of 20 when producing a refinement on the orbital calculations for

Halley's Comet in 1804, using old observation data taken from published tables.

1787 Joseph von Fraunhofer, German physicist and spectroscopist of the Sun's emission and absorption spectrum ("Fraunhofer lines"), famed inventor of new optical glass lenses and achromatic telescope objectives that revolutionized astronomy, born (March 6, 1787 in Straubing, Bavaria).

1790 August Ferdinand Möbius, German mathematician and astronomer, born (November 17, 1790 in Schulpforta near Naumburg at the Saale river).

1787–1799 The French Revolution shook France to the very foundations and directly affected many of her neighboring countries. It reached its first climax in 1789. It denoted the end of the *ancien* regime in France and, in the long run, turned upside down numerous other monarchies in Europe and their ruling classes. The French Revolution overthrew the French monarchy, established a republic, but experienced most violent periods of political anarchy and terror (the "Reign of Terror," *La Terreur* (1793–94)). *La Terreur* was marked by mass executions of "enemies of the revolution" (Robespierre) with a death toll ranging in the tens of thousands. With the ingenious chemist Antoine Lavoisier among the victims of the guillotine (the Revolutionary Tribunal decided: "*The Revolution has no need for geniuses*"). The French Revolution finally culminated in a dictatorship under Napoleon Bonaparte expanding to Western Europe and beyond. What started with the revolutionary motto *liberté, égalité, fraternité* (liberty, equality, fraternity) deteriorated into sheer imperialism and horrendous blood shed throughout Europe. The French Revolution profoundly altered the course of modern history, triggering the global decline in absolute monarchies while replacing them with republics, sometimes leading to states with "democratic" constitutions, sometimes to despotism and barbarism. Great Britain organized—or became active member and financier—as many as seven coalitions of European states to defeat revolutionary and Napoleonic France in her strive for supremacy in Europe. With varying coalition partner states, with victories and defeats oscillating between France and the respective Coalition. In the long run, the Coalitions succeeded in their objective

to tame revolutionary France and blow off imperialistic Napoleon Bonaparte. All in all, this took the Coalitions more than 20 years: The First Coalition (1792–97), the Second Coalition (1799–1802), the Third Coalition (1805–06), the Fourth Coalition (1806–07), the Fifth Coalition (1809), the Sixth Coalition (1812–15), and finally the Seventh Coalition (1815). In 1796, Saxony reluctantly joined the First Coalition (including Prussia) against Revolutionary France, but was defeated by 1796.

1763–1806 Elector Frederick Augustus III of Saxony successfully restructured his country's finances and administration. In foreign policy, he was neutralist but then drifted toward Prussia, whose side he took in the Bavarian succession dispute (1778–79) to prevent Bavaria's cession to Austria. For his cooperation, he received substantial financial compensation from Prussia. In 1785, Frederick Augustus joined the Prussia-sponsored "League of Princes" (*Fürstenbund*) but remained neutral during the Austro–Prussian conflict in 1790. Offered the Polish crown in 1791, he declined as he feared that his "risky" politics may cause further damage to the Polish state, which was already rather weak suffering from the First Partition of Poland in 1772 and the Second Partition of Poland in 1793. The Third Partition of Poland (1795) was the last one in the series of partitions among the Prussian Kingdom, the Austrian Empire, and the Russian Empire. This effectively ended the Polish–Lithuanian national sovereignty until 1918.

1777 Heinrich von Kleist, German poet, dramatist, novelist, journalist, born (October 18, 1777 in Frankfurt at the Oder river).

1797 Heinrich Heine, German poet, writer, journalist, born (December 13, 1797 in Düsseldorf).

1803 Hector Berlioz, French Romantic composer, born (December 11, 1803 near Grenoble). He died on March 8, 1869 in Paris, only a few months after August Ferdinand Möbius. Berlioz is best known for his compositions "*Symphonie fantastique*," "*Grande messe des morts*," or "*La damnation de Faust*." Berlioz called *The Damnation of Faust* a "légende dramatique." He was inspired to this work when reading a translation of Goethe's dramatic poem "*Faust Part I*" and produced a musical work that, like the poetic masterpiece on which it is based, defies easy categorization due to its cosmic perspective.

1803–1806 War of the Third Coalition against Napoleonic France. It was a European conflict; the Third Coalition was mainly made up of the members of the Holy Roman Empire, including Austria and Prussia, Russia, Britain, and Sweden—but not Saxony! At the beginning of the year 1805, Napoleon was planning to cross the English Channel and invade England with 2000 ships and 200,000 soldiers. The next step of escalation was to be expected in the persistent war between the French and the British, the irreconcilable enemies struggling for dominance in Central Europe. Indeed, at the end of summer, Napoleon paraded his soldiers along the Channel shore, but then, surprisingly, changed his mind and ordered his *Grande Armée* to turn around and march eastbound into continental Europe. During the war, France and its client states defeated the alliance of the Third Coalition, most utterly in the Battle of Ulm (1805), the Battle of Austerlitz (1805), and the Battle of Jena-Auerstedt (1806).

1804 Coronation of Napoleon Bonaparte as "Napoleon I, Emperor of the French" on December 2, 1804 at the *Notre Dame* Cathedral in Paris. The staging of the sacred ceremony in the presence of Pope Pius VII (see Fig. 1.3) is considered as a "real masterpiece of modern propaganda" [4]. Napoleon designed this new sacred ceremony because he wanted to establish legitimacy of his imperial reign, with its new royal family and new nobility.

1804 Immanuel Kant, German philosopher and mathematician who is considered a central figure in modern philosophy, died (February 12, 1804 in Königsberg).

1805 Friedrich von Schiller, German poet, philosopher, historian, died (May 9, 1805 in Weimar).

1805 The Battle of Ulm (October 19, 1805). Austria and Russia had joined Britain in an alliance to defeat Napoleon's French Empire once and for all by sheer force of troop numbers. But Napoleon saw at once the flaw in the Allies' strategy. Their forces were widely dispersed across the continent. Napoleon struck the decisive blow: At the battlefield near Ulm, 27,000 Austrian soldiers surrendered on October 19, and almost the entire Austrian army was lost. Now nothing stood between Napoleon and Vienna, the capital of the ancient Austrian Empire! On November 14, Napoleon led his

soldiers into Vienna. Fortunately, there was no fighting in the city. The Emperor Francis II had fled, leaving his palaces and gardens to the enemy. Emperor Napoleon I triumphantly paraded through Vienna. But his triumph had been shadowed by a national disaster.

Figure 1.3 Jacques-Louis David: The Coronation of Napoleon on December 2, 1804 at the Notre Dame Cathedral in Paris (1808, oil on canvas, detail). (Credit: Louvre, Paris). Self-crowned Napoleon I is standing in the center dressed in coronation robes similar to those of Roman emperors. Joséphine de Beauharnais, Napoleon's wife, is kneeling in a submissive position to receive the crown from the hands of her husband, not the Pope. Behind Napoleon I sits Pope Pius VII.

1805 The Naval Battle of Trafalgar (October 21, 1805). On that day the British Admiral Horatio Nelson and his fleet had caught an allied French and Spanish fleet in the Atlantic near Cape Trafalgar off the southwest coast of Spain and utterly destroyed it — at the cost of his own life. Twenty-seven British warships defeated 33 French and Spanish warships. It was the most decisive naval battle of the Napoleonic Wars, conclusively ending French plans to invade England. Nelson's crushing defeat of the French and Spanish Navies established Britain as the dominant world naval power for a century. And Napoleon no longer had a fleet he could count on. But ultimately, the fortunes of war did not (yet) turn against him, on the contrary.

1805 The Battle of Austerlitz (December 2, 1805) was one of the most important and decisive engagements of the Napoleonic Wars. In what is widely regarded as the greatest victory achieved by Napoleon, the *Grande Armée* of France defeated a larger Russian and Austrian army led by Tsar Alexander I and Holy Roman Emperor Francis II. The battle occurred near the village of Austerlitz in the Austrian Empire (modern-day *Slavkov u Brna* in the Czech Republic). Austerlitz brought the War of the Third Coalition to a rapid end, with the Treaty of Pressburg signed by the Austrians later in the month (on December 26, 1805). The treaty confirmed the Austrian loss of lands in Italy and Bavaria to France, and in Germany to Napoleon's German allies. It also imposed an indemnity of 40 million francs on the defeated Habsburg Empire. After the end of the Third Coalition war, France was continuing to be at war with England and Russia.

Critically judged, the victory at Austerlitz prompted Napoleon to create the Confederation of the Rhine, intended as a buffer zone between France and Central Europe. Such achievements, however, did not establish a lasting peace on the continent. Prussia's worries about growing French power in Central Europe sparked the War of the Fourth Coalition in 1806. Saxony again entered the struggle against France on Prussia's side.

1806–1813 The Confederation of the Rhine (*Rheinbund*; *Confédération du Rhin*) was a Confederation of client states of the First French Empire (see Fig. 1.4). The more than 15 million people living within the Confederation provided both a barrier against potential enemies on France's eastern borders and also had to send large contingents of troops to join its armies. No wonder that disaffection, even hostility within the member states over increasingly enforced logistic and military support for France began after Napoleon's inglorious invasion of Russia.

1812 Slowly, new alliances and coalitions were formed, including Britain, Russia, Austria, Sweden, and Prussia, But only after the anti-Napoleonic Allies victory at the Battle of Leipzig.

1813 *("Völkerschlacht")* Many of the Confederation member states switched sides to help the Allies during the ongoing War of German Liberation from Napoleonic yoke.

Figure 1.4 Map of The *Rheinbund* (Confederation of the Rhine) 1808. At its zenith, the Confederation of the Rhine comprised 39 German member states, including large kingdoms and duchies, together with smaller principalities and city states. The key states were Bavaria (3.5 million subjects), Saxony (2 million), Westphalia (2 million), Württemberg (1.5 million), Baden (1 million), and the Duchy of Warsaw (4 million). The *Rheinbund* assembled all of the states of Germany, except Austria and Prussia, under the aegis of Napoleon I.

The Rheinbund was formed initially from 16 German states by Napoleon after he defeated Austria and Russia in the Battle of Austerlitz. The Treaty of Pressburg, in effect, led to the creation of the Confederation. It lasted from 1806 to 1813. The members of the Confederation were German princes (Fürsten) from the Holy Roman

Empire. They were later joined by 19 other rulers of minor German-speaking states. Thus, the members of the Confederation included large kingdoms and duchies, together with smaller principalities and city states. The head of state of the Confederation of the Rhine was Napoleon I himself under the title of *Protector of the Confederation of the Rhine*. Altogether, in the Confederation of the Rhine, over 15 million subjects were assembled. Prussia and Austria were not members of the Confederation. On the one hand, Napoleon sought to consolidate the modernizing achievements of the French Revolution; on the other hand, he wanted the soldiers and supplies that these subject states of the Confederation could provide for his future wars. So, Napoleon ordered the Confederation member states to supply thousands and thousands of fresh troops to his army And to pay for the garrisoning of French troops on their lands. Naturally, the political success of the Confederation depended on Napoleon's success in battle; the Confederation collapsed when Napoleon lost the Battle of Leipzig in 1813.

1806 Twin battles of Jena and Auerstedt (October 14, 1806) in Saxony, commonly called the battle of Jena-Auerstedt, only about 30 km south of Möbius's home town Naumburg. It was fought between 122,000 French troops and 114,000 Prussians and Saxons. In the battle, Napoleon's *Grande Armée* crushed the outdated Prussian army that was inherited from King Frederick II, still dreaming of invincibility. But on October 27, 1806, Napoleon marched into Prussia's capital Berlin, at the head of his troops, just after the Prussian King Friedrich Wilhelm III had fled to Eastern Prussia. The iconic *Quadriga* monument on top of the Brandenburg Gate in Berlin was carried off to Paris by Napoleon's troops. The decisive defeat at Jena-Auerstedt resulted in the reduction of Prussia to half its former size at the Treaty of Tilsit in July 1807. The total defeat by the French had a traumatic effect on Prussia and led to an upsurge of patriotism and cry for comprehensive reforms of the state in terms of civil administration, army, economics, and public education. Curiously, it also led to a renaissance of German culture and renewal of university education and research and intellectual life in public and private circles of great diversity.

1806 *Berlin Decree* (November 21, 1806) enacting the "Continental System" (or "*Continental Blockade*" (*Blocus continental*)) of Napoleon

I in his struggle against Great Britain during the Napoleonic Wars. As a response to the naval blockade of the French coasts enacted by the British government on May 16, 1806, the Continental Blockade brought into effect a large-scale embargo against British trade. The embargo was effective intermittently until April 1814; it ended after Napoleon's first abdication.

The embargo forbade the import of British goods into European countries allied with or dependent upon France, which had installed the Continental System in Europe. All connections were to be cut, even the mail. But British merchants and their continental partners smuggled in many goods. Surely, they became rich, even very rich, and the Continental System lost its significance as a powerful weapon of economic war.

1807 Not surprisingly, after the lost battle at Jena-Auerstedt, Frederick Augustus from Saxony hastened to make peace with Napoleon, who secured the title of King of Saxony for him. Moreover, a year later, Emperor Napoleon secured the title "Grand Duchy of Warsaw" for Frederick Augustus. Although the Duchy of Warsaw was officially created as an independent state by Napoleon, as part of the 1807 Treaty of Tilsit with Prussia, in reality it was only a satellite state of France. Nevertheless, for Polish patriots it became a beacon of hope, a focal point of efforts to restore the Polish nation ("Poland is not yet lost," "Jeszcze Polska nie zginęła"), which had been destroyed by the three "Partitions of Poland" made by Russia, Prussia, and Austria in 1772, 1793, and 1795. It was commonly hoped and believed that with time the Polish nation would be able to regain its former status, not to mention its former borders.

1807–1819 The Prussian reform movement known as the *Stein-Hardenberg-Humboldt Reforms*, with Karl Freiherr vom Stein (1757–1831), Karl August Fürst von Hardenberg (1750–1822), and Wilhelm von Humboldt (1767–1835) acting as their key initiators and leaders. The reform movement was a series of constitutional, administrative, social, educational, and economic reforms in the Kingdom of Prussia. Many historians saw these reforms as the first steps toward the unification of Germany and the foundation of the German Empire in 1871. The reforms were a reaction to the disastrous defeat of the Prussian army by Napoleon I at Jena-Auerstedt in 1806 with the result that Prussia lost about half its

territory and was forced to make massive tribute payments to France. Prussia's defeat and subjection also demonstrated the weaknesses of its absolutistic monarchy model of statehood and excluded it from the leading powers of Europe.

To become a great power again, Prussia initiated reforms from 1807 onward, based on Enlightenment ideas. They led to the reorganization of Prussia's government and administration and drastic changes in its agricultural trade regulations, including the abolition of serfdom and allowing peasants to become landowners. In industry, the reforms aimed to encourage competition by suppressing the monopoly of guilds. The administration was decentralized, and the nobility's privileges were cut back. In parallel, there were also military reforms and, eminently important, far-reaching educational reforms headed by Wilhelm von Humboldt. In Humboldt's thinking, university studies represented the crowning glory of intellectual education, and the expression "Freedom and unity of teaching and research" (*Freiheit und Einheit von Lehre und Forschung*) was a serious cornerstone in it.

For Wilhelm von Humboldt, "the state must treat its universities neither as gymnasia nor as specialist schools and must not serve its Academy as a technical or scientific deputation." Students, in his view, had to learn to think autonomously and work in a scientific way by taking part in research.

It is difficult to ascertain when the reform movement ended; in the fields of the constitution and internal politics in particular, the year 1819 marked a turning point, with restoration tendencies gaining the upper hand over constitutional ones. Though the reforms undoubtedly modernized Prussia, their successes were mixed, as historians continue to debate. Certainly, the educational reforms brought a push forward for the German universities at large in terms of fostering academic talents and excellence in the basic natural sciences and, in particular, "pure" abstract mathematics.

1808 Faust I, the first part of Faust by Johann Wolfgang von Goethe first published as "Faust: A Tragedy" (*Faust. Eine Tragödie*). This *magnum opus* is widely considered the greatest work of drama of German literature.

1809 The War of the Fifth Coalition (Austrian Empire and the United Kingdom against Napoleon's French Empire and Bavaria)

with very high casualty rates for both sides. Britain was already heavily involved in the ongoing Peninsular War, a military conflict between Napoleon's Empire and the allied powers of Spain, Britain, and Portugal for control of the Iberian Peninsula. In part, the Spaniards resorted to guerrilla warfare with armed actions carried out by non-regular troops against Napoleon's *Grand Armée.*

The Austrian futile invasion of Bavaria in 1809 had opened the Franco–Austrian war. This war was one of the few occasions that Napoleon did not initiate the military conflict. All of the impetus toward war came from Austria's leaders. Due to their false assessments of getting quick support by Prussia and Russia, Austria stood alone in this military campaign. The Austrians were defeated in a series of combats against the French and their German allies. Parallel to its involvement in the Peninsular War, Britain sent another military expedition to the Netherlands in order to relieve the Austrians. This effort had little impact, though, on the outcome of the conflict. Vienna was exposed to French attack as a result of the failure of the traumatic siege of Vienna. Actually, the siege lasted only from May 10 to May 13. Then, the Austrian capital fell to Napoleon for the second time in 4 years after a very short attempt to defend the city.

The Battle of Wagram (July 5–6, 1809), in one of Austria's renowned wine regions near the outskirts of Vienna, ended in a decisive victory for Emperor Napoleon's French and allied army against the Austrian army. The battle led to the breakup of the Fifth Coalition, the Austrian and British-led alliance against France. The Battle of Wagram was particularly bloody, mainly due to the extensive use of artillery on a flat battlefield packed with some 300,000 men. The defeat of Austria resulted in the Treaty of Schönbrunn (October 14, 1809), which was the harshest peace treaty that France had ever imposed on Austria in the recent memory. Austria lost over 3 million subjects, i.e., about one-fifth of her total population as a result of extended territorial changes and losses. But, in the long run, this military triumph over Britain-backed Austria did not bring enduring peace to Europe under Napoleon's umbrella. Prussia was threatening that another war with Napoleon's France would be unavoidable to stop her unappeased imperialistic expansion in Europe—and to take revenge for the defeat at Jena-Auerstedt and the subsequent ignoble Tilsit peace treaty in July 1807 with Napoleon. For many Europeans,

it was absolutely frightening: There were more wars looming at the European theater!

Up to 1809, the five anti-Napoleonic Coalitions—1792–97, 1799–1801, 1805, 1806–07, 1809, with varying combinations of various powers—had ended in failure. Napoleonic imperialism appeared to be militarily invincible, largely because of the lack of stringent and constructive cooperation between the monarchic rulers in Vienna, Berlin, St Petersburg, and London. This resulted in no common political goals—sometimes even in contradictory goals—and definitely in no overall military strategy. The predominant foreign political attitudes that characterized these coalitions are defined by deep mistrust, expediency, opportunism, and self-interest, not to mention the urge to expand their territories at the expense of smaller neighboring powers.

After the peace Treaty of Schönbrunn, economic warfare continued in Europe: notably, the French Continental System against British goods as a response to the British naval blockade of French-controlled territories. However, due to an inefficient organization of control measures, an increasing penetrability of the Continental System in French-dominated states occurred. It resulted in tolerated—or even encouraged—concealed trade with the help of British and continental smugglers and contrabandists, particularly from Spain, Portugal, and Russia. Hence, in terms of intended economic damage to Great Britain, the blockade turned out to be largely ineffective. As Napoleon realized that extensive trading, bypassing the Continental System, was executed through Spain and Russia, he invaded those two countries to prevent smuggling. With bad consequences for him: The Iberian conflict began when Portugal continued trade with Britain despite French restrictions. Napoleon tied down his forces in Spain in 1809, and he lost very badly in Russia in 1812.

1809 August Ferdinand Möbius graduated from *Schulpforta College* and became a university student at the University of Leipzig. His family had wanted him to study law, but in the middle of his first year of study, he changed to mathematics, astronomy, and physics.

1809 Hermann Günther Grassmann, Prussian mathematician, linguist, and philosopher (born in Stettin April 15, 1809, died September 26, 1877 in Stettin). His outstanding mathematical work

was largely disregarded by the academic establishment in Germany and France for a long time. Nowadays he is considered the actual inventor of the vector and tensor calculus and forerunner in linear algebra and geometry. He also discussed his early work with his esteemed colleague August Ferdinand Möbius in Leipzig, who was not too appreciative. After the earlier frictions between Möbius and Grassmann, their personal relation later developed into a lasting friendship, one of the few friendships Möbius made with his colleagues.

1810 Foundation of the oldest Berlin University following the pressing initiative of Wilhelm von Humboldt (1767–1835), the German scholar, educator, statesman, political theorist, and philologist. He reformed the Prussian school system. In the course of the Prussian reforms after the crushing defeat at Jena-Auerstedt, King Friedrich Wilhelm III founded the University of Berlin (*Alma Mater Berolinensis*). The foundation concept, which Wilhelm von Humboldt had put forward, made it the "mother" of all modern universities worldwide. It envisaged a *universitas litterarum*, which would achieve a unity of teaching and research and provide students with an all-round humanist education. This concept spread throughout the world and gave rise to the foundation of many universities following the same principles over the following 150 years. Largely due to the influence of the natural scientist Alexander von Humboldt, the university pioneered the introduction of many new academic disciplines: "In time, Berlin should have the first observatory, the first chemistry institute, the first botanical garden, the first school of transcendental mathematics. That is the goal of my efforts for coherence in all I strive for" (Alexander von Humboldt, 1827). The chemist August Wilhelm von Hofmann, the physicist Hermann von Helmholtz, the mathematicians Ernst Kummer, Leopold Kronecker, Karl Theodor Weierstrass (the "triple star of mathematics"), and the medical scientists Johannes Müller and Rudolf Virchow became known in their specialist areas far beyond Berlin University. From 1828 to 1945, the university carried the name "Friedrich-Wilhelm-University"; since 1949 the name has been "Humboldt University in Berlin."

1810 Friedrich Wilhelm Bessel, German astronomer, mathematician, physicist, and geodesist, was appointed director

in January 1810, at the age of 25, of the newly founded Königsberg Observatory by King Frederick William III of Prussia. On the recommendation of the famous mathematician, astronomer, and physicist Carl Friedrich Gauss, Bessel was awarded an honorary doctor degree from the University of Göttingen in 1811.

1810 Heinrich von Kleist, German Romantic poet and dramatist, novelist and journalist, finished his last play "The Prince of Homburg" (in full: *"Prinz Friedrich von Homburg oder Die Schlacht bei Fehrbellin"*), but it was not allowed to be performed in public in Prussia until 1821, 10 years after the author's death. It refers to the historic Battle of Fehrbellin on June 28, 1675 during the war between Sweden and Brandenburg-Prussia. The Prussian forces under Elector Friedrich Wilhelm (Frederick William) ultimately stopped the Swedish invasion of Brandenburg. In the last lines of the play, all the protagonists jointly shout: "Down in the dusts with all the enemies of Brandenburg!" ("*In den Staub mit allen Feinden Brandenburgs!*"), an iconic commitment to Prussian patriotism.

1811 Heinrich von Kleist committed suicide (November 21, 1811 in Berlin).

1812/1813 Although the Fifth Coalition against Napoleon's France had ended with the Schönbrunn peace treaty (1809), Britain, Spain, and Portugal remained at war with France in the still ongoing Peninsular War. In central and eastern Europe, however, there was peace until Napoleon's invasion of Russia in 1812, which led to the formation of the Sixth Coalition in 1812. Toward its end, in 1813, the coalition partners were Great Britain, Russia, Prussia, Sweden, Austria, and numerous minor German states.

1812 The preceding victories against the coalition armies gave Napoleon's *Grande Armée* a sense of invulnerability, especially when it invaded Russia and approached Moscow. Napoleon occupied it after the Russians abandoned it aflame. The Russians had prevented Napoleon from fighting the decisive battle he wanted before penetrating too deeply the vastness of Russian territory in the east. So he ordered his *Grande Armée* to keep marching and marching until they were close to the gates of Moscow, finally meeting the Russian army in Borodino. The Battle of Borodino (*Bataille de la Moskova,* Бородинское сражение) was fought on September 7, 1812, making

the Borodino day to one of the deadliest days to remember of the Napoleonic Wars. The fighting involved around 250,000 troops and left at least 70,000 casualties. In the Battle of Borodino, a large part of Napoleon's cavalry was eliminated. The catastrophic lack of horses turned out to become a major reason for the series of defeats Napoleon's *Grande Armée* suffered in the coming years.

1813/1814 Borodino represented the last Russian effort of stopping the French advance on Moscow, which fell a week later. Napoleon waited for five weeks for a Russian surrender, which would never come. Russia even refused to open peace negotiations with Napoleon after he captured the city. The approaching winter eventually forced the French to retreat from Moscow in October, heralding the ultimate defeat of the French invasion. When retreating from Russia, the French forces were mostly defeated by the bitterly cold winter weather. Chased by heavy snowstorms and harassed by the Cossacks, they suffered extremely high losses of lives and military equipment while running home. Overall, Napoleon's Russian Campaign of 1812 was one of the greatest disasters in military history of the 19th century. He invaded Russia at the head of an army of over 600,000 men, including 20,000 Saxons and a Prussian auxiliary corps of 20,000 soldiers led by general Ludwig Yorck von Wartenburg, then a forced ally of Napoleon's invading army. But by early 1813, only 93,000 of them were still alive and with the French army.

1812/1813 The Tauroggen Convention between Prussia and Russia. During Napoleon's disastrous retreat, Yorck von Wartenburg concluded the Tauroggen Convention with the Russians—against the will of the Prussian King Friedrich Wilhelm III, thereby neutralizing the Prussian corps and, in actual fact, separating it from the French army. At the time of the armistice, Tauroggen was part of Russia, situated 40 km east of the Prussian border. The Convention of Tauroggen was a separate Prussian–Russian armistice signed on December 30, 1812 between General Ludwig Yorck von Wartenburg on behalf of his Prussian auxiliary corps and General Hans Karl von Diebitsch on behalf of the Imperial Russian Army. Yorck's self-determined action is now generally considered a highly significant turning point of Prussian history, triggering an insurgency against Napoleon in the Confederation of the Rhine. Yorck's initiative in

signing a separate neutrality agreement with Russia opened the way for Prussia to join the Allied powers against Napoleon. The retreat from Moscow became one of the defining images of the Napoleonic period, and the disaster in Russia helped convince many of Napoleon's former allies to turn against him, especially in Germany. The final result was that within 2 years, Napoleon went from the master of most of Europe to abdication and his first exile.

1813 French–Prussian battle of Lützen (*La Bataille de Lützen*, *Schlacht von Großgörschen*) in Saxony, May 2, 1813. Following the disaster of Napoleon's invasion of Russia in 1812, a new political and military alliance formed against him. Prussia had declared war on France on March 27, 1813. In response to that, Napoleon hastily assembled an army of just over 200,000 consisting largely of inexperienced, barely trained recruits. They were severely short of horses (a consequence of the Russian invasion, where most of Napoleon's veteran troops and cavalry horses had perished). He crossed the Rhine into Germany near Mainz to link up with remnants of his old *Grande Armée* And to quickly defeat this new alliance before it became too strong (the Sixth Coalition, embedding Russia and Prussia, later also Austria; it was financed by Great Britain). Napoleon and his forces marched from Mainz via Erfurt and Naumburg at the Saale river toward Leipzig and Dresden in Saxony. On April 29, parts of his army reached Naumburg at the Saale river (which is close to Schulpforta where August Ferdinand Möbius was born in 1790). Two days later, Napoleon's troops arrived at Lützen, where Napoleon expected military encounters to happen with nearby allied troops. The Russian commander, attempting to preempt Napoleon's capture of Leipzig, attacked Napoleon's units near Lützen. After a day of heavy fighting, the combined Prussian and Russian force retreated. This halted the advances of the anti-Napoleonic Sixth Coalition. But due to the severe French losses and the lack of a strong cavalry, Napoleon failed to pursue his defeated enemy. Another battle was the unavoidable consequence in these disastrous Napoleonic Wars. And not only one—with Saxony developing into the main theater of war.

1813 The Battle of Dresden was fought on August 26–27, 1813 in the vicinity of Dresden, the capital of Saxony. It resulted in a French victory under Napoleon against the allied forces of the Sixth Coalition.

However, Napoleon's victory was not as complete as it could have been. Substantial pursuit was not undertaken after the battle, in part because the cavalry of the *Grande Armée* had run out of horses. This was because of the disastrous losses in the Russian campaign in 1812 as well as because there had been too many skirmishes with the allied forces within too short a time for replenishment of horses by requisition. With time, the position of Napoleon went on weakening, and the Allies became stronger and stronger. Only Saxony and the Duchy of Warsaw kept faith with Napoleon.

1813 The Battle of Leipzig or "Battle of the Nations" (*Völkerschlacht bei Leipzig*, Битва народов под Лейпцигом, *La Bataille des Nations*) against Napoleon's *Grande Armée* was fought on October 16–19, 1813 at Leipzig in Saxony. The battle was the culmination of the 1813 German "Liberation Campaign" (*Befreiungskriege*) and involved nearly 600,000 soldiers on both sides, making it the largest battle in Europe prior to World War I. The Sixth Coalition armies of Russia, Prussia, Austria, and Sweden decisively defeated the French army, which also contained Polish and Italian auxiliary troops, As well as German auxiliary troops from various states of the Confederation of the Rhine. Being defeated by the coalition armies, Napoleon was compelled to return to France. The coalition armies hurried to keep their momentum, invading France early the next year. Napoleon was forced to abdicate and was exiled to the Isle of Elba in May 1814. And Louis XVIII regained the French Throne (Peace Treaty of Paris in November 1814).

1813 August Ferdinand Möbius finished his studies at Leipzig and traveled to Göttingen, almost 250 km westward, then part of the Kingdom of Westphalia. At that time, Göttingen was probably the most famous university city of Germany, if not of Europe. There he studied astronomy and mathematics under Carl Friedrich Gauss, who was the director of the Göttingen Observatory. In fact, Gauss was (and still is) considered the greatest mathematician of his day. A scholarship in 1813–14 enabled Möbius to complete his two-semester graduate studies with Carl Friedrich Gauss, on which he drew his whole life long.

1813 Georg Büchner, German dramatist and writer of poetry and prose, born (October 17, 1813 in Riedstadt, Grand Duchy of Hesse). He was also a revolutionary and a natural scientist. His

literary achievements, though few in number because of his early death (he died on February 19, 1837 in Zurich), are held in great esteem in Germany. "Danton's Death" (*Dantons Tod*) was the first play written by Georg Büchner, set during the French Revolution. Georges Danton was a leader of the French Revolution, during the period of terror. He created the Revolutionary Tribunal as a strong arm for the Revolutionary Government. To be accused of anything real or imagined meant to be condemned to death without trial, proofs, evidence, or witnesses. Within months, Danton realized that this excessive power was a terrible mistake, and he fought to have it ended. Robespierre stopped him and used the Tribunal to have Danton and all opposition killed. Ultimately, Robespierre followed Danton to the guillotine.

1815 On March 20, 1815, Napoleon returned from forced exile on the Isle of Elba to France, ruling France again for a period of "Hundred Days" (*Les Cent-Jours*). This period marks the time between his arrival in France and the second restoration of Bourbon King Louis XVIII on July 8, 1815. This period saw the War of the Seventh Coalition and included the Battle of Waterloo.

1815 The Battle of Waterloo in present-day Belgium (*La Bataille de Waterloo, Schlacht bei Belle-Alliance*) on June 18, 1815. Napoleon and his French army were again defeated, this time by the Seventh Coalition (including Britain, Russia, Prussia, Sweden, Switzerland, Austria, the Netherlands, and several smaller German states) against France (see Fig. 1.5). On July 7, the Allies entered Paris. This time it was the final defeat of Emperor Napoleon I. This meant the liberation of Central Europe from French domination and suppression, the restoration of the French monarchy for the second time, and the permanent exile of Napoleon Bonaparte to the distant island of Saint Helena, where he died on May 5, 1821.

1806–1918 The Kingdom of Saxony, founded in 1806 by the grace of Emperor Napoleon, then at the height of his power over continental Europe. The new kingdom (King Frederick Augustus I, 1806–27) was an ally of France in all the Napoleonic Wars of the years 1807–13. At the beginning of the great War of German Liberation (1813), King Frederick Augustus I allied neither with Napoleon nor with his allied opponents. Later, he united his troops finally with those of France when Napoleon threatened to treat Saxony as a hostile country.

In the long run, Saxony was always on the wrong side during the Napoleonic Wars: King Frederick Augustus I of Saxony and Duke of Warsaw remained a loyal ally to France even after Napoleon's disastrous Russian campaign (1812–13).

Figure 1.5 Battle of Waterloo, Prussian attack in Plancenoit, June 18, 1815. (Painting by Adolf Northern, Credit: Wikimedia Commons). The village Plancenoit near Waterloo was a key strategic point during the Battle of Waterloo as it was the main focal point of Prussia's successful flank attack on Napoleon's army. Prussia's success here sealed Napoleon's fate. The Battle of Waterloo marks the end of the Napoleonic Wars.

At the Battle of Leipzig (October 16–18, 1813), when Napoleon was completely defeated, the greater part of the Saxon troops deserted to the allied forces. King Frederick Augustus I was taken as a Prussian prisoner to the Palace of *Friedrichsfelde* near Berlin. It was not before June 1815 when he was allowed to return to his depleted kingdom to govern the remains of Saxony. The Congress of Vienna (1814–15) had taken from the Kingdom of Saxony the greater part of its land and gave it to Prussia. Only the intervention of the Austrian Emperor saved Saxony from being entirely absorbed by Prussia. Frederick Augustus I lost three-fifths of his territory to Prussia. Under these conditions, his truncated state became a member of the (North) German Confederation that was founded in

1815. In 1871, Saxony became one of the states of the newly founded German Empire. It lasted until 1918, the end of World War I, after Germany had been totally defeated by the Allied War Coalition.

1814/1815 The Congress of Vienna (September 18, 1814, until June 9, 1815) was a conference of ambassadors of European states that reorganized Europe after the Napoleonic Wars. It was chaired by Austrian Foreign Minister Prince Klemens Wenzel von Metternich (1773–1859) and began 5 months after Napoleon I's first abdication. It completed its "Final Act" in June 1815, shortly before the final defeat of Napoleon in the Battle of Waterloo. The Vienna settlement was the most comprehensive treaty Europe had ever seen. The official objective of the Congress was to provide a long-term peace plan for Europe by settling critical political issues arising from the French Revolutionary and the Napoleonic Wars. The real goal was more a political restauration of the God's grace monarchies and forcible suppression of any republican and democratic ideas that question established forms of monarchic government. Another goal was to reconsider existing political borders in Europe in order to resize the main powers in such a way that they could balance each other off and remain at peace. The leading statesmen were all conservatives with little or no sympathy for republicanism. In their eyes, such movements were threatening to upset the *status quo* in Europe. In the "Final Act," France lost all its recent conquests, while Prussia, Austria, and Russia made major territorial gains. Prussia added smaller German states in the West, and gained Swedish Pomerania and 40% of the Kingdom of Saxony. Austria gained Venice and much of northern Italy. Russia gained parts of Poland.

As a consequence of the Congress of Vienna, the city of Naumburg (at the Saale river) no longer belonged to Saxony but became part of Prussia in 1816. Thereby also Schulpforta, a suburb of Naumburg, where August Ferdinand Möbius was born and went to college until 1806, became Prussian (and stayed so until the end of World War II (1939–45)).

With the wisdom of hindsight, historians are very much in two minds about the implications of the Congress of Vienna: Some historians argue that the Congress of Vienna has to be seen as a reactionary act solely for the benefit of absolute monarchies. Also, the Congress of Vienna has been criticized for causing the subsequent

suppression of the emerging national and liberal movements in Europe. Other historians argue that the Congress of Vienna has to be praised for having created, for a relatively long time, stable and peaceful conditions in most of Europe. In fact, the Congress of Vienna formed the framework for European international politics until the outbreak of World War I in 1914.

Figure 1.6 Map of *The Deutscher Bund* (German Confederation), 1815–66. In the German Confederation, more than 30 sovereign kings and principalities as well as four free cities were represented with equal rights. The *Deutscher Bund* still excluded the German-speaking part of Austria as well as the south German states of Bavaria, Württemberg, Baden, and Hesse. The formation of the German Confederation was a serious setback for Germany's unification efforts. There was neither a uniform system of measurements and coins, nor a common postal system, nor institutions such as a federal court or a united army. The fragmentation of Germany meant a weakening of foreign policy and meant that the economy was hampered by customs and trade barriers.

1815–1866 The German Confederation (*Deutscher Bund*). It was an association of about 35 German states in Central Europe (see Fig. 1.6), created by the Congress of Vienna in 1815 to coordinate the economies of the separated German-speaking countries and to replace the former Holy Roman Empire. The Confederation has been judged by most historians as weak and ineffective, moreover, as an obstacle to the creation of a unified German national state. It collapsed due to the rivalry between Prussia and Austria, the 1848 revolution (see below), and decidedly fell apart after the Prussian victory in the Austro–Prussian War (the "Seven Weeks' War") of 1866. Noteworthily, August Ferdinand Möbius (1790–1868) spent most of his lifetime in the German Confederation.

1815 The Holy Alliance (*Heilige Allianz*; Священный Альянс) signed in Paris on September 26, 1815 by Tsar Alexander I of Russia, Emperor Franz I of Austria, and King Friedrich Wilhelm III of Prussia. It was a coalition created by the monarchist great European powers Russia, Austria, and Prussia after the ultimate defeat of Napoleon. Gradually, most European monarchies acceded to the alliance, with Great Britain being a notable exception. The intention of the alliance was to restrain republicanism and secularism in Europe in the wake of the devastating French Revolutionary Wars. The European borders were regarded inviolable. Members promised each other mutual assistance in case a revolution would break out. The Holy Alliance was used for the purpose of suppressing nationalism and liberalism everywhere in Europe. No wonder that it was condemned by many intellectuals as a reactionary league of kings and princes against their peoples and a conspiracy against liberalism. The Holy Alliance nominally succeeded in this politics until the Crimean War (1853–56) in which the Russian Empire lost to an alliance of France, Britain, the Ottoman Empire, and Sardinia.

1815 Mount Tambora volcano eruption with worldwide consequences. On April 10, 1815, the Indonesian volcano Mount Tambora erupted. It killed thousands of people in the vicinity and cooled the Earth by several degrees. It caused famines and disease outbreaks around the world, leading to the 1816 "Year without Summer," in particular in Central Europe and North America. It remains the largest volcanic eruption on historical record: larger than the 1883 eruption of Krakatoa, and roughly 20 times bigger

than Mount Vesuvius, which wiped the Italian town of Pompeii off the map. Measured in modern units, the power released at the Tambora eruption was estimated to be equivalent to a 30,000 megaton TNT explosion [5–7]. This estimate means that the Tambora eruption was about 2 million times more powerful than the atomic bomb dropped on Hiroshima at the end of World War II! The arguments of the geophysicists and climatologists who link the 1815 Tambora eruption to extreme weather conditions in 1816 in parts of the northern hemisphere with poor harvests and, as a result, sharply increased grain prices in Europe and America, are strong. The economic, demographic, and political upheavals that followed the Napoleonic Wars created conditions that aggravated the agricultural crisis. It led to famine, epidemic diseases, and social unrest in many parts of Europe.

It is surprising that such dramatic observations did not find adequate expression in the biographic notes of our protagonist August Ferdinand Möbius. In contrast, the impact of Tambora's eruption on the atmosphere with subsequent "Year without Summer" has inspired romantic writers and artists alike. Among them was the English author Mary Shelley who was writing her first novel "*Frankenstein or The Modern Prometheus*" when staying with young friends during the cold, rainy summer 1816 at a Mansion near Lake Geneva, which was rented by Lord Byron, the celebrated romantic poet. Or the great landscape painters William Turner in England and Caspar David Friedrich in Germany. Volcano-ash aerosols circulating around in the atmosphere have scattered the Sun's light, creating rich red sunsets that apparently have overwhelmed early 19th-century's painters.

1815 August Ferdinand Möbius finished his doctoral thesis "*The occultation of fixed stars*" ("*De computandis occultationibus fixarum per planetas*") and began to work on his Habilitation thesis. While he was writing this thesis, there was an attempt to draft him into the Prussian army. Möbius could manage to avoid military service at all and completed his Habilitation thesis on *Trigonometrical equations* ("*De peculiaribus quibusdam aequationum trigonometricarum affectionibus disquitio analytica*").

1816 August Ferdinand Möbius was appointed as Extraordinary Professor of Astronomy and Higher Mechanics at the University of

Leipzig. Contrary to his expectations, Möbius did not receive quick promotion to Full Professor in Leipzig.

1816/1819 August Ferdinand Möbius was offered an honorable post as full professor of astronomy at the University of Greifswald in northern Prussia in 1816; only 3 years later, an honorable post as full professor of theoretical and applied mathematics at the University of Dorpat, then in the Russian Tsarist Empire, was offered in 1819. He refused both offers, partly because of his appreciation of the high academic quality of Leipzig University, and partly because of his loyalty to Saxony.

1819 Wilhelm von Humboldt was removed from all his Prussian state offices in 1819 as a consequence of the increasingly reactionary policy of the Prussian government within the Holy Alliance. The apparent reason for his dismissal was his resistance against the "Carlsbad Decrees" (*Karlsbader Beschlüsse*) and his attempt to pass a liberal constitution for Prussia. For Wilhelm von Humboldt, it was unacceptable that the Carlsbad Decrees allowed for the suppression and persecution of liberal ambitions in public life in general and at the universities in particular.

1821 Napoleon Bonaparte died (May 5, 1821 in Saint Helena). How can one judge the actions of Napoleon Bonaparte? Was he a great leader and patriot, or was he a power-hungry dictator? Was he striving for a democratic United Europe or did he commit large-scale crimes against humanity? Even many French historians have mixed feelings when it comes to a fair account of Napoleon's role in history. For example, Thierry Lentz, director of the esteemed *Fondation Napoleon*, an academic think tank devoted to the serious study of the Napoleonic period, said: "*It is a little of many things. It is partly the fact that France has never made up its mind, officially, whether Napoleon was a great hero or a great villain*."

1822 Friedrich Wilhelm (William) Herschel, the famed British astronomer and composer of German origin, died (August 25, 1822 in Slough, England). Born in the Electorate of Hanover (November 15, 1738), Herschel migrated to England in 1757. He constructed a large mirror telescope with unprecedented resolving power in 1774, after which he spent 9 years carrying out sky surveys to investigate double stars. In the course of his telescope observations, in 1781, he realized that one celestial body he had observed was not a star, but a

planet, which was Uranus. This was the first planet to be discovered since antiquity, and Herschel became famous overnight.

1826 Joseph von Fraunhofer, German physicist and spectroscopist of the Sun's emission and absorption spectrum ("Fraunhofer lines"), famed inventor of new optical glass lenses and achromatic telescope objectives that revolutionized astronomy, died (June 7, 1826, aged 39, in Munich, Bavaria).

1827 Ludwig van Beethoven, grand German composer at transition between the Classical and Romantic eras, died (March 26, 1827 in Vienna).

1832 Johann Wolfgang von Goethe, eminent German poet, writer, statesman, died (March 22, 1832 in Weimar).

1833 Carl Friedrich Gauss and the German physics professor Wilhelm Eduard Weber built the first electromagnetic telegraph to be used for regular communication in 1833. In Göttingen, they connected the Observatory and the Institute of Physics with a double wire, covering a distance of about 1.1 km. The transmitter setup consisted of a coil that could be moved up and down over the end of two magnetic steel bars. The resulting induction current was transmitted through two wires to a mirror galvanometer acting as the sensitive receiver. The positive or negative direction of the current could be reversed by commuting the two wires in a manually operated switch acting as the signal generator. Therefore, Gauss and Weber chose to encode the alphabet in a binary code, using positive and negative currents as the two states of information.

1835 Wilhelm von Humboldt, German scholar, writer, and statesman, died (April 8, 1835 in Berlin).

1835 Halley's Comet reappeared in the inner parts of the Solar System. It is arguably the most famous comet; it was observed and described also by August Ferdinand Möbius. It is a "periodic" comet and returns to Earth's vicinity about every 75 years. Halley is the only known short-period comet that is regularly visible to the naked eye from the Earth.

1837 The "*Göttingen Seven*" (*Göttinger Sieben*) were a group of seven professors (out of 41) from the University of Göttingen, all colleagues of famed Carl Friedrich Gauss. In 1837, they protested

against the abolition of the rather liberal constitution of the Kingdom of Hanover by the new king, Ernest Augustus I. They refused to swear an oath to him because of his return to absolutism. The company of the *Göttingen Seven* was led by Friedrich Christoph Dahlmann, historian and politician. The other six were the Germanist brothers Wilhelm Grimm and Jacob Grimm, the jurist Wilhelm Eduard Albrecht, the historian Georg Gottfried Gervinus, the physicist Wilhelm Eduard Weber (a close friend and coworker of Gauss), and the theologian Heinrich Georg August Ewald (the son-in-law of Gauss). The protest's impact on the broad student community prompted King Ernest Augustus I to retaliate: Assisted by a subservient university court, the seven defiant professors were dismissed from their academic posts. Three of them (Dahlmann, Jacob Grimm, and Gervinus) were even expelled from the Kingdom of Hanover, and only 3 days were given them to leave the country. While hundreds of students publicly protested against the arbitrary dismissals of their professors, supported by many Göttingen booksellers and publishers, there was no action of solidarity by their academic colleagues. And whether Carl Friedrich Gauss reacted with indignation about suppression of freedom of opinion also at his university is regrettably not recorded. It would have been appropriate, you would think, after all he had been a public figure as Dean of the Philosophical Faculty (from 1833 to 1834).

Carl Friedrich Gauss had personally experienced all the political uproars in the first half of the 19th century, and it is said that he was interested in the political discourse. However, he avoided any personal positioning in discussions on controversial political and social issues in the public, for whatever reason. Very different from his friend and coworker Wilhelm Weber and his son-in-law Georg Ewald. And rather similar to his former student August Ferdinand Möbius, for whom, however, an important political action in favor of the *Göttingen Seven* is documented (see Chapter 3).

Noticeably, 120 years later, during the early part of the Cold War of the 1950s, the brave professors of the *Göttinger Sieben* were followed by another group of brave Göttingen professors, this time 18 in number—the *Göttinger Achtzehn*, who raised their voice against undue government plans for nuclear armament on April 12, 1957 (the *Göttingen Manifesto*). This declaration of the 18 leading

nuclear scientists of West Germany (among them the Nobel laureates Otto Hahn, Max Born, Werner Heisenberg, and Max von Laue) was a strong statement against arming the West German army with tactical nuclear weapons, as the West German government under chancellor Konrad Adenauer had suggested. The *Göttingen Manifesto* evoked a worldwide response—both positive and negative—as well as local reactions in Göttingen—likewise both positive and negative. The student union at the university showed solidarity with the demands of the 18 scientists.

1837–1901 The Victorian era in Great Britain. During the reign of Queen Victoria, the British Army was called upon to fight in many colonial wars, military expeditions and actions in many parts of the world, primarily outside Europe. In the European international relations, the Victorian era was a long period of peace, known as the *Pax Britannica*, temporarily disrupted by the Crimean War in 1854. At sea, British supremacy remained largely unchallenged throughout the century. The Victorian era was characterized by rapid change and developments in nearly every sphere—from advances in scientific, technological, and medical knowledge to changes in population growth and location. Large-scale relocations from rural to urban areas, often accompanied by social misery and poverty of the lower classes, were facilitated by the rapid expansion of the national railway network. This age saw the birth and spread of political movements, most notably socialism, liberalism, and organized feminism.

1839 The first long-distance railway in Germany between Leipzig and Dresden railway, completed on April 7, 1839 (see Fig. 1.7). Thereby, the travel time between these two major cities in Saxony was reduced to less than 4 hours. Before then, horse-drawn carriages had needed 36 hours. The railways in Germany were given a significant impetus by the development of the first working steam locomotives in England. Notably, the Leipzig–Dresden railway deployed the first German-made steam locomotive, named "Saxonia." German Railway history officially began with the opening of the steam-hauled Bavarian railway between neighboring Nuremberg and Fürth on December 7, 1835. This was followed by the first railway in Prussia between Berlin and Potsdam, a stretch of 11 km. It was opened on September 22, 1838.

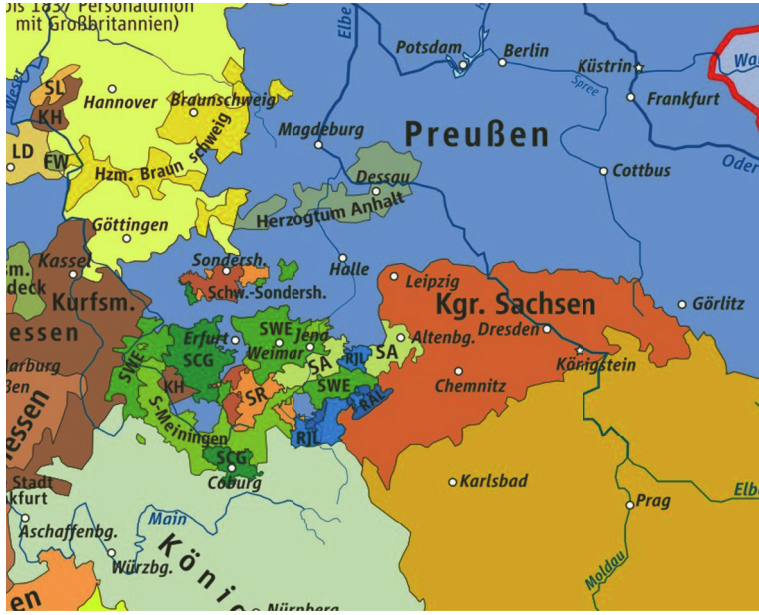

Figure 1.7 Map of German Confederation 1815–66 (zoomed-in detail of Fig. 1.6) with the Kingdom of Saxony and its immediate neighbors. Naumburg (Saale) is located about 45 km south-west of Leipzig, since 1815, no longer part of Saxony but became a town in Prussia.

1842 Friedrich Wilhelm Argelander (1799–1875), German astronomer and director of the new observatory at the University of Bonn, discovered that *Groombridge 1830*, a class G8 sub-dwarf star, had an extremely high proper-motion velocity. For a time, this star was known as *Argelander's Star*. It had been catalogued by British astronomer Stephen Groombridge between 1806 and the 1830s. August Ferdinand Möbius was excited about Argelander's discovery and reported on it in his popular astronomy book "*Astronomie. Größe, Bewegung und Entfernung der Himmelskörper*," Sammlung Göschen 1889 (Astronomy. Magnitude, Motion and Distance of Celestial Bodies). Argelander was responsible for the comprehensive star catalogue known as the *Bonner Durchmusterung* ("*Bonn Screening of the stars*"). It was published between 1852 and 1859 and gave the positions and brightness of more than 324,000 stars, although it did not cover much of the southern half of the sky.

1844 August Ferdinand Möbius was offered a full professorship of mathematics at the University of Jena. At this late stage of his academic career, the University of Leipzig made at last a counteroffer and gave him the Full Professorship in Astronomy, which he accepted.

1844 Weavers' uprising in the Prussian province of Silesia. In June 1844, a revolt of the cotton weavers (*Schlesischer Weberaufstand*) took place, with heavy riots occurring in Silesia, a major center of textile manufacturing. Crowds of weavers attacked homes of rich merchants and warehouses, destroyed machinery, and demanded higher salaries from the local merchants. In response, the Prussian army was called "to restore order." In a brutal confrontation between the weavers and troops, shots were fired into the crowd, killing at least 11 people and wounding many others. The leaders of the revolt were arrested, flagellated, and imprisoned.

The Silesian weavers' uprising was a result of severe social and economic distress in the region. Many rich merchants refused to pay fair salaries to their weavers. Those still used hand-driven weaving looms, and they suffered from British competitors that already used weaving machines, thereby destroying the competitiveness of Silesian linen. As social conditions worsened, growing unrest culminated in the Silesian weavers' uprising. This *Schlesischer Weberaufstand*, on the eve of the German "March 1848" revolution, was closely observed by the German society and treated by several artists, among them Heinrich Heine who wrote the famed *Die armen Weber* poem ("The poor weavers," the so-called "Weaver Song"), and several critical writers for social science journals, among them Karl Marx who regarded the uprising as evidence of the birth of a German workers' movement against exploitation, poverty, and injustice.

The *Schlesischer Weberaufstand* has gained enormous significance in the history of the German labor movements, whether democratic or socialistic. Not surprisingly, as Silesia was a direct neighbor of Saxony. The center of the uprise was only less than 400 km away from Leipzig, the thriving university city.

The Weaver Song was first published on July 10, 1844 in Paris in the German-speaking biweekly exile newspaper *Vorwärts!* ("Forward!"). It was the only uncensored radical paper in the German language published in contemporary Europe. The journal published many polemicists of political emigres to France, such as Heinrich Heine, Mikhail Bakunin, Friedrich Engels, and Karl Marx. He would have an

increasingly important role in editing the journal, particularly from the summer of 1844. The Weaver Song was distributed as a leaflet with a circulation of 50,000 copies in the rebel areas. No later than 1846 it appeared in pamphlets under the current name "The Silesian Weavers." The weavers' rebellion served as an important symbol for later generations concerned about poverty and oppression in early capitalistic societies such as 19th/20th-century Germany and England.

Heinrich Heine wrote the "*Die armen Weber*" poem after the uprising of the Silesian weavers in 1844:

Im düstern Auge keine Thräne,
Sie sitzen am Webstuhl und fletschen die Zähne:
„Altdeutschland, wir weben dein Leichentuch,
Wir weben hinein den dreifachen Fluch!
Wir weben! Wir weben!

„Ein Fluch dem Gotte, dem blinden, dem tauben,
Zu dem wir gebetet mit kindlichem Glauben;
Wir haben vergebens gehofft und geharrt,
Er hat uns geäfft und gefoppt und genarrt.
Wir weben! Wir weben!

„Ein Fluch dem König, dem König der Reichen,
Den unser Elend nicht konnte erweichen,
Der uns den letzten Groschen erpreßt,
Und uns wie Hunde erschießen läßt!
Wir weben! Wir weben!

„Ein Fluch dem falschen Vaterlande,
Wo nur gedeihen Lüg' und Schande,
Wo nur Verwesung und Todtengeruch –
Altdeutschland, wir weben dein Leichentuch!
Wir weben! Wir weben!

1846 Discovery of the eighth planet, Neptune, in the Berlin Observatory during the night of September 23, 1846 by Johann Gottfried Galle (1812–1910) and Heinrich Louis d'Arrest (1822–1875). It was the first planet to be discovered on the basis of mathematical predictions of its location. After the discovery of Uranus in 1781, several astronomers noticed that the planet was being pulled slightly out of its normal orbit. They realized that there had to be some additional planet deeper out in the Solar System that was

influencing the motion of Uranus with its gravity. Two astronomers, Britain's John Couch Adams and France's Urbain Le Verrier worked out the position of the hypothetical eighth planet independently from each other. They figured out not only where the planet was, but also how much mass it had. A young astronomer at the new Berlin Observatory, Johann Gottfried Galle, and his student assistant, Heinrich Louis d'Arrest, decided to search for the predicted planet and observed Neptune for the first time in 1846. Johann Gottfried Galle used the calculations by Le Verrier to find Neptune within just 1° of its predicted location, and just 12° of Adams' predictions. Both astronomers claimed that they were the first to discover the planet, and it led to an international dispute. The international astronomy community agreed that the two astronomers should share credit for the discovery.

1846 Friedrich Wilhelm Bessel, a German astronomer, mathematician, physicist, and geodesist, died (March 17, 1846 in Königsberg). He was the first astronomer who determined reliable values for the distance from the Sun to another star by the method of parallax. Bessel became famous by producing a significant refinement of the calculations of the orbital for the reappearing Halley's Comet in 1804, using 200 years old observation data taken from published astronomical tables. Since 1810, Bessel was appointed Director of the Königsberg Observatory.

1848 August Ferdinand Möbius was appointed Director of the Leipzig Observatory.

1848 The Communist Manifesto published. The Communist Manifesto is a political polemic by the German philosophers Karl Marx and Friedrich Engels. It was commissioned by the Communist League and originally published in London in February 1848 in German as *Manifest der Kommunistischen Partei* (*Manifesto of the Communist Party*), just in time as the March revolutions of 1848 began in several German states and Austria to erupt. The Manifesto was later recognized as one of the world's most influential political manuscripts. It presents an analytical approach to the class struggle, and the problems of capitalism, and the capitalist mode of production. It also briefly features the authors' ideas for how the capitalist society of the time would eventually be replaced by socialism. The much-quoted introduction of the Manifesto begins by proclaiming "*A*

spectre is haunting Europe – the spectre of communism. All the powers of old Europe have entered into a holy alliance to exorcise this spectre."

1848–1849 "March Revolution" (*März Revolution*) in Germany and Austria. Against the background of a revolutionary situation existing since 1844 with and increasingly worsening economic conditions, the German March 1848 revolution began as the immediate consequence of the Parisian February revolution. On account of the fragmentation of the German-speaking countries into a multitude of states, as well as the lack of a national center, the revolution had the character of isolated, yet reciprocally affecting uprisings in the individual German lands and territories.

From the Grand Duchy of Baden, where already on March 1, 1848, a mass rally in front of the Parliament took place—supported by democratic parliamentarians—the revolutionary movement spread to all southern Germany and the larger part of the central German states. Everywhere, up to the middle of March, new governments were established, dominated by moderately liberal representatives of the bourgeoisie, proclaiming programs of liberal reform. In Bavaria, the events resulted in the abdication of the monarch, King Ludwig I. Due to the political support given by Prussia to several princes, the revolution was not victorious in several states of central and northern Germany until the second half of March 1848. The uprisings in Vienna and Berlin were the highlight of the March revolution; in these metropolises, armed riots of the people forced the intransigent pre-March regimes to make concessions or even to surrender.

In Vienna, with its population pressing for political change, a student-organized demonstration on March 13, 1848 escalated into a storm on the *Ständehaus* (House of the Estates). The people put up armed resistance to the troops. After the uprising of the workers in the suburbs and the ultimatum by the liberal bourgeoisie, the Habsburg regime decided to give in. In the evening of March 13, the hitherto omnipotent Prince Wenzel von Metternich was toppled, and the students were given permission to arm themselves. On the following days, repeated actions by the people led to additional concessions: establishment of a National Guard, freedom of the press, vague promises to begin constitutional development in Austria.

The uprising in Berlin on March 18, 1848 was preceded by powerful oppositional governments in almost all Prussian provinces,

especially in the Rhineland. Since March 6, the insubordinate spirit had spread to the capital Berlin. Due to the brutal deployment of Prussian soldiers, the wrath and indignation of the inhabitants grew to demand more and more, ultimately the withdrawal of all troops from Berlin. King Friedrich Wilhelm IV was still unwilling to comply with the demand on a pullout of troops. He even deployed soldiers against a demonstration in front of the Berlin castle, sparking an hour-long street battle that was fought with great embitterment on both sides. Some 4000 insurgents, mostly young workers, craftsmen, and students, were opposed by 14,000 soldiers with 34 cannons. In the fighting, 230 revolutionaries lost their lives. Although the troops were able to conquer almost all barricades erected in the town center, their power was not sufficient to defeat the uprising militarily. Therefore, in the morning of March 19, the king ordered the military to pull out from the town and to agree to the formation of a civic guard. Humiliatingly, he was forced to bow his head to the dead revolutionaries, lying in state in the courtyard. After a transitional government installed on March 19, the way was paved to the transition of Prussia into a constitutional monarchy.

Saxony was among the German states agitated by the revolutionary movement rather late. The famed "May Uprising" took place in Dresden, the capital of the Kingdom of Saxony, in 1849; it was one of the last of the series of rebellions known as the "1848 Revolutions."

1848–1849 The Frankfurt National Assembly (*Frankfurter Nationalversammlung*) was the first freely elected parliament for all of Germany, elected on May 1, 1848 throughout the 39 states of the German Confederation. Generally, there were no free and equal voting rights for all citizens. Some states excluded farm hands, others domestic servants and industry workers. The ballot was not secret, and the elected 583 members were mostly from the middle class, notably judges, lawyers, high state officials, university and high school teachers, merchants and industrialists. Most of the 90 members of the nobility were in the learned professions. The session was held from May 18, 1848 to May 31, 1849, and the *Paulskirche* (St. Paul's church) at Frankfurt (Main) became the short-lived seat of the German National Assembly. Its existence was a concession of the monarchic states of the German Confederation in the face of the "1848 Revolutions."

After long and controversial debates, the assembly produced the so-called Frankfurt Constitution (or *Paulskirchen-Verfassung*, actually the *Verfassung des Deutschen Reiches*), which proclaimed a German Empire, based on the principles of a constitutional monarchy and headed by a hereditary emperor (*Kaiser*). This constitution fulfilled the main demands of the liberal and nationalist movements of the March 1848 revolutions and provided a foundation of basic civil rights.

On April 3, 1849, a group of deputies chosen by the National Assembly offered the Prussian King Friedrich Wilhelm IV the imperial crown of German Emperor. Unexpectedly, he declined, arguing that he could not accept the crown without the agreement of all the crowned heads, princes, and Free Cities of the Confederation. It is reported that when confronted with the king's rejection of the crown, many members of the National Assembly in Frankfurt had tears in their eyes. Everybody felt that the King of Prussia had miserably failed to make the old dream of a united Germany finally come true. In fact, as is also reported that Friedrich Wilhelm IV insisted in the principle of the Divine Right of Kings to be crowned by his peers. Thus, he did not want to accept the "1000 years old crown of the German Nation when touched by the hussy smell of the 1848 revolution." This spelled the final failure of the Frankfurt Parliament's constitution and thus of the German March revolution. The rejection of the crown was understood by the other kings and princes as a signal that the political scales had tipped against the liberals.

The work of the National Assembly was generally judged harshly, or even totally negative, in the immediate aftermath. Many liberals left politics disappointed and retired into private life or focused entirely on their daily professional activities. At that time, the tame *Biedermeier* period reached its culminating point. The term "*Biedermeier*" does not refer solely to the era of time as a whole (roughly from the Congress of Vienna in 1815 until the foundation of the German Empire in 1871), but to a particular mood and trendsetting that grew out of the unique historical consequences of the post-Napoleonic Wars in Central Europe. There were two driving forces for the development of the *Biedermeier* period: The first was the growing urbanization and industrialization leading to a new urban middle class, which created a new kind of audience for the

literature of that day, the arts and handicrafts. The second was the enforced political stability prevalent under the political dominance of Klemens Wenzel von Metternich following the Congress of Vienna. He was one of the most important diplomats of his era, the era of "Restauration" in the member states of the Holy Alliance, serving as the Austrian Empire's Foreign Minister from 1809 and Chancellor from 1821 until the liberal revolutions of 1848 forced his resignation. The overall effect was—for intellectuals and artists in particular, and society in general—to concentrate on the domestic and (at least in public) the non-political affairs. Most intellectuals and artists began to stay in "safer territory," and the emphasis on home life for the growing middle class meant a blossoming of painting, furniture design, and interior decorating.

A positive reception of the 1848 National Assembly's work only came much later, during the Weimar Republic (1918–33) and more so after World War II (1939–45). Then, both the (East) German Democratic Republic and the (Western) Federal Republic of Germany competed for the (mis)use of the democratic *Paulskirche* heritage as a specific tradition of the separate states on the respective sides of the Iron Curtain. After the German reunification in 1989, the National Assembly in the *Paulskirche* is officially highly respected by the constitutional bodies of the Federal Republic of Germany for its achievements in formulating a democratic constitution (*Paulskirchen-Verfassung*), which is considered as a lasting commitment for further democratic development of the German society.

1849 Felix Klein, German mathematician and mathematics educator, born (April 25, 1849 in Düsseldorf). After professorships in Erlangen, Munich, and Leipzig, Klein accepted a chair at the University of Göttingen in 1886. He re-established Göttingen as the world's leading mathematics research center. This center served as a model for such centers of excellence throughout the world. In 1895, Felix Klein hired David Hilbert away from Königsberg University. Hilbert continued Göttingen's glory until his own retirement in 1932. Felix Klein became the editor, along with R. Baltzer and W. Scheibner, of the four-volume "Collected Works of August Ferdinand Möbius" [8].

In 1882, Felix Klein described a novel topologically non-orientable object that soon became popular as the "Klein bottle" (*Klein Flasche*).

Informally, it is a three-dimensional extension of the Möbius strip, which is non-orientable in two dimensions. This means that the Klein bottle is a one-sided surface, which, if traveled upon, could be followed back to the point of origin while flipping the traveler upside down. However, whereas a Möbius strip is a non-orientable surface with boundary, a Klein bottle has a non-orientable surface but no boundary.

1850 Rudolf Clausius (1822–88), German mathematical physicist, enunciates the Second Law of Thermodynamics. It states that all physical systems ultimately succumb to maximum entropy, which means that heat cannot pass from a colder body to a warmer one, but only from a warmer to a colder body. This will prove to be one of the most significant principles of physics, chemistry, and biology. It establishes that even for a perfect physical system, it is impossible to convert thermal energy 100% into mechanical work. The increase in entropy accounts for the irreversibility of natural processes, and the asymmetry between future and past.

1851 The French physicist Léon Foucault made a famous convincing experiment with a long and heavy pendulum to demonstrate the rotation of the Earth. While it had long been known that the Earth rotates, the introduction of the "Foucault pendulum" in 1851 was the first simple proof of the Earth's rotation in an easy-to-see experiment that had a large impact on the public science education of the 19th century and continues to do so up to now. Foucault made his sensational pendulum experiment on March 26, 1851 in the Panthéon in Paris by suspending a 28 kg brass-coated lead orb of 60 cm diameter with a 67 m long wire from the dome of the Panthéon. The plane of the pendulum's swing rotated clockwise approximately 11.3° per hour, making a full circle in approximately 31.8 hours, clearly convincing the curious and patient public.

1855 Carl Friedrich Gauss, eminent German mathematician, astronomer and geophysicist, died (February 23, 1855 in Göttingen).

1856 Heinrich Heine, romantic German poet, writer, political journalist, died (February 17, 1856 in Paris). Heine was buried in the Montmartre cemetery in Paris. His relationships with Germany were characterized by adoration, rejected love, and even hatred for him.

"*Denk ich an Deutschland in der Nacht,*
Dann bin ich um den Schlaf gebracht.
Ich kann nicht mehr die Augen schließen,
Und meine heißen Tränen fließen."
… (1844).

(At night I think of Germany,
And then all sleeping flies from me,
I can no longer close my eyes,
And hot and bitter tears will rise).

1858 August Ferdinand Möbius described the properties of one-sided surfaces, including the "Möbius strip," a two-dimensional surface with only one side. Although this object is known as a Möbius strip today, it was not Möbius who first described it, rather by any criterion, either by publication date or the date of first discovery, precedence goes to the German mathematician Johann Benedict Listing (born 1808 in Frankfurt (Main), died 1882 in Göttingen).

1859 Alexander von Humboldt, German natural scientist and explorer, died (May 6, 1859 in Berlin). He was called the *last universal man*. Charles Darwin described him as "the greatest scientific traveler who ever lived."

1859 Charles Darwin (1809–82), English naturalist and geologist, published (on November 24, 1859 in London) his book *On the Origin of Species by Means of Natural Selection*. This work is considered to be the foundation of evolutionary biology.

1864 Friedrich Georg Wilhelm von Struve, German-Russian astronomer, died (November 23, 1864 in St Petersburg). Born in 1793 in Altona near Hamburg, he entered the Russian Imperial University of Dorpat in 1808. Here, he studied mainly astronomy. From 1813 to 1820, he taught at the Dorpat University and collected astronomical data at the Dorpat Observatory. In 1820, he became a full professor at the Dorpat University and director of the Dorpat Observatory. Struve was occupied with research on double stars and geodesy in Dorpat until 1839, when he founded and became the director of the new Pulkovo Observatory near St Petersburg.

1866 The Austro–Prussian War for hegemony over the German states or "Seven Weeks' War" or "German Civil War of 1866" or "German Brothers War." It was fought between the German

Confederation under the leadership of the Austrian Empire and its German allies on one side and the Kingdom of Prussia with its German allies on the other. It was the first war between two major continental powers in 7 years. It used many of the same technologies as the American Civil War, including railways to concentrate troops during mobilization and telegraphs to enhance long-distance communication. The main campaign of the war occurred in Bohemia. Among the various battles, that of Königgrätz (*Hradec Králové*) at the Elbe river on July 3, 1866 brought the decisive Prussian victory against the Austrians and their allies. Ultimately, Königgrätz resulted in Prussian dominance over the German states. There was a shift in power among the German states away from Austrian and toward Prussian hegemony. On top of that, the Prussian victory at Königgrätz provided the decisive impetus toward an alliance of all of the northern German states with nearly 30 million inhabitants. It was the "small German solution" ("*kleindeutsche Lösung*") of the notorious German division that still excluded the German-speaking part of Austria as well as the south German states of Bavaria, Württemberg, Baden, and Hesse.

During the 1866 Austro–Prussian War, Saxony allied with Austria, and the Saxon army is generally seen as the only ally to bring substantial aid to the Austrian cause. It abandoned even the defense of Saxony itself to join up with the Austrian army in Bohemia. This effective loyalty to Austria probably allowed Saxony to escape the fate of other north German states allied with Austria—notably the Kingdom of Hanover—which were annexed by Prussia after the war. In the peace negotiations, the Austrians insisted—probably as a point of honor—that Saxony must be spared from annexation, and the Prussians acquiesced. In return, Saxony joined the Prussia-led northern German Confederation the next year.

This northern German Confederation only lasted from 1867 to 1871. It was the first modern German nation-based state and formed the basis for the later German Empire (1871–1918) to which also the south German states affiliated. Despite its short lifetime of less than 4 years, the north German Confederation is historically important as a milestone on the long way to the political, economic, and judicial unification of Germany. Many of its laws inspired those which were introduced by the German Empire, for example free movement

of the citizens within the territory of the Confederation (1867), a common postal system (1867/1868), common passports (1867), equal rights for the different denominations (1869), including Jewish emancipation.

1867 Volume I of "*Das Kapital, Kritik der politischen Ökonomie*" ("Capital: Critique of Political Economy") by Karl Marx (1818–83) was published. The complete opus comprises three volumes; Vol. II and Vol. III were edited by Friedrich Engels (1820–95), the personal and political friend of Karl Marx. Karl Marx's *Das Kapital* is considered a theoretical foundation text in communist philosophy, economics, and politics. Karl Marx aimed to reveal the economic patterns underpinning the capitalist mode of production, in contrast to classical political economists of his time.

1868 August Ferdinand Möbius, German mathematician and astronomer, died (September 26, 1868 in Leipzig).

1870 The German–French War (*Deutsch-Französischer Krieg*, *Guerre franco-allemande*), a conflict between the Second French Empire of Napoleon III and the German states of the northern German Confederation led by the Kingdom of Prussia. The conflict was caused by Prussian ambitions to advance German unification and French fears of the shift in the European balance of power at the expense of France that would result if Prussia succeeded. France started the conflict, but some historians argue that the Prussian Chancellor Otto von Bismarck (1815–98) deliberately provoked a French attack in order to draw the southern German states—Baden, Württemberg, Bavaria, and Hesse-Darmstadt—into an alliance with the north German Confederation dominated by Prussia. Anyway, on July 16, 1870, the French parliament voted to declare war on the Kingdom of Prussia, and hostilities began 3 days later. The German coalition was able to mobilize its troops much more quickly than the French and rapidly invaded north-eastern France. The allied German forces were superior in numbers, had better training and leadership, and made more effective use of modern technology, particularly railroads and artillery. A series of swift Prussian and allied German victories in eastern France, culminating in the Siege of Metz (August 20 to October 27, 1870) and the Battle of Sedan (September 1 and 2, 1870), ended with Napoleon III captured by the army of the Second

Empire decisively defeated. A "Government of National Defense" declared the "Third Republic" in Paris on September 4, 1870, and continued the war for another 5 months.

1871 After the victories over the French army at Metz and Sedan, the German Reich was founded when the new constitution took effect on January 1, 1871.

Until March 19, 1871, Napoleon III and his entourage were held in comfortable captivity in Germany in a castle near Kassel. When peace was arranged between France and Germany, Bismarck released Napoleon III. He decided to go into exile in England. He died there on January 9, 1873.

1871 The German forces defeated the new French armies of the Third Republic in northern France. Following the Siege of Paris (from September 19, 1870 to January 28, 1971), the French capital surrendered on January 28, 1871. While the French government deteriorated, and chaos was spreading in the country, Prussian Chancellor Bismarck succeeded in achieving his main goal of the war, the German unification, on January 18, 1871. King Wilhelm I of Prussia was proclaimed Emperor of the new German Empire (*Kaiser des Deutschen Reiches*) (see Fig. 1.8). For this solemn proclamation to duly celebrate (and to humiliate the war loser France), a pompous act was initiated in the famed *Hall of Mirrors* in the Versailles Palace near Paris. The staging of this opulent ceremony reminds strongly of the staging of the sacred ceremony of the coronation of "Napoleon I, Emperor of the French" on December 2, 1804 at the *Notre Dame* Cathedral in Paris, which was realized as a "masterpiece of modern propaganda" (see Fig. 1.3, above).

The new German military command structure headed by General Field Marshal Hellmuth von Moltke wanted to enforce an advantageous peace treaty right away with hard claims to exploit France's momentary weakness. Chancellor Bismarck, however, insisted on an immediate armistice first and a peace treaty later, as his primary reason for this war, German unification, had already been accomplished. He was concerned that further violence would render many more German casualties and draw strong French resentment. He was also fearing other European nations to intervene if the new German Empire appeared to be too power hungry.

Figure 1.8 Proclamation of the German Emperor in the Palace of Versailles near Paris, January 18, 1871. Oil painting by Anton von Werner (1843–1915). (Credit: German Historical Museum, Berlin). In the center stands Chancellor Otto von Bismarck, to his left stands General Field Marshal Hellmuth von Moltke, both of whom look sternly at King Wilhelm I of Prussia (standing in the center of the podium), who has just been proclaimed "Emperor of Germany."

After lengthy peace-treaty negotiations, the Treaty of Frankfurt was signed on May 10, 1871: France was obliged to pay a war indemnity of 5 billion gold francs in 5 years. German troops remained in parts of France until the last installment of the indemnity was paid, actually already in September 1873, ahead of schedule. France gave Germany most of Alsace and some parts of Lorraine, which became the German imperial territory of Alsace-Lorraine (*Reichsland Elsass-Lothringen*). Bismarck opposed further annexations because he did not wish to make France a permanent enemy of Germany (see Fig. 1.9).

The German conquest of parts of France and the unification of Germany clearly upset the European balance of power that had existed since the 1815 Congress of Vienna. French determination to regain Alsace-Lorraine, along with British apprehensions about the balance of power in Europe, became important factors for the outbreak of World War I in 1914.

Figure 1.9 Map of the *Deutsches Reich* (German Empire) 1871–1918.

1871 In Paris, a revolutionary uprising, called the *Paris Commune*, seized power in the capital and held it for 2 months, until it was bloodily suppressed by the regular French army at the end of May 1871. The Paris Commune of 1871 was one of the great traumas that shaped the 19th-century France. It stands alongside the 1789 Revolution and the final Napoleonic campaigns that led to Waterloo, 1815. After the French surrender to the German army, in the February of 1871, new legislative elections were held throughout France, and a majority returned in favor of a form of "republican royalism." The assembly, led by the aging statesman Adolphe Thiers, soon declared itself the Third Republic. The people of Paris, always farther to the left than the rest of France, feared that the new republic would be republican in name only, and began organizing their own, alternative people's regime in the capital. A confrontation between what remained of the regular French army, called the *Versaillais*, and Paris's popular militia, known as the *National Guard*, ended with a

bloodshed. The royalist-minded government Adolphe Thiers fled Paris for Versailles, the traditional seat of the French kings. In Paris, a left-wing Communard government, protected by the National Guard, rose up and seized power. For about 2 months, the Communard government tried to rule on radical socialist and revolutionary principles. It made various statements that still seem prophetically advanced—particularly on self-organization and feminism. Women played a central role in the Paris Commune, building barricades and chairing committees, generally raising hell: "*The social revolution will not be realized until women are equal to men. Until then, you have only the appearance of revolution.*" The Communards also insulted and threatened the clergy and the few remaining rich people and committed mostly disorganized acts of looting and reprisal against its royalist political enemies, including tearing down Thiers's house and toppling the Place Vendôme column with its statue of Napoleon Bonaparte. In reaction, the Versaillais invaded Paris and, with minimal military difficulty, though at maximal human cost, reconquered the city. The Communards, as they were crushed by the advancing and brutal Versaillais, set fire to much of the city, including the Tuileries Palace, which burned to the ground. Whether all the fires were the result of sinister female "*pétroleuses*" activists with their hidden petroleum cans ready for setting a next fire or of general chaos and violence is still a hotly debated issue.

1872 The unified German Empire assigned Berlin as its official capital. Berlin remained the capital of the Kingdom of Prussia, the leading state of the new Empire.

1914–1918 World War I, also called the First World War, was an international conflict that embroiled most of the nations of Europe along with Russia, the United States, the Middle East, and other regions. The war split the world into two blocks of enemies bitterly fighting each other: the Central Powers—mainly Germany, Austria-Hungary, and Turkey—against the Allies—mainly France, Great Britain, Russia, Italy, Japan, and, from 1917, the United States of America. World War I ended with the total defeat of the Central Powers. This war was virtually unprecedented in the slaughter, carnage, and destruction it caused, mainly because both sides made ruthless use of modern technology in large-scale industrial production of weaponry.